# Spin-1 quarkonia in a rotating frame and their spin contents

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**Summary.** — We propose a new way of studying the spin content of a hadron by looking at its response in a rotating frame. By collecting all responses of quarks and gluons in a rotating frame, we describe the spin-rotation coupling of spin-1 quarkonia and thereby reveal their spin contents in a relativistic formalism.

## 1. – Introduction

The reaction of particles with spin in a rotating frame has been of interest for a long time since the mid 1910s when it was realized that mechanical rotation can polarize particle spins through the Barnett effect [1]. This effect is understood by spin-rotation coupling and recently led to new wave of active research field such as measurement of the spin polarization of hadrons in heavy-ion collisions. In this work, we propose a new method to study the spin content of a hadron by examining its response in a rotating frame, and as a first step, we investigate the spin contents of spin-1 quarkonia.

Let us first consider two reference frames: an inertial frame and a (non-inertial) rotating frame which rotates with an angular velocity $\boldsymbol{\Omega}$ with respect to the inertial frame. Then, for a classical particle, the Hamiltonian in the inertial frame($H_i$) and the Hamiltonian in the rotating frame($H_r$) are related by $H_r = H_i - \boldsymbol{L}\cdot\boldsymbol{\Omega}$ where $\boldsymbol{L}$ is the orbital angular momentum of the particle. For a particle with intrinsic spin, it seems natural to generalize this relation to $H_r = H_i - (\boldsymbol{L}+\boldsymbol{S})\cdot\boldsymbol{\Omega}$. For spin-1/2 Dirac particles, this relation can be explicitly derived from the Dirac equation in a rotating frame,

$$[i\gamma^\mu D_\mu - m + \gamma^0(\hat{L}_q + \hat{S}_q)\cdot\boldsymbol{\Omega}]\Psi = 0, \tag{1}$$

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where $D_\mu = \partial_\mu + igA_\mu$ is the covariant derivative and $\hat{L}_q = \boldsymbol{x}\times(-i\boldsymbol{D})$ and $\hat{S}_q = \frac{1}{2}\gamma^0\boldsymbol{\gamma}\gamma^5$ are the orbital and spin angular momentum operator for Dirac fields, respectively. On the other hand, recent research by Kapusta *et al.* [2] explored spin-rotation coupling for massive spin-1 particles using the Proca equation in a rotating frame, revealing an unexpected reduction of the Hamiltonian to $H_r = H_i - (\boldsymbol{L}+\frac{1}{2}\boldsymbol{S})\cdot\boldsymbol{\Omega}$ in the non-relativistic limit. Consequently, it becomes crucial to establish the strength of spin-rotation coupling for spin-1 particles in a model independent way based on Quantum Chromodynamics(QCD).

## 2. – Method

In this work, we introduce a free parameter $g_\Omega$, so-called gravitomagnetic moment in [3], which represents the strength of the spin-rotation coupling for spin-1 system composed of a heavy quark and its anti-quark. To derive the value of $g_\Omega$ on the basis of quarks and gluons degrees of freedom, let us first consider a two-point correlation function for the vector or axial vector current,

$$\Pi^{\mu\nu}(q) = i\int d^4x e^{iqx}\langle 0|\mathrm{T}\big[j^\mu(x)j^\nu(0)\big]|0\rangle. \tag{2}$$

For simplicity, we put the system at the center of the rotation and pick out a right circularly polarized state with $\epsilon^+_\mu = (0,1,i,0)/\sqrt{2}$ in the rotating frame in which the polarization axis and the angular velocity are along the same $z$-direction, *i.e.* $q_\mu = (\omega, 0)$ and $\boldsymbol{\Omega} = (0,0,\Omega)$. Since we are mainly interested in the terms linear in $\Omega$, the relevant component is defined as $\Pi^+(\omega) = \Pi^{\mu\nu}(\omega,0)\epsilon^+_\mu\epsilon^{+*}_\nu = \omega^2\Pi^{\mathrm{vac}}(\omega^2) + \omega\Omega\Pi^{\mathrm{rot}}(\omega^2)$ where $\Pi^{\mathrm{vac}}(\omega^2)$ is the vacuum invariant function and $\Pi^{\mathrm{rot}}(\omega^2)$ is a new function appearing in the rotating frame.

Now let us discuss the phenomenological structure of $\Pi^+(\omega)$. In an inertial frame, $\Pi^+(\omega) = \omega^2\Pi^{\mathrm{vac}}(\omega^2)$. Once we turn on the rotation, the energy of a right circularly polarized state is shifted by $-g_\Omega\Omega$. Therefore $\Pi^+(\omega) \to \Pi^+(\omega + g_\Omega\Omega)$ in the rotating frame. Then we can infer a simple relation between $\Pi^{\mathrm{vac}}(\omega^2)$ and $\Pi^{\mathrm{rot}}(\omega^2)$,

$$\Pi^{\mathrm{rot}}_{\mathrm{phen}}(\omega^2) = 2g_\Omega\Big\{\Pi^{\mathrm{vac}}(\omega^2) + \omega^2\frac{\partial\Pi^{\mathrm{vac}}(\omega^2)}{\partial\omega^2}\Big\}. \tag{3}$$

To compute the OPE in the rotating frame, we first need quark propagators in that frame. Referring to Eq.(1), we use the following expansion of the quark propagator,

$$\begin{aligned} S(x,0) = S^{(0)}(x) + \sum_{n=1}^{\infty}(-1)^n\int dz_1\ldots dz_n S^{(0)}(x-z_1) \\ \times\big[\Delta\mathcal{I}(z_1)\big]S^{(0)}(z_1-z_2)\ldots\big[\Delta\mathcal{I}(z_n)\big]S^{(0)}(z_n), \end{aligned} \tag{4}$$

where $S^{(0)}(x)$ is the free quark propagator and $\Delta\mathcal{I} = g\not{A} + \gamma^0((\hat{L}_q)_z + (\hat{S}_q)_z)\Omega$ includes all the interaction terms. For convenience, we also distinguish the orbital angular momentum operator into two pieces, $\hat{L}_q = \hat{L}_k + \hat{L}_p$ where $\hat{L}_k = \boldsymbol{x}\times\boldsymbol{p}$ is the kinetic part and $\hat{L}_p = \boldsymbol{x}\times(-g\boldsymbol{A}(x))$ is the potential part, respectively. Furthermore, gluon fields appearing in the interaction terms are also modified by the rotation. Within Fock-Schwinger gauge($x^\mu A_\mu(x)=0$), the gluon field in the rotating frame is expressed as

$$A_\mu(x) = -\frac{1}{2}x^\nu G_{\mu\nu}(0) - \frac{1}{3}x^\nu x^\alpha(\Gamma^\rho_{\alpha\mu}G_{\rho\nu}(0)+\Gamma^\rho_{\alpha\nu}G_{\mu\rho}(0))+\ldots, \tag{5}$$

where $\Gamma^\rho_{\mu\nu}$ denotes the Christoffel symbol. We then denote the contribution of $\Omega$ linear terms in Eq.(5) as $\hat{J}_g$ because we later found that the sum gives the same contribution with the gluon's total angular momentum operator, $\hat{J}_g = \boldsymbol{x}\times(\boldsymbol{E}\times\boldsymbol{B})$, refering to [4]. By collecting terms linear in $\Omega$, we compute the OPE upto operators of dimension 4,

$$\Pi^{\rm rot}_{\rm OPE}(Q^2) = \sum_{i=S_q,L_k,L_p,J_g} \Pi^{\rm rot}_{{\rm I},i}(Q^2) + \Pi^{\rm rot}_{G_0,i}(Q^2), \tag{6}$$

where $\Pi^{\rm rot}_{{\rm I},i}$ denotes the leading perturbative part and $\Pi^{\rm rot}_{G_0,i} = C_i(Q^2)\cdot G_0$ denotes the leading non-perturbative part with $C_i(Q^2)$ as the Wilson coefficients for the scalar gluon condensates $G_0 \equiv \langle \frac{\alpha_s}{\pi} G^a_{\mu\nu}G^{a,\mu\nu}\rangle = (0.35\,{\rm GeV})^4$. Conventionally, these coefficients are expressed using $J_N(y)=\int_0^1 dx[1+x(1-x)y]^{-N}$ where $y=Q^2/m^2$.

## 3. – OPE results

For vector channel,

$$\mathrm{Im}\Pi^{\rm rot}_{{\rm I},S_q}(s) = \frac{3m^2}{2\pi\sqrt{s(s-4m^2)}}, \tag{7}$$

$$\mathrm{Im}\Pi^{\rm rot}_{{\rm I},L_k}(s) = \frac{(s-m^2)\sqrt{s(s-4m^2)}}{2\pi s^2}, \tag{8}$$

$$C_{S_q} = -\frac{2-y+12J_2-26J_3+12J_4}{12Q^4}, \tag{9}$$

$$C_{L_k} = \frac{11-4y-(8+y)J_1+13J_2-16J_3}{72Q^4}, \tag{10}$$

$$C_{L_p} = -\frac{13+2y-(12+5y)J_1+3J_2-4J_3}{72Q^4}, \tag{11}$$

$$C_{J_g} = \frac{13-(2+2y)J_1-23J_2+12J_3}{36Q^4}. \tag{12}$$

For axial vector channel,

$$\mathrm{Im}\Pi^{\rm rot}_{{\rm I},S_q}(s) = \frac{3m^2\sqrt{s(s-4m^2)}}{2\pi s^2}, \tag{13}$$

$$\mathrm{Im}\Pi^{\rm rot}_{{\rm I},L_k}(s) = \frac{(s-m^2)\sqrt{s(s-4m^2)}}{2\pi s^2}, \tag{14}$$

$$C_{S_q} = -\frac{6-y-12J_2+6J_3}{12Q^4}, \tag{15}$$

$$C_{L_k} = \frac{7-4y-(8+y)J_1+25J_2-24J_3}{72Q^4}, \tag{16}$$

$$C_{L_p} = -\frac{9+2y+(4-5y)J_1-17J_2+4J_3}{72Q^4}, \tag{17}$$

$$C_{J_g} = \frac{1+(6-2y)J_1-3J_2-4J_3}{36Q^4}. \tag{18}$$

By comparing these results with the counterparts of the vacuum OPE, we indeed find $g_\Omega = 1$ for both channels. In fact, what we computed in the OPE is equivalent to the expectation value of the total angular momentum operator in QCD, $\boldsymbol{J}_{\rm QCD} = \int d^3x(\frac{1}{2}\bar{\psi}\boldsymbol{\gamma}\gamma_5\psi + \psi^\dagger(\boldsymbol{x}\times(-i\boldsymbol{D}))\psi + \boldsymbol{x}\times(\boldsymbol{E}\times\boldsymbol{B}))$ [5]. Similarly, what we considered in the phenomenological side is nothing else but the expectation value of $g_\Omega \boldsymbol{S}\cdot\boldsymbol{\Omega}$ where $\boldsymbol{S}$ is the spin-1 operator. Therefore, our finding, *i.e.* $g_\Omega = 1$, just confirms that the total spin of the system is equal to the total angular momentum of quarks and gluons.

## 4. – Spin contents of spin-1 quarkonia

The above findings can be utilized to calculate the spin contents of spin-1 quarkonia such as $J/\psi$, $\Upsilon(1\mathrm{S})$, $\chi_{c1}$, and $\chi_{b1}$. $\mathrm{Im}\Pi^{\rm vac}(s)$ represents the spectral density in which all physical states that can couple to the vector or axial vector current are involved. In order to pick out the ground state, it is often modeled as $\mathrm{Im}\Pi^{\rm vac}(s) = \pi f_0\delta(s - m_0^2) + \theta(s-s_0)\mathrm{Im}\Pi^{\rm vac}_{\rm I}(s)$ where $f_0$ is the residue, $m_0$ is the ground state mass, $s_0$ is the continuum threshold. Then the fraction of $g_\Omega$ for the ground state can be extracted from the following equations,

$$g_\Omega(M,s_0) = -\frac{M^2}{2}\frac{\bar{\mathcal{M}}^{\rm rot}}{\partial\bar{\mathcal{M}}^{\rm vac}/\partial(1/M^2)}, \tag{19}$$

$$\bar{\mathcal{M}}^{\rm vac} = \mathcal{B}\big[\Pi^{\rm vac}_{\rm OPE}(Q^2)\big] - \int_{s_0}^{\infty} ds e^{-s/M^2}\mathrm{Im}\Pi^{\rm vac}_{\rm I}(s), \tag{20}$$

$$\bar{\mathcal{M}}^{\rm rot} = \mathcal{B}\big[\Pi^{\rm rot}_{\rm OPE}(Q^2)\big] - \int_{s_0}^{\infty} ds e^{-s/M^2}\mathrm{Im}\Pi^{\rm rot}_{\rm I}(s). \tag{21}$$

Here, Borel transformation is defined by

$$\mathcal{B} \equiv \lim_{\substack{Q^2/n\to M^2,\\ n,Q^2\to\infty}} \frac{\pi(Q^2)^{n+1}}{n!}\left(-\frac{d}{dQ^2}\right)^n. \tag{22}$$

In an actual analysis, we need to specify an effective threshold($\bar{s}_0$) and a reliable range of Borel mass($M$), so-called Borel window. The values of $\bar{s}_0$ and Borel window used in this work are listed in Table I; See [6-8] for more details. For input parameters, we use $m_c(p^2=-m_c^2) = 1.262\,\mathrm{GeV}$, $\alpha_s(8m_c^2) = 0.21$ for charmonia and $m_b(p^2=-m_b^2) = 4.12\,\mathrm{GeV}$, $\alpha_s(8m_b^2) = 0.158$ for bottomonia following [6].

Then we finally estimate the spin contents of the spin-1 quarkonia by averaging the contribution of each angular momentum operator in $g_\Omega(M,\bar{s}_0)$ over the given Borel window. We also calculate the variance of each contribution to estimate the uncertainty. The average values and their uncertainties(subscript) are listed in Table II. In all cases the sum of the four components is exactly 1 as we expected, but the spin contents are quite different from each other. While the bottomonia results are somewhat comparable with the non-relativistic quark model picture, the spin contents start to deviate from

TABLE I.: $\sqrt{s_0}$ and Borel window for spin-1 quarkonia

| | $J/\psi$ | $\chi_{c1}$ | $\Upsilon(1S)$ | $\chi_{b1}$ |
|---|---|---|---|---|
| $\sqrt{s_0}$ [GeV] | 3.5 | 4.0 | 10.3 | 11 |
| ($M_{\rm min}$,$M_{\rm max}$) [GeV] | (1,2.3) | (1.4,2.3) | (3,5.5) | (3.6,4.9) |

TABLE II.: The spin contents of spin-1 quarkonia

| | $J/\psi$ | $\Upsilon(1S)$ | $\chi_{c1}$ | $\chi_{b1}$ |
|---|---|---|---|---|
| $S_q$ | $0.88_{1.8e\text{-}4}$ | $0.92_{7.6e\text{-}5}$ | $0.40_{8.2e\text{-}5}$ | $0.43_{1.1e\text{-}5}$ |
| $L_k$ | $0.11_{4.9e\text{-}4}$ | $0.076_{7.8e\text{-}5}$ | $0.61_{5.8e\text{-}6}$ | $0.57_{1.0e\text{-}5}$ |
| $L_p$ | $2.0e\text{-}3_{2.9e\text{-}6}$ | $3.5e\text{-}5_{3.0e\text{-}10}$ | $8.2e\text{-}4_{2.3e\text{-}8}$ | $-1.0e\text{-}5_{3.4e\text{-}10}$ |
| $J_g$ | $8.0e\text{-}3_{5.9e\text{-}5}$ | $1.5e\text{-}4_{7.3e\text{-}9}$ | $-0.015_{5.2e\text{-}5}$ | $-5.2e\text{-}5_{2.3e\text{-}8}$ |

this picture as the quark mass becomes lighter. For example, the $J/\psi$ is traditionally considered as an S-wave particle but now we find that the quark spin does not carry all of the total spin as in the case of the proton spin [9].

## 5. – Summary

We have proven that $g_\Omega = 1$ for heavy composite particles with spin-1 and simultaneously have identified how the angular momenta of quarks and gluons add up to their total spin in a relativistic way. The most crucial finding in this work is the discovery of the universal formula in the OPE which is given by a simple relation between the rotating frame part and the corresponding inertial frame part. Because this relation indicates that the total spin of the system is equal to the total angular momentum of its constituents, the methodology outlined in this study holds the potential to be extended to other systems, offering insights into their spin contents.

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