Isoscalar axial-vector bottom-charm tetraquarks from QCD

Archana Radhakrishnan(1), M. Padmanath(2) and Nilmani Mathur(1)

(1) *Department of Theoretical Physics, Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400005, India*

(2) *The Institute of Mathematical Sciences, a CI of Homi Bhabha National Institute, Chennai, 600113, India*

Summary. — The increasing number of discovered heavy quark exotic hadrons call for immediate theoretical investigations based on first principles. Our study focuses on tetraquark states made up of a bottom and charm quark in the axial-vector (1^+) channel with isospin I=0, using Lattice Quantum Chromodynamics. These computations were conducted on the state-of-the-art MILC ensembles using dynamical up/down, strange, and charm quark fields implemented with a highly improved staggered quark action. The valence quarks were implemented using an overlap action, with quark masses ranging from light to the charm sector, while the evolution of the bottom quark was studied within a non-relativistic QCD framework. We observe strong evidence of an energy level beneath the elastic threshold, which imply an attractive interaction between the bottom and charm mesons, indicating the possible existence of bound charmed-bottomed tetraquarks.

1. – Introduction

The recent discovery of a doubly charmed tetraquark, denoted as T_{cc}, represents a significant step in the study of hadronic spectroscopy [1]. Due to recent advancements, similar doubly heavy tetraquarks, like the one comprised of a bottom and a charm quark, with a valence quark configuration $T_{bc} \equiv bc\bar{u}\bar{d}$, has become one of the highly sought-after hadrons [2]. From a phenomenological perspective, doubly heavy tetraquarks, particularly in the heavy quark limit, have long been speculated to form deeply bound states, with binding energies on the order of $\mathcal{O}(100\ \text{MeV})$ relative to the elastic strong decay threshold. In this study(1), we employ lattice QCD calculations to present compelling evidence of an attractive interaction between the D and B^* mesons, of sufficient strength to support the existence of a real bound state denoted as T_{bc}.

(1) for more details see [3]

2. – Calculation details

We used four different lattice QCD ensembles with $N_f = 2+1+1$ dynamical quarks using the Highly Improved Staggered Quark (HISQ) formulation, which were generated by the MILC collaboration [4]. In these ensembles, the charm and strange quark masses in the sea were meticulously tuned to match their physical values. Meanwhile, the dynamical light quarks were selected such that the ratio of strange to light quark masses, denoted as $m_s/m_l \sim 5$.

For the valence quark fields up to the charm quark masses, we employed an overlap fermion action, consistent with previous work [5, 6]. For the bottom quark, a non-relativistic QCD (NRQCD) Hamiltonian [7] was utilized. In accordance with the Fermilab prescription [8], the bare charm and bottom quark masses was tuned on each ensemble, with the kinetic mass of spin-averaged $1S$ quarkonia, denoted as $\{a\overline{M}^{\bar{Q}Q}_{kin} = \frac{3}{4}aM_{kin}(V) + \frac{1}{4}aM_{kin}(PS)\}$, determined on the respective lattice setups. Further information about the fine-tuning of heavy quark masses can be found in the references [9] for charm quarks and references [10] for bottom quarks. Additionally, the bare strange quark mass was established by equating the lattice-based estimate of the hypothetical pseudoscalar $\bar{s}s$ meson mass to 688.5 MeV [11].

We considered five distinct light quark mass $m_{u/d}$. These included three unphysical quark mass values, corresponding to approximate pseudoscalar meson masses M_{ps} of around 0.5, 0.6, and 1.0 GeV. In addition the case with physical strange quark mass M_{ps} around 0.7 GeV and the physical charm quark mass case with M_{ps} around 3.0 GeV were also studied. We systematically evaluated the finite-volume spectrum for all five quark masses across all four lattice ensembles. Subsequently, we investigated the scattering of D and B^* mesons under each of these scenarios and extracted the dependence of the scattering parameters on the light quark mass $m_{u/d}$. Throughout our calculations, we applied a wall-smearing technique to enhance the quality of our quark propagator measurements[12, 13, 14].

2˙1. *Finite Volume Spectrum.* – We determined the finite-volume spectrum by examining Euclidean two-point correlation functions, denoted as $\mathcal{C}_{ij}(t)$, which involve interpolating operators $\mathcal{O}_i(\mathbf{x})$ with specific quantum numbers. In our analysis, we utilize a set of operators within the T_1^+ irreducible representation of the finite-volume lattice, expressed as:

$$\begin{aligned}
\mathcal{O}_1(x) &= [\bar{u}(x)\gamma_i b(x)][\bar{d}(x)\gamma_5 c(x)] - [\bar{d}(x)\gamma_i b(x)][\bar{u}(x)\gamma_5 c(x)] \\
\mathcal{O}_2(x) &= [\bar{u}(x)\gamma_5 b(x)][\bar{d}(x)\gamma_i c(x)] - [\bar{d}(x)\gamma_5 b(x)][\bar{u}(x)\gamma_i c(x)] \\
\mathcal{O}_3(x) &= (\bar{u}(x)^T\Gamma_5\bar{d}(x) - \bar{d}(x)^T\Gamma_5\bar{u}(x))(b(x)\Gamma_i c(x)).
\end{aligned}$$

Here, $\Gamma_k = C\gamma_k$, with $C = i\gamma_y\gamma_t$ representing the charge conjugation matrix. The diquarks (antidiquarks) are in the color antitriplet (triplet) representations. In this analysis, we neglect other high-lying two-meson (B^*D^*) and three-meson ($DB\pi$) interpolators, as they have energies sufficiently high to affect the extracted ground states. We also compute two-point correlation functions for B, B^*, D, and D^* mesons, using standard local quark bilinear interpolators ($\overline{Q}\ \Gamma\ q$), with spin structures $\Gamma \sim \gamma_5$ and γ_i for pseudoscalar and vector quantum numbers, respectively. Assuming isospin symmetry, we consider two significant low-lying two-meson thresholds in ascending energy order: $E_{DB^*} = M_{B^*} + M_D$ and $E_{BD^*} = M_B + M_{D^*}$. To derive the energy levels of the low-lying spectrum, we analyze the correlation matrices using a variational approach [15] by solving the generalized eigenvalue problem (GEVP).

The extraction of the energy spectrum involves fitting the eigenvalue correlators, $\lambda_n(t)$, or the ratios $R^n(t) = \lambda^n(t)/\mathcal{C}_{m_1}(t)\mathcal{C}_{m_2}(t)$, which exhibit expected asymptotic exponential behavior. Here, $\mathcal{C}_{m_i}$ is the two-point correlation function for the meson m_i. [16]. Our final results are based on fitting the ratio correlators $R^n(t)$.

In Figure 1, we provide estimates of the ground state energy, expressed in units relative to the elastic threshold energy E_{DB^*}, across various values of M_{ps} for all the ensembles used in our study. In cases where bottom meson is involved, we incorporate the corresponding adjusted mass $\tilde{M}_{m_i}$, considering the quark mass correction in NRQCD. This correction is calculated as $\tilde{M}_{B^{(*)}} = M_{B^{(*)}} - 0.5\overline{M}^{\bar{b}b}_{lat} + 0.5\overline{M}^{\bar{b}b}_{phys}$, where $\overline{M}^{\bar{b}b}_{lat}$ ($\overline{M}^{\bar{b}b}_{phys}$) signifies the spin-averaged mass of the $1S$ bottomonium measured on the lattice (in experiments).

The spectrum clearly exhibits a pattern of decreasing energy splitting, indicative of a diminishing interaction strength, with increasing M_{ps}. Another noteworthy observation is the presence of non-zero lattice spacing (a) dependence in the ground state energies for similar volume ensembles. We address this dependence by introducing an a-related factor into the parametrized amplitude.

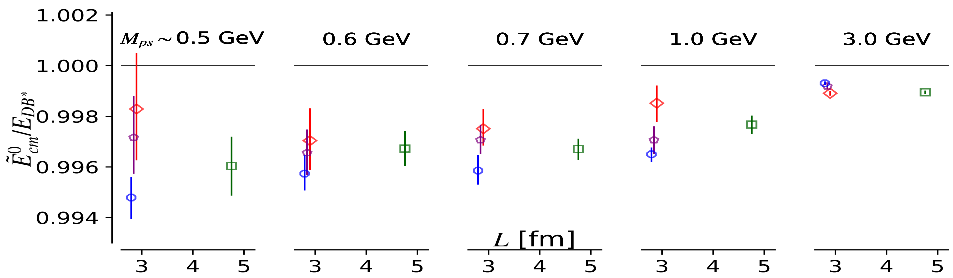

Fig. 1. – The ground state energies in units of the elastic threshold (DB^*) on all ensembles for all M_{ps} values (different vertical panes).

2˙2. *Amplitude analysis-.* – We follow Lüscher's finite-volume prescription [17, 18] to constrain the amplitudes from the finite volume spectrum by assuming elastic DB^* scattering. For the scattering of D and B^* mesons in the S-wave with total angular momentum and parity $J^P = 1^+$, the scattering phase shifts $\delta_{l=0}(k)$ can be related to the finite-volume energy spectrum through $kcot[\delta_0(k)] = 2Z_{00}[1;(\frac{kL}{2\pi})^2)]/(L\sqrt{\pi})$. Here, $k(E_{cm} = \sqrt{s})$ is the momentum in the center of momentum frame such that $4sk^2 = (s-(M_D+M_{B^*})^2)(s-(M_D-M_{B^*})^2)$. We follow Appendix B of Ref. [19] to constrain the amplitude, which is then used to find poles in the complex energy plane. A sub-threshold pole in the S-wave scattering amplitude $t = (\cot\delta_0 - i)^{-1}$ occurs when $k\cot\delta_0 = \pm\sqrt{-k^2}$ for scattering in S-wave. We parametrize the elastic DB^* scattering amplitude with a term related to the scattering length a_0, along with a term that describes the lattice spacing dependence.

By fitting across various quark masses, we find that the scattering length of the DB^* system at the physical light quark mass ($m^{phys}_{u/d}$), corresponding to $M_{ps} = M^{phys}_\pi$), to be

$$a^{phys}_0 = 0.57(^{+4}_{-5})(17)\ \text{fm} \quad (1)$$

The asymmetric errors show the statistical uncertainties, while the second parenthesis indicates the systematic uncertainties with the most dominant contribution arising from the chiral extrapolation fit forms. The positive value of the scattering length at $M_{ps} = M_{\pi}^{phys}$, is an evidence for the ability of the hadron-hadron interaction potential to host a real $bc\bar{u}\bar{d}$ tetraquark bound state.

3. – Summary

We conducted a lattice QCD calculation that focused on the scattering of DB^* in the isoscalar axialvector $(I(J^P) = 0(1^+))$ channel. Our approach involved a meticulous analysis of finite-volume energy spectra and the extrapolation of elastic DB^* scattering amplitudes across a range of five light quark masses. This allowed us to establish the relationship between the elastic DB^* scattering length, denoted as a_0, and the varying light quark masses.

We observe a negative shift in the ground state energy with respect to the threshold and a positive value for the physical scattering length, a_0^{phys}, possibly signifying an attractive force between the D and B^* mesons. In future calculations, we extended this work further to look for poles in the scattering amplitude to check for bound states.

References

[1] Roel Aaij et al. "Observation of an exotic narrow doubly charmed tetraquark". In: *Nature Phys.* 18.7 (2022), pp. 751–754. DOI: 10.1038/s41567-022-01614-y. arXiv: 2109.01038 [hep-ex].

[2] Ivan Polyakov. "Doubly charmed tetraquarks T_{cc} and new road it opens". In: *20th International Conference on Hadron Spectroscopy and Structure*. 2023.

[3] M. Padmanath, Archana Radhakrishnan, and Nilmani Mathur. "Bound isoscalar axial-vector $bc\bar{u}\bar{d}$ tetraquark T_{bc} in QCD". In: (July 2023). arXiv: 2307.14128 [hep-lat].

[4] A. Bazavov et al. "Lattice QCD Ensembles with Four Flavors of Highly Improved Staggered Quarks". In: *Phys. Rev. D* 87.5 (2013), p. 054505. DOI: 10.1103/PhysRevD.87.054505. arXiv: 1212.4768 [hep-lat].

[5] Y. Chen et al. "Chiral logarithms in quenched QCD". In: *Phys. Rev.* D70 (2004), p. 034502. DOI: 10.1103/PhysRevD.70.034502. arXiv: hep-lat/0304005 [hep-lat].

[6] A. Li et al. "Overlap Valence on 2+1 Flavor Domain Wall Fermion Configurations with Deflation and Low-mode Substitution". In: *Phys. Rev. D* 82 (2010), p. 114501. DOI: 10.1103/PhysRevD.82.114501. arXiv: 1005.5424 [hep-lat].

[7] G. Peter Lepage et al. "Improved nonrelativistic QCD for heavy quark physics". In: *Phys. Rev.* D46 (1992), pp. 4052–4067. DOI: 10.1103/PhysRevD.46.4052. arXiv: hep-lat/9205007 [hep-lat].

[8] Aida X. El-Khadra, Andreas S. Kronfeld, and Paul B. Mackenzie. "Massive fermions in lattice gauge theory". In: *Phys. Rev. D* 55 (1997), pp. 3933–3957. DOI: 10.1103/PhysRevD.55.3933. arXiv: hep-lat/9604004.

[9] S. Basak et al. "Hadron spectra from overlap fermions on HISQ gauge configurations". In: *PoS* LATTICE2013 (2014), p. 243. arXiv: 1312.3050 [hep-lat].

[10] Nilmani Mathur, M. Padmanath, and Randy Lewis. “Charmed-Bottom Mesons from Lattice QCD”. In: *PoS* LATTICE2016 (2016), p. 100. arXiv: 1611.04085 [hep-lat].

[11] Bipasha Chakraborty et al. “High-precision quark masses and QCD coupling from $n_f = 4$ lattice QCD”. In: *Phys. Rev. D* 91.5 (2015), p. 054508. DOI: 10.1103/PhysRevD.91.054508. arXiv: 1408.4169 [hep-lat].

[12] Nilmani Mathur, M. Padmanath, and Sourav Mondal. “Precise predictions of charmed-bottom hadrons from lattice QCD”. In: *Phys. Rev. Lett.* 121.20 (2018), p. 202002. DOI: 10.1103/PhysRevLett.121.202002. arXiv: 1806.04151 [hep-lat].

[13] Parikshit Junnarkar, Nilmani Mathur, and M. Padmanath. “Study of doubly heavy tetraquarks in Lattice QCD”. In: *Phys. Rev.* D99.3 (2019), p. 034507. DOI: 10.1103/PhysRevD.99.034507. arXiv: 1810.12285 [hep-lat].

[14] Nilmani Mathur, M. Padmanath, and Debsubhra Chakraborty. “Strongly Bound Dibaryon with Maximal Beauty Flavor from Lattice QCD”. In: *Phys. Rev. Lett.* 130.11 (2023), p. 111901. DOI: 10.1103/PhysRevLett.130.111901. arXiv: 2205.02862 [hep-lat].

[15] Christopher Michael. “Adjoint Sources in Lattice Gauge Theory”. In: *Nucl. Phys.* B259 (1985), pp. 58–76. DOI: 10.1016/0550-3213(85)90297-4.

[16] Jeremy R. Green et al. “Weakly Bound H Dibaryon from SU(3)-Flavor-Symmetric QCD”. In: *Phys. Rev. Lett.* 127.24 (2021), p. 242003. DOI: 10.1103/PhysRevLett.127.242003. arXiv: 2103.01054 [hep-lat].

[17] Martin Luscher. “Two particle states on a torus and their relation to the scattering matrix”. In: *Nucl. Phys. B* 354 (1991), pp. 531–578. DOI: 10.1016/0550-3213(91)90366-6.

[18] Raul A. Briceno. “Two-particle multichannel systems in a finite volume with arbitrary spin”. In: *Phys. Rev. D* 89.7 (2014), p. 074507. DOI: 10.1103/PhysRevD.89.074507. arXiv: 1401.3312 [hep-lat].

[19] M. Padmanath and S. Prelovsek. “Signature of a Doubly Charm Tetraquark Pole in DD* Scattering on the Lattice”. In: *Phys. Rev. Lett.* 129.3 (2022), p. 032002. DOI: 10.1103/PhysRevLett.129.032002. arXiv: 2202.10110 [hep-lat].