Scattering of $J = 0, 2$ glueballs and their thermodynamic properties

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Overview

Quantum chromodynamics (QCD)

- Non-Abelian gauge theory [gauge group $SU(N_C = 3)$]
- Theory for strong interactions between gluons and quarks

$$\mathscr{L}_{QCD} = -\frac{1}{4}\sum_{a=1}^{8} F^{a\mu\nu}F^{a}_{\mu\nu} + \sum_{j=1}^{n_f} \bar{q}_j(i\gamma^\mu D_\mu - m_j)q_j$$

$D_\mu = \partial_\mu - igA_\mu$
$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu;\ a, b, c = 1, \ldots, N_C^2 - 1$

- The green part is called Yang-Mills (YM) Lagrangian

QCD bound states of q and g

Baryon number $\mathfrak{B}_q = 1/3$, $\mathfrak{B}_{\bar{q}} = -1/3$, $\mathfrak{B}_g = 0$

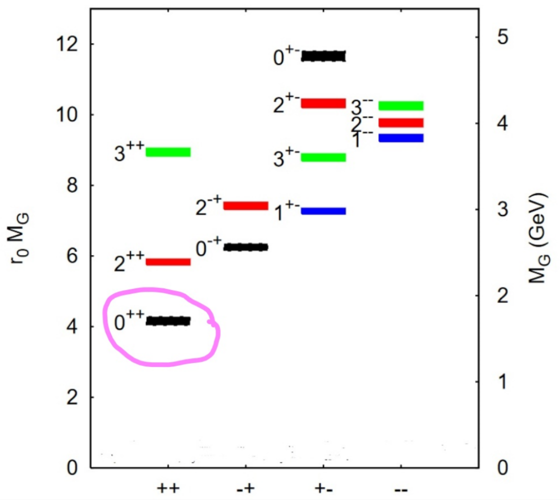
- **Conventional**

$$Hadrons \begin{cases} Baryons & [qqq] & \mathfrak{B} = 1 \\ Mesons & [\bar{q}q] & \mathfrak{B} = 0 \end{cases}$$

- **NON Conventional**
  Baryons: pentaquarks $(\bar{q}qqqq)$,... $\mathfrak{B} = 1$
  Mesons: tetraquarks $(\bar{q}\bar{q}qq)$, glueballs,... $\mathfrak{B} = 0$

Glueballs

- Self interaction gluons → bound state (not yet observed)
- Several lattice QCD simulations of glueballs
- Scalar glueball stable in pure YM



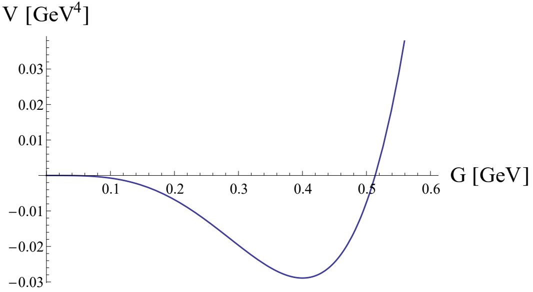
Y. Chen et al, Phys. Rev. D73, 014516 (2006)

What about the scattering between two scalar glueballs (in YM)?

Dilaton Lagrangian

$$\mathcal{L}_{dil} = \frac{1}{2}(\partial_\mu G)^2 - V(G) = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda_G^2}\left(G^4 \ln\left|\frac{G}{\Lambda_G}\right| - \frac{G^4}{4}\right)$$ [1]

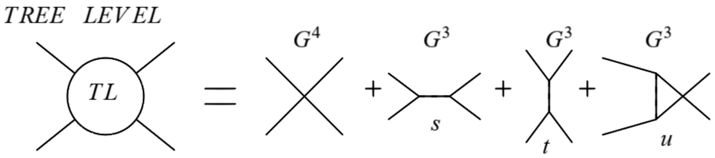
This form reproduces the trace anomaly in YM



[1] A. A. Migdal and M. A. Shifman, "Dilaton Effective Lagrangian In Gluodynamics," Phys. Lett.B114, 445 (1982)

Tree-level scattering EPJC 82 (2022)

$$V(G) \approx -\frac{1}{16}\Lambda_G^4 + \frac{1}{2}m_G^2 G^2 + \frac{1}{3!}\left(5\frac{m_G^2}{\Lambda_G}\right)G^3 + \frac{1}{4!}\left(11\frac{m_G^2}{\Lambda_G^2}\right)G^4 + \ldots$$



$$\mathcal{A}(s,t,u) = -11\frac{m_G^2}{\Lambda_G^2} - \left(5\frac{m_G^2}{\Lambda_G}\right)^2 \frac{1}{s-m_G^2} - \left(5\frac{m_G^2}{\Lambda_G}\right)^2 \frac{1}{t-m_G^2} - \left(5\frac{m_G^2}{\Lambda_G}\right)^2 \frac{1}{u-m_G^2}$$

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$$\begin{aligned} s &= (p_1+p_2)^2\,, \\ t &= (p_1-p_3)^2 = -2k^2(1-\cos\theta)\,, \\ u &= (p_1-p_4)^2 = -2k^2(1+\cos\theta)\,, \end{aligned}$$

$$\mathcal{A}(s,t,u) = \mathcal{A}(s,\cos\theta) = \sum_{\ell=0}^{\infty}(2\ell+1)\mathcal{A}_\ell(s)P_\ell(\cos\theta)$$

$$\boxed{\mathcal{A}_0(s) = -11\frac{m_G^2}{\Lambda_G^2} - 25\frac{m_G^4}{\Lambda_G^2}\frac{1}{s-m_G^2} + 50\frac{m_G^4}{\Lambda_G^2}\frac{\log\left(1+\frac{s-4m_G^2}{m_G^2}\right)}{s-4m_G^2}}$$

$\mathcal{A}_0(s)$

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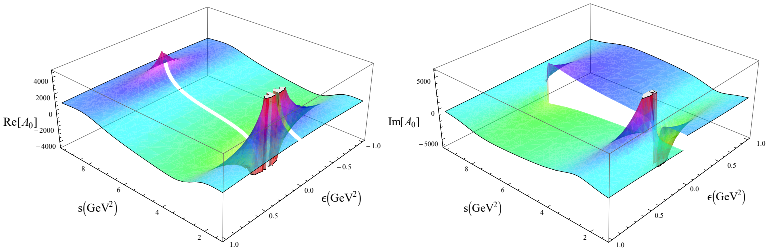
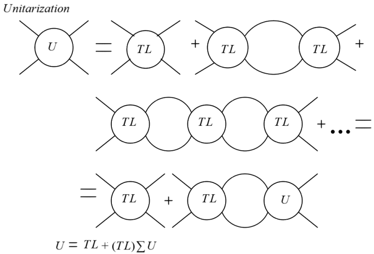


$$\Delta\delta_0 := \delta_0(s\rightarrow\infty) - \delta_0(4m_G^2) \neq n\pi \rightarrow \text{unitarization}$$

Unitarization



Let $\Sigma(s)$ be a properly subtracted glueball-glueball self-energy loop function, then:

$\mathcal{A}_0^U(s) = [\mathcal{A}_0^{-1}(s) - \Sigma(s)]^{-1}$

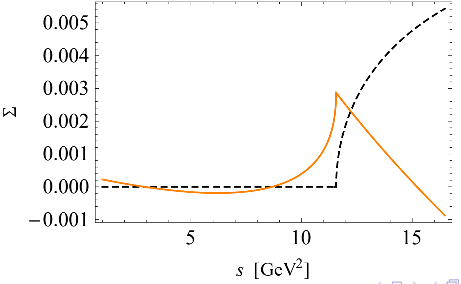
-preserve the pole corresponding to $s = m_G^2$

-$\mathcal{A}_0^U(s = 3m_G^2) = \infty$: the branch point $s = 3m_G^2$ is generated by the single particle pole for m_G^2 along the t and u channels.

$$\Sigma(s) = \frac{(s - m_G^2)(s - 3m_G^2)}{\pi} \int_{4m_G^2}^{\infty} \frac{\mathrm{Im}\Sigma(s')}{(s' - s - i\varepsilon)(s' - m_G^2)(s' - 3m_G^2)} ds'$$

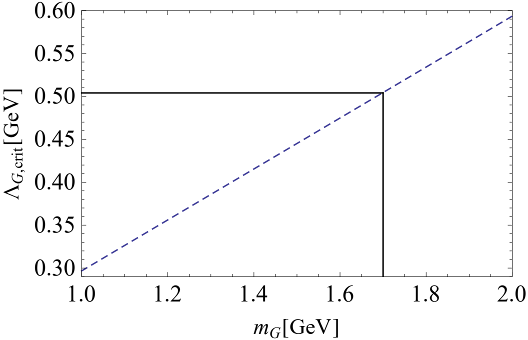
The imaginary part is fixed by (optical theorem)

$$\mathrm{Im}\Sigma(s) = \theta(s - 4m_G^2)\frac{1}{2}\frac{1}{16\pi}\sqrt{1 - \frac{4m_G^2}{s}}$$



$\Lambda_{G,crit}$

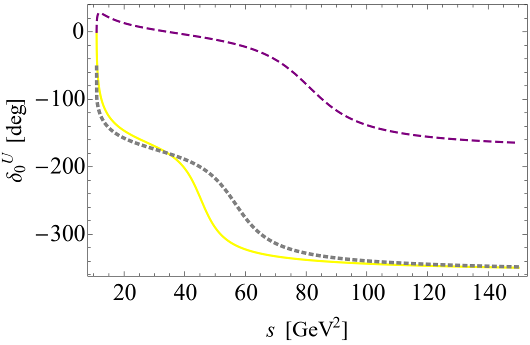
$$\mathcal{A}_0^U(s) = [\mathcal{A}_0^{-1}(s) - \Sigma(s)]^{-1} \xrightarrow{s=s_{th}} \left.\frac{3\Lambda_G^2}{92m_G^2} - \Sigma(4m_G^2)\right|_{\Lambda_G=\Lambda_{G,crit}} = 0$$



Overall result: attraction $\longrightarrow$ decreases up to $\Lambda_{G,crit}$

Phase-shift

$$\frac{e^{2i\delta_\ell^U(s)}-1}{2i}=\frac{1}{2}\cdot\frac{k^{2\ell+1}}{8\pi\sqrt{s}}\mathcal{A}_\ell^U(s)$$

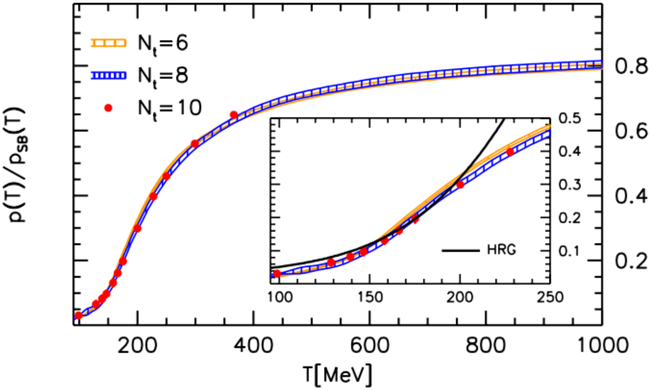


$\Delta\delta_0^U \to -\pi,\ -2\pi$ (Levinson theorem fulfilled)

Can we use these results?

Hadron Resonance Gas model

- All baryons and mesons (m < 2.5 GeV) from PDG [Borsnayi et al.JHEP11(2010)077]



Glueball thermodynamics EPJC83 (2023)

Pressure

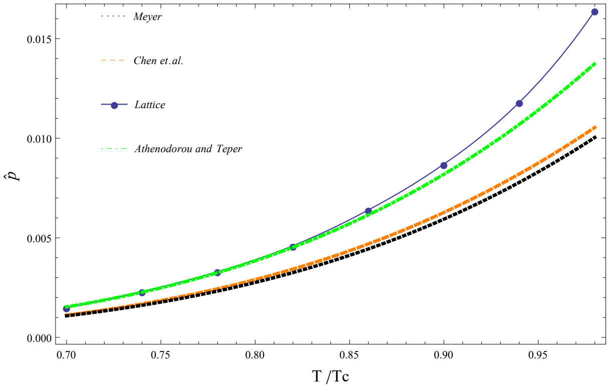
$$p_i = -(2J_i + 1)T \int_0^\infty \frac{k^2}{2\pi^2} \ln\left(1 - e^{-\frac{\sqrt{k^2+m_i^2}}{T}}\right) dk$$

Lattice calculated p from Borsanyi et al.JHEP07(2012)056
VS
GRG model with masses from:
Chen et al. Phys.Rev.D 69 (2004) 076003
Athenodorou, Teper JHEP 11 (2020) 172
Meyer hep-lat/0508002 [hep-lat]

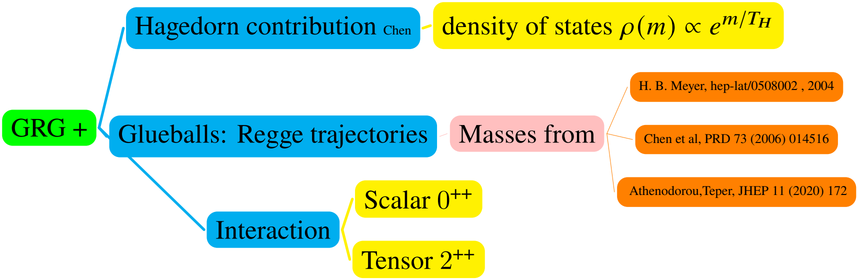
$n\,J^{PC}$	M[MeV]			$n\,J^{PC}$	M[MeV]		
	Chen et al.	Meyer	A & T		Chen et al.	Meyer	A & T
$1\,0^{++}$	1710(50)(80)	1475(30)(65)	1653(26)	$1\,1^{--}$	3830(40)(190)	3240(330)(150)	4030(70)
$2\,0^{++}$		2755(30)(120)	2842(40)	$1\,2^{--}$	4010(45)(200)	3660(130)(170)	3920(90)
$3\,0^{++}$		3370(100)(150)		$2\,2^{--}$		3740(200)(170)	
$4\,0^{++}$		3990(210)(180)		$1\,3^{--}$	4200(45)(200)	4330(260)(200)	
$1\,2^{++}$	2390(30)(120)	2150(30)(100)	2376(32)	$1\,0^{+-}$	4780(60)(230)		
$2\,2^{++}$		2880(100)(130)	3300(50)	$1\,1^{+-}$	2980(30)(140)	2670(65)(120)	2944(42)
$1\,3^{++}$	3670(50)(180)	3385(90)(150)	3740(70)	$2\,1^{+-}$			3800(60)
$1\,4^{++}$		3640(90)(160)	3690(80)	$1\,2^{+-}$	4230(50)(200)		4240(80)
$1\,6^{++}$		4360(260)(200)		$1\,3^{+-}$	3600(40)(170)	3270(90)(150)	3530(80)
$1\,0^{-+}$	2560(35)(120)	2250(60)(100)	2561(40)	$2\,3^{+-}$		3630(140)(160)	
$2\,0^{-+}$		3370(150)(150)	3540(80)	$1\,4^{+-}$			4380(80)
$1\,2^{-+}$	3040(40)(150)	2780(50)(130)	3070(60)	$1\,5^{+-}$		4110(170)(190)	
$2\,2^{-+}$		3480(140)(160)	3970(70)				
$1\,5^{-+}$		3942(160)(180)					
$1\,1^{-+}$			4120(80)				
$2\,1^{-+}$			4160(80)				
$3\,1^{-+}$			4200(90)				

LQCD papers	Number of glueballs	Lattice Parameter	T_c
Chen et al.	12	$r_0^{-1} = 410(20)$ MeV	317 ± 23 MeV
Meyer	22	$\sqrt{\sigma} = 440(20)$ MeV	277 ± 13 MeV
Athenodorou and Teper	20	$r_0^{-1} = 418(5)$ MeV	323 ± 18 MeV

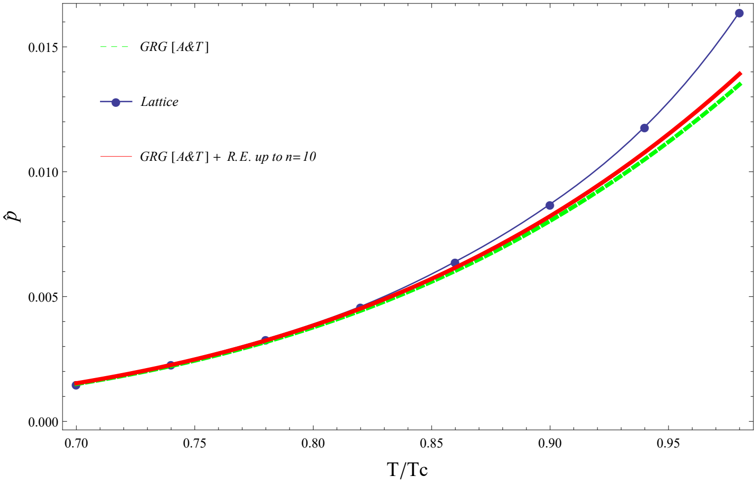
Lattice comparison [Borsnayi et al.JHEP07(2012)056]



with Chen et al.: Hagedorn term $\propto e^{m/T_H}$ needed (Meyer PRD 80 (2009))
with A and T: better agreement without Hagedorn term → hint of accuracy.

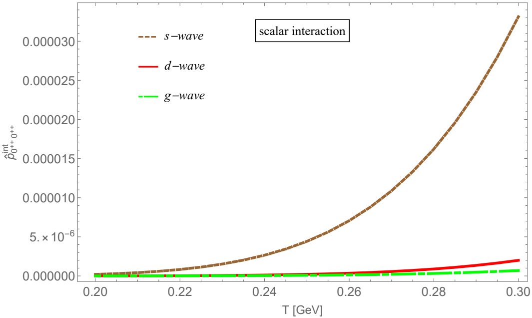

GRG +
Hagedorn contribution Chen
density of states $\rho(m) \propto e^{m/T_H}$
Glueballs: Regge trajectories
Masses from
H. B. Meyer, hep-lat/0508002 , 2004
Chen et al, PRD 73 (2006) 014516
Athenodorou,Teper, JHEP 11 (2020) 172
Interaction
Scalar 0^{++}
Tensor 2^{++}

Additional glueballs



Scalar glueball

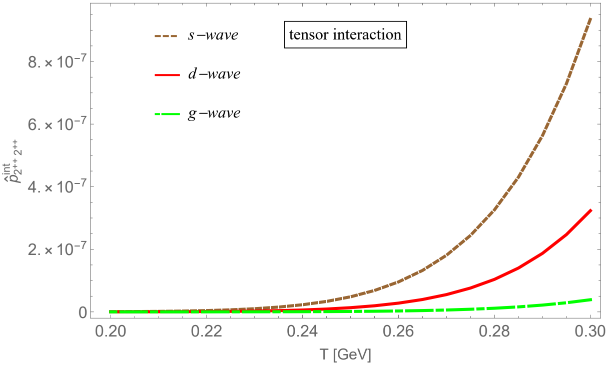
$$\hat{p}^{\text{int}}_{0^{++}0^{++}} = -\frac{1}{T^3}\sum_{l=0}^{\infty}\int_{2m_{0^{++}}}^{\infty} dx \frac{2l+1}{\pi}\frac{d\delta_l^{0^{++}0^{++}}(x)}{dx}\int \frac{d^3k}{(2\pi)^3}\ln\left(1-e^{-\frac{\sqrt{k^2+x^2}}{T}}\right)+\hat{p}_B$$



Samanta, Giacosa PRD, 102 (2020)

Tensor glueball

$$\hat{p}^{\text{int}}_{2^{++}2^{++}} = -\frac{1}{T^3}\sum_{J=0}^{4}\sum_{l=0}^{\infty}\int_{2m_2^{++}}^{\infty} dx(2J+1)\frac{2l+1}{\pi}\frac{d\delta_l^{2^{++}2^{++},J}(x)}{dx}\int\frac{d^3k}{(2\pi)^3}\ln\left(1-e^{-\frac{\sqrt{k^2+x^2}}{T}}\right)$$



- Emergence of a 2 scalar glueball bound state, that we named ”glueballonium”. If existent, could be seen in experiments -G mix, unstable- (PANDA, BesIII, BelleII,LHCb,TOTEM) and in lattice (Yamanaka et al. PRD102 (2020)).
- Free glueball gas with lightest states: sufficient for TMD description of LQCD results for pure YM.
- The critical temperature in YM turns out to be $T_C = 323 \pm 18$ MeV.
- Effect of heavier glueballs and of interactions are very small.
- GRG works well with the masses of Athenodorou & Teper $\longrightarrow$ those masses are favoured.

Thank you for your attention

Phase shift

$$A_\ell(s) = \frac{1}{2}\int_{-1}^{1} d\cos\theta A(s,\cos\theta)P_\ell(\cos\theta)\,.$$

where $P_\ell(\cos\theta)$ are the Legendre polynomials.

$$\hat{A}_\ell(s) = \frac{1}{k^{2l}}A_\ell(s)\,.$$

Scattering lengths:

$$a_\ell = \frac{\hat{A}_\ell(4m_G^2)}{32\pi\, m_G}\,.$$

hence

$$\delta_\ell(s) = \frac{1}{2}\arg\left[1 + 2i\rho_\ell(s)\hat{A}_\ell(s)\right],$$

$$\frac{1}{2}\cdot\frac{k^{2\ell+1}}{8\pi\sqrt{s}} = \rho_\ell(s)$$

YM field

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}, \qquad G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_0 f^{abc} A^b_\mu A^c_\nu$$

gluonic quantum fluctuation: dilatatation symmetry

$$g_0 \xrightarrow[\text{anomaly}]{\text{trace}} g(\mu)$$

$$\Rightarrow \partial_\mu J^\mu_{\text{dil}} = (T^\mu_\mu)_{YM} = \frac{\beta(g)}{2g} G^a_{\mu\nu} G^{a,\mu\nu} \neq 0, \beta(g) = \partial g / \partial \ln \mu$$

At one loop ($\beta(g) = -bg^3$ & $b = 11N_c/(48\pi^2)$):

$$g^2(\mu) = \frac{1}{2b \ln(\mu/\Lambda_{\text{YM}})}, \Lambda_{\text{YM}} \approx 250 \text{ MeV}$$

Nonvanishing expectation value of the trace anomaly:

$$\langle T^\mu_\mu \rangle_{YM} = -\frac{11N_c}{24} \left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a,\mu\nu} \right\rangle = -\frac{11N_c}{24} C^4$$

Glueball field

Gluons $\xrightarrow[\text{confinement}]{}$ not the asymptotic states of the theory

Scalar field G describing scalar glueball & trace anomaly at the composite level

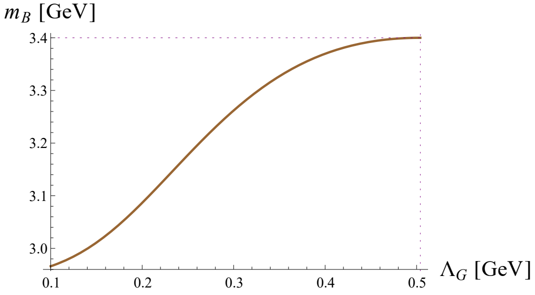
$\mathcal{L}_{\text{dil}} = \frac{1}{2}(\partial_\mu G)^2 - V(G)$

$\partial_\mu J^\mu_{\text{dil}} = 4V - G\,\partial_G V \propto G^4$ only if $V(G) = \dfrac{1}{4}\dfrac{m_G^2}{\Lambda_G^2}\left(G^4 ln\left|\dfrac{G}{\Lambda_G}\right| - \dfrac{G^4}{4}\right)$

$\Lambda_G \simeq 0.4$ GeV: $\left[\partial_\mu J^\mu_{\text{dil}} = -\frac{1}{4}\,\frac{m_G^2}{\Lambda_G^2}\,G^4\right]\langle T^\mu_\mu\rangle_{YM}$: Dilaton field saturates the trace of the dilatation current.

Mass of the glueballonium as function of Λ_G

For $\Lambda_G = 0.4$ GeV: bound state (glueballonium) with a mass of 3.37 GeV.
For $m_B = 2m_G = 3.4$ GeV one has the critical value $\Lambda_{G,crit} \sim 0.5$ GeV.



$$T_c = 1.26(7) \cdot \Lambda_{MS} = 1.26(7) \cdot 0.614(2) \cdot r_0^{-1} \,, \quad (1)$$

$$T_c = 0.629(3) \cdot \sqrt{\sigma} \,. \quad (2)$$

LQCD papers	Number of glueballs	Lattice Parameter	T_c (using Eqs. (1)-(2))
Chen et.al	12	$r_0^{-1} = 410(20)$ MeV	317 ± 23 MeV
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Additional glueballs

$$\chi^2(a, b_{0^{++}}, b_{2^{++}}, b_{0^{-+}}, b_{2^{-+}}, b_{1^{+-}}) = \sum_{J^{PC}} \left(\frac{M(n, J^{PC}) - M^{\text{lat}}(n, J^{PC})}{\delta M^{\text{lat}}(n, J^{PC})} \right)^2$$

Glueball spectrum compared to the fit				Parameters [GeV^2]
$n\, J^{PC}$	m [GeV] (A and T)	Fit [GeV]	χ_i^2	
1 0^{++}	1.653(26)	1.647(25)	0.04	$b_{0^{++}} = -2.78 \pm 0.21$
2 0^{++}	2.842(40)	2.865(30)	0.3	
1 2^{++}	2.376(32)	2.367(30)	0.08	$b_{2^{++}} = -10.87 \pm 0.57$
2 2^{++}	3.30(5)	3.33(3)	0.38	
1 0^{-+}	2.561(40)	2.572(38)	0.08	$b_{0^{-+}} = 1.12 \pm 0.27$
2 0^{-+}	3.54(8)	3.48(4)	0.57	
1 2^{-+}	3.07(6)	3.11(5)	0.52	$b_{2^{-+}} = -6.79 \pm 0.66$
2 2^{-+}	3.97(7)	3.90(4)	1.10	
1 1^{+-}	2.944(42)	2.955(37)	0.07	$b_{1^{+-}} = -2.25 \pm 0.45$
2 1^{+-}	3.80(6)	3.77(3)	0.23	
			χ^2_{tot}=3.38	$a = 5.49 \pm 0.17$

Trace anomaly