Pion−nucleon σ term by the pion deep bound states

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Summary. — The pion deep bound states are one of the most important systems to deduce the pion properties in nuclear medium. The pion−nucleon σ term can be determined, in principle, in the bound states by observing the modification of the pion properties at finite density. We investigate the possibility to determine the value of the pion−nucleon σ term precisely by the experimental observables of the deeply bound pionic atoms. We discuss the sensitivity of the observables to the σ term, and find that the gap of the binding energies and the width of the deep bound states are good quantities for the σ term determination.

1. – Introduction

The study of the pion deep bound states in nucleus, which can not be populated by the X−ray spectroscopy because of the nuclear absorption, began in the 1980s [1, 2]. The production of the deep bound states by the (d,^{3}He) reactions [3, 4] is now well established and the bound states are studied seriously to deduce the information on the QCD chiral symmetry at finite density from the data with high precision [5].

In this article, we discuss briefly the possibility to determine the value of the pion −nucleon σ term precisely by the experimental observables of the deeply bound pionic atoms based on Ref. [6]. The accurate value of the σ term has not be determined yet and still controversial [7]. Hence, it is very interesting to study the possibility to determine the value of the pion−nucleon σ term using the pion deep bound states. First, we consider the sensitivity of the observables to the σ term. Then, by taking into account the typical errors in the up−to−date experiments, we find out the observables of the deeply bound pionic atoms by which we can determine the value of the σ term with the minimal uncertainty [6].

2. – Formalism

We solve the Klein−Gordon equation written as;

$$(1)\qquad \left[-\nabla^2+\mu^2+2\mu V_{\rm opt}(r)\right]\phi(\vec{r})=\left[E_\pi-V_{\rm em}(r)\right]^2\phi(\vec{r}),$$

to obtain the complex eigen energy E_π and the wave function $\phi(\vec{r})$ of the pion bound states. The strong interaction effects are described by the pion−nucleus optical potential $V_{\rm opt}(r)$ expressed as;

$$(2)\qquad 2\mu V_{\rm opt}(r)=-4\pi[b(r)+\varepsilon_2 B_0\rho^2(r)]+4\pi\nabla\cdot[c(r)+\varepsilon_2^{-1}C_0\rho^2(r)]L(r)\nabla,$$

with

$$(3)\qquad b(r)=\epsilon_1[b_0\rho(r)+b_1[\rho_n(r)-\rho_p(r)]],$$

$$(4)\qquad c(r)=\epsilon_1^{-1}[c_0\rho(r)+c_1[\rho_n(r)-\rho_p(r)]],$$

$$(5)\qquad L(r)=\left\{1+\frac{4}{3}\pi\lambda[c(r)+\epsilon_2^{-1}C_0\rho^2(r)]\right\}^{-1},$$

where ϵ_1 and ϵ_2 are defined as $\epsilon_1=1+\dfrac{\mu}{M}$ and $\epsilon_2=1+\dfrac{\mu}{2M}$ with the pion−nucleus reduced mass μ and the nucleon mass M. The pion−nucleus electromagnetic interaction is taken into account by the potential $V_{\rm em}(r)$ [6].

The σ term is included in the density dependence of the optical potential parameters as [8, 9];

$$(6)\qquad b_1(\rho)=b_1^{\rm free}\left(1-\frac{\sigma}{m_\pi^2 f_\pi^2}\rho\right)^{-1},$$

with the σ term, the pion mass m_π and the pion decay constant f_π. The b_0 parameter has also the density dependence because of the double scattering term with $b_1(\rho)$ [6].

By solving the Klein−Gordon equation, we can study the sensitivities of the observables to the value of the σ term. The binding energies and the widths of the pionic atoms are the real and imaginary parts of the eigen energies E_π, and the formation cross sections of the bound states by the (d,^{3}He) reactions can be reliably calculated using the wave function $\phi(\vec{r})$.

3. – Numerical results and discussions

To express the sensitivities of the observables to the σ value, we consider the average sizes of the shifts of observables due to the variation of the value of σ [6]. In Table I, we show the shifts of the binding energies B_π and the widths Γ_π of the $1s$ pionic states, and the shifts of the difference of the binding energies between the $1s$ and $2p$ states ($B_\pi(1s)-B_\pi(2p)$) in ^{123}Sn and ^{111}Sn for the 1 MeV change of the σ value. We can see the shifts of the observables are larger for the lighter isotope ^{111}Sn because of the weaker repulsive effects of the optical potential and the larger fraction of pion inside the nucleus. Hence, the observables of the deep pionic atoms in the lighter Sn isotope are expected to be more sensitive to the value of σ.

In Table I, the sizes of the typical experimental errors are also shown for up−to−date experiments for deeply bound pionic atom formation by the (d,^{3}He) reaction. We can

TABLE I. – *The calculated average shifts of the observables of the deeply bound pionic states in* ^{123}*Sn and* ^{111}*Sn are shown in* keV *for the* 1 MeV *change of the* σ *value.* $B_\pi(1s)$, $\Gamma_\pi(1s)$, *and* $(B_\pi(1s) - B_\pi(2p))$ *indicate the binding energy and the width of the* 1*s* *state, and the difference of the binding energies between the* 1*s* *and* 2*p* *states. Typical errors of up–to–date experiments for deeply bound pionic atom observables in the Sn region are also shown. Numbers are from Ref. [6].*

in keV	^{123}Sn	^{111}Sn	Typical Exp. error
$\lvert\Delta B_\pi(1s)\rvert$	6.2	7.5	∼80
$\lvert\Delta\Gamma_\pi(1s)\rvert$	5.9	12.9	∼40
$\lvert\Delta(B_\pi(1s) - B_\pi(2p))\rvert$	4.5	5.8	10–15

see that the errors for the differences of the binding energies $(B_\pi(1s) - B_\pi(2p))$ are significantly smaller than those of B_π and Γ_π because there exists the large cancellation of the systematic errors appeared in the energy calibration [6]. Hence, the absolute values of the binding energies are not appropriate to determine the value of the σ term because of the large experimental errors, even they have high sensitivities to the σ value. By considering the expected experimental errors together with the sensitivities of the observables, we conclude that the width of the deeply bound 1*s* state in the lighter Sn isotope and the energy difference $(B_\pi(1s) - B_\pi(2p))$ are suited for the determination of the σ term value.

In Fig. 1, we show examples of the calculated (d,^{3}He) reaction spectra for the formation of the pionic bound states in ^{123}Sn to see the sensitivity of the spectra to the

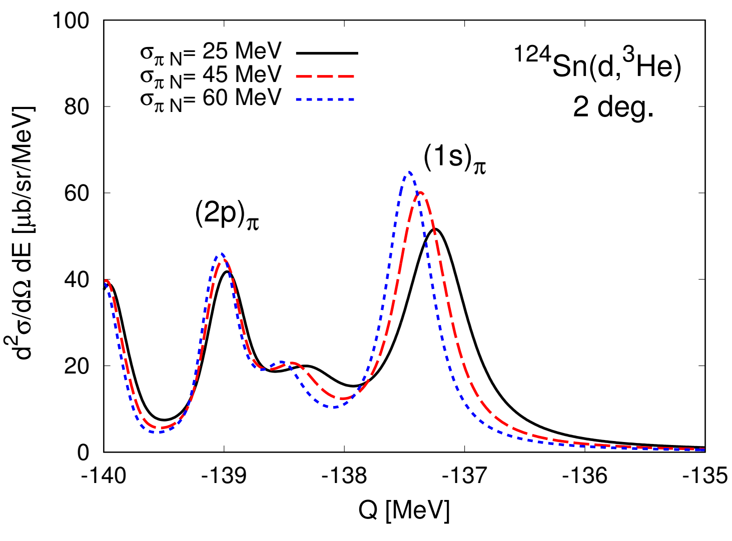

Fig. 1. – Formation cross sections of the deeply bound pionic atoms in ^{123}Sn by the (d,^{3}He) reaction for three different values of σ calculated in Ref. [6]. The scattering angle of the emitted ^{3}He nucleus is fixed to be 2^o in the laboratory frame. The experimental energy resolution is assumed to be 150 keV.

value of the σ term. We can clearly see the σ value dependence of the peak position and the peak width in the spectrum, which are usually fitted by the theoretical results to deduce the binding energies and the widths of the pion bound states in the processes of the analyses of observed results [5]. Hence, we can expect to have the constraints to σ from the spectrum. To obtain more information in addition to B_π and Γ_π from the spectra, we need to study the detailed structure of the spectra such as the σ dependence seen in the spectrum at $Q = -138.5 \sim -138$ MeV in Fig. 1, which will be studied in future.

4. – Conclusion

In this article, we investigate the feasibilities of the observables of the deeply bound pionic atoms to determine the σ value precisely based on Ref. [6], by studying the sensitivities of them to the σ value and by taking into account the typical errors of them in the up−to−date experiments. The σ term is implemented in the calculations of the structure of the deeply bound pionic atoms through the potential parameters.

We consider the pionic atoms in Sn isotopes and study the various observables. We find that the gap of the binding energies of the $1s$ and $2p$ pionic states, and the width of the $1s$ state for the lighter Sn isotope are expected to be most important observables to determine the σ value by the pionic atom data. The uncertainty of the σ value determination is estimated to be $2 \sim 3$ MeV [6]. The detailed structure of the formation spectra is also expected to provide the additional information on σ in future.

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This work was partly supported by JSPS KAKENHI Grant Number JP19K14709 and JP23K03417.

REFERENCES

[1] Yamazaki T., Hirenzaki S., Hayano R. S. and Toki H., *Phys. Rept.*, **514** (2012) 1-87; doi:10.1016/j.physrep.2012.01.003.

[2] Hirenzaki S. and Ikeno N., *Handbook of Nuclear Physics*, **Springer, Singapore** (2022) eds Tanihata, I., Toki, H., Kajino, T.; https://doi.org/10.1007/978-981-15-8818-1_35-1.

[3] Hirenzaki S., Toki H. and Yamazaki T., *Phys. Rev. C*, **44** (1991) 2472-2479; doi:10.1103/PhysRevC.44.2472.

[4] Ikeno N., Nagahiro H. and Hirenzaki S., *Eur. Phys. J. A*, **47** (2011) 161; doi:10.1140/epja/i2011-11161-9 [arXiv:1112.2450 [nucl-th]].

[5] T. Nishi *et al.* [piAF], *Nature Phys.*, **19** (2023) no.6 788-793; doi:10.1038/s41567-023-02001-x [arXiv:2204.05568 [nucl-ex]].

[6] Ikeno N., Nishi T., Itahashi K., Nose-Togawa N., Tani A. and Hirenzaki S., *PTEP*, **2023** (2023) no.3 033D03; doi:10.1093/ptep/ptad031 [arXiv:2204.09211 [nucl-th]].

[7] Gupta R., Park S., Hoferichter M., Mereghetti E., Yoon B.and Bhattacharya T., *Phys. Rev. Lett.*, **127** (2021) 24.

[8] Friedman E. and Gal A.,, *Phys. Lett. B.*, **792** (2019) 340; doi:10.1016/j.physletb.2019.03.036.

[9] Friedman E. and Gal A.,, *Acta Phys. Polon. B.*, **51** (2020) 45; doi:10.5506/APhysPolB.51.45.