# Spectroscopy of Heavy Baryons and Roles of Diquarks

**Makoto Oka**

**Nishina Center for Accelerator-Based Science, RIKEN**

**and**

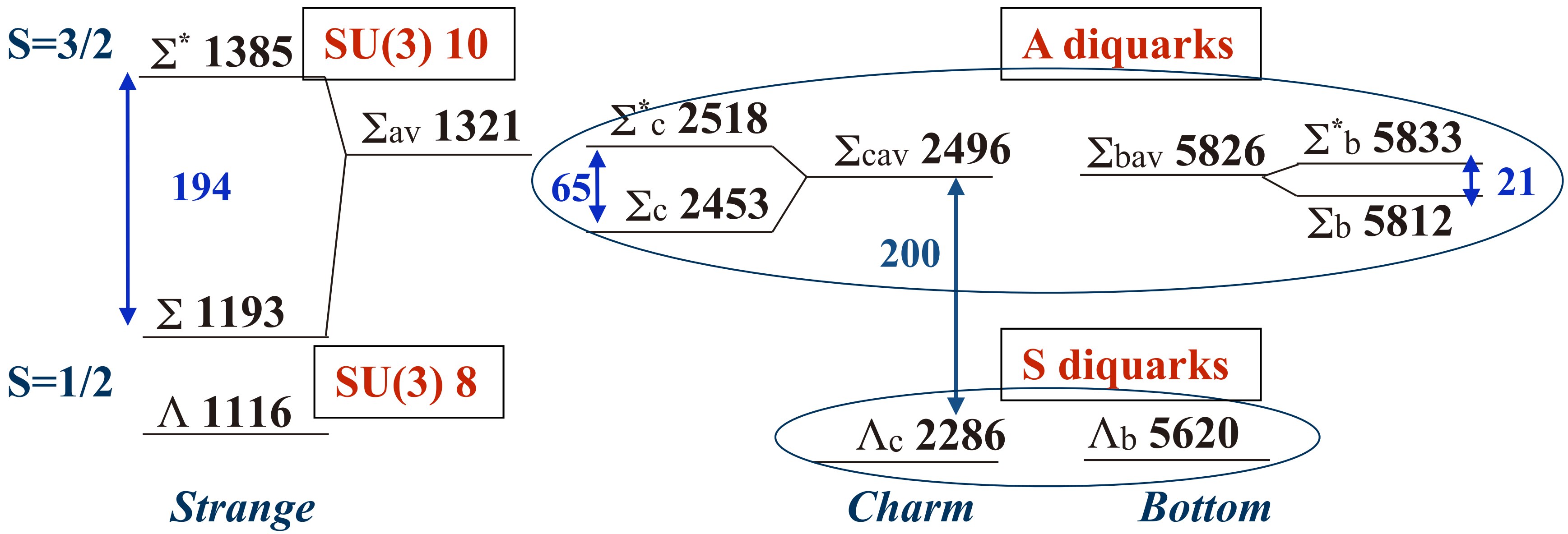
**Advanced Science Research Center, JAEA**

***HADRON2023, Genova, June 05,2023***

# Contents

# Introduction
# Diquarks in Heavy Baryons

# Diquarks in Heavy Baryons

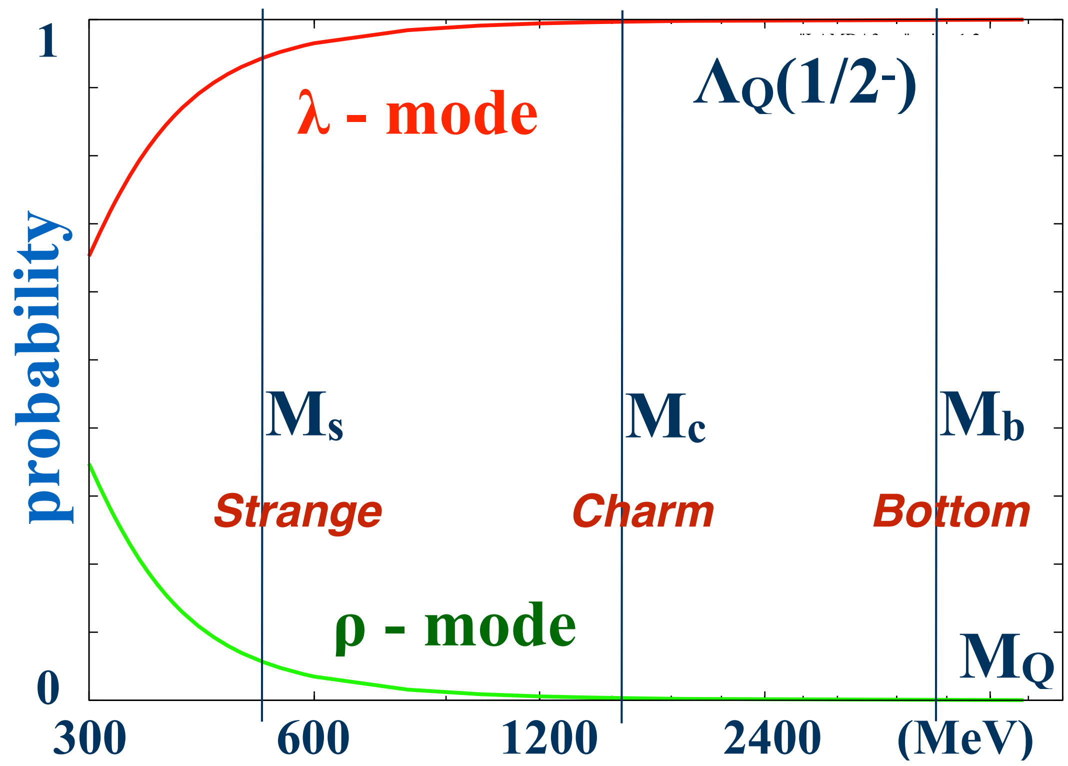


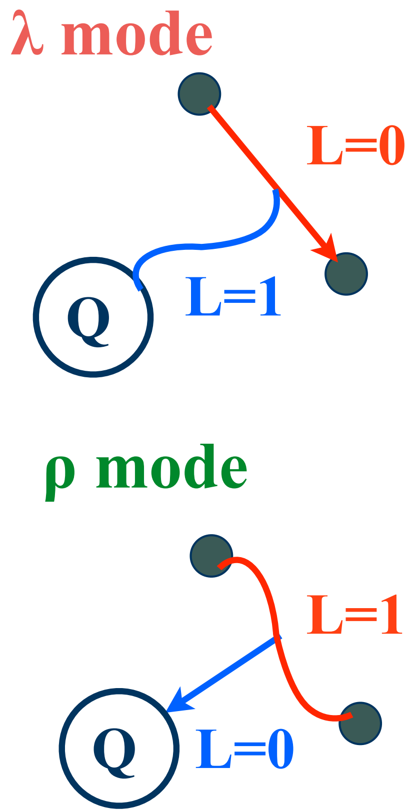

- **For light baryons, the spin-spin force splits SU(3) 8 and 10.**
- **For heavy baryons, the diquark structure is dominant due to the weak spin-spin force (heavy-quark spin symmetry).**

# Diquarks in Heavy Baryons

- P-wave excited states from s → c → b: the λ and ρ modes are split.

Probabilities of the λ and ρ modes
in the lowest P-wave $\Lambda_Q(1/2^-)$ state

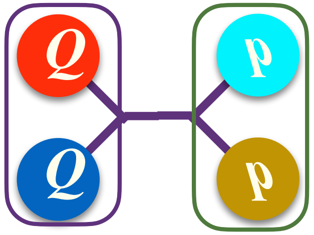


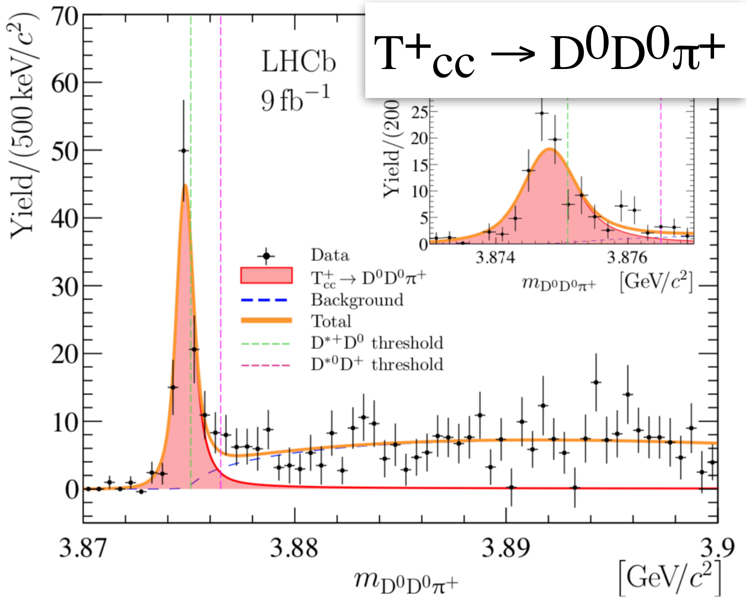




*Quark model calculation by Yoshida, et al., PRD 92, 114029 (2015)*

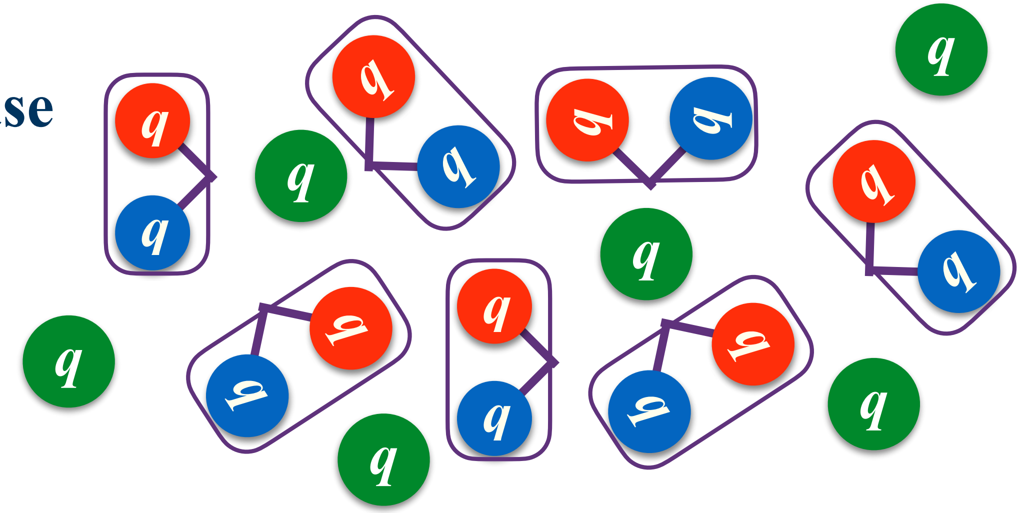
# Diquarks in exotic hadrons/matter

- **Diquark may form doubly heavy tetraquark bound states.**
  **$T_{QQ} = QQ\bar{q}\bar{q}$**



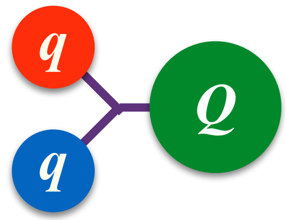



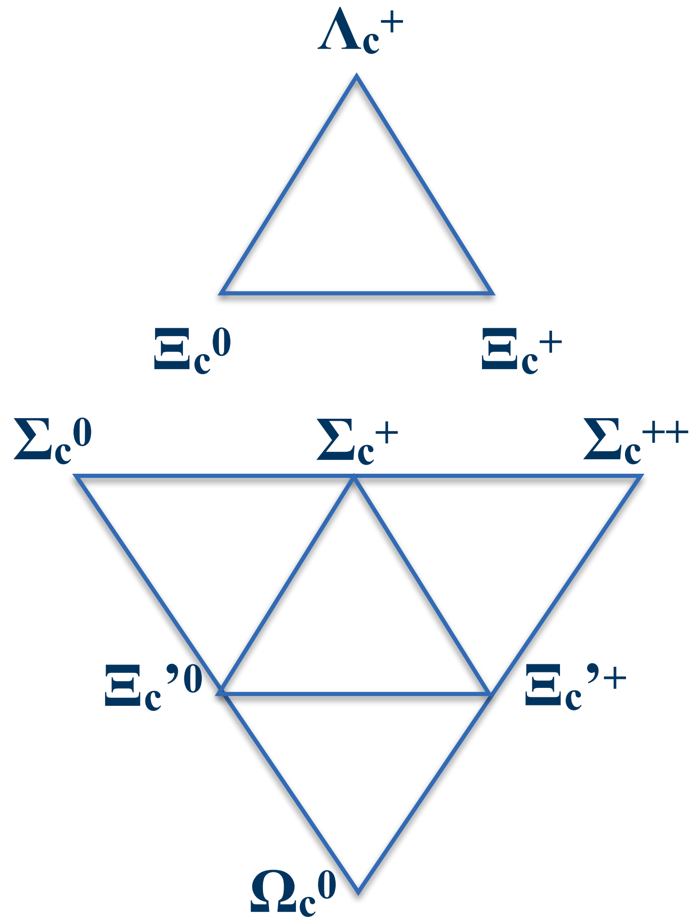

- **Diquarks may form BE condensate in dense hadronic matter.**
  **=> color-superconducting phase**



# Diquarks in Heavy Baryons

- **Scalar diquark S($0^+$)**
  **L=0, S=0, color $\bar{3}$ → flavor SU(3)$_f$ $\bar{3}$ (antisym) :**
  **[ud]=(ud-du), [ds]=(ds-sd), [su]=(su-us)**
  **⇒ flavor $\bar{3}$ HQ baryons: $\Lambda_Q$, $\Xi_Q$**




- **Axial vector diquark A($1^+$)**
  **L=0, S=1, color $\bar{3}$ → SU(3)$_f$ 6 (sym)**
  **uu, {ud}, dd, {us}, {ds}, ss**
  **⇒ flavor 6 HQ baryons: $\Sigma_Q$, $\Xi'_Q$, $\Omega_Q$**

- **SU(3)$_f$ symmetry breaking is suppressed**
  **due to the *isospin symmetry* for $\Lambda_Q - \Sigma_Q$**
  **due to the *HQ spin symmetry* for $\Xi_Q - \Xi'_Q$**

# Diquarks in QCD

| | Local diquark operators | $J^\pi$ | color | flavor |
|---|---|---|---|---|
| Pseudoscalar | $\epsilon_{abc}(u_a^T C d_b)$ | $0^-$ | $\bar{3}$ | $\bar{3}\ (I=0)$ |
| Scalar | $\epsilon_{abc}(u_a^T C\gamma^5 d_b)$ | $0^+$ | $\bar{3}$ | $\bar{3}\ (I=0)$ |
| Vector | $\epsilon_{abc}(u_a^T C\gamma^\mu\gamma^5 d_b)$ | $1^-$ | $\bar{3}$ | $\bar{3}\ (I=0)$ |
| Axial Vector | $\epsilon_{abc}(u_a^T C\gamma^\mu d_b)$ | $1^+$ | $\bar{3}$ | $6\ (I=1)$ |
| | $\epsilon_{abc}(u_a^T C\sigma^{\mu\nu} d_b)$ | $1^+, 1^-$ | $\bar{3}$ | $6\ (I=1)$ |
| color 6 | $(u_a^T C d_b) + (a \leftrightarrow b)$ | $0^-$ | $6$ | $6\ (I=1)$ |
| | $(u_a^T C\gamma^5 d_b) + (a \leftrightarrow b)$ | $0^+$ | $6$ | $6\ (I=1)$ |
| | $(u_a^T C\gamma^\mu\gamma^5 d_b) + (a \leftrightarrow b)$ | $1^-$ | $6$ | $6\ (I=1)$ |
| | $(u_a^T C\gamma^\mu d_b) + (a \leftrightarrow b)$ | $1^+$ | $6$ | $\bar{3}\ (I=0)$ |
| | $(u_a^T C\sigma^{\mu\nu} d_b) + (a \leftrightarrow b)$ | $1^+, 1^-$ | $6$ | $\bar{3}\ (I=0)$ |

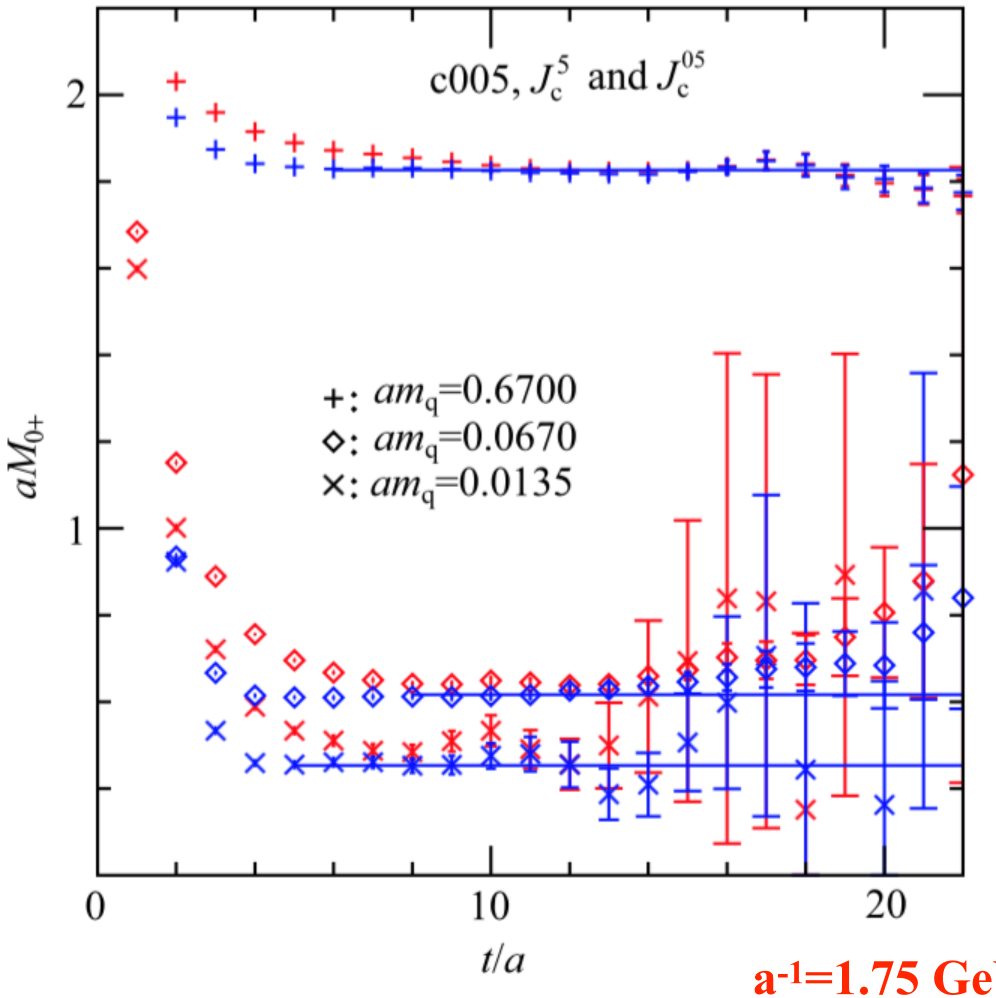
# Diquark in Lattice QCD

- Hess, Karsch, Laermann, Wetzorke, PR D58, 111502 (1998)
  quench, Landau gauge fixed
  $M(0^+) \sim 694$ MeV, $M(1^+) \sim 810$ MeV
- Alexandrou, de Forcrand, Lucini, PRL 97, 222002 (2006)
  From Qqq system, quench, gauge invariant
  $M(1^+) - M(0^+) \sim 200$-$220$ MeV, $R(S) \sim 1$ fm
- Babich, et al., PR D76, 074021 (2007)
  quench, Landau gauge
  $M(1^+) - M(0^+) \sim 162$ MeV, $M(0^+) - 2m_q \sim -200$ MeV
- Yujiang Bi, et al., Chinese Physics C40 (2016) 073106
  full QCD, Landau gauge
  $M(1^+) - M(0^+) \sim 290$ MeV, $M(0^+) - m_q \sim 310$ MeV
- K. Watanabe, Phys. Rev. D105 (2022) 074510
  quark-diquark potential and diquark mass

Chinese Physics C Vol. 40, No. 7 (2016) 073106

# Diquark mass differences from unquenched lattice QCD

Yujiang Bi(毕玉江)[1;1)] Hao Cai(蔡浩)[1;2)] Ying Chen(陈莹)[2] Ming Gong(宫明)[2]
Zhaofeng Liu(刘朝峰)[2;3)] Hao-Xue Qiao(乔豪学)[1] Yi-Bo Yang(杨一玻)[3]




**$a^{-1}$=1.75 GeV**

**$M(1^+)-M(0^+) \sim 290$ MeV**

**$M(0^-)- M(0^+) \sim 540$ MeV**

**$M(1^-)- M(1^+) \sim 510$ MeV**

**$M(0^+) \sim 720$ MeV**

**$M(0^-) \sim 1260$ MeV**

**$M(1^+) \sim 1010$ MeV**

**$M(1^-) \sim 1520$ MeV**

# Chiral Effective Theory of Diquarks

# Chiral Effective Theory of Diquarks

- **Goal: to explore properties of *light diquarks* under SU(3)×SU(3) chiral symmetry and answer questions such as**

**What are the chiral partners of diquarks and their implications to hadron spectroscopy?**

**How can we observe the chiral properties of diquarks?**

**What are the roles of $U(1)_A$ anomaly in diquark interactions?**

**How do diquarks decay strongly?**

**How do diquarks behave in matter, where chiral symmetry is partially restored?**

- ***Chiral effective Lagrangian based on the linear representation of diquarks (and S+PS mesons)***

# Chiral Effective Theory of Diquarks

**M. Harada, Y.R. Liu, M.O., K. Suzuki, "*Chiral effective theory of diquarks and $U_A(1)$ anomaly*", Phys. Rev. D 101, 054038 (2020)**

**Y. Kim, E. Hiyama, M.O., K. Suzuki, "*Spectrum of singly heavy baryons from a chiral effective theory of diquarks*", Phys. Rev. D 102, 014004 (2020)**

**Y. Kawakami, M. Harada, M.O., K. Suzuki, "*Suppression of decay widths in singly heavy baryons induced by the $U_A(1)$ anomaly*", Phys. Rev. D 102, 114004 (2020)**

**Y. Kim, Y.R. Liu, M.O., K. Suzuki, "*Heavy baryon spectrum with chiral multiplets of scalar and vector diquarks*", Phys. Rev. D 104, 054012 (2021)**

**Y. Kim, M.O., K. Suzuki, "*Doubly heavy tetraquarks in a chiral-diquark picture*", Phys. Rev. D 105, 074021 (2022)**

**Y. Kim, M.O., D. Suenaga, K. Suzuki, "*Strong decays of singly heavy baryons from a chiral effective theory of diquarks*", Phys. Rev. D 107, 074015 (2023)**

**D. Suenaga, M.O., "*Axial anomaly effect to the chiral-partner structure of diquarks at high temperature*", ArXiv:2305.09730**

**H. Takada, D. Suenaga, M. Harada, A. Hosaka, M.O., in preparation**

# Chiral Effective Theory of Diquarks

- **Some previous works on ChET for diquarks:**
  **D.K. Hong, Y.J. Sohn, I. Zahed, Phys. Lett. B596 (2004) 191, Int. J. Mod. Phys. A27, 1250051 (2012), *Non-linear chiral diquark effective theory for penta/tetraquarks***
  **T. Hatsuda, M. Tachibana, N. Yamamoto, G. Baym, Phys. Rev. Lett. 97, 122001 (2006), Phys. Rev. D76, 074001 (2007), *Chiral effective theory and the axial anomaly in dense QCD***
- **Chiral effective theories of single heavy baryons**
  **D. Ebert, T. Feldmann, C. Kettner, H. Reinhardt, Zeit. Phys. C71, 329–335 (1996), *Diquark model for single heavy baryons***
  **Y. Kawakami, M. Harada, Phys. Rev. D97, 114024 (2018), Phys. Rev. D99, 094016 (2019), *Chiral effective theory of single heavy baryons***
  **D. Suenaga, A. Hosaka, PR D104 (2021) 034009, Phys. Rev. D105, 074036 (2022), *Pentaquark picture for singly heavy baryons***

# Chiral Effective Theory of Diquarks

- **Linear representation of chiral SU(3)$_R$ x SU(3)$_L$**

$$q^a_{\alpha i} \quad a \text{ (color)},\ \alpha \text{ (Dirac)},\ i \text{ (flavor)}$$

$$q^a_{iR} = P_R\, q^a_i, \quad q^a_{iL} = P_L\, q^a_i \qquad P_{R,L} \equiv \frac{1 \pm \gamma_5}{2}$$

$$q_R \to U_R\, q_R = (U_R)_{ij} q_{jR}, \quad U_R \in SU(3)_R$$

$$q_L \to U_L\, q_L = (U_L)_{ij} q_{jL}, \quad U_L \in SU(3)_L$$

- **Scalar diquarks (color $\bar{3}$)**

$$d^a_{iR} \equiv \epsilon_{ijk}(q^T_{jR}\, C\, q_{kR})^{\bar{3}} \quad \text{Right scalar diquark, chiral } (\bar{3},1)\text{, color } \bar{3}$$

$$d^a_{iL} \equiv \epsilon_{ijk}(q^T_{jL}\, C\, q_{kL})^{\bar{3}} \quad \text{Left scalar diquark, chiral } (1,\bar{3})\text{, color } \bar{3}$$

- **Parity eigenstates: $0^+$, $0^-$ diquarks**

$$S^a_i = d^a_{iR} - d^a_{iL} = \epsilon_{ijk}(q^T_j\, C\gamma_5\, q_k)^{\bar{3}}$$

$$P^a_i = d^a_{iR} + d^a_{iL} = \epsilon_{ijk}(q^T_j\, C\, q_k)^{\bar{3}}$$

$$(\bar{3},1) + (1,\bar{3})$$

# Scalar/Pseudoscalar Diquarks

- **The effective Lagrangian with $SU(3)_R \times SU(3)_L$ symmetry**

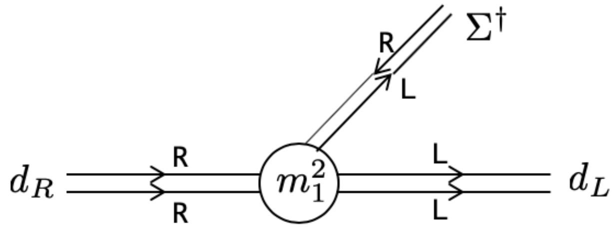
**M. Harada, Y.R. Liu, M.O., K. Suzuki, PR D101, 054038 (2020)**

$$
\begin{aligned}
\mathcal{L} = {} & \mathcal{D}_\mu d_{R,i} (\mathcal{D}^\mu d_{R,i})^\dagger + \mathcal{D}_\mu d_{L,i} (\mathcal{D}^\mu d_{L,i})^\dagger \\
& - m_0^2 (d_{R,i} d_{R,i}^\dagger + d_{L,i} d_{L,i}^\dagger) \\
& \boxed{- \frac{m_1^2}{f} (d_{R,i} \Sigma_{ij}^\dagger d_{L,j}^\dagger + d_{L,i} \Sigma_{ij} d_{R,j}^\dagger)} \\
& - \frac{m_2^2}{2f^2} \epsilon_{ijk} \epsilon_{\ell mn} (d_{R,k} \Sigma_{\ell i} \Sigma_{mj} d_{L,n}^\dagger + d_{L,k} \Sigma_{\ell i}^\dagger \Sigma_{mj}^\dagger d_{R,n}^\dagger) \\
& + \frac{1}{4} \mathrm{Tr} \left[ \partial^\mu \Sigma^\dagger \partial_\mu \Sigma \right] + V(\Sigma).
\end{aligned}
$$

**chiral invariant**

**$U_A(1)$ anomalous**

**SCSB mass**




$$\Delta Q_5 = \pm 6 \, (= \pm 2N_f)$$

**Scalar and PS nonets mesons**

$$\Sigma_{ij} \equiv \sigma_{ij} + i\pi_{ij}$$

# Scalar/Pseudoscalar Diquarks

- **For the SSB vacuum** $\langle \Sigma_{ij} \rangle = f\delta_{ij}$
  **the mass term of the right and left diquarks are given by**

$$M^2 = \begin{pmatrix} m_0^2 & m_1^2 + m_2^2 \\ m_1^2 + m_2^2 & m_0^2 \end{pmatrix}$$

- **The mass eigenstates are**
  **Scalar diquark** $S_i^a = \frac{1}{\sqrt{2}}(d_{R,i}^a - d_{L,i}^a)$

$$\longrightarrow M(0^+) = \sqrt{m_0^2 - m_1^2 - m_2^2},$$

**Pseudo-scalar diquark**

$$P_i^a = \frac{1}{\sqrt{2}}(d_{R,i}^a + d_{L,i}^a)$$

$$\longrightarrow M(0^-) = \sqrt{m_0^2 + m_1^2 + m_2^2},$$

# Scalar/Pseudoscalar Diquarks

## SU(3) breaking and inverse mass hierarchy

$$A \equiv 1 + \epsilon \equiv \frac{f_s}{f_\pi}\left(1 + \frac{m_s}{g_s f_s}\right) \sim \frac{5}{3}$$

***i*=3 (ud)**

$$M_3(0^+) = \sqrt{m_0^2 - A m_1^2 - m_2^2}, \qquad M_3(0^-) = \sqrt{m_0^2 + A m_1^2 + m_2^2}.$$

***i*=1,2 (ds), (us)**

$$M_1(0^+) = M_2(0^+) = \sqrt{m_0^2 - m_1^2 - A m_2^2}, \quad M_1(0^-) = M_2(0^-) = \sqrt{m_0^2 + m_1^2 + A m_2^2},$$

**Inverse Mass Hierarchy due to $U_A(1)$ anomaly**

$$M_1(0^+)^2 + M_1(0^-)^2 = M_3(0^+)^2 + M_3(0^-)^2$$

$$M_1(0^+)^2 - M_3(0^+)^2 = M_3(0^-)^2 - M_1(0^-)^2 > 0$$

$$M_3(0^-) > M_1(0^-)$$

**(ds), (us)   (ud)**

**$0^-$(ud)**

**$0^-$(su)**

**$0^+$(su)**

**$0^+$(ud)**

*M. Harada, Y.R. Liu, M.O., K. Suzuki, Phys. Rev. D101, 054038 (2020)*

# Axialvector/Vector Diquarks

Y. Kim, Y.R. Liu, M.O., K. Suzuki, Phys. Rev. D 104, 054012 (2021)

- **The $1^+/1^-$ diquarks in (3,3) representation**

$$d^{\mu a}_{ij} \equiv \epsilon_{abc}(q^{bT}_{iL}\, C\gamma^\mu\, q^c_{jR}) = \epsilon_{abc}(q^{bT}_{jR}\, C\gamma^\mu\, q^c_{iL}) \quad \text{chiral (3,3) vector diquark}$$

$$d^{\mu a}_{V[ij]} = d^{\mu a}_{ij} - d^{\mu a}_{ji} = \epsilon_{abc}(q^{bT}_i\, C\gamma^\mu\gamma^5\, q^c_j) \quad \text{Vector } 1^- \text{ diquark, flavor } \bar{3}$$

$$d^{\mu a}_{A\{ij\}} = d^{\mu a}_{ij} + d^{\mu a}_{ji} = \epsilon_{abc}(q^{bT}_i\, C\gamma^\mu\, q^c_j) \quad \text{Axial-vector } 1^+ \text{ diquark, flavor } 6$$

$$d^\mu \longrightarrow U_L d^\mu U_R^T, \quad (3,3) \qquad d^{\mu\dagger} \longrightarrow U_R^{T\dagger} d^\mu U_L^\dagger \quad (\bar{3},\bar{3})$$

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}[F^{\mu\nu}F^\dagger_{\mu\nu}] + m_0^2\mathrm{Tr}[d^\mu d^\dagger_\mu] + \frac{m_1^2}{f_\pi^2}\mathrm{Tr}[\Sigma^\dagger d^\mu \Sigma^T d^{\dagger T}_\mu] + \frac{2m_2^2}{f_\pi^2}\mathrm{Tr}[\Sigma^\dagger\Sigma d^{\mu T} d^{\dagger T}_\mu]$$

$$F^{\mu\nu} = D^\mu d^\nu - D^\nu d^\mu$$

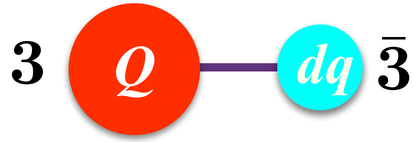
- **All the terms are chiral and $U_A(1)$ invariant.**

# Spectrum of Single Heavy Baryons

# Diquark-Heavy-Quark model

## ⌗ Single-Heavy-Baryon with a *Q-dq* potential:

$$V(r) = -\frac{\alpha}{r} + \lambda r + C,$$



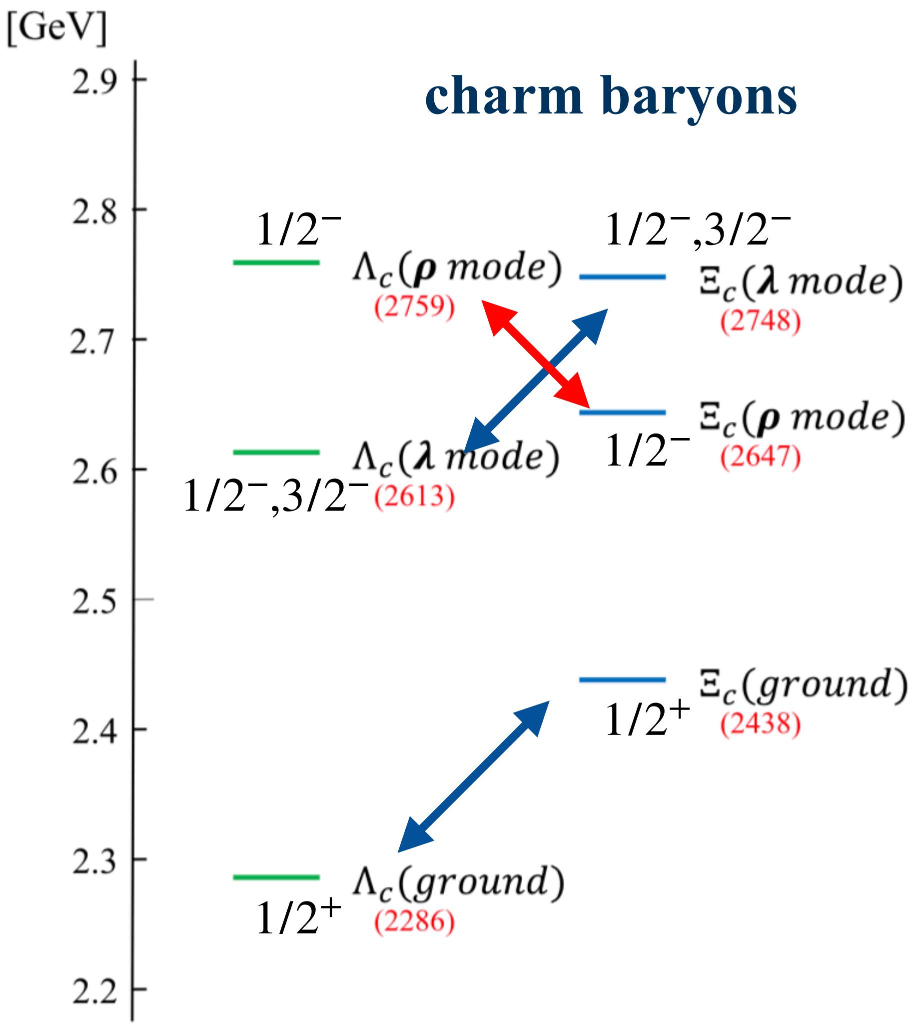

| $\alpha$ | $\lambda(\mathrm{GeV}^2)$ | $C_c(\mathrm{GeV})$ | $C_b(\mathrm{GeV})$ | $M_c(\mathrm{GeV})$ | $M_b(\mathrm{GeV})$ |
|---|---|---|---|---|---|
| $(2/3)\times 90/\mu$ | 0.165 | $-0.58418362$ | $-0.58829590$ | 1.750 | 5.112 |

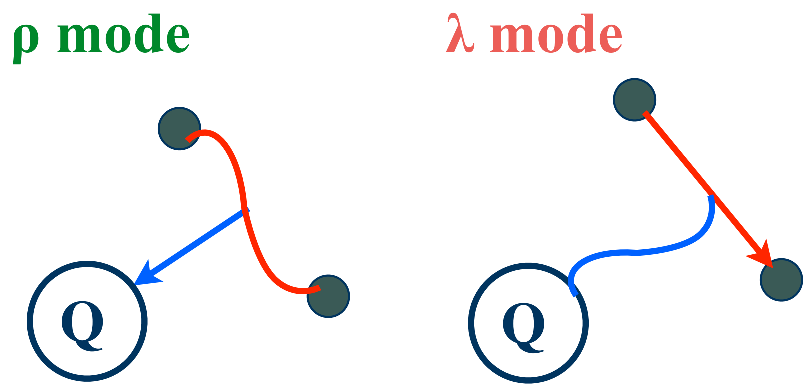
*B. Silvestre-Brac, C. Semay, Z. Phys. C 59, 457 (1993)*
*T. Yoshida, E. Hiyama, A. Hosaka, M. Oka, K. Sadato, PR D 92, 114029 (2015)*

$$M_{(ud)}(0^+) = 725\ \mathrm{MeV} \qquad M_{(ud)}(0^-) = 1265\ \mathrm{MeV}$$
$$M_{(us)}(0^+) = 906\ \mathrm{MeV} \qquad M_{(us)}(0^-) = 1142\ \mathrm{MeV}$$
$$M_{(qq)}(1^+) = 974\ \mathrm{MeV} \qquad M_{(qq)}(1^-) = 1447\ \mathrm{MeV}$$
$$M_{(qs)}(1^+) = 1116\ \mathrm{MeV} \qquad M_{(ss)}(1^+) = 1242\ \mathrm{MeV}$$

# Inverse mass hierarchy for Baryons

Y. Kim, E. Hiyama, M. O., K. Suzuki, Phys. Rev. D 102, 014004 (2020)

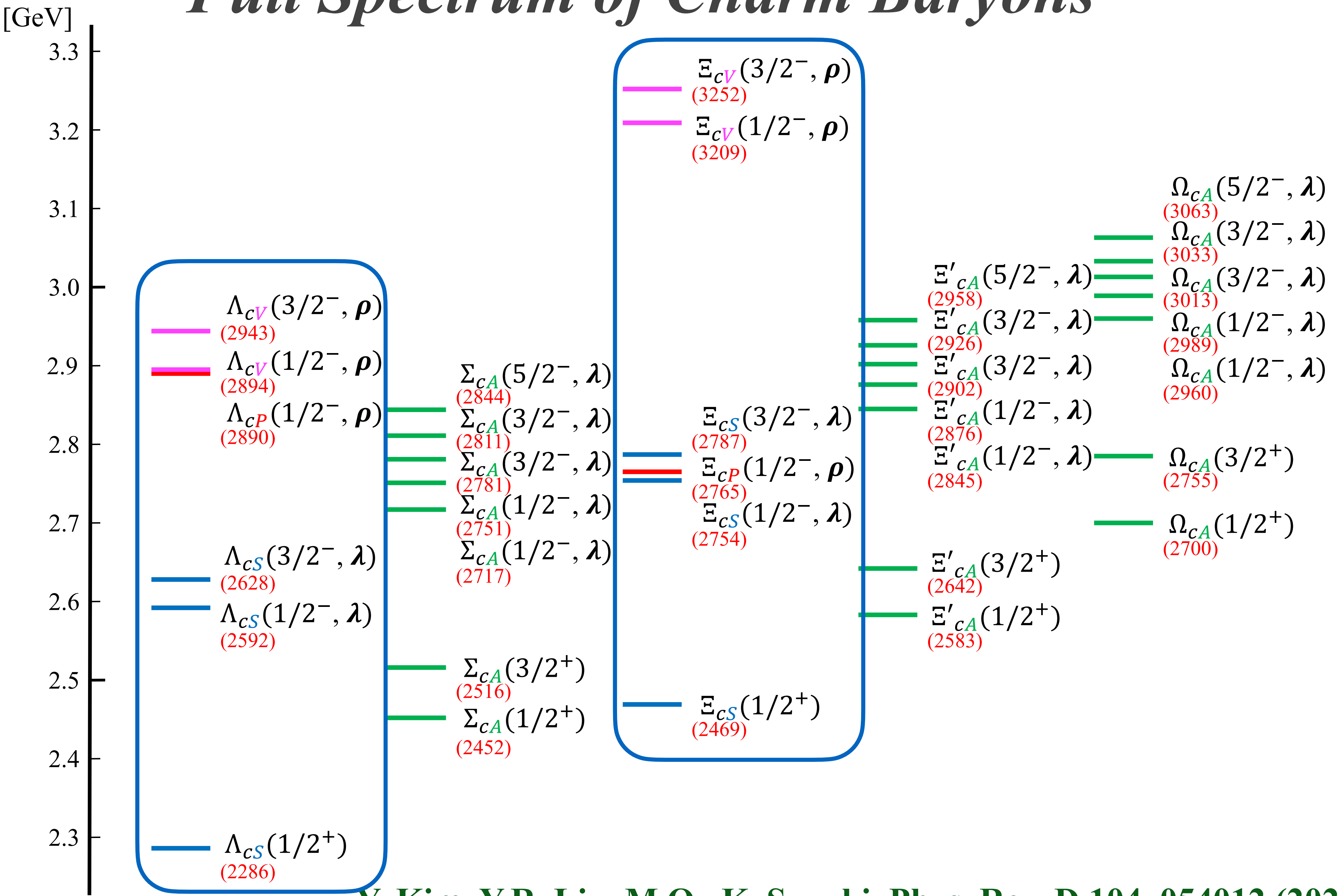



ρ mode (1/2-) vs λ mode (1/2-, 3/2-)

**Inverse mass hierarchy for ρ mode**

# *Full Spectrum of Charm Baryons*




**Y. Kim, Y.R. Liu, M.O., K. Suzuki, Phys. Rev. D 104, 054012 (2021)**

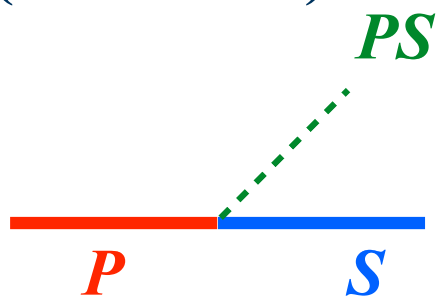
# Decays of Single Heavy Baryon Resonances

# Strong decays of SHB resonances

- transition within a chiral representation: P → S (and V → A)

$$\mathcal{L}_{\pi SP} = \frac{i(\boxed{m_1^2} + m_2^2)}{f}\pi_p(S\lambda_p P^\dagger - P\lambda_p S^\dagger) \quad \textit{octet + singlet}$$

$$-\frac{3im_2^2}{f}\pi_0(S\lambda_0 P^\dagger - P\lambda_0 S^\dagger) \quad \textit{singlet}$$




- transition across different chiral representations: A → S

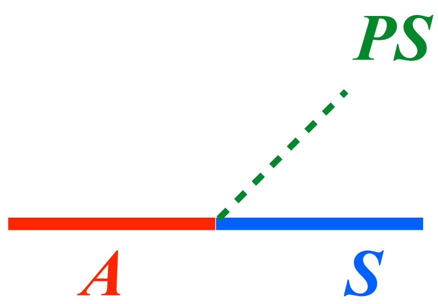
$$\mathcal{L}_{A\to S\pi} = -\sqrt{2}iG_1\left[A^\mu_{\{uu\}}(\partial_\mu\pi^-)S^\dagger_{[ud]} - A^\mu_{\{ud\}}(\partial_\mu\pi^0)S^\dagger_{[ud]} - A^\mu_{\{dd\}}(\partial_\mu\pi^+)S^\dagger_{[ud]}\right]$$

$$-iG_2\left[A^\mu_{\{ds\}}(\partial_\mu\pi^+)S^\dagger_{[su]} - A^\mu_{\{us\}}(\partial_\mu\pi^-)S^\dagger_{[ds]}\right]$$

$$+i\frac{G_2}{\sqrt{2}}\left[A^\mu_{\{us\}}(\partial_\mu\pi^0)S^\dagger_{[su]} + A^\mu_{\{ds\}}(\partial_\mu\pi^0)S^\dagger_{[ds]}\right]$$

$$G_1 = g_1 + \boxed{\alpha g_2} \qquad G_2 = g_1 + \boxed{g_2}$$

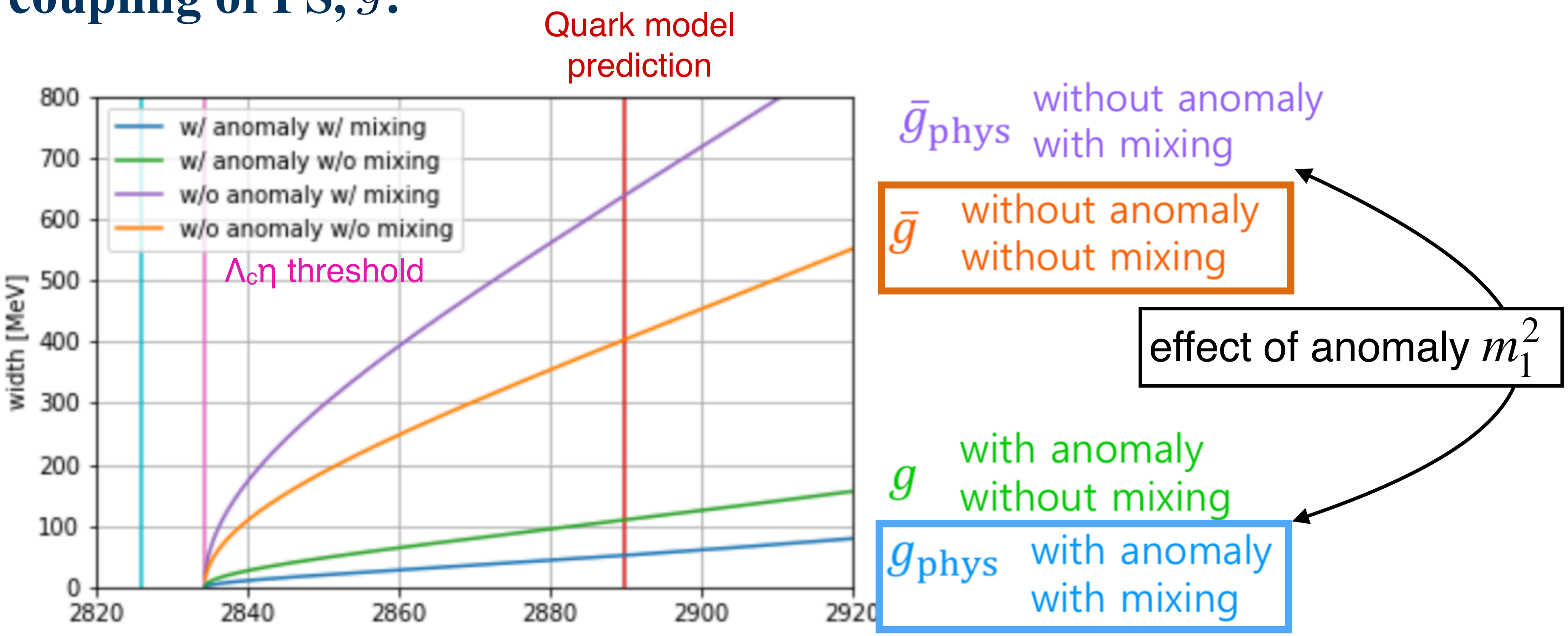
$$\mathcal{M}_{\text{eff}} \simeq g_s f_\pi \text{diag}(1,1,\alpha), \quad \alpha = \frac{f_s}{f_\pi}\left(1 + \frac{m_s}{g_s f_s}\right)$$




- $\boxed{m_1^2}$ and $\boxed{g_2}$ terms are **U$_A$(1) anomalous.**

# $B_{\bar{3}}(1/2^-) \to B_{\bar{3}}(1/2^+) + PS$

- **The generalized Goldberger-Treiman relation determines the coupling of PS, $g$.**




Unknown mass of the chiral partner $\Lambda_c(1/2^-)$

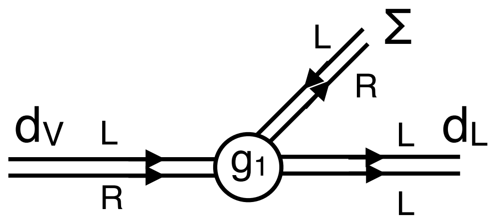
**Y. Kawakami, M. Harada, M.O., K. Suzuki, *PR D102, 114004 (2020)***

- **The decay coupling constant is suppressed by the $U_A(1)$ anomaly as well as the mixing of $\eta_1$ and $\eta_8$.**

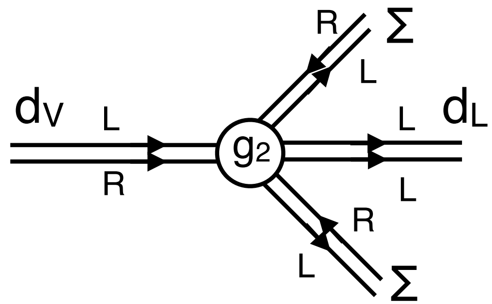
# $B_6$ (1/2⁺, 3/2⁺) → $B_{\bar{3}}$ (1/2⁺) + PS

- **U(1)$_A$ anomaly in diquark couplings**

$$\mathcal{L}_{SV}^{(1)} = g_1 \epsilon_{ijk} \left[ d^\mu_{ni} (\partial_\mu \Sigma^\dagger)_{jn} d^\dagger_{R,k} + d^\mu_{in} (\partial_\mu \Sigma)_{jn} d^\dagger_{L,k} \right]$$




$$\mathcal{L}_{SV}^{(2)} = \frac{g_2}{f_\pi} \epsilon_{ijk} \left[ d^\mu_{in} \{ \Sigma_{jn} (\partial_\mu \Sigma)_{km} - (\partial_\mu \Sigma)_{jn} \Sigma_{km} \} d^\dagger_{R,m} + d^\mu_{ni} \{ \Sigma^\dagger_{jn} (\partial_\mu \Sigma^\dagger)_{km} - (\partial_\mu \Sigma^\dagger)_{jn} \Sigma^\dagger_{km} \} d^\dagger_{L,m} \right]$$

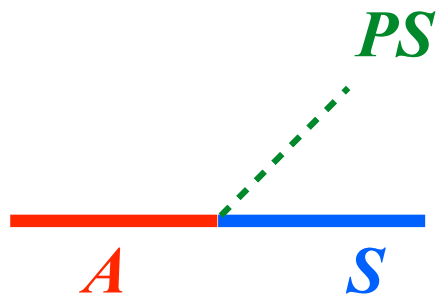



**U(1)$_A$ anomalous coupling**

- **Coupling constant fitted to the decay widths**

**charm** $(g_1^c, g_2^c) = (30.26, -3.36)$

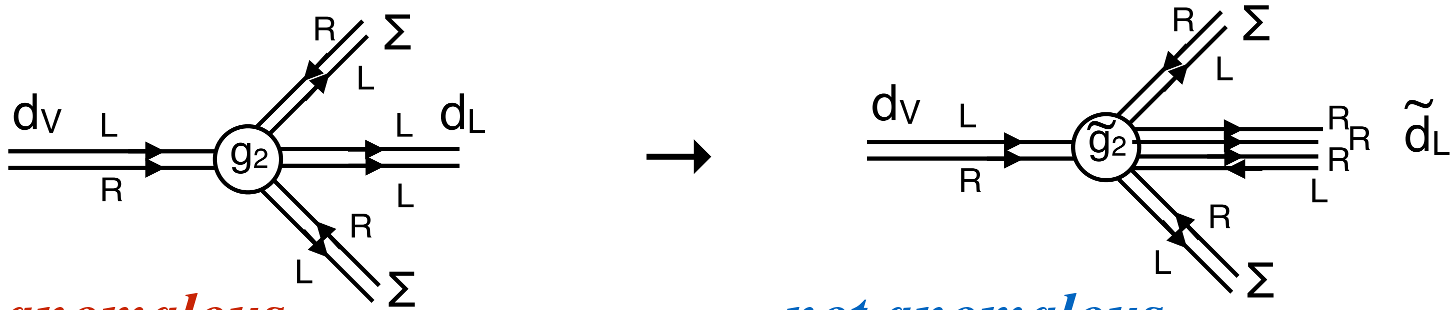
**bottom** $(g_1^b, g_2^b) = (32.78, -5.62)$




- **The anomalous term $g_2$ is suppressed, because it has a higher dimension.**

# Tetra-diquark for U(1)$_A$ anomaly

- **D. Suenaga, A. Hosaka, Phys. Rev. D 104, 034009 (2021), and Phys. Rev. D 105, 074036 (2022)**
  **"Pentaquark picture and U(1)$_A$ anomaly in Single-Heavy Baryon"**
  **→ tetra-diquark** $q_R q_R q_R \bar{q}_L (Q_5 = +4) \sim q_L q_L (Q_5 = -2)$
  **in the chiral representation**




***anomalous*** ***not anomalous***

- **Further studies of the SHB spectrum and decay widths with mixings of 3- and 5-quark structures**
  ***H. Takada, D. Suenaga, M. Harada, A. Hosaka, M.O., in preparation***

# Diquarks and Heavy Baryons for Restoration of Chiral Symmetry

# Scalar-Pseudoscalar Diquarks

- **D. Suenaga and MO, *ArXiv:2305.09730***
  **Analysis of the chiral-partner structure of diquarks in the $N_f = 3$ Nambu-Jona-Lasinio model**

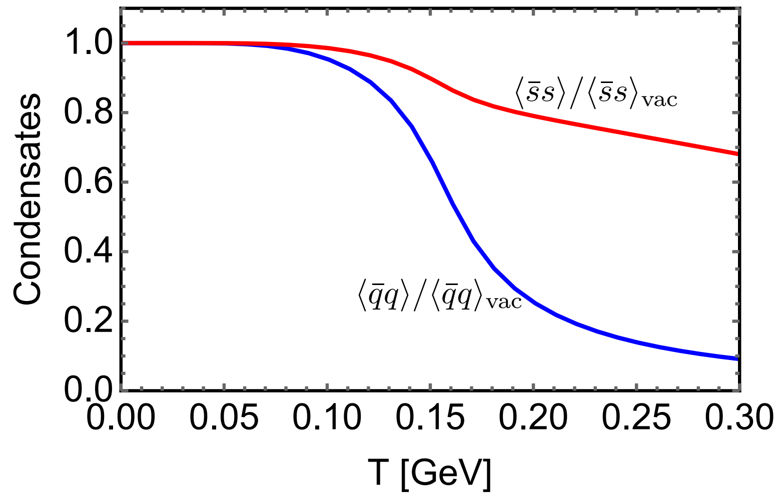
***mesons*** $\phi_{ij} = (\bar{\psi}_R)^a_j (\psi_L)^a_i$

***diquarks***
$$(\eta_L)^a_i = \epsilon_{ijk}\epsilon^{abc}(\psi_L^T)^b_j C(\psi_L)^c_k$$
$$(\eta_R)^a_i = \epsilon_{ijk}\epsilon^{abc}(\psi_R^T)^b_j C(\psi_R)^c_k$$

$$\mathcal{L}_{4q} = 8G\mathrm{tr}[\phi^\dagger\phi] + 2H(\eta_L^T\eta_L^* + \eta_R^T\eta_R^*)$$

$$\mathcal{L}_{6q}^{\mathrm{anom.}} = -8K(\mathrm{det}\phi + \mathrm{det}\phi^\dagger) + K'(\eta_L^T\phi\eta_R^* + \eta_R^T\phi^\dagger\eta_L^*)$$

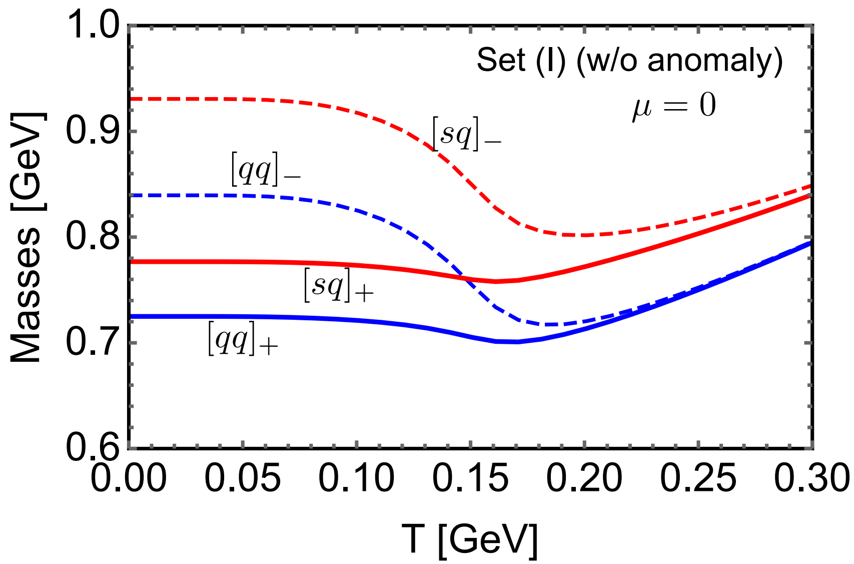
- **Chiral order parameters at finite T**



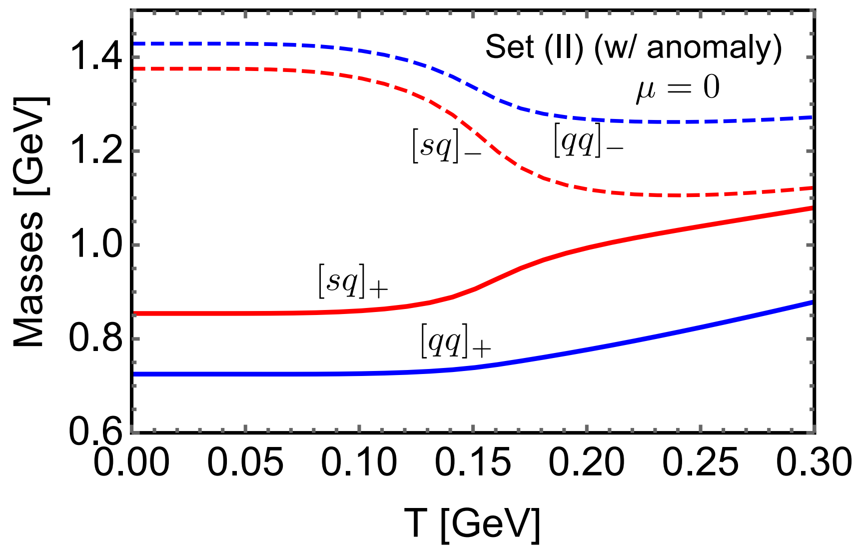
# Scalar-Pseudoscalar Diquarks

- **Finite temperature behaviors of the diquark masses**

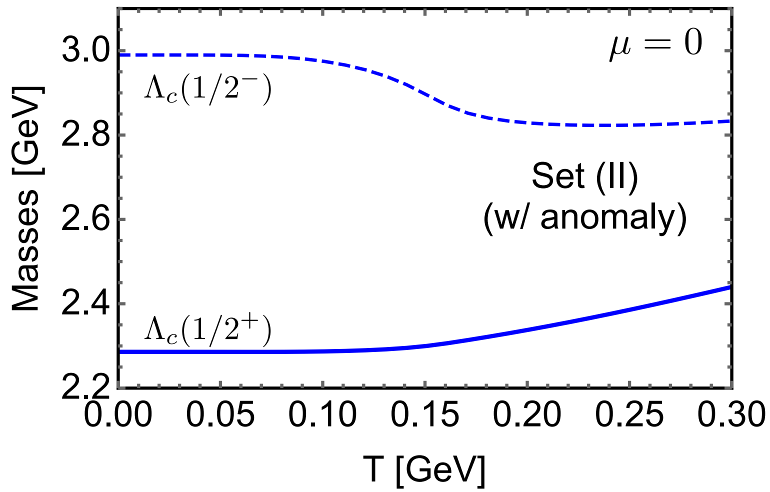
**without anomaly**




**with anomaly**




- **Expect significant effects of U(1) anomaly on the decay widths of the ρ-mode negative-parity states**



# Scalar and Axialvector Diquarks

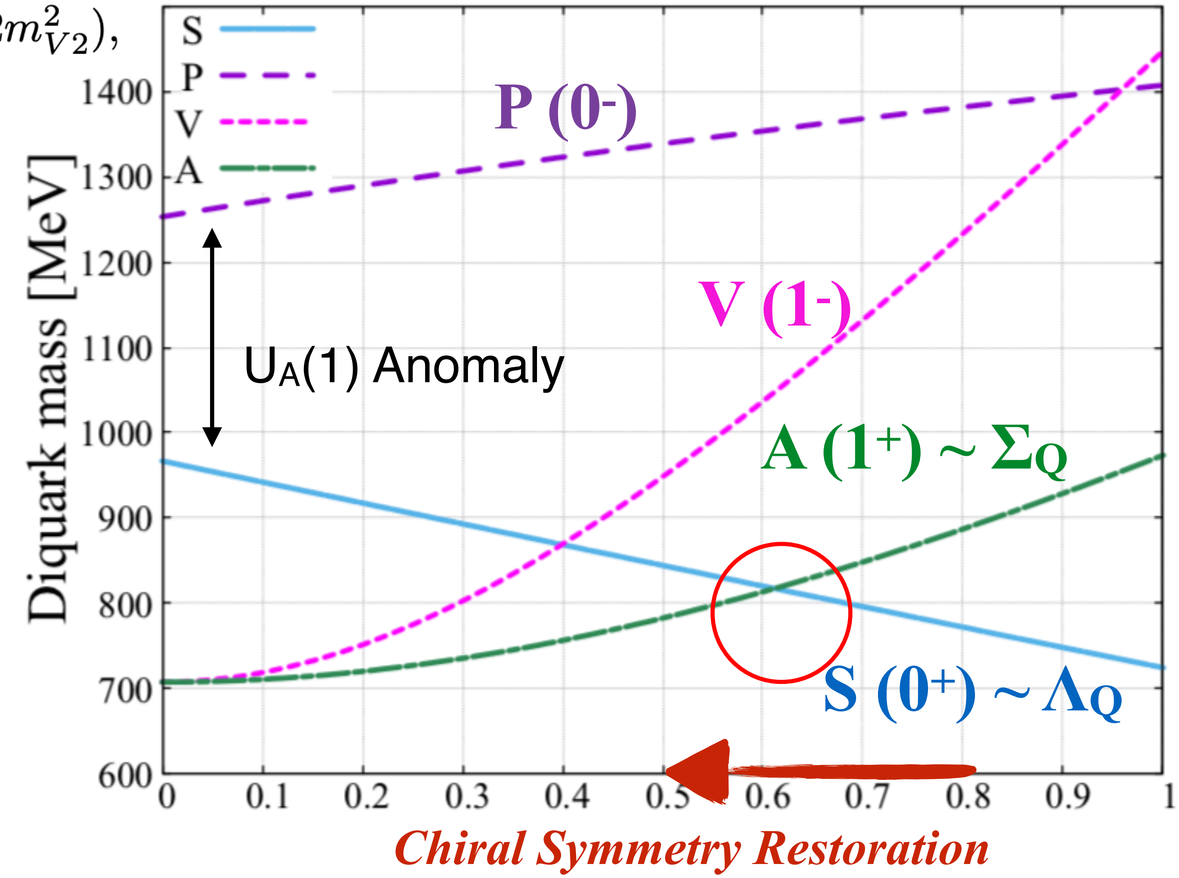
- **Masses of the 0+ and 1+ diquarks under chiral restoration**

$$m_{[ud]} = \sqrt{m_{S0}^2 - (x+\alpha-1)m_{S1}^2 - x^2 m_{S2}^2},$$

$$m_{\{uu/ud/dd\}} = \sqrt{m_{V0}^2 + x^2(m_{V1}^2 + 2m_{V2}^2)},$$

$$m_{S0}^2 = (1031\ \mathrm{MeV})^2$$
$$m_{S1}^2 = (606\ \mathrm{MeV})^2$$
$$m_{S2}^2 = -(274\ \mathrm{MeV})^2$$

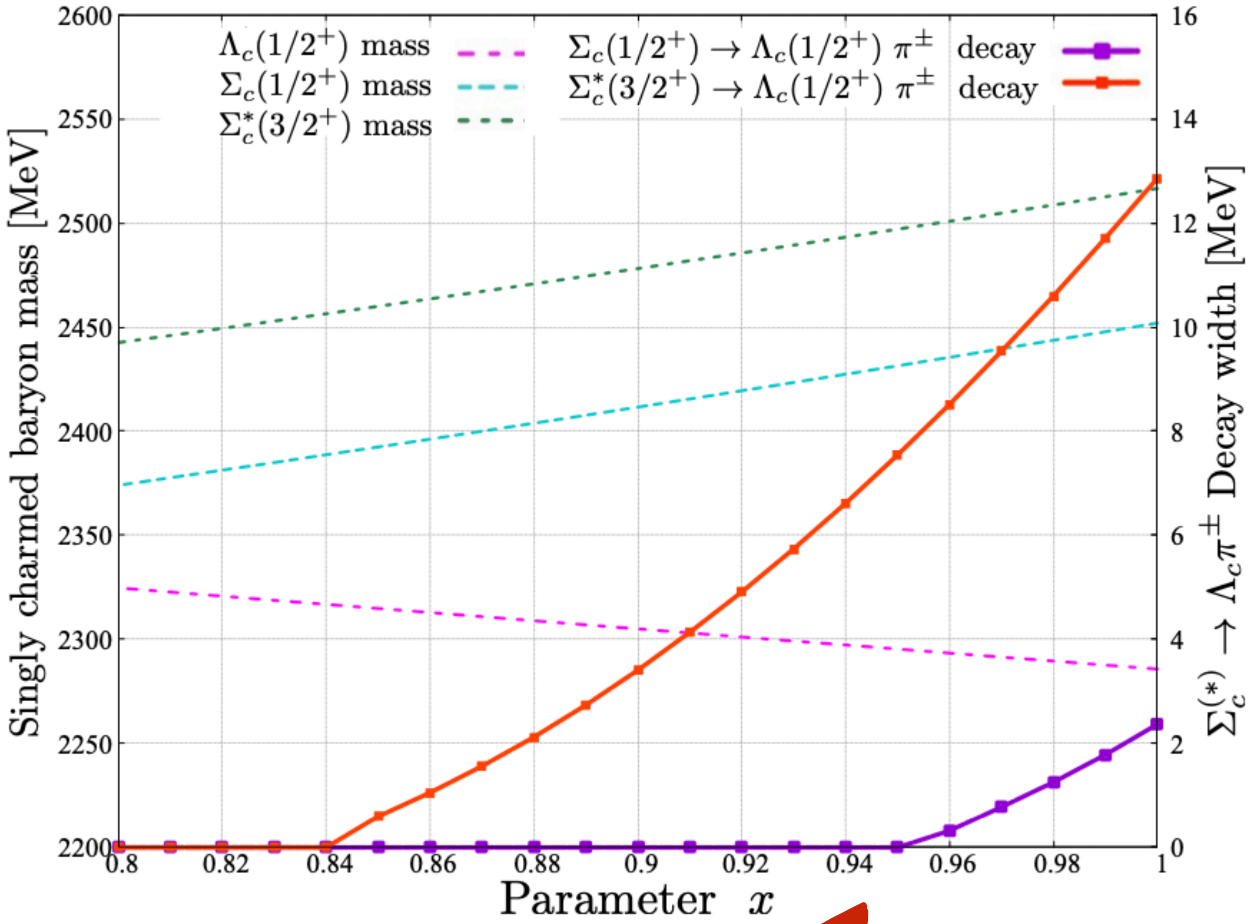
$$m_{V0}^2 = (708\ \mathrm{MeV})^2$$
$$m_{V1}^2 = -(760\ \mathrm{MeV})^2$$
$$m_{V2}^2 = (714\ \mathrm{MeV})^2$$




**Y. Kim, Y.R. Liu, M.O., K. Suzuki, Phys. Rev. D 104, 054012 (2021)**

# Decays of SHB in matter

- **Decays of $\Sigma_c, \Sigma_c^*$ baryons under chiral symmetry restoration**




*Chiral Symmetry Restoration*

# Summary

- Chiral effective theories of Scalar/Pseudoscalar diquarks and Axialvector/Vector diquarks are formulated.
- $U_A(1)$ anomaly is found to give the inverse mass hierarchy in the pseudoscalar diquark spectrum.

  $0^-(ud)$

  $0^-(su)$
- Spectrum of Single Heavy Baryon (SHB) is calculated based on the chiral picture of diquarks. Inverse mass hierarchy appears in ρ-mode excited states, and make the $\Lambda_Q$ and $\Xi_Q$ spectra largely different.
- Effects of $U_A(1)$ anomaly in the strong decays of excited SHB are studied. The GT coupling is found to be suppressed by the anomaly, while the anomalous coupling of $1^+$ and $0^+$ diquarks is suppressed.
- Under chiral restoration, we find the mass crossing of the $1^+$ and $0^+$ diquarks, which may give significant effects on the behaviors of SHB in hot/dense matter.

# Summary

**M. Harada, Y.R. Liu, M.O., K. Suzuki, "*Chiral effective theory of diquarks and $U_A(1)$ anomaly*", Phys. Rev. D 101, 054038 (2020)**

**Y. Kim, E. Hiyama, M.O., K. Suzuki, "*Spectrum of singly heavy baryons from a chiral effective theory of diquarks*", Phys. Rev. D 102, 014004 (2020)**

**Y. Kawakami, M. Harada, M.O., K. Suzuki, "*Suppression of decay widths in singly heavy baryons induced by the $U_A(1)$ anomaly*", Phys. Rev. D 102, 114004 (2020)**

**Y. Kim, Y.R. Liu, M.O., K. Suzuki, "*Heavy baryon spectrum with chiral multiplets of scalar and vector diquarks*", Phys. Rev. D 104, 054012 (2021)**

**Y. Kim, M.O., K. Suzuki, "*Doubly heavy tetraquarks in a chiral-diquark picture*", Phys. Rev. D 105, 074021 (2022)**

**Y. Kim, M.O., D. Suenaga, K. Suzuki, "*Strong decays of singly heavy baryons from a chiral effective theory of diquarks*", Phys. Rev. D 107, 074015 (2023)**

**D. Suenaga, M.O., "*Axial anomaly effect to the chiral-partner structure of diquarks at high temperature*", ArXiv:2305.09730**

**H. Takada, D. Suenaga, M. Harada, A. Hosaka, M.O., in preparation**