Mass spectrum of three-quark and five-quark singly heavy baryons from a chiral model

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Summary. — We construct a chiral effective model for three-quark and five-quark singly heavy baryons (SHBs) which is heavy-quark spin singlet, focusing on the $U(1)_A$ axial anomaly effects. Based on the model, we find that the anomaly effects induce the inverse mass hierarchy of negative-parity three-quark SHBs where Λ_c becomes heavier than Ξ_c. On the contrary, the anomaly effect is found to provide no effects for the mass spectrum of five-quark SHBs. We also present a predicted mass spectrum of the SHBs in the presence of the mixing between three-quark and five-quark states. The predicted five-quark dominant $\Lambda_c(-)$ whose mass is approximately 2700 MeV is expected to be a useful evidence to check our description.

1. – Introduction

Singly heavy baryons (SHBs) composed of a heavy quark and a diquark provide us with useful testing ground to explore the diquark dynamics, since the heavy quark can be regarded as a spectator due to its large mass. In this write-up, we review our recent studies on the heavy-quark spin-singlet SHBs, quark contents of which are Qqq and $Qqq\bar{q}q$, i.e., three-quark states and five-quark states, where the Roper-like $\Lambda_c(2765)$ and $\Xi_c(2970)$ are assumed to be the five-quark dominant SHBs [1, 2, 3]. In particular, we focus on effects of $U(1)_A$ axial anomaly on the SHBs based on a three-flavor chiral model.

This article is organized as follows. In Sec. **2** we introduce our chiral mode describing the three-quark and five-quark SHBs. Then, in Sec. **3** and Sec. **4**, we investigate masses of the SHBs with and without mixings between the three-quark and five-quark states. Finally, in Sec. **5** we conclude the present work .

2. – Model

In this section, we introduce our chiral model to describe the three-quark and five-quark SHBs.

Toward the model construction, we invent the following four building blocks:

$$B_{R,a} \sim Q^\alpha d^\alpha_{R,a}\ , \quad B_{L,i} \sim Q^\alpha d^\alpha_{L,i}\ , \quad B'^\alpha_{R,i} \sim Q^\alpha d'_{R,i}\ , \quad B'_{L,a} \sim Q^\alpha d'^\alpha_{L,a} \tag{1}$$

where $d^{\alpha}_{R,a}$, $d^{\alpha}_{L,a}$, $d'^{\alpha}_{R,a}$ and $d'^{\alpha}_{L,a}$ represent the conventional diquarks and the newly invented *tetra-diquarks* defined by [1, 3]

$$
\begin{aligned}
(d_R)^{\alpha}_a &\sim \epsilon_{abc}\epsilon^{\alpha\beta\gamma}(q_R^T)^{\beta}_b C(q_R)^{\gamma}_c \ , \\
(d_L)^{\alpha}_i &\sim \epsilon_{ijk}\epsilon^{\alpha\beta\gamma}(q_L^T)^{\beta}_j C(q_L)^{\gamma}_k \ , \\
(d'_R)^{\alpha}_i &\sim \epsilon_{abc}\epsilon^{\alpha\beta\gamma}(q_R^T)^{\beta}_b C(q_R)^{\gamma}_c[(\bar{q}_L)^{\delta}_i(q_R)^{\delta}_a] \ , \\
(d'_L)^{\alpha}_a &\sim \epsilon_{ijk}\epsilon^{\alpha\beta\gamma}(q_L^T)^{\beta}_j C(q_L)^{\gamma}_k[(\bar{q}_R)^{\delta}_a(q_L)^{\delta}_i] \ ,
\end{aligned}
\tag{2}
$$

respectively. In these equations the subscripts "$a, b, \cdots$" and "$i, j, \cdots$" stand for the left-handed and right-handed chiral indices, respectively, within a three-flavor description, while the superscripts "$\alpha, \beta, \cdots$" indicate the color indices. Thus, in Eq. (1), $B_{R,a}$ and $B_{L,i}$ are regarded as the three-quark SHBs, while $B'_{R,i}$ and $B'_{L,a}$ are the five-quark ones. The chiral representation for those diquarks reads

$$
d_R \sim (\mathbf{1}, \bar{\mathbf{3}})_{+2} \ , \quad d_L \sim (\bar{\mathbf{3}}, \mathbf{1})_{-2} \ , \quad d'_R \sim (\bar{\mathbf{3}}, \mathbf{1})_{+4} \ , \quad d'_L \sim (\mathbf{1}, \bar{\mathbf{3}})_{-4} \ , \tag{3}
$$

where the number attached to the respective bracket, e.g., +2 for d_R, stands for their $U(1)_A$ axial charges. The corresponding SHBs take the identical symmetry properties. We note that the chiral representations carried by d_R and d'_L are the same, likewise, those by d_L and d'_R are the same. That is, those states are distinguished by the $U(1)_A$ axial charges.

From the chiral representation (3) with definition of the SHB fields (1), one can construct a chiral model for the SHBs interacting with a light-meson nonet $\Sigma = S + iP$ whose chiral representation is $\Sigma \sim (\mathbf{3}, \bar{\mathbf{3}})_{-2}$. Our Lagrangian is based on the following counting scheme: First we include all possible terms which is invariant under both $U(1)_A$ axial and $SU(3)_L \times SU(3)_R$ chiral transformations, and next, we additionally incorporate contributions which only violate $U(1)_A$ axial symmetry to take into account the anomalous contributions with the minimal number of $\Sigma^{(\dagger)}$. Then, our chiral Lagrangian within the heavy-baryon effective theory is constructed as [3]

$$
\mathcal{L}_{\rm SHB} = \mathcal{L}_{3q} + \mathcal{L}_{5q} + \mathcal{L}_{\rm mix} \ , \tag{4}
$$

where

$$
\begin{aligned}
\mathcal{L}_{3q} = &\sum_{\chi=L,R}(\bar{B}_\chi iv\cdot\partial B_\chi - \mu_1\bar{B}_\chi B_\chi) - \frac{\mu_3}{f_\pi^2}\Big[\bar{B}_L(\Sigma\Sigma^\dagger)^T B_L + \bar{B}_R(\Sigma^\dagger\Sigma)^T B_R\Big] \\
&- \frac{g_1}{2f_\pi}\left(\epsilon_{ijk}\epsilon_{abc}\bar{B}_{L,k}\Sigma_{ia}\Sigma_{jb}B_{R,c} + \text{h.c.}\right) - g'_1(\bar{B}_L\Sigma^* B_R + \text{h.c.}) \ ,
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
\mathcal{L}_{5q} = &\sum_{\chi=L,R}(\bar{B}'_\chi iv\cdot\partial B'_\chi - \mu_2\bar{B}'_\chi B'_\chi) - \frac{\mu_4}{f_\pi^2}\Big[\bar{B}'_R(\Sigma\Sigma^\dagger)^T B'_R + \bar{B}'_L(\Sigma^\dagger\Sigma)^T B'_L\Big] \\
&- \frac{g_2}{6f_\pi^3}\Big[(\epsilon_{abc}\epsilon_{ijk}\Sigma^\dagger_{ci}\Sigma^\dagger_{bj}\Sigma^\dagger_{ak})(\bar{B}'_R\Sigma^* B'_L) + \text{h.c.}\Big] \\
&- \frac{g_3}{2f_\pi^3}\Big(\epsilon_{abc}\epsilon_{ijk}\bar{B}'_{R,l}\Sigma^\dagger_{cl}\Sigma^\dagger_{bi}\Sigma^\dagger_{aj}\Sigma^\dagger_{dk}B'_{L,d} + \text{h.c.}\Big) + g'_2\left(\bar{B}'_R\Sigma^* B'_L + \bar{B}'_L\Sigma^T B'_R\right) \ ,
\end{aligned}
\tag{6}
$$

and

$$
\begin{aligned}
\mathcal{L}_{\rm mix} = & -\mu_1'(\bar{B}_R B_L' + \bar{B}_L' B_R + \bar{B}_L B_R' + \bar{B}_R' B_L) \\
& - g_4(\bar{B}_R' \Sigma^* B_R + \bar{B}_L \Sigma^* B_L' + \text{h.c.}) \ , \qquad (7)
\end{aligned}
$$

with v^μ and f_π being a velocity of the SHBs and a pion decay constant. In this Lagrangian, $\mathcal{L}_{3q}$ describes interactions among the three-quark SHBs and light-meson nonet. Similarly, $\mathcal{L}_{5q}$ describes those among the five-quark SHBs and the nonet. The last piece, $\mathcal{L}_{\rm mix}$, is responsible for interplay between the three-quark and five-quark SHBs mediated by the light mesons. We note that only g_1', g_2' and μ_1' terms correspond to the anomalous contributions. Meanwhile, all the remaining terms proportional to μ_1, μ_2, μ_3, μ_4, g_1, g_2, g_3 and g_4 are $U(1)_A$ invariant although some of them are of fourth order of $\Sigma^{(\dagger)}$.

3. – Masses of the pure three-quark and five-quark SHBs

In this section, we examine the $U(1)_A$ axial anomaly effects on masses of the pure three-quark SHBs and five-quark SHBs.

First, we investigate the mass spectrum of the pure three-quark SHBs. The parity eigenstates, i.e., the mass eigenstates of them are defined by $B_{\pm,i} = (B_{R,i} \mp B_{L,i})/\sqrt{2}$, with the diagonal parts of left- and right-handed indices: $i = a$, where the subscript "$\pm$" stands for the parity eigenvalues. Here, the SHBs with $i = 1, 2$ and $i = 3$ represent $\Xi_c^{[\mathbf{3}]} \sim cus\,(cds)$ and $\Lambda_c^{[\mathbf{3}]} \sim cud$ for the charm sector, where the superscript "$[\mathbf{3}]$" is attached to emphasize that those states are three-quark SHBs. Thus, the mass formulas for $B_{\pm,i}$ are read off by quadratic terms of B_R and B_L in $\mathcal{L}_{3q}$ in Eq. (5), which yields [3]

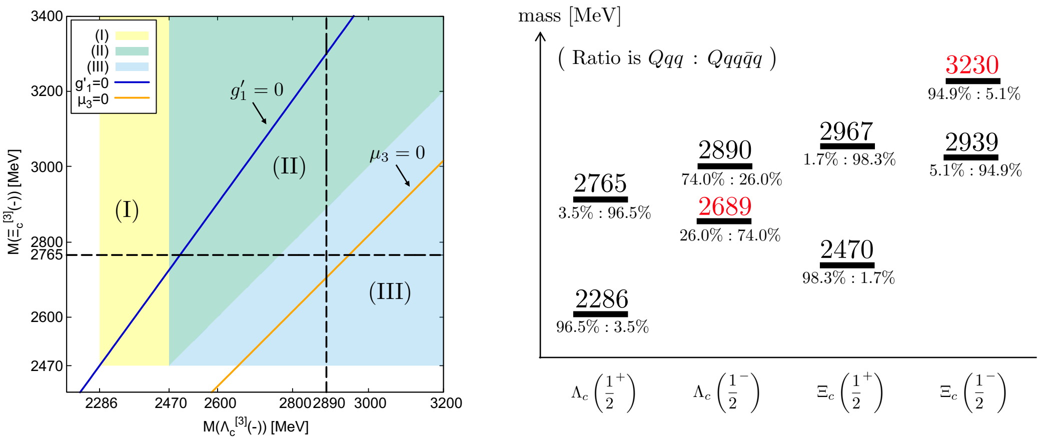
$$
\begin{aligned}
M[\Lambda_c^{[\mathbf{3}]}(\pm)] &= m_B + \mu_1 + \mu_3 \mp f_\pi(g_1 + A g_1') \ , \\
M[\Xi_c^{[\mathbf{3}]}(\pm)] &= m_B + \mu_1 + A^2\mu_3 \mp f_\pi(A g_1 + g_1') \ . \qquad (8)
\end{aligned}
$$

In this equation the sign "$\pm$" again indicates the parity eigenvalue. An arbitrary constant m_B is additionally included to defined the masses to incorporate the universal "heavy mass". Besides, in deriving the mass formulas (8), the chiral-symmetry breaking effects are taken into account by replacing the light meson nonet Σ by its vacuum expectation value: $\langle\Sigma\rangle = f_\pi {\rm diag}(1, 1, A)$, where $f_\pi = 93$ MeV and $A = (2f_K - f_\pi)/f_\pi = 1.38$.

When the ground-state SHBs are regarded as the three-quark SHBs, $M\big[\Lambda_c^{[\mathbf{3}]}(+)\big] = 2286$ MeV and $M\big[\Xi_c^{[\mathbf{3}]}(+)\big] = 2470$ MeV are satisfied, then two free parameters are left. When taking $M\big[\Lambda_c^{[\mathbf{3}]}(-)\big]$ and $M\big[\Xi_c^{[\mathbf{3}]}(-)\big]$ to be free, one can draw mass hierarchies in $M\big[\Lambda_c^{[\mathbf{3}]}(-)\big]$ - $M\big[\Xi_c^{[\mathbf{3}]}(-)\big]$ plane as in Fig. 1 (a) with $g_1' = 0$ exhibited by the blue line. In this figure, the three colored regions represent the following possible mass orderings:

$$
\begin{aligned}
&\text{(I)} \ \ M[\Lambda_c^{[\mathbf{3}]}(+)] < M[\Lambda_c^{[\mathbf{3}]}(-)] < M[\Xi_c^{[\mathbf{3}]}(+)] < M[\Xi_c^{[\mathbf{3}]}(-)] \ , \\
&\text{(II)} \ \ M[\Lambda_c^{[\mathbf{3}]}(+)] < M[\Xi_c^{[\mathbf{3}]}(+)] < M[\Lambda_c^{[\mathbf{3}]}(-)] < M[\Xi_c^{[\mathbf{3}]}(-)] \ , \\
&\text{(III)} \ \ M[\Lambda_c^{[\mathbf{3}]}(+)] < M[\Xi_c^{[\mathbf{3}]}(+)] < M[\Xi_c^{[\mathbf{3}]}(-)] < M[\Lambda_c^{[\mathbf{3}]}(-)] \ . \qquad (9)
\end{aligned}
$$

The regions (I) and (II) indicate that the negative-parity SHBs satisfy the normal mass hierarchy $M\big[\Lambda_c^{[\mathbf{3}]}(-)\big] < M\big[\Xi_c^{[\mathbf{3}]}(-)\big]$ as naively expected from their quark contents. On the other hand, in the region (III) those masses read $M\big[\Lambda_c^{[\mathbf{3}]}(-)\big] > M\big[\Xi_c^{[\mathbf{3}]}(-)\big]$ despite



(a) The three mass hierarchies in Eq. (9).

(b) The mass spectrum with the mixing effects.

Fig. 1. – (a) The mass orderings for the pure three-quark SHBs listed in Eq. (9) where the normal [(I) and (II)] and inverse (III) mass hierarchies are explicitly shown. (b) The mass spectrum in the presence of the mixing between three-quark and five-quark SHBs. The figures are taken from Ref. [3]. Copyright 2203, American Physical Society.

the quark contents. For this reason we call this ordering the *inverse mass hierarchy*. Since the blue line in Fig. 1 (a) indicates the masses with $g_1' = 0$ where the $U(1)_A$ axial anomaly effects are absent. Thus, we can conclude that the inverse mass hierarchy is induced by the $U(1)_A$ anomaly. This is the main finding of influence of the anomaly on mass spectrum of the pure three-quark SHBs.

Next, as for the pure five-quark SHBs, from $\mathcal{L}_{5q}$ in (6) the mass eigenvalues read [3]

$$
\begin{aligned}
M[\Lambda_c^{[\mathbf{5}]}(\pm)] &= m_B + \mu_2 + A^2\mu_4 \pm A f_\pi\left[A(g_2+g_3)+g_2'\right] , \\
M[\Xi_c^{[\mathbf{5}]}(\pm)] &= m_B + \mu_2 + \mu_4 \pm f_\pi\left[A(g_2+g_3)+g_2'\right] ,
\end{aligned} \tag{10}
$$

where $\Xi_c^{[\mathbf{5}]} \sim csdu\bar{u}\,(csud\bar{d})$ and $\Lambda_c^{[\mathbf{5}]} \sim cuds\bar{s}$ and the other notations follow Eq. (8). In this case, the common piece of $h \equiv A(g_2+g_3)+g_2'$ appears in $M[\Lambda_c^{[\mathbf{5}]}(\pm)]$ and $M[\Xi_c^{[\mathbf{5}]}(\pm)]$, and thus, the anomalous contributions from g_2' effectively disappear for the mass formulas. Therefore, in contrast to the pure three-quark SHBs, for the pure five-quark SHBs the $U(1)_A$ axial anomaly has no influence on the mass spectrum.

4. – Masses in the presence of the three-quark and five-quark SHBs

In this section, we present a typical predictions of the mass spectrum with mixing effects triggered by $\mathcal{L}_{\rm mix}$ in Eq. (7).

When the mixing is present, in general, the mass eigenstates read, e.g.,

$$
\begin{pmatrix} \Lambda_c^L(\pm) \\ \Lambda_c^H(\pm) \end{pmatrix} = \begin{pmatrix} \cos\theta_{\Lambda_c(\pm)} & \sin\theta_{\Lambda_c(\pm)} \\ -\sin\theta_{\Lambda_c(\pm)} & \cos\theta_{\Lambda_c(\pm)} \end{pmatrix} \begin{pmatrix} \Lambda_c^{[\mathbf{3}]}(\pm) \\ \Lambda_c^{[\mathbf{5}]}(\pm) \end{pmatrix} , \tag{11}
$$

for Λ_c sector with $\theta_{\Lambda_c(\pm)}$ being a mixing angle. Similar equation follows for Ξ_c sector. The superscript "L/H" represents the eigenstate whose mass is lower/higher, and those mass eigenvalues $M[\Lambda_c^{L/H}(\pm)]$ and $M[\Xi_c^{L/H}(\pm)]$ are evaluated by diagonalizing the corresponding mass matrix from Eqs. (5), (6) and (7). In the following analysis, we will assume the $U(1)_A$ anomaly effect is absent for a transparent demonstration with the mixing: $g_1' = g_2' = \mu_1' = 0$. Hence, there remain seven free parameters to be fixed: μ_1, μ_2, μ_3, μ_4, g_1, $h = A(g_2 + g_3)$ and g_4.

For positive-parity states, denote the ground-state and Roper-like SHBs as Λ_c^L (Ξ_c^L) and Λ_c^H (Ξ_c^H), respectively. Then the following four inputs are employed: $M[\Lambda_c^L(+)] = 2286\,\text{MeV}$, $M[\Xi_c^L(+)] = 2470\,\text{MeV}$, $M[\Lambda_c^H(+)] = 2765\,\text{MeV}$ and $M[\Xi_c^L(+)] = 2967\,\text{MeV}$. For the negative-parity SHBs, first, we employ a quark-model prediction for $\Lambda_c(-)$ provided in Ref. [4] as another input: $M[\Lambda_c^L(-)] = 2890\,\text{MeV}$. Next, the experimentally observed $\Xi_c(2930)$ would be regarded as $\Xi_c^L(-)$, hence, the last input can be $M[\Xi_c^L(-)] = 2939\,\text{MeV}$. Here, we have attached "L" for both the inputs $\Lambda_c(-)$ and $\Xi_c(-)$, since as shown below those states will be found to be the lower mass-eigenvalue states.

From the above six inputs, only one free parameter is left. By fixing this last parameter at which $\theta_{\Xi_c(-)}$ becomes the largest value allowed by the decay width of $\Xi_c(2930)$, one can obtain the mass spectrum in the presence of the mixing as Fig. 1 (b). In this figure the mass values indicated in black and red correspond to the inputs and outputs, respectively. Also, the ratios shown below the mass values represent $Qqq : Qqq\bar{q}q$ for the respective states. Then, from this figure one can see that the ground-state (Roper-like) negative-parity SHBs are dominated by the five-quark (three-quark) components, while the positive-parity SHBs exhibits the opposite tendency by the assumption. One notable prediction is the presence of five-quark dominant $\Lambda_c(-)$ whose mass is 2689 MeV. This SHB decays only through the heavy-quark spin-symmetry breaking processes, and the resultant width reads of order a few MeV. Meanwhile, the three-quark dominant $\Xi_c(-)$ whose mass is 3230 MeV has a catastrophically large decay width. We note that, even when the $U(1)_A$ axial anomaly effects are present, our main prediction of $\Lambda_c(-)$ whose mass is approximately 2700 MeV dose not change as shown in Ref. [3].

5. – Conclusions

In this write-up, we have unveiled effects of the $U(1)_A$ axial anomaly on the three-quark and five-quark SHBs based on a chiral model, and presented a prediction of the mass spectrum of those SHBs. The predicted five-quark dominant $\Lambda_c(-)$ whose mass is approximately 2700 MeV is expected to be a useful prove to check our description.

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