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Understanding the nature of the controversial $\rho(1250)$ meson through the covariant representation of hadrons

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Outline

1. Introduction
2. A framework for covariant description of hadrons
3. Possible assignments for excited ρ states based on linear Regge trajectories
4. Study of pion emission decays
5. Summary

1. Introduction

PDG listings and conventional quark model

- In the PDG summary table (including only established states), the low-lying excited states of $\rho(770)$ are $\rho(1450)$ and $\rho(1700)$, in the energy region below 2 GeV.

- These masses are fairly agreement with the calculated results for $2S$, $1D$ levels of constituent quark model by Godfrey and Isgur.

- Note that reported masses and widths of $\rho(1450/1700)$ listed in PDG seems to be scattered in each experiments.

→ **See, C. Amsler et al., "Spectroscopy of Light Meson Resonances" in Ref. [1].**

GI quark model predictions vs. PDG listings

${n_r}^{2S+1}L_J$	GI [1]	PDG [2]
$1\,{}^3S_1$	0.77	$\rho(770)$
$2\,{}^3S_1$	1.45	$\rho(1450)$
$1\,{}^3D_1$	1.66	$\rho(1700)$

[1] S. Godfrey and N. Isgur, PRD32 (1985) .

[2] R.L. Workman et al. (Particle Data Group),PTEP 2022 (2022).

Preceding work and our confirmation

- Results of analysis of $\pi\pi$ scattering phase shift and inelasticity

[3] N. Hammoud, R. Kaminski, V. Nazari, G. Rupp, PRD102 (2020).
implemented unitary of three-channel S-matrix parametrization and (approximate) crossing symmetry

  - **In addition to $\rho(1450)$, an extra $\rho(1250)$ exists.**
  - **$\rho(1700)$(referred in PDG) seems to be separated into $\rho(1600)$ and $\rho(1800)$.**

- As shown in the previous talk, we have confirmed the existence of $\rho(1250)$ in the $e^+e^- \to \omega\pi$ decay process.

Obtained parameters in Ref. [3] and significance in $e^+e^- \to \omega\pi$ in our analysis. (in MeV)

states	$\rho(1250)$		$\rho(1450)$		$\rho(1600)$		$\rho(1800)$	
Ref. [3]	Mass	Width/2	Mass	Width/2	Mass	Width/2	Mass	Width/2
	1264.1(33)	146.7(12)	1424.7(26)	104.9(24)	1595.1(5)	69.5(4)	1779.2(14)	121.9(16)
Significance in $\omega\,\pi$	$4.92\ \sigma$		$> 10\ \sigma$		$1.74\ \sigma$		0.74σ	

Difficulty of $\rho(1250)$

- However, the existence of extremely low-mass $\rho(1250)$ leads to serious difficulties for the classification of states based on the standard nonrelativistic quark model.
  In fact, it is difficult to make the predicted mass of the $2\,{}^3S_1$ state in the relevant mass region smaller than the value of GI model.

- One explanation for the light $\rho(1250)$ state is that it is realized by the effect of channel coupling by meson loops.

bare ρ′ mass = 1.48 GeV → physical ρ′ mass = 1.29 GeV.

[4] G. Rupp, S. Coito, E. van Beveren, APPB Proc. Suppl.(2012).

It should be noted, however, this approach would lead to change whole parameters, including the ground-state.

- Taking the above results of analyses seriously, we try to explain these excited ρ states, especially focusing the $\rho(1250)$, in the framework of relativistically extended quark model. We propose a novel classification that incorporates a negative energy components, which is necessarily appeared in relativistic framework.

2. A framework for covariant description of hadrons

Covariant representation scheme

- In adopting the quark model approach, its relativistic extension is essential to handle light quark mesons. Since the LS coupling plays an effective role in the classification of excited hadrons, we extend the WF covariantly keeping the spacetime-part and spin-part, separately.

$$\Psi(x,X)^{(\pm)b,\beta}_{a,\alpha} = \underbrace{\frac{1}{\sqrt{2P_0}}e^{\mp iPX}}_{\text{External}}\underbrace{\Phi(x,P)^{b,\beta}_{a,\alpha}}_{\text{Internal}}$$

[5] Feynman, Kislinger and Ravndal (1971),
Namiki et al. (1970),
Ishida et al. (1971),
Kim, Noz, PRD8 (1973).

Internal WF

$$\Phi(x,P)^{b,\beta}_{a,\alpha} = \overbrace{f(x,P)}^{\text{spacetime-part}}\,\overbrace{W(P)^{\beta}_{\alpha}}^{\text{spin-part}}\,\phi^{(\text{Flavor})b}_{a}$$

$$\sim u(P)_\alpha\,\bar{v}(P)^\beta$$

$$O(3,1)_L \otimes \tilde{U}(4)_\sigma \quad \rightarrow O(3)_L \otimes SU(2)_\sigma$$

In this work we apply the above covariant bilocal- and bispinor-WF to describe the light composite $q\bar{q}$ systems.

Linear rising Regge trajectory

- Our covariant WF is set as a solution of the master Klein-Gordon equation with 4 dimensional SHO potential.

$$\begin{cases} \left(\frac{\partial^2}{\partial X_\mu \partial X^\mu} + \mathcal{M}^2(x)\right)\Psi(X,x)_\alpha{}^\beta = 0, \\ \mathcal{M}(x)^2 = -4m\left(-\frac{1}{2\mu}\frac{\partial^2}{\partial x_\mu \partial x^\mu} + \frac{1}{2}K x_\mu x^\mu\right) + \text{const.} \end{cases}$$

[6] Takabayasi,NC33(1964),Ishida,Otokozawa,PTP47(1972), Ishida, Yamada, PRD35 (1987).

($X^\mu = (x_1^\mu + x_2^\mu)/2$, $x^\mu = x_1^\mu - x_2^\mu$, $\mu = m/2$, m: quark mass.)

- In order to freeze the degrees of freedom for the excitations corresponding to oscillations in the relative-time direction at rest frame of meson, the definite-metric auxiliary condition $P^\mu\ {a_\mu}^\dagger f_{4\mathrm{dSHO}}(x,P) = 0$ (, where ${a_0}^\dagger$ is the annihilation operator,) is imposed.

- With above treatment of relative time, it can be derived that the eigenvalue of the mass-squared operator is proportional to the orbital and radial quantum numbers, respectively (so-called linear rising Regge trajectory). In this work, Ω_l and Ω_r are treated as phenomenological parameters.

$$\boxed{M_{n_r,l}^2 = l\Omega_l + n_r\Omega_r + \text{const.}} \qquad (n_r, l = 0,1,2,\cdots)$$

Covariant spin WF including negative ρ_3 components

- The bi-spinor WF consists of the direct product of the Dirac spinors boosted by the center-of-mass velocity. It has 16 independent components since it necessarily includes a negative energy component, which does not exist in a non-relativistic framework.

$$W(P)_\alpha^\beta = \{1, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu}\}_\alpha^\beta$$

Two types of WF with $J^{PC} = 1^{--}$ (apart from excitation of spacetime part) can be constructed from γ_i and σ_{jk} and recombining them so that they have the same sign of ρ_3 yields

$$W_{V++}^{(h)}(P) = -\frac{\gamma^\rho \epsilon^{(h)}(P)_\rho}{2\sqrt{2}}(1 - \not{v}), \quad W_{V--}^{(h)}(P) = \frac{\gamma^\rho \epsilon^{(h)}(P)_\rho}{2\sqrt{2}}(1 + \not{v})$$

ρ spin: $(\rho_3 = +, \rho_3' = +)$

ρ spin: $(\rho_3 = -, \rho_3' = -)$

Here, the ρ spin represents a positive (+) or negative (-) energy components of respective covariant Dirac spinors. Note that both of spin WFs have a correctly positive norm.

Covariant spin WF including negative ρ_3 components

- In addition, spin WF with one positive and the other negative combinations lead to the quantum numbers $J^{PC} = 0^{++}$ and 1^{++};

$$W_{S_A^-}(P) = \frac{1}{2}, \qquad W_{A^-}^{(h)}(P) = -\frac{\gamma^\rho \epsilon^{(h)}(P)_\rho}{2}\gamma_5$$

ρ spin: $(\rho_3 = \pm, \rho_3' = \mp)$ $\qquad$ ρ spin: $(\rho_3 = \pm, \rho_3' = \mp)$

By coupling the above spin WF with the P-wave excited space-time WF, another types of $J^{PC} = 1^{--}$ WF can be constructed. However, note that these spin WFs have negative norms and cannot be physical states independently.

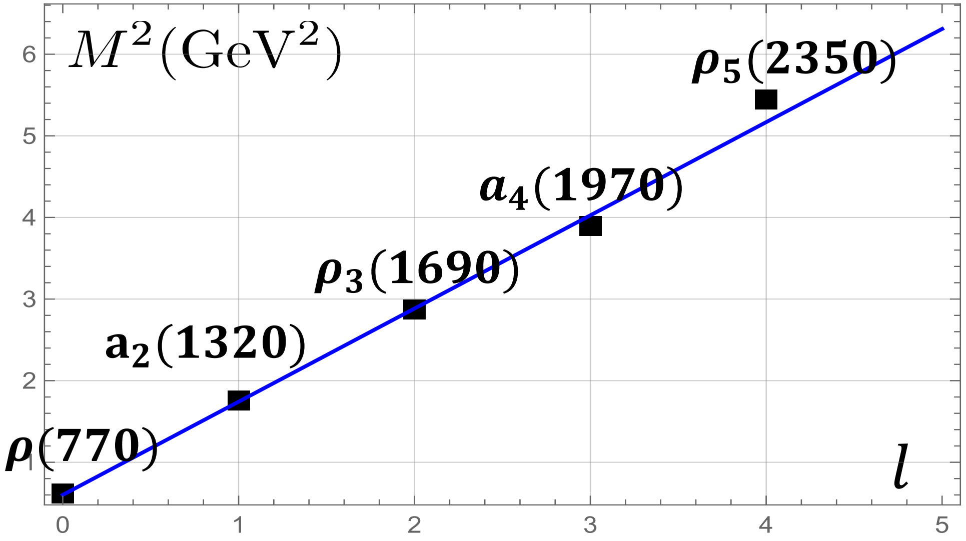
- In a relativistic scheme, solutions with negative energy components are necessarily included as part of the complete set for meson WF. We shall define a physical state as a state with a positive definite WF norm. However, states in which one of the constituents has negative energy (having negative norm) may also contribute as components to the other (positive norm) state.

3. Possible assignments for observed ρ states based on linear Regge trajectories

Squared mass spectra and possible assignments

- Based on our classification scheme, we make possible assignments of excited ρ states including states reported in Ref. [3].

- We can determine the slope of the orbital trajectory as $\Omega_l = 1.14\ \mathrm{GeV}^2$. In the following analysis, we assume the ``universal slope", $\Omega_l = \Omega_{n_r}$.

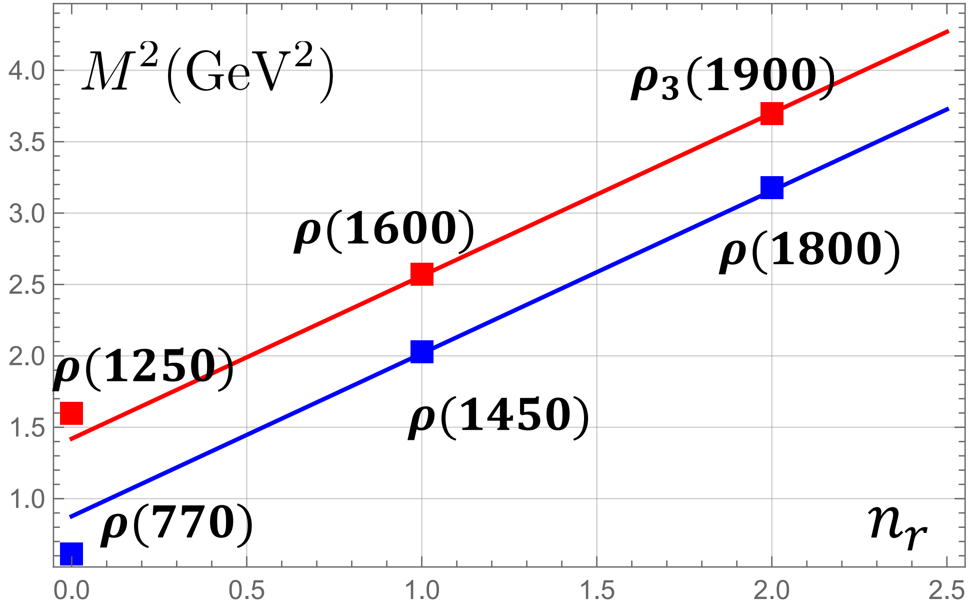


	$(+,+)$		$(-,-)$	
1^3S_1	$\rho(770)$	**0.775**	$\rho(1250)$	**1.26**
2^3S_1	$\rho(1450)$	**1.42**	$\rho(1600)$	**1.60**
1^3D_1	-	**~1.65**	-	**~1.87**
3^3S_1	$\rho(1800)$	**~1.78**	$\rho(1900)$	**~1.92**
2^3D_1	$\rho(2000)^*$	**~1.97**	$\rho(2150)$	**~2.15**
3^3D_1	$\rho(2270)^*$	**~2.24**	-	**~2.40**

(Mass values are given in GeV; * denotes ``Further states" referred in PDG.)

Squared mass spectra and possible assignments

- Next, the radial S-wave trajectory is shown. We observe that $\rho(1800)$ is on the trajectory as ``$(+,+)$'' states including $\rho(770)$ and $\rho(1450)$ (as in the GI model). Accordingly, we identify $\rho(1800)$ as the 3S state. In addition, $\rho(1250)$, $\rho(1600), \rho(1900)$ can be plotted on another radial S-wave trajectory. Hence, we assign these states relativistic ``$(-,-)$'' states.



	$(+,+)$		$(-,-)$	
1^3S_1	$\rho(770)$	**0.775**	$\rho(1250)$	**1.26**
2^3S_1	$\rho(1450)$	**<u>1.42</u>**	$\rho(1600)$	**<u>1.60</u>**
1^3D_1	-	**~1.65**	-	**~1.87**
3^3S_1	$\rho(1800)$	**~1.78**	$\rho(1900)$	**~1.92**
2^3D_1	$\rho(2000)^*$	**~1.97**	$\rho(2150)$	**~2.15**
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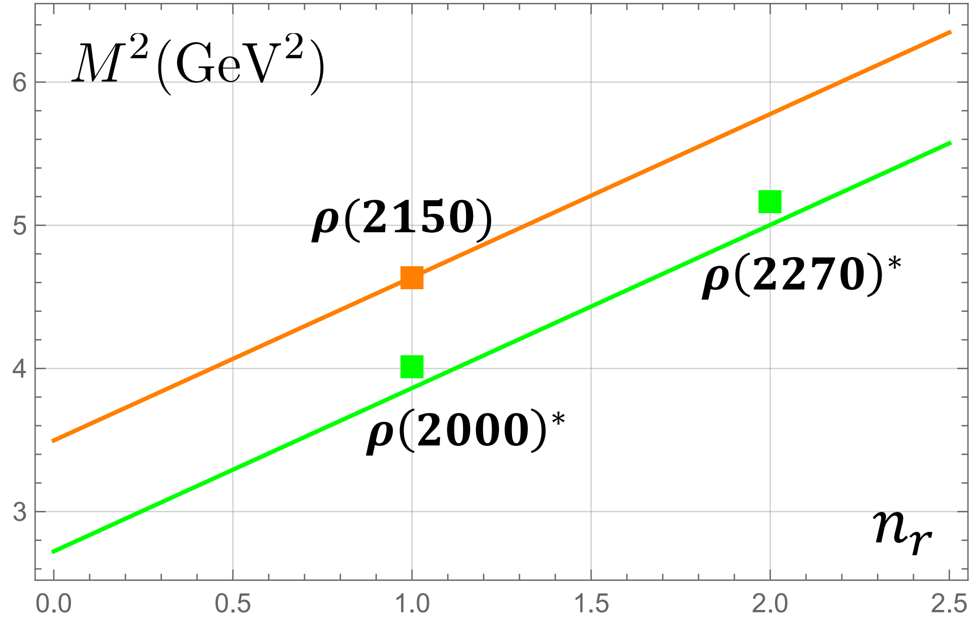
(Mass values are given in GeV; * denotes ``Further states'' referred in PDG.)

Squared mass spectra and possible assignments

- Finally, the radial D-wave trajectories derived by inputting the S(P)-wave mass values are shown. Although there are some ambiguity on how the inputs to be taken, $\rho(2000)$, $\rho(2270)$, $\rho(2150)$ successfully assigned as $2D^{++}$, $3D^{++}$, $2D^{--}$ states, respectively. Note that two more states are expected in the 1D locations.



	$(+,+)$		$(-,-)$	
1^3S_1	$\rho(770)$	**0.775**	$\rho(1250)$	**1.26**
2^3S_1	$\rho(1450)$	**<u>1.42</u>**	$\rho(1600)$	**<u>1.60</u>**
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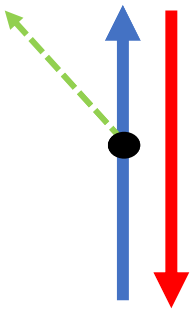
(Mass values are given in GeV; * denotes ``Further states" referred in PDG.)

4. Study of pion emission decays

Effective quark-pion coupling

- In order to clarify the nature of $\rho(1250)$, we study the pion decay properties using the covariant WF of our model. In this study, we treat the emitted pions as point-like Nambu-Goldstone particles.

- The effective coupling satisfying the PCAC condition, equivalent to that used in ``Feynman, Kislinger and Ravndal (1971)”, is given by



$$\int d^4X\mathcal{L} = -2i\int d^4x_1\int d^4x_2 \mathrm{tr}\left(\bar{\Psi}(x_1,x_2)\frac{g_A}{\sqrt{2}f_\pi}\gamma_5\sigma_{\mu\nu}(\overrightarrow{\partial_1^\nu}-\overleftarrow{\partial_1^\nu})\Psi(x_1,x_2)\right)\partial_1^\mu\phi_\pi$$

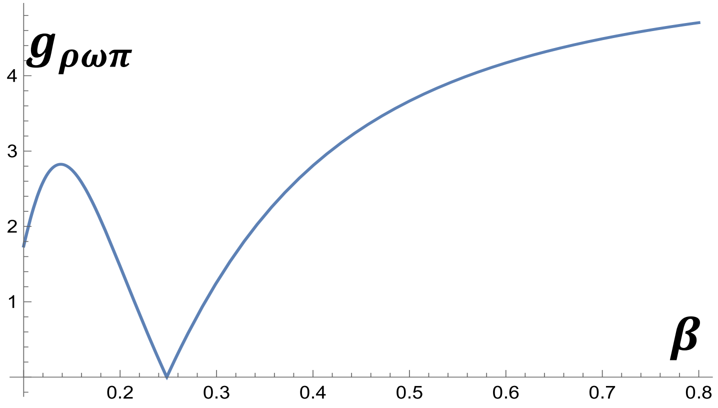
Remark: Spin WF of ``composite pion” required in $\pi\pi$ decay is taken to be $W = \gamma_5/2$. This treatment leads correctly reproduce KSRF rel. $g_{\rho\pi\pi} = M_\rho/(\sqrt{2}f_\pi) \simeq 5.9$ with following parametrization.

- The parameters used in the calculations are $f_\pi = 93\ \mathrm{MeV},\ g_A \sim 1.$
Note that the Gaussian parameter of the 4d SHO WF is uniquely determined to be $\beta = 0.38\ \mathrm{GeV}$ by study of the Regge trajectory.

Coupling to $\pi\pi/\omega\pi$ channel

- Using the above amplitude formula, we show the calculated results of the coupling to $\pi\pi$ and $\omega\pi$ channel for each possible components that can be coupled to physical $\rho(1250)$ state.

- Due to the WF nodes and boosting effects, the couplings to both mode are quite small, which makes it difficult to explain $\rho(1250)$ as conventional $2S^{++}$.



β dependence for $g_{\rho(2S^{++})\omega\pi}$

Effective couplings

$$\rho(1250) \to \pi\pi : \quad \mathcal{L}_{\rho\pi\pi} = g_{\rho\pi\pi}\varepsilon_{abc}\rho_\mu^a \pi^b \partial^\mu \pi^c$$

$$\rho(1250) \to \omega\pi : \quad \mathcal{L}_{\rho\omega\pi} = \frac{g_{\rho\omega\pi}}{M}\delta_{ab}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu \rho_\nu^a \partial_\alpha \omega_\beta \pi^b$$

Initial ρ	$g_{\rho\pi\pi}$	$g_{\rho\omega\pi}$
$2^3S_1^{++}(1.260)$	1.08	2.57
$1^3S_1^{--}(1.260)$	6.98	0.949
$1^3P_1^{-}(1.260)$	0	6.90
$1^1P_1^{-}(1.260)$	0	15.9

Coupling to $\pi\pi/\omega\pi$ channel

- Instead, the states with $1S^{--}$ core are found to couple strongly to $\pi\pi$ and to be broad, consistent with the result in Ref. [3].

- On the other hand, as confirmed by our analysis, there seems to be a certain coupling $\rho(1250) \to \omega\pi$. Interestingly, the P-wave components with negative norm couple only to the $\omega\pi$ mode. Thus, the inclusion of such contributions would allow us to fully reproduce the results of analysis.

e. g. pure $1^3S_1^{--}(1.260)$;

$$\Gamma(1^3S_1^{--}(1.260)) \simeq \Gamma(\pi\pi) + \Gamma(\omega\pi)$$
$$\sim 378 \text{ MeV} + 1 \text{ MeV}$$

Effective couplings

$$\rho(1250) \to \pi\pi : \quad \mathcal{L}_{\rho\pi\pi} = g_{\rho\pi\pi}\varepsilon_{abc}\rho^a_\mu \pi^b \partial^\mu \pi^c$$

$$\rho(1250) \to \omega\pi : \quad \mathcal{L}_{\rho\omega\pi} = \frac{g_{\rho\omega\pi}}{M}\delta_{ab}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu \rho^a_\nu \partial_\alpha \omega_\beta \pi^b$$

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$1^1P_1^{-}$ (1.260)	0	15.9

$$|\rho(1250)_{\text{phys}}\rangle \simeq c_1|1^3S_1^{--}\rangle + c_2|1^1P_1^{-}\rangle + c_3|1^3P_1^{-}\rangle + \cdots$$

Dominant
$\to \pi\pi$ strong

Negative norm contirib.
$\to \omega\pi$ strong and enhance $1^3S_1^{--} \to \omega\pi$

5. Summary

Summary and significant results

1. In this work we have studied excited ρ mesons in the covariant representation scheme to elucidate the property of recently reported (and have confirmed by us) enigmatic $\rho(1250)$ in the relativistically extended quark model point of view.

2. There are two S-wave radial trajectories in our novel classification, the conventioal $(+,+)$ states to which $\rho(770), \rho(1450)$ and $\rho(1800)$ belong, and the other ``relativistic $(-,-)$ core" states where $\rho(1250)$, $\rho(1600)$ and $\rho(1900)$ can be assigned.

3. A closer look at the data listings for $\rho(1700)$ in PDG shows a clear difference in mass and width between about 1600 and 1800. In addition, we point out that the possible existence of $\rho(\sim 1.66)$ and $\rho(\sim 1.87)$ as the $1D^{++}$ and $1D^{--}$ - states near 1700 MeV.

Remark: $\rho(1570)$ (ref. in PDG) may be identified as $\rho(1600)$ due to reported M, Γ.

4. Using the covariate WF of our model to study the pion emission decay, we conclude that $\rho(1250)$ could be considered to consist primarily of the state $1S^{--}$.

$$\left|\rho(1250)_{\text{phys}}\right\rangle \simeq c_1\left|1^3S_1^{--}\right\rangle + c_2\left|1^1P_1^{-}\right\rangle + c_3\left|1^3P_1^{-}\right\rangle + \cdots$$

Dominant
$\rightarrow \pi\pi$ strong

Negative norm contirib.
$\rightarrow \omega\pi$ strong and enhance $1^3S_1^{--} \rightarrow \omega\pi$