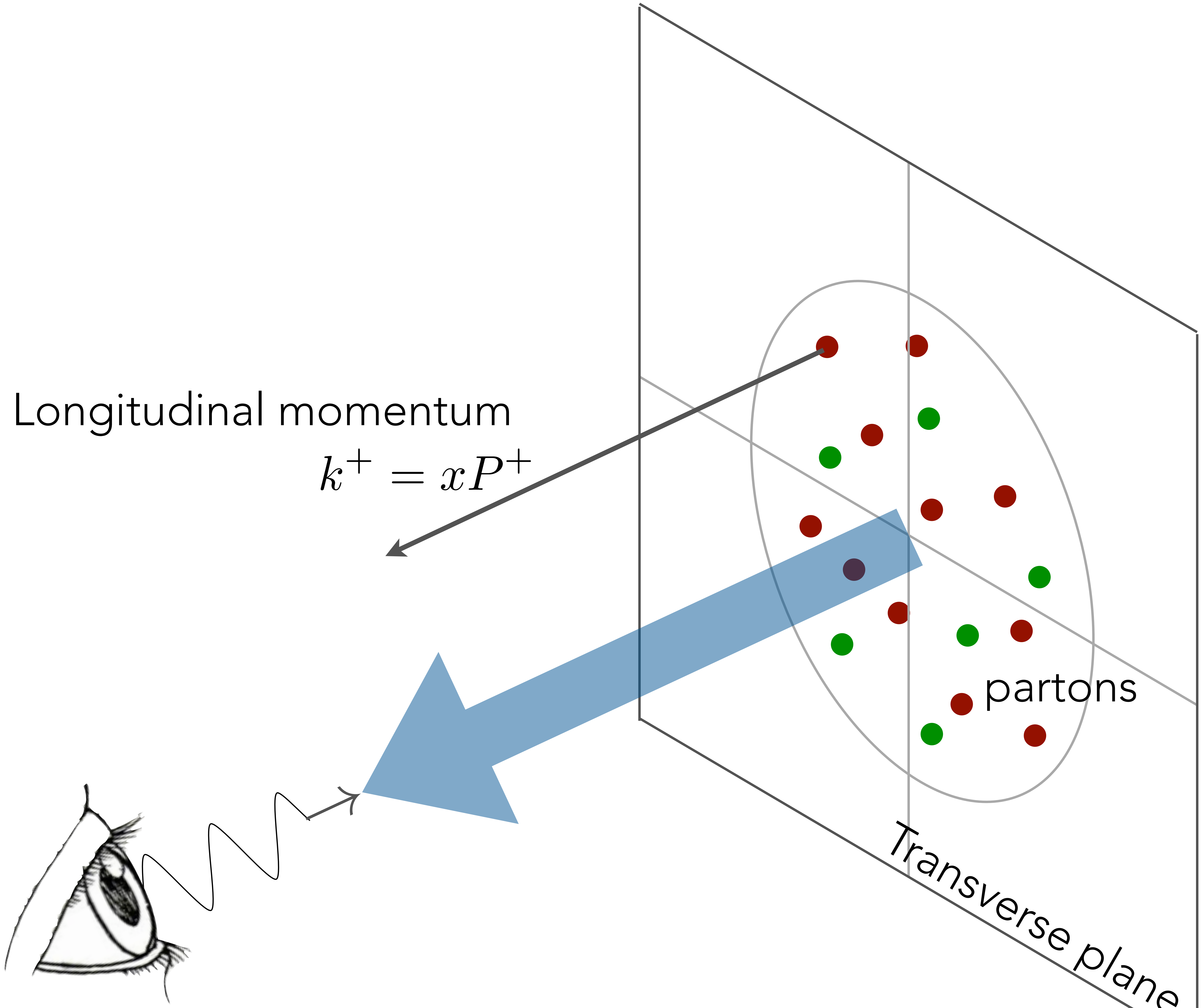
ALESSANDRO BACCHETTA, PAVIA U. AND INFN

PRESENT KNOWLEDGE OF TMDS

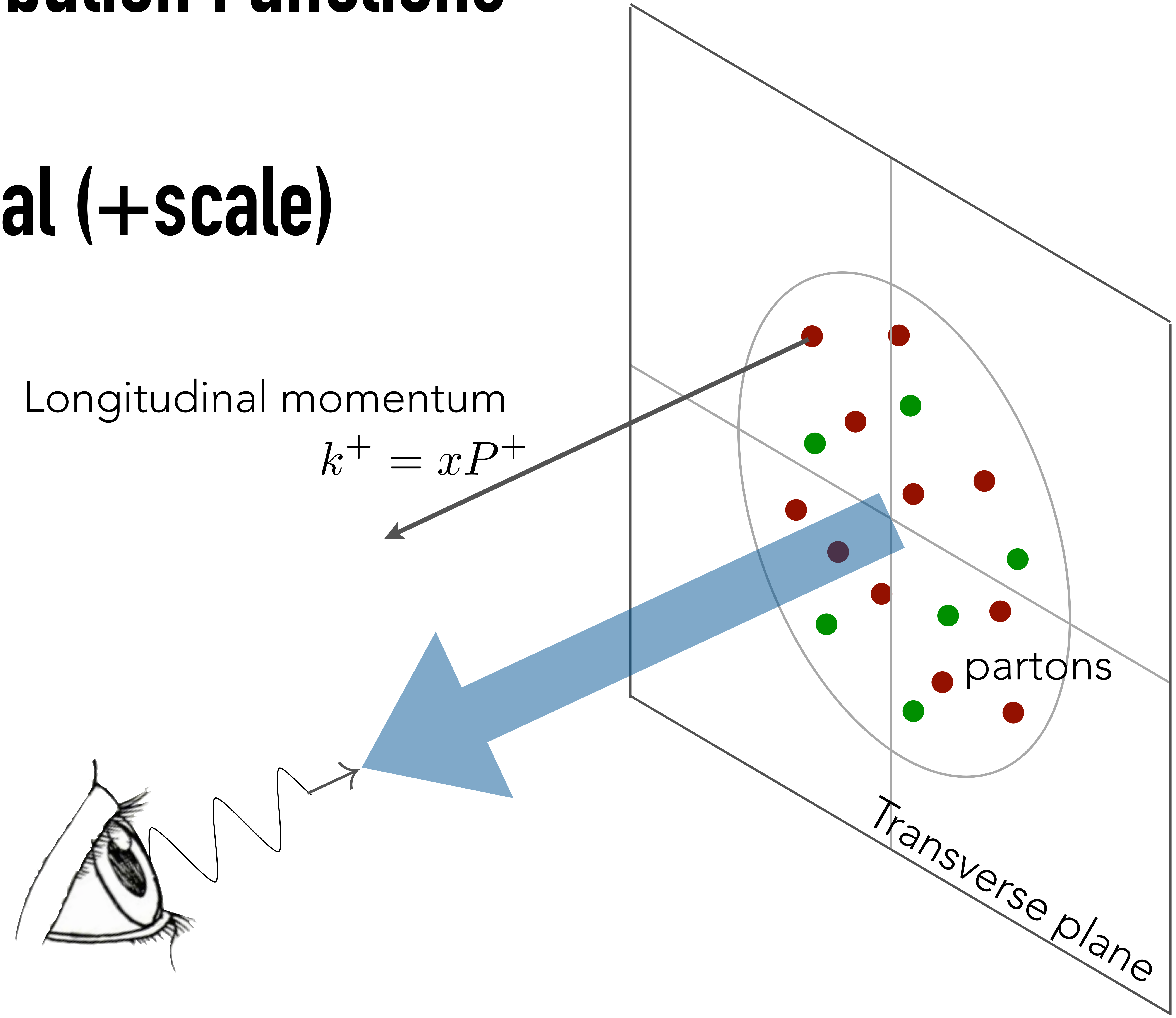


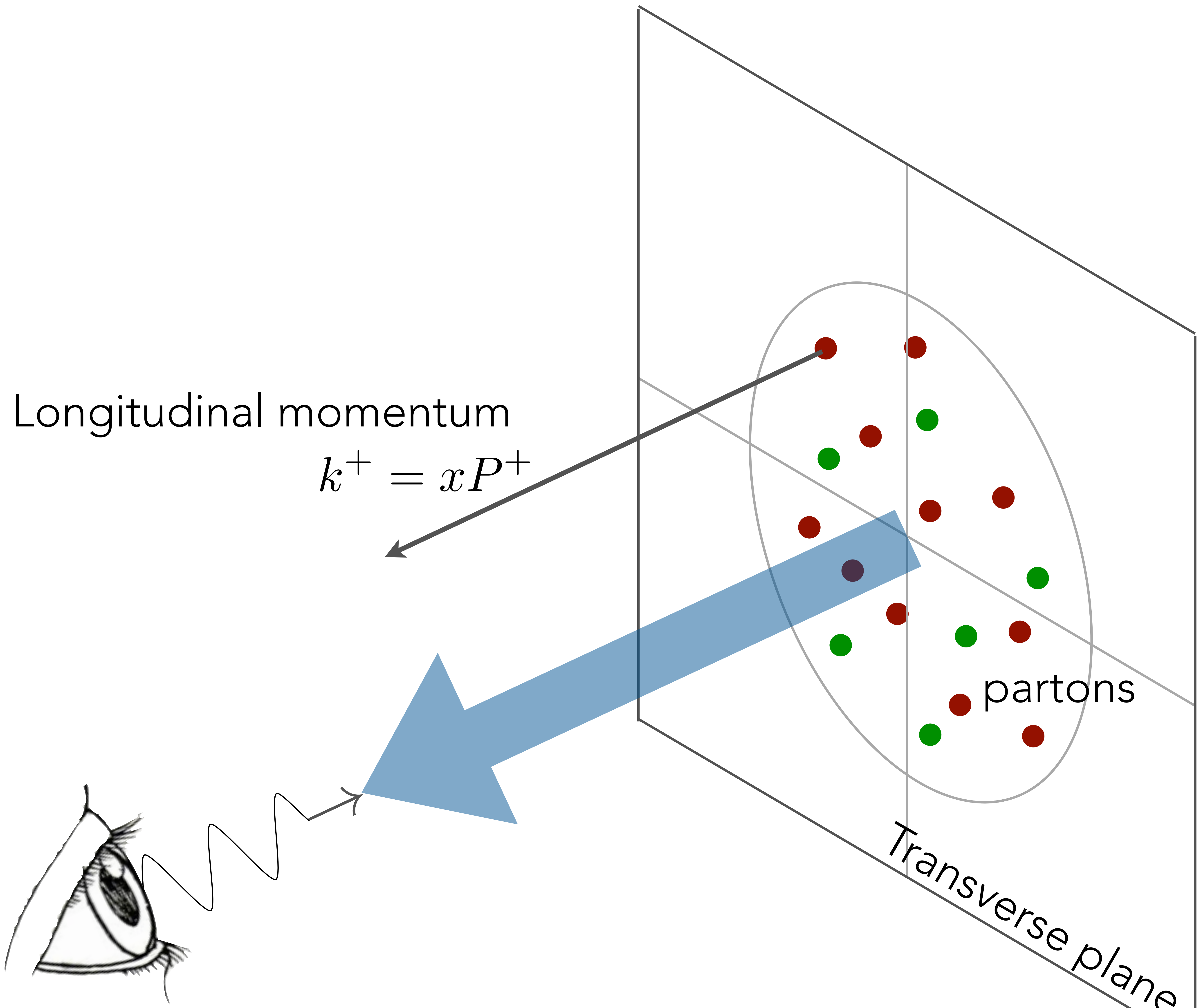
Longitudinal momentum
$k^+ = xP^+$
partons
Transverse plane

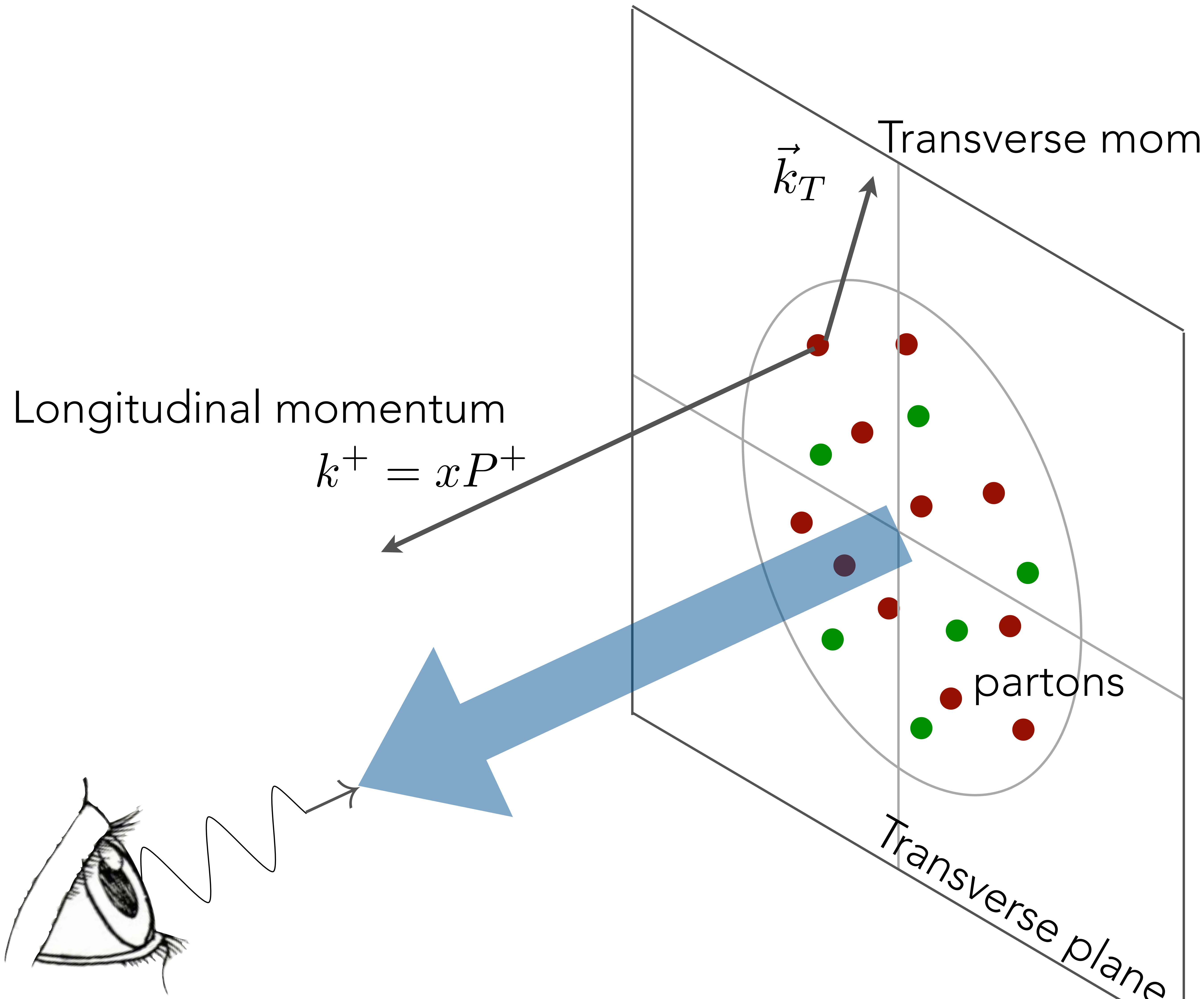
Parton Distribution Functions

$f(x)$

1 dimensional (+scale)



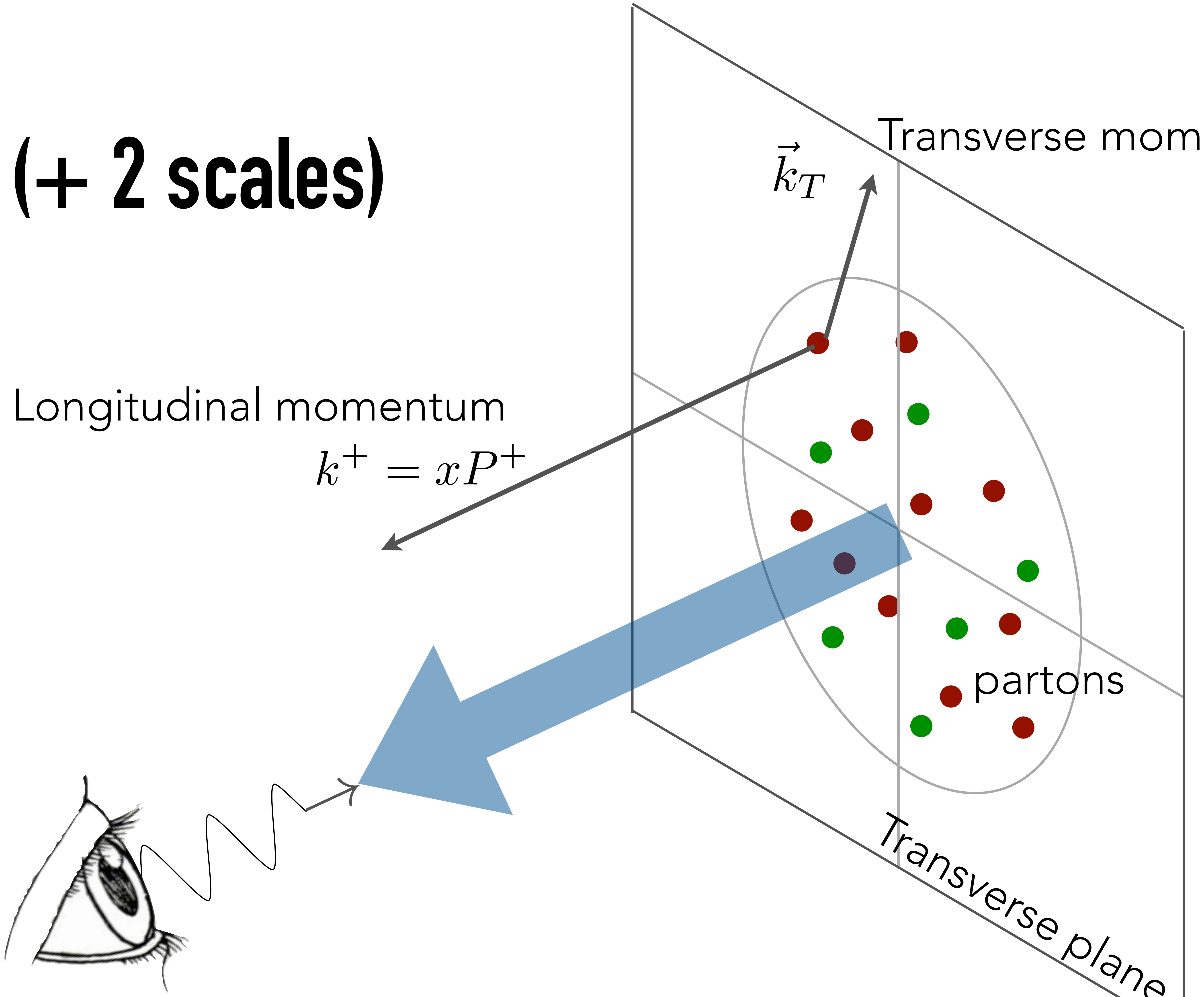

Longitudinal momentum
$k^+ = xP^+$
partons
Transverse plane


Transverse momentum
$\vec{k}_T$
Longitudinal momentum
$k^+ = xP^+$
partons
Transverse plane

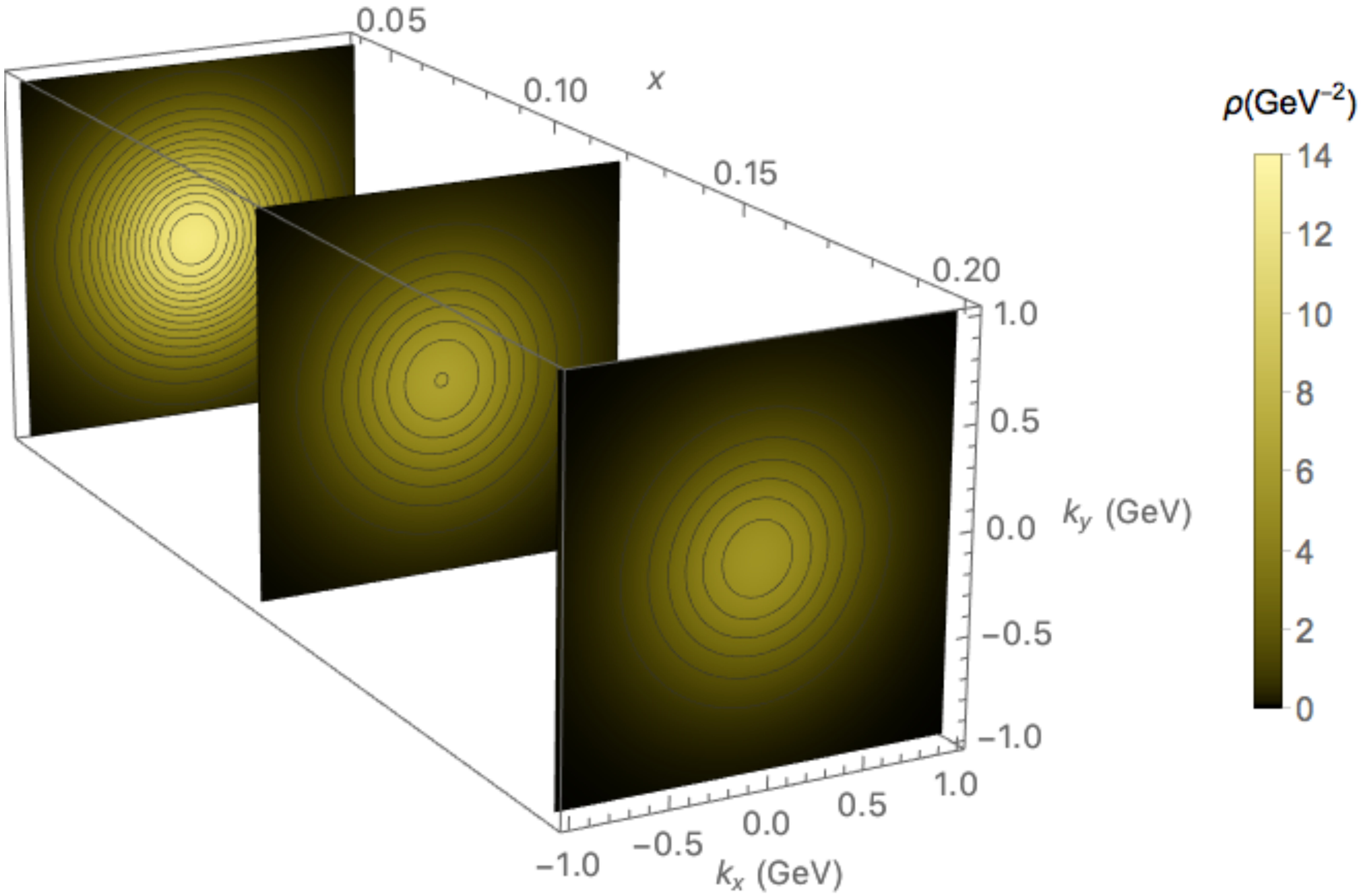
Transverse-Momentum Distributions

$f(x, \vec{k}_T)$

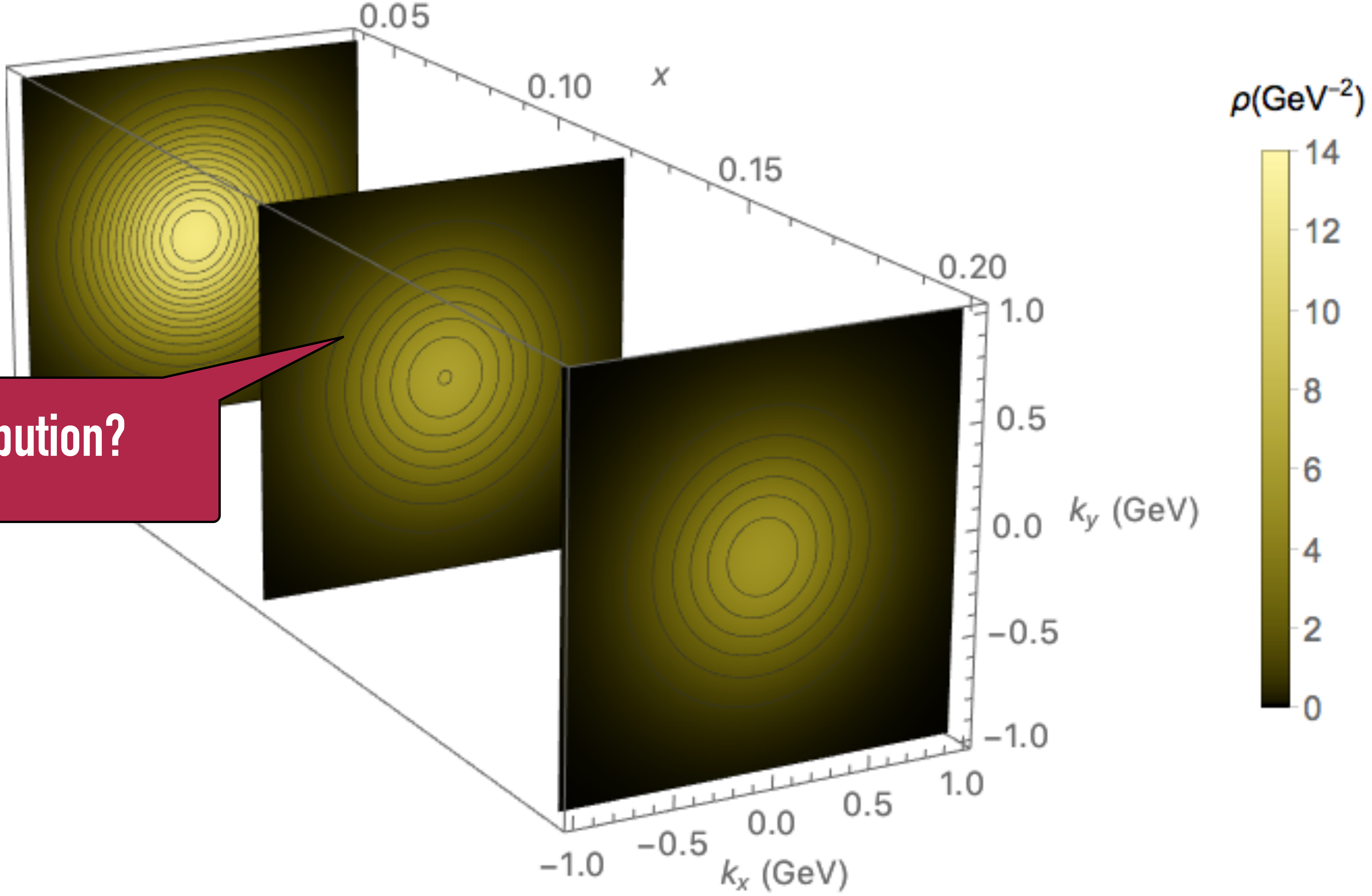
3 dimensional (+ 2 scales)



PRESENT KNOWLEDGE (OR LACK OF KNOWLEDGE)?

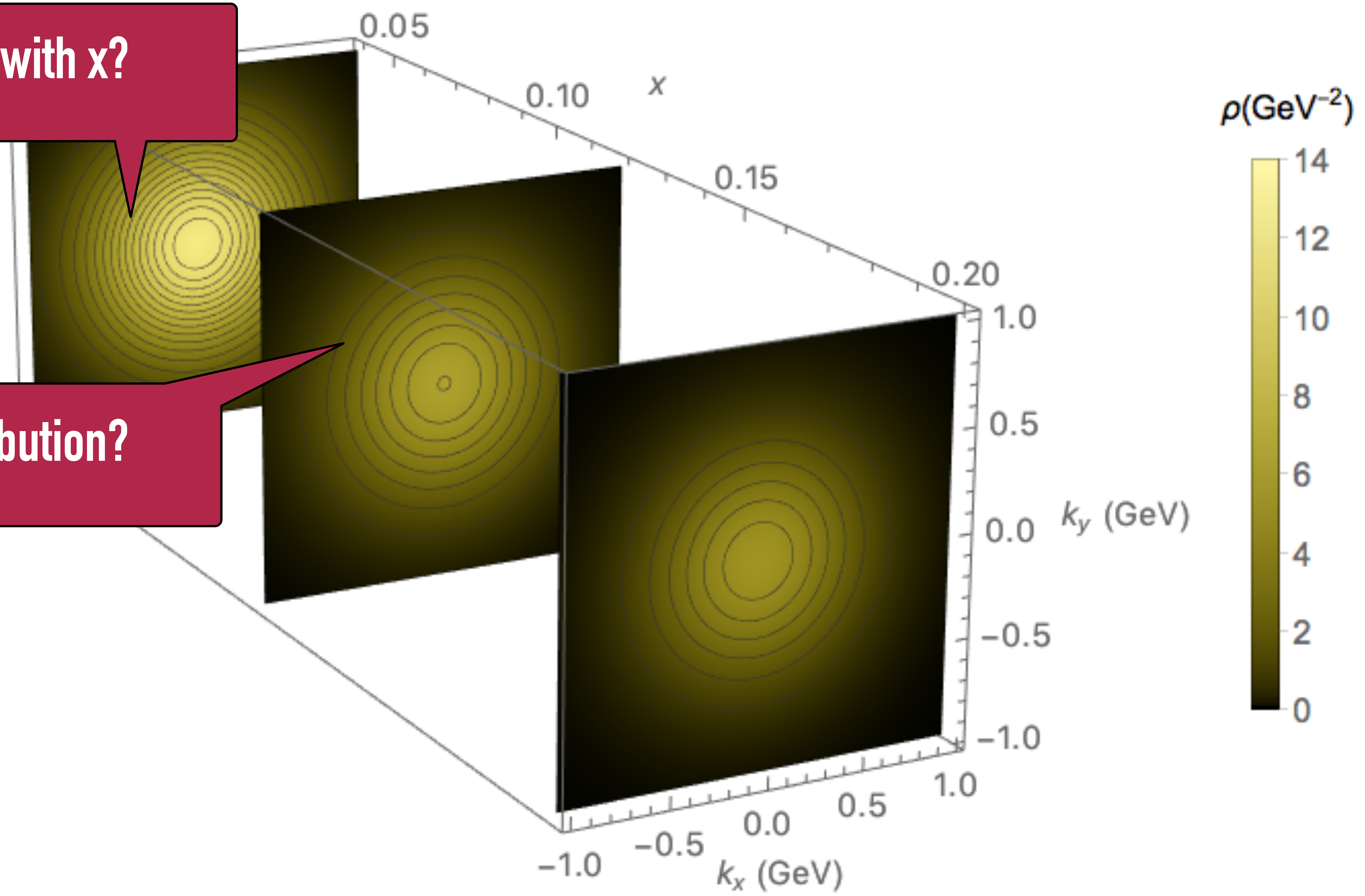


PRESENT KNOWLEDGE (OR LACK OF KNOWLEDGE)?



How “wide” is the distribution?

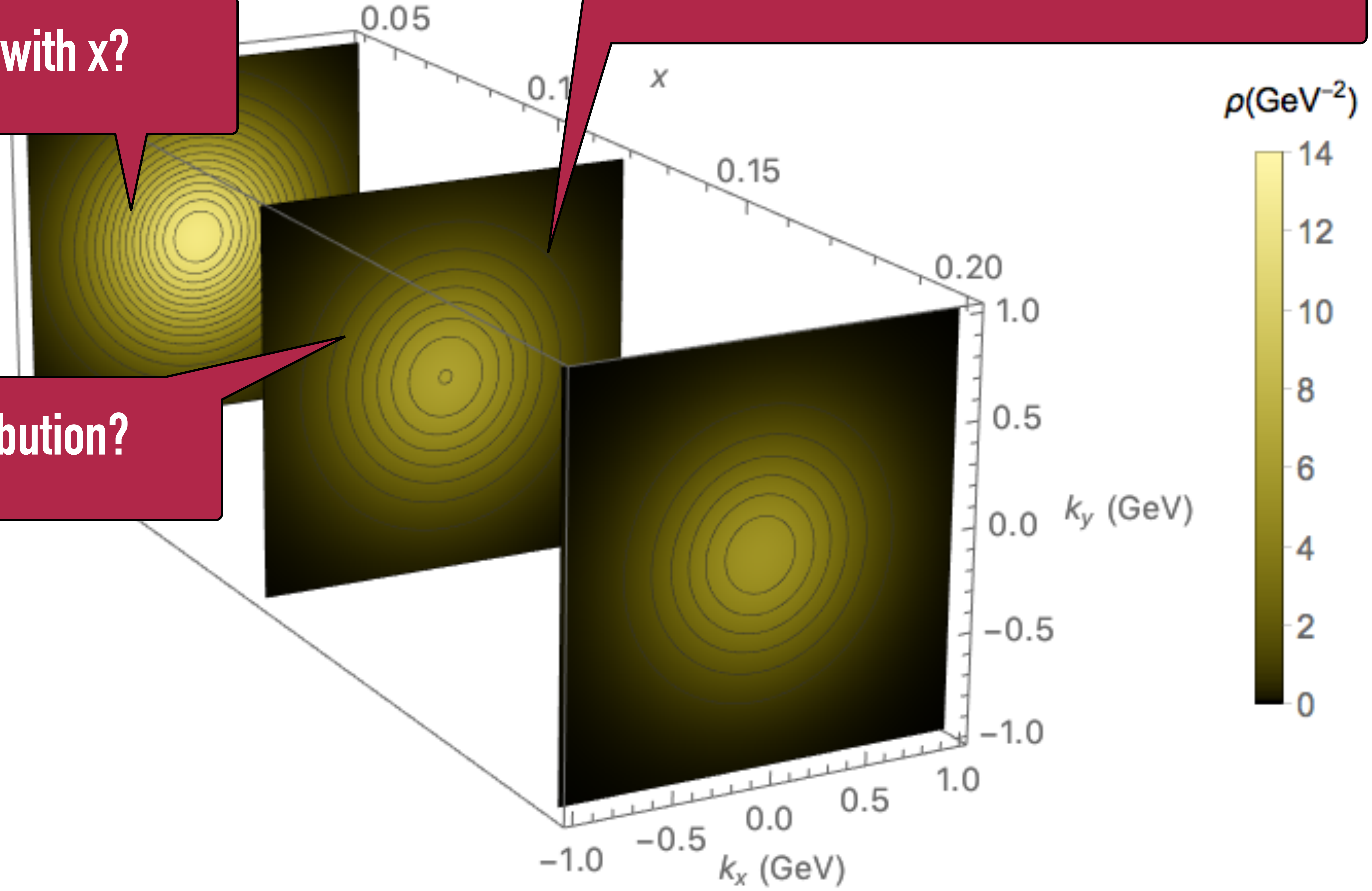
PRESENT KNOWLEDGE (OR LACK OF KNOWLEDGE)?



How does it change with x?

How “wide” is the distribution?

PRESENT KNOWLEDGE (OR LACK OF KNOWLEDGE)?

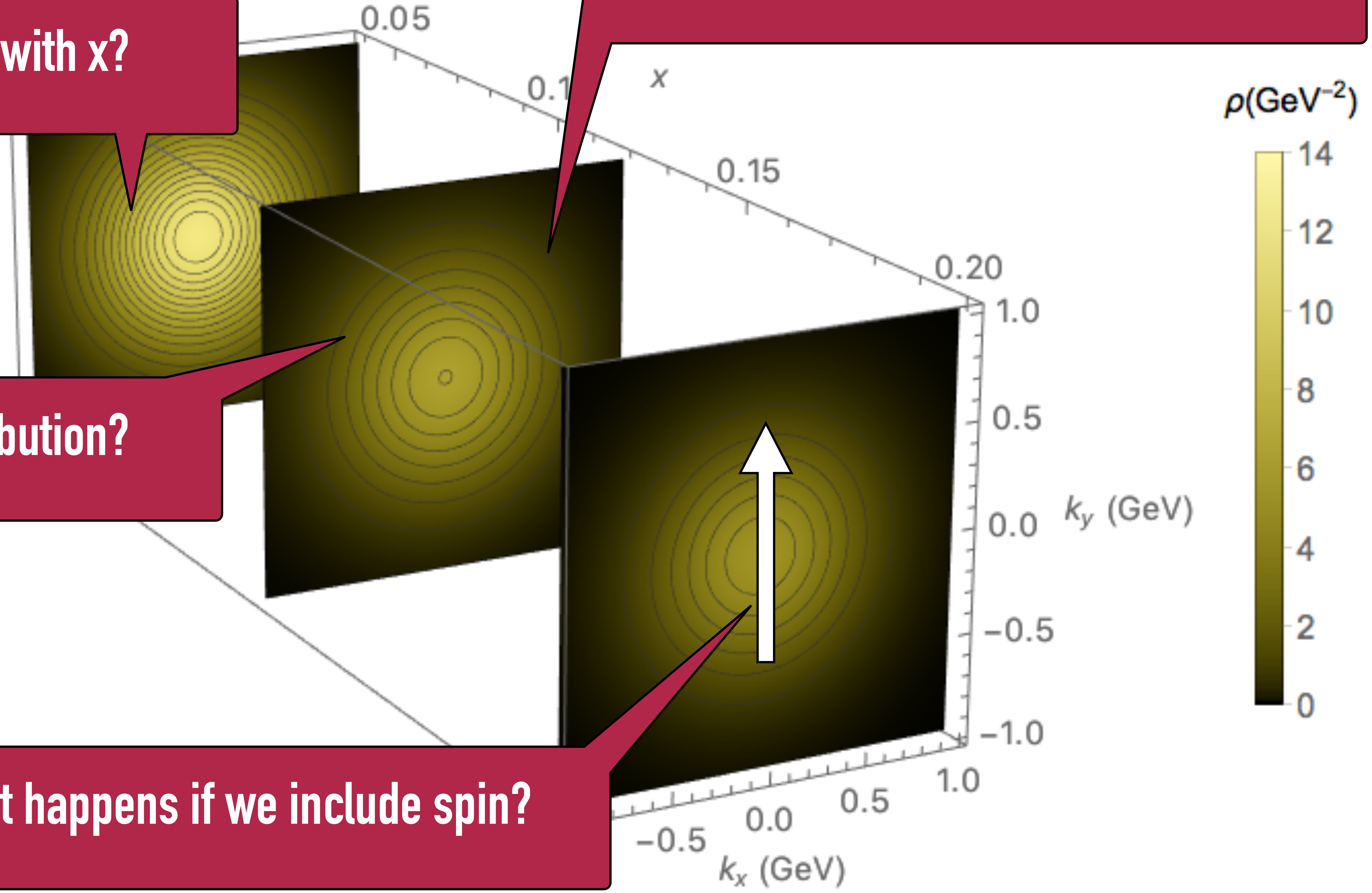


How does it change with x?

Is there a difference between flavors?

How “wide” is the distribution?

PRESENT KNOWLEDGE (OR LACK OF KNOWLEDGE)?



How does it change with x?

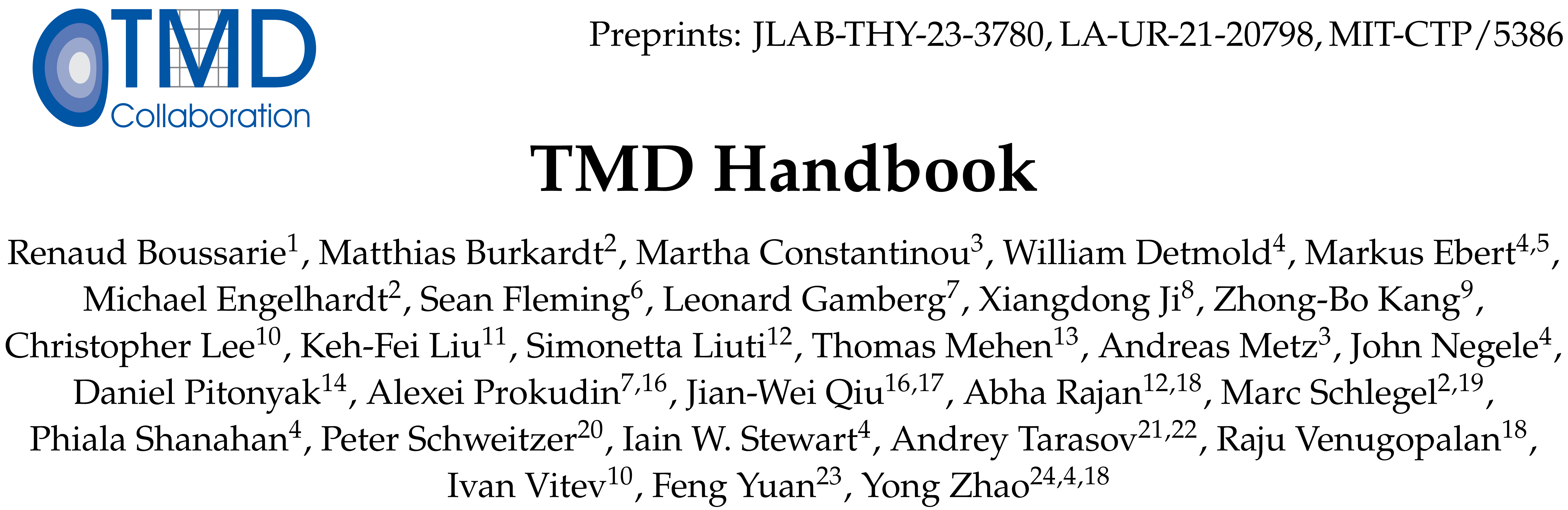
Is there a difference between flavors?

How “wide” is the distribution?

What happens if we include spin?

RECENT REVIEW

TMD collaboration, "TMD Handbook," arXiv:2304.03302



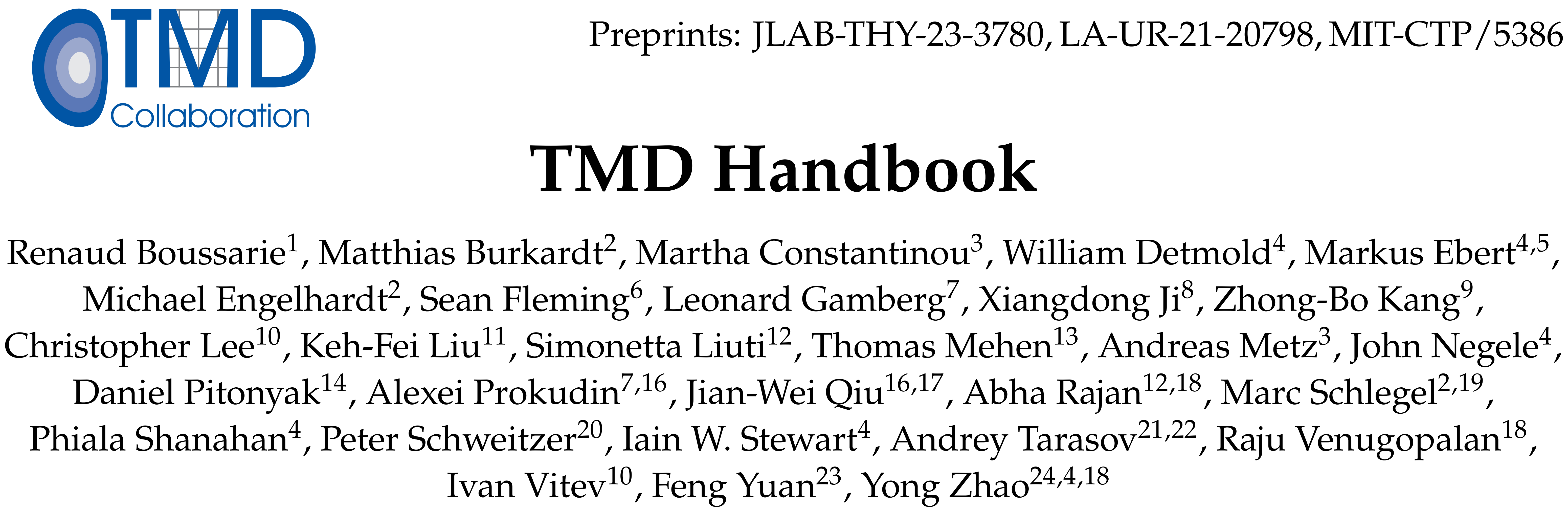
Preprints: JLAB-THY-23-3780, LA-UR-21-20798, MIT-CTP/5386

TMD Handbook

Renaud Boussarie[1], Matthias Burkardt[2], Martha Constantinou[3], William Detmold[4], Markus Ebert[4,5], Michael Engelhardt[2], Sean Fleming[6], Leonard Gamberg[7], Xiangdong Ji[8], Zhong-Bo Kang[9], Christopher Lee[10], Keh-Fei Liu[11], Simonetta Liuti[12], Thomas Mehen[13], Andreas Metz[3], John Negele[4], Daniel Pitonyak[14], Alexei Prokudin[7,16], Jian-Wei Qiu[16,17], Abha Rajan[12,18], Marc Schlegel[2,19], Phiala Shanahan[4], Peter Schweitzer[20], Iain W. Stewart[4], Andrey Tarasov[21,22], Raju Venugopalan[18], Ivan Vitev[10], Feng Yuan[23], Yong Zhao[24,4,18]

RECENT REVIEW

TMD collaboration, "TMD Handbook," arXiv:2304.03302



Preprints: JLAB-THY-23-3780, LA-UR-21-20798, MIT-CTP/5386

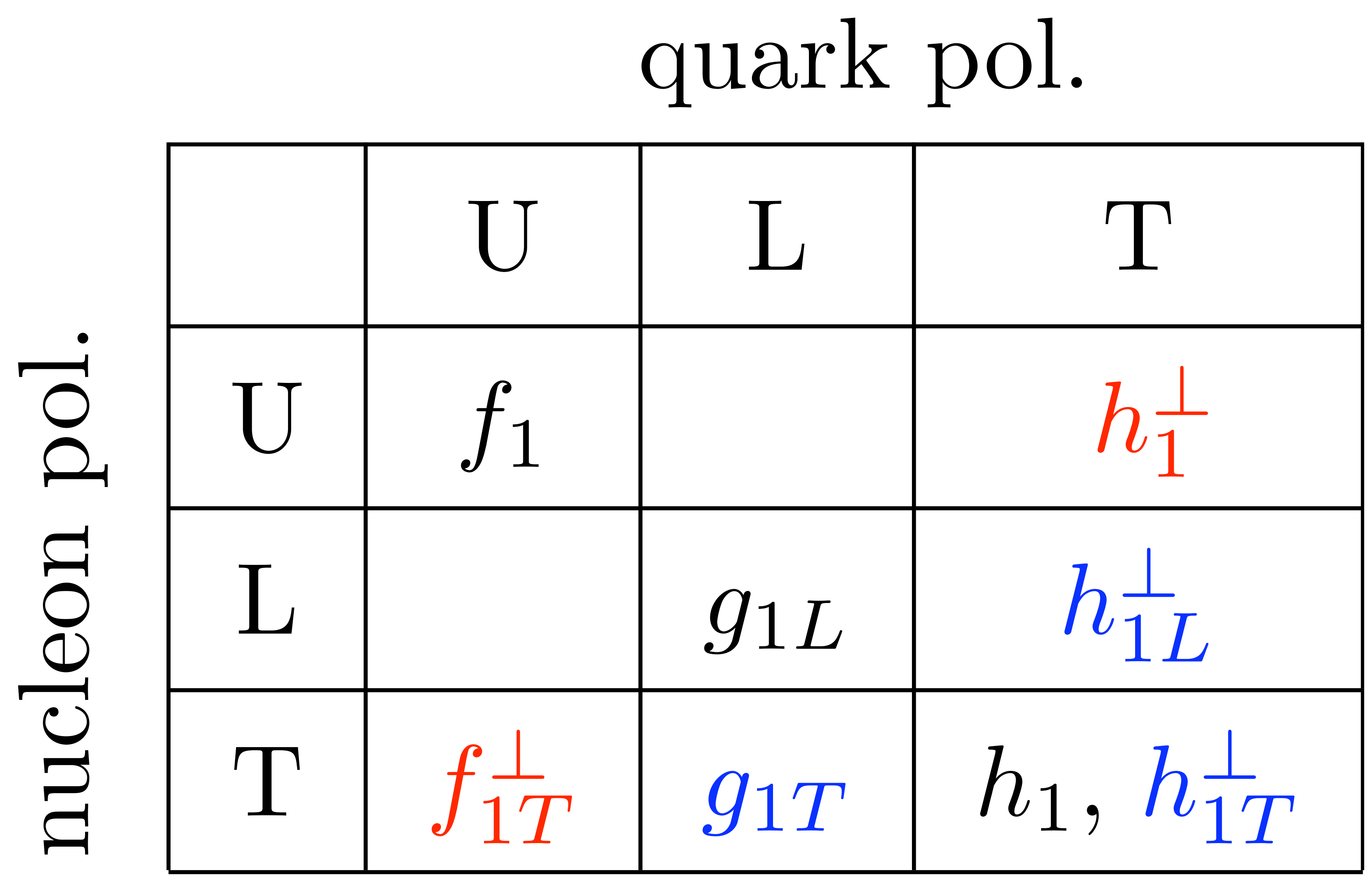
TMD Handbook

Renaud Boussarie[1], Matthias Burkardt[2], Martha Constantinou[3], William Detmold[4], Markus Ebert[4,5],
Michael Engelhardt[2], Sean Fleming[6], Leonard Gamberg[7], Xiangdong Ji[8], Zhong-Bo Kang[9],
Christopher Lee[10], Keh-Fei Liu[11], Simonetta Liuti[12], Thomas Mehen[13], Andreas Metz[3], John Negele[4],
Daniel Pitonyak[14], Alexei Prokudin[7,16], Jian-Wei Qiu[16,17], Abha Rajan[12,18], Marc Schlegel[2,19],
Phiala Shanahan[4], Peter Schweitzer[20], Iain W. Stewart[4], Andrey Tarasov[21,22], Raju Venugopalan[18],
Ivan Vitev[10], Feng Yuan[23], Yong Zhao[24,4,18]

471 PAGES

TMD TABLES: QUARK, LEADING TWIST

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)



quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

TMDs in **black** survive integration over transverse momentum
TMDs in **red** are time-reversal odd

TMD TABLES: QUARK, LEADING TWIST

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)

quark pol.

nucleon pol.

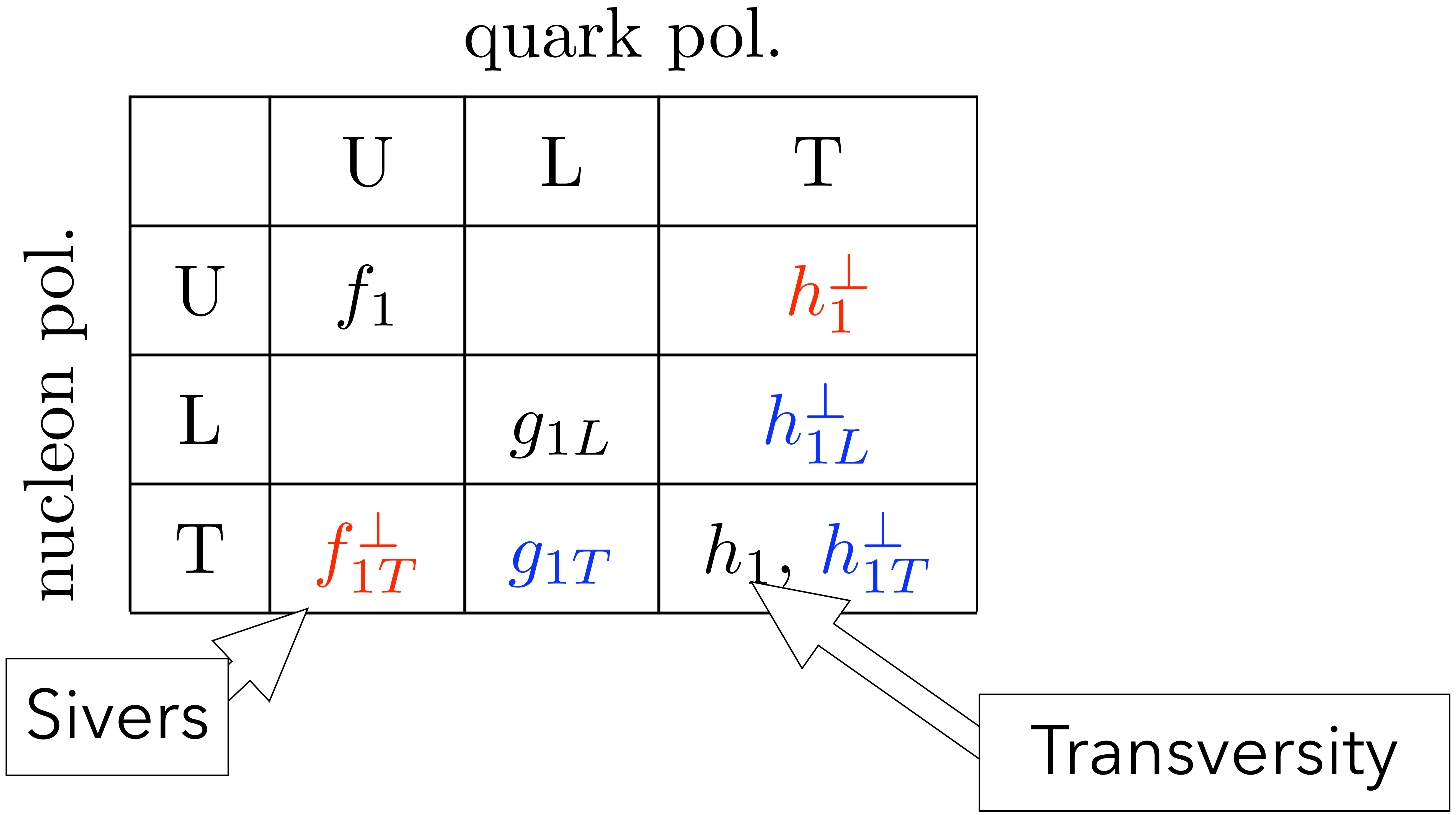
	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	$h_1,\ h_{1T}^{\perp}$

Sivers

TMDs in **black** survive integration over transverse momentum
TMDs in **red** are time-reversal odd

TMD TABLES: QUARK, LEADING TWIST

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)



quark pol.

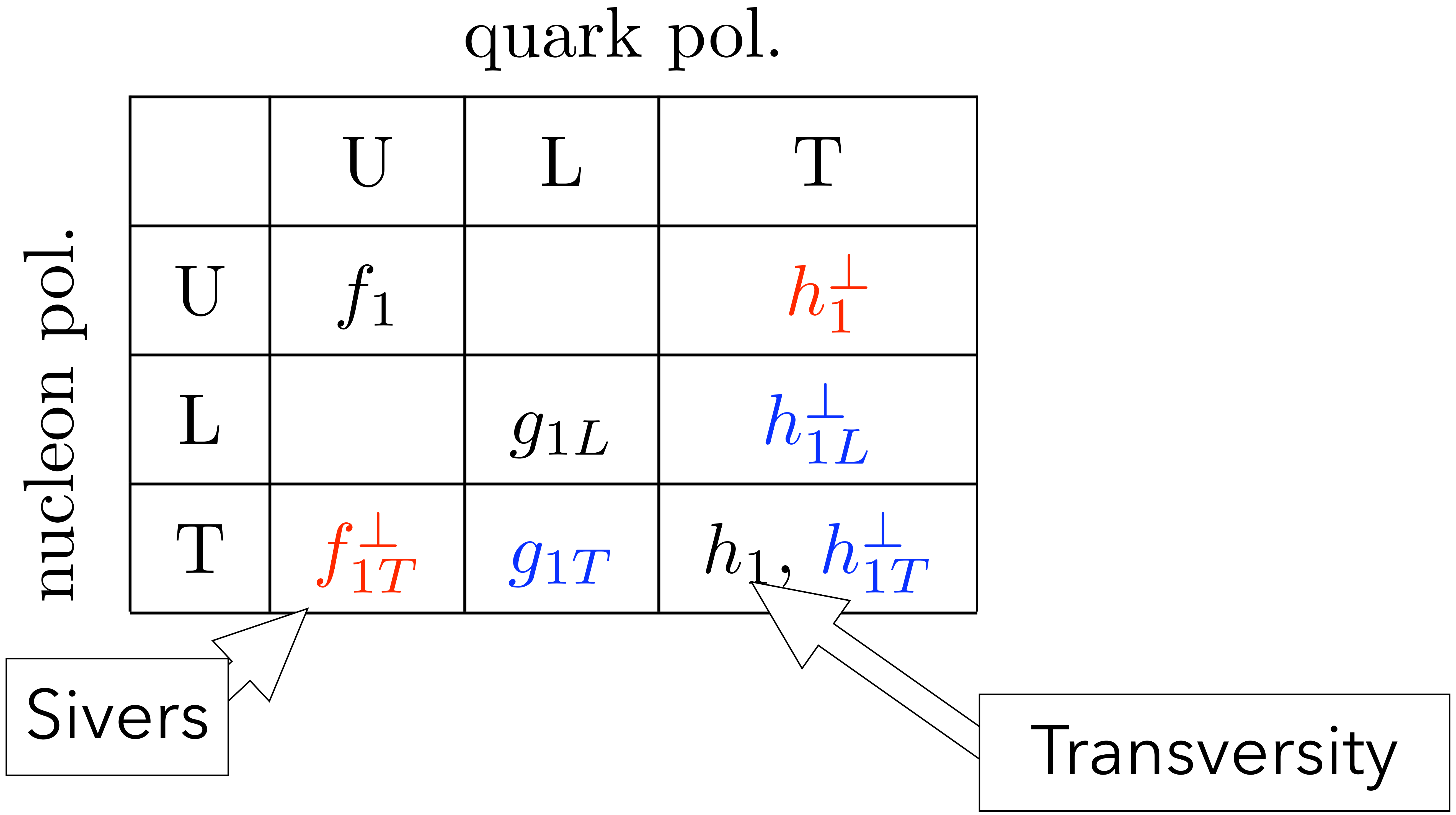
nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

TMDs in **black** survive integration over transverse momentum
TMDs in **red** are time-reversal odd

TMD TABLES: QUARK, LEADING TWIST

[Mulders-Tangerman, NPB 461 (96)](#)
[Boer-Mulders, PRD 57 (98)](#)



quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1, $h_{1T}^{\perp}$

Sivers

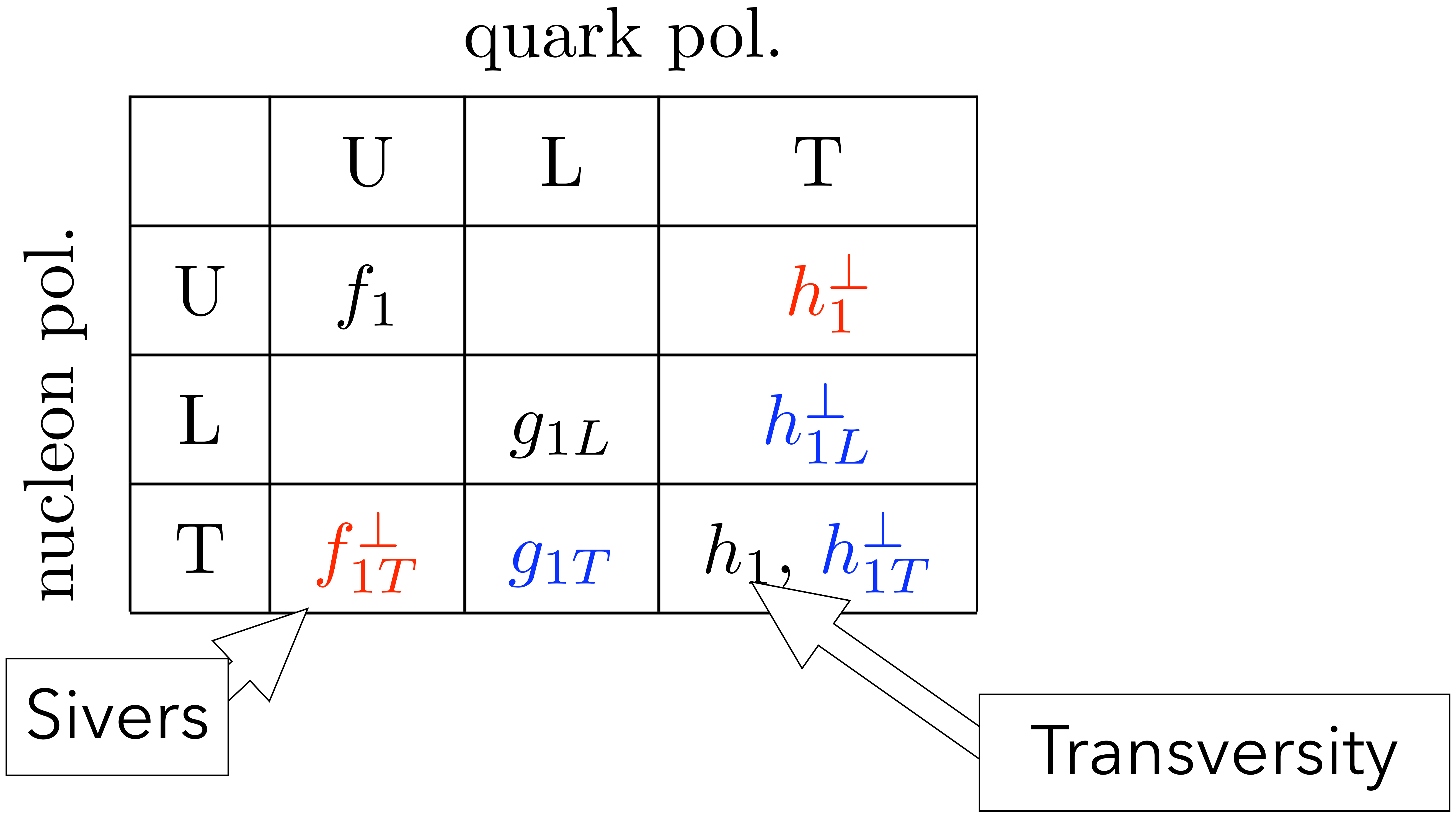
Transversity

- Very good knowledge of x dependence of f_1 and g_{1L}

TMDs in **black** survive integration over transverse momentum
TMDs in **red** are time-reversal odd

TMD TABLES: QUARK, LEADING TWIST

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)



quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

- Very good knowledge of x dependence of f_1 and g_{1L}
- Good knowledge of the k_T dependence of f_1 (also for pions)

TMDs in **black** survive integration over transverse momentum
TMDs in **red** are time-reversal odd

TMD TABLES: QUARK, LEADING TWIST

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)

quark pol.

nucleon pol.		U	L	T
	U	f_1		$h_1^\perp$
	L		g_{1L}	$h_{1L}^\perp$
	T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

Sivers

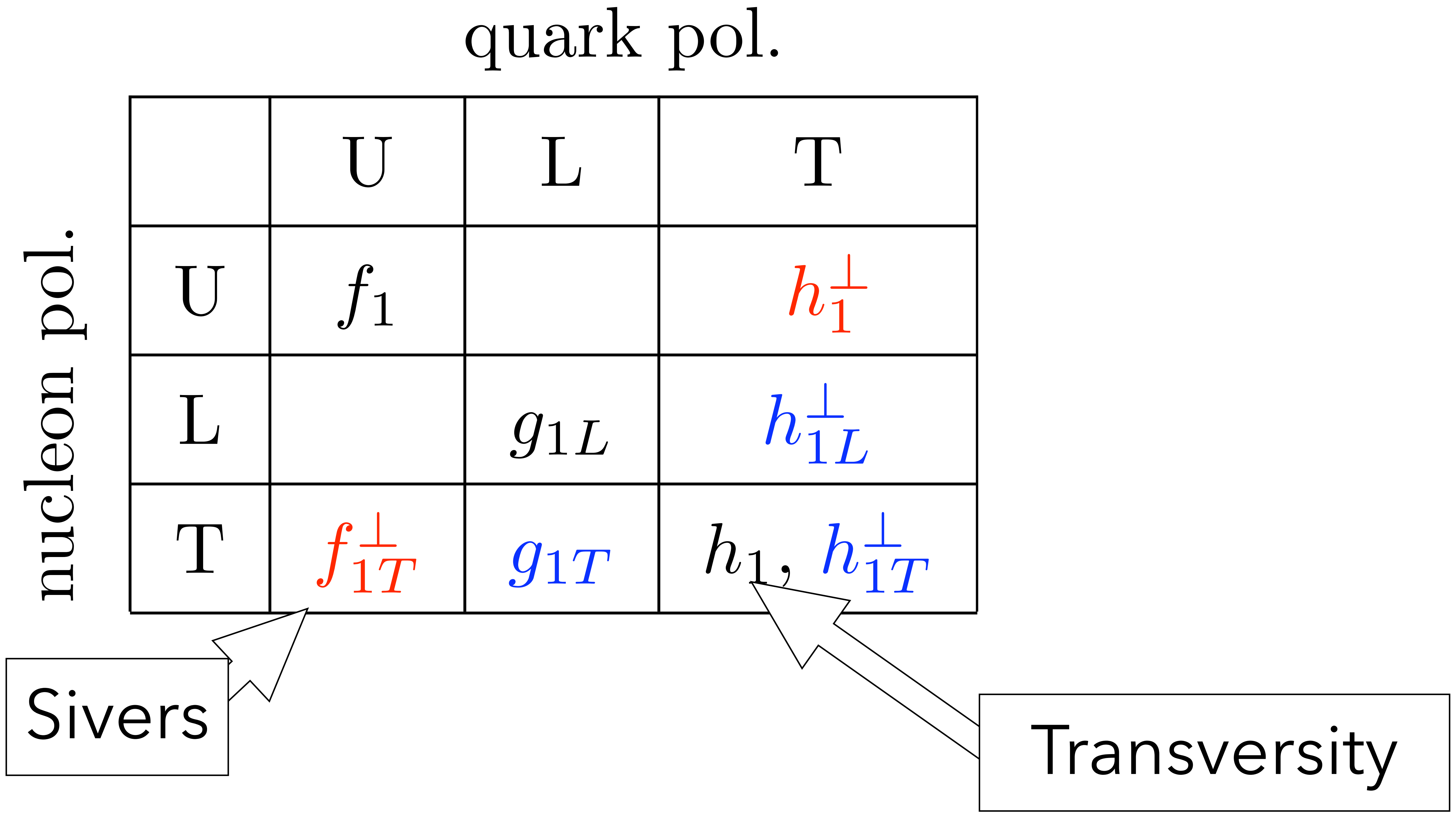
Transversity

TMDs in **black** survive integration over transverse momentum
TMDs in **red** are time-reversal odd

- Very good knowledge of x dependence of f_1 and g_{1L}
- Good knowledge of the k_T dependence of f_1 (also for pions)
- Fair knowledge of Sivers and transversity (mainly x dependence)

TMD TABLES: QUARK, LEADING TWIST

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)



quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

Sivers

Transversity

TMDs in **black** survive integration over transverse momentum
TMDs in **red** are time-reversal odd

- Very good knowledge of x dependence of f_1 and g_{1L}
- Good knowledge of the k_T dependence of f_1 (also for pions)
- Fair knowledge of Sivers and transversity (mainly x dependence)
- Some hints about all others

QUARK, SUBLEADING TWIST

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)
Bacchetta, Mulders, Pijlman, hep-ph/0405154
Goeke, Metz, Schlegel, hep-ph/0504130

quark pol.

		U	L	T	
nucleon pol.	U	$f^{\perp}$	$g^{\perp}$	e	h
	L	$f_L^{\perp}$	$g_L^{\perp}$	h_L	e_L
	T	$f_T,\ f_T^{\perp}$	$g_T,\ g_T^{\perp}$	$h_T,\ h_T^{\perp}$	$e_T,\ e_T^{\perp}$

TMDs in **black** survive integration over transverse momentum
TMDs in **red** are time-reversal odd

QUARK, SUBLEADING TWIST

		quark pol.			
		U	L	T	
nucleon pol.	U	$f^{\perp}$	$g^{\perp}$	e	h
	L	$f_L^{\perp}$	$g_L^{\perp}$	h_L	e_L
	T	$f_T,\ f_T^{\perp}$	$g_T,\ g_T^{\perp}$	$h_T,\ h_T^{\perp}$	$e_T,\ e_T^{\perp}$

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)
Bacchetta, Mulders, Pijlman, hep-ph/0405154
Goeke, Metz, Schlegel, hep-ph/0504130

- Lots of progress from the theory side

TMDs in **black** survive integration over transverse momentum
TMDs in **red** are time-reversal odd

QUARK, SUBLEADING TWIST

		quark pol.			
		U	L	T	
nucleon pol.	U	$f^{\perp}$	$g^{\perp}$	e	h
	L	$f_L^{\perp}$	$g_L^{\perp}$	h_L	e_L
	T	$f_T,\ f_T^{\perp}$	$g_T,\ g_T^{\perp}$	$h_T,\ h_T^{\perp}$	$e_T,\ e_T^{\perp}$

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)
Bacchetta, Mulders, Pijlman, hep-ph/0405154
Goeke, Metz, Schlegel, hep-ph/0504130

- Lots of progress from the theory side
- Some knowledge of g_T x-dependence

TMDs in **black** survive integration over transverse momentum
TMDs in **red** are time-reversal odd

QUARK, SUBLEADING TWIST

		quark pol.			
		U	L	T	
nucleon pol.	U	$f^{\perp}$	$g^{\perp}$	e	h
	L	$f_L^{\perp}$	$g_L^{\perp}$	h_L	e_L
	T	$f_T,\ f_T^{\perp}$	$g_T,\ g_T^{\perp}$	$h_T,\ h_T^{\perp}$	$e_T,\ e_T^{\perp}$

TMDs in **black** survive integration over transverse momentum
TMDs in **red** are time-reversal odd

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)
Bacchetta, Mulders, Pijlman, hep-ph/0405154
Goeke, Metz, Schlegel, hep-ph/0504130

- Lots of progress from the theory side
- Some knowledge of g_T x-dependence
- First hints about e x-dependence

QUARK, SUBLEADING TWIST

		quark pol.			
		U	L	T	
nucleon pol.	U	$f^{\perp}$	$g^{\perp}$	e	h
	L	$f_L^{\perp}$	$g_L^{\perp}$	h_L	e_L
	T	$f_T,\ f_T^{\perp}$	$g_T,\ g_T^{\perp}$	$h_T,\ h_T^{\perp}$	$e_T,\ e_T^{\perp}$

TMDs in **black** survive integration over transverse momentum
TMDs in **red** are time-reversal odd

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)
Bacchetta, Mulders, Pijlman, hep-ph/0405154
Goeke, Metz, Schlegel, hep-ph/0504130

- Lots of progress from the theory side
- Some knowledge of g_T x-dependence
- First hints about e x-dependence
- All others unknown

GLUONS, LEADING TWIST

Mulders, Rodrigues, PRD63, 2001

gluon pol.

nucleon pol.

	U	L	linear
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g, $h_{1T}^{\perp g}$

TMDs in **black** survive integration over transverse momentum

TMDs in **red** are time-reversal odd

GLUONS, LEADING TWIST

Mulders, Rodrigues, PRD63, 2001

gluon pol.

nucleon pol.

	U	L	linear
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g, $h_{1T}^{\perp g}$

- Good knowledge of x-dependence of f_1 and g_{1L}

TMDs in **black** survive integration over transverse momentum
TMDs in **red** are time-reversal odd

GLUONS, LEADING TWIST

Mulders, Rodrigues, PRD63, 2001

gluon pol.

nucleon pol.

	U	L	linear
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_1^g,\ h_{1T}^{\perp g}$

- Good knowledge of x-dependence of f_1 and g_{1L}
- Some hints on the k_T dependence of f_1

TMDs in **black** survive integration over transverse momentum

TMDs in **red** are time-reversal odd

GLUONS, LEADING TWIST

Mulders, Rodrigues, PRD63, 2001

gluon pol.

nucleon pol.

	U	L	linear
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_1^g,\ h_{1T}^{\perp g}$

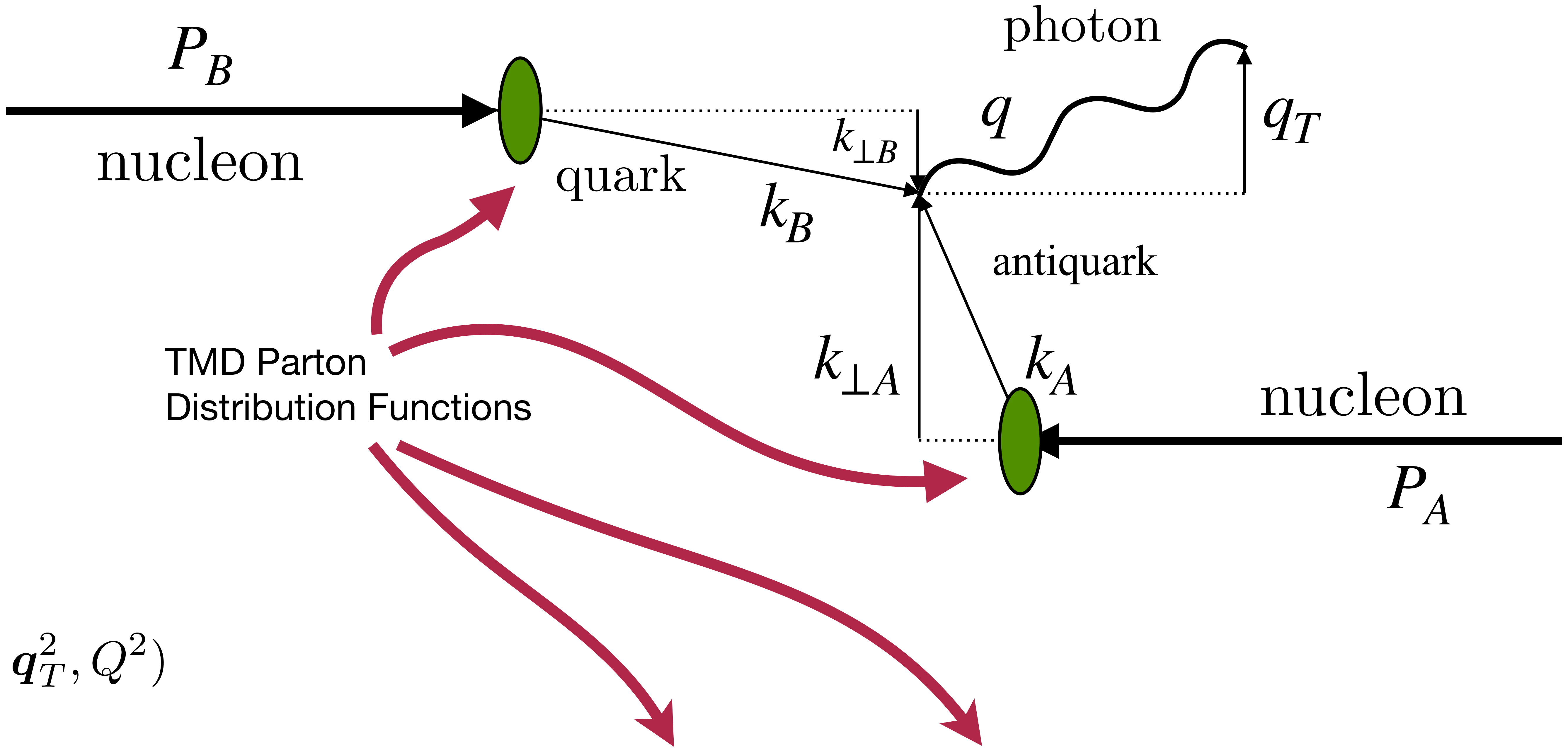
- Good knowledge of x-dependence of f_1 and g_{1L}
- Some hints on the k_T dependence of f_1

TMDs in **black** survive integration over transverse momentum

TMDs in **red** are time-reversal odd

QUARKS

TMDS IN DRELL-YAN PROCESSES

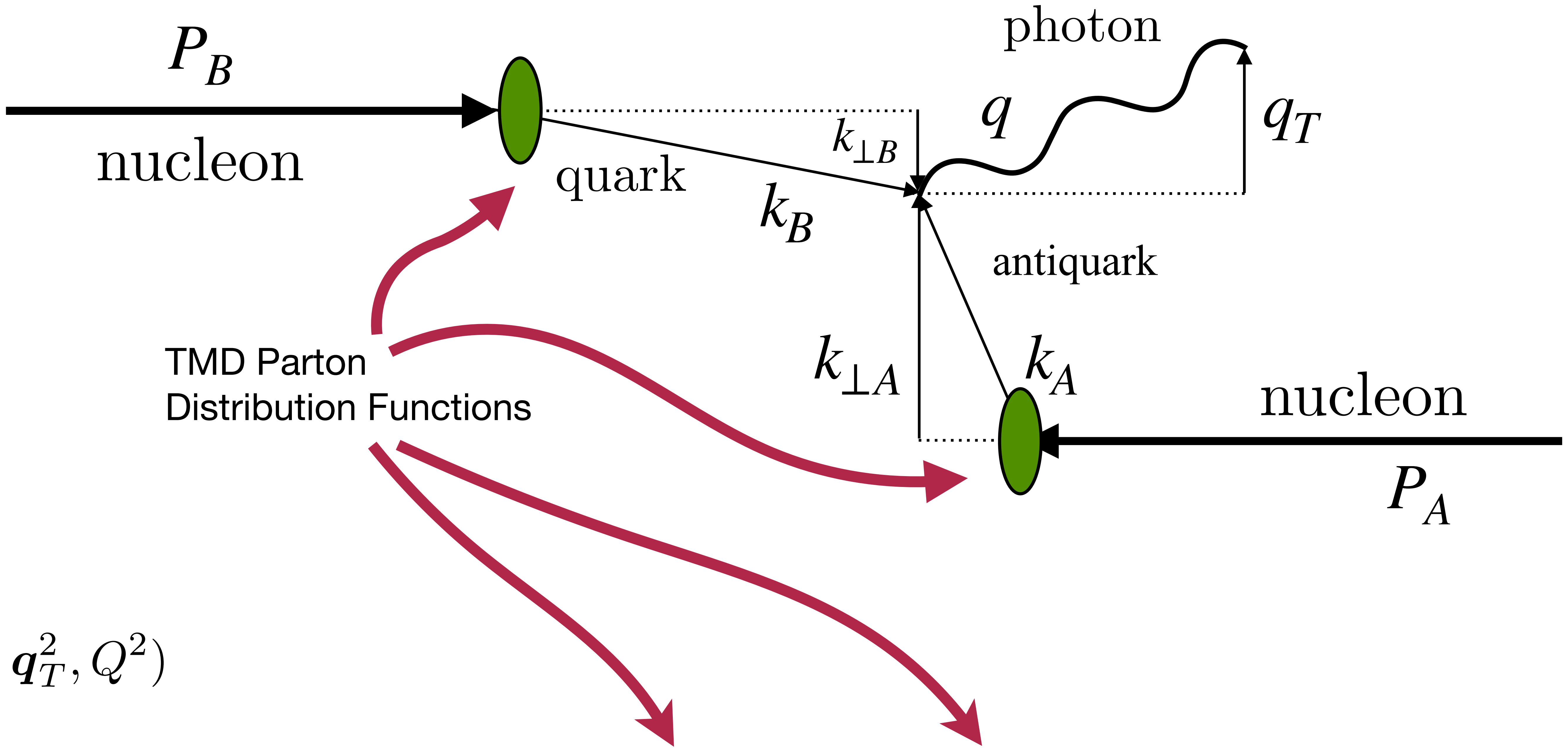


TMD Parton
Distribution Functions

$$F^1_{UU}(x_A, x_B, \boldsymbol{q}_T^2, Q^2)$$

$$\approx \sum_q \mathcal{H}^{1q}_{UU}(Q^2, \mu^2) \int d^2\boldsymbol{k}_{\perp A}\, d^2\boldsymbol{k}_{\perp B}\, f_1^q\big(x_A, \boldsymbol{k}^2_{\perp A}; \mu^2\big)\, f_1^{\bar{q}}\big(x_B, \boldsymbol{k}^2_{\perp B}; \mu^2\big)\, \delta^{(2)}\big(\boldsymbol{k}_{\perp A} - \boldsymbol{q}_T + \boldsymbol{k}_{\perp B}\big)$$

TMDS IN DRELL–YAN PROCESSES

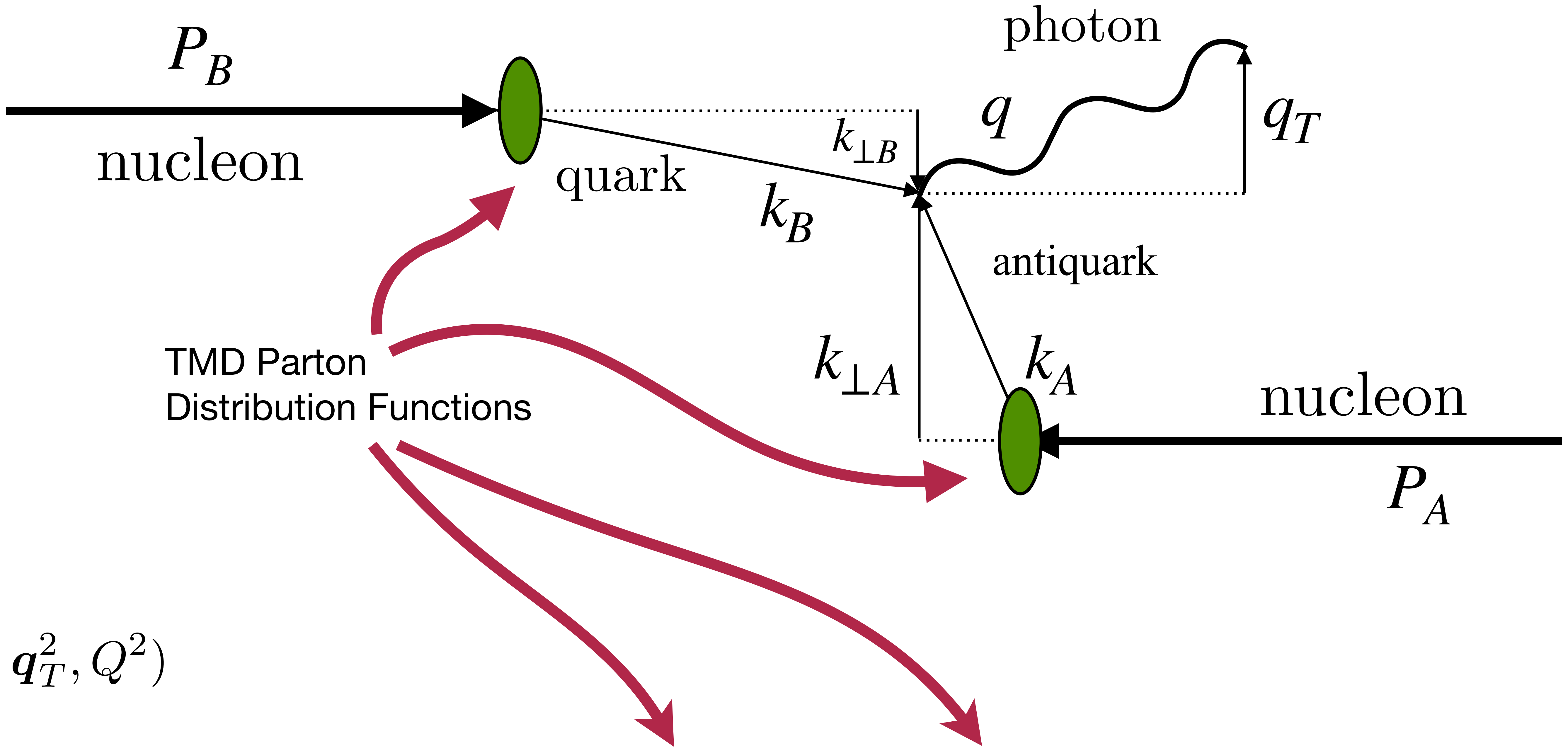


TMD Parton
Distribution Functions

$$
\begin{aligned}
&F^1_{UU}(x_A, x_B, \boldsymbol{q}_T^2, Q^2) \\
&\approx \sum_q \mathcal{H}^{1q}_{UU}(Q^2,\mu^2) \int d^2\boldsymbol{k}_{\perp A}\, d^2\boldsymbol{k}_{\perp B}\, f_1^q(x_A, \boldsymbol{k}^2_{\perp A};\mu^2)\, f_1^{\bar{q}}(x_B, \boldsymbol{k}^2_{\perp B};\mu^2)\, \delta^{(2)}(\boldsymbol{k}_{\perp A} - \boldsymbol{q}_T + \boldsymbol{k}_{\perp B}) \\
&= \sum_q \mathcal{H}^{1q}_{UU}(Q^2,\mu^2) \int db_T b_T J_0(b_T|\boldsymbol{q}_T|) \hat{f}_1^q(x_A, b_T^2;\mu^2)\, \hat{f}_1^{\bar{q}}(x_B, b_T^2;\mu^2)
\end{aligned}
$$

The analysis is usually done in Fourier-transformed space

TMDS IN DRELL-YAN PROCESSES



TMD Parton
Distribution Functions

$$F^1_{UU}(x_A, x_B, \boldsymbol{q}_T^2, Q^2)$$

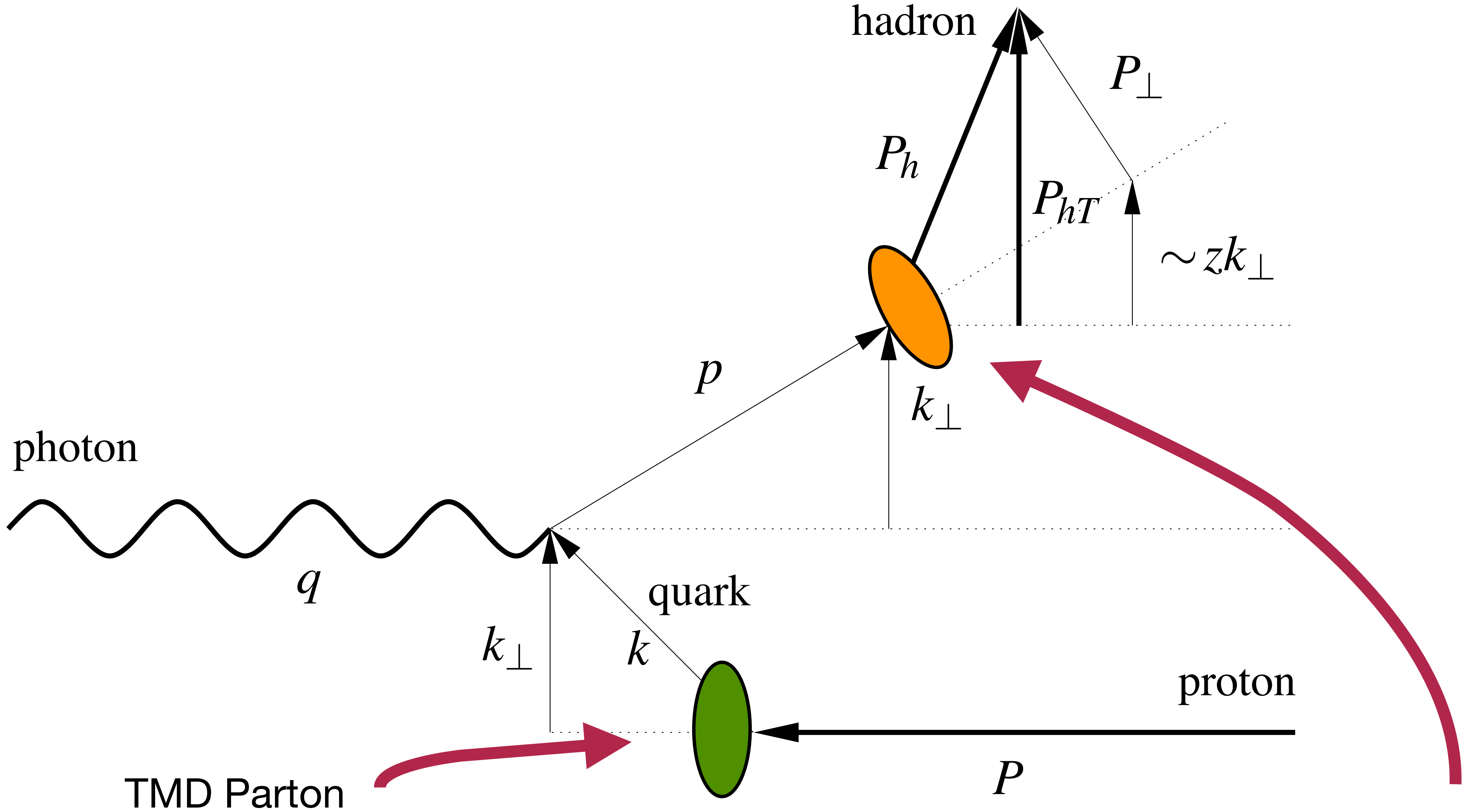
$$\approx \sum_q \mathcal{H}^{1q}_{UU}(Q^2, \mu^2) \int d^2\boldsymbol{k}_{\perp A}\, d^2\boldsymbol{k}_{\perp B}\, f_1^q(x_A, \boldsymbol{k}^2_{\perp A}; \mu^2)\, f_1^{\bar{q}}(x_B, \boldsymbol{k}^2_{\perp B}; \mu^2)\, \delta^{(2)}(\boldsymbol{k}_{\perp A} - \boldsymbol{q}_T + \boldsymbol{k}_{\perp B})$$

$$= \sum_q \mathcal{H}^{1q}_{UU}(Q^2, \mu^2) \int db_T b_T J_0(b_T|\boldsymbol{q}_T|) \hat{f}_1^q(x_A, b_T^2; \mu^2)\, \hat{f}_1^{\bar{q}}(x_B, b_T^2; \mu^2)$$

The analysis is usually done in Fourier-transformed space

TMDs formally depend on two scales, but we set them equal.

TMDS IN SEMI-INCLUSIVE DIS (SIDIS)



TMD Parton
Distribution Functions

TMD Parton
Fragmentation Functions

$$F_{UU,T}(x,z,\boldsymbol{P}_{hT}^2,Q^2)$$

$$= x\sum_q \mathcal{H}^q_{UU,T}(Q^2,\mu^2)\int d^2\boldsymbol{k}_\perp\, d^2\boldsymbol{P}_\perp\, f_1^a(x,\boldsymbol{k}_\perp^2;\mu^2)\, D_1^{a\to h}(z,\boldsymbol{P}_\perp^2;\mu^2)\,\delta(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp)$$

$$= x\sum_a \mathcal{H}^q_{UU,T}(Q^2,\mu^2)\int db_T b_T J_0(b_T|\boldsymbol{P}_{h\perp}|)\hat{f}_1^q(x,z^2b_\perp^2;\mu^2)\,\hat{D}_1^{a\to h}(z,b_\perp^2;\mu^2)$$

TMD STRUCTURE

$$\hat{f}_1^a(x, |\boldsymbol{b}_T|; \mu, \zeta) = \int d^2\boldsymbol{k}_\perp \, e^{i\boldsymbol{b}_T \cdot \boldsymbol{k}_\perp} \, f_1^a(x, \boldsymbol{k}_\perp^2; \mu, \zeta)$$

see, e.g., Collins, "Foundations of Perturbative QCD" (11)
TMD collaboration, "TMD Handbook," arXiv:2304.03302

TMD STRUCTURE

$$\hat{f}_1^a(x, |\boldsymbol{b}_T|; \mu, \zeta) = \int d^2\boldsymbol{k}_\perp \, e^{i\boldsymbol{b}_T \cdot \boldsymbol{k}_\perp} \, f_1^a(x, \boldsymbol{k}_\perp^2; \mu, \zeta)$$

$$\hat{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = [C \otimes f_1](x, \mu_{b_*}) \; e^{\int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu} \left(\gamma_F - \gamma_K \, \ln \frac{\sqrt{\zeta_f}}{\mu}\right)} \left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}}\right)^{K_{\mathrm{resum}} + g_K}$$

see, e.g., Collins, "Foundations of Perturbative QCD" (11)
TMD collaboration, "TMD Handbook," arXiv:2304.03302

TMD STRUCTURE

$$\hat{f}_1^a(x, |\boldsymbol{b}_T|; \mu, \zeta) = \int d^2\boldsymbol{k}_\perp\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{k}_\perp}\, f_1^a(x, \boldsymbol{k}_\perp^2; \mu, \zeta)$$

$$\hat{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = [C \otimes f_1](x, \mu_{b_*})\; e^{\int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu}\left(\gamma_F - \gamma_K\ \ln\frac{\sqrt{\zeta_f}}{\mu}\right)} \left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}}\right)^{K_{\mathrm{resum}}+g_K}$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

see, e.g., Collins, "Foundations of Perturbative QCD" (11)
TMD collaboration, "TMD Handbook," arXiv:2304.03302

TMD STRUCTURE

$$\hat{f}_1^a(x,|\boldsymbol{b}_T|;\mu,\zeta) = \int d^2\boldsymbol{k}_\perp\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{k}_\perp}\, f_1^a(x,\boldsymbol{k}_\perp^2;\mu,\zeta)$$

perturbative Sudakov
form factor

$$\hat{f}_1^a(x,b_T^2;\mu_f,\zeta_f) = [C \otimes f_1](x,\mu_{b_*})\; e^{\int_{\mu_{b_*}}^{\mu_f}\frac{d\mu}{\mu}\left(\gamma_F-\gamma_K\,\ln\frac{\sqrt{\zeta_f}}{\mu}\right)}\left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}}\right)^{K_{\mathrm{resum}}+g_K}$$

collinear PDF

Collins-Soper kernel
(perturbative and
nonperturbative)

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

matching coefficients
(perturbative)

see, e.g., Collins, "Foundations of Perturbative QCD" (11)
TMD collaboration, "TMD Handbook," arXiv:2304.03302

TMD STRUCTURE

$$\hat{f}_1^a(x,|\boldsymbol{b}_T|;\mu,\zeta) = \int d^2\boldsymbol{k}_\perp\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{k}_\perp}\, f_1^a(x,\boldsymbol{k}_\perp^2;\mu,\zeta)$$

perturbative Sudakov
form factor

$$\hat{f}_1^a(x,b_T^2;\mu_f,\zeta_f) = [C \otimes f_1](x,\mu_{b_*})\; e^{\int_{\mu_{b_*}}^{\mu_f}\frac{d\mu}{\mu}\left(\gamma_F-\gamma_K\,\ln\frac{\sqrt{\zeta_f}}{\mu}\right)}\left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}}\right)^{K_{\rm resum}+g_K}$$

collinear PDF

matching coefficients
(perturbative)

Collins-Soper kernel
(perturbative and
nonperturbative)

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

$$\Downarrow$$

$$\mu_{b*} = \frac{2e^{-\gamma_E}}{\bar{b}_*}$$

see, e.g., Collins, "Foundations of Perturbative QCD" (11)
TMD collaboration, "TMD Handbook," arXiv:2304.03302

TMD STRUCTURE

$$\hat{f}_1^a(x,|\boldsymbol{b}_T|;\mu,\zeta) = \int d^2\boldsymbol{k}_\perp\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{k}_\perp}\, f_1^a(x,\boldsymbol{k}_\perp^2;\mu,\zeta)$$

perturbative Sudakov form factor

$$\hat{f}_1^a(x,b_T^2;\mu_f,\zeta_f) = [C \otimes f_1](x,\mu_{b_*})\, e^{\int_{\mu_{b_*}}^{\mu_f}\frac{d\mu}{\mu}\left(\gamma_F-\gamma_K\ \ln\frac{\sqrt{\zeta_f}}{\mu}\right)} \left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}}\right)^{K_{\text{resum}}+g_K} f_{1\,NP}(x,b_T^2;\zeta_f,Q_0)$$

collinear PDF

matching coefficients (perturbative)

Collins-Soper kernel (perturbative and nonperturbative)

nonperturbative part of TMD

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

⬇

$$\mu_{b*} = \frac{2e^{-\gamma_E}}{\bar{b}_*}$$

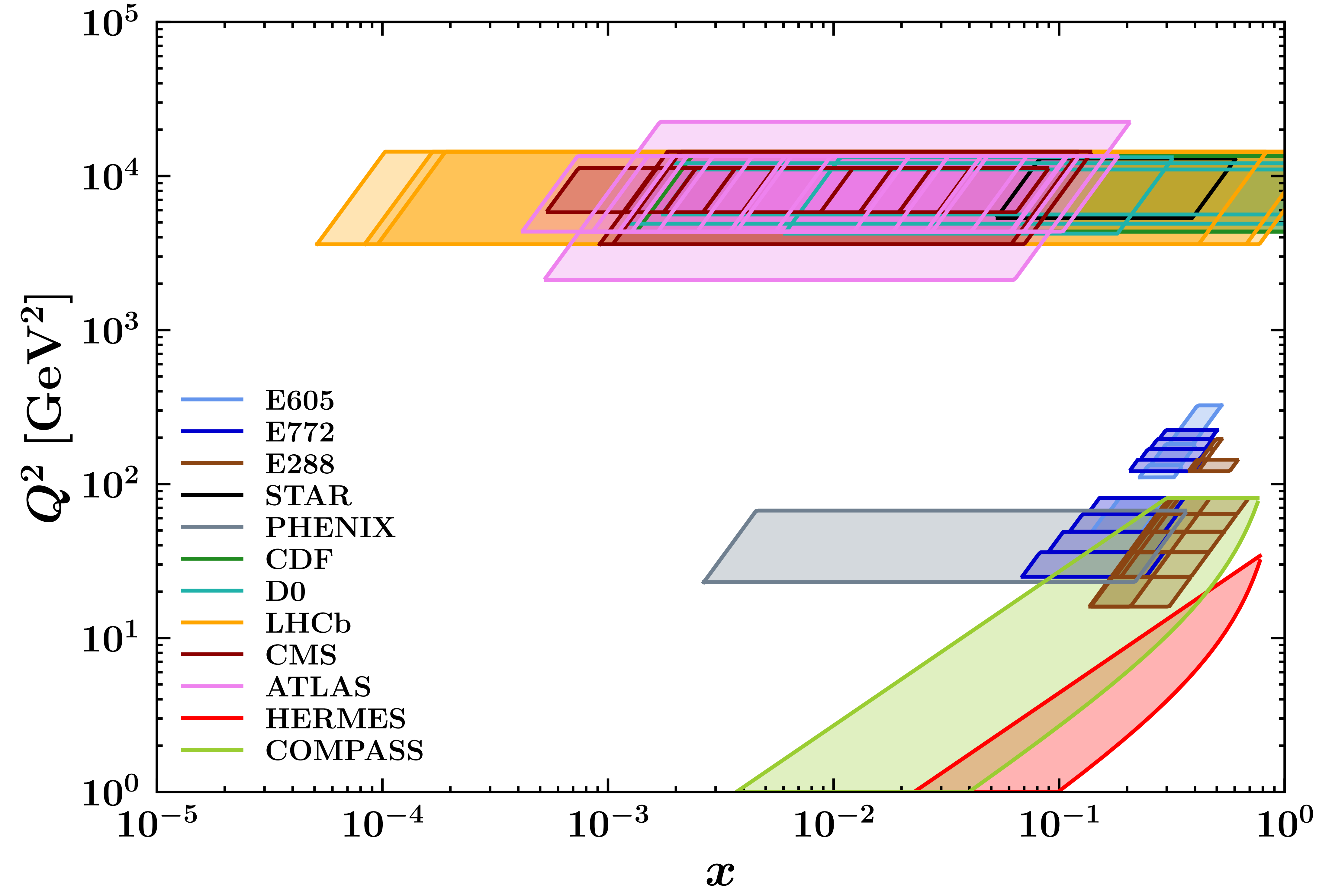
see, e.g., Collins, "Foundations of Perturbative QCD" (11)

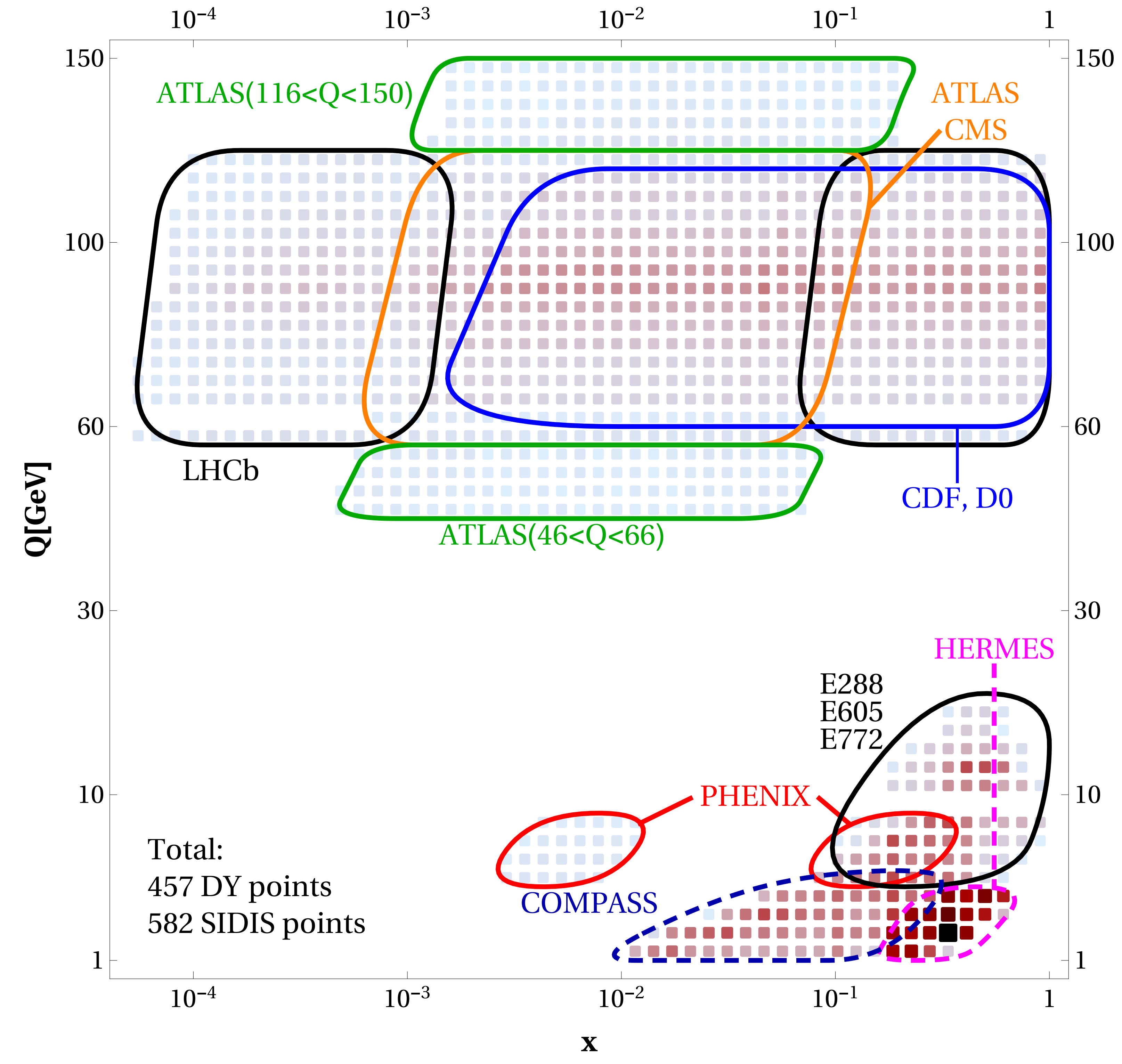
TMD collaboration, "TMD Handbook," arXiv:2304.03302

TMD GLOBAL FITS

	Accuracy	SIDIS HERMES	SIDIS COMPASS	DY fixed target	DY collider	N of points	χ^2/N_{points}
Pavia 2017 arXiv:1703.10157	NLL	✔	✔	✔	✔	8059	1.55
SV 2019 arXiv:1912.06532	N^3LL^-	✔	✔	✔	✔	1039	1.06
MAP22 arXiv:2206.07598	N^3LL^-	✔	✔	✔	✔	2031	1.06

x–Q^2 COVERAGE

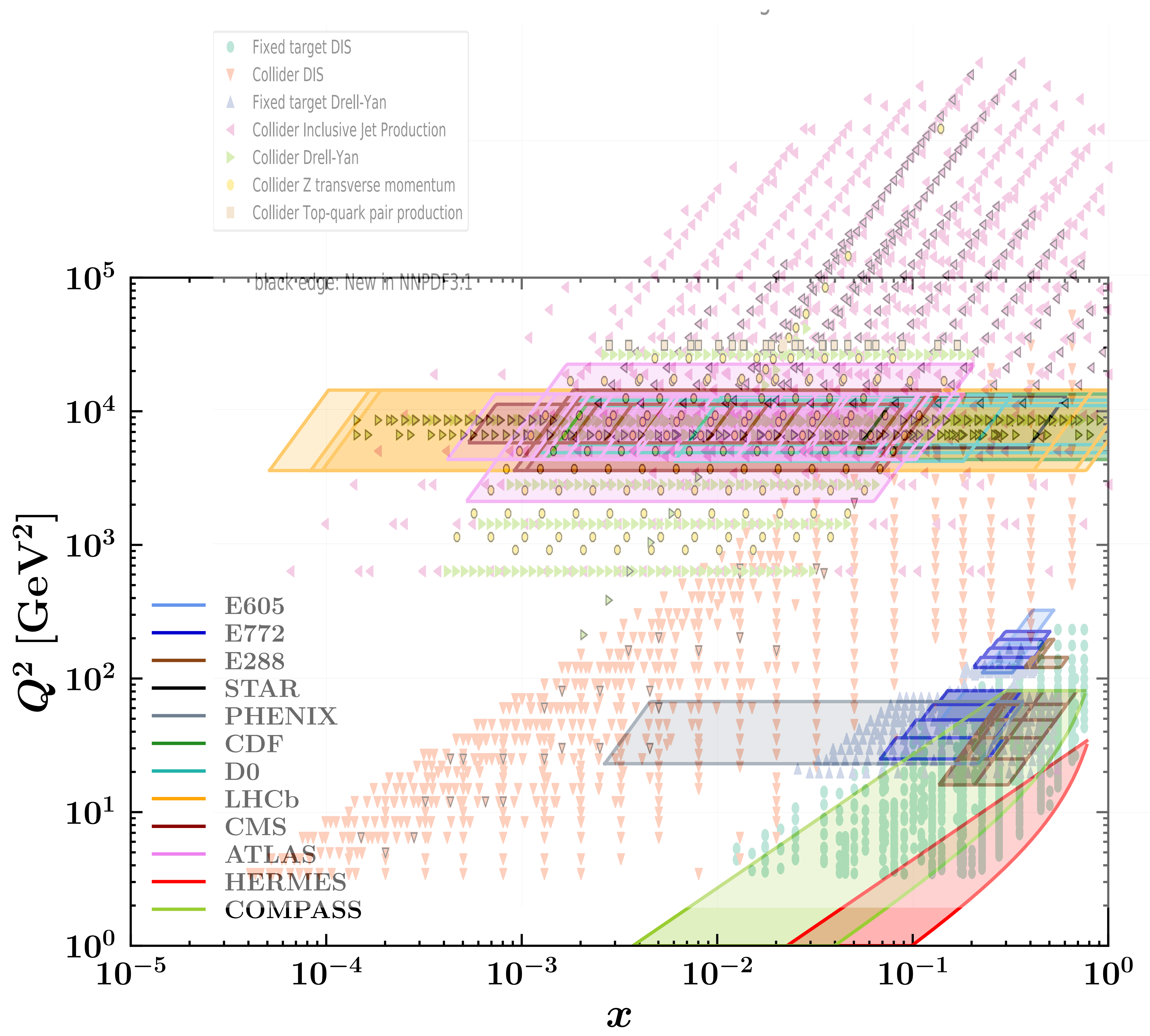


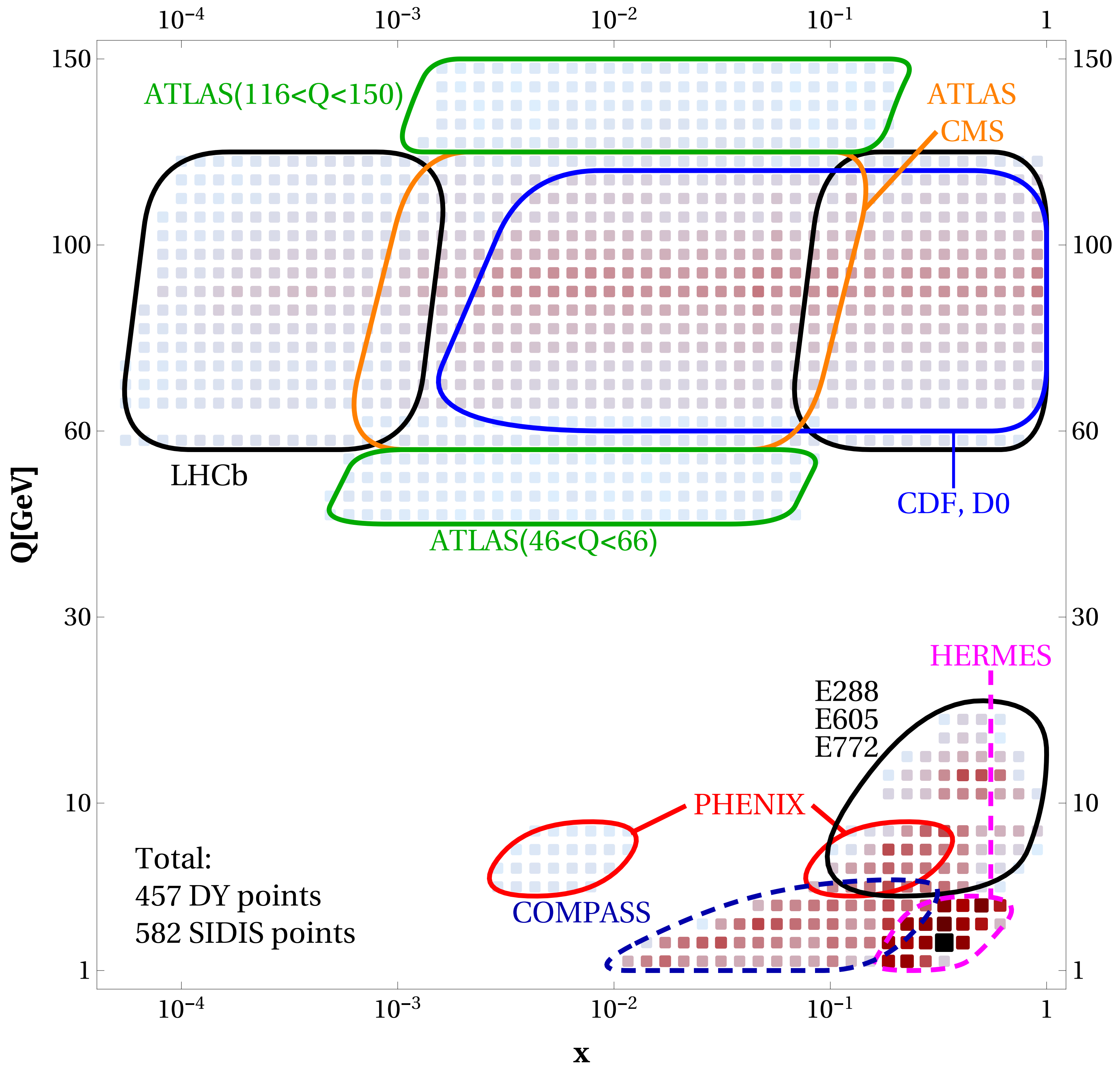


MAP Collaboration
Bacchetta, Bertone, Bissolotti, Bozzi, Cerutti,
Piacenza, Radici, Signori, arXiv:2206.07598

Scimemi, Vladimirov,
arXiv:1912.06532

x–Q^2 COVERAGE





MAP Collaboration
Bacchetta, Bertone, Bissolotti, Bozzi, Cerutti, Piacenza, Radici, Signori, arXiv:2206.07598

Scimemi, Vladimirov, arXiv:1912.06532

EXAMPLE OF RESULTING TMDS

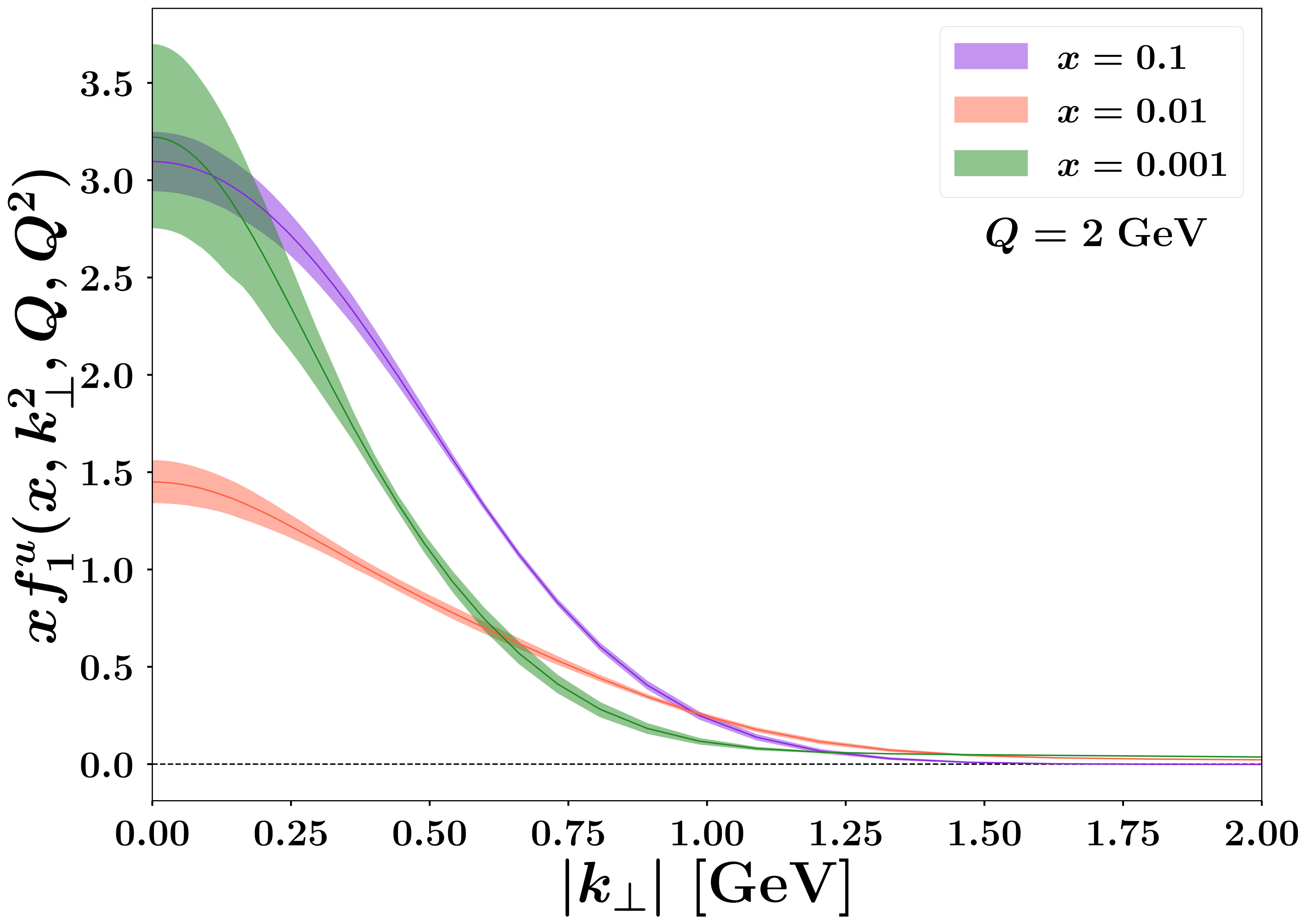


FIG. 13: The TMD PDF of the up quark in a proton at $\mu = \sqrt{\zeta} = Q = 2$ GeV (left panel) and 10 GeV (right panel) as a function of the partonic transverse momentum $|\boldsymbol{k}_\perp|$ for $x = 0.001$, 0.01 and 0.1. The uncertainty bands represent the 68% CL.

EXAMPLE OF RESULTING TMDS

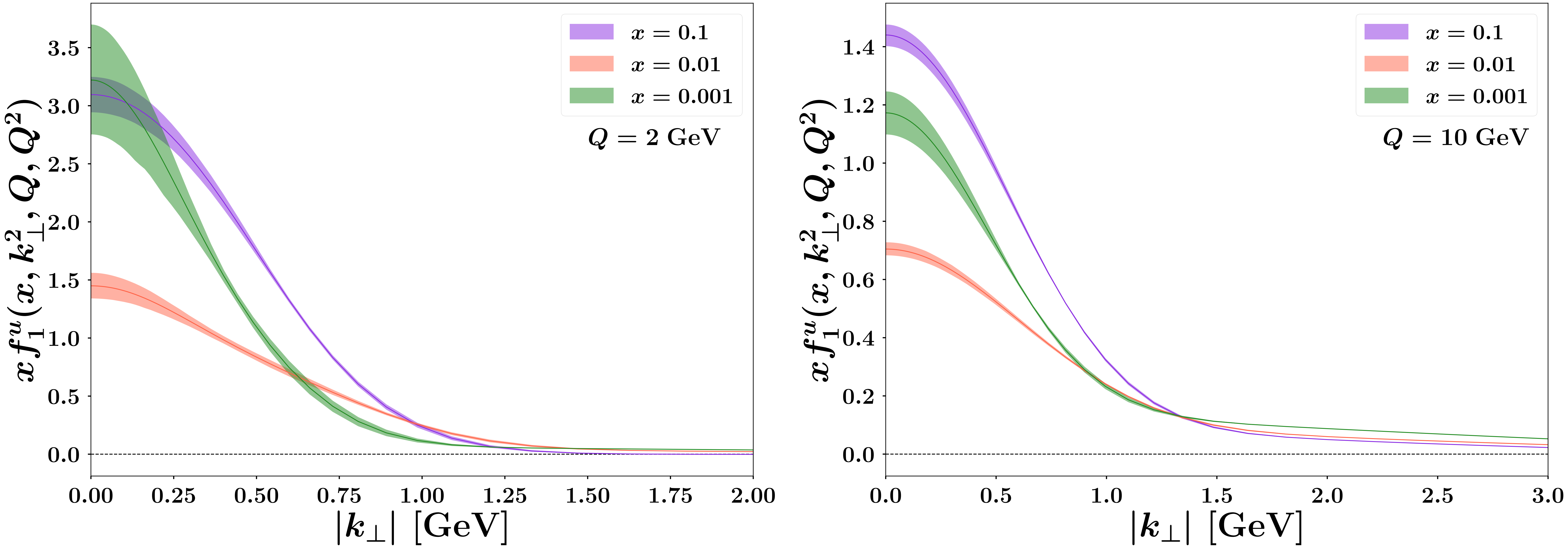
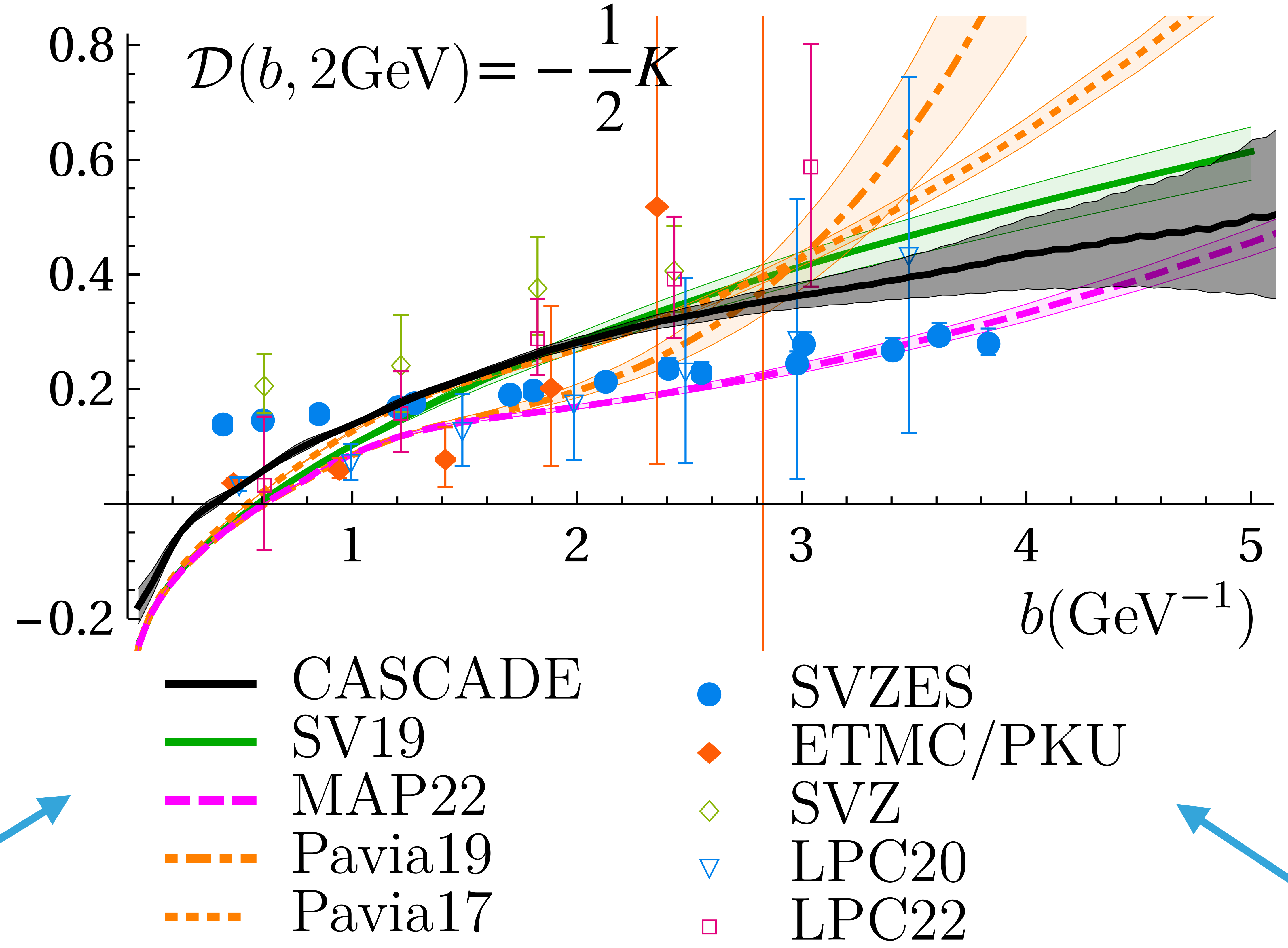


FIG. 13: The TMD PDF of the up quark in a proton at $\mu = \sqrt{\zeta} = Q = 2$ GeV (left panel) and 10 GeV (right panel) as a function of the partonic transverse momentum $|\boldsymbol{k}_\perp|$ for $x = 0.001$, 0.01 and 0.1. The uncertainty bands represent the 68% CL.

CONNECTIONS WITH LATTICE QCD: COLLINS–SOPER KERNEL

Bermudez Martinez, Vladimirov, arXiv:2206.01105

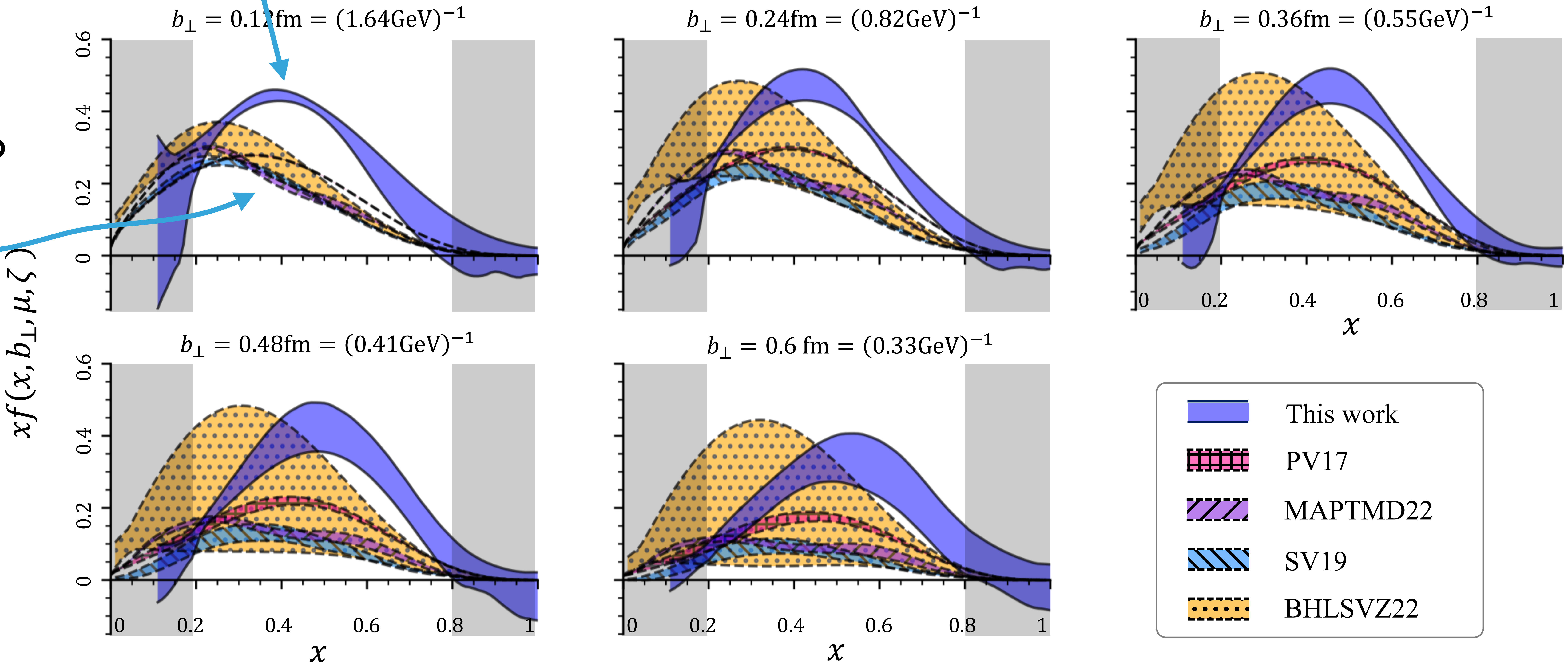


TMD phenomenology

Lattice QCD

CONNECTION WITH LATTICE QCD: TMDS

[LPC collaboration, arxiv:2211.02340](https://arxiv.org/abs/2211.02340)



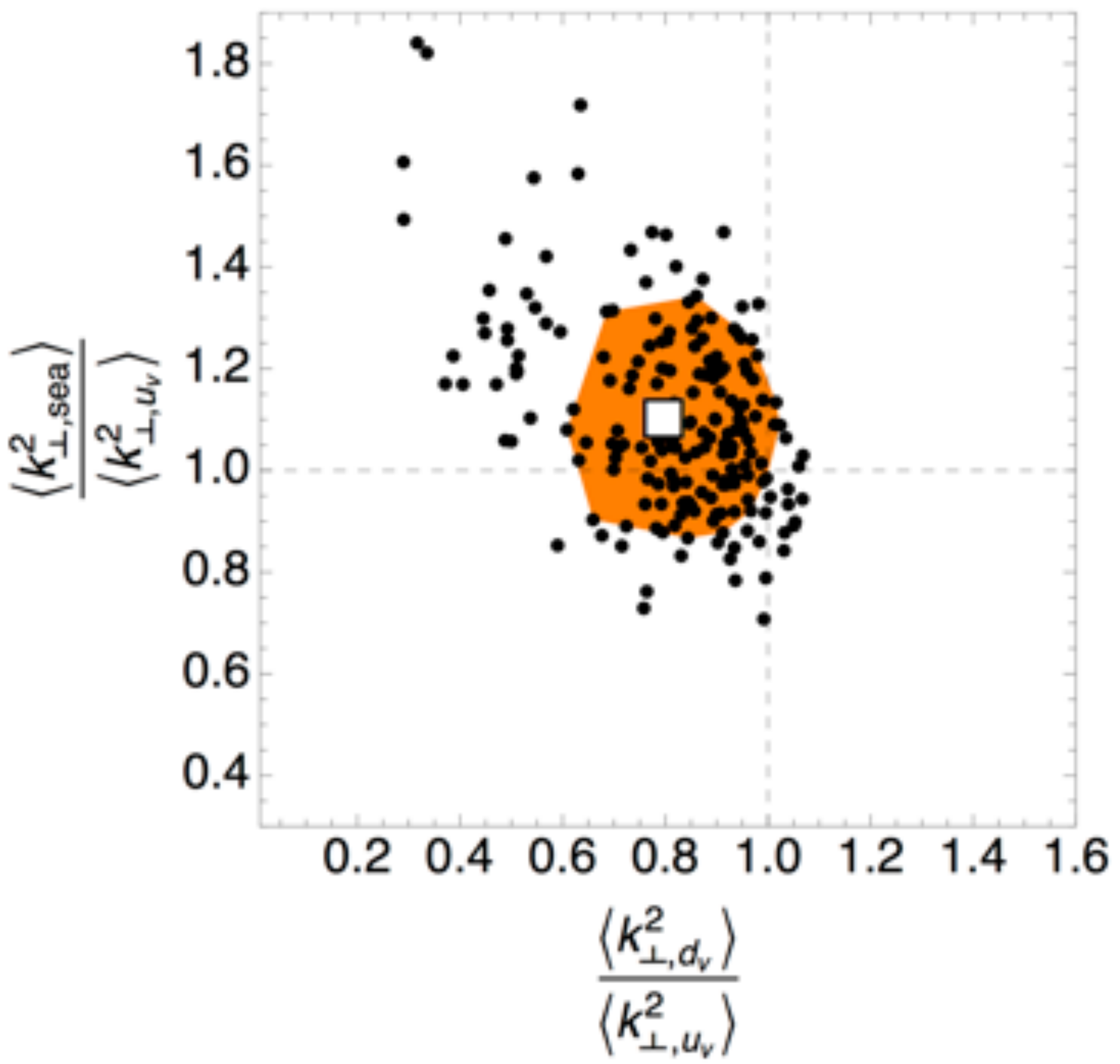
Lattice QCD

TMD pheno

FLAVOR DEPENDENCE OF TMDS

Signori, Bacchetta, Radici, Schnell JHEP 1311 (13)

Ratio of width of sea / width of up valence

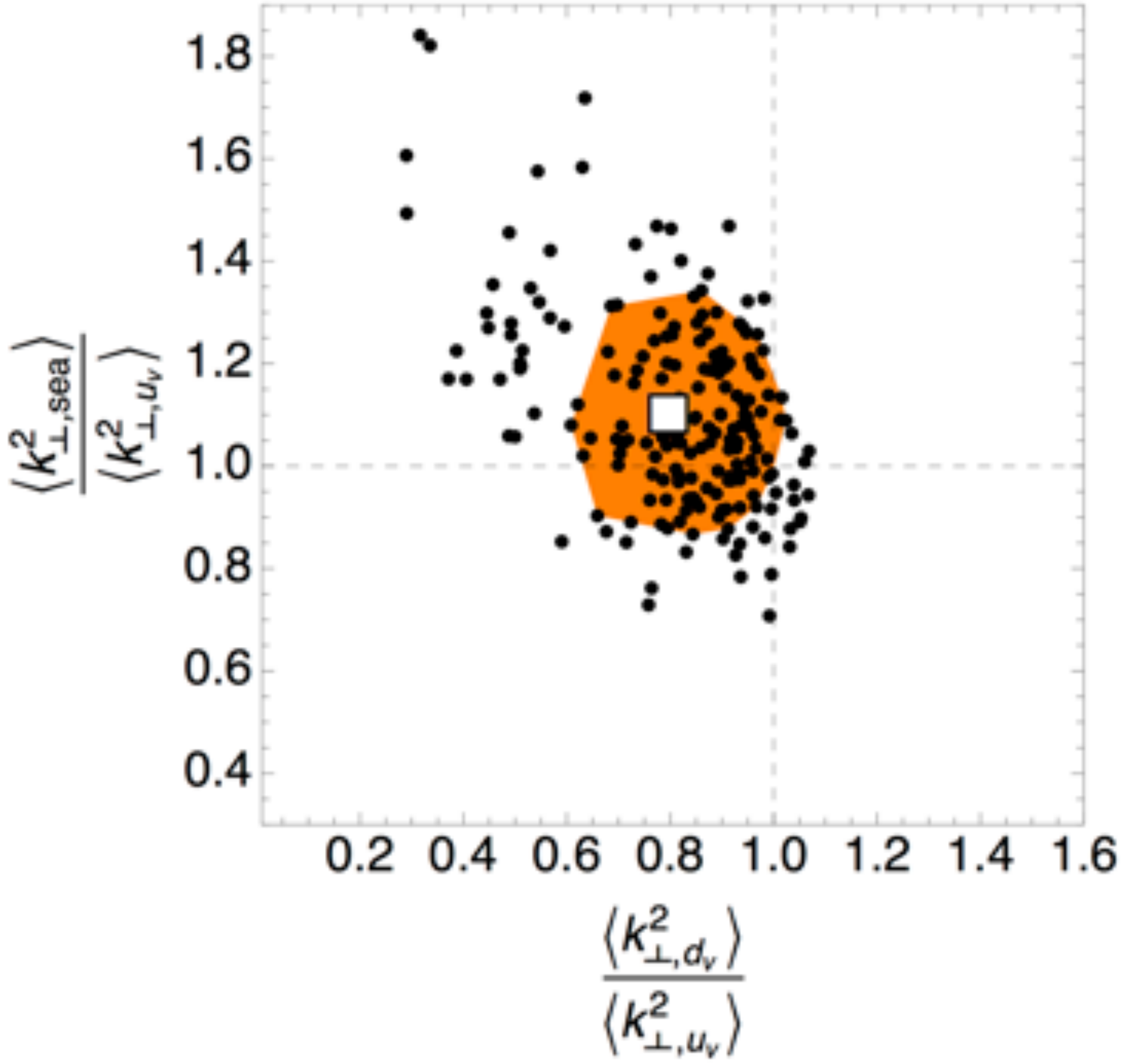


Ratio width of down valence/
width of up valence

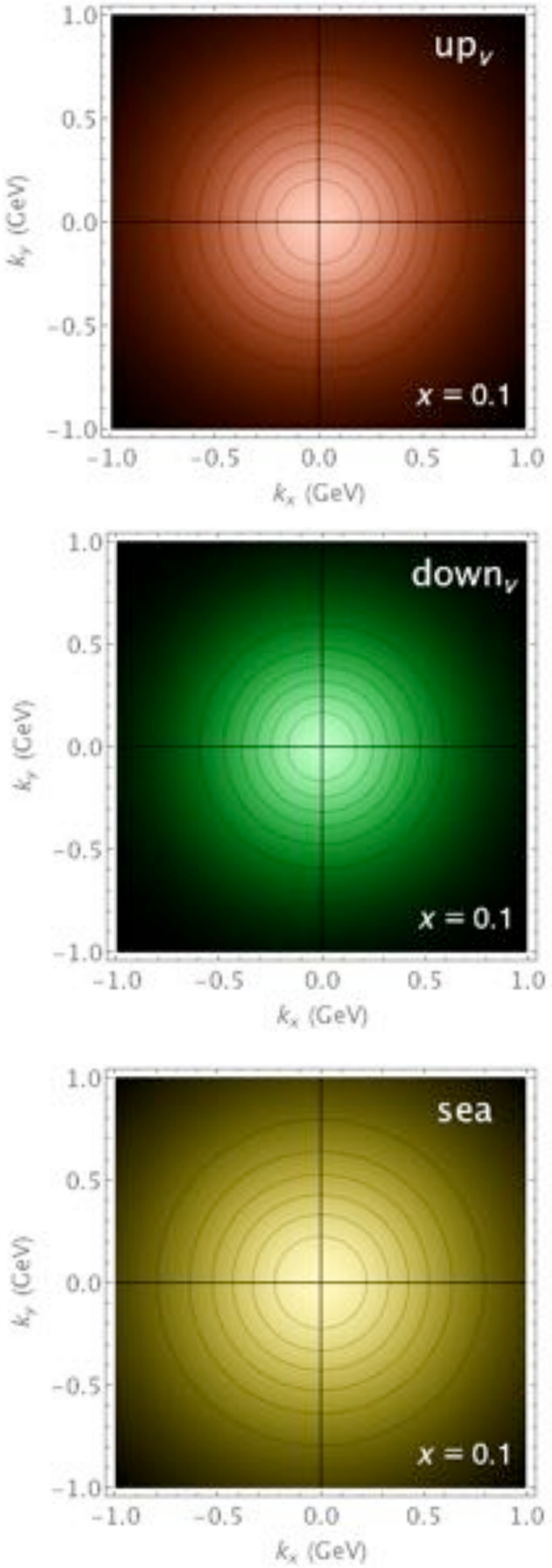
FLAVOR DEPENDENCE OF TMDS

Signori, Bacchetta, Radici, Schnell JHEP 1311 (13)

Ratio of width of sea / width of up valence



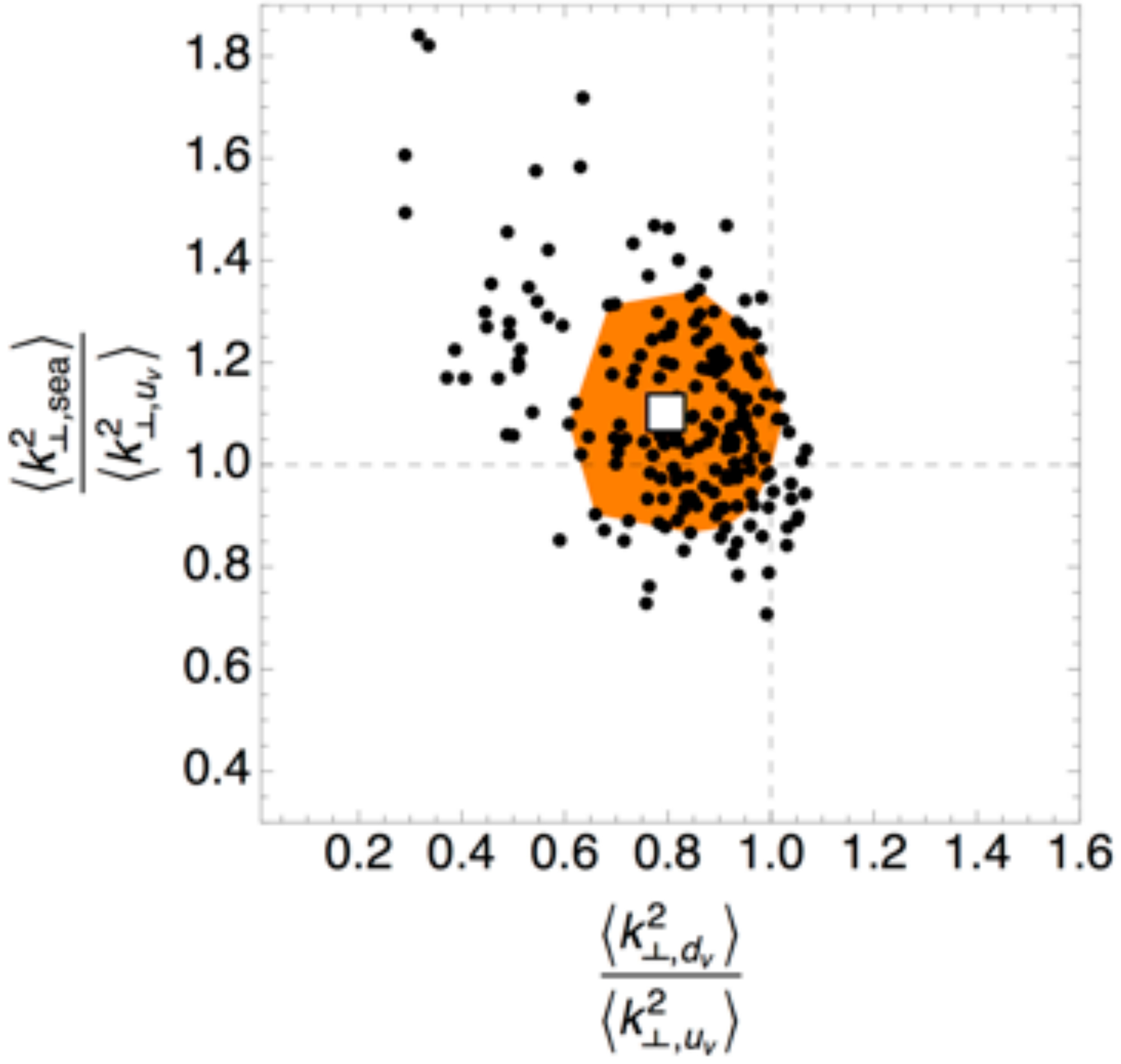
Ratio width of down valence/ width of up valence



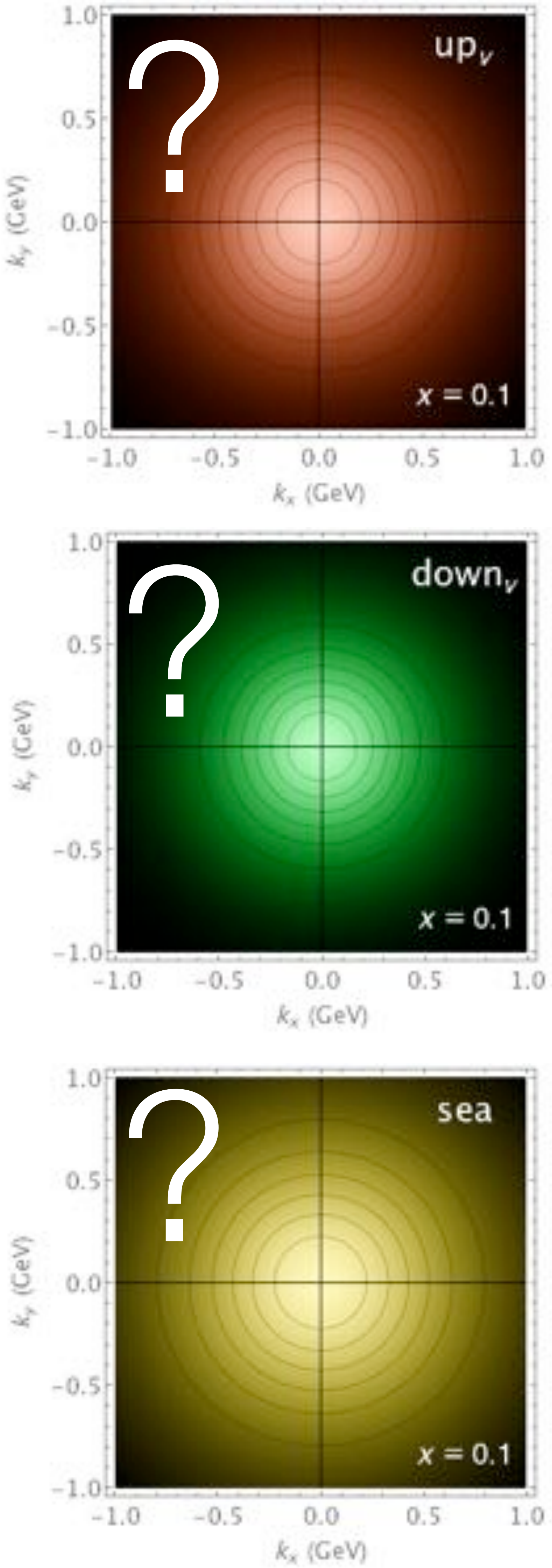
FLAVOR DEPENDENCE OF TMDS

Signori, Bacchetta, Radici, Schnell JHEP 1311 (13)

Ratio of width of sea / width of up valence



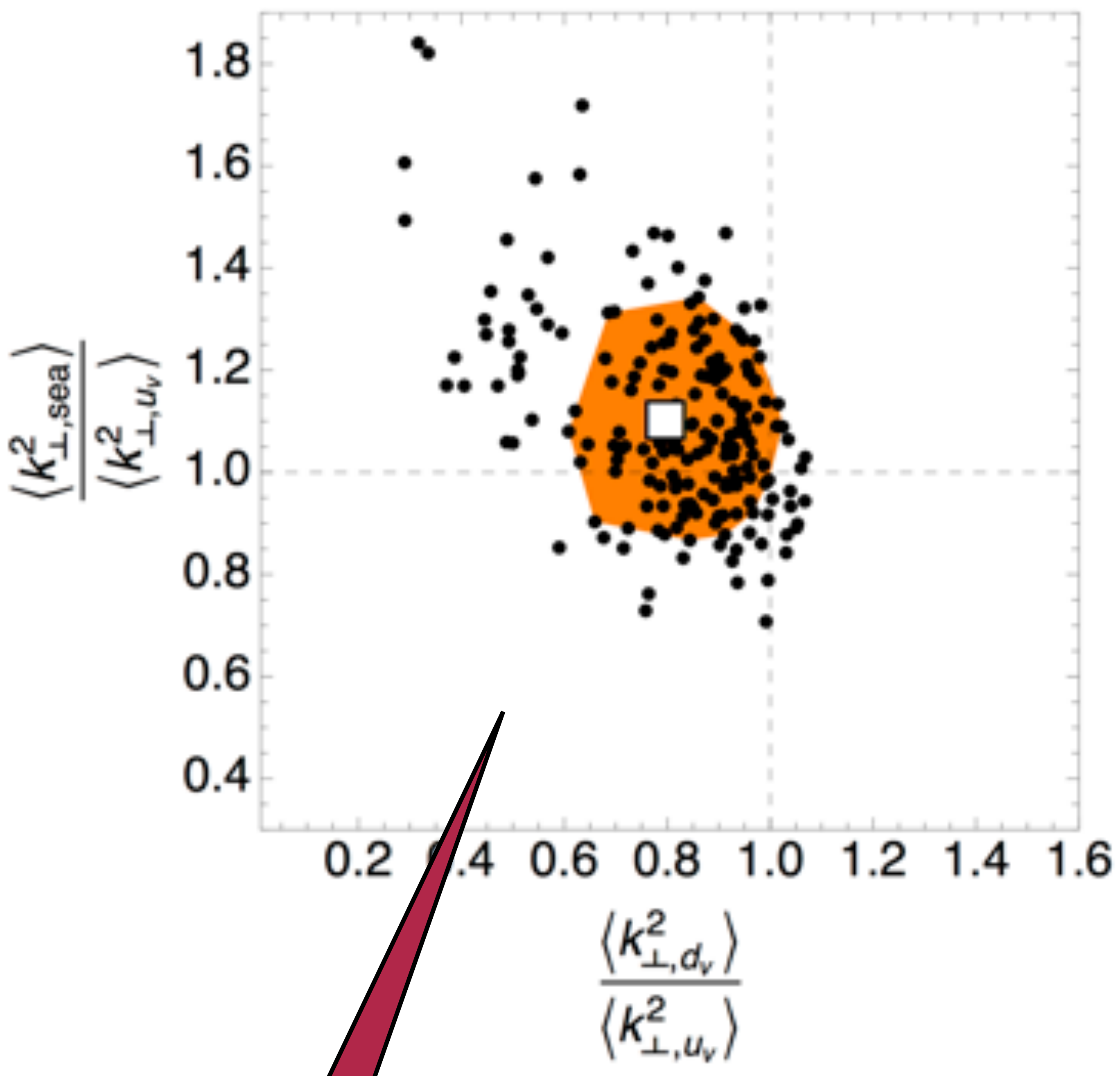
Ratio width of down valence/ width of up valence



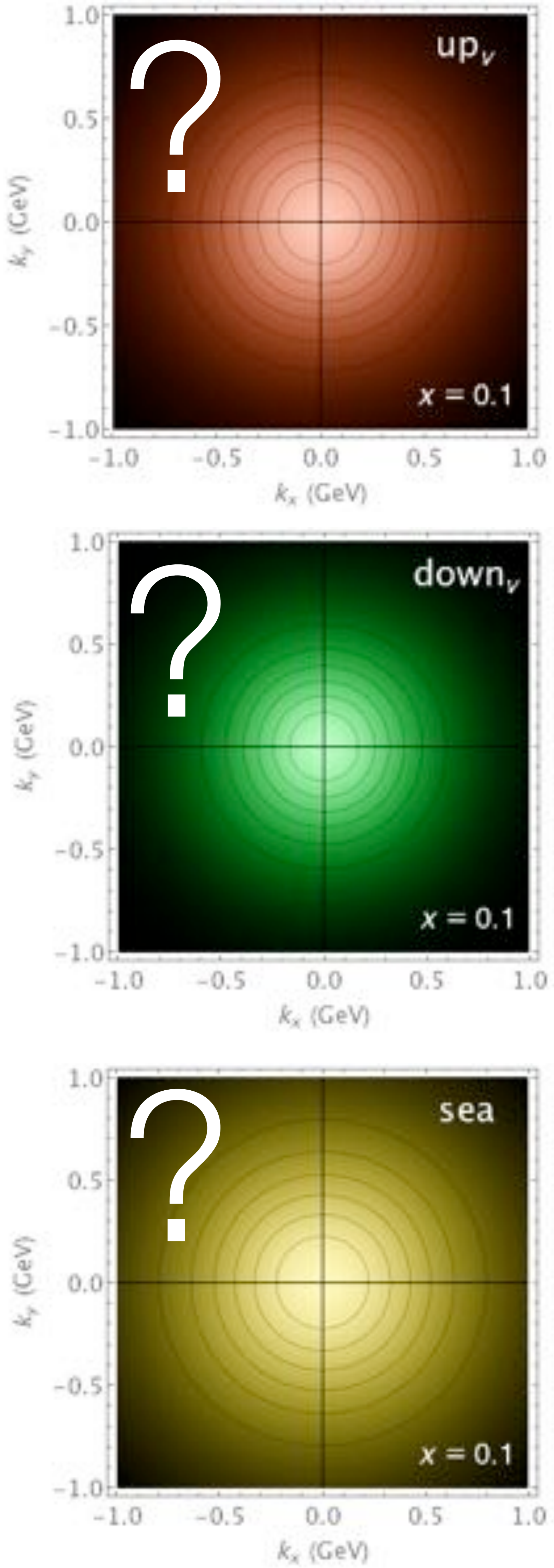
FLAVOR DEPENDENCE OF TMDS

Signori, Bacchetta, Radici, Schnell JHEP 1311 (13)

Ratio of width of sea / width of up valence



Ratio vidth of down valence/
vidth of up valence



First investigation 10 years ago:
there is room for flavor dependence,
but we don't control it well

MOST RECENT EXTRACTION

ART23

Moos, Scimemi, Vladimirov, Zurita, 2305.07473

N^4LL^- accuracy

Drell-Yan only

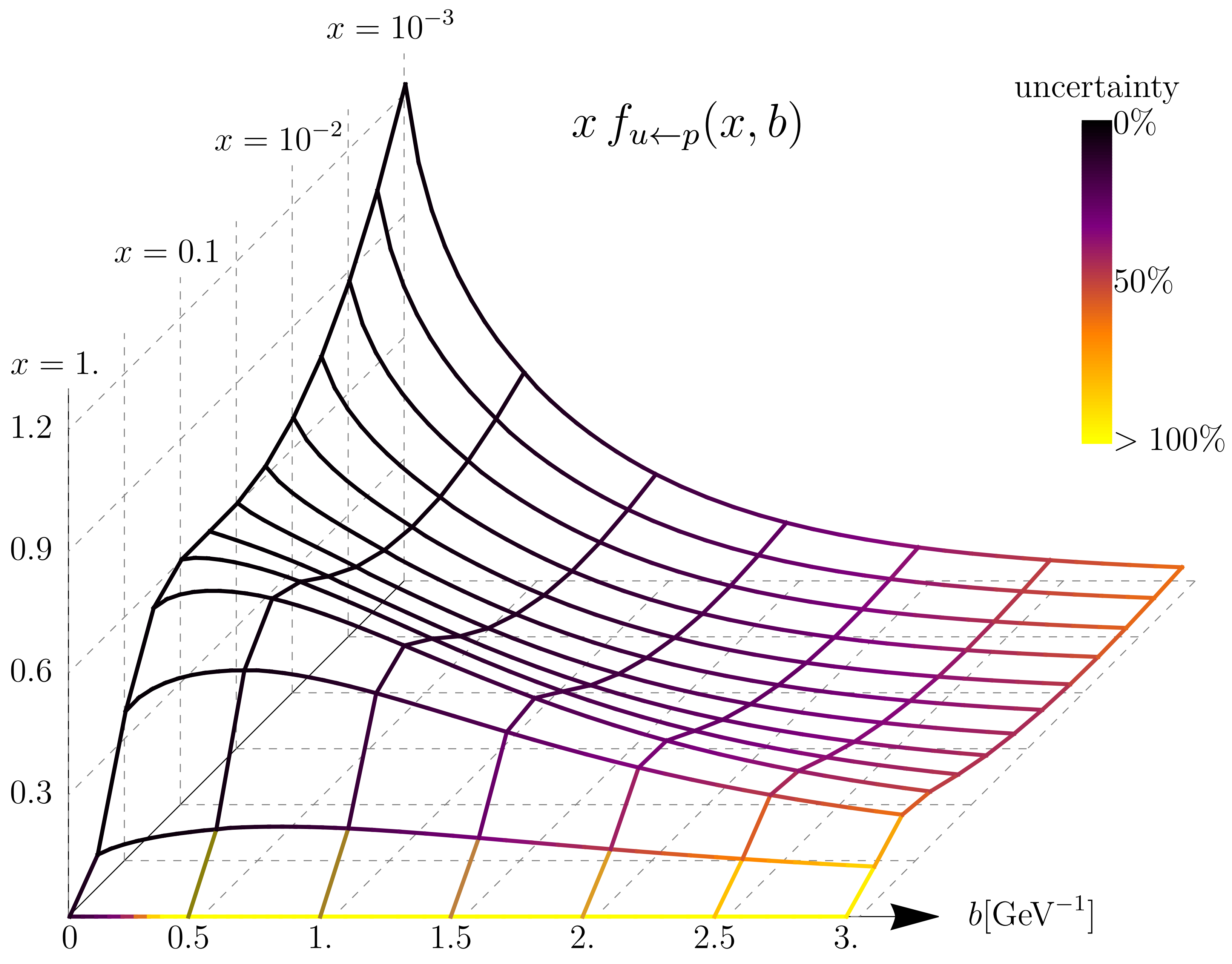
MOST RECENT EXTRACTION

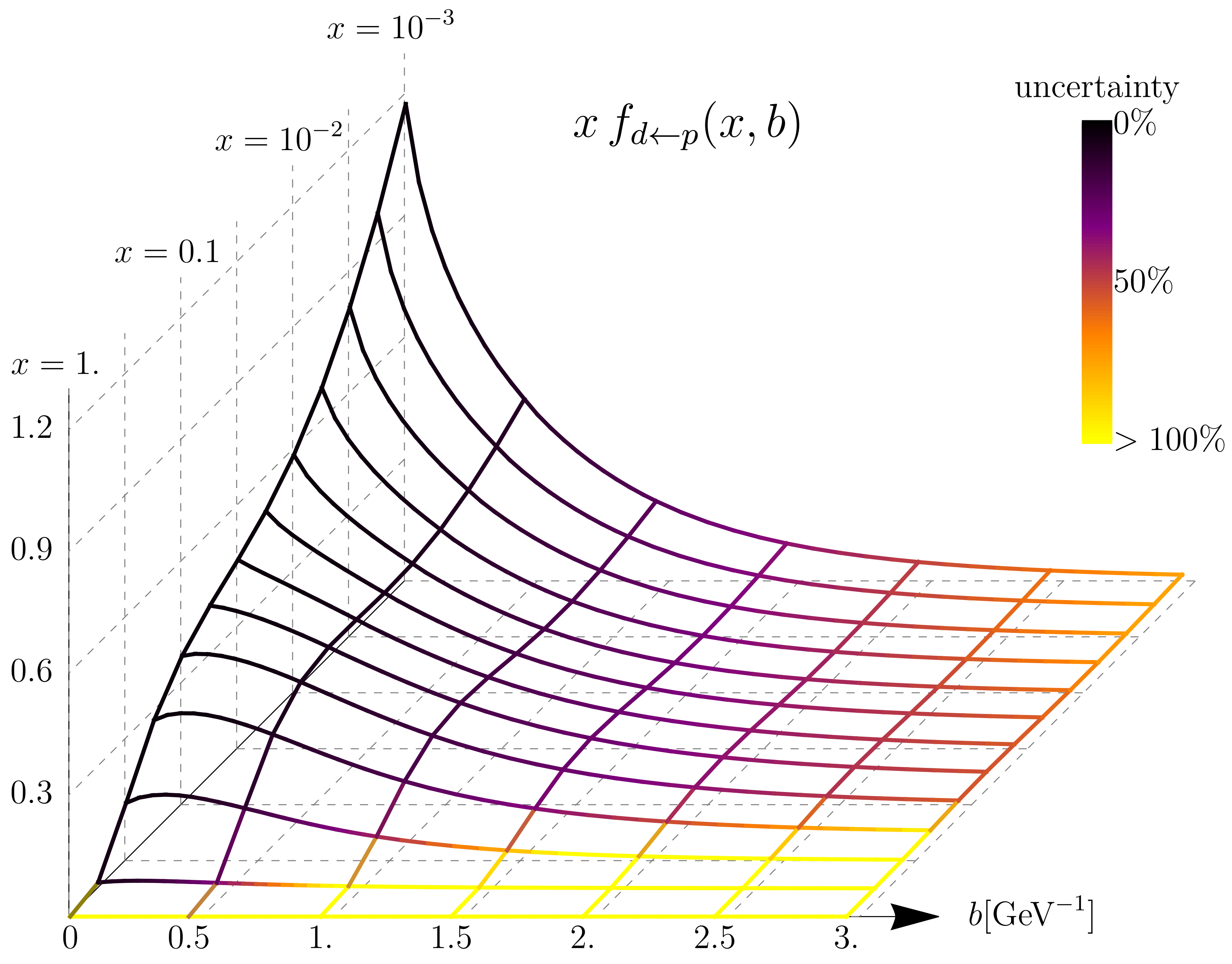
ART23

Moos, Scimemi, Vladimirov, Zurita, 2305.07473

N^{4}LL$^-$ accuracy

Drell-Yan only





MOST RECENT EXTRACTION

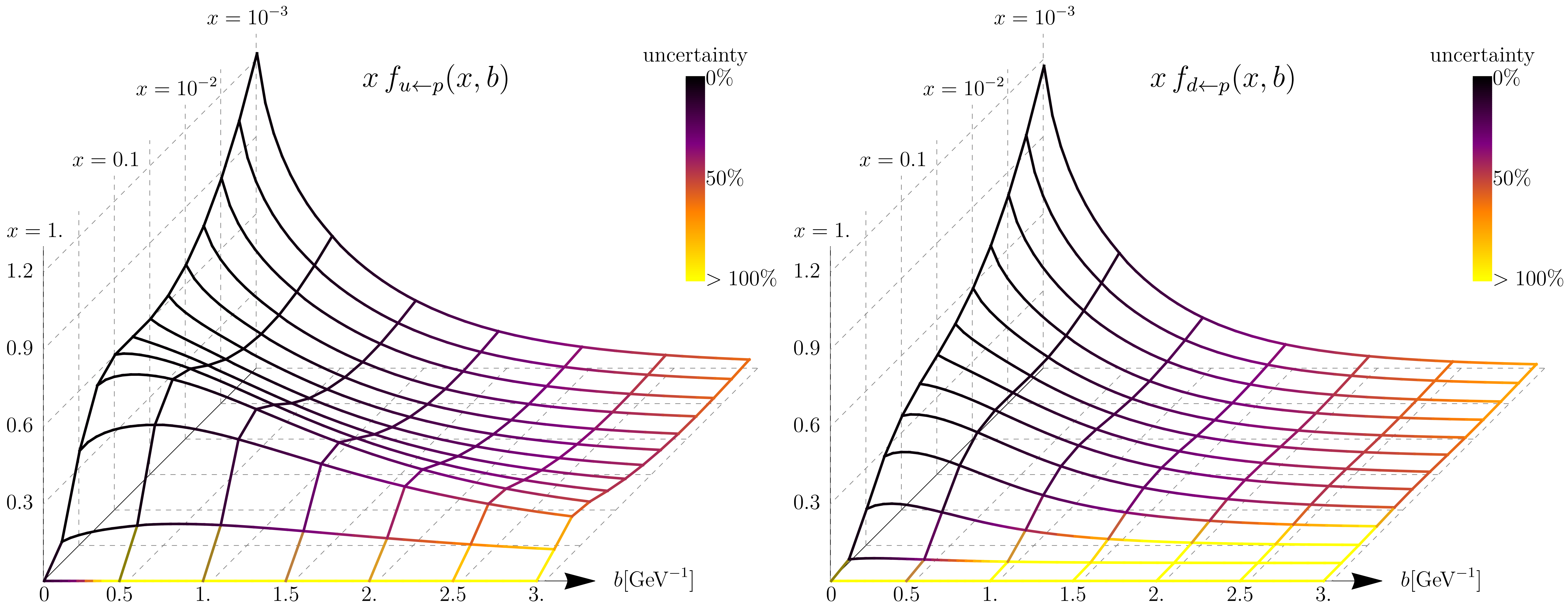
ART23

Moos, Scimemi, Vladimirov, Zurita, 2305.07473

N^{4}LL$^-$ accuracy

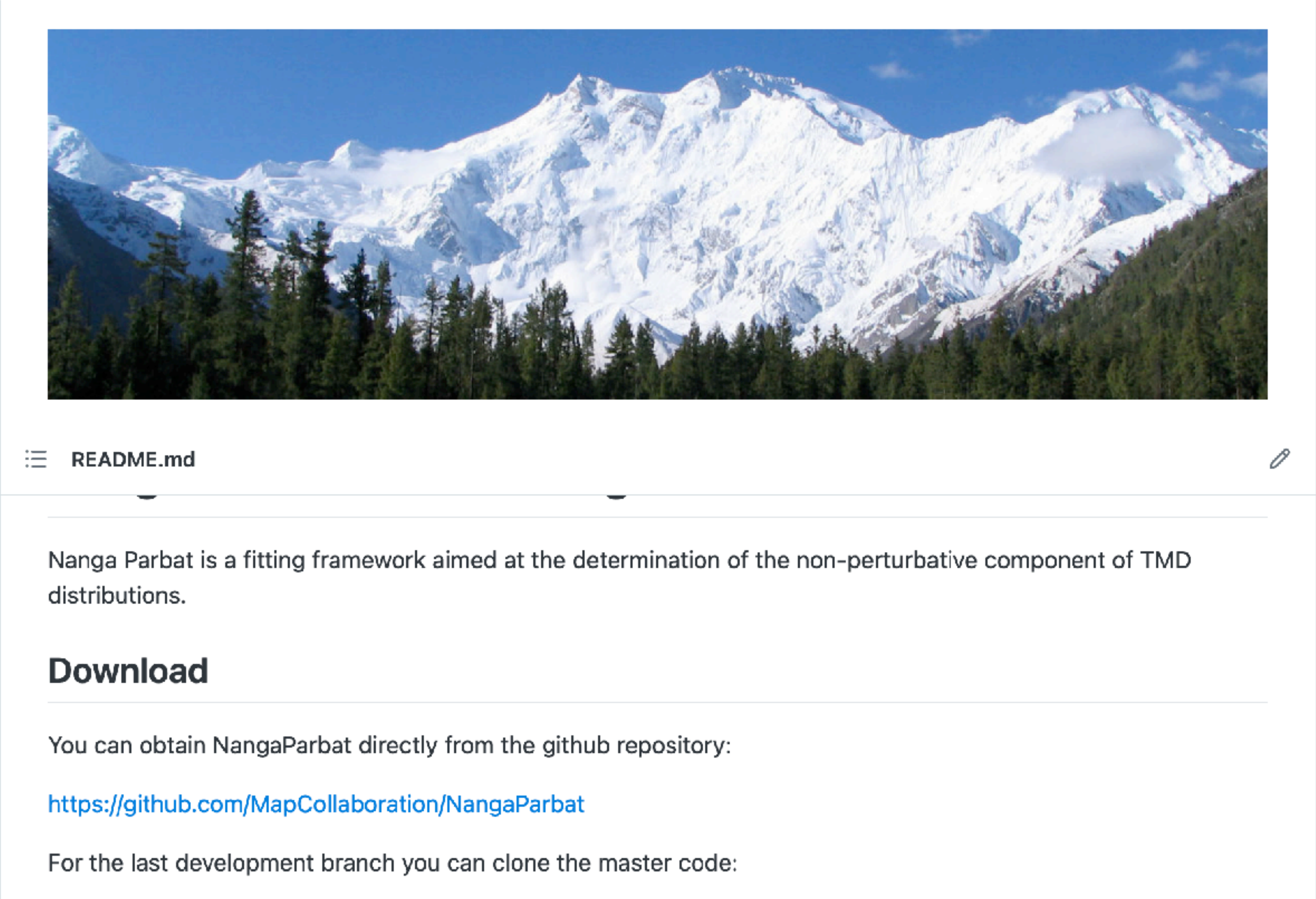
Drell-Yan only

Different up and down TMDs



AVAILABLE TOOLS: NANGA PARBAT

https://github.com/MapCollaboration/NangaParbat



Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

Download

You can obtain NangaParbat directly from the github repository:

https://github.com/MapCollaboration/NangaParbat

For the last development branch you can clone the master code:

AVAILABLE TOOLS: ARTEMIDE

https://teorica.fis.ucm.es/artemide/

arTeMiDe

News

12 Dec 2019: Version 2.02 released (+manual update).

23 Feb 2019: Version 1.4 released (+manual update).

21 Jan 2019: Artemide now has a repository.

Archive of older links/news.

Articles, presentations & supplementary materials

Extra pictures for the paper arXiv:1902.08474

Seminar of A.Vladimirov in Pavia 2018 on TMD evolution.

Link to the text in Inspire.

Archive of older links/news.

Download

Recent version/release can be found in repository

About us & Contacts

If you have found mistakes, or have suggestions/questions,

please, contact us.

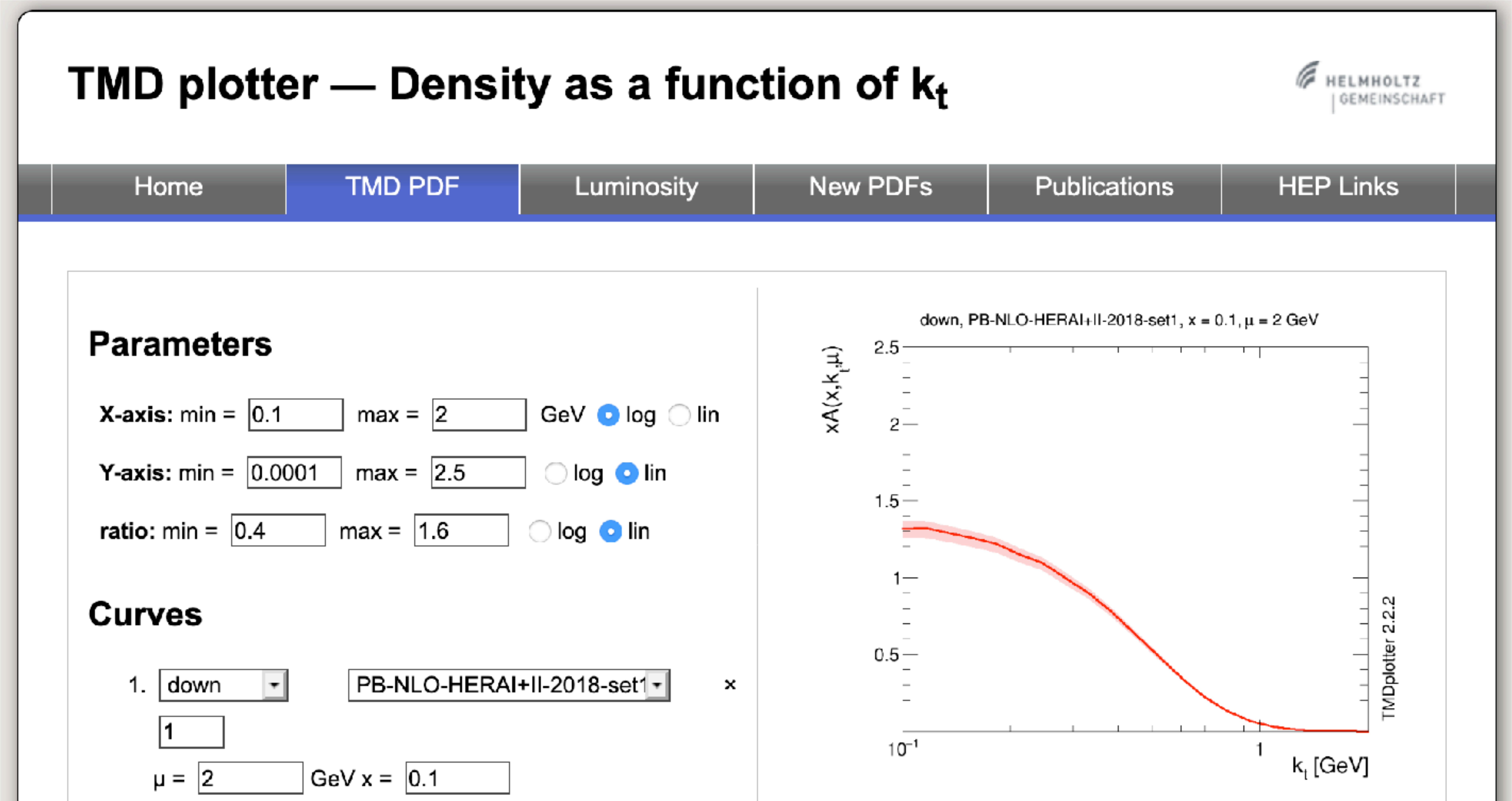
Some extra materials can be found on Alexey's web-page

Alexey Vladimirov Alexey.Vladimirov@physik.uni-regensburg.de

Ignazio Scimemi ignazios@fis.ucm.es

AVAILABLE TOOLS: TMDLIB AND TMDPLOTTER

https://tmdlib.hepforge.org/



TMD plotter — Density as a function of k_t

Parameters

X-axis: min = 0.1 max = 2 GeV ● log ○ lin

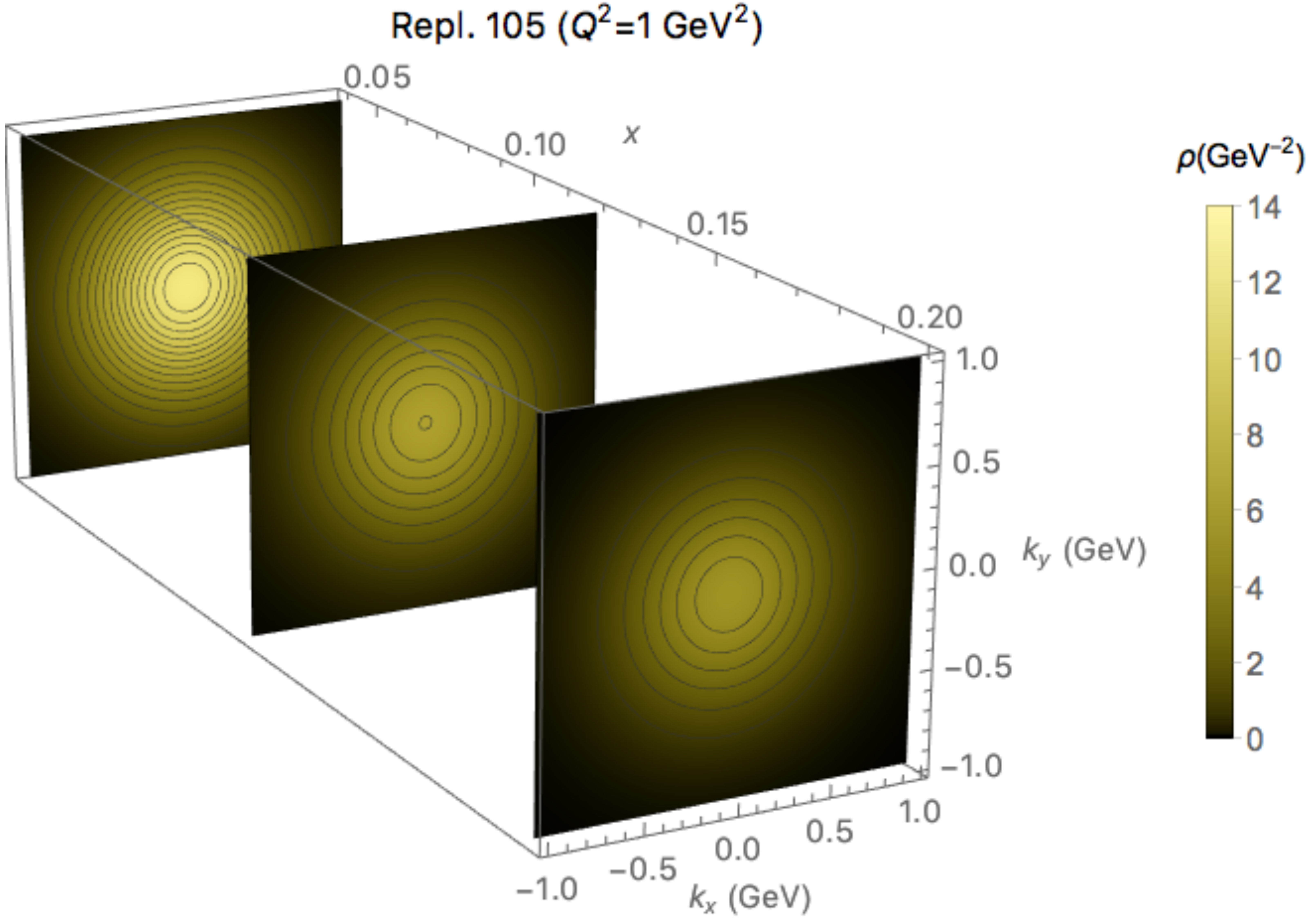
Y-axis: min = 0.0001 max = 2.5 ○ log ● lin

ratio: min = 0.4 max = 1.6 ○ log ● lin

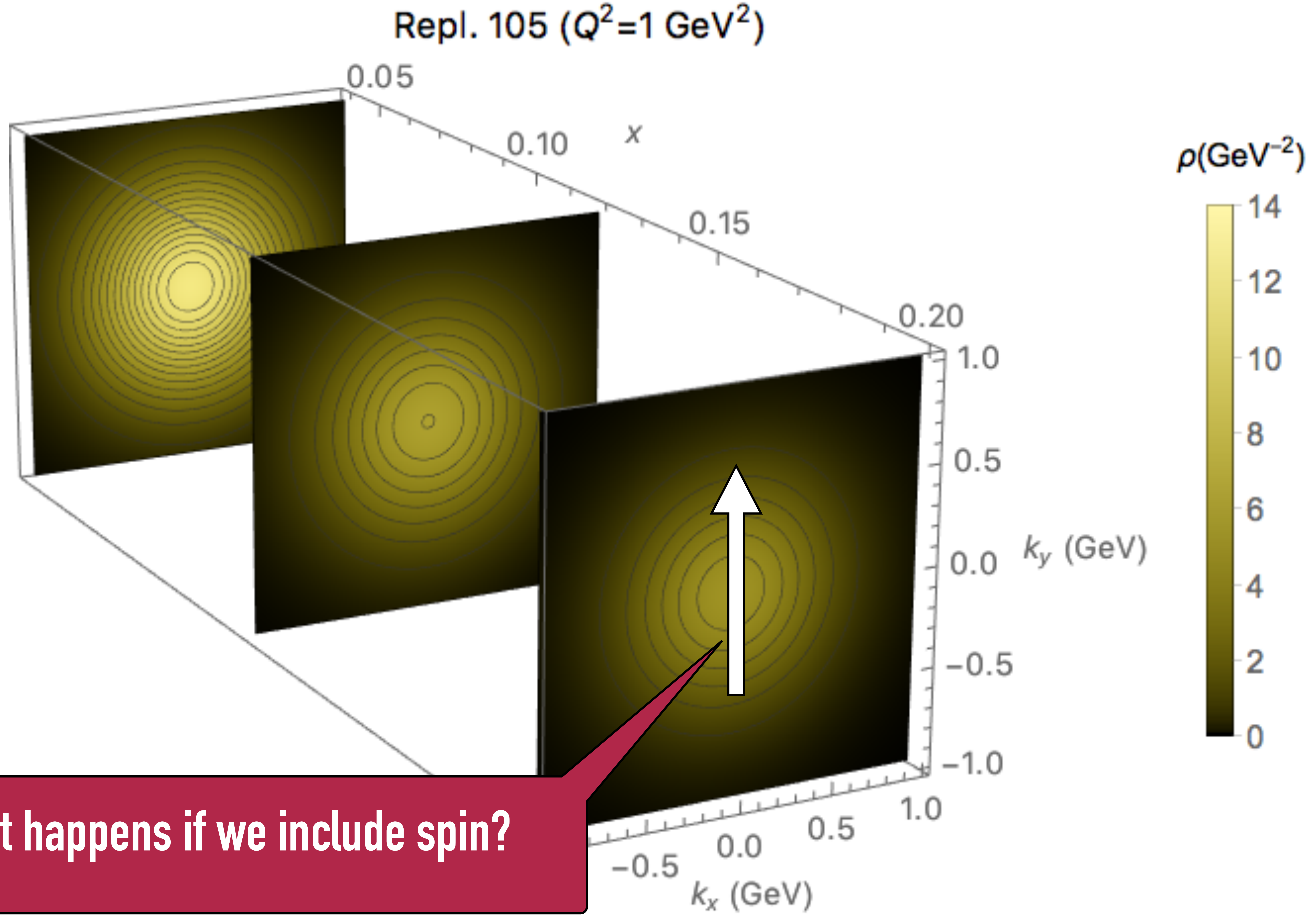
Curves

1. down PB-NLO-HERAI+II-2018-set1 ×
1
μ = 2 GeV x = 0.1

PRESENT KNOWLEDGE (OR LACK OF KNOWLEDGE)?



PRESENT KNOWLEDGE (OR LACK OF KNOWLEDGE)?



What happens if we include spin?

SIVERS FUNCTION

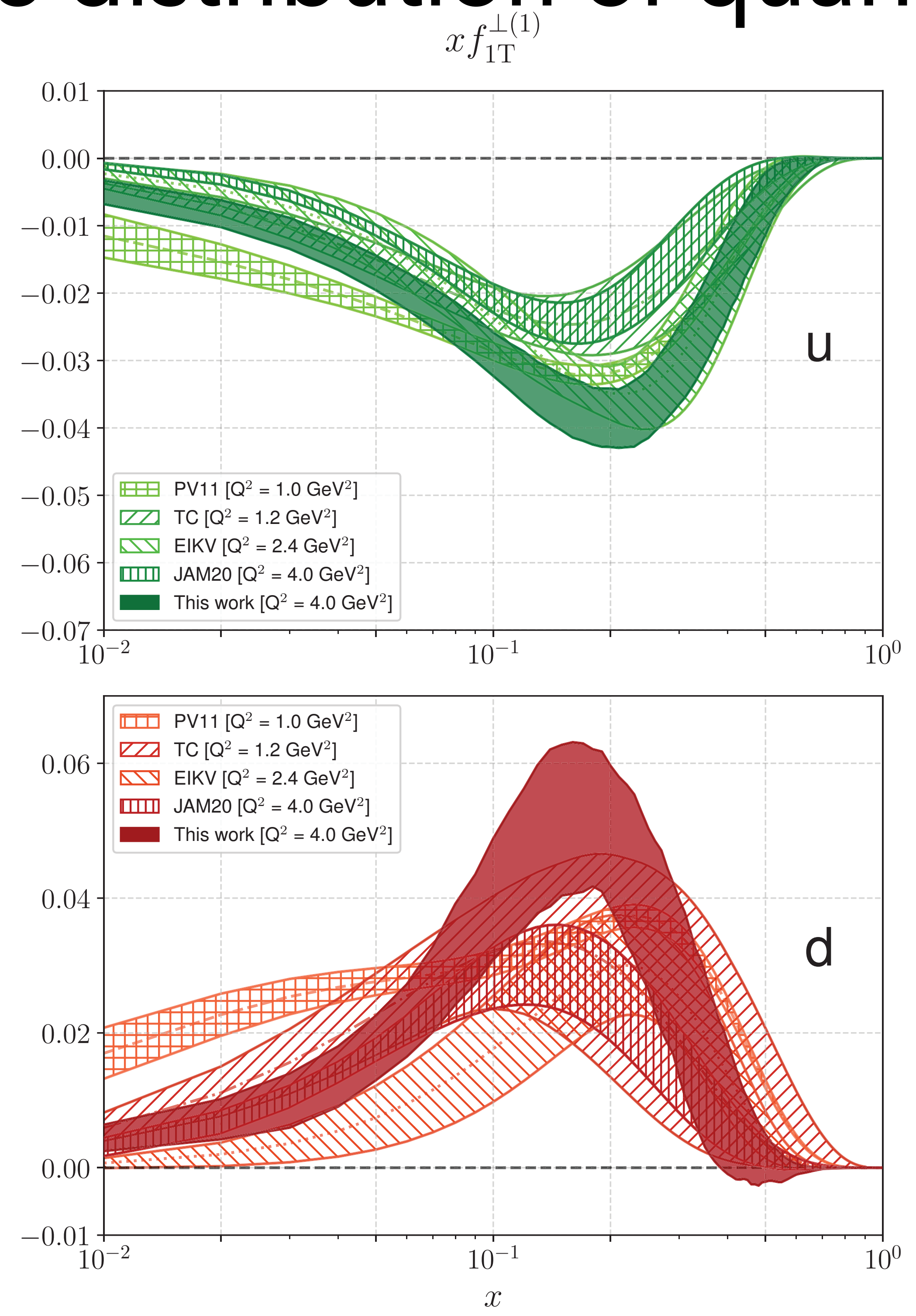
$$\rho^{q}_{N^{\uparrow}}(x, k_x, k_y; Q^2) = f^{q}_{1}(x, k_T^2; Q^2) - \frac{k_x}{M} f^{\perp q}_{1T}(x, k_T^2; Q^2)$$

In a nucleon polarized in the +y direction,
the distribution of quarks can be distorted in the x direction

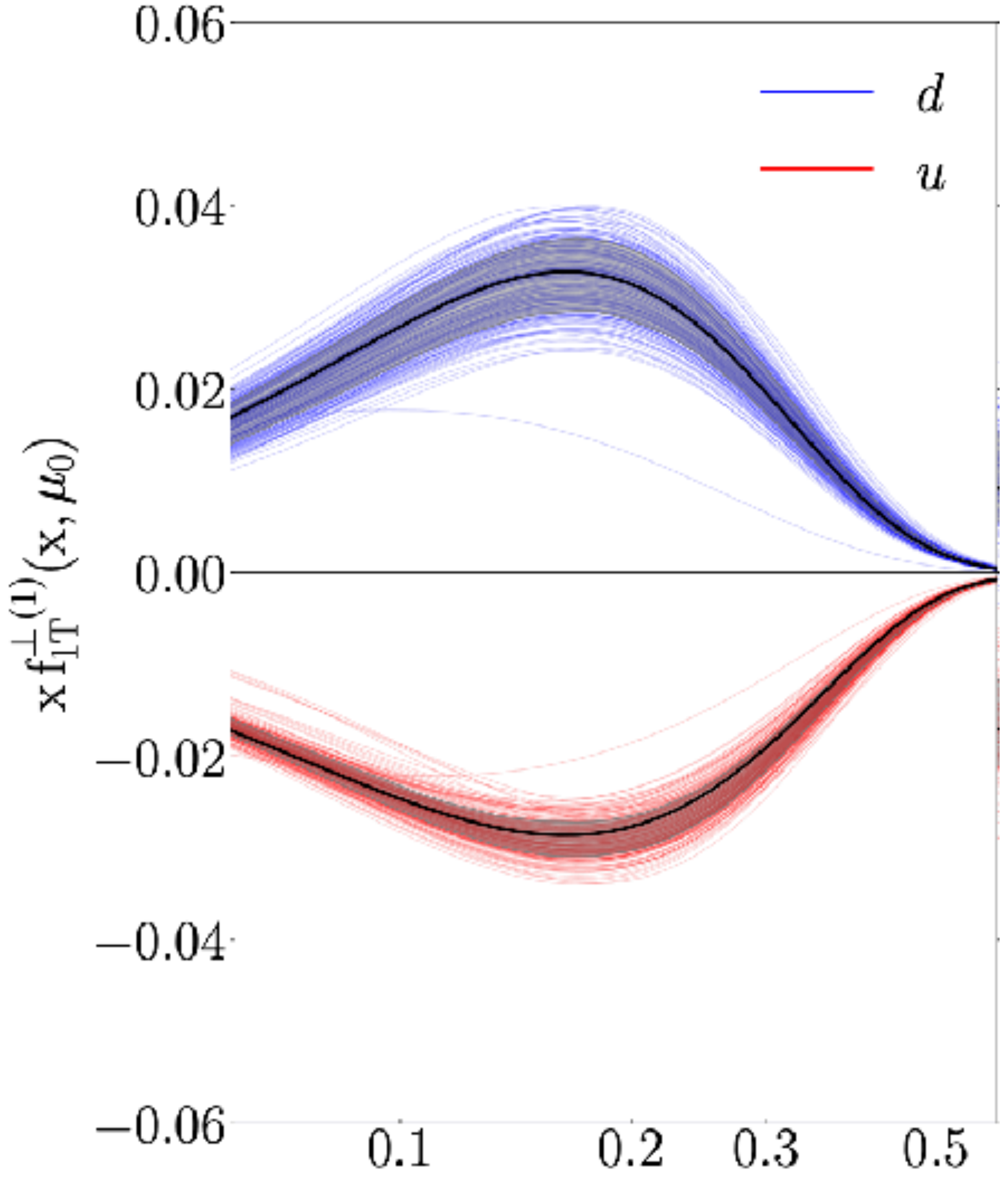
SIVERS FUNCTION

$$\rho^q_{N^\uparrow}(x, k_x, k_y; Q^2) = f_1^q(x, k_T^2; Q^2) - \frac{k_x}{M} f_{1T}^{\perp q}(x, k_T^2; Q^2)$$

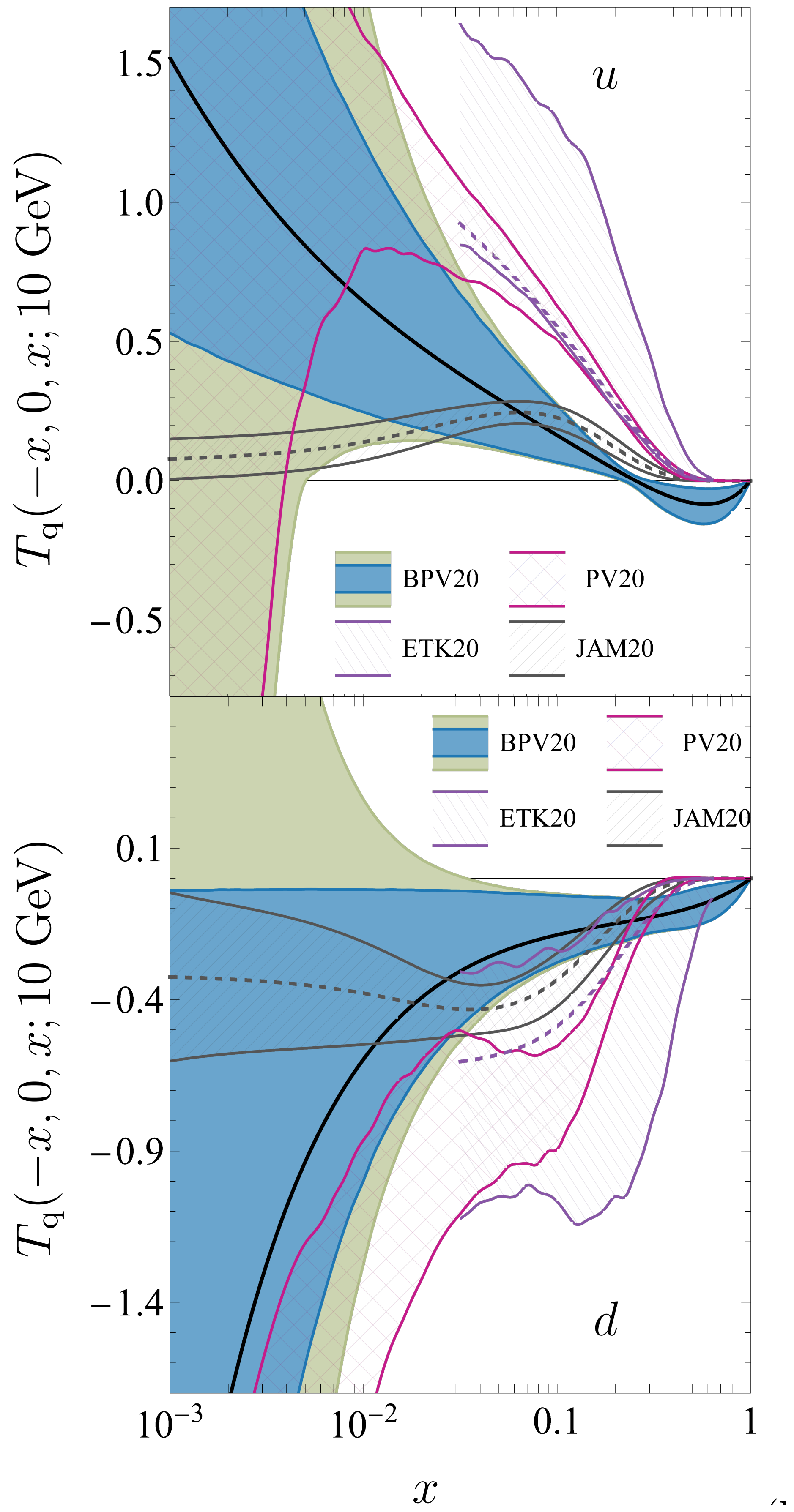
In a nucleon polarized in the +y direction,
the distribution of quarks can be distorted in the x direction



Bacchetta, Delcarro, Pisano, Radici, arXiv:2004.14278

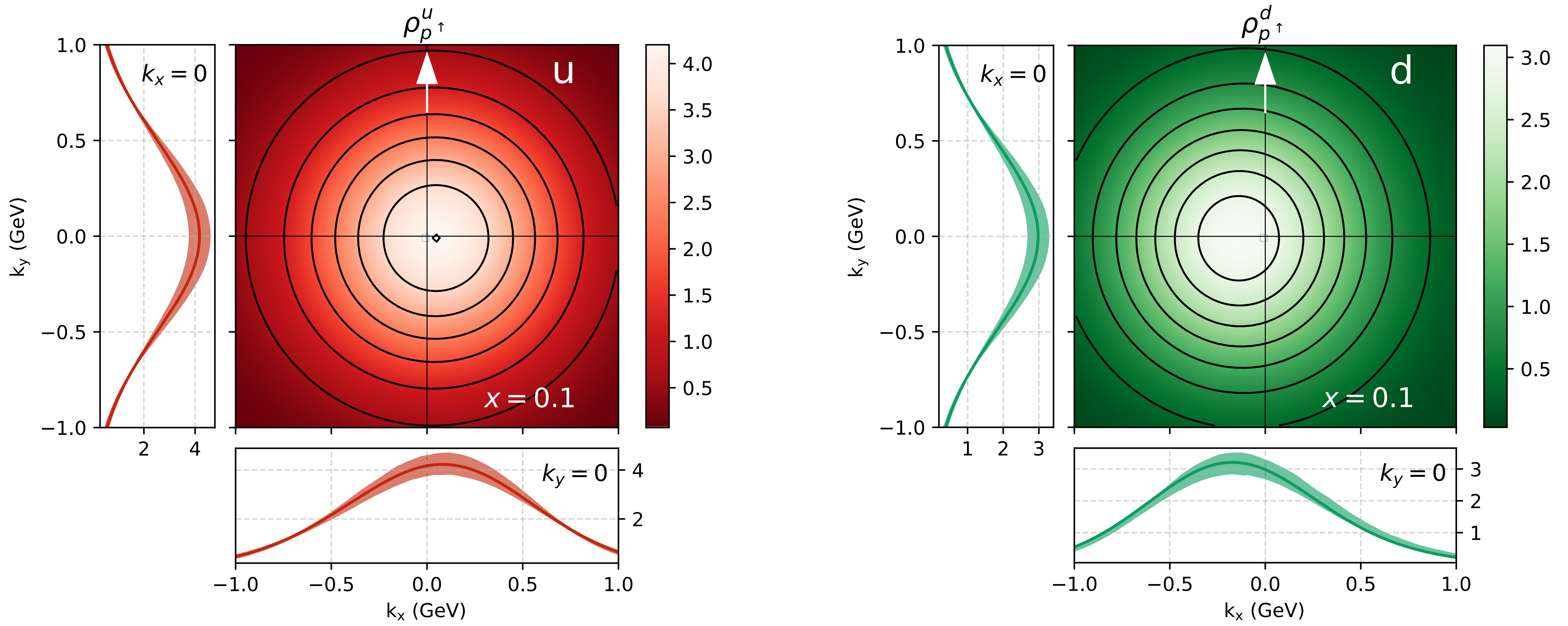


Echevarria, Kang, Terry, arXiv:2009.10710



Bury, Prokudin, Vladimirov, arXiv:2103.03270

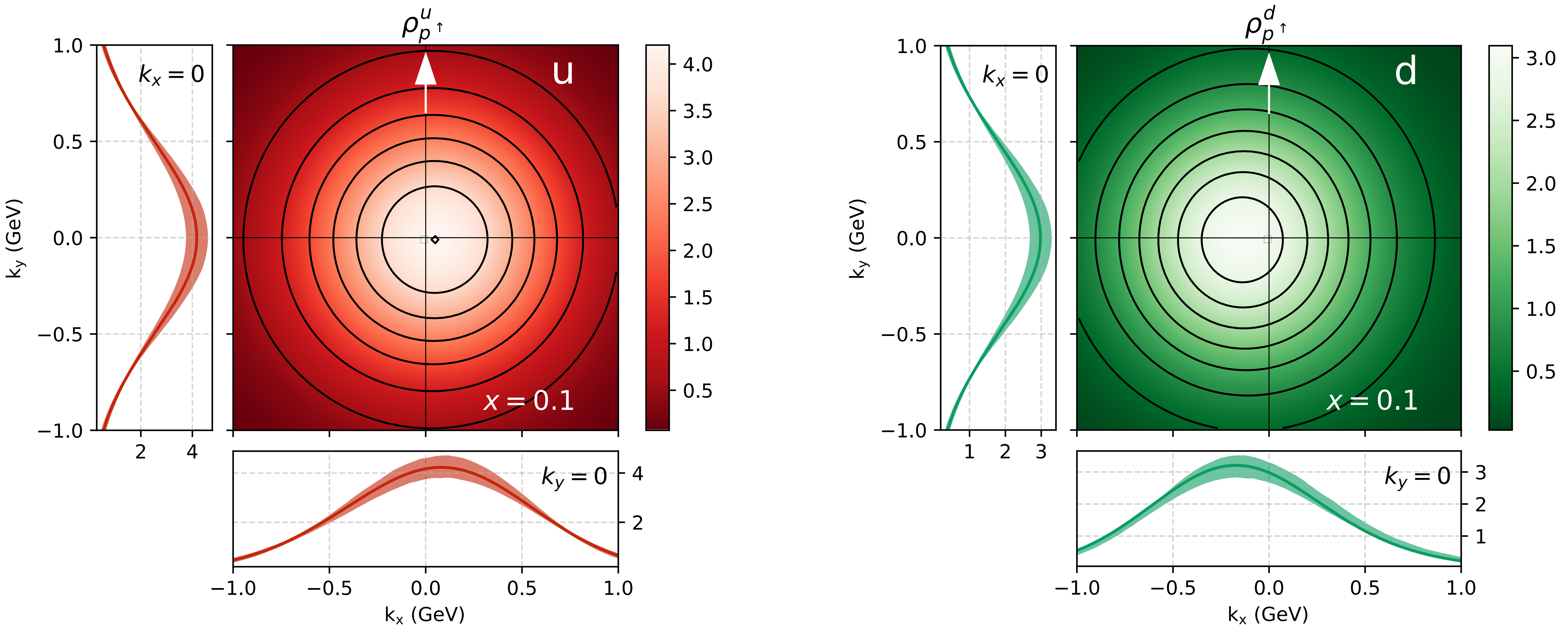
3D STRUCTURE IN MOMENTUM SPACE



Q= 2GeV

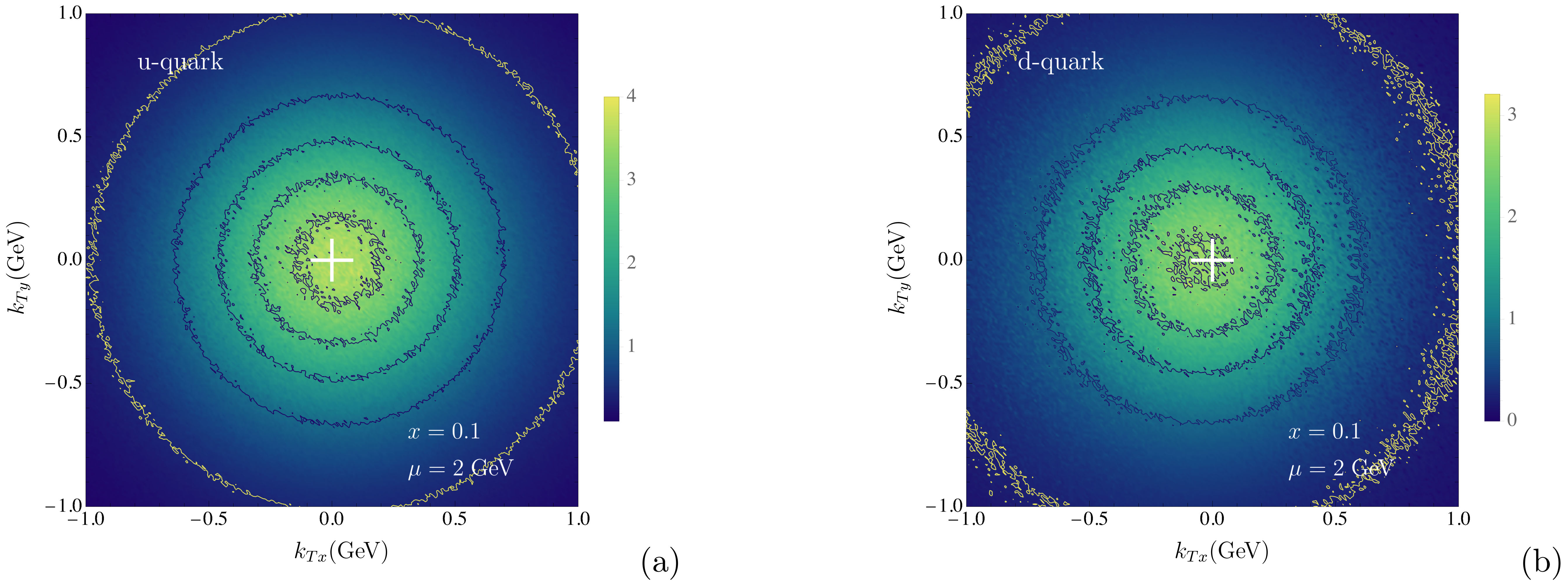
[Bacchetta, Delcarro, Pisano, Radici, arXiv:2004.14278](https://arxiv.org/abs/2004.14278)

3D STRUCTURE IN MOMENTUM SPACE



Q= 2GeV

Bacchetta, Delcarro, Pisano, Radici, arXiv:2004.14278



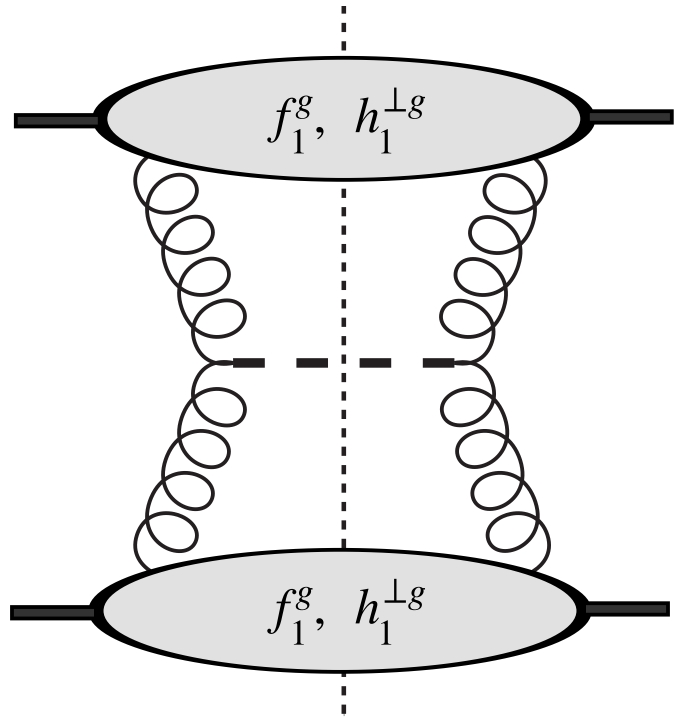
Bury, Prokudin, Vladimirov, arXiv:2103.03270

GLUONS

GLUON TMDS

Higgs production

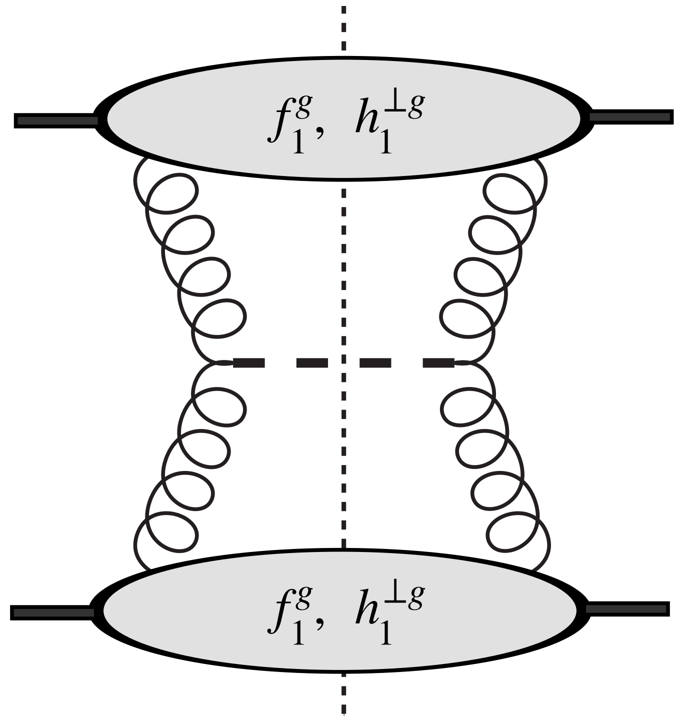
Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov, arXiv:1907.03780



GLUON TMDS

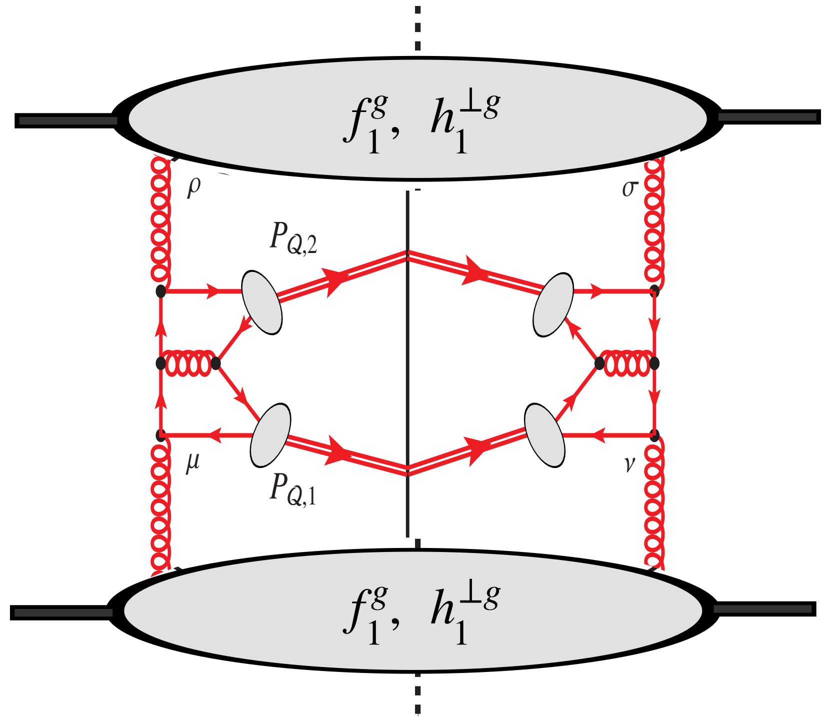
Higgs production

[Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov, arXiv:1907.03780](#)



Quarkonium-pair production

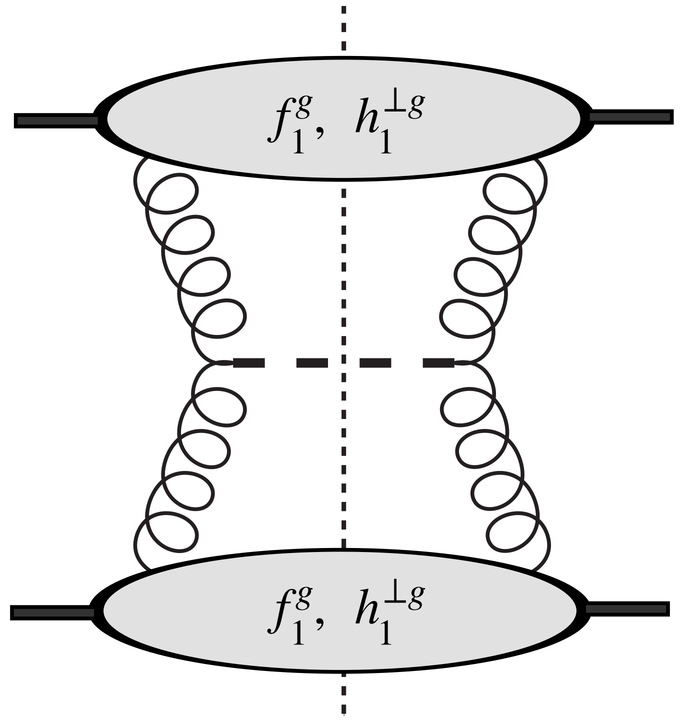
[Scarpa, Boer, Echevarria, Lansberg, Pisano, Schlegel, arXiv:1909.05769](#)

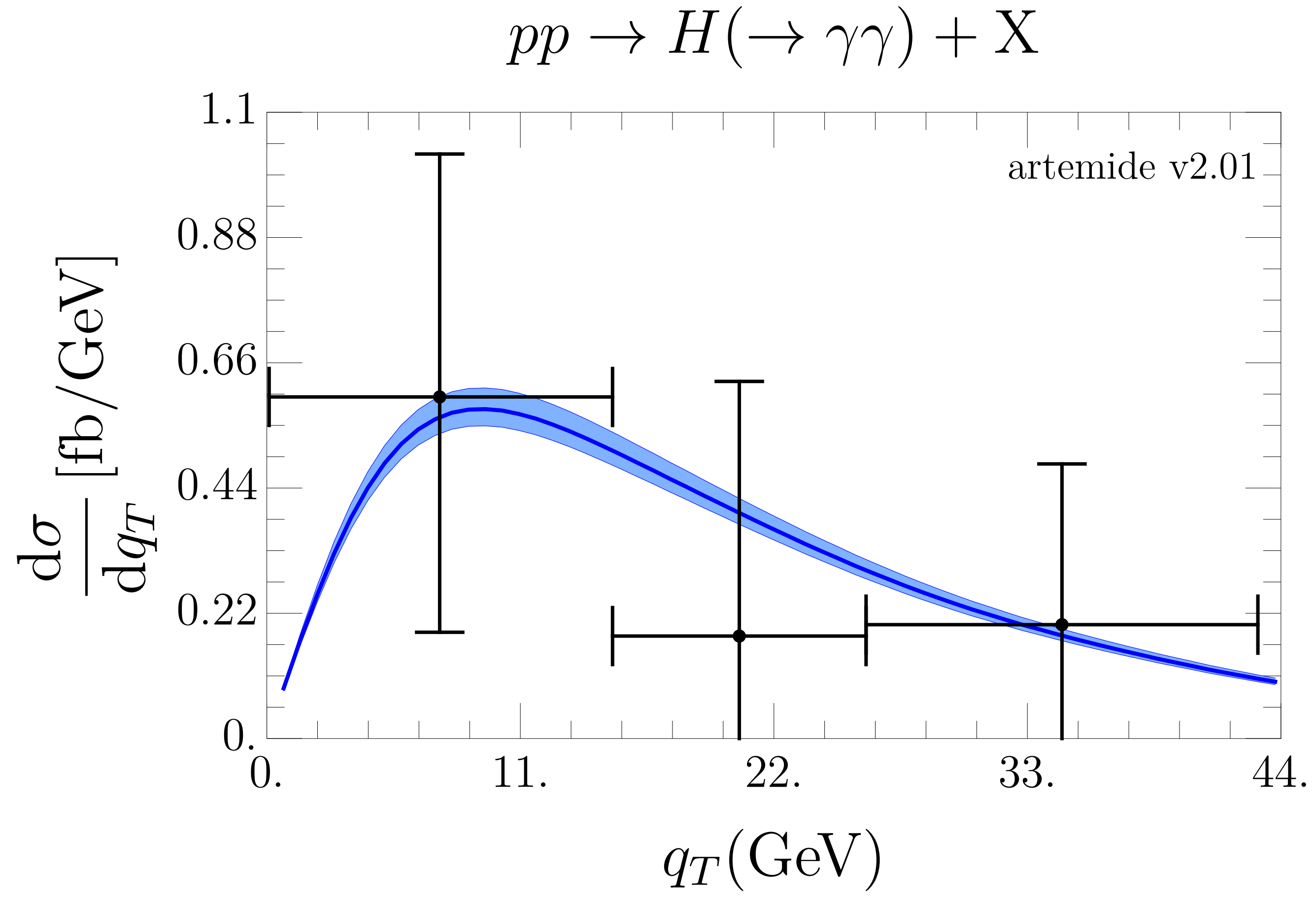


GLUON TMDS

Higgs production

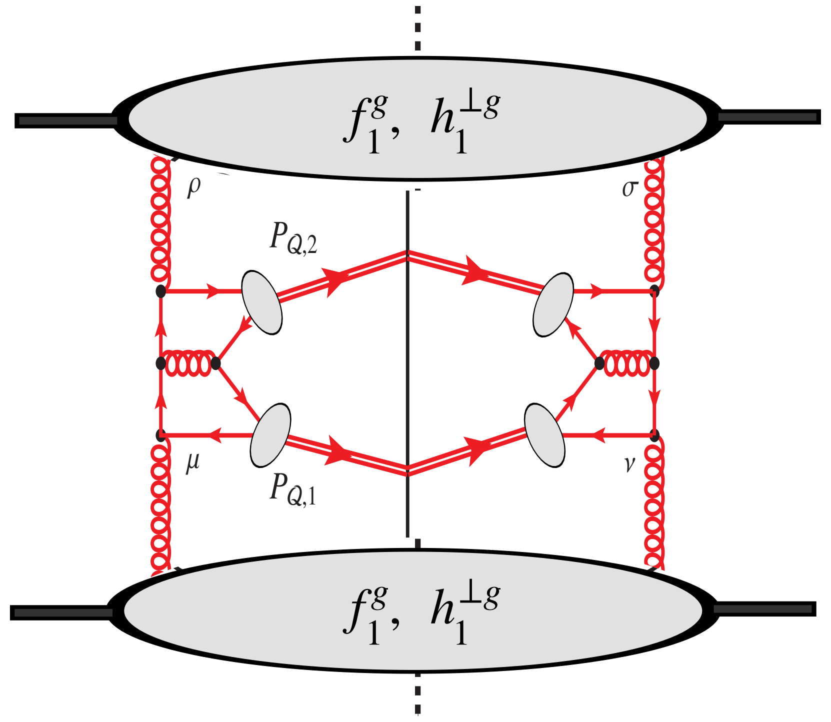
Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov, arXiv:1907.03780





Quarkonium-pair production

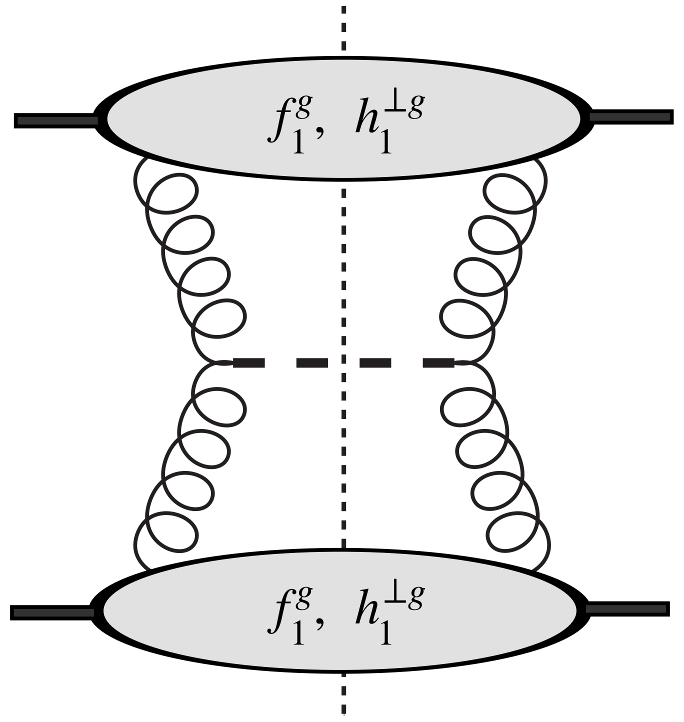
Scarpa, Boer, Echevarria, Lansberg, Pisano, Schlegel, arXiv:1909.05769

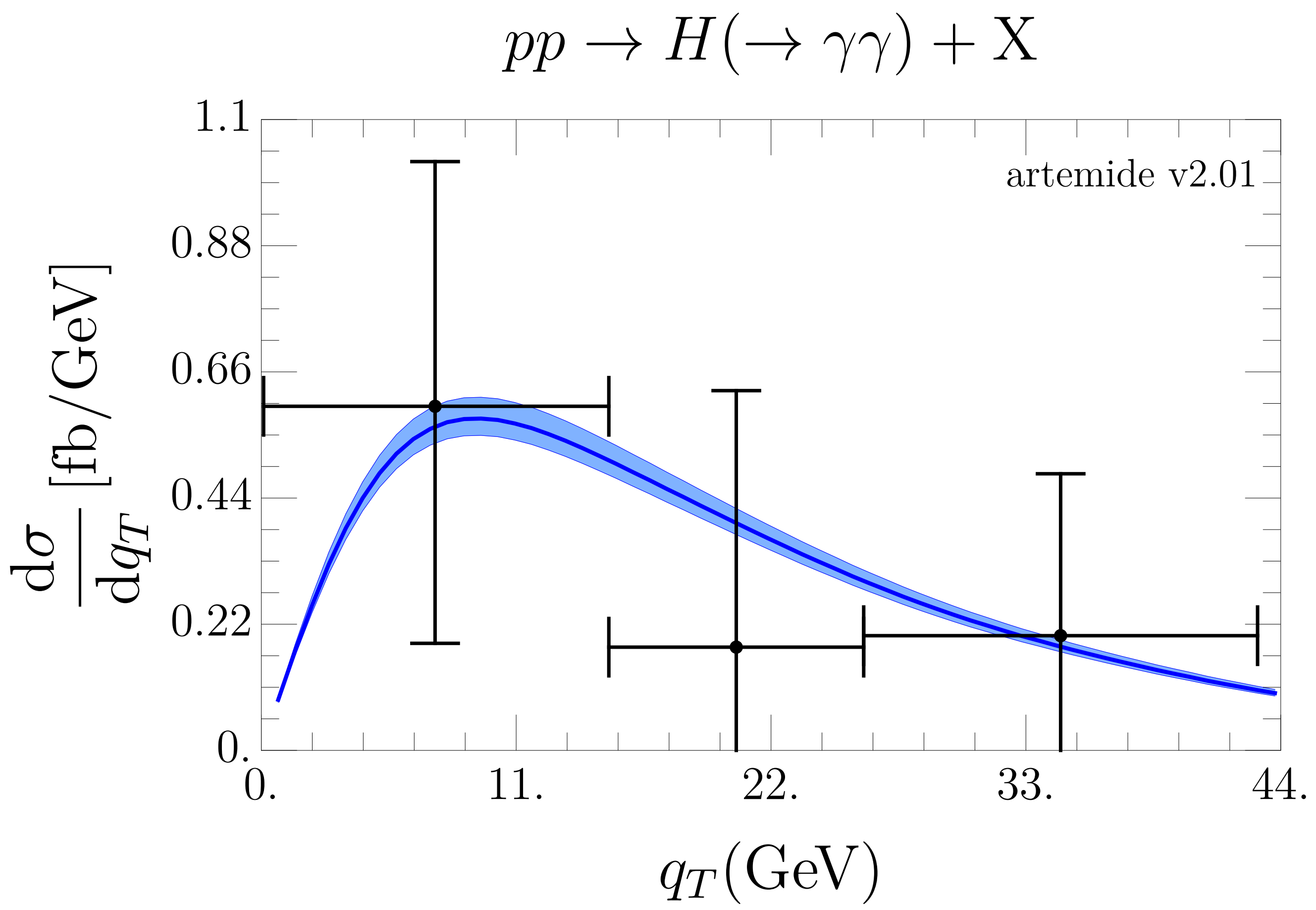


GLUON TMDS

Higgs production

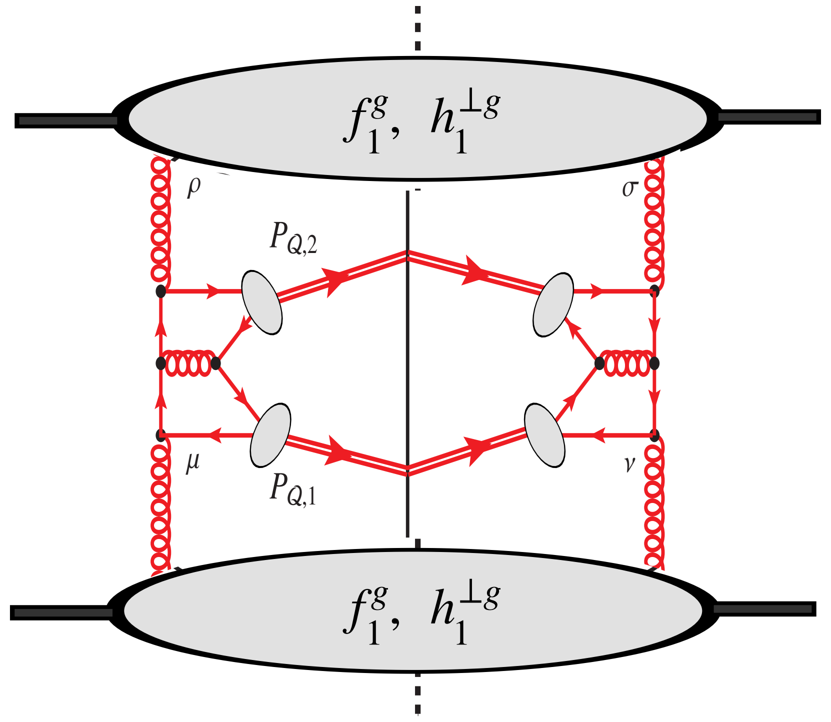
Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov, arXiv:1907.03780

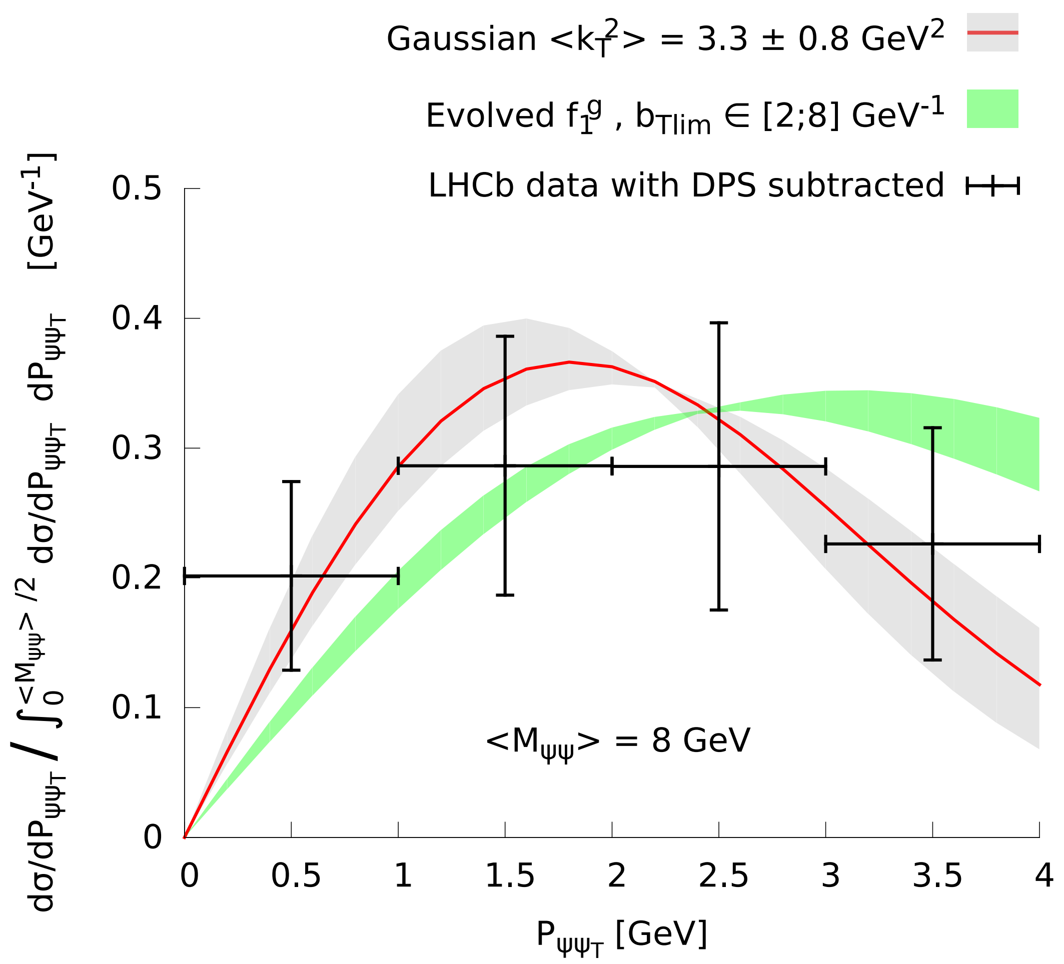




Quarkonium-pair production

Scarpa, Boer, Echevarria, Lansberg, Pisano, Schlegel, arXiv:1909.05769

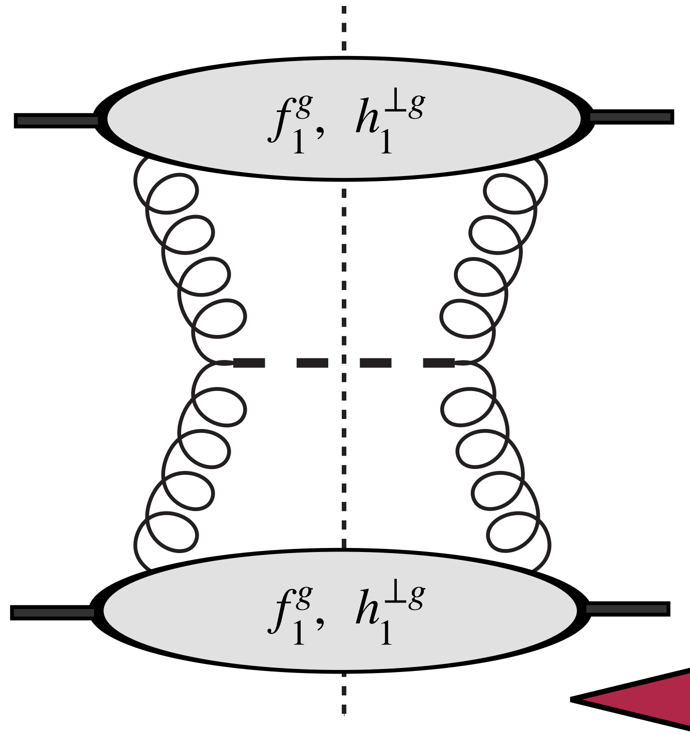


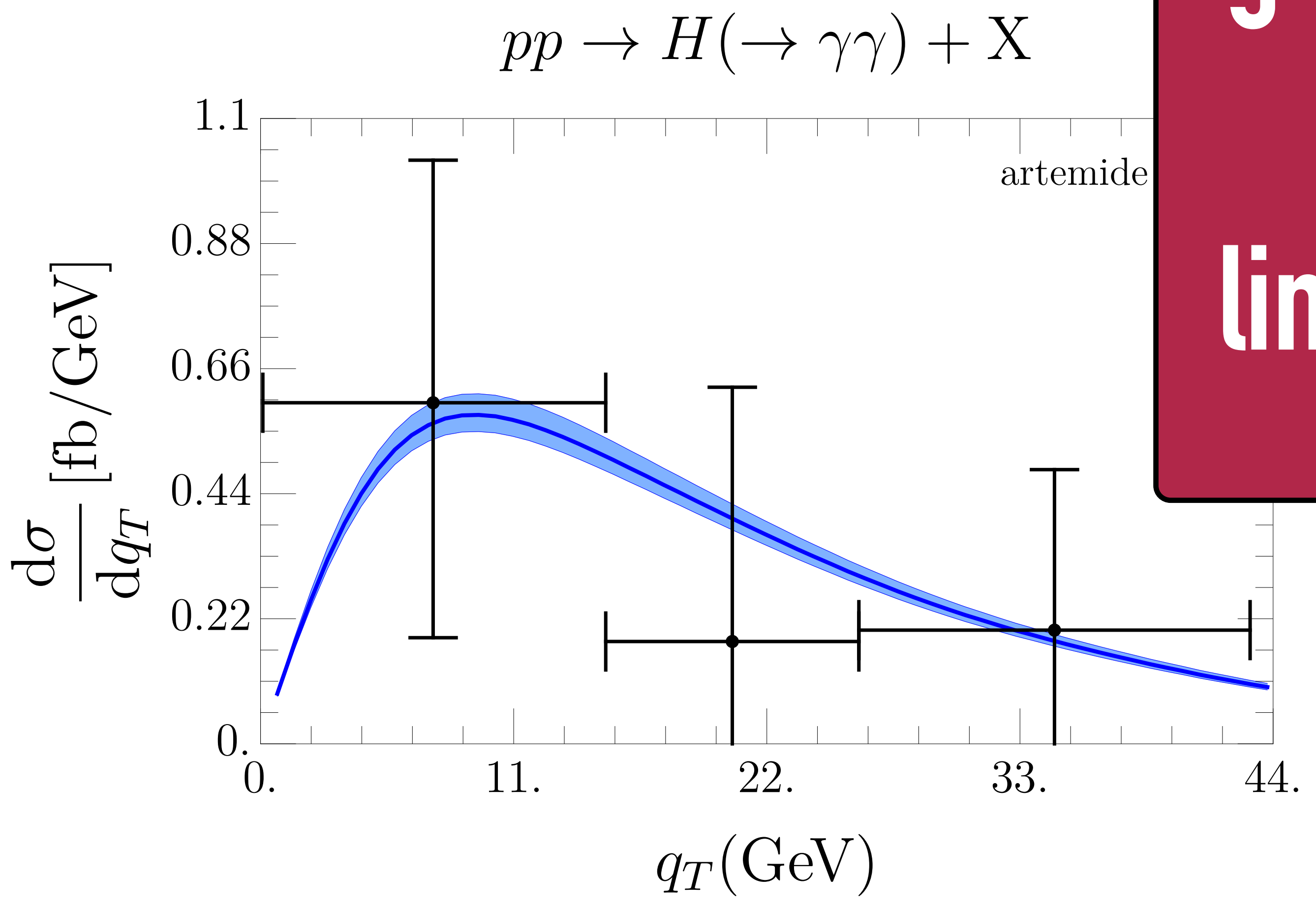


GLUON TMDS

Higgs production

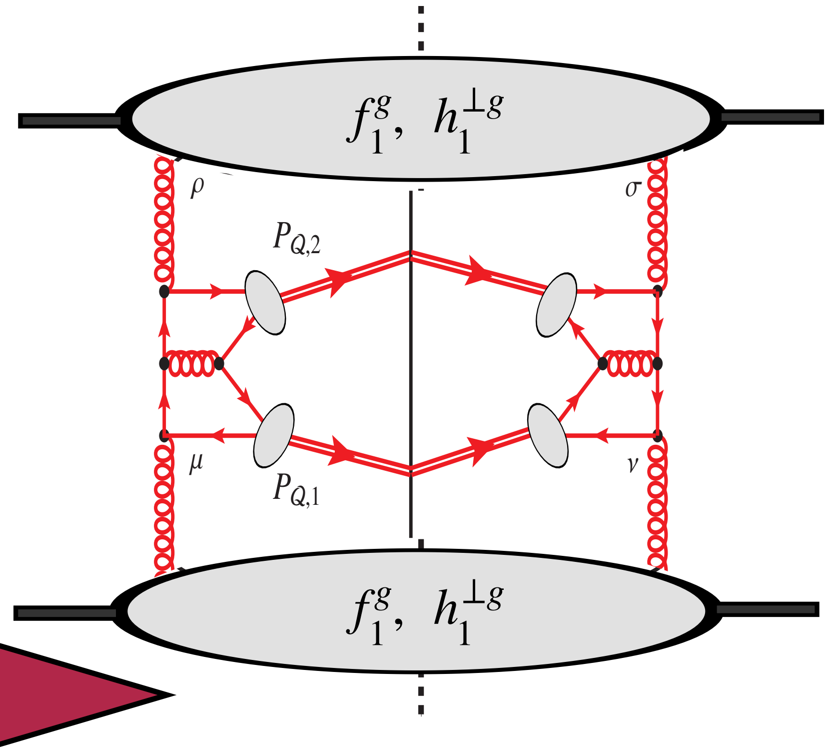
Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov, arXiv:1907.03780

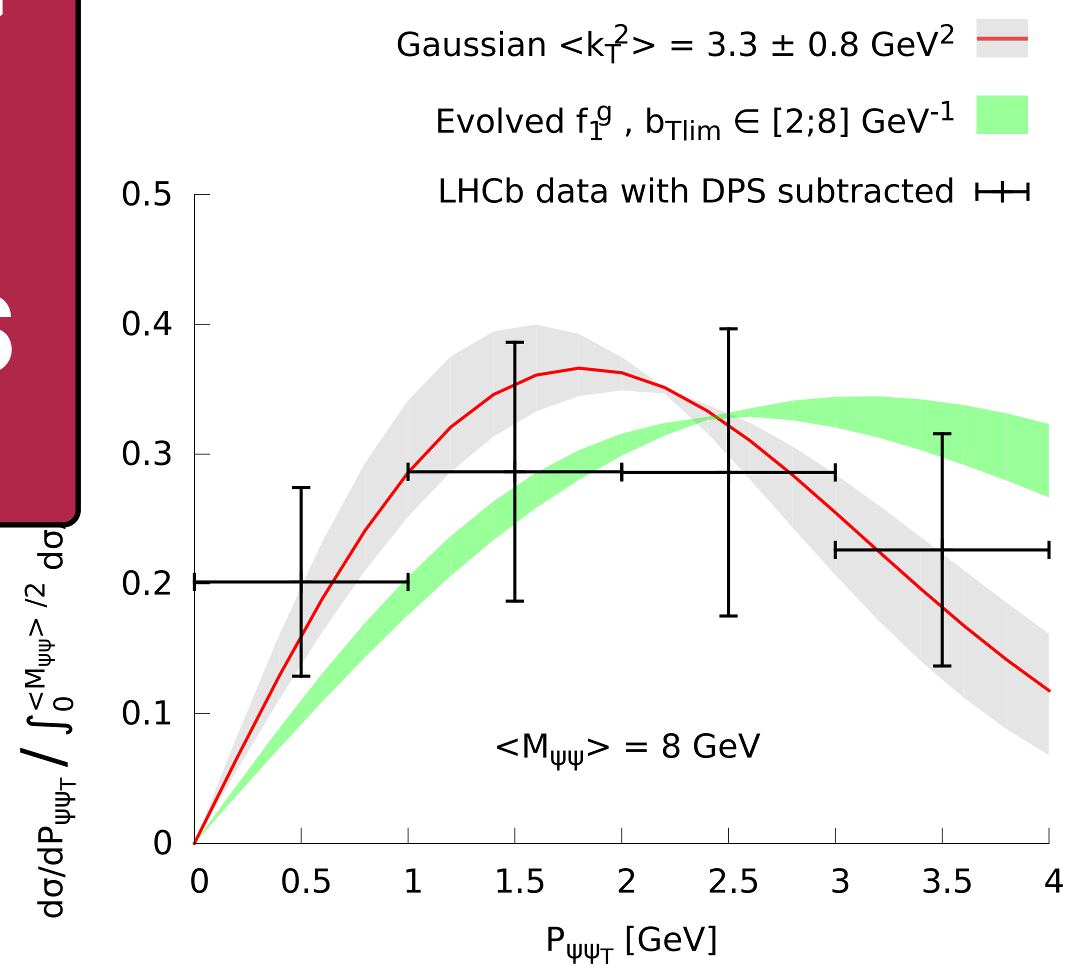




Quarkonium-pair production

Scarpa, Boer, Echevarria, Lansberg, Pisano, Schlegel, arXiv:1909.05769

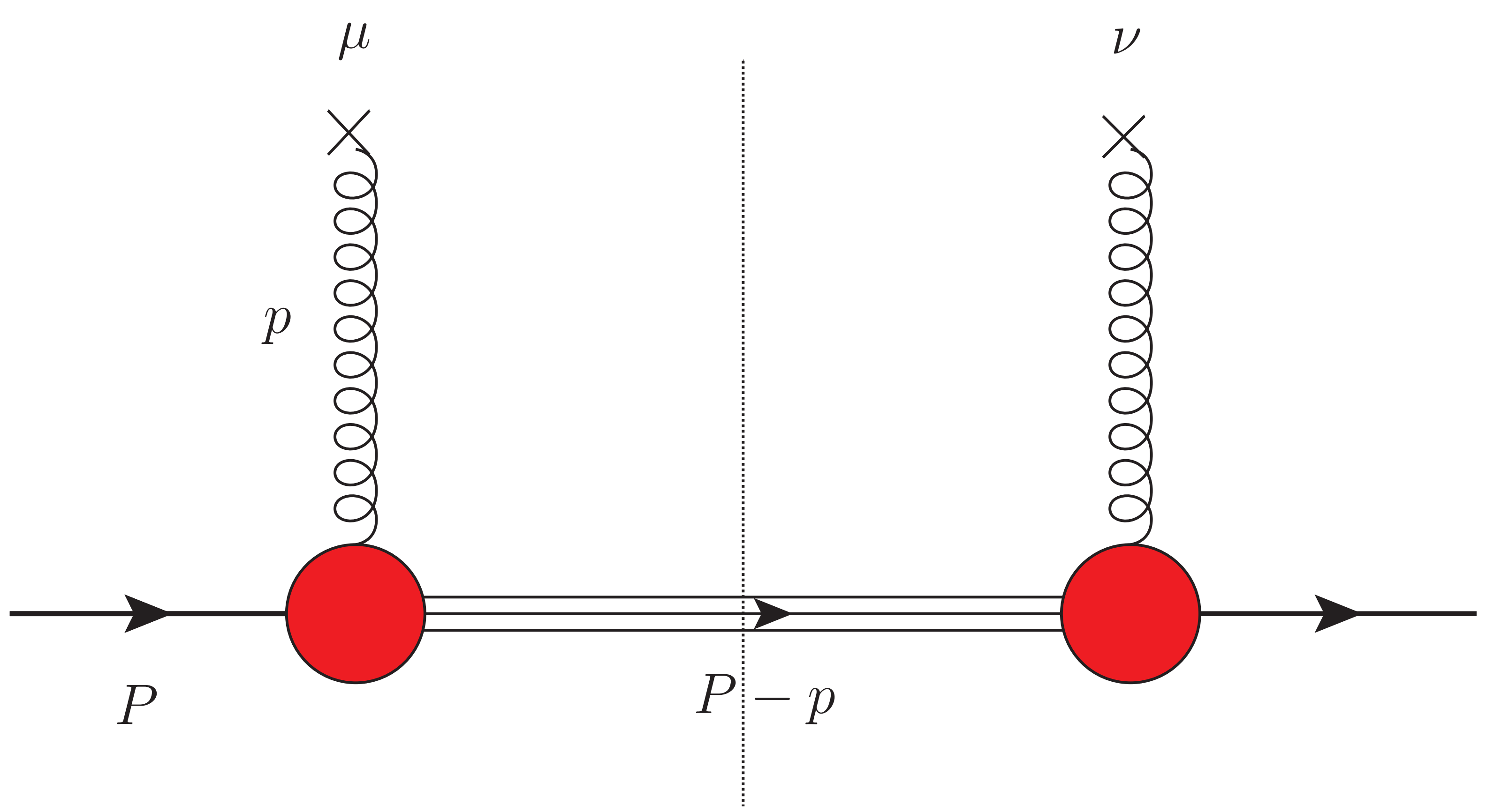




Several complications: factorization breaking, generalized universality, shape functions, linearly polarized gluons

GLUON TMD MODELING

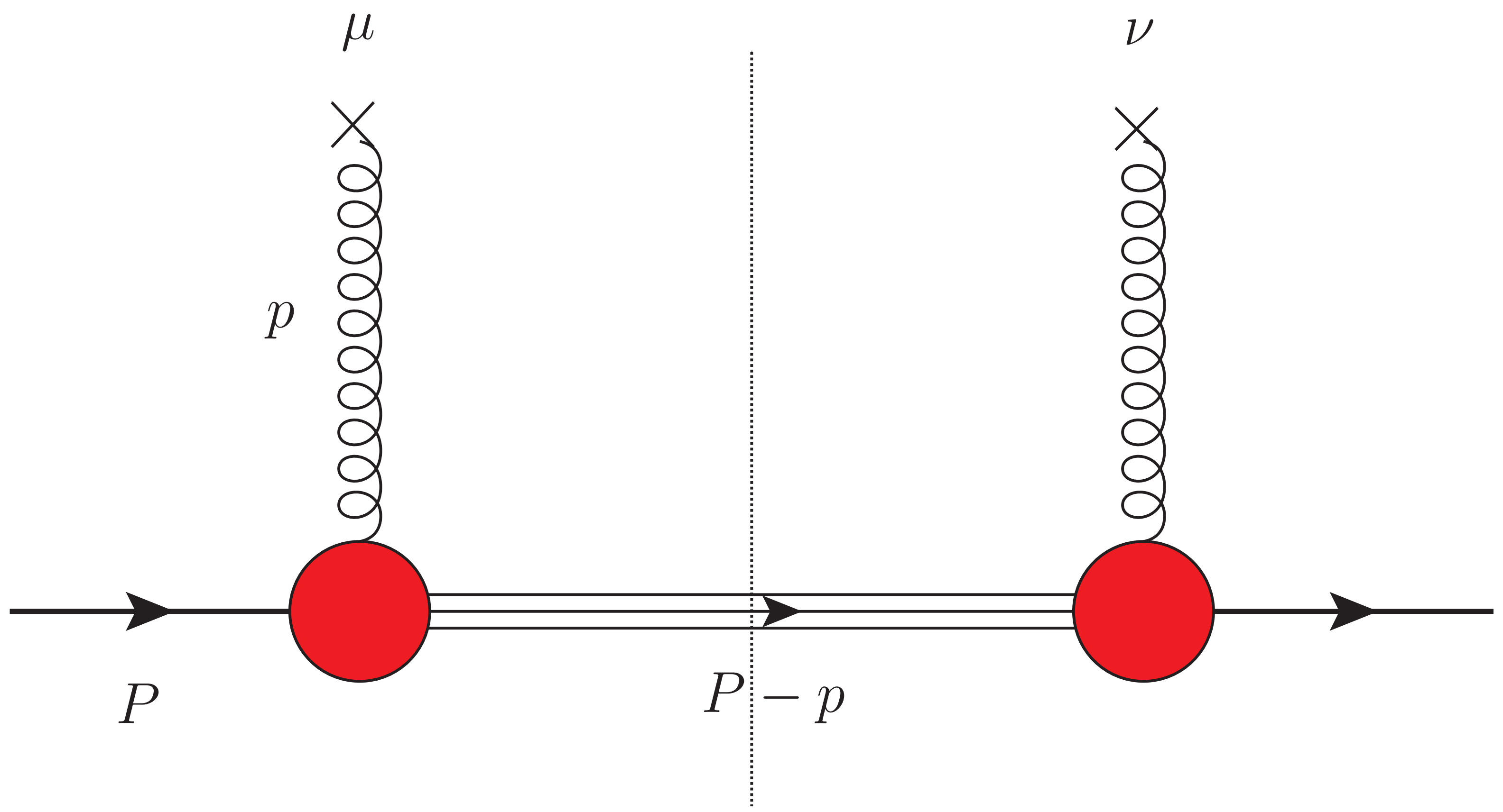
Bacchetta, Celiberto, Radici, Taels, arxiv:2005.02288



Spectator model

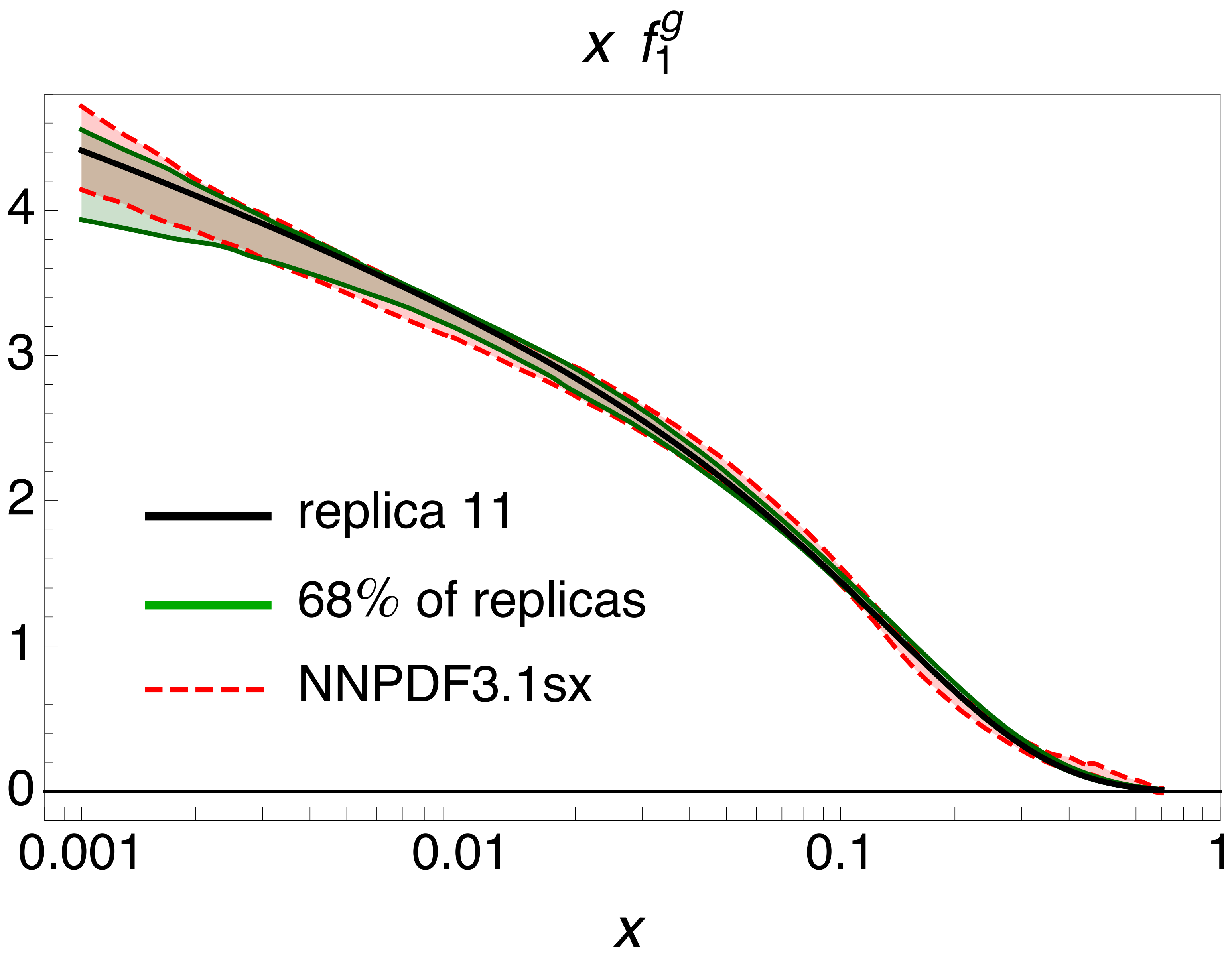
GLUON TMD MODELING

Bacchetta, Celiberto, Radici, Taels, arxiv:2005.02288



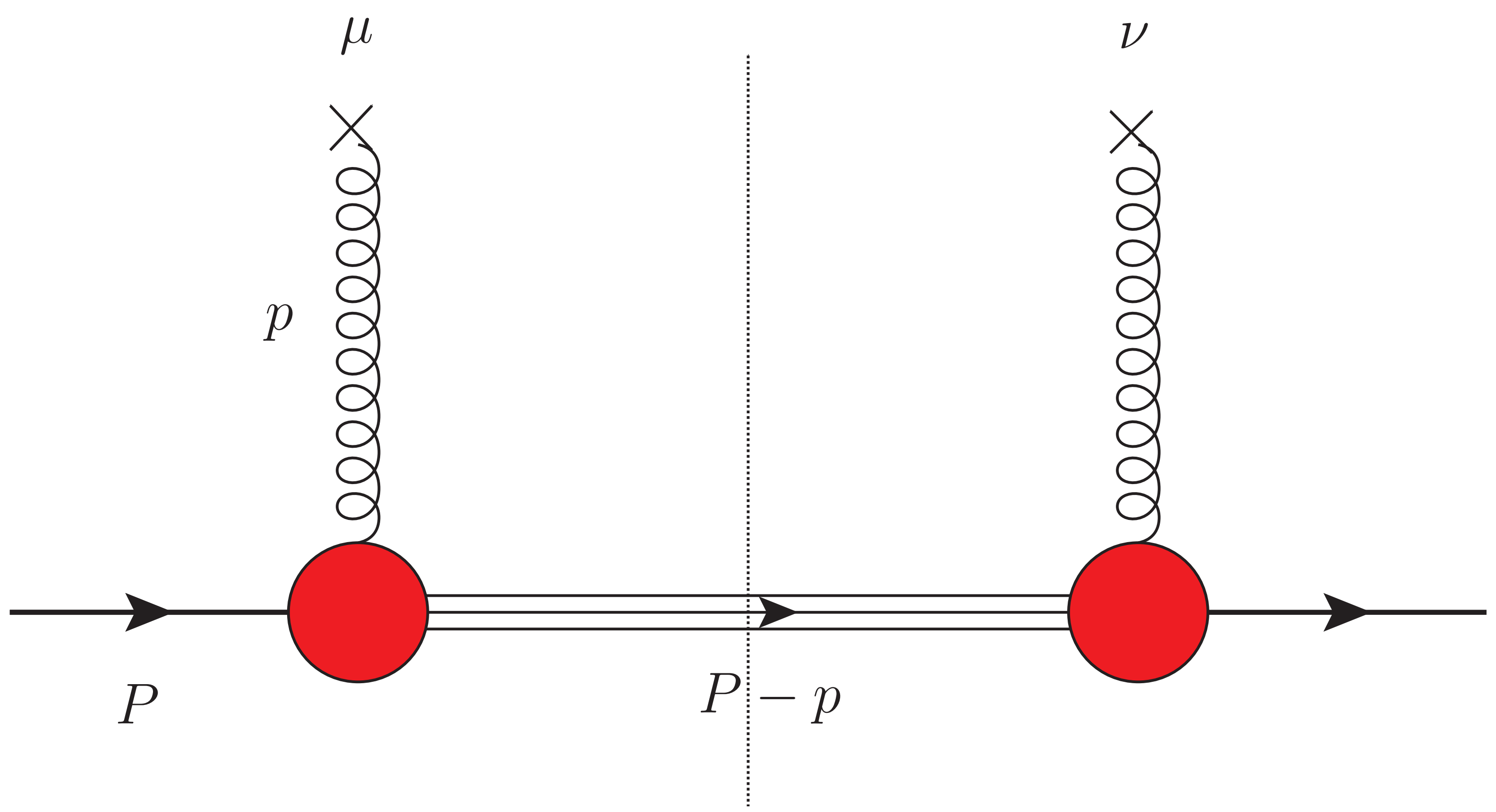
Spectator model

Reproduces collinear gluon PDFs



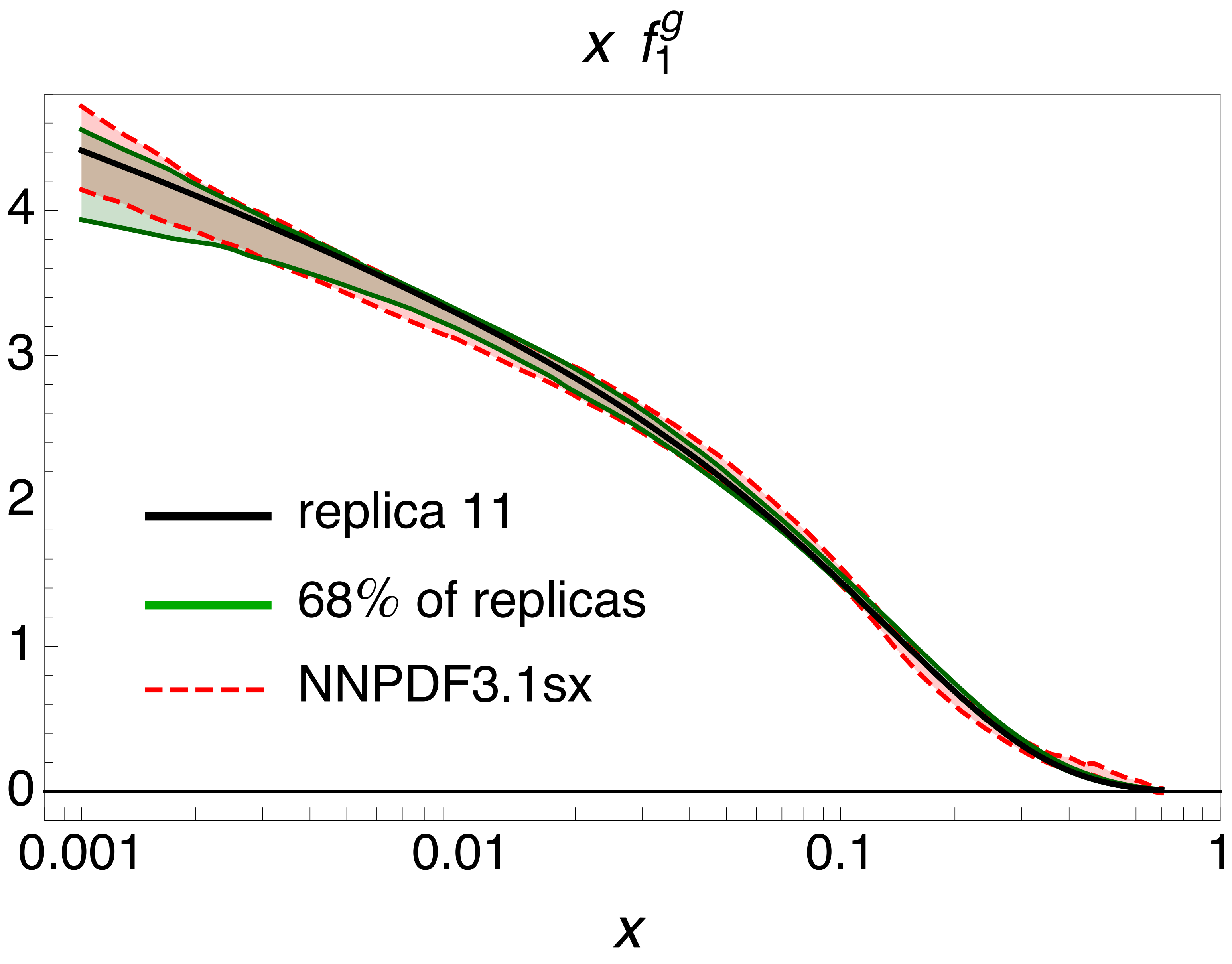
GLUON TMD MODELING

[Bacchetta, Celiberto, Radici, Taels, arxiv:2005.02288](https://arxiv.org/abs/2005.02288)

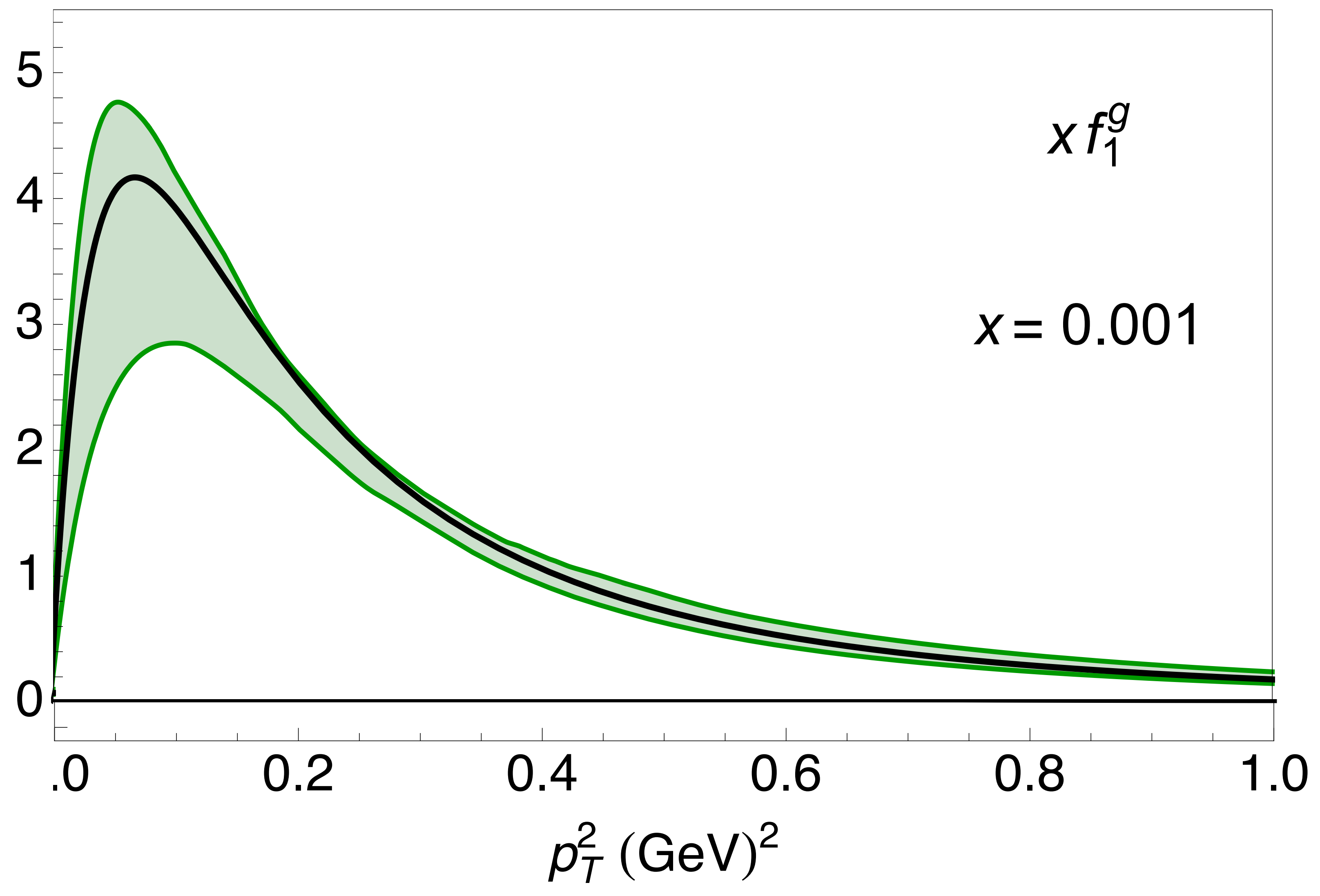


Spectator model

Reproduces collinear gluon PDFs

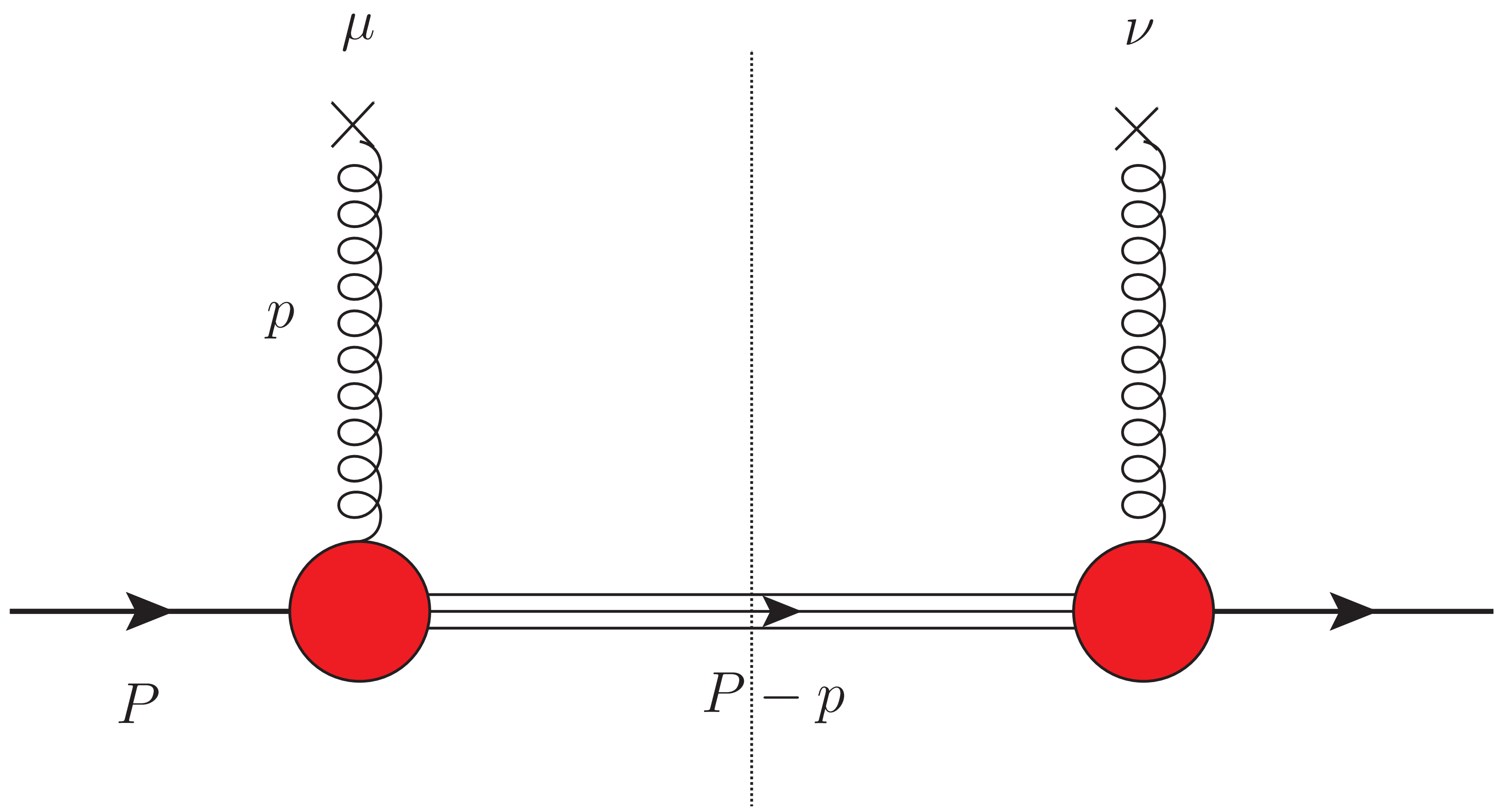


Generates nontrivial and widely different TMDs



GLUON TMD MODELING

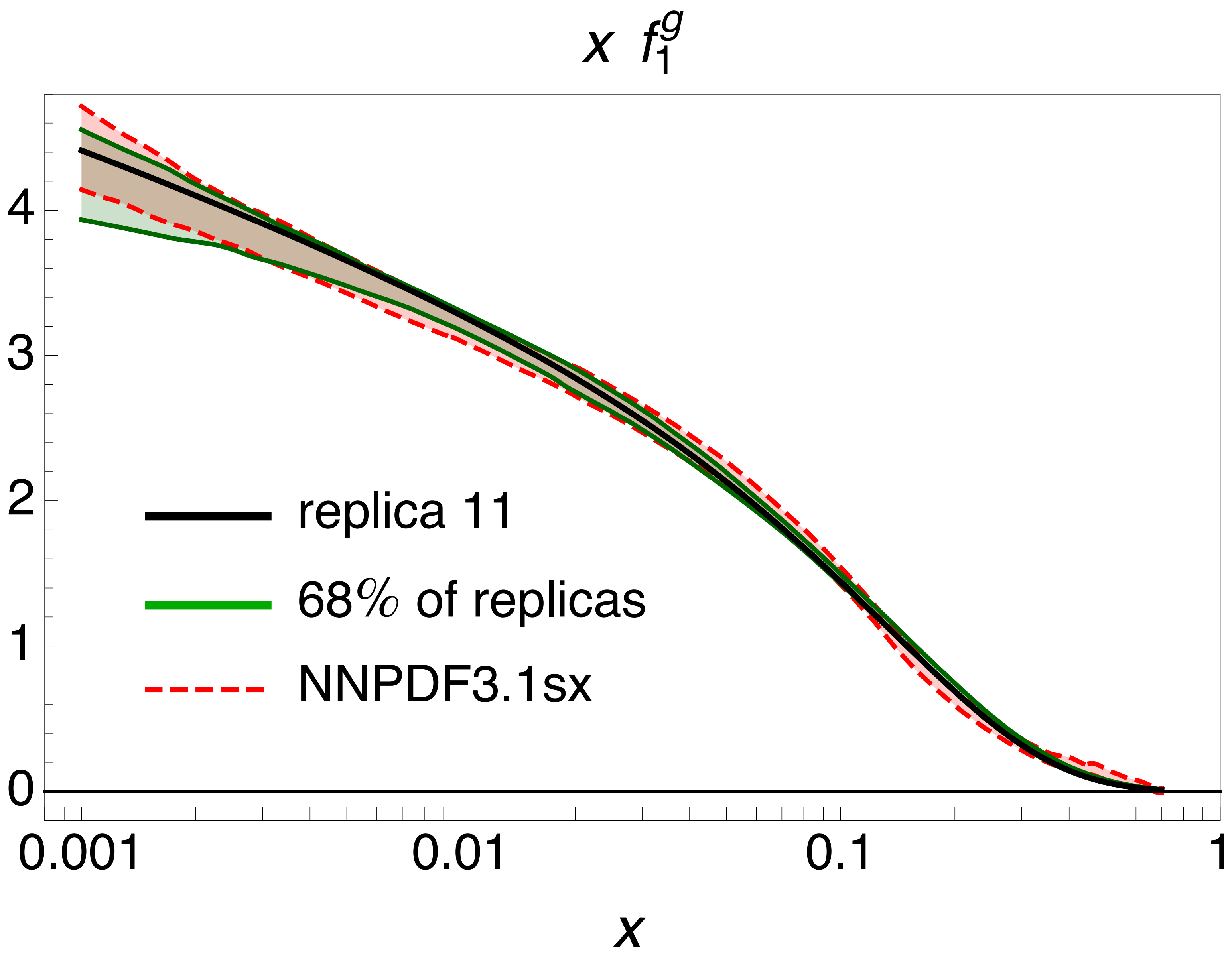
Bacchetta, Celiberto, Radici, Taels, arxiv:2005.02288



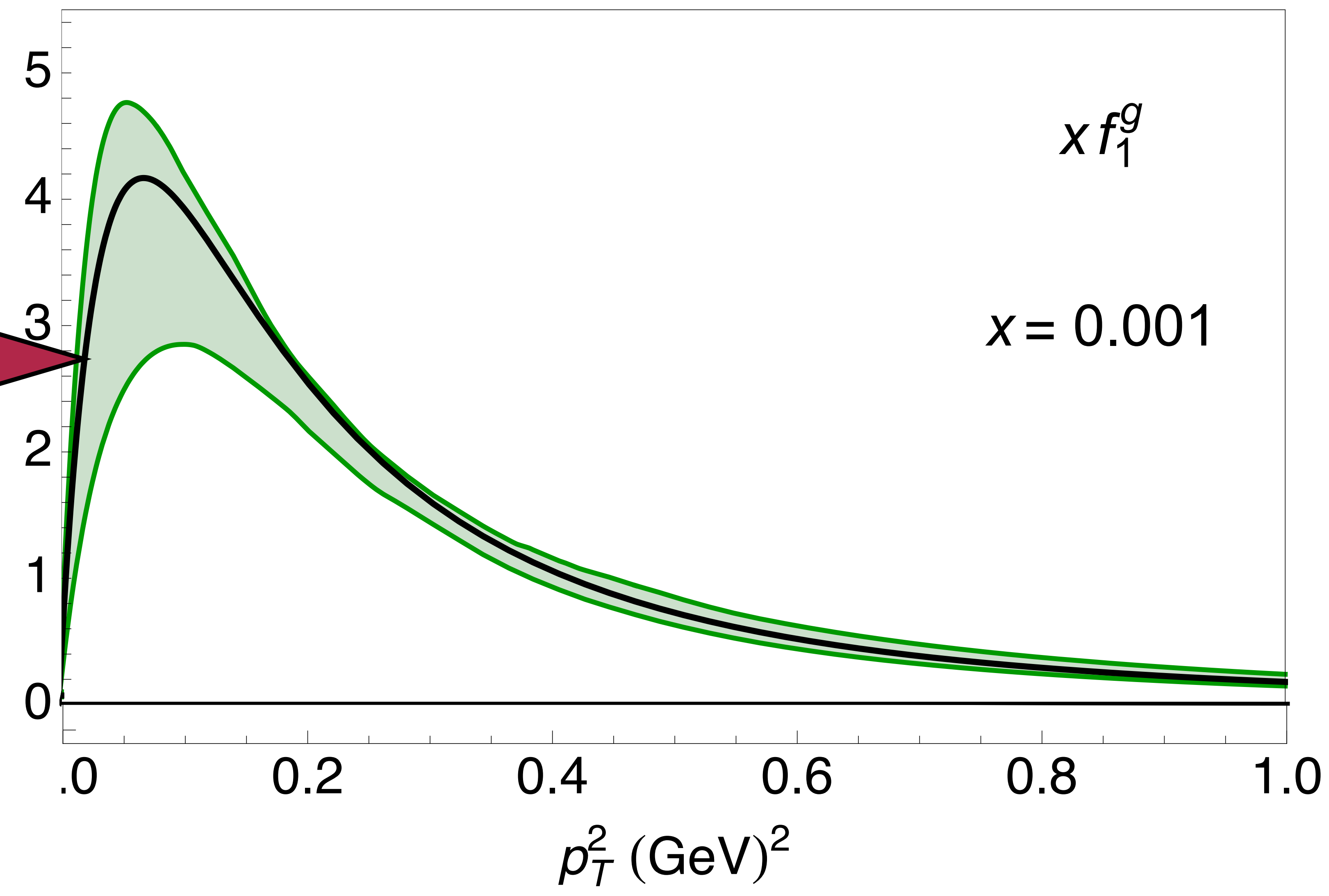
Spectator model

Reproduces collinear gluon PDFs

Generates nontrivial and widely different TMDs

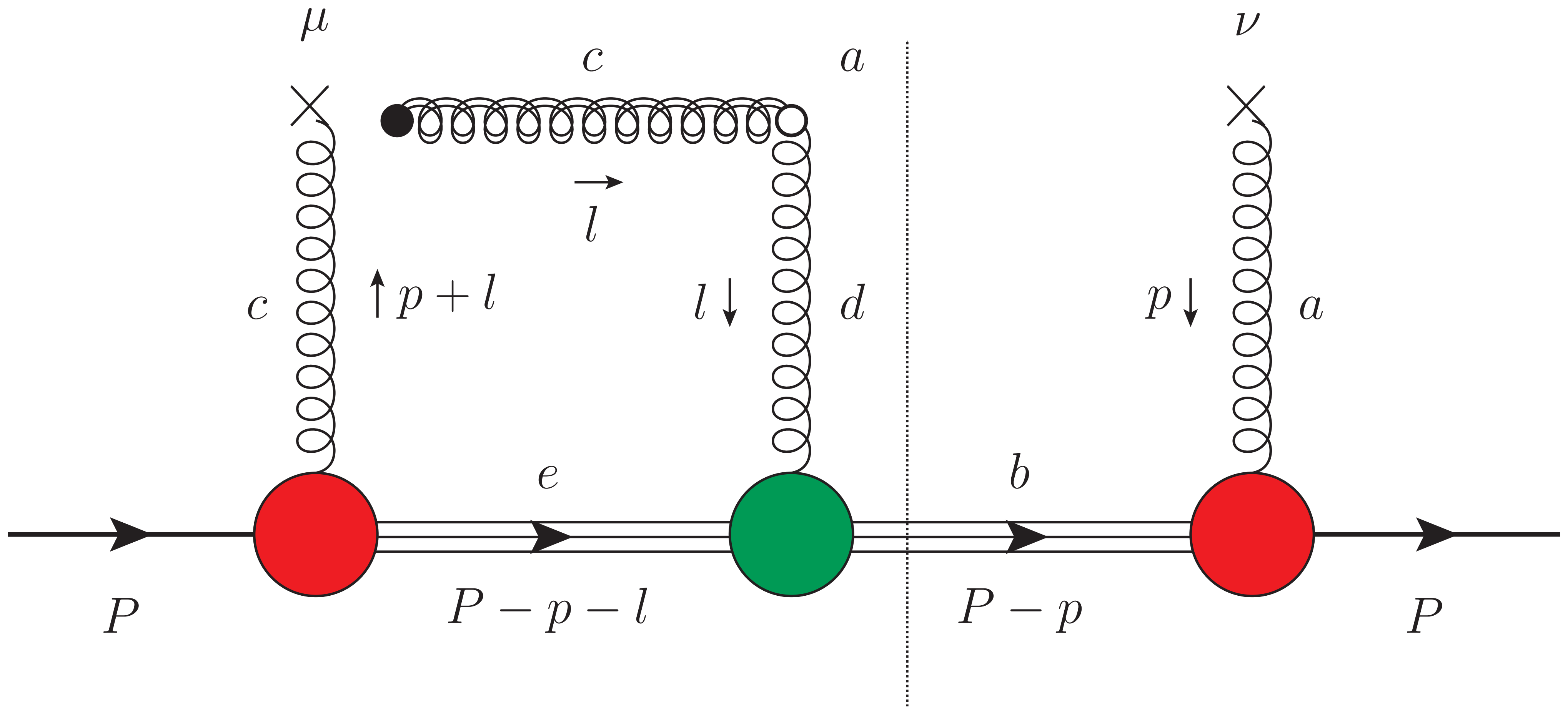


Qualitatively different from quarks



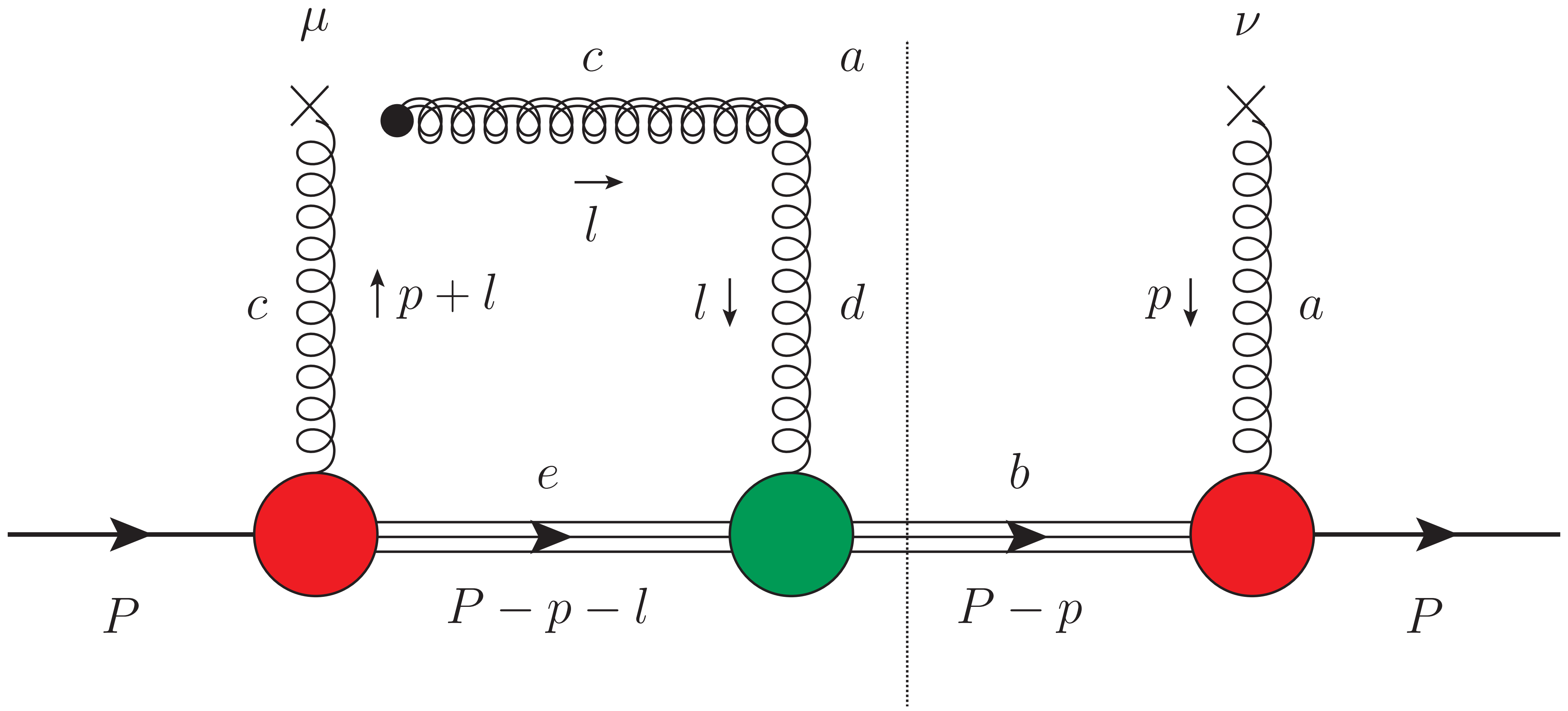
GLUON SIVERS TMD MODELING

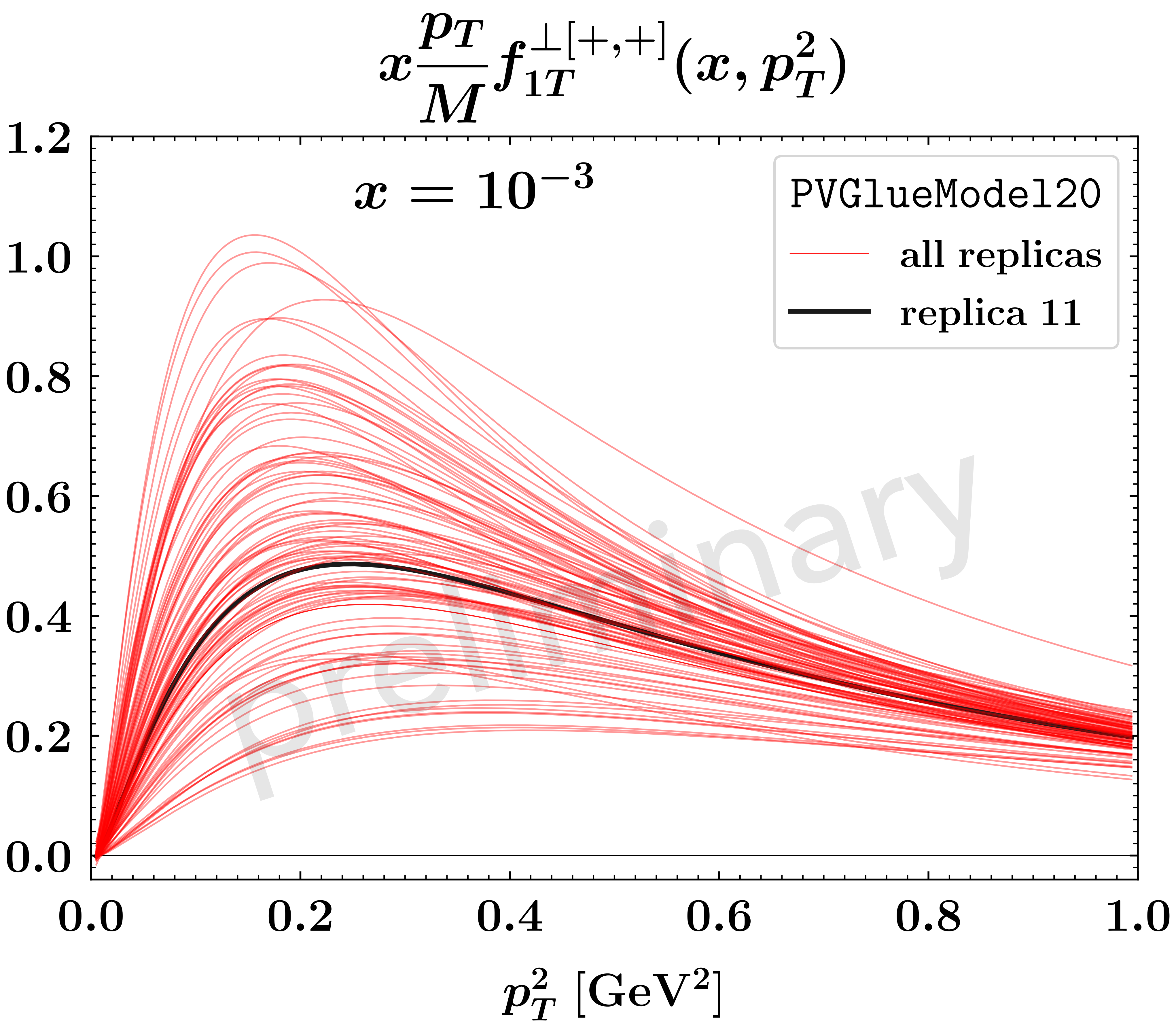
Bacchetta, Celiberto, Radici, in preparation



GLUON SIVERS TMD MODELING

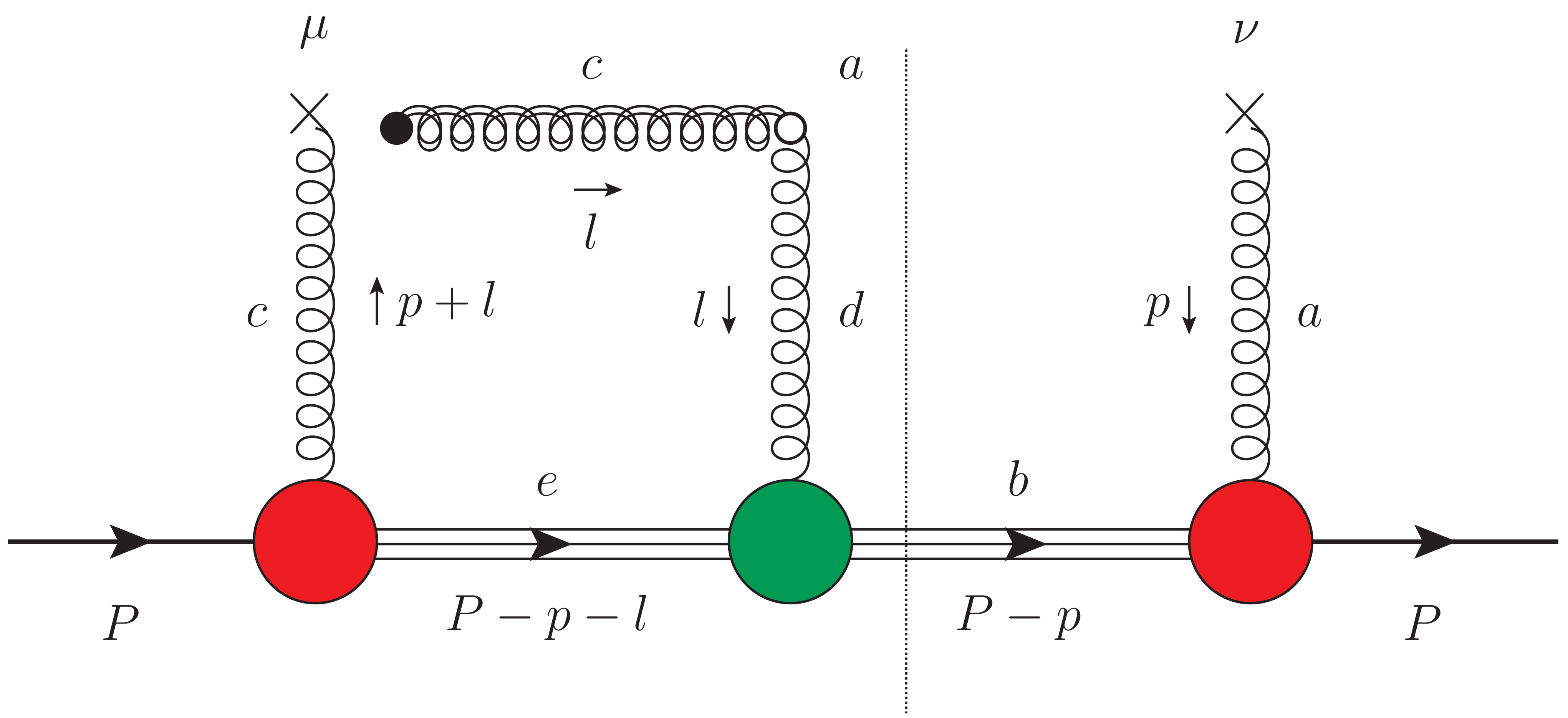
Bacchetta, Celiberto, Radici, in preparation

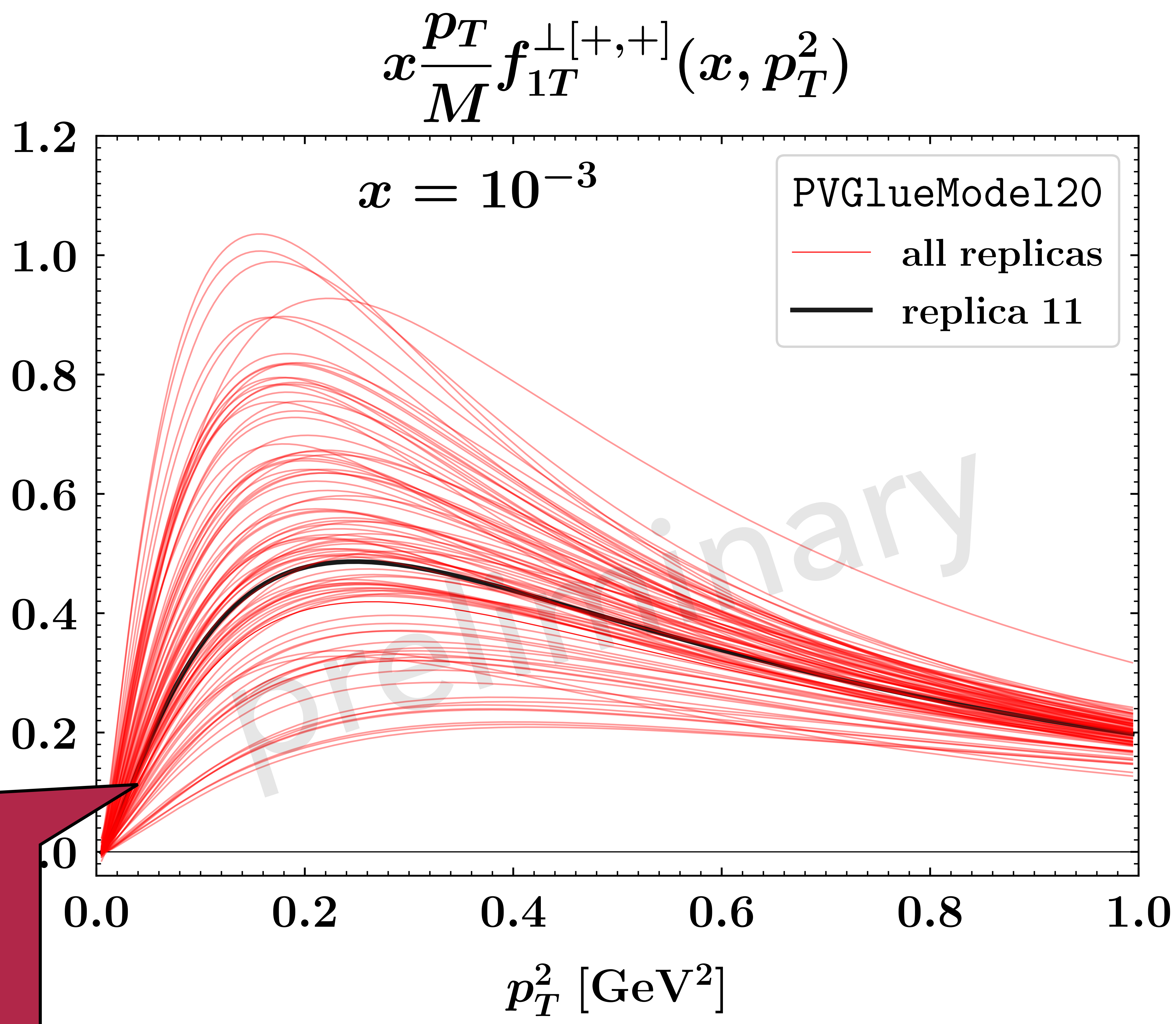




GLUON SIVERS TMD MODELING

Bacchetta, Celiberto, Radici, in preparation



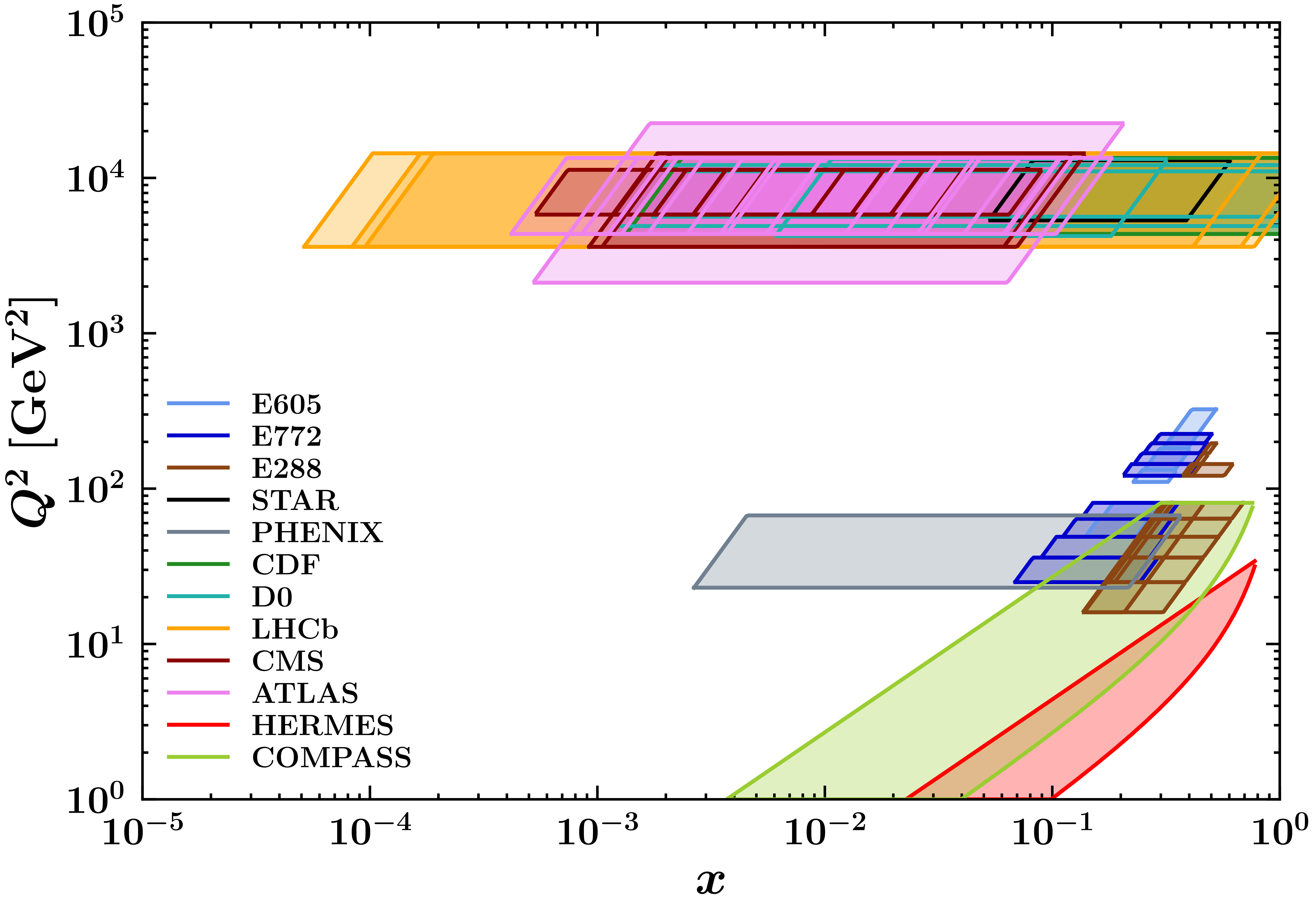


Large size.

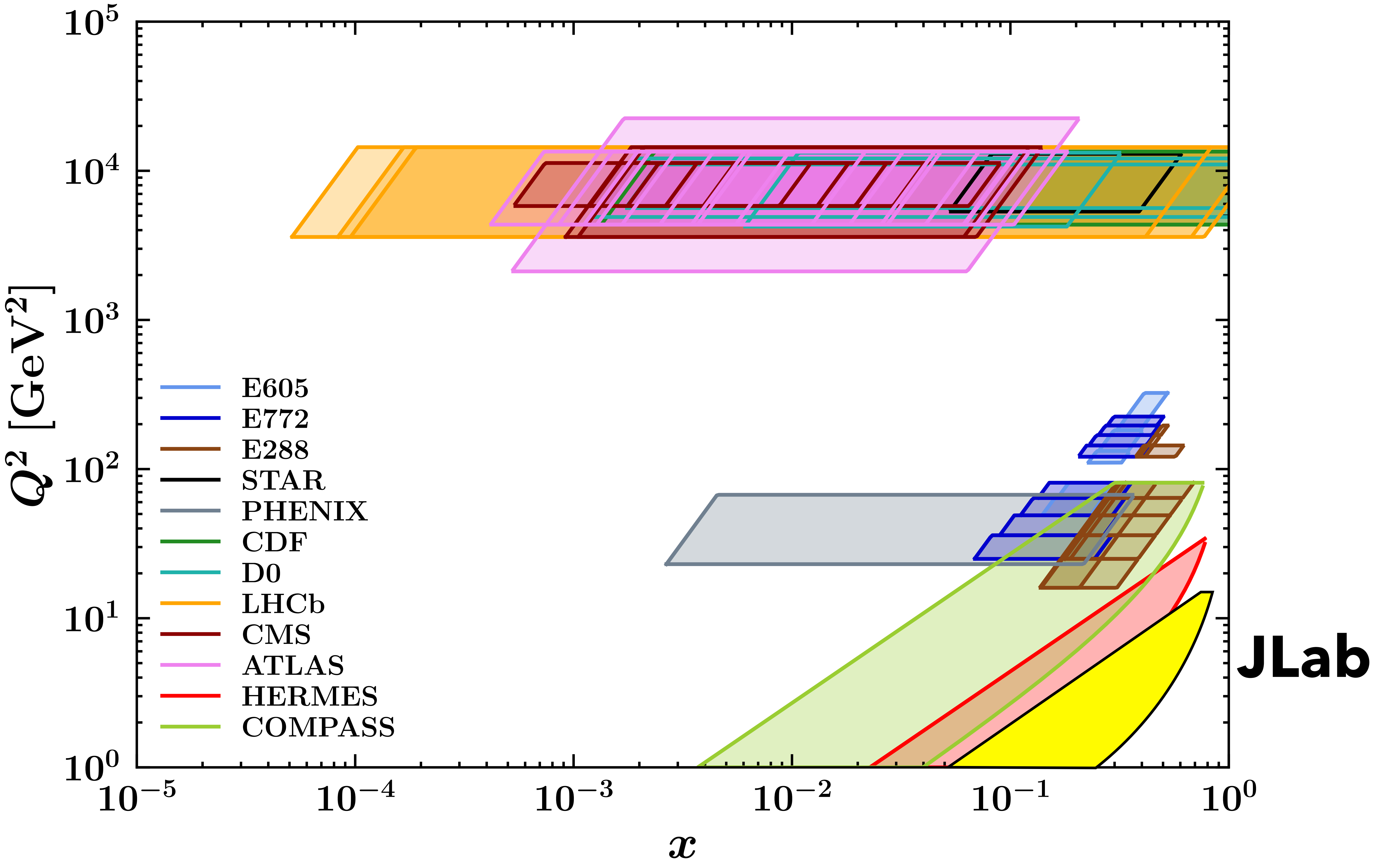
High sensitivity to model parameters.

FUTURE

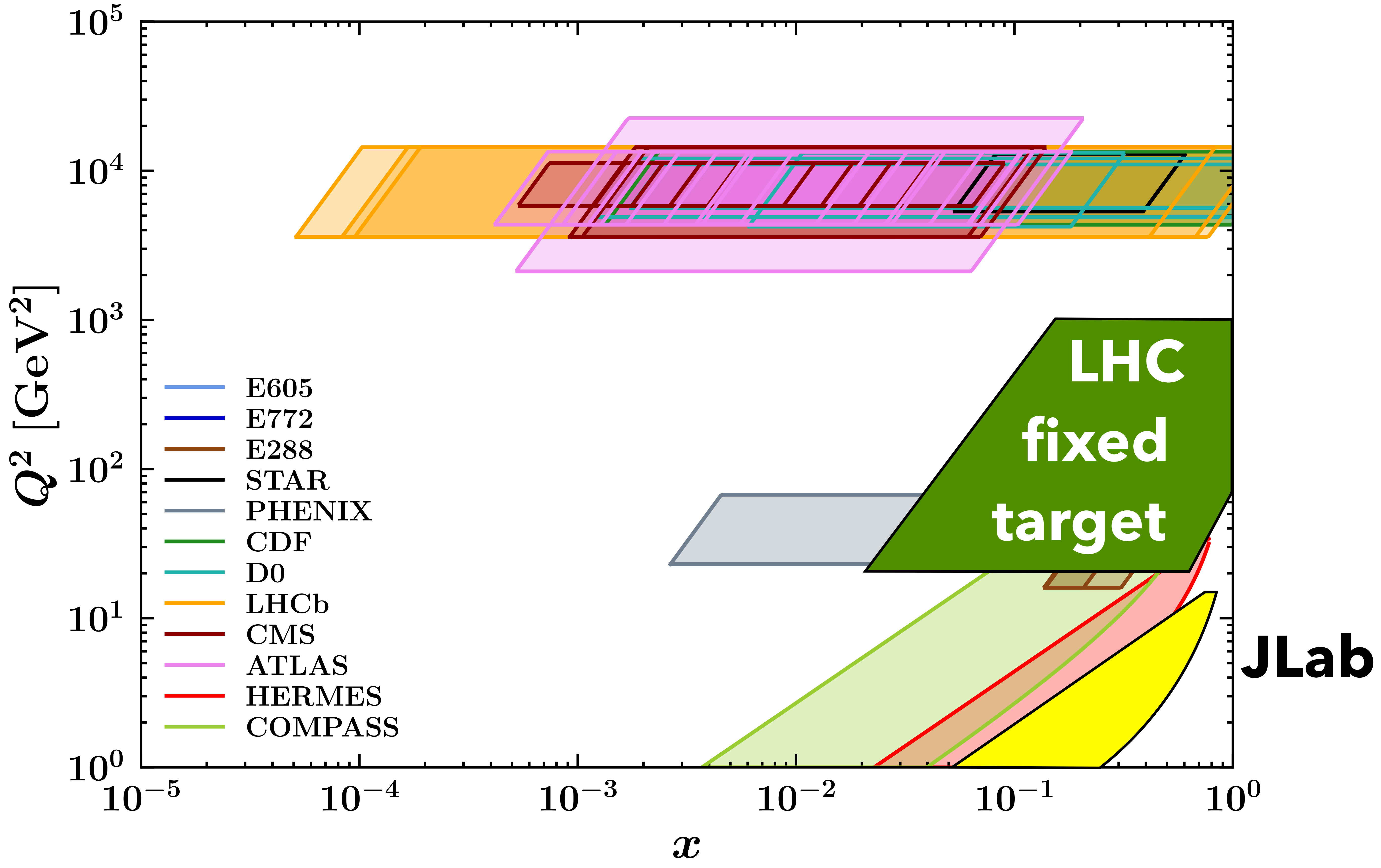
WE NEED MORE DATA



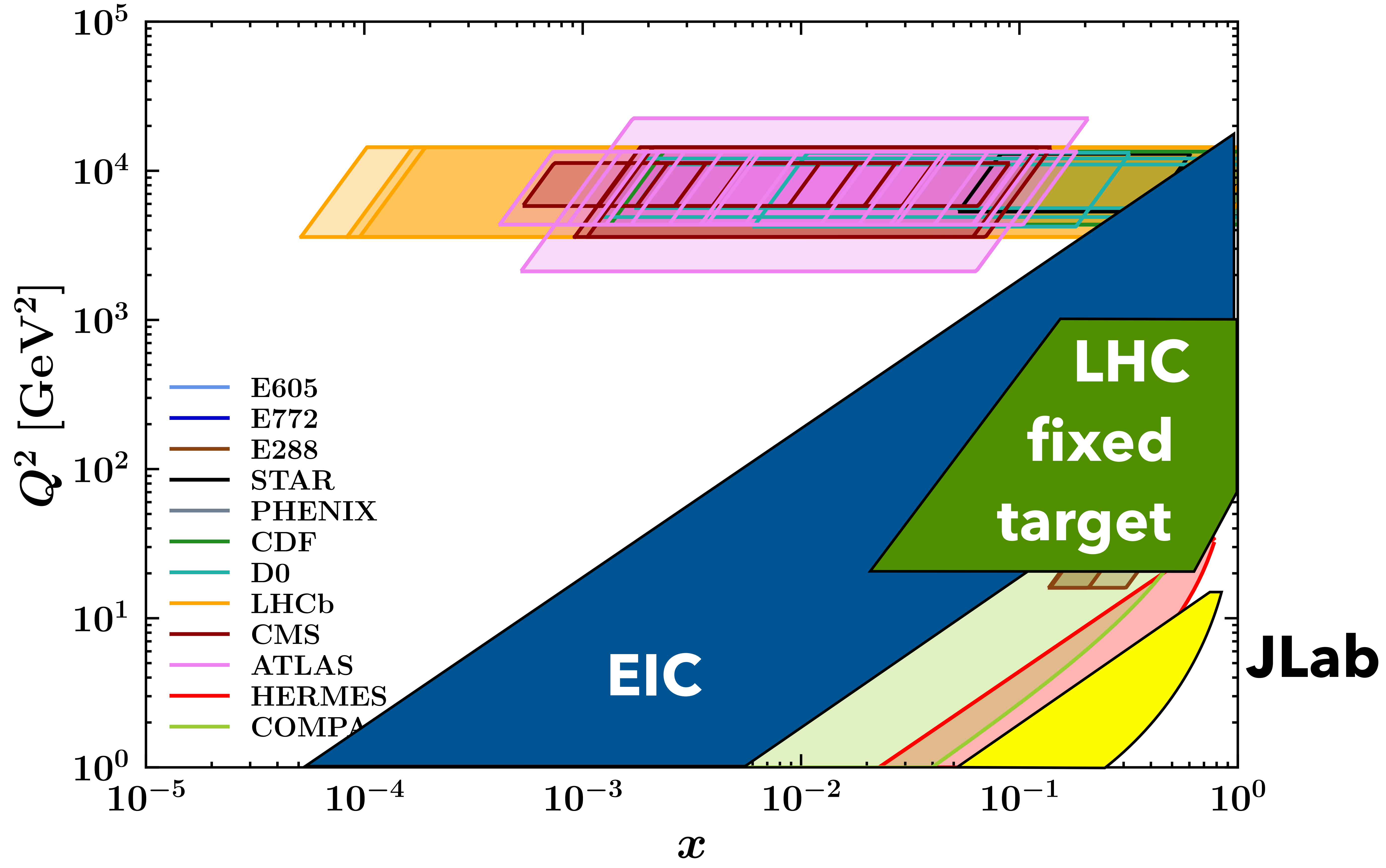
WE NEED MORE DATA



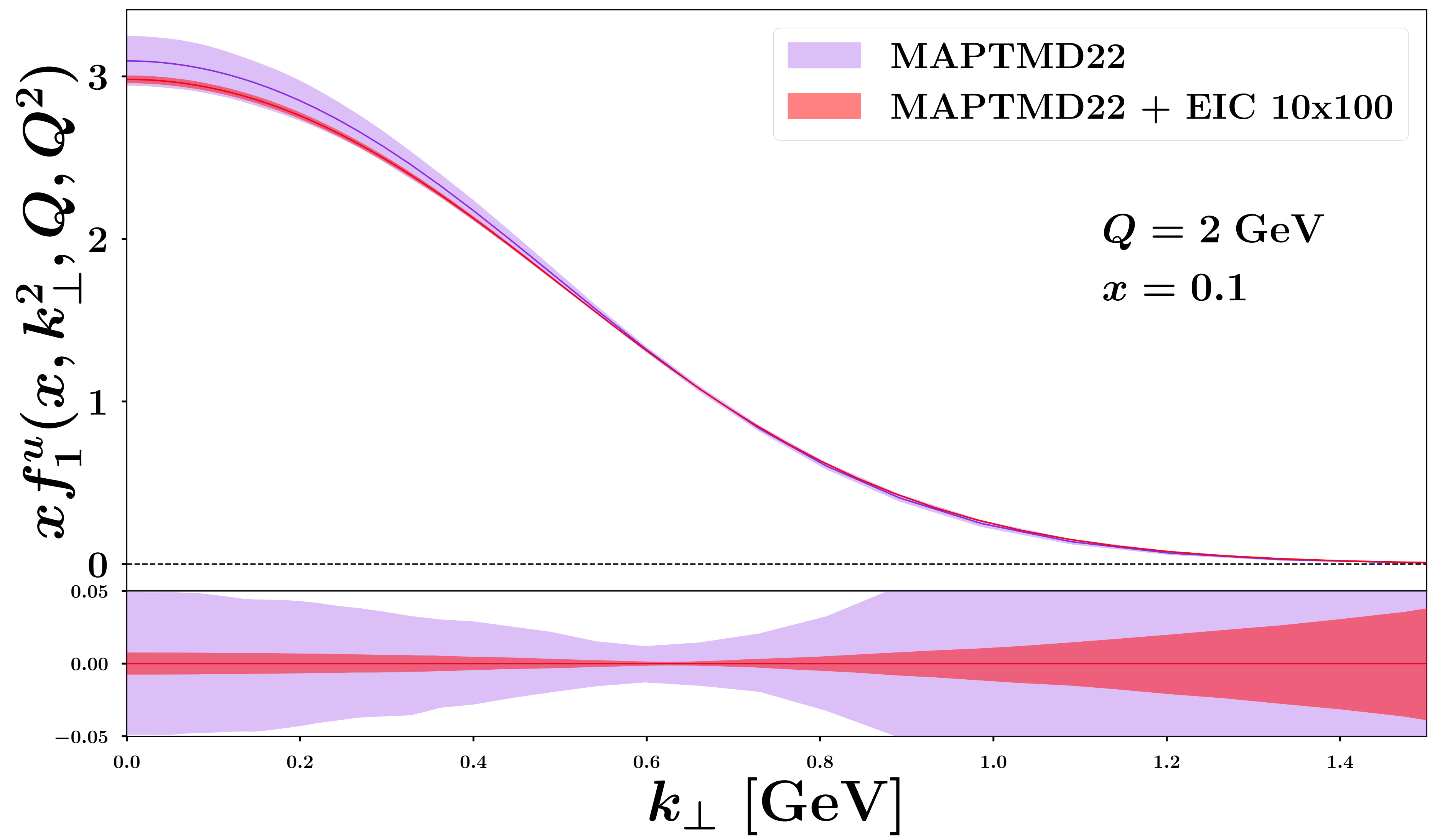
WE NEED MORE DATA

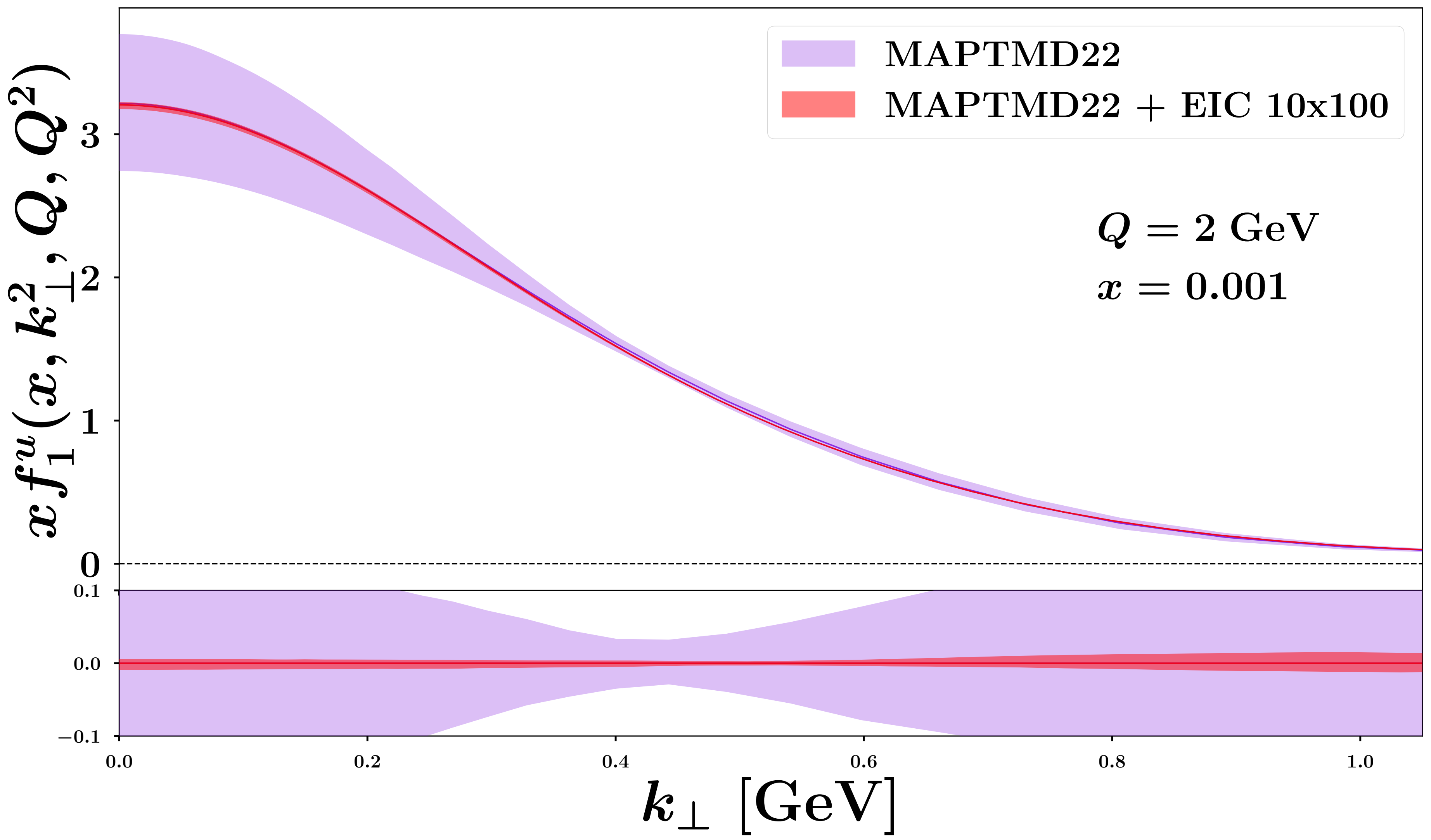


WE NEED MORE DATA



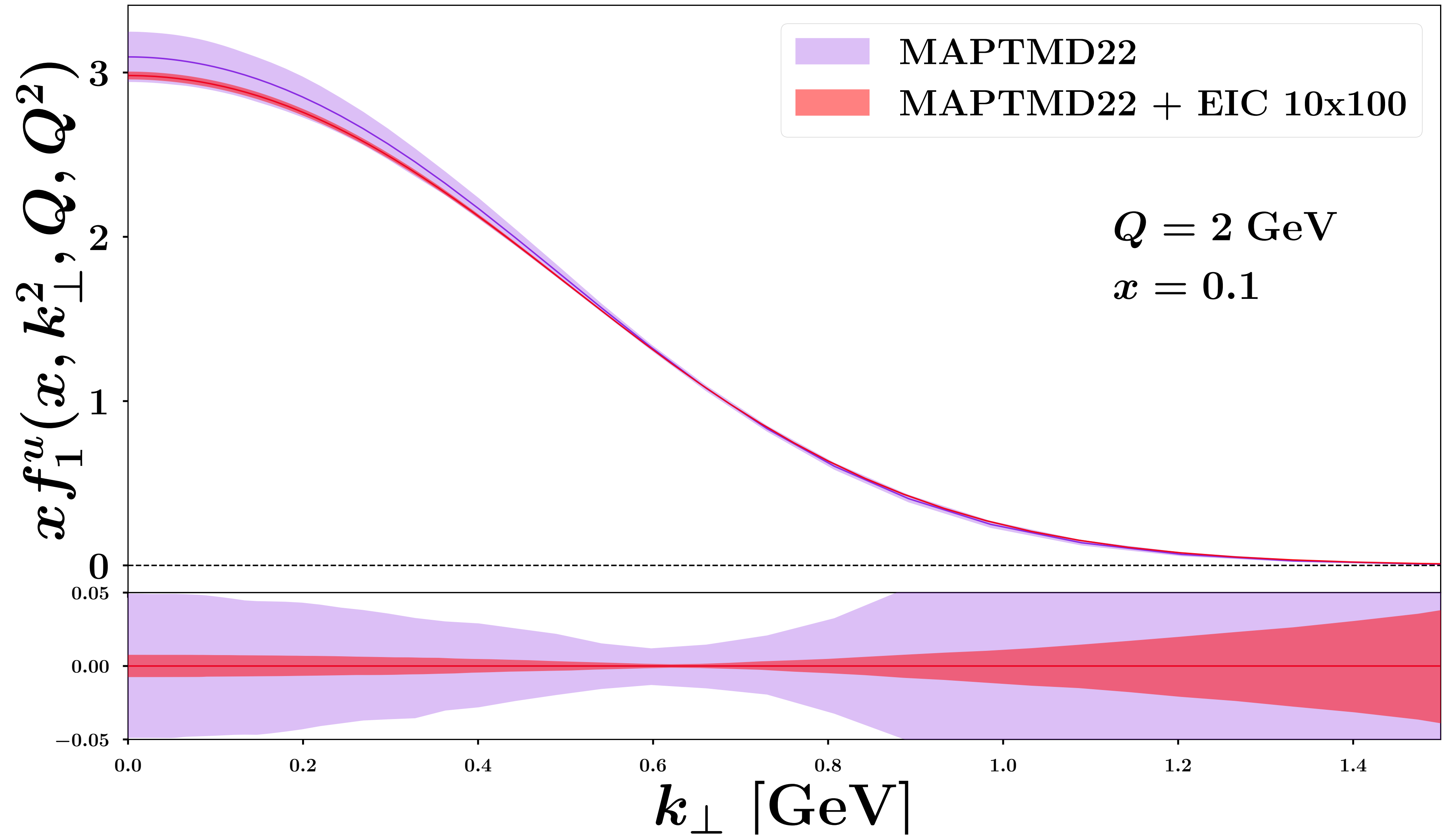
EIC AND JLAB22 IMPACT

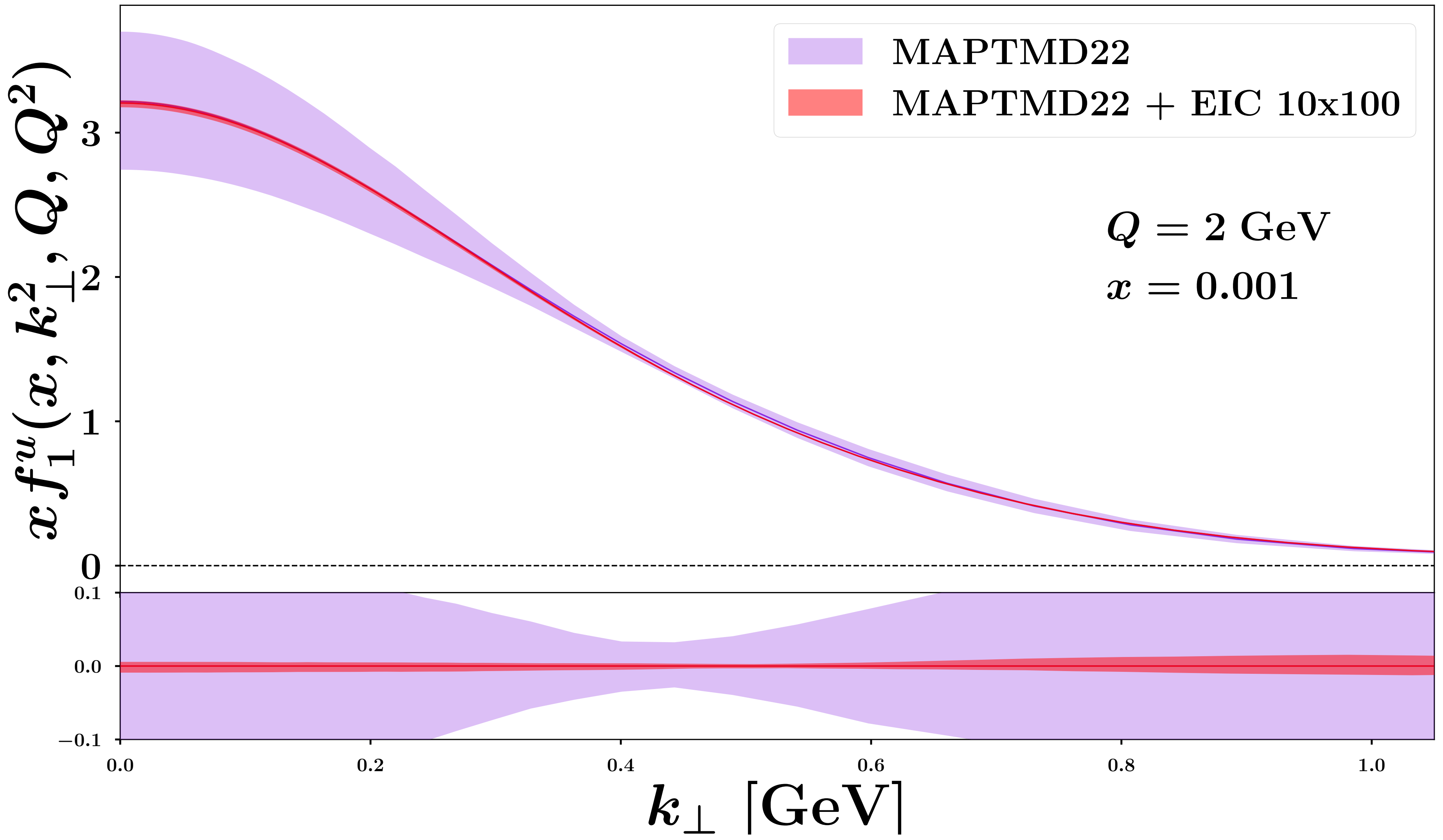




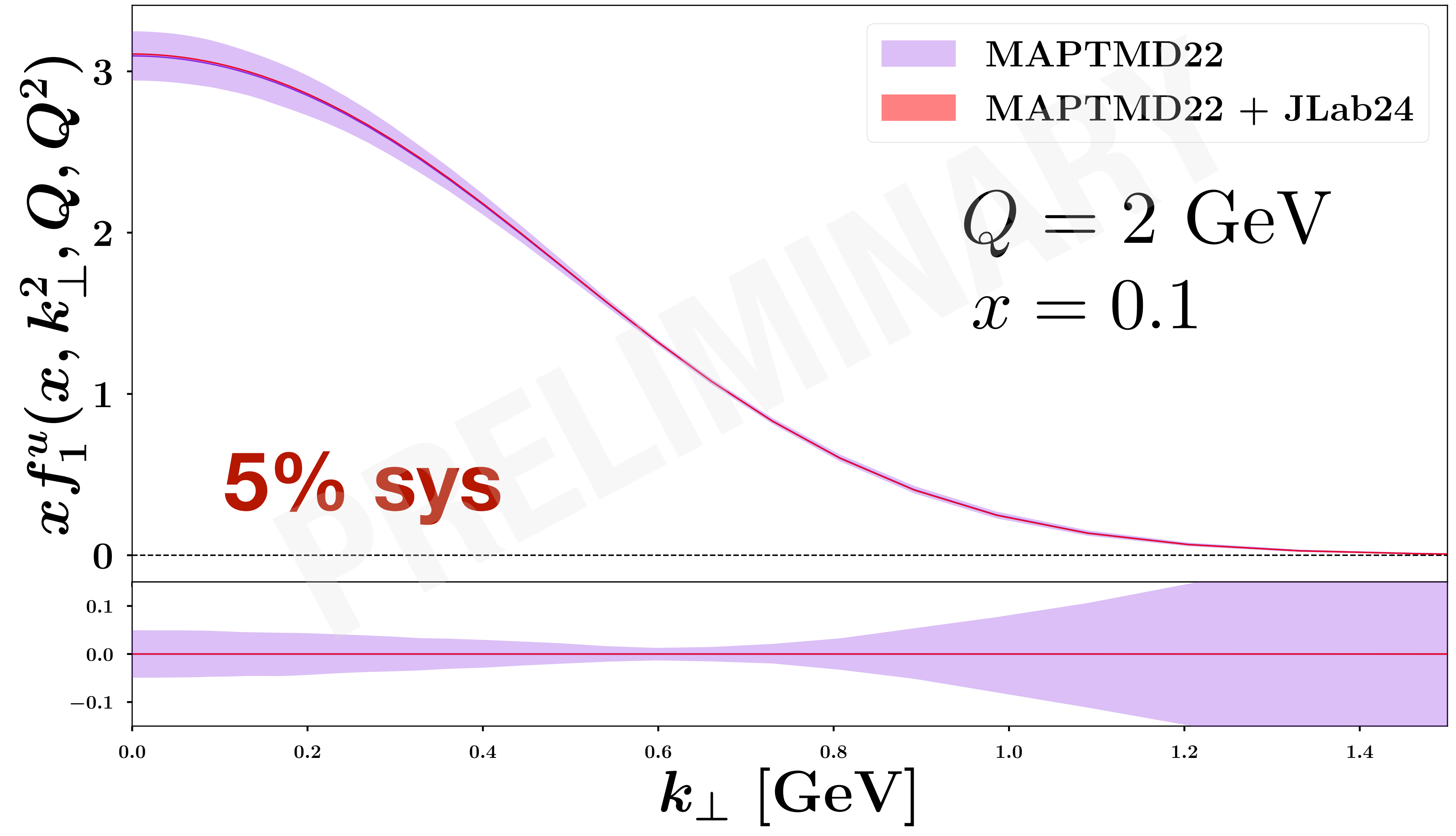
EIC

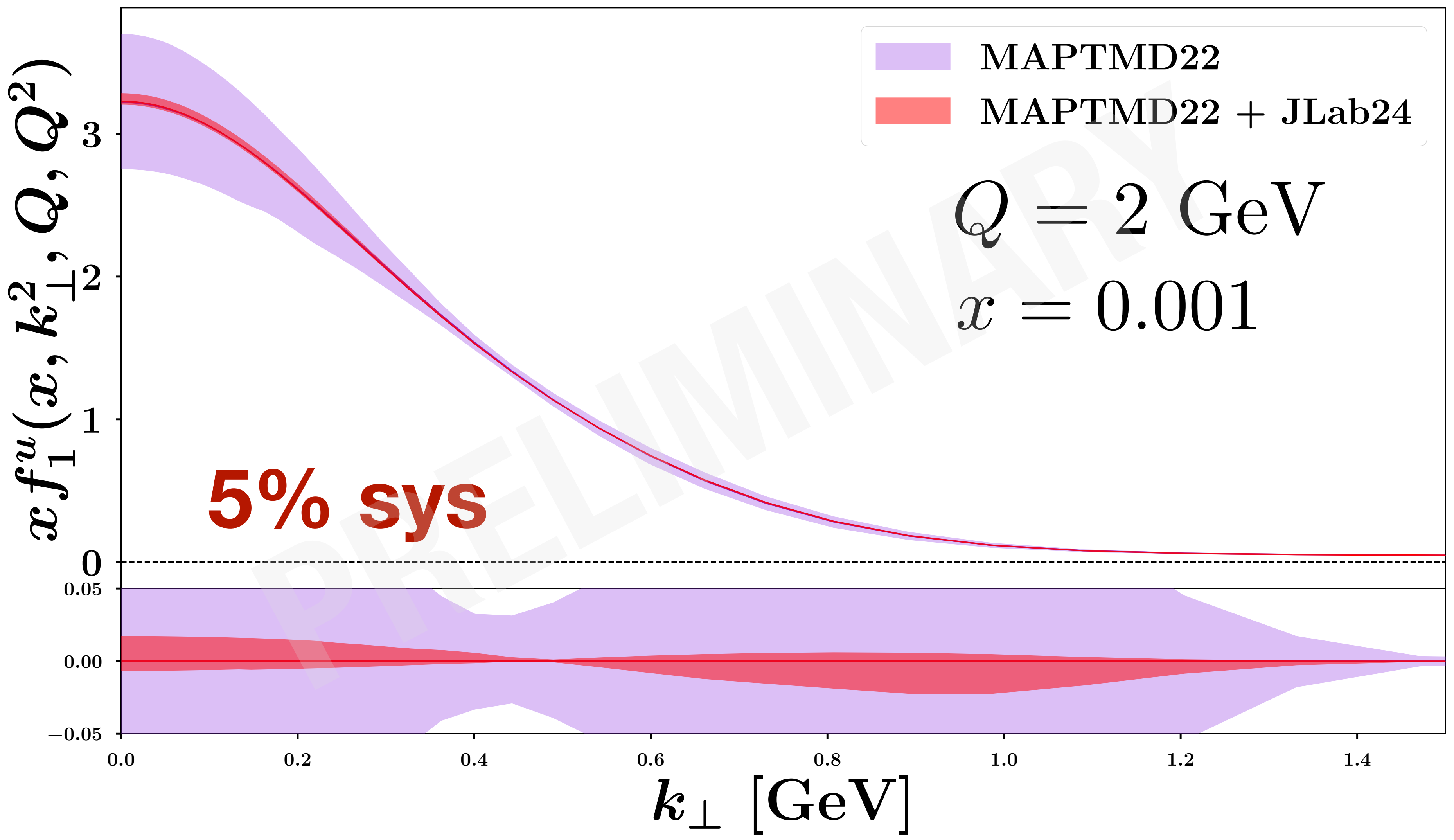
EIC AND JLAB22 IMPACT





EIC





JLAB

CONCLUSIONS

CONCLUSIONS

- From the theoretical side, the formalism to study TMDs is well known, for quarks and gluons at leading twist

CONCLUSIONS

- From the theoretical side, the formalism to study TMDs is well known, for quarks and gluons at leading twist
- Improvements are still needed, e.g., subleading twist and other power corrections, increase of perturbative accuracy

CONCLUSIONS

- From the theoretical side, the formalism to study TMDs is well known, for quarks and gluons at leading twist
- Improvements are still needed, e.g., subleading twist and other power corrections, increase of perturbative accuracy
- From the phenomenological side, we have a good knowledge of the unpolarized TMD, some knowledge of the Sivers function, and some sparse information about other TMDs.