

QCD confronts heavy flavor and exotic hadrons

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Hadrons

$G = \text{gluon}, q = \text{quark} = u, d, s, c, b$

Today we know (from exp and theory) that hadrons with the following minimal quark and gluon contents. There may be more categories, but these are not reliably confirmed yet.

minimal quark (q) and gluon (G) contents

conventional hadrons

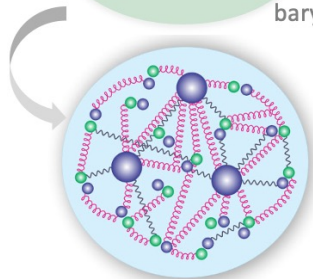
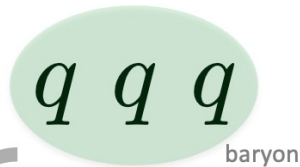
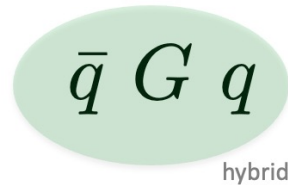
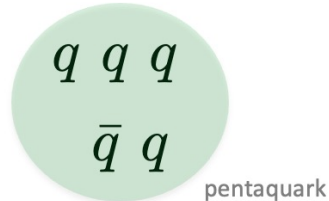
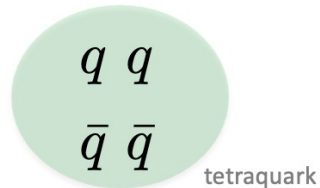


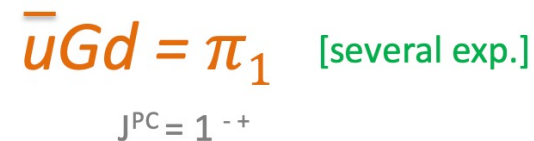
image:

<https://physicstoday.scitation.org/doi/10.1063/PT.5.7167/full/>

exotic hadrons



experimentally discovered example



LHCb discoveries:
talk by Mikhasenko

This talk: how well QCD confronts (heavy flavor and exotic) hadrons

- intro on theory approaches with few examples
- journey to various hadron sectors
- T=0 thought

QCD: $\mathcal{L}_{QCD} = \frac{1}{4} G_a^{\mu\nu} G_a^{\mu\nu} + \bar{q} i \gamma_\mu (\partial^\mu + i g_s G_a^\mu T^a) q - m_q \bar{q} q$

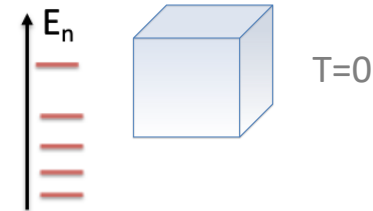
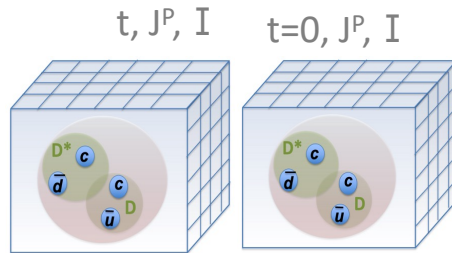
$g_s \ll 1$ at hadronic energy scale

Non-perturbative theory approaches to QCD

Lattice QCD Main quantity extracted: eigen-energy E_n

$$C_{ij}(t) = \langle 0 | \mathcal{Q}_i(t) \mathcal{Q}_j^\dagger(0) | 0 \rangle = \sum_n \langle 0 | \mathcal{Q}_i | n \rangle e^{-E_n t} \langle n | \mathcal{Q}_j^\dagger | 0 \rangle$$

Euclidian time ↓



$$\langle C \rangle = \int DG Dq D\bar{q} C e^{-S_{QCD}^E/\hbar}$$

often “non-precision” studies: single a, $m_{u/d} > m_{u/d}^{phy}$, $m_\pi > 140$ MeV

E_n allow to extract stable hadrons and decaying resonances (as I’ll explain later)

This talk: mostly
lattice, lattice + EFT

recent reviews:

[N. Brambilla et al. 1907.07583, Phys. Rept](#)

[M. Mai, U. Meissner, C. Urbach, 2206.01477](#)

[N. Brambilla, 2111.10788](#)

Effective Field Theory (EFT)

- Lagrangian for effective d.o.f. based on symmetries of QCD
- Taylor expansion in a small quantity ($1/m_Q, p, E, \dots$)
- drawback: dependence on (unknown) parameters

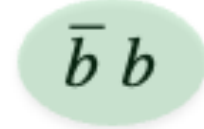
Functional methods (Dyson-Schwinger & Bethe-Salpeter equations): parallel talk by C. Fischer

...

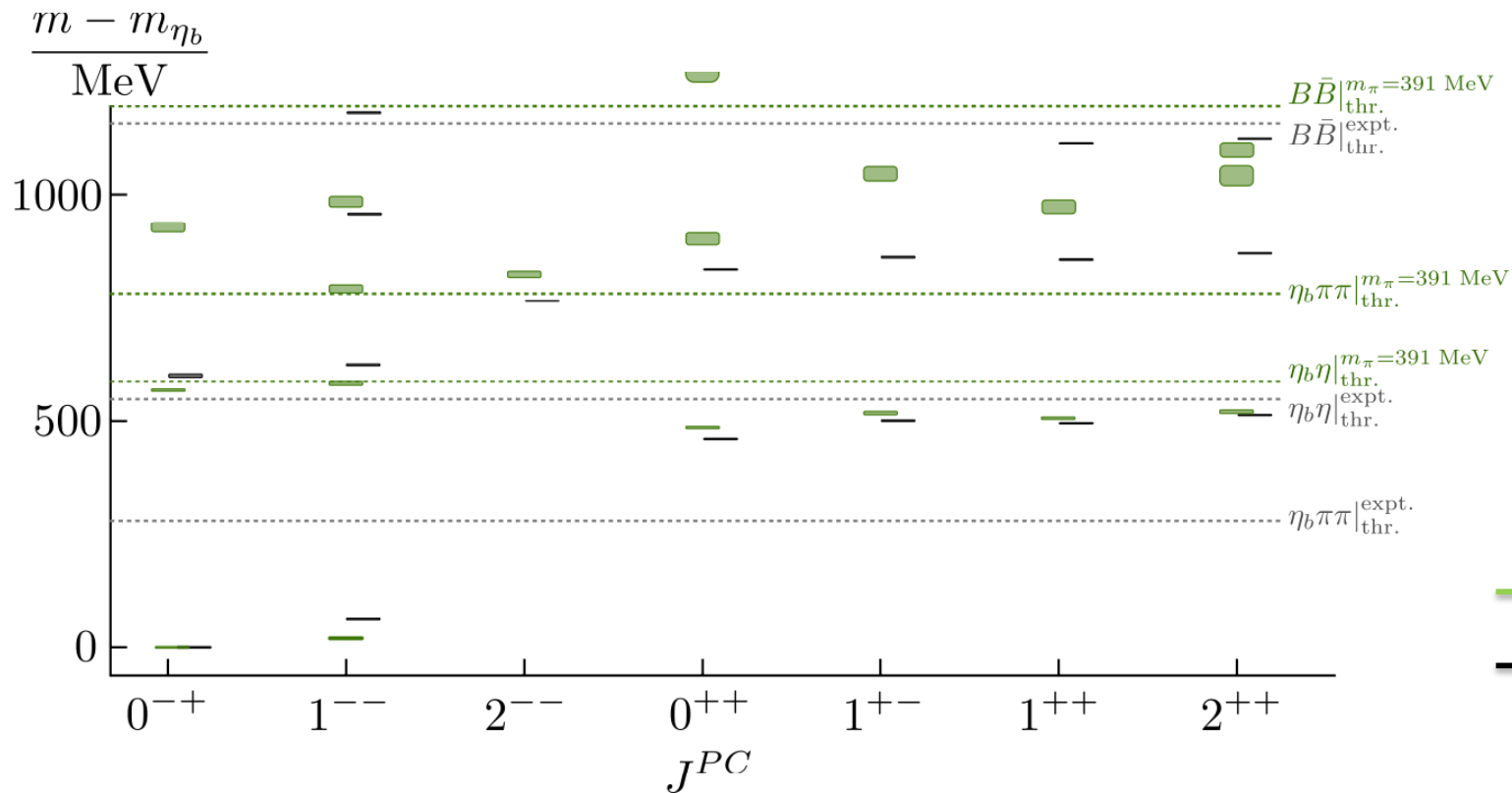
Strongly stable hadrons (well below threshold)

$$m = E (\vec{p}=0)$$

Example: bottomonia : reachest spectrum of (almost) strongly stable hadrons



Ryan & Wilson (HadSpec)
2008.02656, JHEP



lowest lying $\underline{b}\bar{b}$ and $\underline{c}\bar{c}$ with m_{π}^{phy} and $a \rightarrow 0$, QCD+QED

HPQCD, 2101.08103, JHEP, 2204.02137: also other observables

	lat	exp
$m(Y) - m(\eta_b)$ [MeV]	57.5 (2.3) (1.0)	62.3 (3.2)

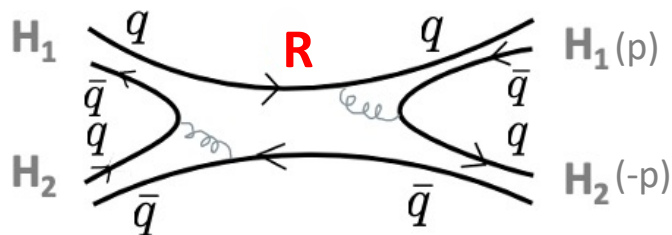
systematic error from omitted $\underline{b}\bar{b}$ annihilation

General conclusion on
strongly stable hadrons:

- many precision lattice studies
- QCD confronts exp data well !

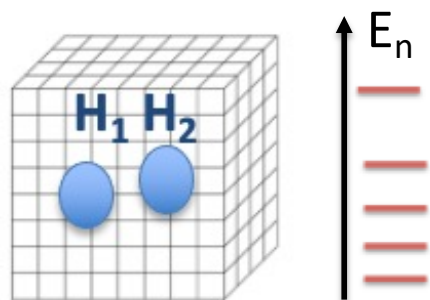
Resonances $R \rightarrow H_1 H_2$, bound states near threshold

see also: plenay by Pelaez

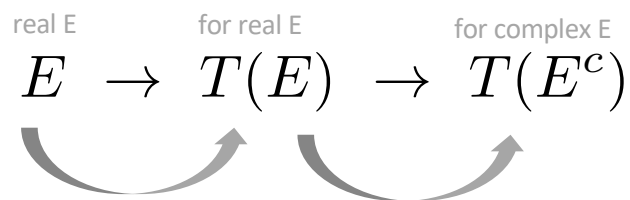


scattering amplitude $T(E)$

Scattering amplitude $T(E)$ from lattice QCD



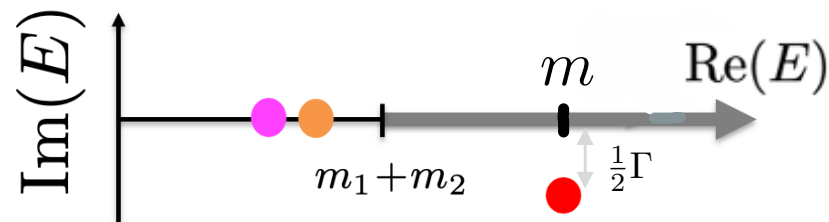
many resonances and bound states extracted in this way by now (apologies for not covering all)



analytic relation:
Luscher 1991

generalizations by many authors

analytic contin.
to complex E



Virtual bound st.

$$p = -i |p|$$

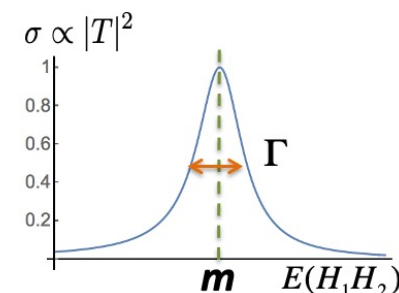
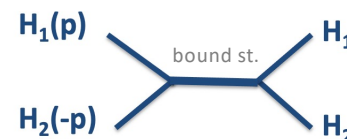
Bound st.

$$p = i |p|$$

Resonance

$$T(E) \propto \frac{1}{E^2 - m^2}$$

$$T(E) \propto \frac{1}{E^2 - m^2 + iE\Gamma}$$



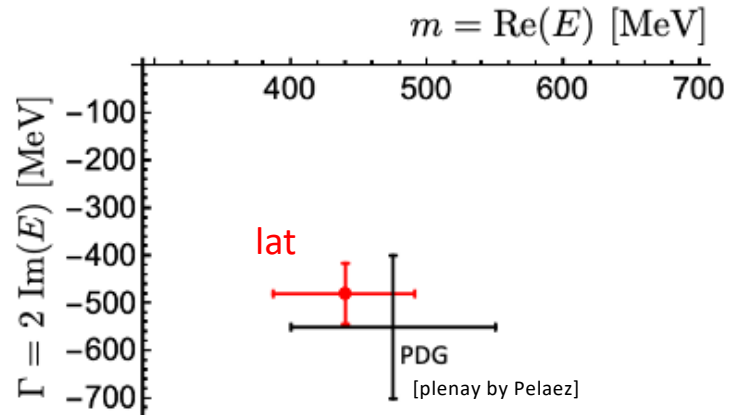
Resonances $R \rightarrow H_1 H_2$

examples

GWU, 1803.02897

$m_\pi = 227,315 \text{ MeV} \rightarrow m_\pi^{phy}$
UChPT

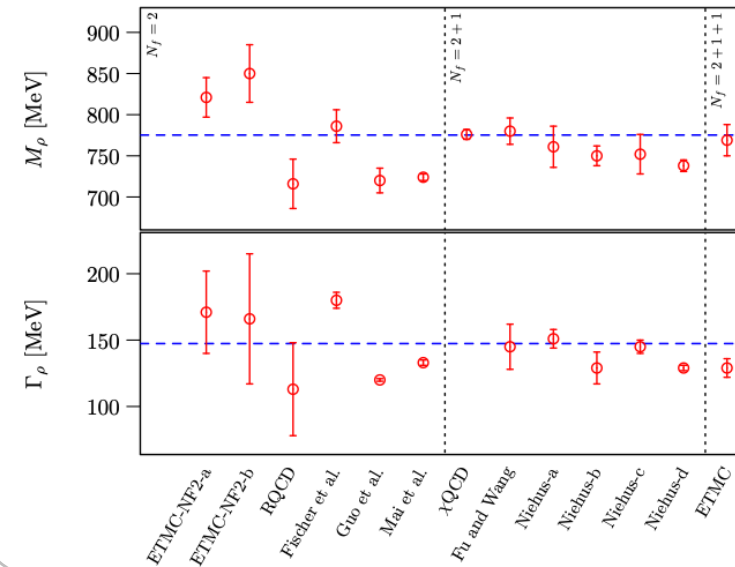
$$\sigma \rightarrow \pi\pi$$



$$\rho \rightarrow \pi\pi$$

from review

Mai, Meissner, Urbach 2206.01477



HadSpec, 2102.04973 JHEP

$$D_0^* \rightarrow D\pi$$

$m = \text{Re}\sqrt{s_0}/\text{MeV}$

$$J^P = 0^+$$

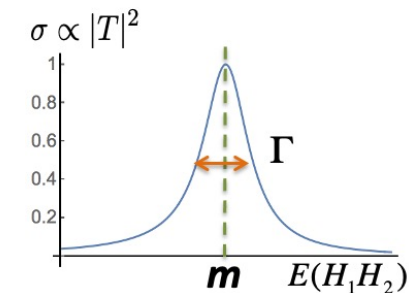
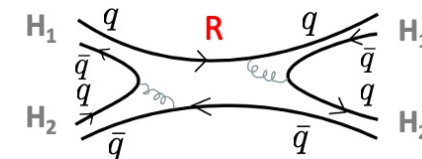
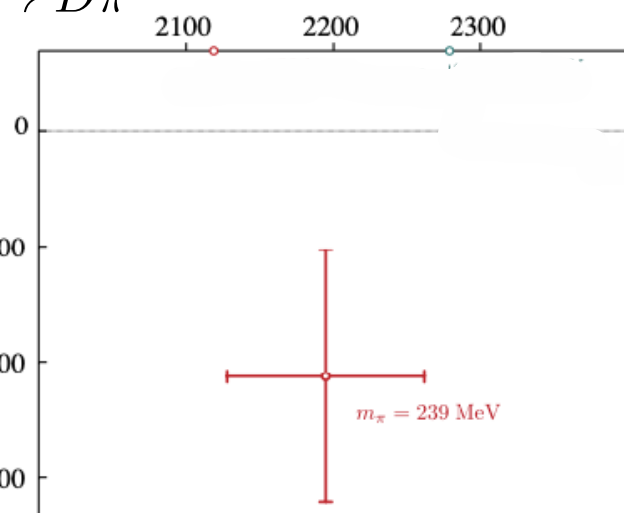
PDG: puzzle

$$m[D_0^*] \simeq m[D_{s0}^*(2317)]$$

lat: OK

$$m[D_0^*] < m[D_{s0}^*(2317)]$$

$$\Gamma = -2 \text{Im}\sqrt{s_0}/\text{MeV}$$



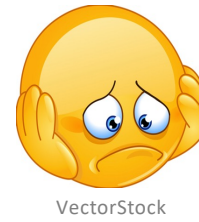
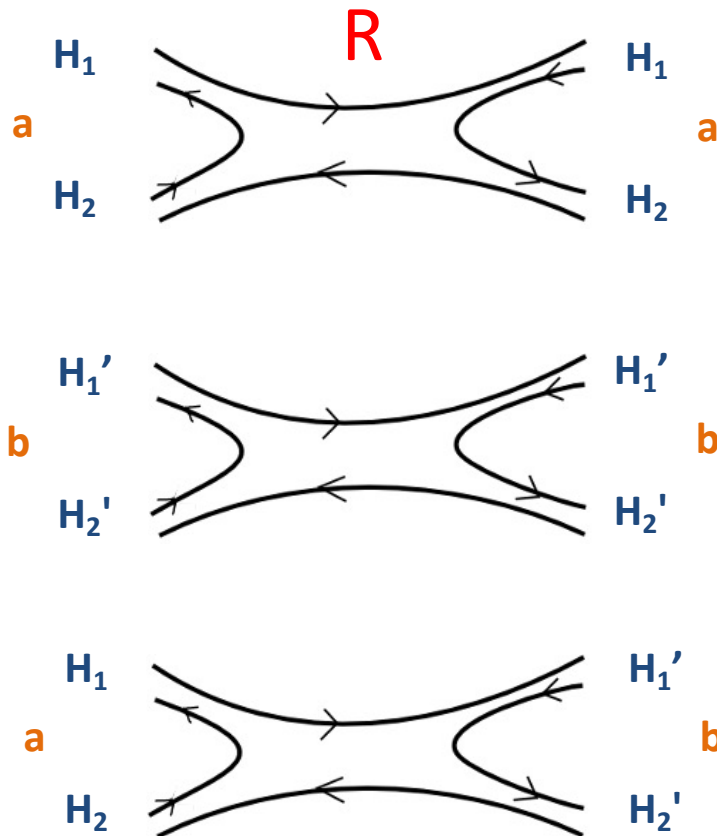
see also: plenay by Pelaez

Resonances from coupled-channel scattering

$$R \rightarrow H_1 H_2, H_1' H_2', \dots$$

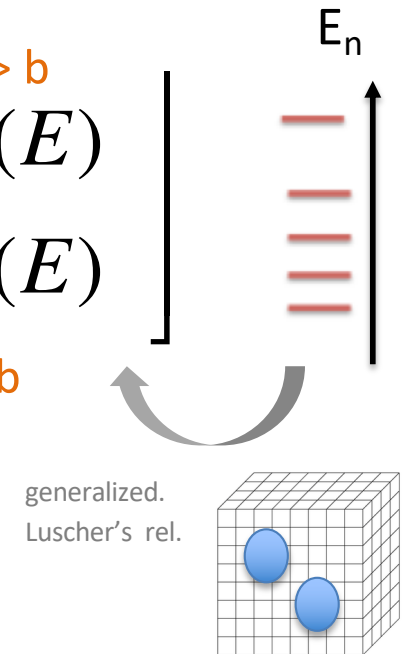
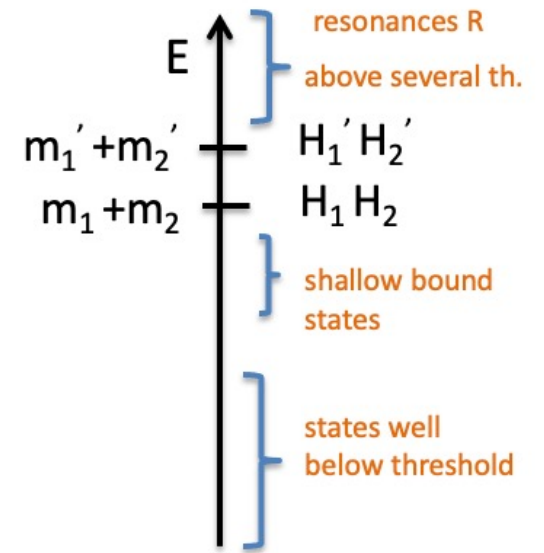
channel a : $H_1 H_2$

channel b : $H_1' H_2'$



$$T(E) = \begin{bmatrix} \overset{a \rightarrow a}{T_{aa}(E)} & \overset{a \rightarrow b}{T_{ab}(E)} \\ \underset{b \rightarrow a}{T_{ab}(E)} & \underset{b \rightarrow b}{T_{bb}(E)} \end{bmatrix}$$

- lattice QCD studies extracted $T(E)$ for several resonances

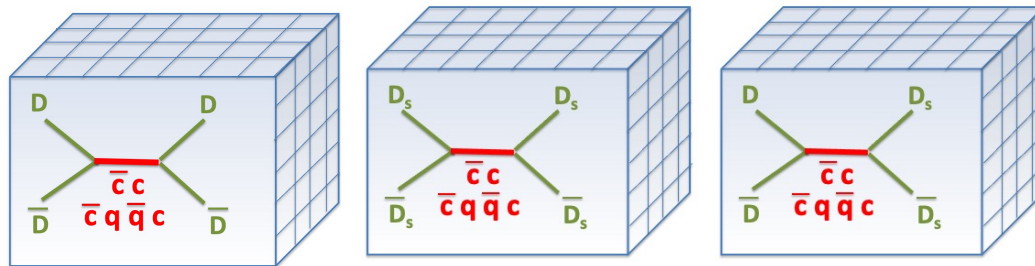


Resonances from coupled-channel scattering

- most results by HadSpec. coll.: mostly light meson sector

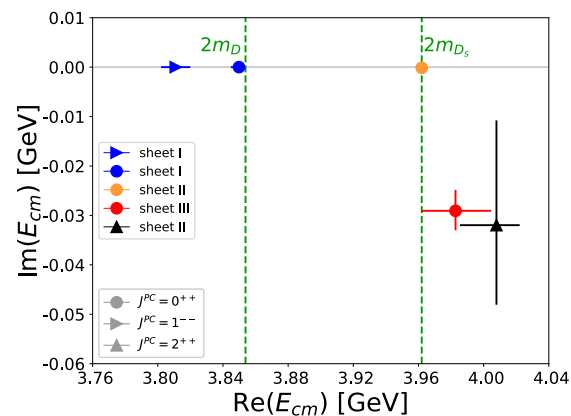
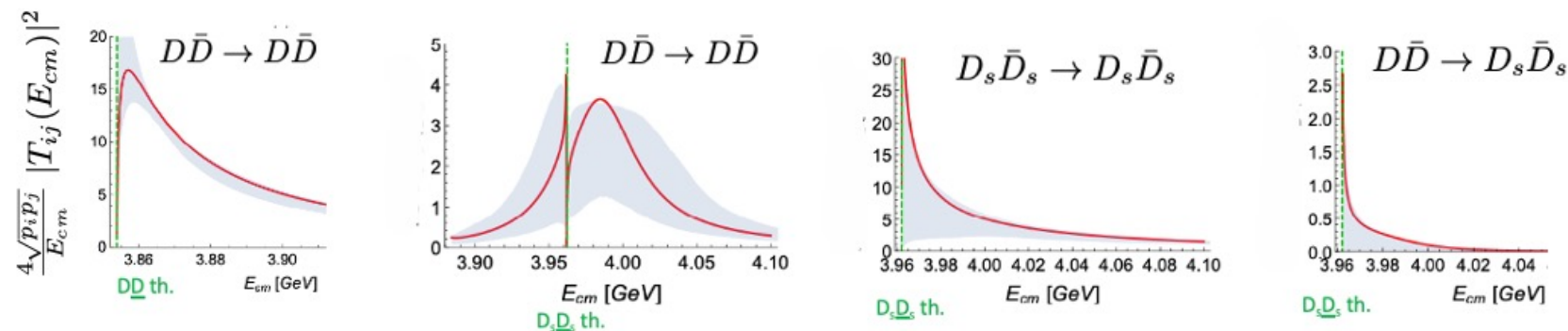
- example in heavy-quark sector

$$\bar{c}c, \bar{c}q\bar{q}c \quad q=u,d,s \quad I=0$$



two coupled channels

$$D\bar{D} - D_s\bar{D}_s$$



S.P. , Collins, Padmanath, Mohler, Piemonte
2011.02542 JHEP

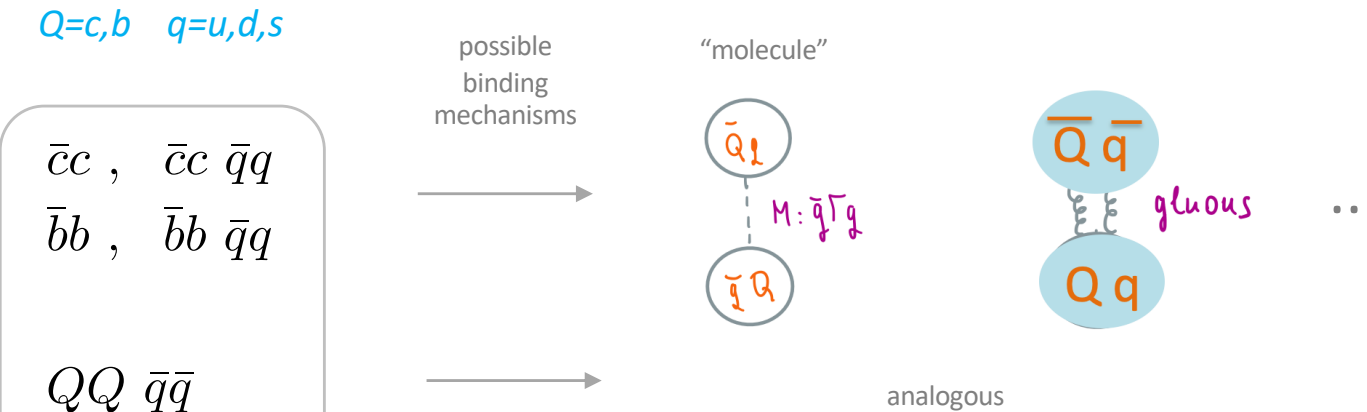
physics interpretation: two slides later

journey to various hadron sectors

(with heavy quarks)

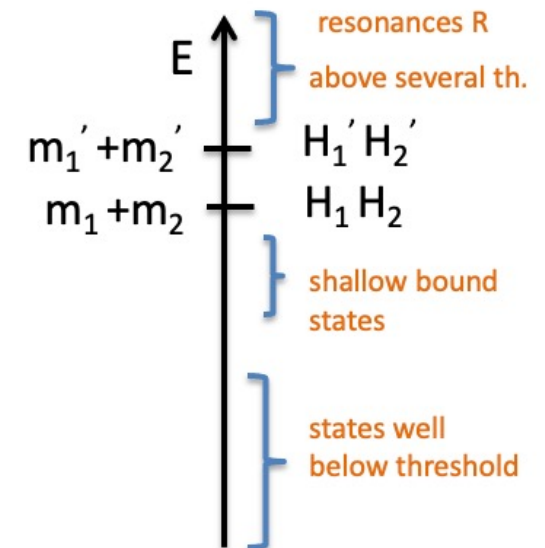
most of discovered exotic hadrons contain heavy quarks:

(these are more likely to be quasi-bound due to small kinetic energy of Q)



exciting experimental discoveries by:

LHCb, Belle, BESIII, Babar, ...



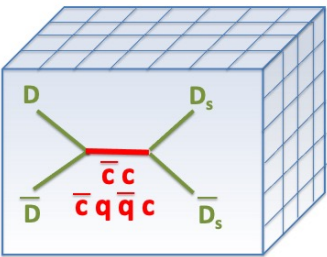
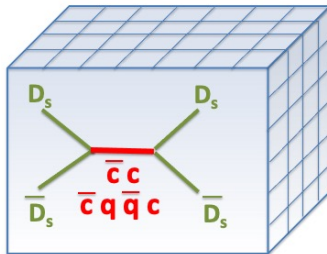
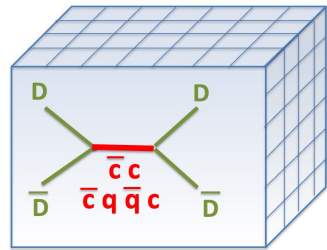
How challenging is a given state for ab-initio study? General rule:
more strong decay channels -> more challenging

charmonium(like) sector

$\bar{c}c$, $\bar{c}q\bar{q}c$

Charmonium(like) resonances and bound states

$$\bar{c}c, \bar{c}q\bar{q}c \quad q=u,d,s \quad I=0$$



$$\bar{D}_s D_s \quad J^P=0^+ \text{ state}$$

likely related to $X(3915) / \chi_{c0}(3930) / X(3960)$
[BaBar, LHCb 2009.00026, LHCb 2022 [indico..../1176505/](#)]
explaining why it has narrow width to $\underline{D}\underline{D}$.
Supported by some pheno studies:
Lebed, Polosa 1602.08421, Oset et al. 2207.08490,
Guo et al, 2101.01021,

$$\bar{D} D \quad J^P=0^+ \text{ state}$$

predicted in models [Oset et al,
0612179 PRD, Hildago Duque et al
1305.4487, Baru et al 1605.09649 PLB]

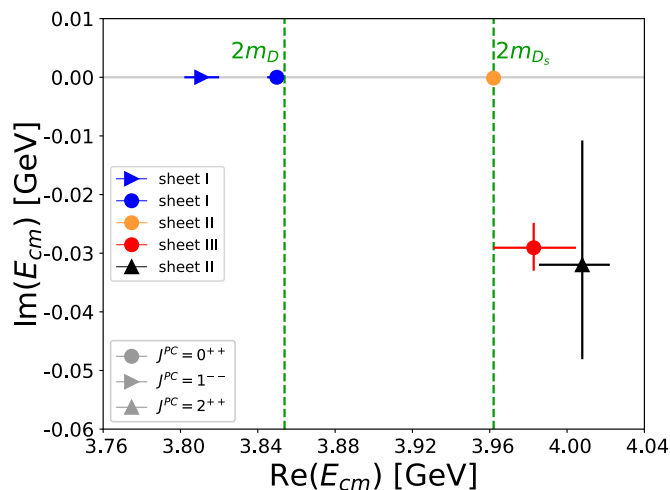
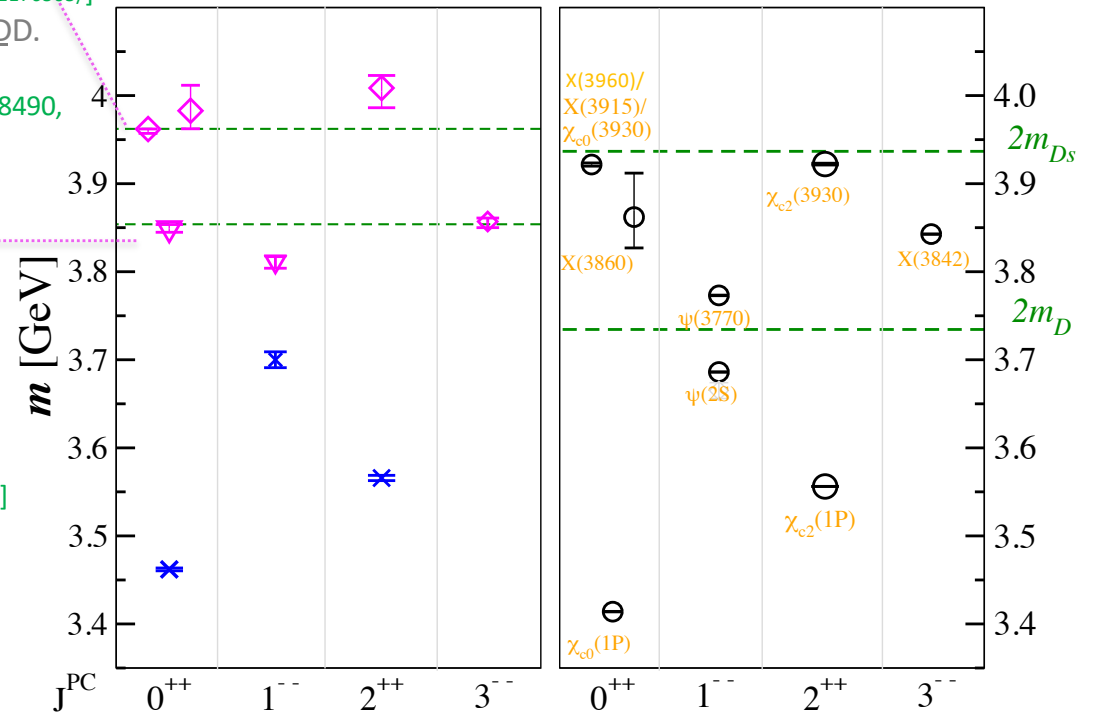
seen in dispersive re-analysis of exp.
[Deineka, Danilkin et al 2111.15033, poster]

+ expected conventional charmonia

$$m_\pi \simeq 280 \text{ MeV}$$

Lat

Exp

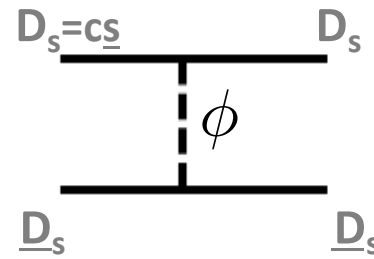
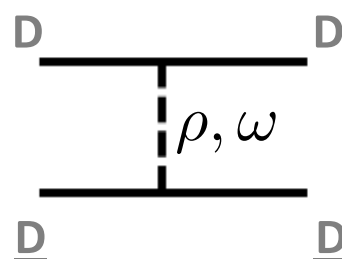
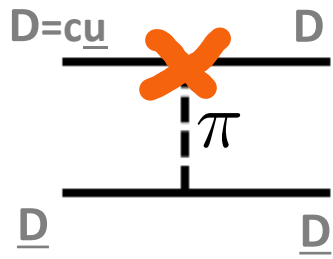
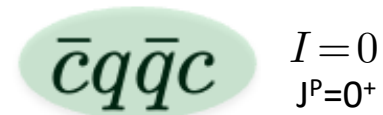


S.P., Collins, Padmanath, Mohler, Piemonte
2011.02542 JHEP, 1905.03506 PRD, 2111.02934

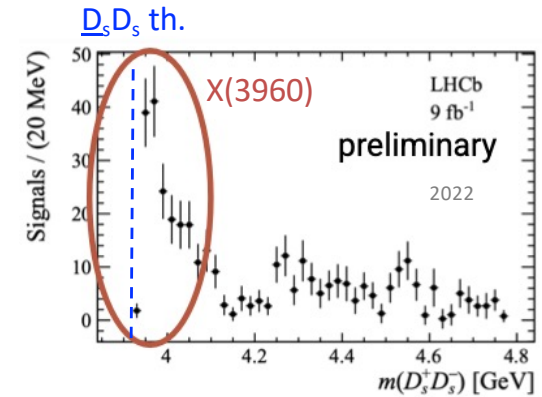
Likely interpretation of some near-threshold states: “molecules” attracted by V exchange

a number of pheno studies
 Oset et al, 0612179 PRD,
 Wu, Molina, Oset, Zou, 1007.0573, PRL
 Guo et al, 2101.01021,...

indico.cern.ch/event/1176505/, july 2022

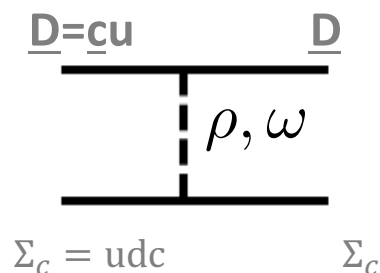
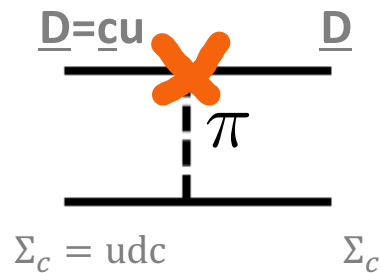


now support also from lattice

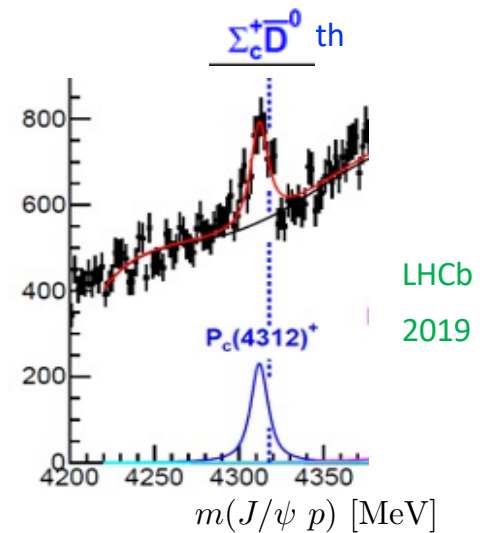


LHCb discoveries:

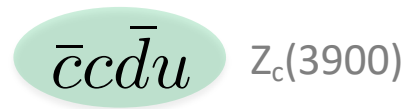
talk by Mikhasenko



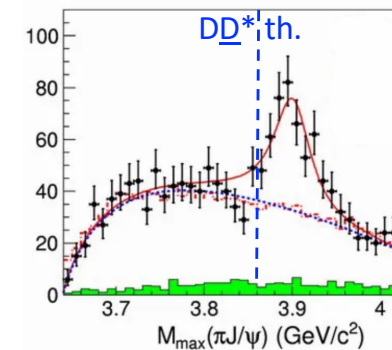
currently to challenging for lattice



Charmonium(like) resonances



[BessIII & Belle 2013]

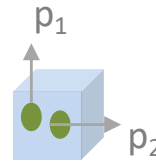


$$\bar{c}c\bar{d}u \rightarrow D\bar{D}^*, J/\psi\pi$$

status from lattice QCD: still challenging, giving perhaps puzzling results

- E_n close to the non-interacting E_n^{ni}

$$E^{n,i}(H_1 H_2) = \sqrt{\vec{p}_1^2 + m_1^2} + \sqrt{\vec{p}_2^2 + m_2^2} \quad \vec{p}_i = \vec{n}_i \frac{2\pi}{L}$$



- very small interaction between mesons observed
- this should already constrain some models and interpretations
- perhaps consistent with virtual bound st. found by some approaches?
[talk by Rodriguez Entem]

consistent conclusions from studies via Luscher's method

[Leskovec Mohler Lang SP: 1308.2097, 1405.7623

HadSpec 1709.01417

Liuming Liu et al. 1907.03371, 1911.08560,

M. Sadl, SP, Padmanath, Collins @ lattice 2022]

HALQCD method

Ikeda et al., HALQCD, 1602.03465, PRL

- indication of large coupling between both channels
that could be responsible for $Z_c(3900)$ peak in exp

bottomonium(like) sector

$$\bar{b}b \quad \bar{b}b\bar{q}q$$

Bottomonia and bottomonium hybrids, I=0

$\bar{b} b$

$\bar{b} G b$

omitting effects from
strong decays and thresholds

Lattice QCD

relativistic b quarks

Ryan & Wilson (HadSpec)

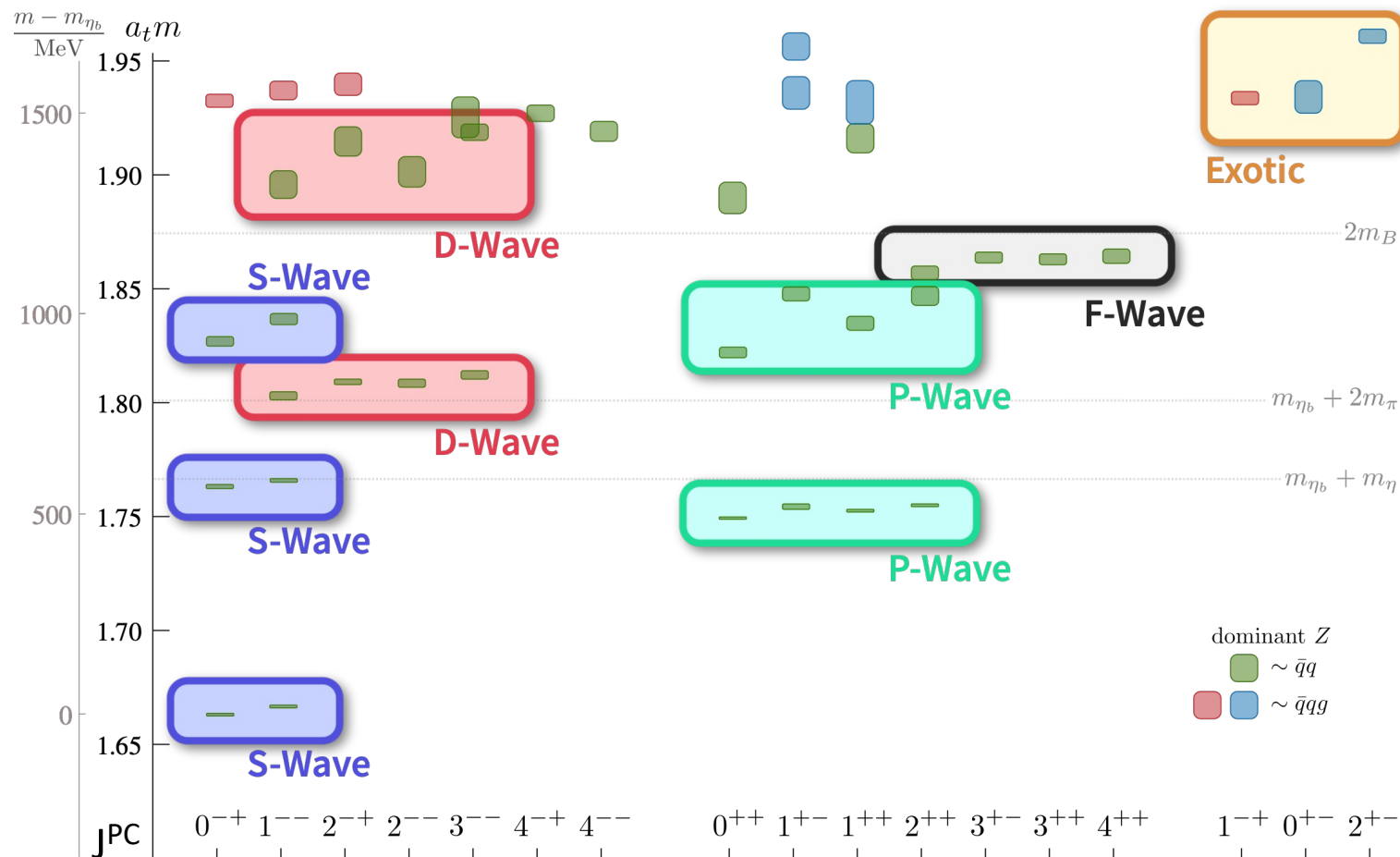
2008.02656, JHEP

(same result as shown earlier)

$$m = E_n(p=0)$$

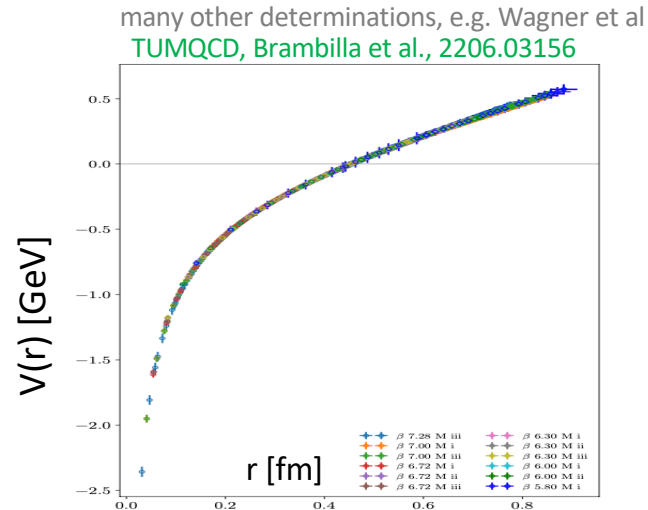
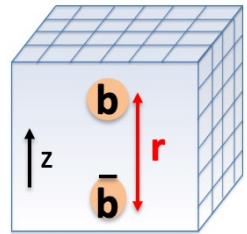
$$\frac{m - m_{\eta_b}}{\text{MeV}}$$

$$\frac{m - m_{\eta_b}}{\text{MeV}} a_t m$$



$$m_{\text{hybrid}} \geq 10.9 \text{ GeV}$$

Bottomonia ($I=0$) from static potentials & Born Oppenheimer approximation & EFTs



masses of low-lying
confinement
determination of α_s

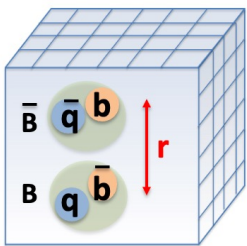
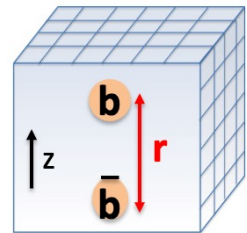
$\bar{b} b$

e.g. works of TUMQCD,
Leino et al. [2020-2022]

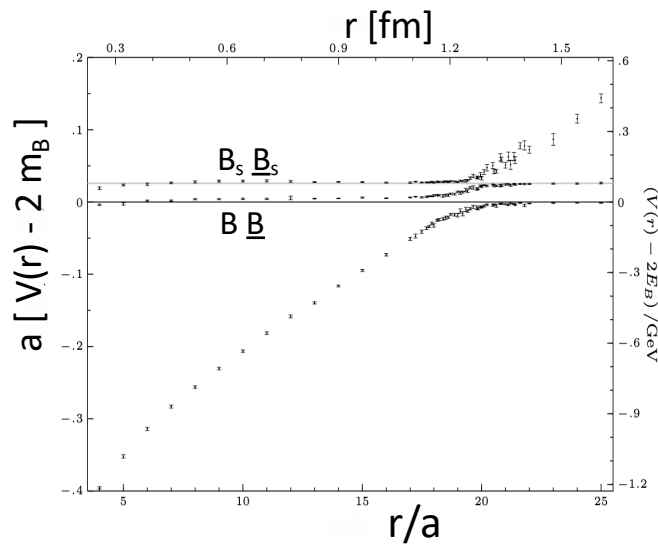
string breaking

SESAM, Bali et al, 2005, 0505012

Bulava et al, 1902.04006, PLB



$q=u,d,s$



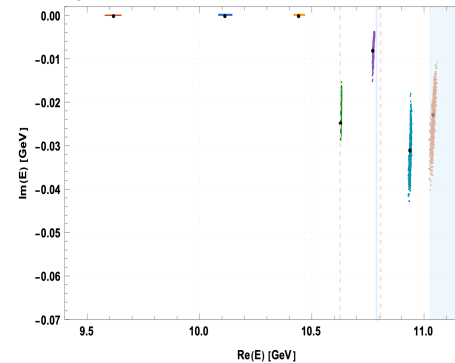
Bicudo, Cardoso, Mueller, Wagner, 2205.11475,

$\tilde{J}=0$: here; $J=1,2,3$ in the paper;

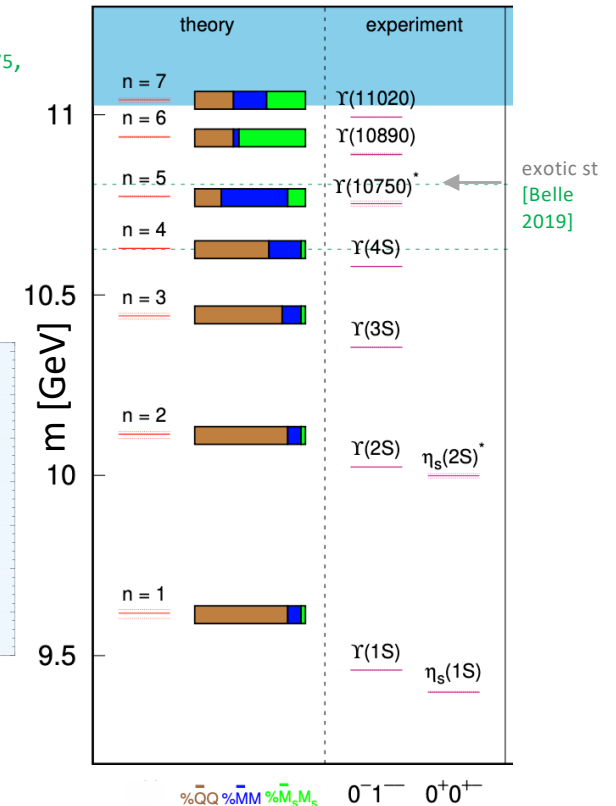
\tilde{J} =total ang. mom. without heavy quark spins

$\bar{b} b$ $\bar{b} b q \bar{q}$

poles



see also: Castella, 2207.09365

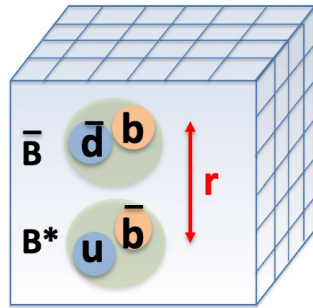


Bottomonium-like states from static potentials & Born Oppenheimer approximation & EFTs

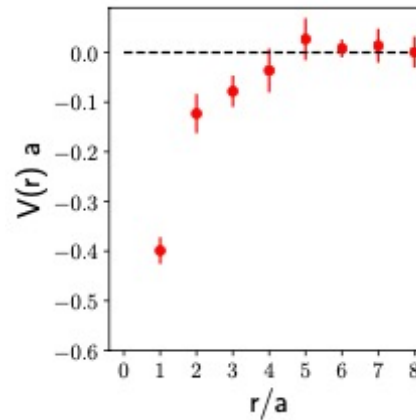
$$I=1, J^P=1^+$$

$\bar{b}b\bar{d}u$

$$\bar{b}b\bar{d}u \rightarrow B\bar{B}^* \Upsilon\pi$$



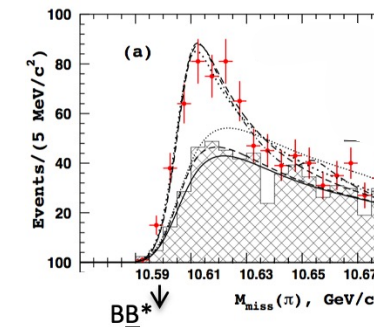
Peters, Wagner, Bicudo
SP, Bahtiyar, Petkovic, Sadl 2019, 2020



bound state just below $B\bar{B}^*$ th.

likely related to

Z_b [Belle, 2011, 2016]



Brambilla, 2111.10788;

Soto & Castella, 2005.00552

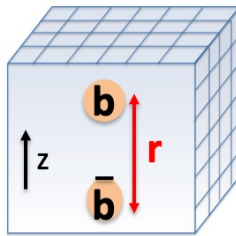
Juge, Kuti, Morningstar, 1997, 1998

$$I=0, \text{ various } J^P$$

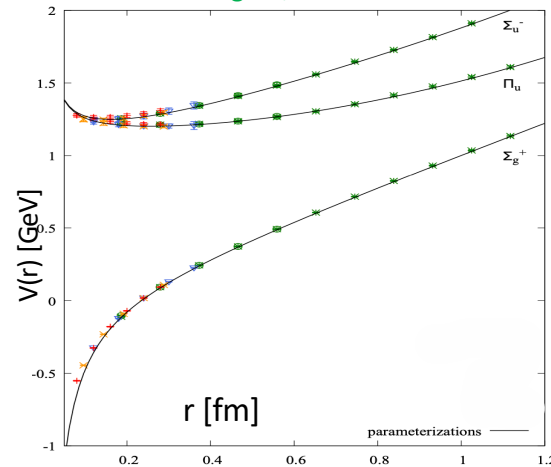
$\bar{b}b$

$\bar{b}Gb$

omit strong decays
quenched

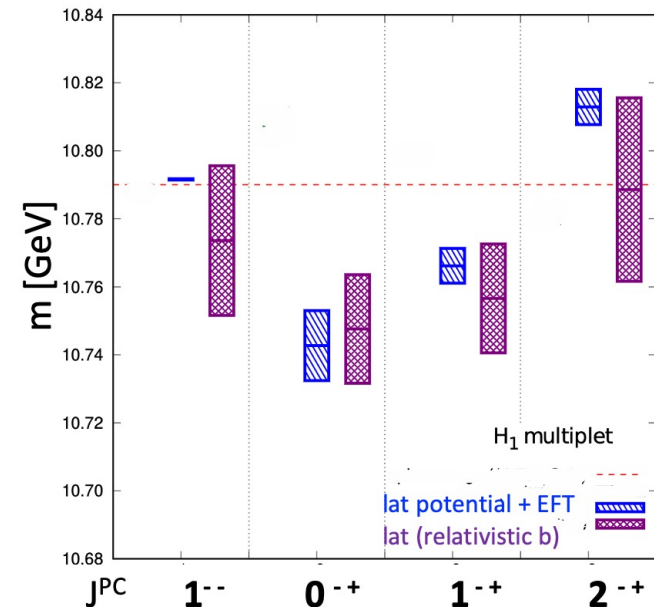


Schlosser & Wagner, 2111.00741



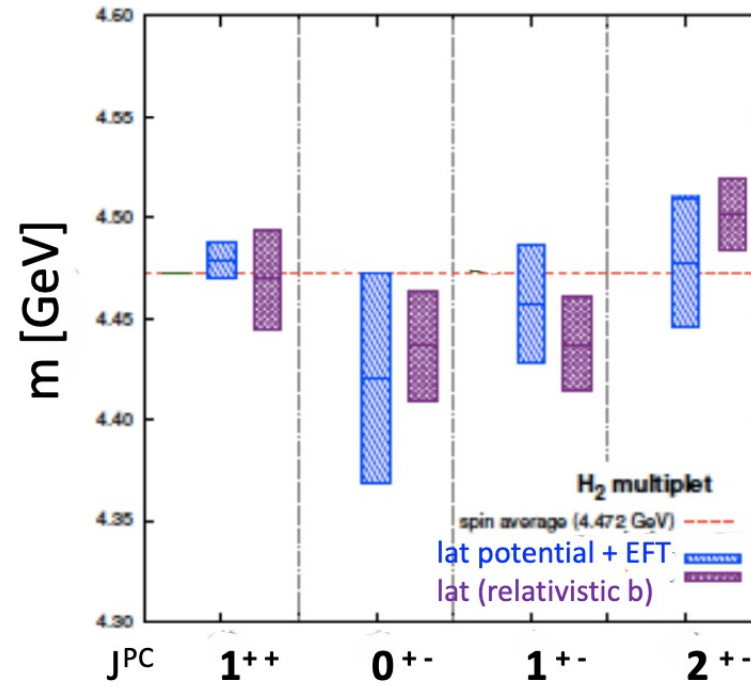
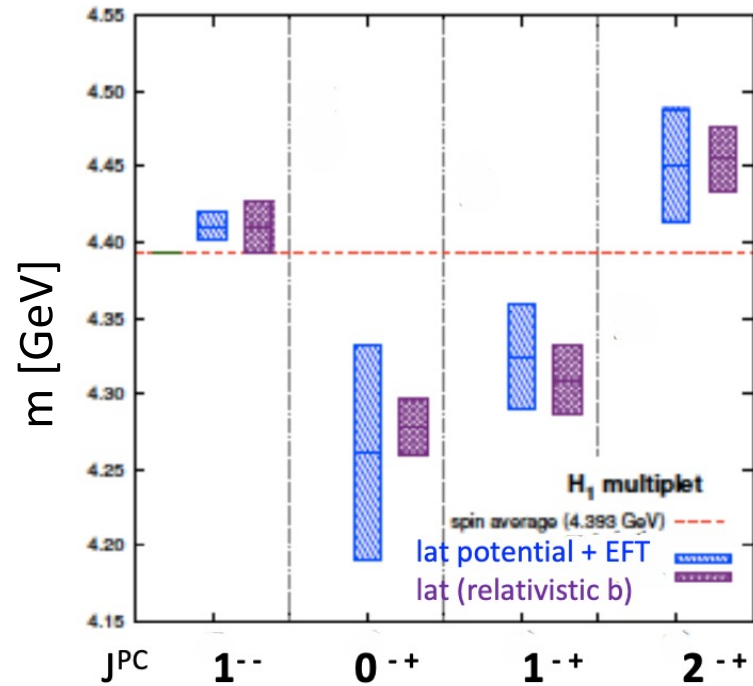
$\bar{b}Gb$

Segovia, Tarrus; Brambilla MITP 2022
(unpublished, with 1/mb spin effects)



Ryan & Willson
2020

see also: Brambilla et al, 1805.07713, 1908.11699, PRD



TUMQCD, Brambilla et al.
Cheung et al, HadSpec, 2016

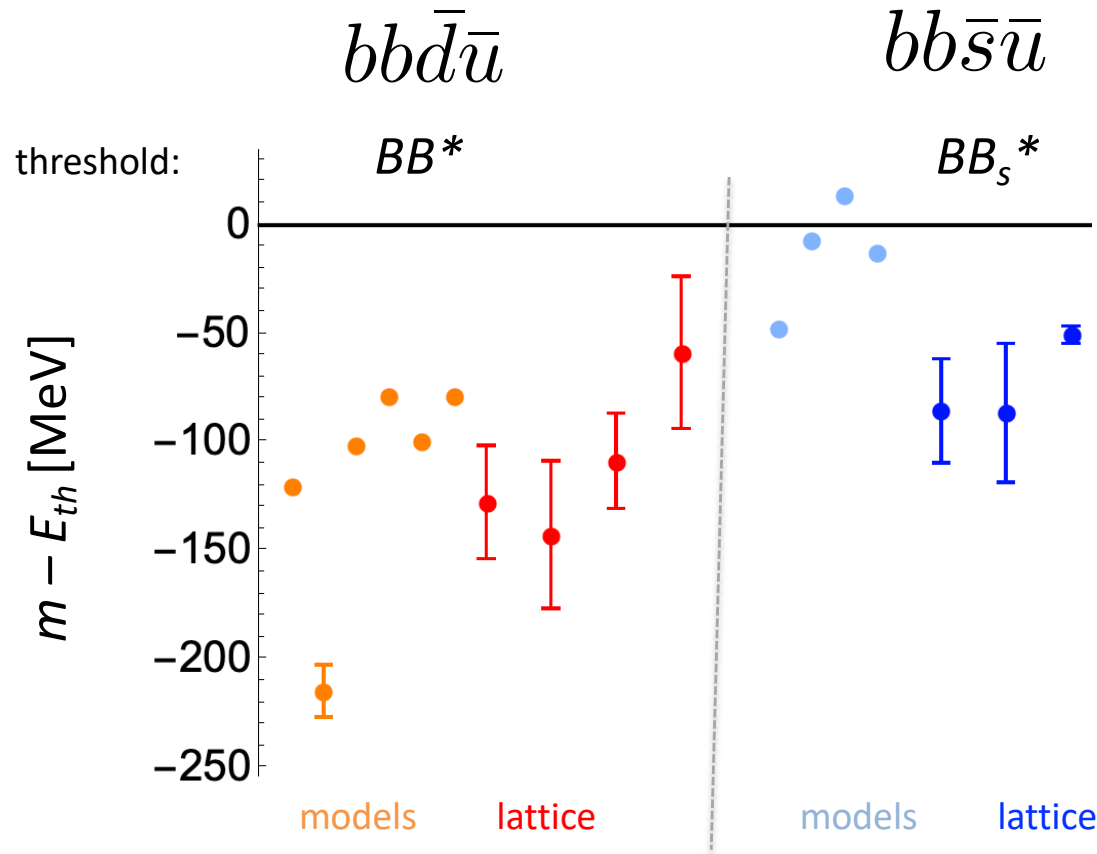
Doubly heavy tetraquarks

$$QQ'\bar{q}\bar{q}'$$

$$Q=c,b \quad q=u,d,s$$

Doubly bottom tetraquarks

not found in exp, difficult to find

 $bb\bar{d}\bar{u}$
 $bb\bar{s}\bar{u}$
 $I=0, J^P=1^+$


references from left to right

models (many more references):

Eichten and Quigg (2017) 1707.09575 PRL

Karliner and Rosner (2017) 1707.07666 PRL

Ebert et al. (2007) 0706.3853

Silvestre-Brac and Semay (1993)

Janc and Rosina (2004) hep-ph/0405208

lattice: most updated results

[Leskovec, Meinel, Pflaumer, Wagner \(2019\) 1904.04197](#)

Junnarkar, Mathur, Padmanth (2018) 1810.12285

Frances, Colquhoun, Hudspith, Maltman (2021) preliminary

Bicudo, Wagner et al. 1612.02758 static potentials

models (many more references)

Eichten and Quigg (2017) 1707.09575 PRL

Parket al. (2018) 1809.05257

Ebert et al. (2007) 0706.3853

Silvestre-Brac and Semay (1993)

lattice: most updated results

[Meinel, Pflaumer, Wagner \(2022\) 2205.13982](#)

Junnarkar, Mathur, Padmanth (2018) 1810.12285

Frances, Colquhoun, Hudspith, Maltman (2021) preliminary

lattice	$m_{u/d}$	a [fm]
Leskovec, Pflaumer, Wagner, Meinel	$m_{u/d} \rightarrow m_{u/d}^{\text{phy}}$	0.08-0.11
Junnarkar et al.	$m_{u/d} \rightarrow m_{u/d}^{\text{phy}}$	$a \rightarrow 0$
Francis et al	$m_{u/d} \rightarrow m_{u/d}^{\text{phy}}$	0.09
Bicudo et al.	$m_{u/d} \rightarrow m_{u/d}^{\text{phy}}$	0.08

extracted also

$BB_{(s)}^*$ scattering amplitudes

earlier results of Frances et al in 1810.10550, 1607.05214 PRL

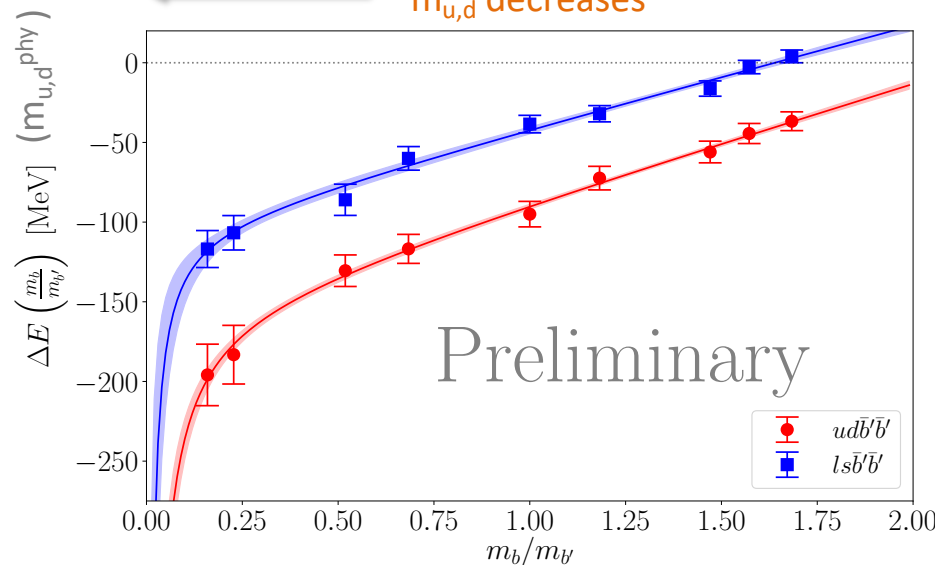
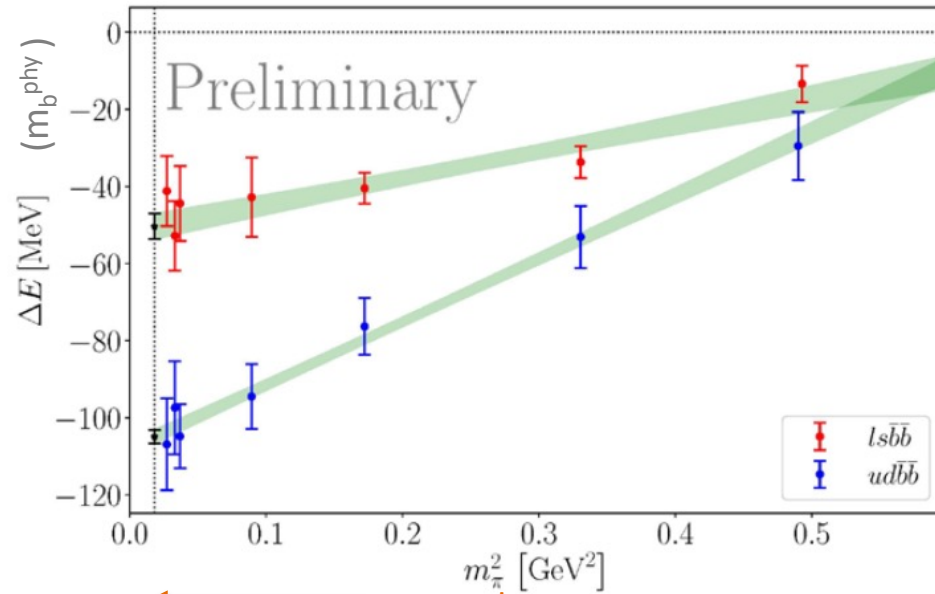
Doubly bottom tetraquarks

$b\bar{b}d\bar{u}$

$b\bar{b}s\bar{u}$

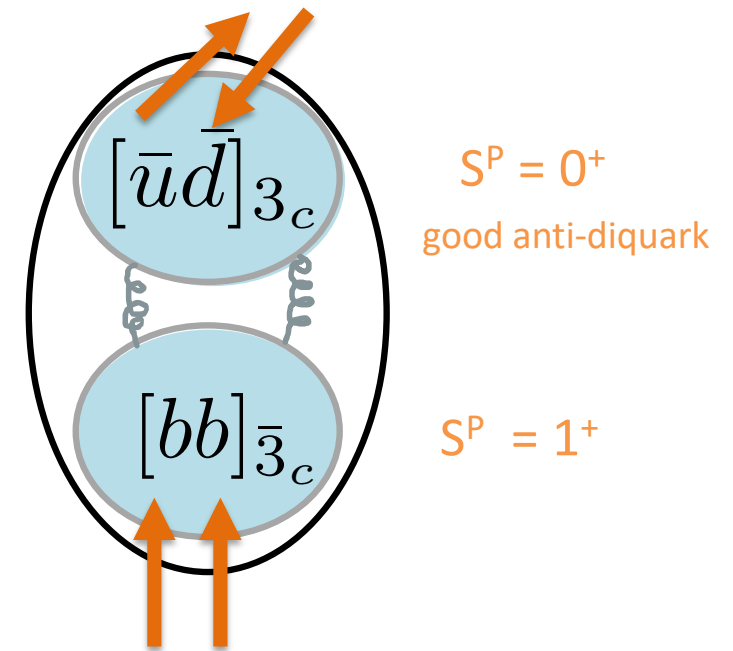
$I=0, J^P=1^+$

lattice: dependence on m_b and $m_{u,d}$



Colquhoun, Francis, Hudspith, Maltman, Lewis
1810.10550, PoS LATTICE2021 (2022) 144
supports internal structure below

supported also by almost all model studies
Karliner and Rosner (2017), Janc and Rosina (2004), ...



these are the only tetraquarks where lattice
finds support for significant $[qq][\bar{q}\bar{q}]$

good and bad diquark properties:

Francis et al, 2201.03332

Other $QQ'\bar{q}\bar{q}'$ and J^P

$bcd\bar{u}$ $ccd\bar{u}$

Theoretically expected near or above threshold

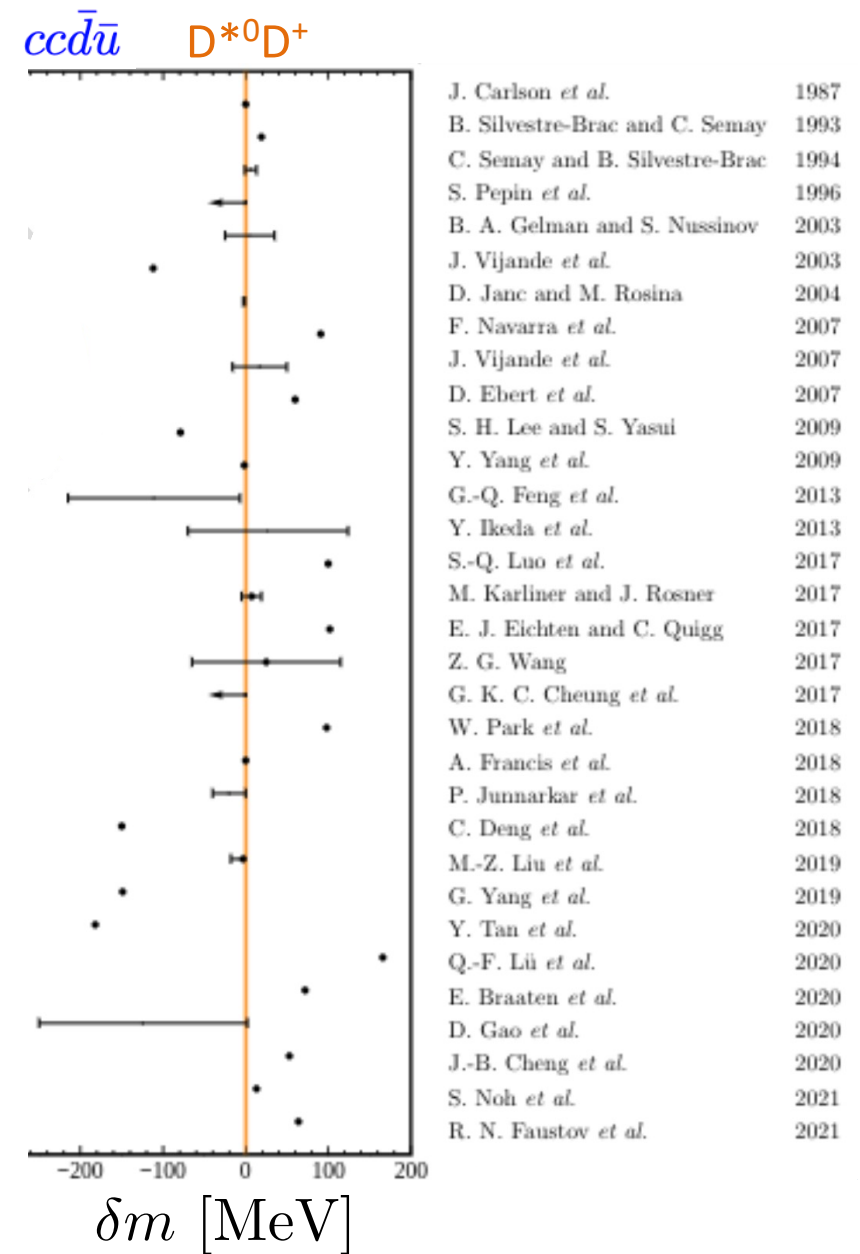
States near or above threshold have to be identified as poles in scattering $T(E)$

Other $QQ'\bar{q}\bar{q}'$ and J^P

$bcd\bar{u}$ $ccd\bar{u}$

Theoretically expected near or above threshold

States near or above threshold have to be identified as poles in scattering $T(E)$



Theoretical predictions for T_{cc} mass ($I=0, J^P=1^+$)

[compilation by Polyakov]

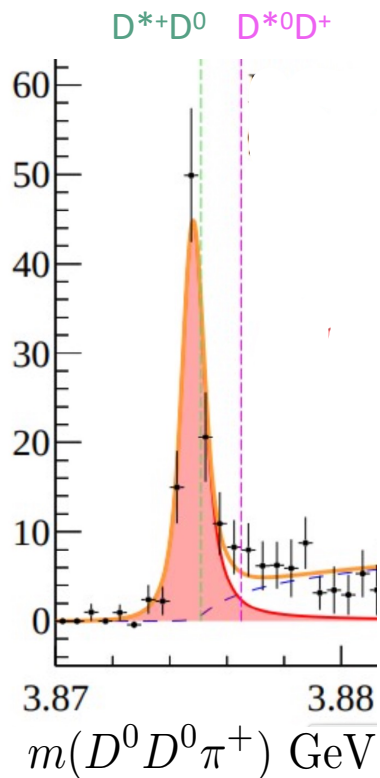
Other $QQ'\bar{q}\bar{q}'$ and J^P

$bcd\bar{u}$ $ccd\bar{u}$

Theoretically expected near or above threshold

States near or above threshold have to be identified as poles in scattering $T(E)$

Doubly charm tetraquark T_{cc} $ccd\bar{u}$

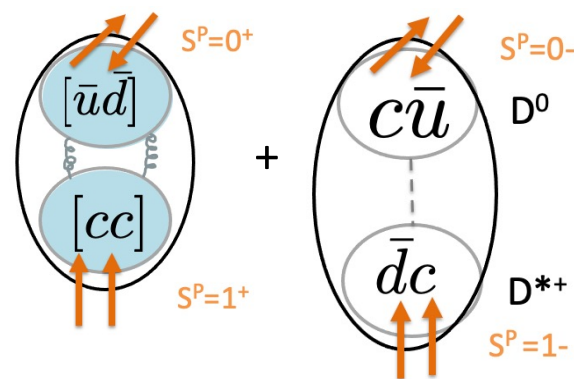


Sasa Prelovsek

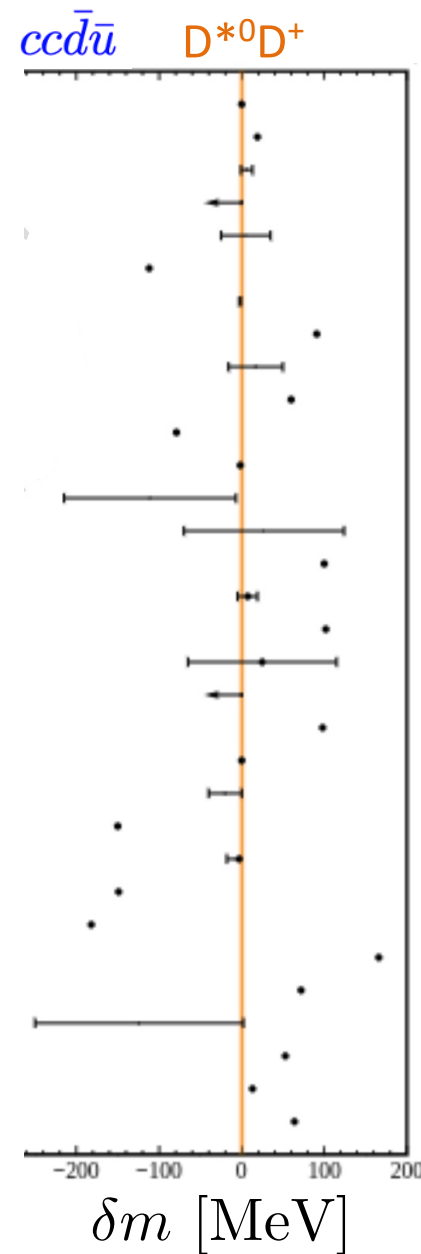
$$\delta m = m - (m_{D^{*+}} + m_{D^0})$$

$$\delta m_{pole} = -0.36 \pm 0.04 \text{ MeV}$$

LHCb 2109.01038, 2109.01056, Nature Physics



likely dominant



Theoretical predictions for T_{cc} mass ($I=0, J^P=1^+$)

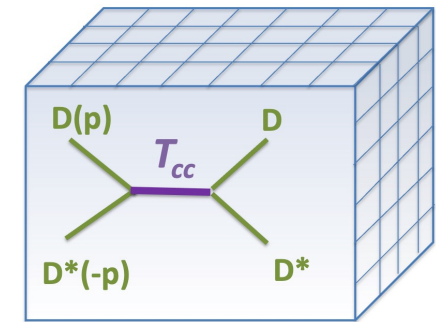
[compilation by Polyakov]

J. Carlson <i>et al.</i>	1987
B. Silvestre-Brac and C. Semay	1993
C. Semay and B. Silvestre-Brac	1994
S. Pepin <i>et al.</i>	1996
B. A. Gelman and S. Nussinov	2003
J. Vijande <i>et al.</i>	2003
D. Janc and M. Rosina	2004
F. Navarra <i>et al.</i>	2007
J. Vijande <i>et al.</i>	2007
D. Ebert <i>et al.</i>	2007
S. H. Lee and S. Yasui	2009
Y. Yang <i>et al.</i>	2009
G.-Q. Feng <i>et al.</i>	2013
Y. Ikeda <i>et al.</i>	2013
S.-Q. Luo <i>et al.</i>	2017
M. Karliner and J. Rosner	2017
E. J. Eichten and C. Quigg	2017
Z. G. Wang	2017
G. K. C. Cheung <i>et al.</i>	2017
W. Park <i>et al.</i>	2018
A. Francis <i>et al.</i>	2018
P. Junnarkar <i>et al.</i>	2018
C. Deng <i>et al.</i>	2018
M.-Z. Liu <i>et al.</i>	2019
G. Yang <i>et al.</i>	2019
Y. Tan <i>et al.</i>	2020
Q.-F. Lü <i>et al.</i>	2020
E. Braaten <i>et al.</i>	2020
D. Gao <i>et al.</i>	2020
J.-B. Cheng <i>et al.</i>	2020
S. Noh <i>et al.</i>	2021
R. N. Faustov <i>et al.</i>	2021

Doubly charm tetraquark T_{cc} from lattice QCD

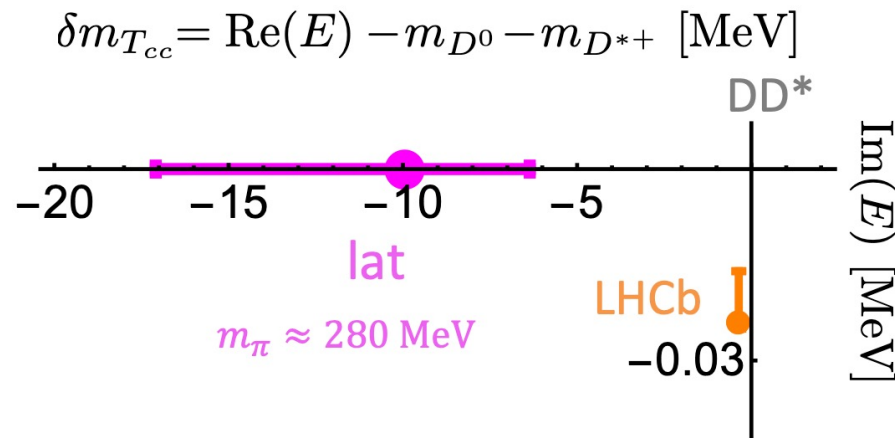


$$J^P = 1^+, I=0$$



What is virtual bound state? See supplemental of [2202.10110, PRL](#)

Pole of $T(E)$ ● virtual bound state pole $p = -i|p|$

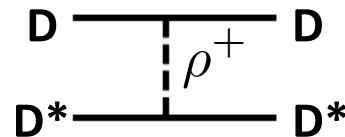


- ❖ The first lat. study that extracts $T(E)$:
← [\[Padmanath, SP: 2202.10110, PRL\]](#) $m_\pi \approx 280$ MeV
- ❖ Evidence for pole related to T_{cc}
- ❖ For $m_{u,d} > m_{u,d}^{\text{phy}}$ one expects decreased attraction
 T_{cc} : bound state becomes virtual bound state
this is indeed found on the lattice

another recent lattice study: [CLQCD, Chen et al. 2206.06185](#)

comparison of $I=0,1$: attraction in $I=0$ channel arises mainly from

$$m_\pi \simeq 350 \text{ MeV}$$

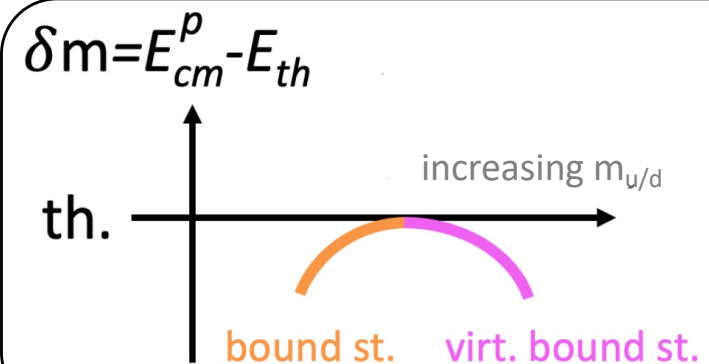


previous simulations extracted only eigen-energies:

[\[Junnarkar, Mathur, Padmanath, 1810.12285 PRD ; HadSpec 1709.01417 JHEP\]](#)

Dyson Schwinger: [Fischer et al. 2111.15310](#): $I=0$ state near threshold found

Sketch of expected binding energy



Scalar heavy-light mesons

$$J^P = 0^+$$

Scattering on the lattice

S=1 Mohler et al, 1308.3175, PRL

Lang et al, 1403.8103, PRD

RQCD, 1706.01247, PRD

HadSpec 2008.06432, JHEP

S=0 Mohler et al. 1208.4059, PRD

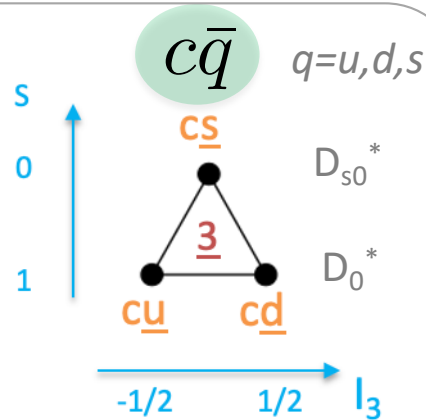
HadSpec, 1607.07093, JHEP

HadSpec 2102.04973, JHEP

HadSpec 2205.05026

S=-1 HadSpec, 2008.06432, JHEP

Conventional
quark model



new paradigm supported by:

- lattice
- effective models ChPT+HQET
- reanalysis of exp data
- states circled by blue seem to feature in the spectrum

New paradigm

Du et al, 1712.07957, PRD

Albaladejo et al, 1610.06727, PLB

Lutz et al (2003), 0307133, PLB

$$c\bar{q} + c\bar{q} q\bar{q} \quad q=u,d,s \quad n=u,d$$

$$\underline{3} \otimes 8 = \underline{3} \oplus \underline{6} \oplus 15 \quad \text{SU}(3)_F$$

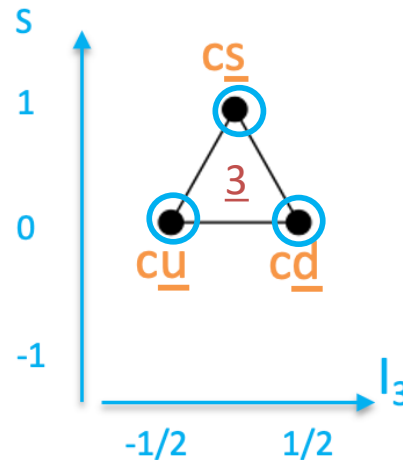
Beveren, Rupp; Dmitrasinovic

2.3 GeV

lat: 2.1-2.2 GeV (pole)

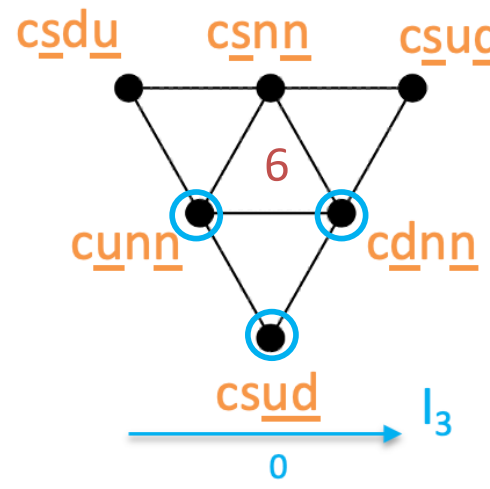
PDG: 2.3 GeV (BW)

(see backup)



attractive

mix
~~SU(3)_F~~



attractive

mixes with repulsive 15

2.4-2.5 GeV

reanalysis of lat
1607.07093 by
Albaladejo 1610.06727

virtual bound state
HadSpec 2008.06432
partner of X(2900)
[LHCb 2009.00025] ?

Di-baryons

binding energy $\Delta E = m - m_{B1} - m_{B2}$

H-dibaryon

many lattice studies:

NPLQCD, CalLAT, Mainz, ...

mostly at $m_u=m_d=m_s$

[Mainz group, 2103.01054 PRL]

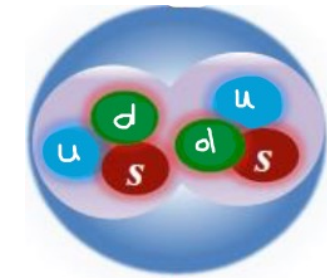
$$m_\pi = m_K \simeq 420 \text{ MeV}$$

$$\Delta E = -4.56 \pm 1.13_{\text{stat}} \pm 0.63_{\text{syst}} \text{ MeV.}$$

significant discretization effects on ΔE found

physical m_q : ??

experimentally not confirmed (yet)



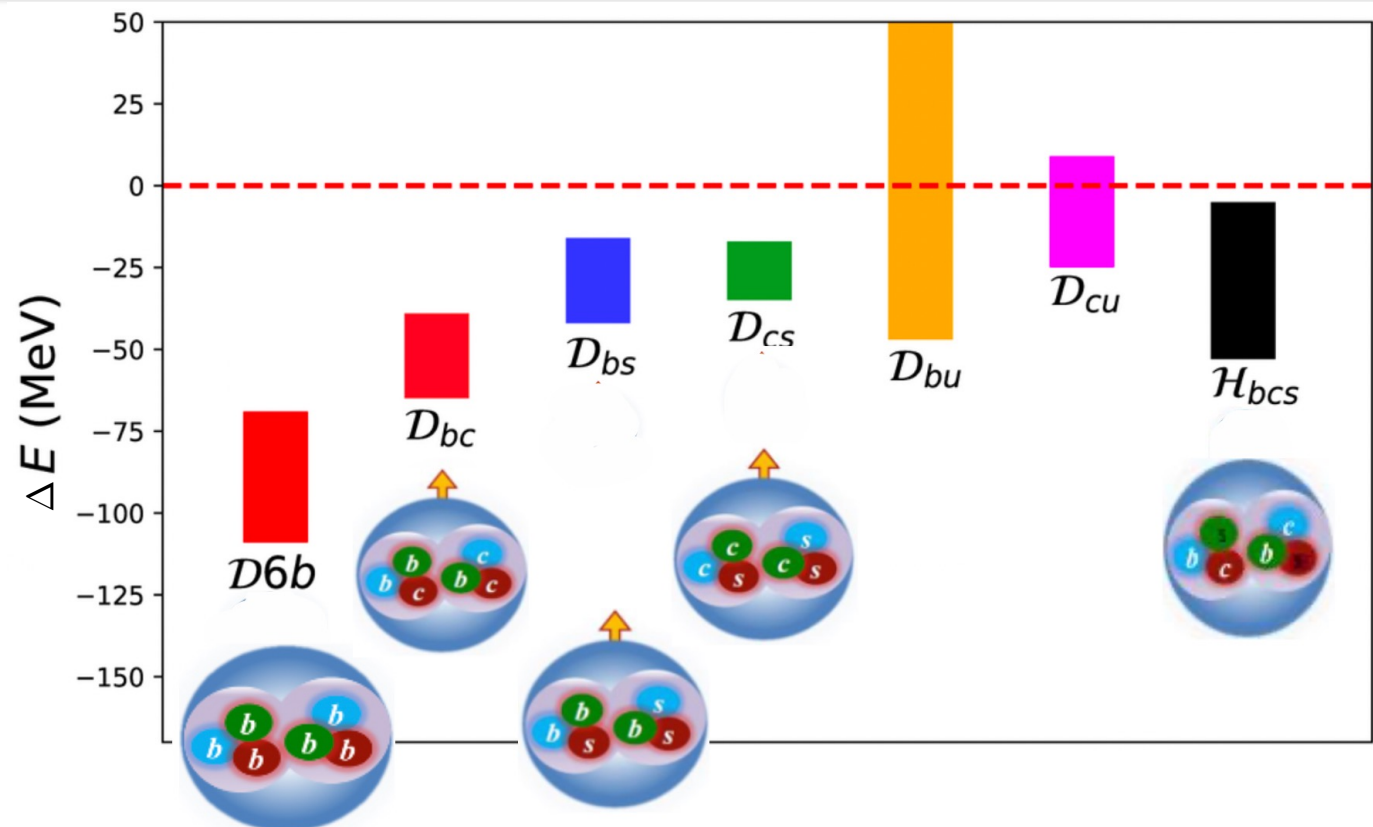
$\Lambda \Lambda$

dibaryons with heavy quarks

Junnarkar Mathur
1906.06054, PRL 2019

Mathur, Padmanath, Chakraborty
2205.02862

Junnarkar, Mathur, 2206.02942



Conclusions

Compliments to experiments for GREAT results !!

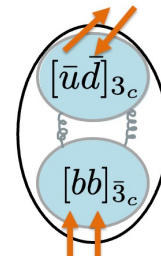
Status on hadrons from Lattice , Lattice + EFTs:

- hadrons that are not resolved (yet)
strongly decay via many decay channels: P_c , $Z_c(4430)$, $X(6900)$,...
- available: valuable results on conventional and exotic hadrons
strongly stable ; strongly decaying to 1,2,3 channels

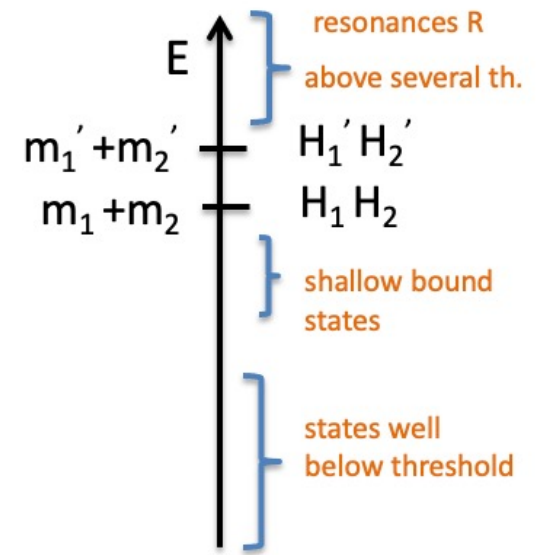
support for specific binding mechanisms

one picture can not explain all exotic hadrons

for each exotic hadron there is at least one viable picture



...



A direction that might lead to a valuable insight into dynamics of hadrons:

identify and inspect states that can be rigorously studied by theory and well explored by experiments
quark-mass dependence of these states in theory ... could lead to further clues about their nature

backup

Hadrons

$$\bar{Q}Q\bar{q}q,$$

$$\bar{Q}Qqqq$$

$$Q=c,b \quad q=u,d,s$$

$$Z_b = \bar{b}b\bar{d}u \quad [Belle \ 2011] \quad P_c = \bar{c}cuud \quad [LHCb \ 2015]$$

$$Z_c = \bar{c}c\bar{d}u \quad [BessII, Belle, 2013]$$

GRRR!

challenging for ab-initio study
due to many decay channels:

$$\bar{Q}Q\bar{q}q \rightarrow \bar{Q}q + \bar{q}Q, \quad \bar{Q}Q + \bar{q}q$$

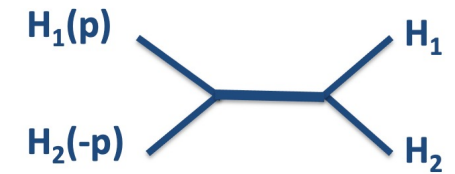
$$\bar{Q}Qqqq \rightarrow \bar{Q}q + Qqq, \quad \bar{Q}Q + qqq$$

Only partial conclusions are available
from ab-initio approaches
[reviewed e.g. in S.P. 2001.01767]
I'll discuss other approaches
(also due to lack of time).

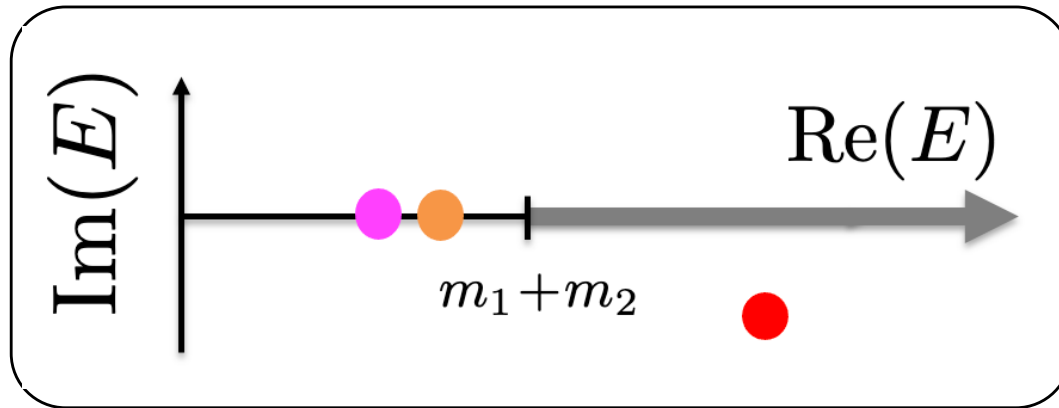
Definitions: bound state, virtual bound state & resonance

$$T(E) \propto \frac{1}{E^2 - m^2}$$

$$T(E) \propto \frac{1}{E^2 - m^2 + iE\Gamma}$$



Poles of $T(E)$, $E=E_{cm}$



Virtual bound st. Bound st. Resonance

$$p = -i|p|$$

$$p = i|p|$$

$$E = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + (-p)^2} < m_1 + m_2$$

Bound and virtual bound state: simplest example scattering in square-well potential in QM

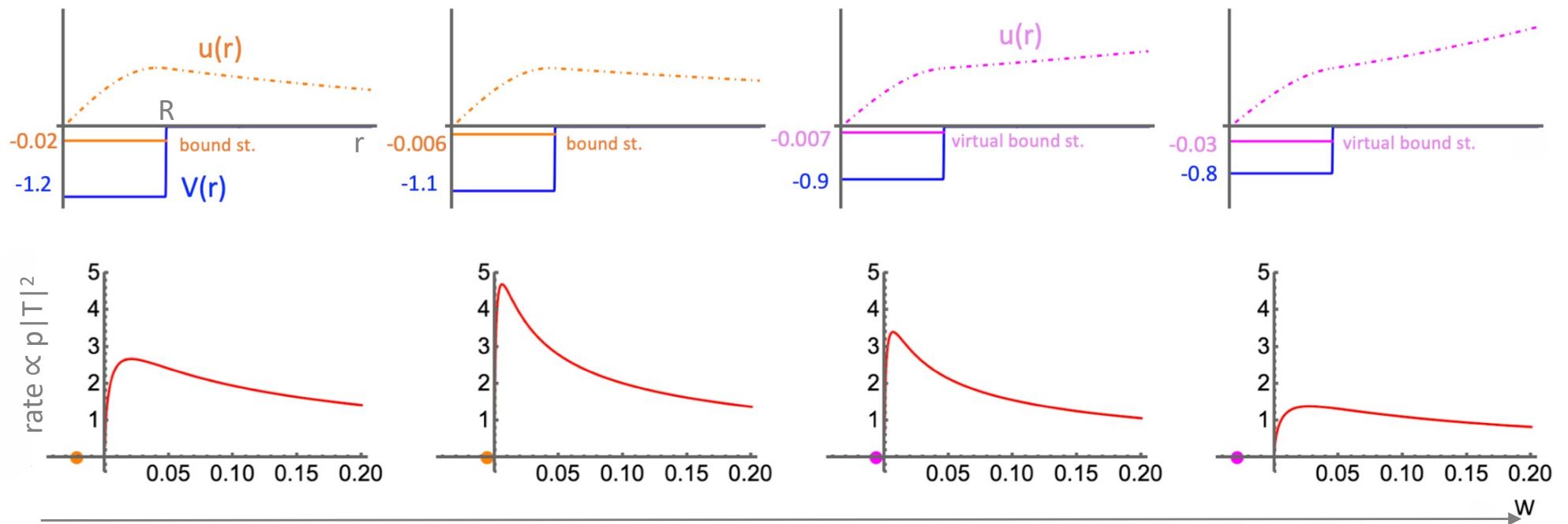
partial wave $l=0$

$$\delta = \arctan[\tan(qR)\frac{p}{q}] - pR$$

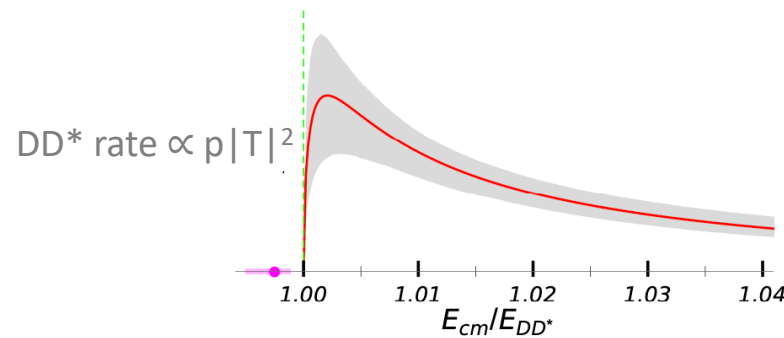
$$u(r) = A \sin(qr) \quad u(r) = B \sin(pr + \delta)$$

$$p=i|p| \quad e^{ipr} = e^{-|p|r}$$

$$p=-i|p| \quad e^{ipr} = e^{|p|r}$$



increasing $m_{u/d}$, decreasing attraction

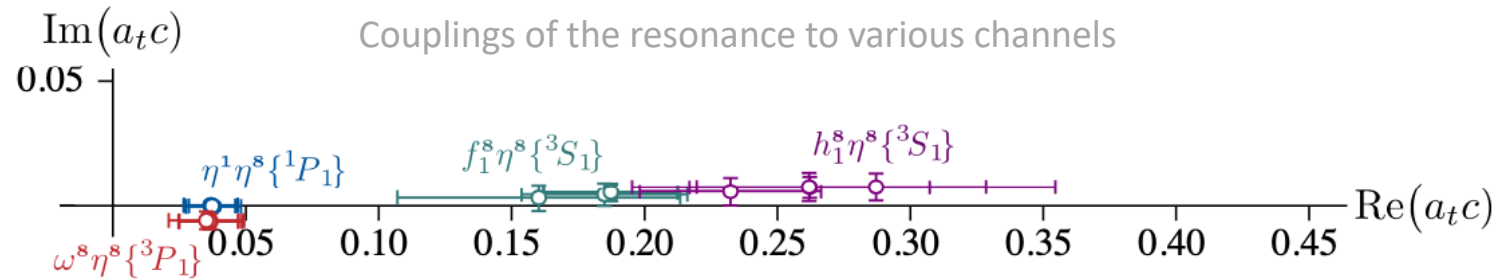
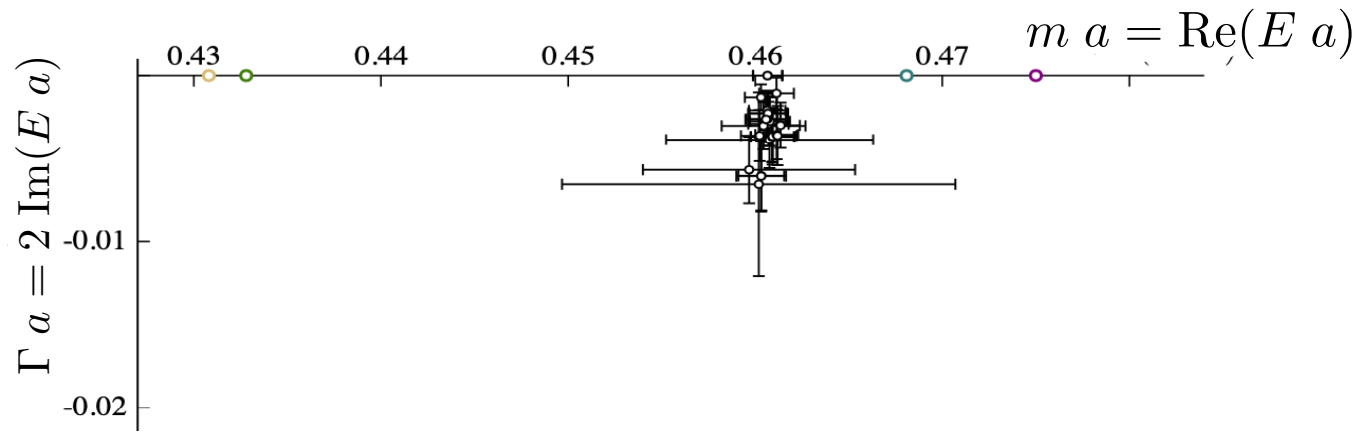


for T_c extracted on lattice

light hybrid meson π_1 from lattice

$\bar{d}Gu$

$$J^{PC} = 1^{-+}$$



$$T_{ij} \sim \frac{c_i c_j}{E_p^2 - E^2}$$

Woss et al. (HadSpec)
2009.10034

$$m_u = m_d = m_s, m_\pi \approx 700 \text{ MeV}$$

pheno

analysis

physical world

$\rho \pi$

$\eta' \pi$

$f_1 \pi$

$b_1 \pi$

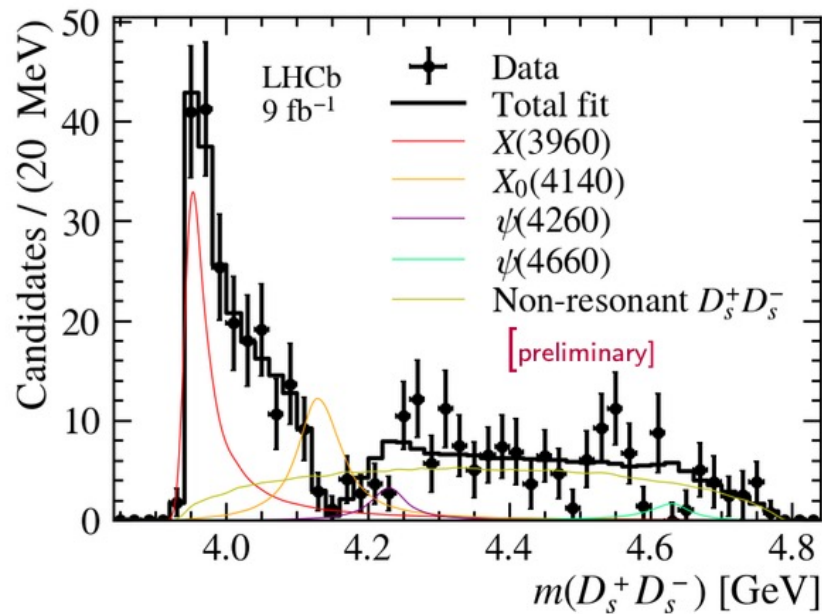
dominant coupling

resemblance to experimental $\pi_1(1564)$: COMPASS+JPAC Rodas 1810.04171 [PRL]

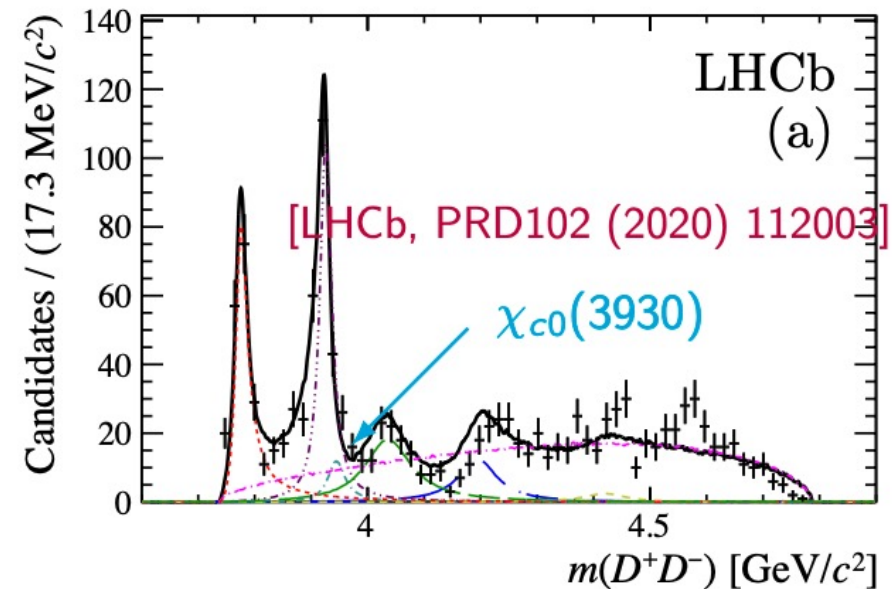
$\pi_1(1564)$ in COMPASS+JPAC replaces two older resonances $\pi_1(1400)$ and $\pi_1(1600)$

Is $X(3960)$ the same as $\chi_{c0}(3930)$ from $D^+ D^-$?

$B^+ \rightarrow (D_s^+ D_s^-) K^+$ by LHCb:



$B^+ \rightarrow (D^+ D^-) K^+$ by LHCb:



- Assuming to be the same, $\mathcal{B}(\chi_{c0} \rightarrow D^+ D^-) / \mathcal{B}(\chi_{c0} \rightarrow D_s^+ D_s^- P) \sim 0.3$
large molecular component, or large tetraquark component, $T_{\psi\phi}$
- [JHEP 06 (2021) 035] finds a state coupled to $D_s^+ D_s^-$ on the lattice