# flavour anomalies,

# correlations,

# hadronic uncertainties and all that

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# emerging anomalies in the flavour sector

in SM ($SU(3)_c \times SU(2)_L \times U(1)_Y$) the SSB & Yukawa sectors arbitrary → no theory of flavour

→ no insight on

- ✓ neutrino masses
- ✓ CKM and PPMN textures
- ✓ number of families
- ✓ ….
- ✓ fermion mass hierarchy
- ✓ matter-antimatter abundances
- ✓ …
- ✓ SSB
- ✓ Higgs mass stability againts rad corrections
- ✓ …

BSM must exist

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BSM must exist

«sicut verum per prius invenitur
in intellectu quam in rebus»
Thomas de Aquino (1225-1274)

«sometimes the truth is at first found in the mind,
and then in the observations»

## emerging anomalies in the flavour sector

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- ✓ ….
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- ✓ …
- ✓ SSB
- ✓ Higgs mass stability againts rad corrections
- ✓ …

**BSM must exist - is it around the corner?**

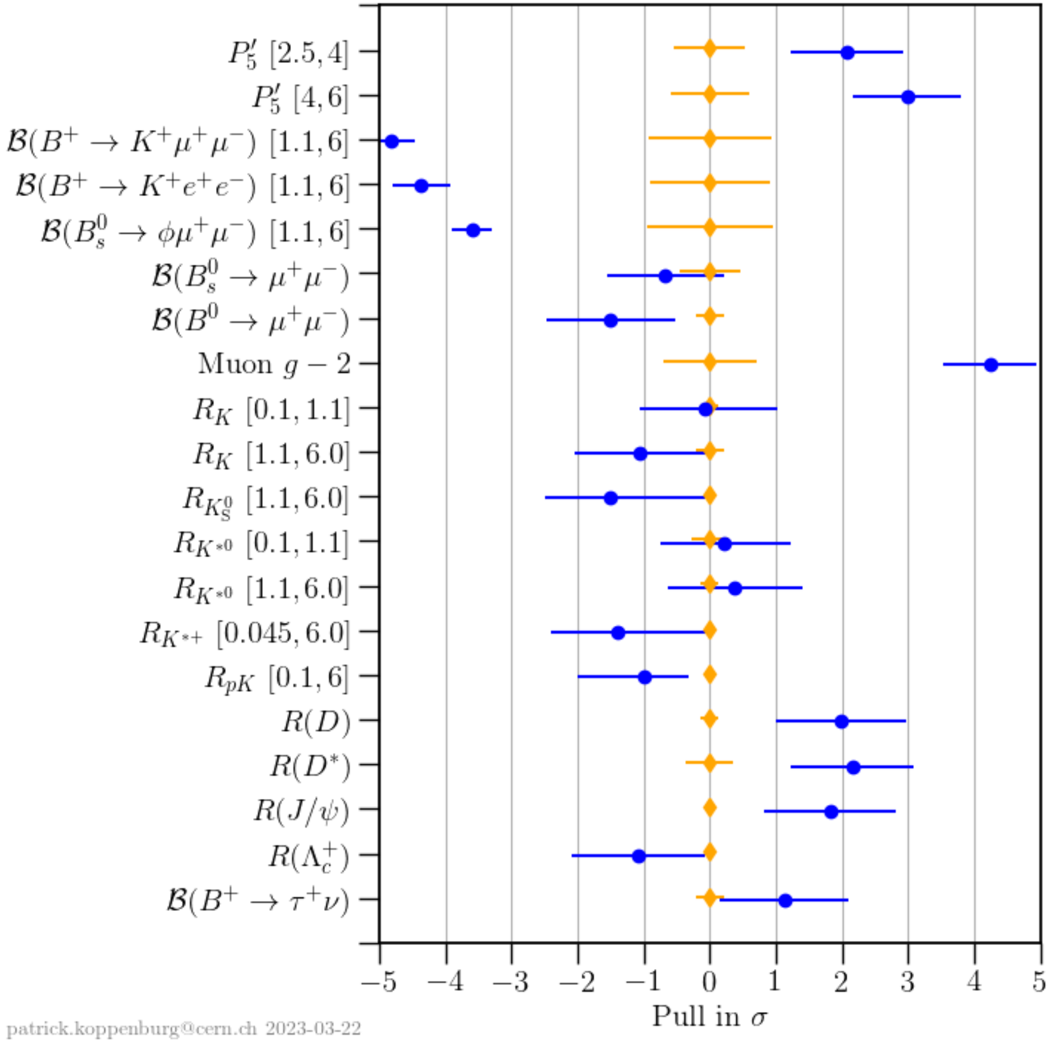
### BSM in low-energy experiments

- deviations of SM allowed processes wrs predictions
  - tree level processes → $R(D^{(*)})$ , …
  - loop-induced processes → $P'_5$ , …
  - inconsistency in CKM, $V_{cb}$, $V_{ub}$, $\varepsilon'/\varepsilon$, $(g-2)_\mu$…
  - precision meas. of SM parameters

    $\sin^2\theta_W$ → Maas Jones Grazzi
- observations of processes forbidden (or strongly suppressed) in SM
  - LFV processes $\tau \to 3\mu$, $\mu \to e\gamma$, $\mu \to 3e$ …

- several tensions in different observables
- hints of LFU violation in the third generation

# emerging anomalies in the flavour sector



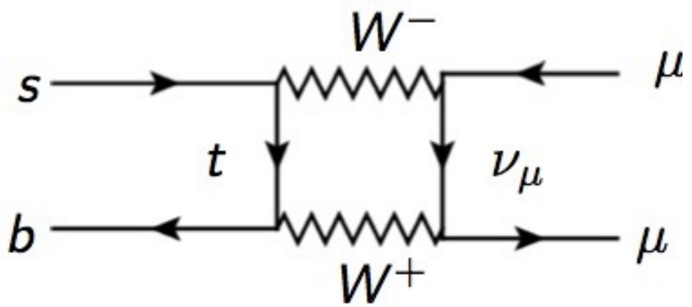
$b \to s\mu^+\mu^-$

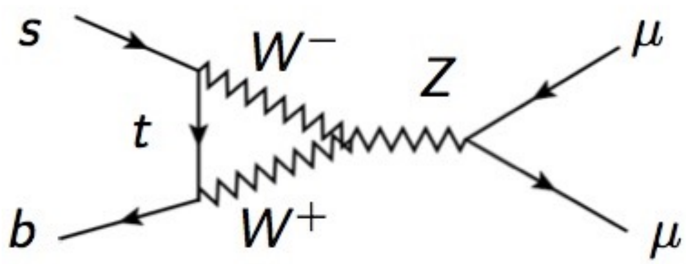
FCNC process suppressed in SM by

- CKM elements
- electroweak scale
- loop-factors

Wilson coefficients precisely known

Bobeth et al PRL (2014) 101801




sensitivity to BSM

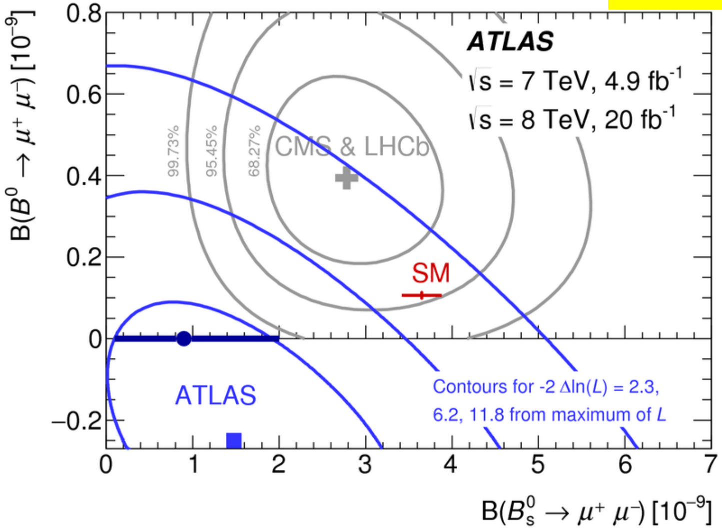
heroic efforts in exp and th

# emerging anomalies in the flavour sector

$B_s \longrightarrow \mu^+\mu^-$

ATLAS-CONF-2020-049
CMS-PAS-BPH-20-003
LHCb-CONF-2020-002

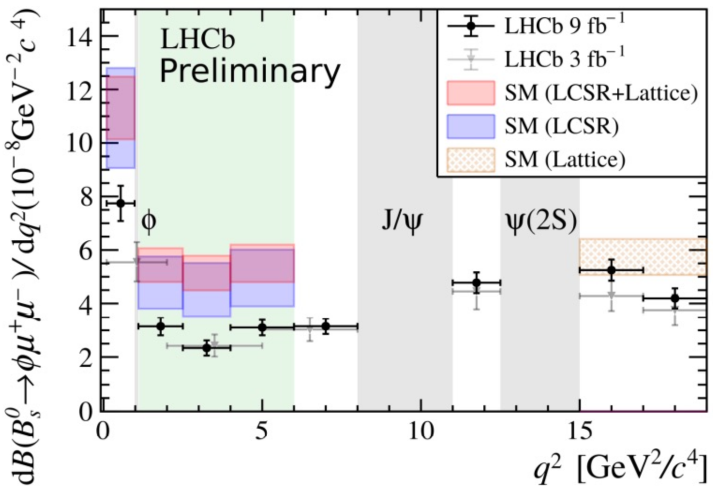
chiral suppression
small statistics




new results → Turkikhin
no significant deviation from SM

$B_s \longrightarrow \phi\mu^+\mu^-$

arXiv:2105.14007




low $q^2$ - tension wrs SM

local & nonlocal form factor uncertainty → Virto

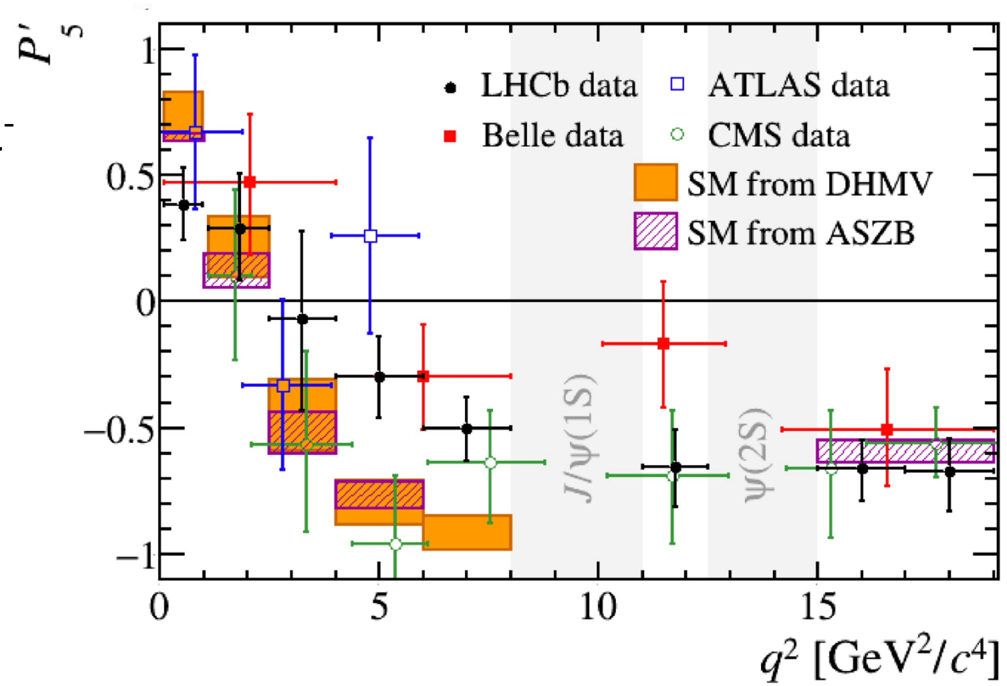
new LHCb measurement of $R_{K(*)} = \Gamma(B \to K^{(*)}\mu^+\mu^-) / \Gamma(B \to K^{(*)}e^+e^-)$ in agreement with SM arXiv:2212.09153.

# $P_5'$ anomaly

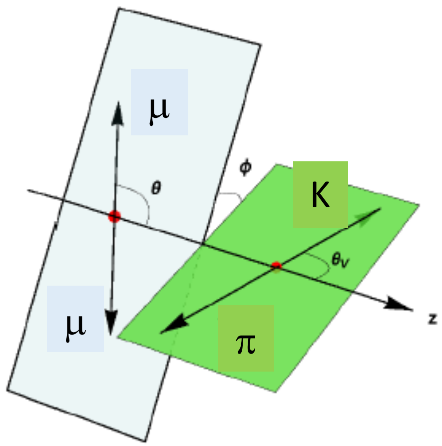
- angular distribution of $B \rightarrow K^*(K\pi)\mu^+\mu^-$
- $P_5'$ angular observable

  minimized form factor uncertainty

  Descotes-Genon Hurth Matias Virto JHEP 05 (2013) 137




LHCb: tension in the charged mode

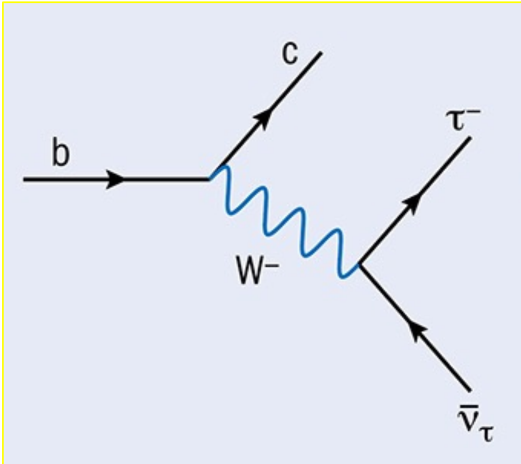



3σ from SM

## b→clν

- tree level
- form factors uncertainty in exclusive modes $B \to D^{(*)} \tau \nu$ ….
- measurement of $|V_{cb}|$ using electrons and $\mu$

→ Benane Penalva

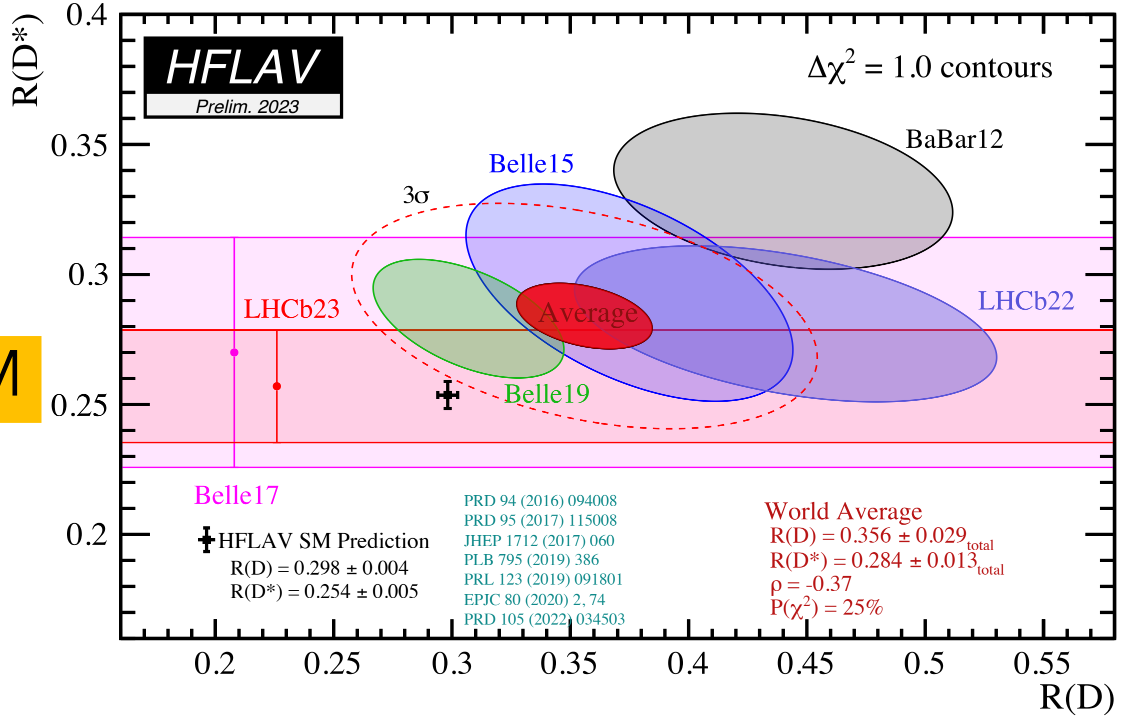


# emerging anomalies in the flavour sector

b→clν

$$R(D^{(*)}) = \frac{\Gamma(B \longrightarrow D^{(*)}\tau\,\nu_\tau)}{\Gamma(B \longrightarrow D^{(*)}\ell\,\nu_\ell)}$$

>3σ from SM




$$R(J/\Psi) = \frac{\Gamma(B_c \longrightarrow J/\Psi\,\tau\,\nu_\tau)}{\Gamma(B_c \longrightarrow J/\Psi\;\ell\,\nu_\ell)}$$

$$R(J/\Psi)_{LHCb} = 0.71 \pm 0.17 \pm 0.18$$
$$R(J/\Psi)_{SM} = 0.25 - 0.28$$

form factors uncertainty

$$R(\Lambda_c) = \frac{\Gamma(\Lambda_b \longrightarrow \Lambda_c\tau\,\nu_\tau)}{\Gamma(\Lambda_b \longrightarrow \Lambda_c\ell\,\nu_\ell)}$$

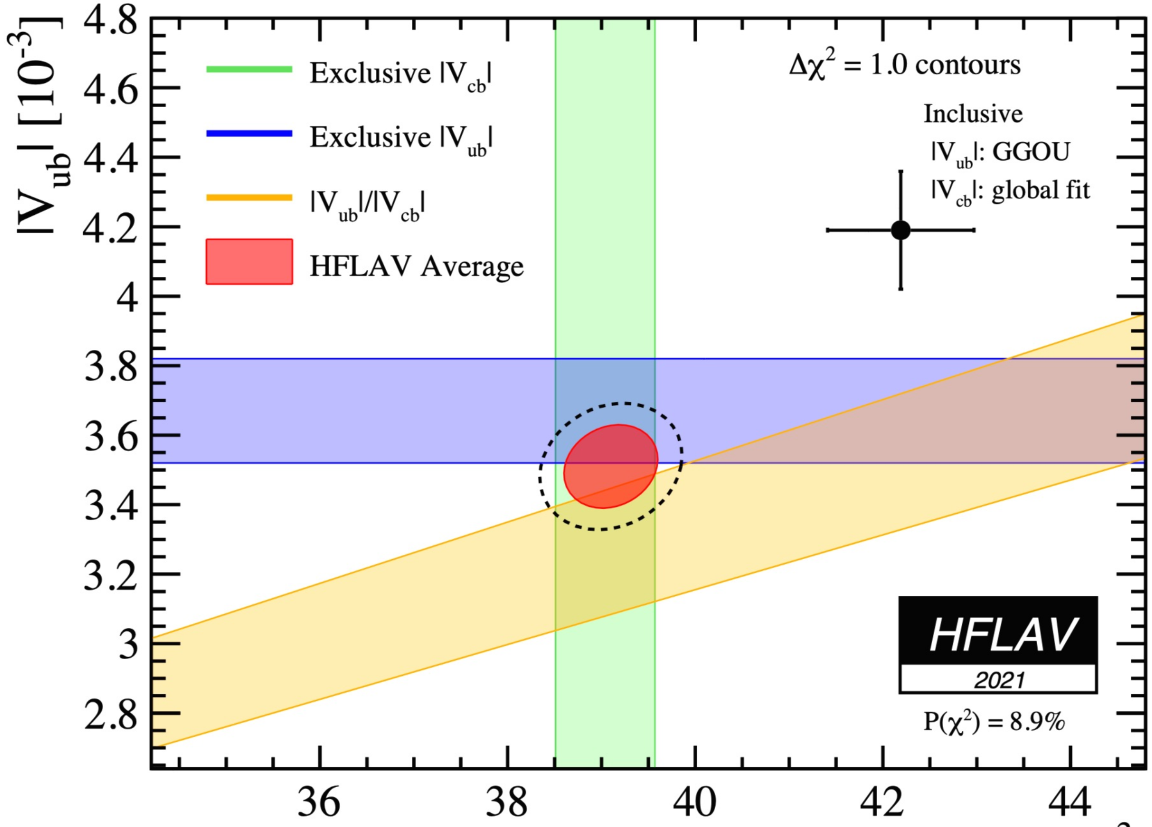
$$R(\Lambda_c)_{LHCb} = 0.242 \pm 0.026 \pm 0.040 \pm 0.059$$
$$R(\Lambda_c)_{SM} = 0.324 \pm 0.004$$

## $|V_{ub}|$ & $|V_{cb}|$

tension in

inclusive vs exclusive

measurements



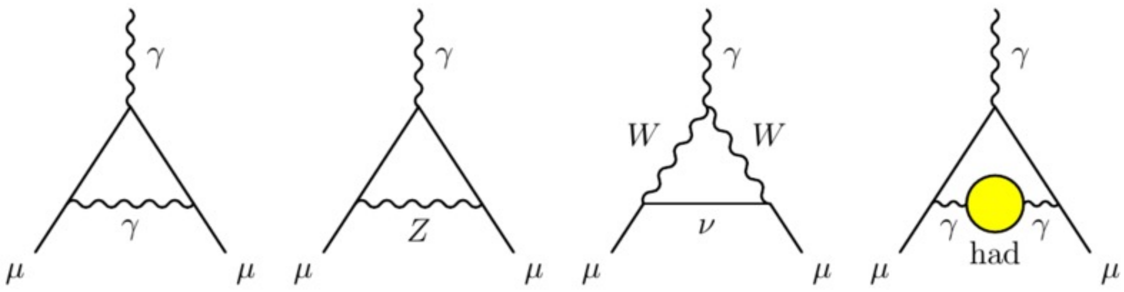

is the solution of the puzzle related to other tensions?

$|V_{cb}|_{incl}$ vs $|V_{cb}|_{excl}$ vs anomalies in $b \rightarrow c$ semileptonic modes De Fazio PC PRD 95 (17) 011701

$R_{Xe/\mu}$ compatible with SM → Merola (BelleII)

# $(g\text{-}2)_\mu$

- uncertainty in the hadronic contribution (HVP & HLbL)
- need NP of the order of the SM EW contribution




$$\Delta a_\mu := a_\mu^{\rm exp} - a_\mu^{\rm SM} = 279(76) \times 10^{-11}$$

**3.7 → 4.2 σ**

White Paper (WP): Aoyama et al. arXiv:2006.04822

Abi et al. arXiv:2104.03281

efforts to improve the evaluation of HVP → Redmer Rojas Bulava Bruno Saccardi

and HLbL → Roig

$a_\mu^{HLbL;P}$ =$(85.1\pm 4.7_{stat} \pm 2.3)$x$10^{-11}$ lattice QCD - Gerardin et al. arXiv:2305.04570

$a_\mu^{HLbL;\pi^0\eta\,\eta'}$ =$(93.8\pm 4)$x$10^{-11}$ WP: mainly data driven dispersive approach

$a_\mu^{HLbL;total}$ =$(92\pm 19)$x$10^{-11}$ WP: P + meson loops, axial vectors , scalars, light quark loops

results also from holographic methods

D'Ambrosio Rebhan Giannuzzi…

**Cabibbo anomaly**

deficit in first row and first column CKM unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985 \pm 0.0005$$

**3 σ**

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 0.9970 \pm 0.0018$$

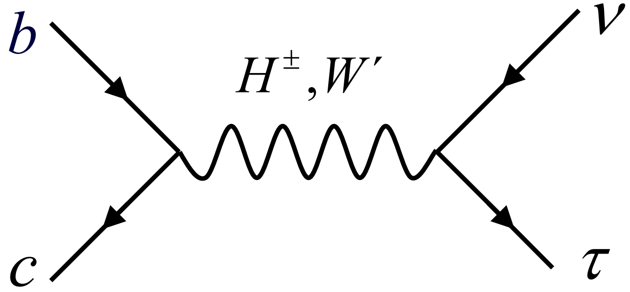
→ Gorshtein

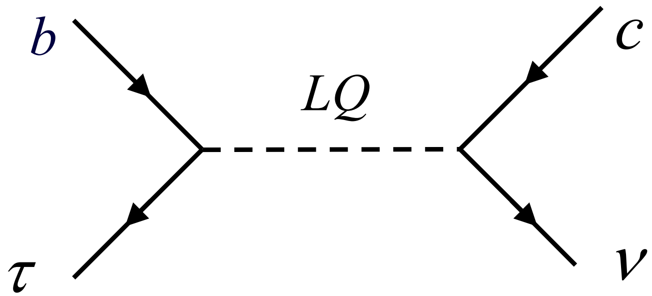
BSM :

- LQ
- W' and W-W' mixing
- Z'
- singly charged scalar
- RH currents in ud and us

# NP scenarios

## $b \to cl\nu$




- charged scalars → troubles with $\tau(B_c)$ and distributions Celis Jung Li Pich PLB 2017, Alonso Grinstein Camalich PRL 2017
- W′ → constrained by LHC searches Buttazzo Greljo Isidori Marzocca JHEP 2017
- Leptoquark (LQ) also searched in qq→ ττ CMS 1809.05558; ATLAS 1902.08103

$$L = y_{ij}\bar{Q}_i S_3 L_j + z_{ij}\bar{Q}_i S_3 Q_j + h.c.$$

$$\frac{1}{\Lambda^2}\left(\bar{c}\gamma^\mu P_L b\right)\left(\tau\gamma^\mu P_L \upsilon\right) + tensor + ..$$

➢ Spin 0, 1 LQ → predicted in GUT/compositeness frameworks coupled to quarks and leptons

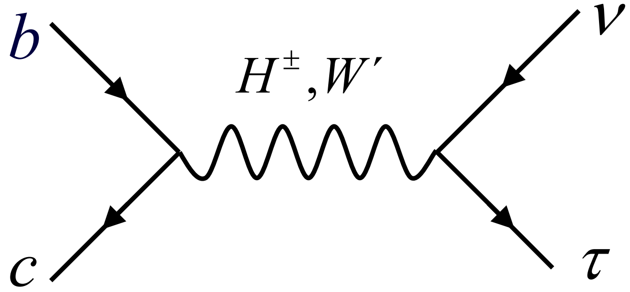
➢ SU(2) singlet vector LQ $U_1$

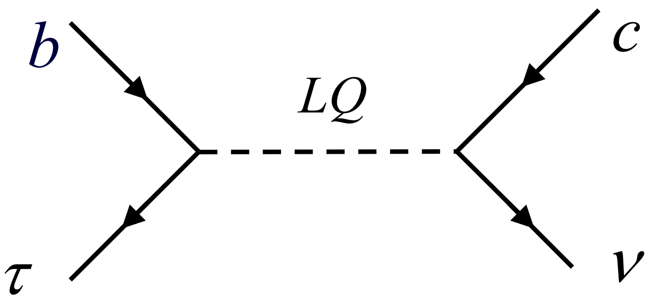
➢ SU(2) triplet scalar LQ $S_3$

Aebisher et al, Alonso et al, Barbieri et al, Calibbi et al, Fajfer et al, Hiller et al, Bhattacharaya et al, Buttazzo et al.,… Kowalska et al, Dorsnee et al, Becirevic et al,…

## NP scenarios

**$b \to c l \nu$**

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**SMEFT approach (model independent)**

- general low energy weak Hamiltonian
- parameter space from measurements → global fits
- effects in new observables → role of the hadronic uncertainties

# going beyond SM

BSM at a high scale $\Lambda >> M_W$ - BSM gauge group G ⊃ $SU(3)_C \times SU(2)_L \times U(1)_Y$

$\Longrightarrow$ SM effective theory at the scale $M_W$

energy scale: $\Lambda$, $M_W$, $\mu_b$

$\Lambda$: $\mathcal{L}_{NP} \xrightarrow{\text{matching}} \mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda} C_{\nu\nu}^{(5)} O_{\nu\nu}^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} O_k^{(6)}$

RGE $\downarrow$

$M_W$: $\mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda} C_{\nu\nu}^{(5)} O_{\nu\nu}^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} O_k^{(6)} \xrightarrow{\text{matching}} H_{eff} = -\frac{4G_F^2}{\sqrt{2}} \sum_k C_k O_k$

RGE $\downarrow$

$\mu_b$: $H_{eff} = -\frac{4G_F^2}{\sqrt{2}} \sum_k C_k O_k$

- coefficients in the low-energy $H_{eff}$ related to the high-scale ones
- different processes related
- basis of effective operators, ordered by dimension, comprising SM fields

Buchmuller and Wyler, NPB 268 (1986) 621
Grzadkowski et al., JHEP 10 (2010) 085

# going beyond SM

BSM at a high scale $\Lambda >> M_W$ - BSM gauge group $G \supset SU(3)_C \times SU(2)_L \times U(1)_Y$

$\Longrightarrow$ SM effective theory at the scale $M_W$

Weinberg opertor: $\nu$ mass

$$\mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda} C_{\nu\nu}^{(5)} O_{\nu\nu}^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} O_k^{(6)}$$

$$\underbrace{\mathcal{L}^{kin} + \mathcal{L}^{gauge} + \mathcal{L}^{Higgs}}_{\text{accidental symmetries}} + \mathcal{L}^{Yukawa}$$

D=6

- violates accidental symmetries
- fermion masses and mixings
- source of CP violation

$b \to c\ell\nu_\ell$

SM low-energy Hamiltonian + D=6 operators and LH neutrinos

U=c,u

$$H_{\rm eff}^{b\to U\ell\nu} = \frac{G_F}{\sqrt{2}} V_{Ub} \Big[ \, (1+\epsilon_V^\ell)\, \big(\bar{U}\gamma_\mu(1-\gamma_5)b\big)\, \big(\bar{\ell}\gamma^\mu(1-\gamma_5)\nu_\ell\big) + \epsilon_S^\ell\, (\bar{U}b)\, \big(\bar{\ell}(1-\gamma_5)\nu_\ell\big) + \epsilon_P^\ell\, \big(\bar{U}\gamma_5 b\big)\, \big(\bar{\ell}(1-\gamma_5)\nu_\ell\big) + \epsilon_T^\ell\, \big(\bar{U}\sigma_{\mu\nu}(1-\gamma_5)b\big)\, \big(\bar{\ell}\sigma^{\mu\nu}(1-\gamma_5)\nu_\ell\big) + \epsilon_R^\ell\, \big(\bar{U}\gamma_\mu(1+\gamma_5)b\big)\, \big(\bar{\ell}\gamma^\mu(1-\gamma_5)\nu_\ell\big) \Big] + h.c. \; .$$

$\varepsilon_i^l$ lepton flavour dependent contrained by measurements

$\varepsilon_i^{l.} \to 0 \to$ SM

# semileptonic b decays

$b \to c\ell\nu_\ell$

- correlations between meson (B, $B_s$, $B_c$) and baryon ($\Lambda_b$, $\Xi_b$, $\Omega_b$) observables

- effects in inclusive and exclusive modes
- different hadronic uncertainties to deal with

two examples:

- inclusive $\Lambda_b$

- exclusive $B_c$

$b \to c\ell\nu_\ell$

## inclusive $H_b$ modes: optical theorem + heavy quark expansion (HQE)

Chay Georgi Grinstein, PLB 247 (1990) 399, Bigi Shifman Uraltsev Vainshtein PRL 71 (1993) 496

## observables as a double expansion in $1/m_Q$ and $\alpha_s(m_Q)$

b → u: De Fazio Neubert 9905351

$$H_{\rm eff}^{b\to U\ell\nu} = \frac{G_F}{\sqrt{2}} V_{Ub} \sum_{i=1}^{5} C_i^\ell \, J_M^{(i)} \, L^{(i)M} + h.c.$$

leptonic currents

hadronic currents

$$C_1^\ell = (1+\epsilon_V^\ell)$$

$$C_{2,3,4,5}^\ell = \epsilon_{S,P,T,R}^\ell$$

SM: $\epsilon^\ell_{V,S,P,T,R} = 0$

$$J_\mu^{(1)} = \overline{U}\gamma_\mu(1-\gamma_5)b$$

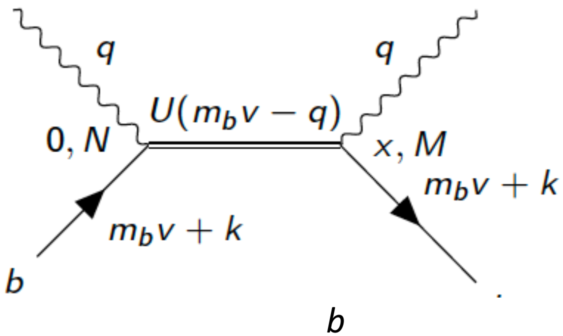
$$L^{\mu(1)} = \overline{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell$$

$$d\Gamma = d\Sigma \, \frac{G_F^2 |V_{Ub}|^2}{4 m_H} \sum_{i,j} C_i^* C_j \underbrace{(W^{ij})_{MN}}_{\text{hadronic tensor}} (L^{ij})^{MN}$$

$$(W^{ij})_{MN} = \frac{1}{\pi} {\rm Im}(T^{ij})_{MN}$$

$$(T^{ij})_{MN} = i \int d^4x \, e^{i\,(m_b v - q)\cdot x} \langle H_b(v,s)| T[\hat{J}_M^{(i)\dagger}(x) \, \hat{J}_N^{(j)}(0)] |H_b(v,s)\rangle$$

OPE



# b-baryon inclusive semileptonic decays

$$\mathcal{O}\left(\frac{1}{m_b^n}\right)\dots\begin{cases}\mathcal{O}\left(\frac{1}{m_b^3}\right)\begin{cases}\mathcal{O}\left(\frac{1}{m_b^2}\right)\begin{cases}-2\,M_H\,\hat{\mu}_\pi^2 = \langle H_b|\bar{b}_v\, iD^\mu\, iD_\mu\, b_v|H_b\rangle\\ 2\,M_H\,\hat{\mu}_G^2 = \langle H_b|\bar{b}_v\,(-i\sigma_{\mu\nu})\, iD^\mu\, iD^\nu\, b_v|H_b\rangle\end{cases}\\ 2\,M_H\,\hat{\rho}_D^3 = \langle H_b|\bar{b}_v\, iD^\mu\,(iv\cdot D)\, iD_\mu\, b_v|H_b\rangle\\ 2\,M_H\,\hat{\rho}_{LS}^3 = \langle H_b|\bar{b}_v\,(-i\sigma_{\mu\nu})\, iD^\mu\,(iv\cdot D)\, iD^\nu\, b_v|H_b\rangle\end{cases}\\ \dots\end{cases}$$

$\hat{\mu}_\pi^2$ matrix element of the kinetic energy operator

$$\boxed{\mu_\pi^2(B)-\mu_\pi^2(\Lambda_b)=\frac{2m_b m_c}{m_b-m_c}\left[(m_{\Lambda_b}-m_{\Lambda_c})-(\overline{m}_B-\overline{m}_D)\right]\left(1+\mathcal{O}(1/m_{b,c}^2)\right)}$$

$$\boxed{\hat{\mu}_\pi^2(\Lambda_b)=(0.50\pm 0.1)\,\mathrm{GeV}^2}$$

Dominguez Nardulli Paver PC
PRD 54 (96) 4622

$\hat{\mu}_G^2$ matrix element of the chromomagnetic operator $\boxed{\hat{\mu}_G^2(\Lambda_b)=0}$

$\hat{\rho}_D^3$ Darwin term $\boxed{\rho_D^3(\Lambda_b)\simeq\rho_D^3(B)}$ $\boxed{\rho_D^3(\Lambda_b)\ =\ (0.17\pm 0.08)\,\mathrm{GeV}^3}$

$\hat{\rho}_{LS}^3$ spin-orbit term $\boxed{\hat{\rho}_{LS}^3(\Lambda_b)=0}$

$$\boxed{\mathcal{M}_{\mu_1\dots\mu_n}=\langle H_b(v,s)|(\bar{b}_v)_a(iD_{\mu_1})\dots(iD_{\mu_n})(b_v)_b|H_b(v,s)\rangle}$$

general parametrization
including dependence on the spin to $O(1/m_b^3)$

## fully differential decay width

$$\frac{d^4\Gamma}{dE_\ell\, dq^2\, dq_0\, d\cos\theta_P}$$

$p_l=(E_l, \mathbf{p}_l)$ charged lepton
$q=(q_0, \mathbf{q})$
$\theta_P$ angle between the hadron spin **s** and **p** $_l$

O($1/m_b^3$) with all NP operators and $m_l \neq 0$ De Fazio Loparco PC JHEP 11 (2020) 032

$$\Gamma(H_b \to X_U \ell^- \bar{\nu}_\ell) = \Gamma_0 \sum_{i,j} g_i^* g_j \left[ \mathcal{C}_0^{(i,j)} + \frac{\hat{\mu}_\pi^2}{m_b^2} \mathcal{C}_{\hat{\mu}_\pi^2}^{(i,j)} + \frac{\hat{\mu}_G^2}{m_b^2} \mathcal{C}_{\hat{\mu}_G^2}^{(i,j)} + \frac{\hat{\rho}_D^3}{m_b^3} \mathcal{C}_{\hat{\rho}_D^3}^{(i,j)} + \frac{\hat{\rho}_{LS}^3}{m_b^3} \mathcal{C}_{\hat{\rho}_{LS}^3}^{(i,j)} \right]$$

$$\Gamma_0 = \frac{G_F^2 |V_{Ub}|^2 m_b^5}{192\pi^3}$$

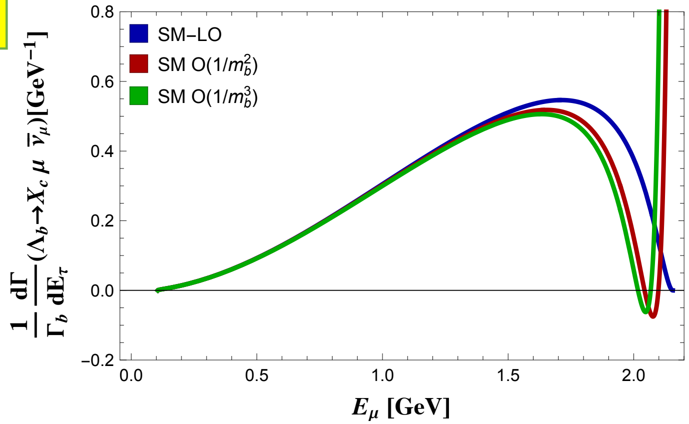
NP effects through the ε couplings

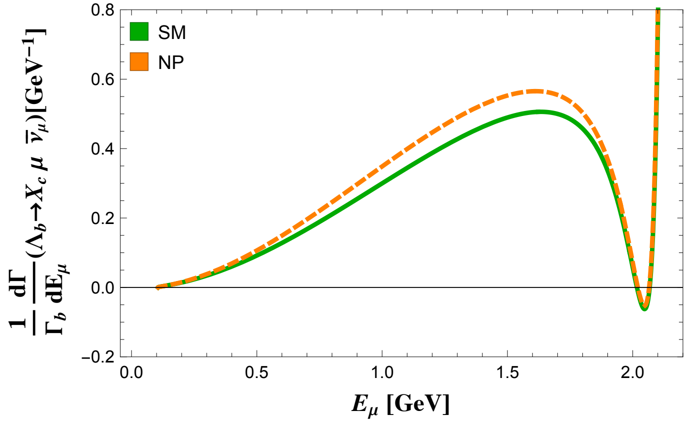
perturbative QCD corrections in SM
$O((\alpha_s/\pi)^3)$ for $\Gamma_0$ Fael Schonwald Steinhauser, 2011.13654 2005.06487 2011.11655
$O(\alpha_s/\pi)$ for $\Gamma_\pi$ Alberti Gambino Nandi 1311.7381
Mannel Pivovarov Rosenthal 1405.5072 Mannel Pivovarov 1907.09187
Capdevilla Gambino Nandi 2102.03343

ongoing new measurements for B → $|V_{cb}|_{incl}$ $|V_{ub}|_{incl}$

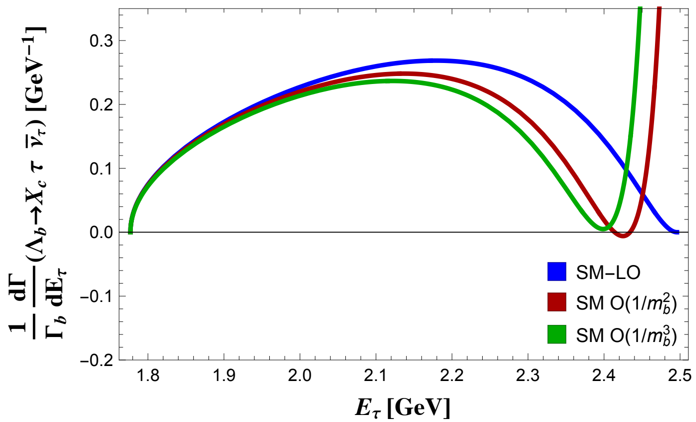
# $\Lambda_b$ inclusive semileptonic decays

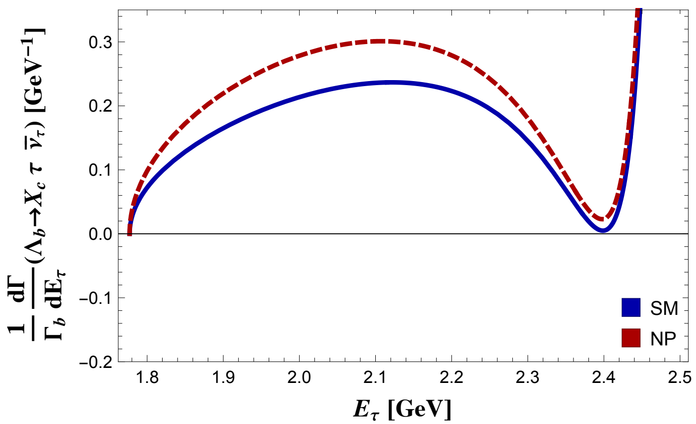
$\Lambda_b \longrightarrow X_c\ l\ \nu_l$




$\Lambda_b \longrightarrow X_c\ \tau\ \nu_\tau$

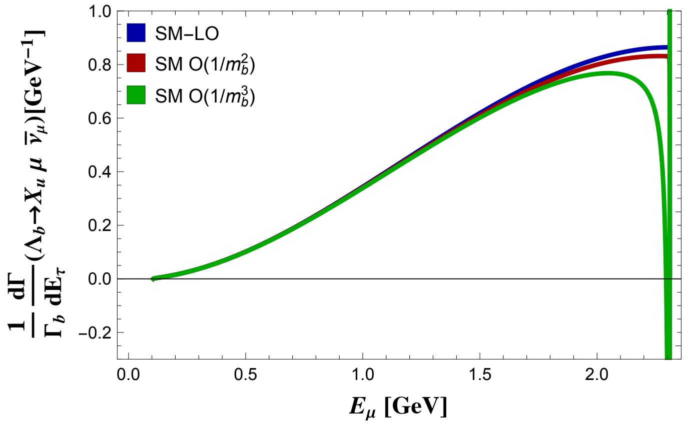


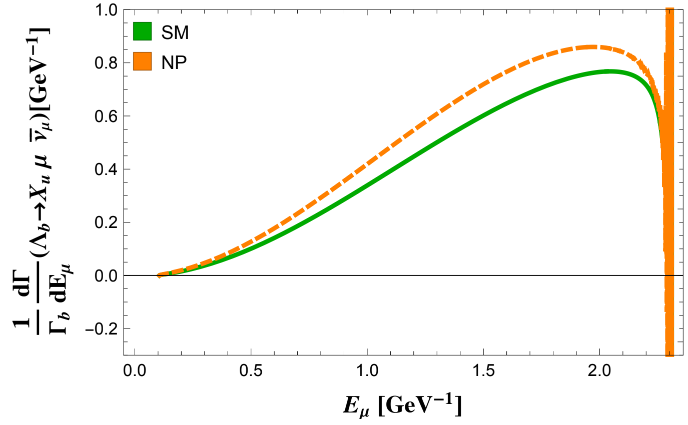




NP benchmark point: $b \to c$ De Fazio PC JHEP 06 (18) 082, Shi Geng Grinstein Jager Camalich JHEP 12 (19) 065
$b \to u$ De Fazio Loparco PC PRD 100 (19) 075037

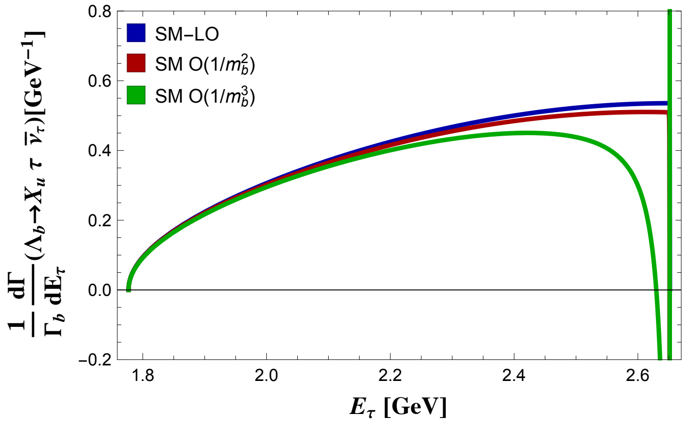
# b-baryon inclusive semileptonic decays

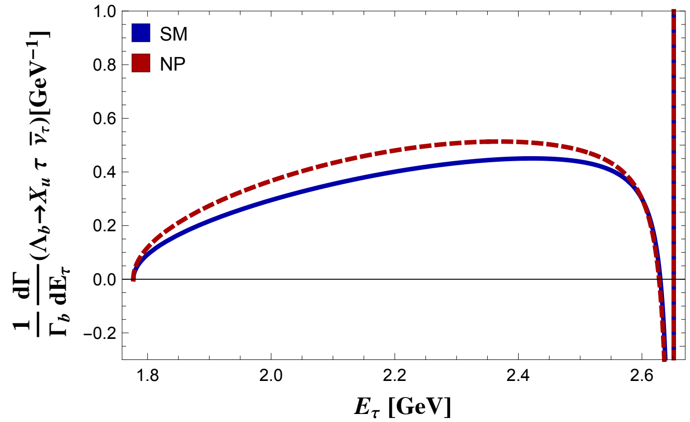
$\Lambda_b \to X_u\, l\, \nu_l$




$\Lambda_b \to X_u\, \tau\, \nu_\tau$

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observables sensitive to $\Lambda_b$ polarization and BSM contributions
longitudinal polarization measured for $\Lambda_b$ from b quark produced in $Z^0$ decays (LEP)

De Fazio Loparco PC
JHEP 11 (2020) 032

$$\boxed{\frac{d\Gamma(\Lambda_b \to X_U \ell \bar{\nu}_\ell)}{d\cos\theta_P} = A_\ell^U + B_\ell^U \cos\theta_P}$$

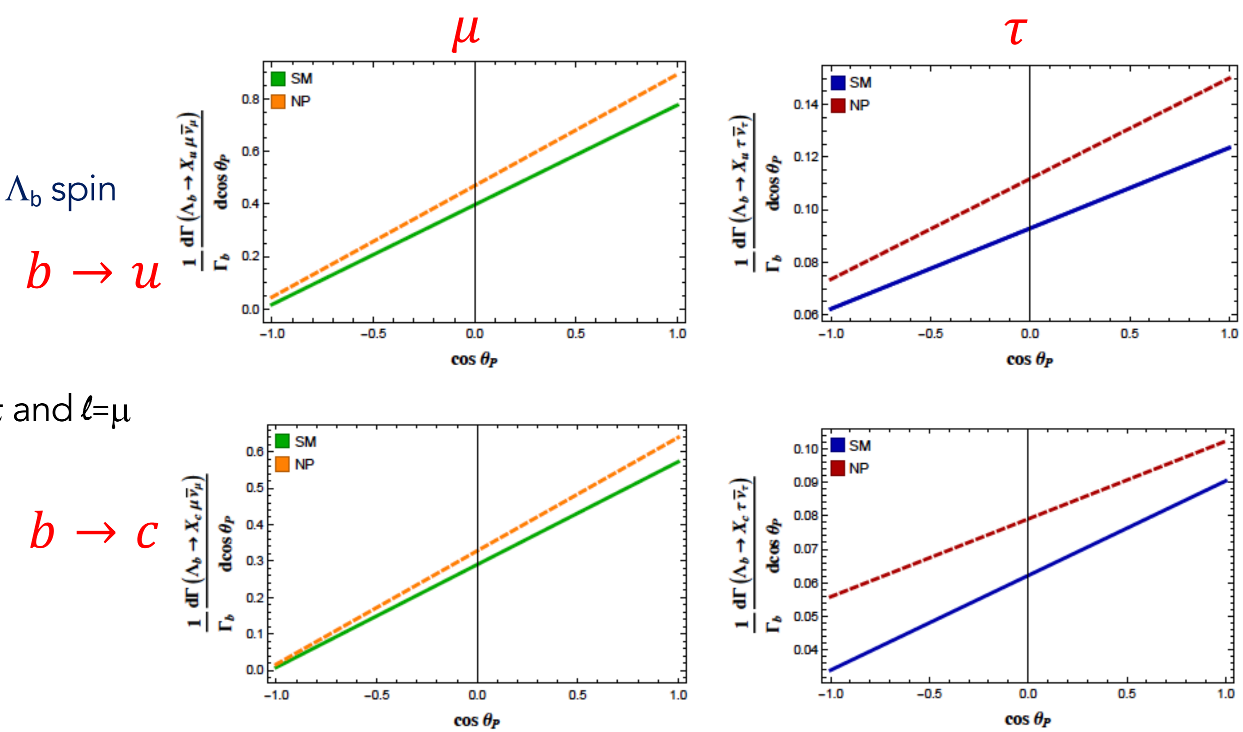
angle between
$\ell$ direction and $\Lambda_b$ spin

$$\boxed{R_{\Lambda_b}(X_U) = \frac{A_\tau^U}{A_\mu^U}}$$

analogous to $R(D^{(*)})$
no polarization

$$\boxed{R_S^U = \frac{B_\tau^U}{B_\mu^U}}$$

slope ratios for $\ell=\tau$ and $\ell=\mu$



# b-baryon inclusive semileptonic decays

observables sensitive to $\Lambda_b$ polarization and BSM contributions
polarization expected for $\Lambda_b$ from b quark from top or $Z^0$ decays

De Fazio Loparco PC
JHEP 11 (2020) 032

$$\frac{d\Gamma(\Lambda_b \to X_U \ell \bar{\nu}_\ell)}{d\cos\theta_P} = A_\ell^U + B_\ell^U \cos\theta_P$$

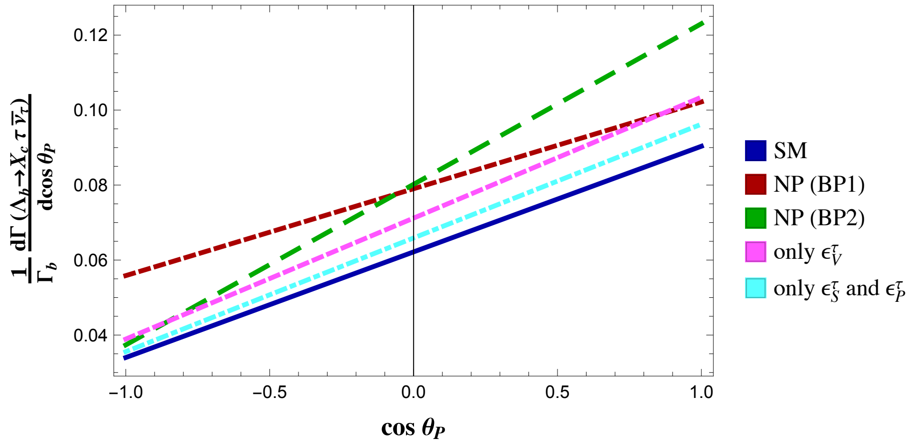
angle between
$\ell$ direction and $\Lambda_b$ spin

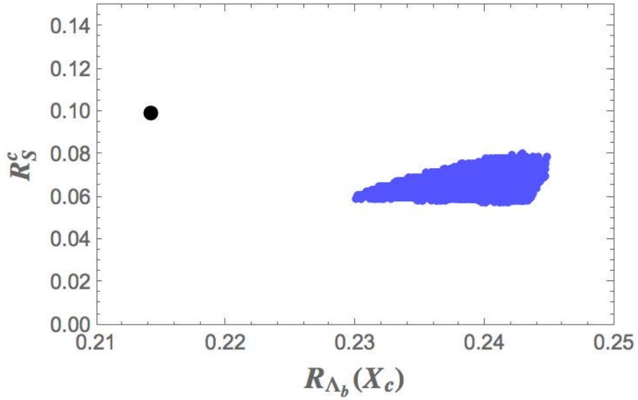
$$R_{\Lambda_b}(X_U) = \frac{A_\tau^U}{A_\mu^U}$$

$$R_S^U = \frac{B_\tau^U}{B_\mu^U}$$

slope ratios for $\ell=\tau$ and $\ell=\mu$

$$R_{\Lambda_b}(X_u)^{SM} = 0.234$$
$$R_{\Lambda_b}(X_c)^{SM} = 0.214$$




difficult to measure
$\Lambda_b$ unpolarized at LHC
accessible with polarized $\Lambda_b$ from Z or top decays

physics programs of future lepton colliders

$b \to c \ell \nu_\ell$

$B_c$ quarkonium-like state with weak decays → Mastrapasqua Belov

semileptonic modes

- $B_c \to \eta_c$ , $J/\psi$ — 1S charmonium $J^{PC}=(0^{--},1^{--})$
- $B_c \to \chi_{c0}, \chi_{c1}, \chi_{c2}, h_c$ — 1P — $J^{PC}=(0^{++}, 1^{++}, 2^{++}, 1^{+-})$
- $B_c \to \chi_{c0}$ (2P), $\chi_{c1}$ (2P), $\chi_{c2}$ (2P), $h_c$ (2P) — 2P — $J^{PC}=(0^{++}, 1^{++}, 2^{++}, 1^{+-})$

1. scrutinize BSM effects
2. probe the structure of the produced state
   → nature of X(3872) → Spadaro Novella Guo Polosa Tanida Mitchell Gershon Giannuzzi Nerling Pelizaus ….

$\chi_{c1}(3872)$ vs $\chi_{c1}(2P)$

# $B_c \to \chi_{ci}$

NRQCD & spin symmetry

doublet of negative parity states

$(B_c, B_c^*) \longrightarrow$ $\boxed{\mathcal{M}(v) = P_+(v)\,[B_c^{*\mu}\,\gamma_\mu - B_c\,\gamma_5]\,P_-(v)}$

$(\eta_c,\ J/\psi) \longrightarrow$ $\boxed{\mathcal{M}'(v') = P_+(v')\Big[\Psi^{*\mu}\gamma_\mu - \eta_c\,\gamma_5\Big]P_-(v')}$

4-plet of positive parity states

$(\chi_{c0,1,2}, h_c) \longrightarrow$ $\boxed{\mathcal{M}'^{\mu}(v') = P_+(v')\Big[\chi_{c2}^{\mu\nu}\,\gamma_\nu + \frac{1}{\sqrt{2}}\chi_{c1,\gamma}\,\epsilon^{\mu\alpha\beta\gamma}\,v'_\alpha\,\gamma_\beta + \frac{1}{\sqrt{3}}\chi_{c0}\,(\gamma^\mu - v'^\mu) + h_c^\mu\,\gamma_5\Big]P_-(v')}$

universal functions: the same for all members of the multiplet of final states
relations among the various modes

LO

$$\langle M'(v')|J_0|M(v)\rangle = -\Xi(w)\,v_\mu\,\mathrm{Tr}\big[\overline{\mathcal{M}'}^{\mu}\,\Gamma\,\mathcal{M}\big]$$

a single universal function

$O(1/m_Q)$

$$\langle M'(v')|\bar\psi'_+\,\Gamma\,i\,\overrightarrow{D}_\alpha\,\psi_+|M(v)\rangle = -\mathrm{Tr}\big[\Sigma^{(b)}_{\mu\alpha}\,\overline{\mathcal{M}'}^{\mu}\,\Gamma\,\mathcal{M}\big]$$

$$\langle M'(v')|\bar\psi'_+\,(-i\,\overleftarrow{D}_\alpha)\,\Gamma\,\psi_+|M(v)\rangle = -\mathrm{Tr}\big[\Sigma^{(c)}_{\mu\alpha}\,\overline{\mathcal{M}'}^{\mu}\,\Gamma\,\mathcal{M}\big]$$

$O(1/m_Q)^2$

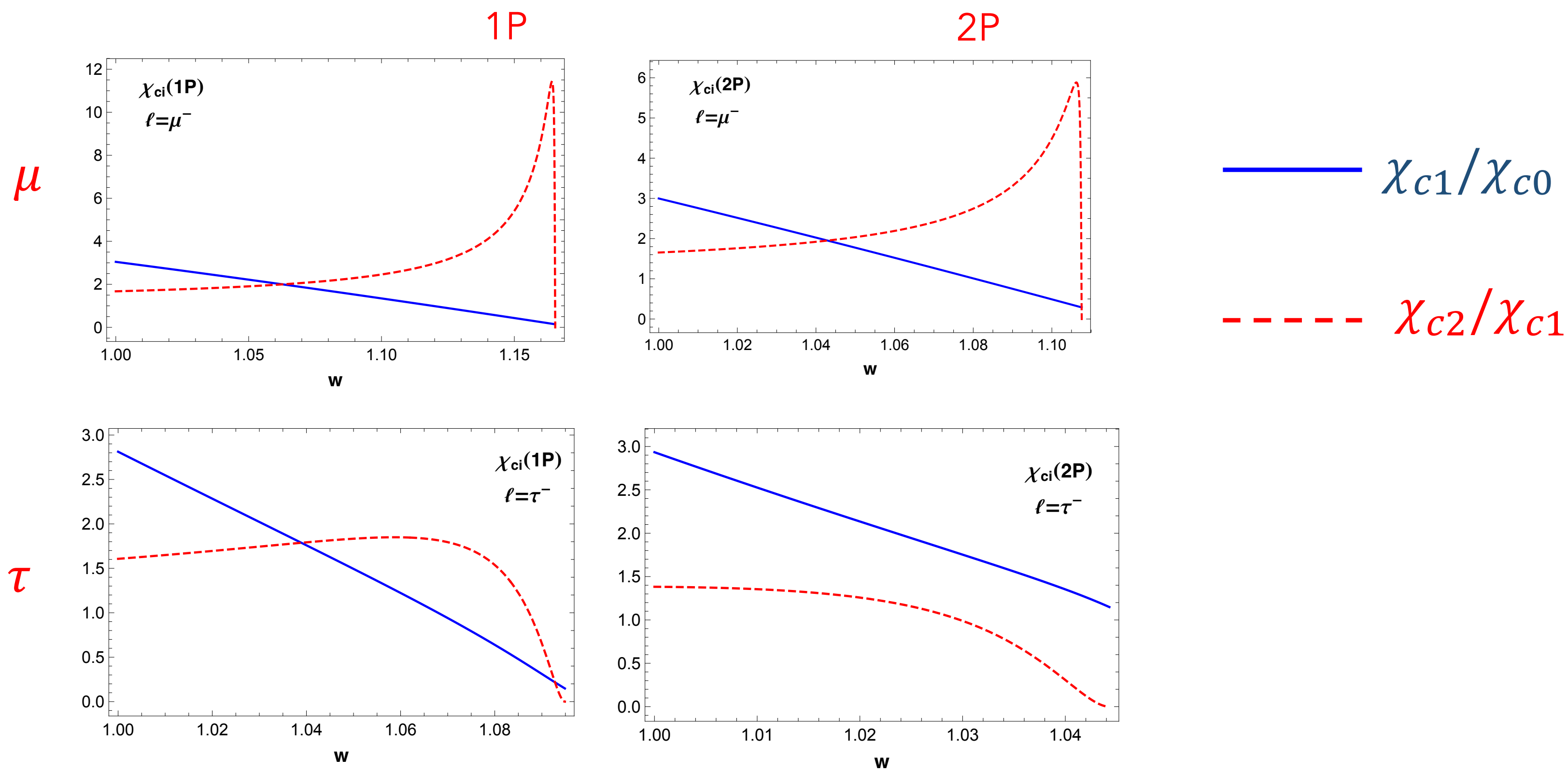
$$\langle M'(v')|\bar\psi'_+\,\Gamma\,i\,\overrightarrow{D}_\alpha\,i\,\overrightarrow{D}_\beta\,\psi_+|M(v)\rangle = -\mathrm{Tr}\big[\Omega^{(b)}_{\mu\alpha\beta}\,\overline{\mathcal{M}'}^{\mu}\,\Gamma\,\mathcal{M}\big]$$

$$\langle M'(v')|\bar\psi'_+\,i\,\overleftarrow{D}_\alpha\,i\,\overleftarrow{D}_\beta\,\Gamma\,\psi_+|M(v)\rangle = -\mathrm{Tr}\big[\Omega^{(c)}_{\mu\alpha\beta}\,\overline{\mathcal{M}'}^{\mu}\,\Gamma\,\mathcal{M}\big]$$

# $B_c \to (\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c)$ with FF relations at LO

$$\frac{d\Gamma(B_c\to\chi_{c1}\ell\bar{\nu})/dw}{d\Gamma(B_c\to\chi_{c0}\ell\bar{\nu})/dw} \qquad \frac{d\Gamma(B_c\to\chi_{c2}\ell\bar{\nu})/dw}{d\Gamma(B_c\to\chi_{c1}\ell\bar{\nu})/dw}$$

→ cancellations of ff in ratios



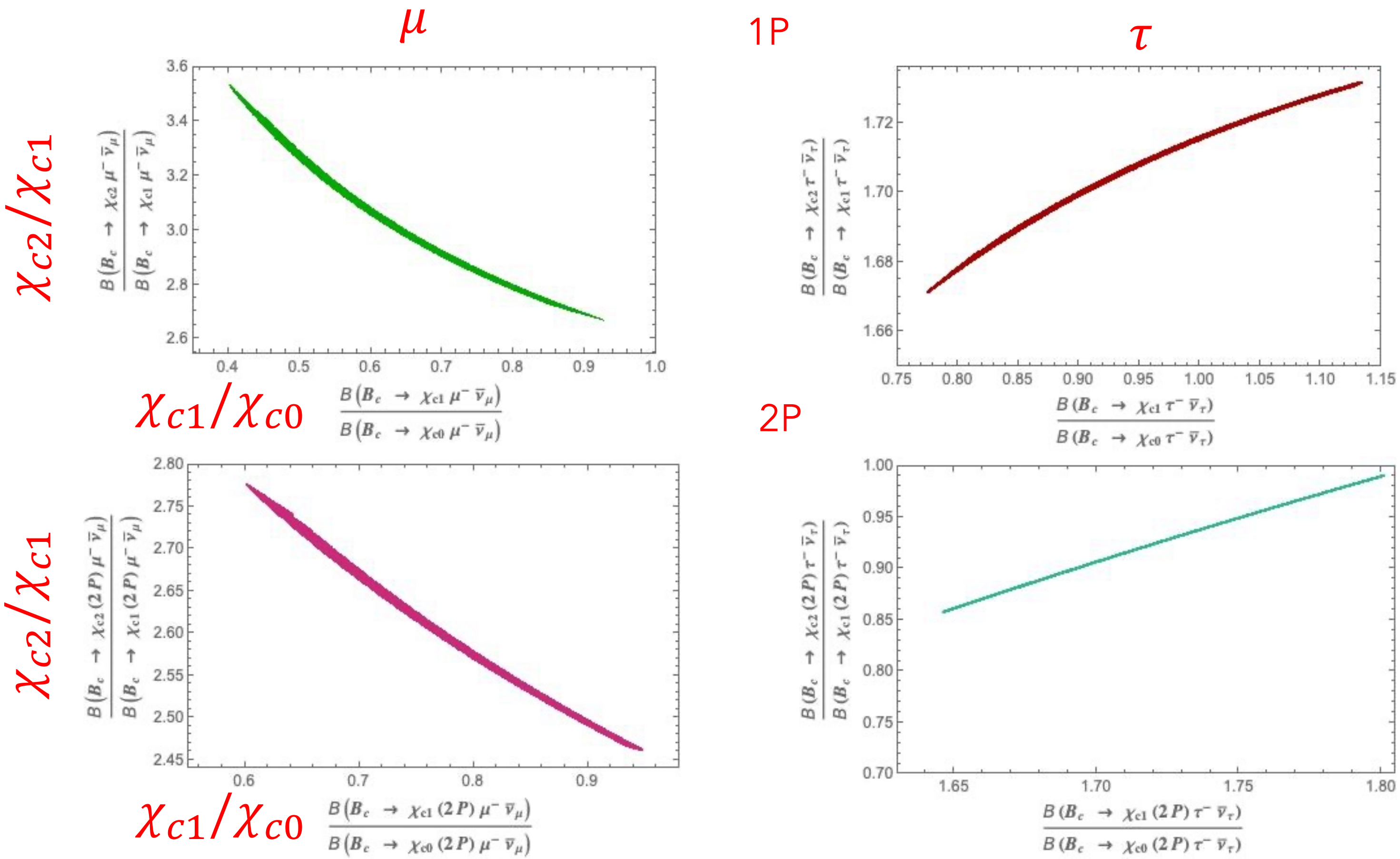

$$\boxed{2\frac{d\Gamma}{dw}(B_c\to\chi_{c0}\ell\bar{\nu}_\ell)+\frac{d\Gamma}{dw}(B_c\to\chi_{c1}\ell\bar{\nu}_\ell)-\frac{d\Gamma}{dw}(B_c\to\chi_{c2}\ell\bar{\nu}_\ell)=0.}$$

satisfied by three members of the 4-plet

# $B_c \to (\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c)$ with FF relations at LO

$$\mathcal{B}(B_c^+ \to \chi_{c0}\pi^+) = (2.4\pm^{0.9}_{0.8}) \times 10^{-5}$$

correlations



# $B_c \to (\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c)$ with FF relations at LO

$$\mathcal{B}(B_c^+ \to \chi_{c0}\pi^+) = (2.4\pm^{0.9}_{0.8}) \times 10^{-5}$$

naive factorization $\mathcal{B}(B_c^+ \to \chi_{c1}\pi^+)\sim 0$

| | $\frac{\mathcal{B}(B_c^+\to\chi_{c0}\,\pi^+)}{\mathcal{B}(B_c^+\to\chi_{c2}\,\pi^+)}$ | $\frac{\mathcal{B}(B_c^+\to h_c\,\pi^+)}{\mathcal{B}(B_c^+\to\chi_{c0}\,\pi^+)}$ | $\frac{\mathcal{B}(B_c^+\to h_c\,\pi^+)}{\mathcal{B}(B_c^+\to\chi_{c2}\,\pi^+)}$ |
|---|---|---|---|
| $1P$ | 0.658 | 2.429 | 1.597 |
| $2P$ | 0.583 | 2.746 | 1.601 |

| | $\frac{\mathcal{B}(B_c^+\to\chi_{c0}\,K^+)}{\mathcal{B}(B_c^+\to\chi_{c2}\,K^+)}$ | $\frac{\mathcal{B}(B_c^+\to h_c\,K^+)}{\mathcal{B}(B_c^+\to\chi_{c0}\,K^+)}$ | $\frac{\mathcal{B}(B_c^+\to h_c\,K^+)}{\mathcal{B}(B_c^+\to\chi_{c2}\,K^+)}$ |
|---|---|---|---|
| $1P$ | 0.663 | 2.482 | 1.645 |
| $2P$ | 0.586 | 2.845 | 1.668 |

| | $\frac{\mathcal{B}(B_c^+\to\chi_{c1}\rho^+)}{\mathcal{B}(B_c^+\to\chi_{c0}\rho^+)}$ | $\frac{\mathcal{B}(B_c^+\to\chi_{c1}\rho^+)}{\mathcal{B}(B_c^+\to\chi_{c2}\rho^+)}$ | $\frac{\mathcal{B}(B_c^+\to\chi_{c0}\rho^+)}{\mathcal{B}(B_c^+\to\chi_{c2}\rho^+)}$ |
|---|---|---|---|
| $1P$ | 0.206 | 0.122 | 0.590 |
| $2P$ | 0.315 | 0.159 | 0.503 |
| | $\frac{\mathcal{B}(B_c^+\to h_c\rho^+)}{\mathcal{B}(B_c^+\to\chi_{c0}\rho^+)}$ | $\frac{\mathcal{B}(B_c^+\to h_c\rho^+)}{\mathcal{B}(B_c^+\to\chi_{c1}\rho^+)}$ | $\frac{\mathcal{B}(B_c^+\to h_c\rho^+)}{\mathcal{B}(B_c^+\to\chi_{c2}\rho^+)}$ |
| $1P$ | 2.226 | 10.790 | 1.312 |
| $2P$ | 2.449 | 7.770 | 1.232 |

| | $\frac{\mathcal{B}(B_c^+\to\chi_{c1}K^{*+})}{\mathcal{B}(B_c^+\to\chi_{c0}K^{*+})}$ | $\frac{\mathcal{B}(B_c^+\to\chi_{c1}K^{*+})}{\mathcal{B}(B_c^+\to\chi_{c2}K^{*+})}$ | $\frac{\mathcal{B}(B_c^+\to\chi_{c0}K^{*+})}{\mathcal{B}(B_c^+\to\chi_{c2}K^{*+})}$ |
|---|---|---|---|
| $1P$ | 0.276 | 0.157 | 0.570 |
| $2P$ | 0.422 | 0.203 | 0.481 |
| | $\frac{\mathcal{B}(B_c^+\to h_cK^{*+})}{\mathcal{B}(B_c^+\to\chi_{c0}K^{*+})}$ | $\frac{\mathcal{B}(B_c^+\to h_cK^{*+})}{\mathcal{B}(B_c^+\to\chi_{c1}K^{*+})}$ | $\frac{\mathcal{B}(B_c^+\to h_cK^{*+})}{\mathcal{B}(B_c^+\to\chi_{c2}K^{*+})}$ |
| $1P$ | 2.159 | 7.834 | 1.231 |
| $2P$ | 2.350 | 5.568 | 1.131 |

Losacco
arXiv:2302.12534

# BSM realization : 331 model → correlations between FCNC decays in c and b, s sectors

$SU(3)_c \times SU(3)_L \times U(1)_X$ $\longrightarrow$ $N_{generations} = N_{colors}$

Frampton Pisano Valle…

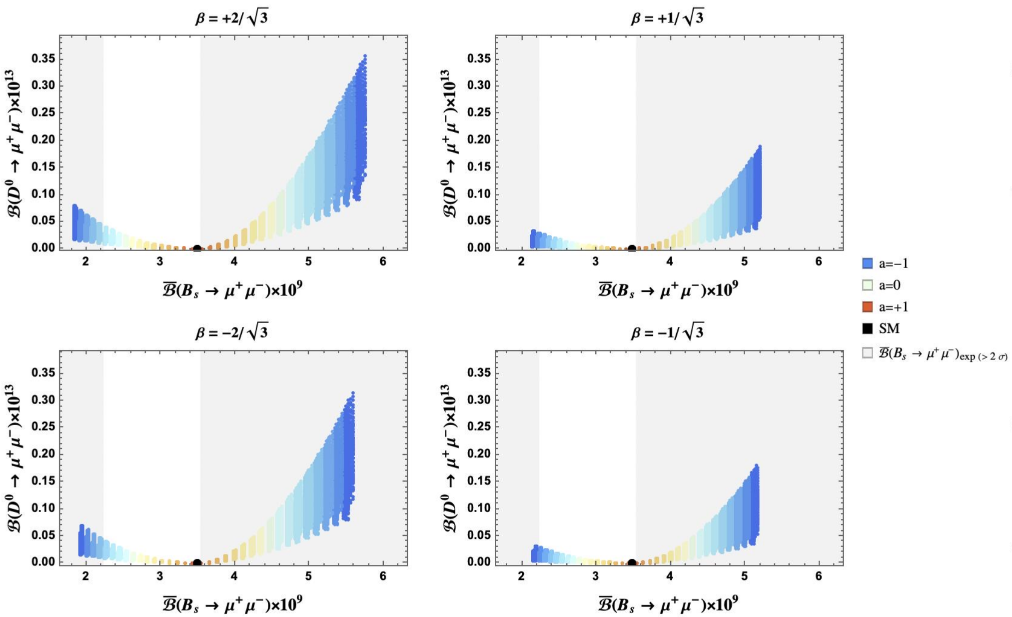
correlations between rare FCNC transitions in the up and down quark sectors

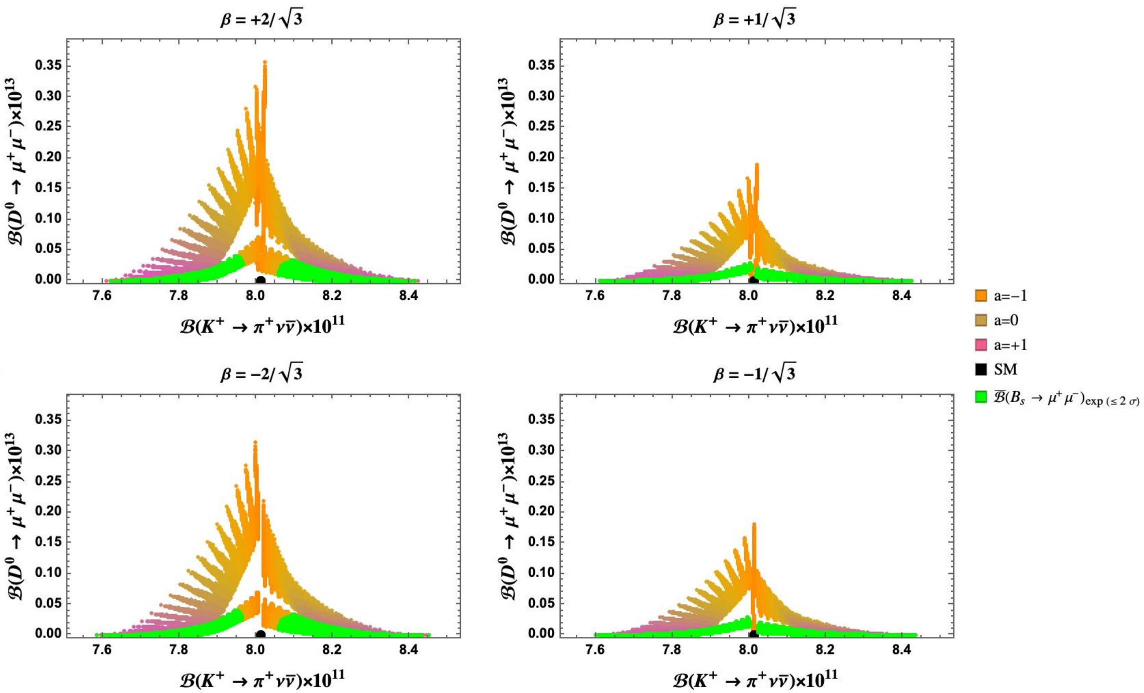
Buras De Fazio Loparco PC JHEP 10 (2021) 021
De Fazio Loparco PC PRD 104 (2021) 115024

$D^0 \to \mu^+\mu^-$ vs $B_s \to \mu^+\mu^-$

$D^0 \to \mu^+\mu^-$ vs $K^+ \to \pi^+\nu\,\bar{\nu}$

discussions in parallel session «hadron production and decays»



# summary & perspectives

unprecedented situation:
we do not know which significant deviation wrs SM will be found at first

in spite of this

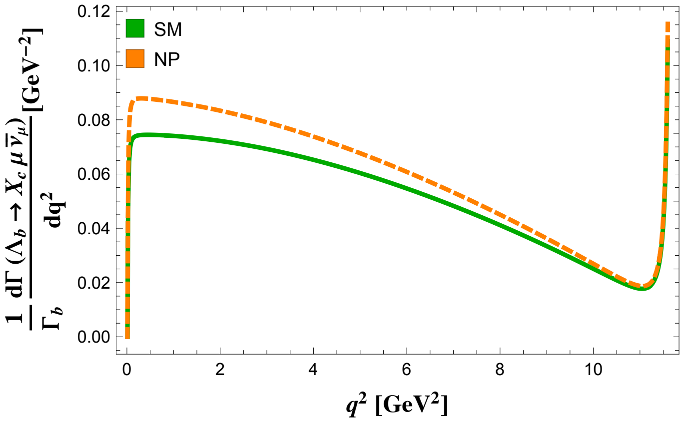
- hints to BSM from various measurements of low energy observables
- SMEFT $\rightarrow$ correlations among observables
- effective QCD theories useful to deal with hadronic quantities/uncertainties
- predictions for low energy observables in particular BSM realizations
- input for the physics programs of next facilities

exciting time in th and exp ahead of us

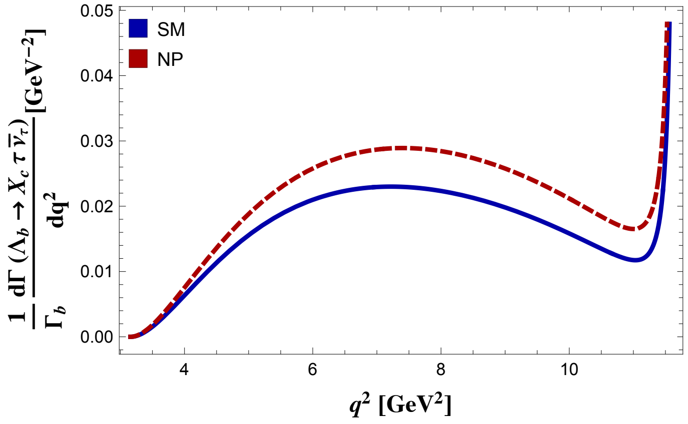
# spares

# $\Lambda_b$ inclusive semileptonic decays

$\Lambda_b \longrightarrow X_c\, l\, \nu_l$




$\Lambda_b \longrightarrow X_c\, \tau\, \nu_\tau$

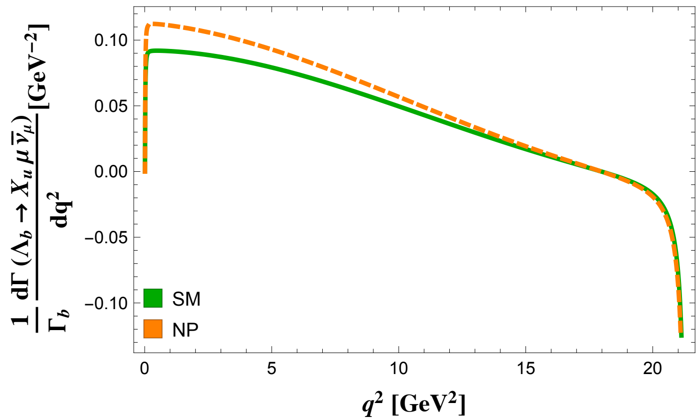



NP benchmark points: $b \to u$ De Fazio Loparco PC PRD 100 (19) 075037

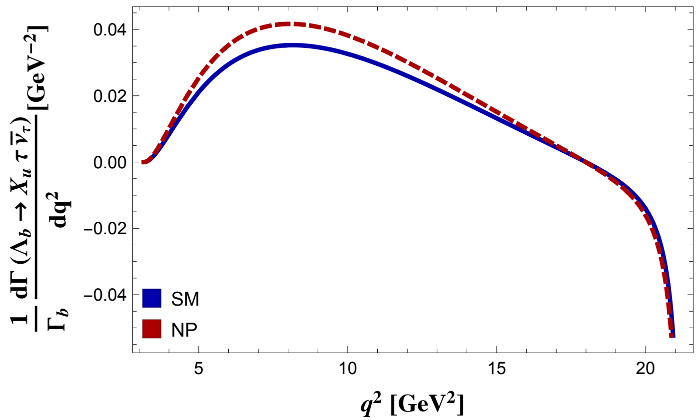
$b \to c$ De Fazio PC JHEP 06 (18) 082, Shi Geng Grinstein Jager Camalich JHEP 12 (19) 065

# $\Lambda_b$ inclusive semileptonic decays

$\Lambda_b \to X_u \, l \, \nu_l$




$\Lambda_b \to X_u \, \tau \, \nu_\tau$




NP benchmark points: b → u  De Fazio Loparco PC PRD 100 (19) 075037
b → c  De Fazio PC JHEP 06 (18) 082, Shi Geng Grinstein Jager Camalich JHEP 12 (19) 065

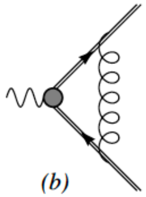
HQ limit: decoupling of the HQ

heavy-light mesons $\rightarrow$ HQ spin & flavour symmetry

heavy-heavy mesons $\rightarrow$ HQ spin symmetry

**relations among the FF in selected kinematical ranges**

heavy-heavy meson decays




IR divergent for two HQs with same v

Thacker Lepage PRD43 (1991) 196

- Infrared divergences regulated in the HQ limit by the kinetic energy operator $O_p$
- $O_p$ breaks flavour symmetry ⇢ only spin symmetry

non relativistic quarks with relative velocity **v**

NRQCD Lagrangian: expansion in $1/m_Q$

terms further organized: expansion in powers of **v**

Caswell Lepage PLB 167 (86) 437

Bodwin Braaten Lepage PRD51 (95) 1125

# EFT

- expansion parameters for a system with 2 heavy quarks: relative HQ velocity (hadron rest-frame) (NRQCD)
  inverse HQ mass $1/m_Q$ (HQET)

- HQ field:

$$Q(x) = e^{-im_Q v\cdot x}\psi(x) = e^{-im_Q v\cdot x}\Big(\psi_+(x) + \psi_-(x)\Big) \qquad \psi_\pm(x) = P_\pm\psi(x) = \frac{1\pm \not{v}}{2}\psi(x)$$

$$\Rightarrow \quad Q(x) = e^{-im_Q v\cdot x}\left(1 + \frac{i\not{D}_\perp}{2m_Q} + \frac{(-iv\cdot D)}{2m_Q}\frac{i\not{D}_\perp}{2m_Q} + \dots\right)\psi_+(x) \qquad D_{\perp\mu} = D_\mu - (v\cdot D)v_\mu$$

$$\Rightarrow \quad \boxed{\mathcal{L}_{QCD} = \bar{\psi}_+(x)\left(\underbrace{iv\cdot D + \frac{(iD_\perp)^2}{2m_Q}}_{\mathcal{O}(\tilde{v}^2)\ \text{LO}} + \underbrace{\frac{g}{4m_Q}\sigma\cdot G_\perp + \frac{i\not{D}_\perp}{2m_Q}\frac{(-iv\cdot D)}{2m_Q}(i\not{D}_\perp)}_{\mathcal{O}(\tilde{v}^4)\ \text{NLO}} + \dots\right)\psi_+(x)}$$

$$\boxed{\mathcal{L}_0 = \bar{\psi}_+(x)\left(iv\cdot D + \frac{(iD_\perp)^2}{2m_Q}\right)\psi_+(x)} \qquad \boxed{\mathcal{L}_1 = \mathcal{L}_{1,1} + \mathcal{L}_{1,2}}$$

power counting in NRQCD

Lepage et al.PRD46 (92) 4052

$$\psi_+ \sim \tilde{v}^{3/2}$$

$$D_\perp \sim \tilde{v} \qquad D_t \sim \tilde{v}^2$$

$$E_i = G_{0i} \sim \tilde{v}^3 \qquad B_i = \frac{1}{2}\epsilon_{ijk}G^{jk} \sim \tilde{v}^4$$

# $B_c \to (\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c)$ with FF relations at LO

tests of LFU

$$R(C) = \frac{\Gamma(B_c \to C\tau\bar{\nu}_\tau)}{\Gamma(B_c \to C\mu\bar{\nu}_\mu)}$$