Calculations of Nucleon EDMs on a Lattice with Background Field

Sergey N. Syritsyn, with

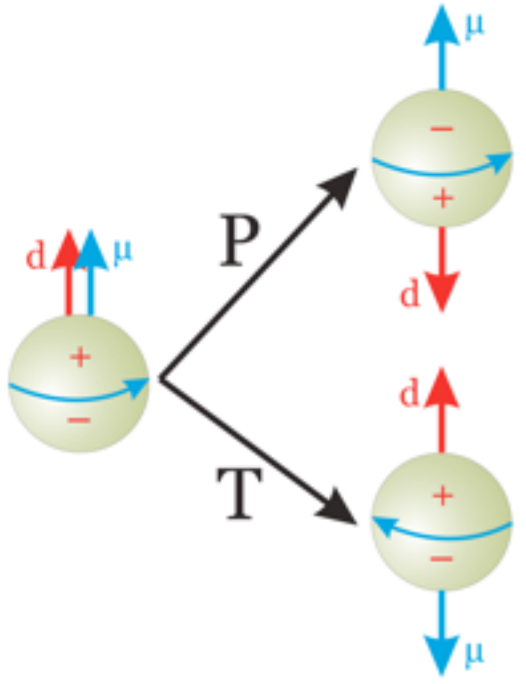
Stony Brook University & RIKEN / BNL Research Center

with M. Abramczyk, T. Blum, F. He, T. Izubuchi, H. Ohki,

(RBC collaboration)

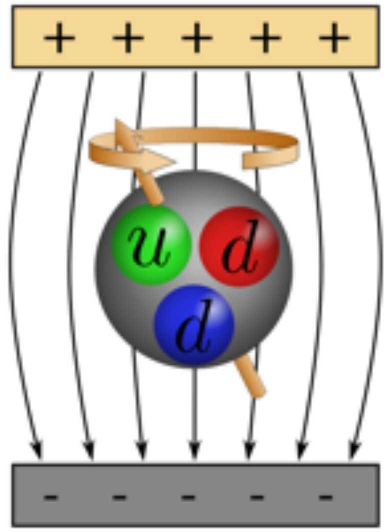
HADRON 2023

Nucleon Electric Dipole Moments



$$\vec{d}_N = d_N \frac{\vec{S}}{S}$$

$$\mathcal{H} = -\vec{d}_N \cdot \vec{E}$$



EDMs are the most sensitive probes of CPv:

- Signals for beyond SM physics

  (SM = 10^{-5} of the current exp.bound)
- Prerequisite for Baryogenesis
- Strong ~~CP~~ problem : θ_{QCD}-induced EDM?

A.Sakharov's conditions for baryon asymmetry in the Universe [JETP letters, 1967]

- ~~P~~, ~~CP~~ symmetry violation
- Baryon number violation
- non-equilibrium transition

$$\langle N_{p'}|J^\mu|\bar{N}_p\rangle_{\cancel{CP}} = \bar{u}_{p'}\left[F_1\gamma^\mu + (F_2 + iF_3\gamma_5)\frac{\sigma^{\mu\nu}(p'-p)_\nu}{2m_N}\right]u_p$$

Dirac

Pauli (anom.magnetic)

Electric dipole

Experimental Outlook

Current nEDM limits:

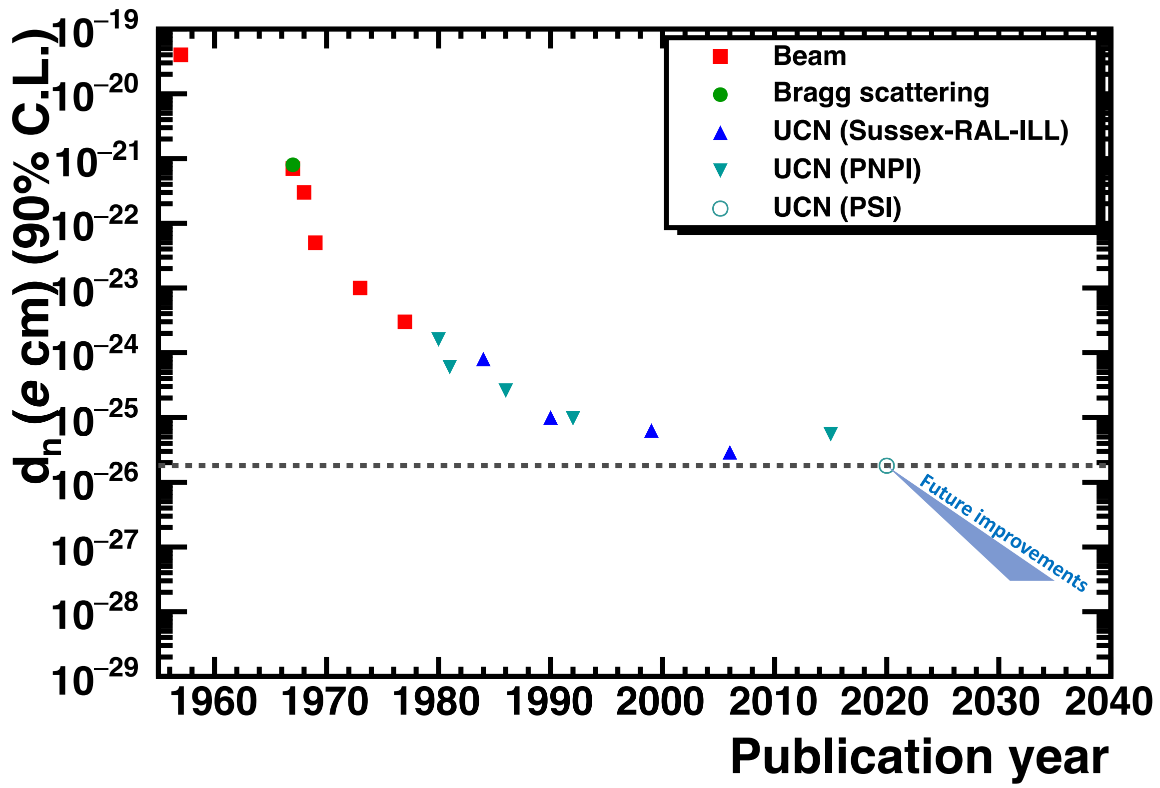
- $|d_n| < 2.9 \times 10^{-26}\, e \cdot \mathrm{cm}$ (stored UC neutrons)
  [Baker et al, PRL97: 131801(2006)]
- $|d_n| < 1.6 \times 10^{-26}\, e \cdot \mathrm{cm}$ (^{199}Hg)
  [Graner et al, PRL116:161601(2016)]

Future nEDM sensitivity :

- 1–2 years : next best limit?
- 3–4 years : x10 improvement
- 7-10 years : x100 improvement

	10^{-28} e cm
CURRENT LIMIT	<300
Spallation Source @ORNL	**< 5**
Ultracold Neutrons @LANL	~30
PSI EDM	<50 (I), **<5 (II)**
ILL PNPI	<10
Munich FRMII	**< 5**
RCMP TRIUMF	<50 (I), **<5 (II)**
JPARC	**< 5**
Standard Model (CKM)	*< 0.001*

[Snowmass EDM workshop report, arXiv:2203.08103]



Nucleon EDMs: a Window into New Physics

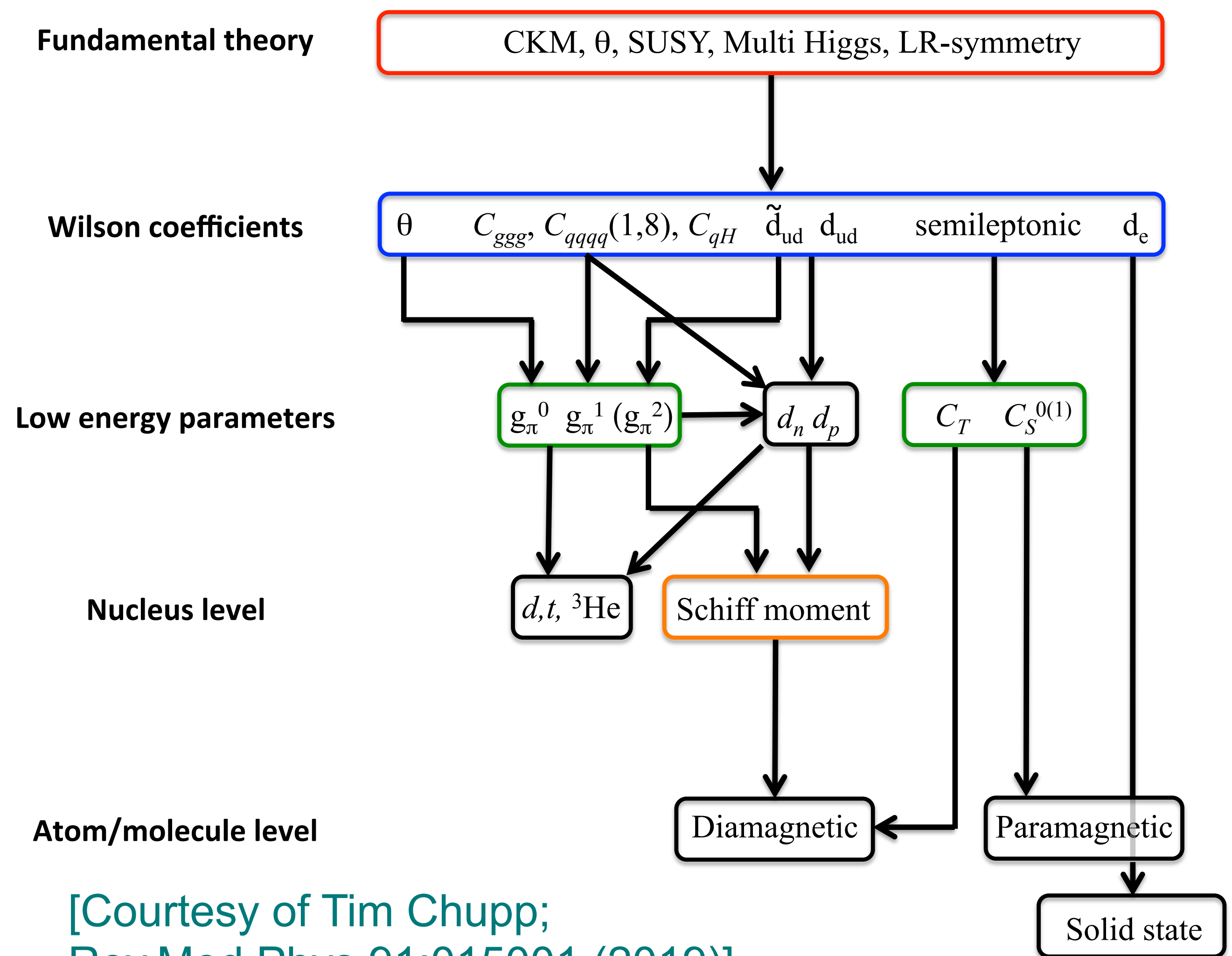
- Effective quark-gluon CPv interactions: dimension $\Longleftrightarrow$ scale of BSM physics
  [Engel, Ramsey-Musolf, van Kolck, Prog.Part.Nucl.Phys. 71:21 (2013)]

$$\mathcal{L}_{eff} = \sum_i \frac{c_i}{[\Lambda_{(i)}]^{d_i-4}} \mathcal{O}_i^{[d_i]}$$

d=4 : θ_{QCD}

d=5(6) : quark EDM, chromo-EDM

d=6 : 4-fermion CPv, 3-gluon (Weinberg)



[Courtesy of Tim Chupp; Rev.Mod.Phys 91:015001 (2019)]

$$d_{n,p} = d^{\theta}_{n,p}\theta_{\rm QCD} + d^{cEDM}_{n,p} c_{cEDM} + \dots$$

$$\boxed{c_i \iff d_{n,p} \quad ?}$$

- Nonperturbative QCD on a Lattice:
  Quark-gluon CPv interactions $\Longrightarrow$ nucleon EDMs , CPv πNN couplings

Determination of Nucleon EDM on a Lattice

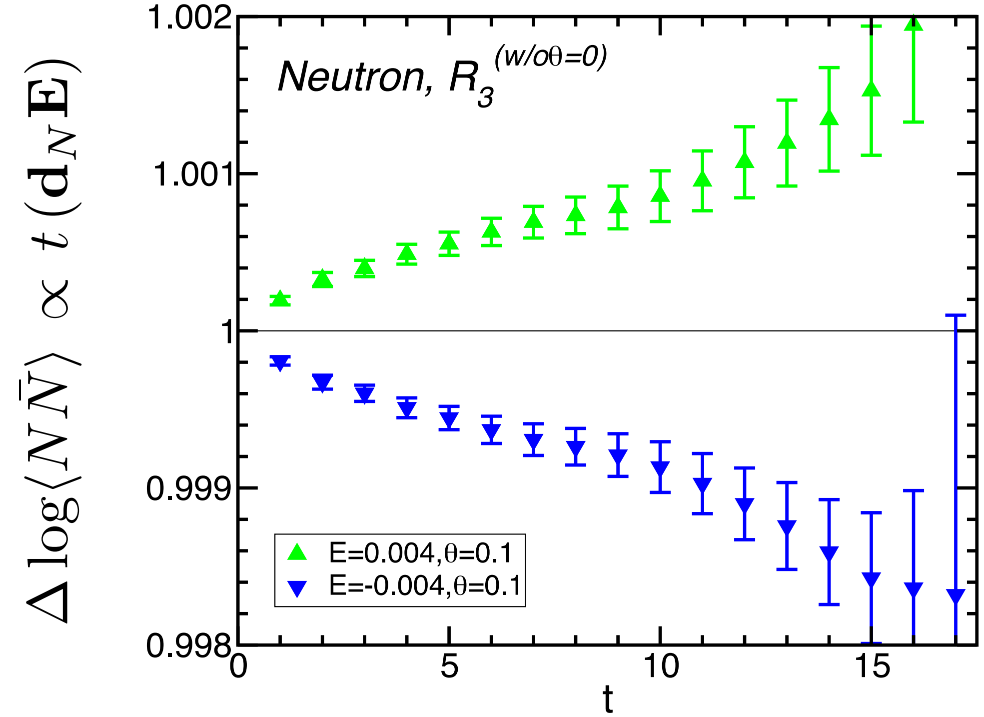
- Energy-Shift method (**uniform electric field**)
  [S.Aoki et al '89 ; E.Shintani et al '06;
  E.Shintani et al, PRD75, 034507(2007)]

$$\langle N(t)\bar N(0)\rangle_{\theta,\vec E} \sim e^{-(E\pm\vec d_N\cdot\vec E)t}$$

Euclidean lattice:

Real-valued **E** $\Longrightarrow$ violate time-BC

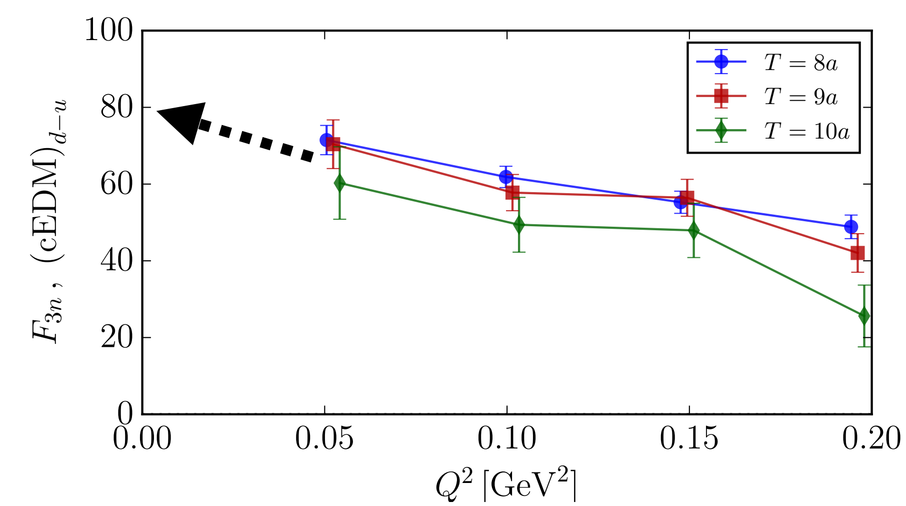
Imag-valued **E** $\Longrightarrow$ imaginary shift in m_N



- Electric dipole Form-Factor method (**EDFF**): $d_N=F_3(Q^2\to 0)\,/\,(2m_N)$
  [(everybody else, almost)]

$$\langle N_{p'}|\bar q\gamma^\mu q|N_p\rangle_{\cancel{CP}} = \bar u_{p'}\left[F_1\gamma^\mu + (F_2 + i\boxed{F_3}\gamma_5)\frac{i\sigma^{\mu\nu}(p'-p)_\nu}{2m_N}\right]u_p$$

- pre-2017 : spurious $\mu_n\leftrightarrow d_n$ mixing
- Dragos et al(2019) $d_n/\theta = -0.0015(7)\ e\cdot\mathrm{fm}$
- Alexandrou et al(2020) $d_n/\theta = 0.0009(24)\ e\cdot\mathrm{fm}$
- Bhattacharya et al (2021) $|d_n/\theta| \lesssim 0.01\ e\cdot\mathrm{fm}$
- Liang et al (2023) $d_n/\theta = -0.0015(1)(3)\ e\cdot\mathrm{fm}$



Need extrapolation to forward-limit $F_3(Q^2\to 0)$

EDFF: Nucleon "Parity Mixing"

CPv interaction induces a chiral phase in nucleon wave functions on a lattice

$$\langle \mathrm{vac}|N|p,\sigma\rangle_{\cancel{CP}} = e^{i\alpha\gamma_5} u_{p,\sigma} = \tilde{u}_{p,\sigma}$$

$$N \downarrow \; u\,[u^T C\gamma_5 d]$$

$$\tilde{u}_{p,\sigma} \downarrow \; \sum_\sigma \tilde{u}_{p,\sigma}\bar{\tilde{u}}_{p,\sigma} \sim \left(-i\not{p}_{\mathcal{E}} + m_N e^{2i\alpha\gamma_5}\right)$$

[M.Abramczyk, S.Aoki, S.N.S, et al (2017) arXiv:1701.07792]
EDM and MDM are defined with positive-parity spinors

$$\langle N_{p'}|\bar{q}\gamma^\mu q|N_p\rangle_{\cancel{CP}} = \bar{u}_{p'}\left[F_1\gamma^\mu + (F_2 + i\boxed{F_3}\gamma_5)\frac{i\sigma^{\mu\nu}(p'-p)_\nu}{2m_N}\right]u_p \quad , \quad \text{with} \quad \begin{aligned} \gamma_4 u &= +u \\ \bar{u}\gamma_4 &= +\bar{u} \end{aligned}$$

Θ-nEDM		m_π [MeV]	m_N [GeV]	F_2	α	$\tilde{F}_3$	F_3
[ETMC 2016]	n	373	1.216(4)	−1.50(16)[a]	−0.217(18)	−0.555(74)	0.094(74)
[Shintani et al 2005]	n	530	1.334(8)	−0.560(40)	−0.247(17)[b]	−0.325(68)	−0.048(68)
	p	530	1.334(8)	0.399(37)	−0.247(17)[b]	0.284(81)	0.087(81)
[Berruto et al 2006]	n	690	1.575(9)	−1.715(46)	−0.070(20)	−1.39(1.52)	−1.15(1.52)
	n	605	1.470(9)	−1.698(68)	−0.160(20)	0.60(2.98)	1.14(2.98)
[Guo et al 2015]	n	465	1.246(7)	−1.491(22)[c]	−0.079(27)[d]	−0.375(48)	−0.130(76)[d]
	n	360	1.138(13)	−1.473(37)[c]	−0.092(14)[d]	−0.248(29)	0.020(58)[d]

After removing the spurious contribution,

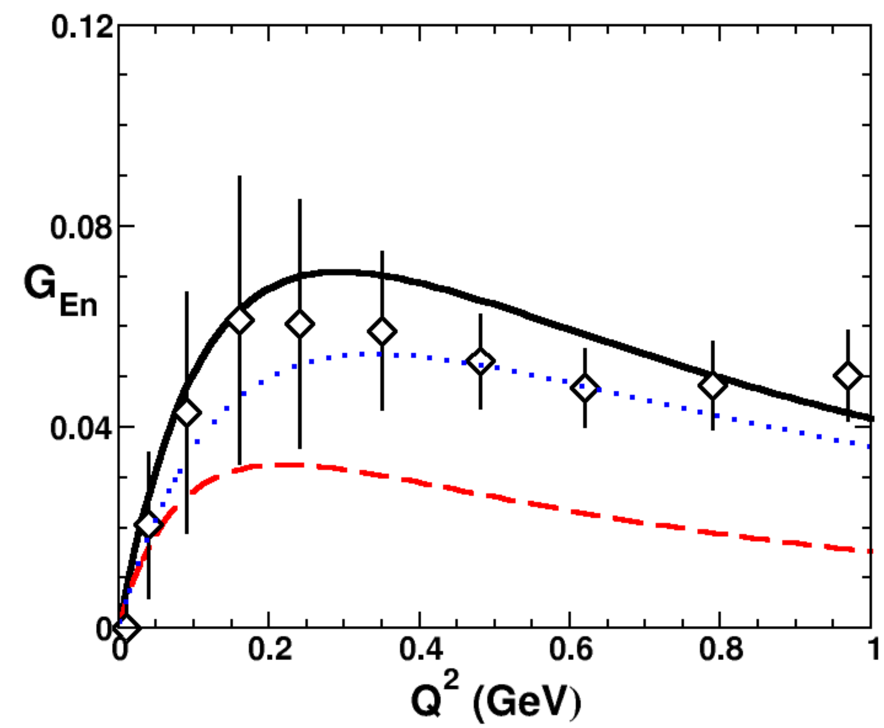
- *no lattice signal for θ_{QCD}-induced nEDM*
- *RESOLVED conflict with pheno. values, lack of $d_N \sim m_q$ scaling*

EDFF: Parity Mixing Correction

Exact value of α_5 is critical for correct determination of EDM:

$$F_3^{\text{lat}}(Q^2) \approx \frac{m}{q_3} \underbrace{\langle N_\uparrow(0)|\bar{q}\gamma_4 q|N_\uparrow(-q_3)\rangle_{\cancel{CP}}}_{\text{CPv matrix element}} - \underbrace{\alpha_5 G_E(Q^2)}_{\text{Sachs form factor subtraction}}$$

- Proton ($G_{Ep}(0)=1$) : Correction ~ α_5
- Neutron ($G_{En}(0)=0$) : No correction at $Q^2=0$
  However, $Q^2\to 0$ extrapolation may be skewed by neutron electric form factor $\sim\alpha_5\, G_{En}(Q^2)$



[Punjabi et al, 1503.01452]

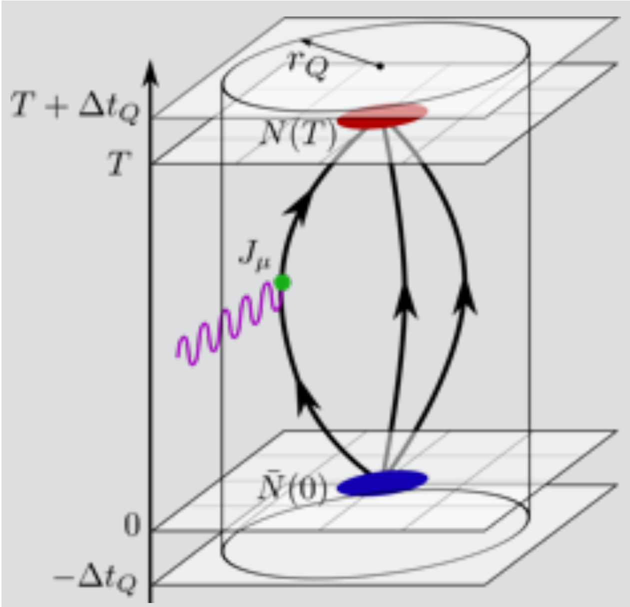
Controlling Noise from θ-Term

Variance of lattice θ-induced nEDM signal ~ $(Volume)_{4d}$:

$$d_N \sim \langle Q \cdot (N J_\mu \bar{N}) \rangle$$

Top. charge $Q \sim \int_{V_4} (G\tilde{G}) \sim \text{integer}$

Fluctuation $\langle |Q|^2 \rangle \sim V_4$

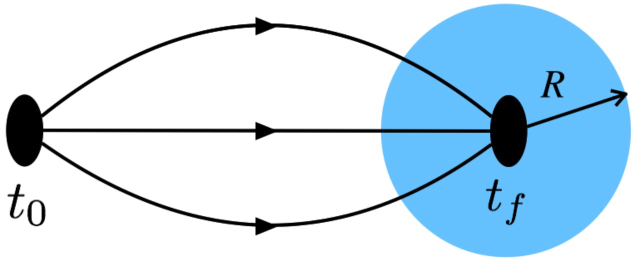


$\Longrightarrow$Need to constrain Q integral to the volume around N, $\bar{N}$, J_μ

- in time around current, $|t_Q - t_J| < \Delta t$

  [E.Shintani et al (2015); Yoon et al (2019)]

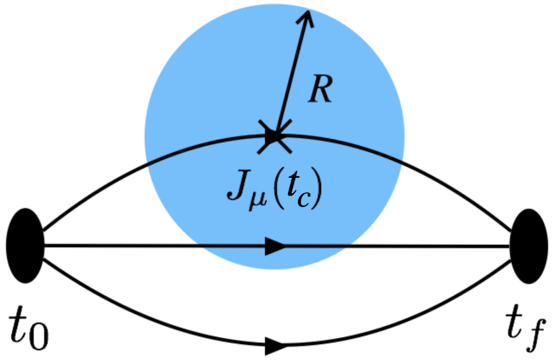
- in time around source, $|t_Q - t_{source}| < \Delta t$ [Dragos et al (2019)]



- 4-d sphere around sink or current,

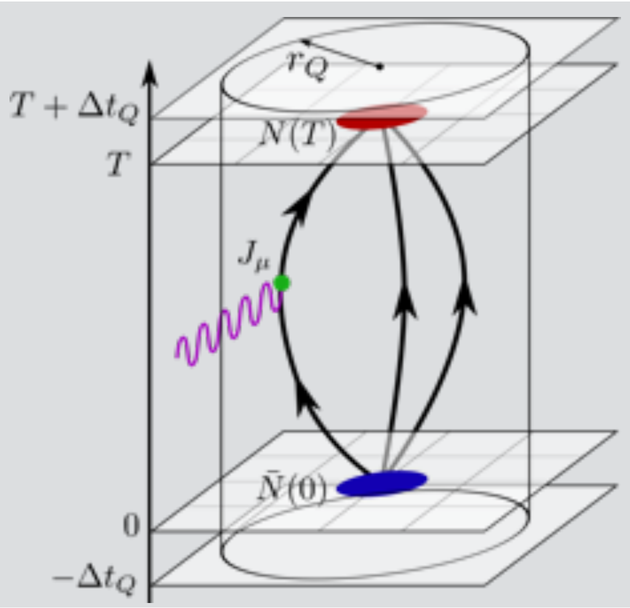
  $|x_Q - x_{sink}| < R$ [K.-F. Liu et al (2023)]:

  $d_n^\theta/\theta = -0.0015(1)(3)\ e \cdot \text{fm}$ (chiral extrapolation)

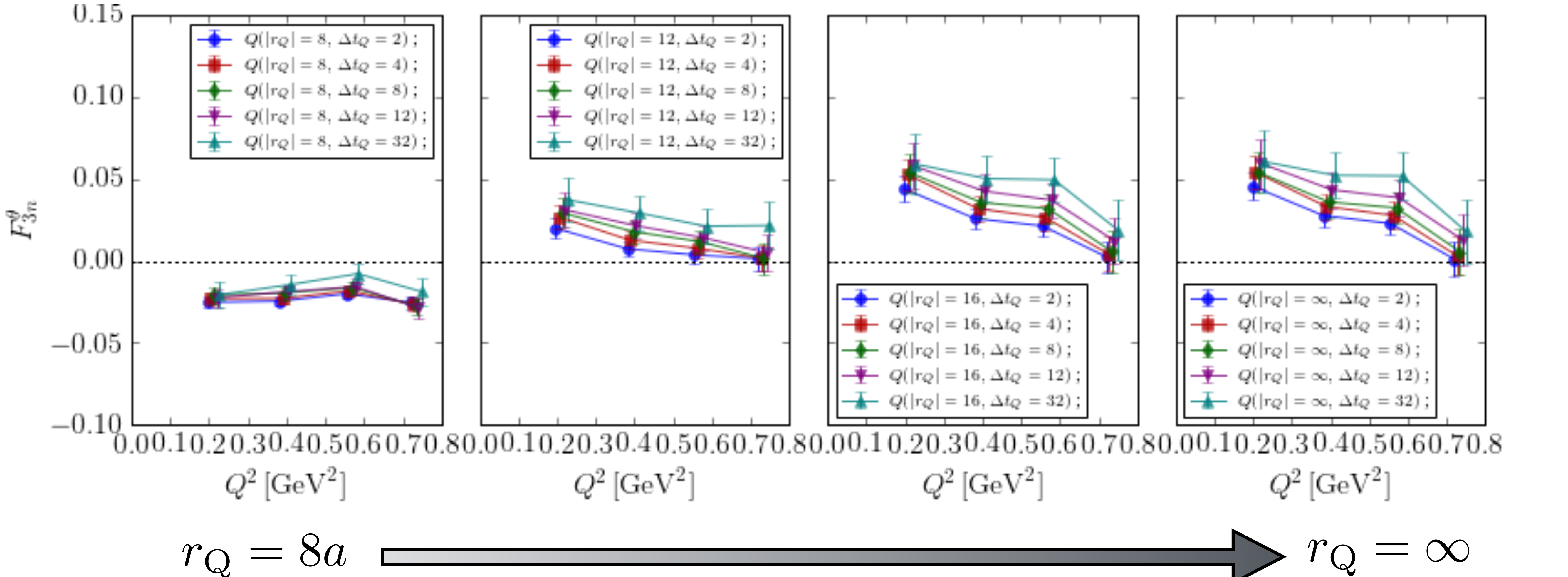


EDFF: Effect of $G\tilde{G}$ cuts

- 24^3x64 a = 0.114 fm mπ=330 MeV (N_f=2+1 chiral-symmetric quarks)
- 1400 confiigs $\Longrightarrow$ 89.6k stat.
- $G\tilde{G}$: Wilson-flowed (t=8a^2) gauge field [M.Luscher, 1006.4518]
  5-loop improved $G\tilde{G}$ [P. de Forcrand et al '97]
- Cuts in space $r \le r_Q$, time Δt_Q



$$F_3^{\text{lat}}(Q^2) \approx \frac{m}{q_3} \underbrace{\langle N_\uparrow(0)|\bar{q}\gamma_4 q|N_\uparrow(-q_3)\rangle_{\cancel{CP}}} - \underbrace{\alpha_5 G_E(Q^2)}$$



EDM from Energy Shift / Feynman-Hellman Thm

- FH theorem :
  Energy shift $\Longleftrightarrow$ Perturbation's matrix element

$$\frac{\partial E_\lambda}{\partial \lambda} = \left\langle \phi_\lambda \middle| \frac{\partial \hat{H}_\lambda}{\partial \lambda} \middle| \phi_\lambda \right\rangle \qquad \text{with} \qquad (-\delta H) = \frac{\theta g^2}{32\pi^2} \int d^3x \, (G^a_{\mu\nu} \widetilde{G}^a_{\mu\nu})$$

$$m'_N = m_N - (d_N^\theta \theta)\, \vec{\Sigma} \cdot \vec{\mathcal{E}}$$

sample $G\tilde{G}$ on only one time slice
$\Longrightarrow$ noise reduction

EDM $\Leftrightarrow$ Density of Top.charge in doubly (spin & electric) polarized nucleon

$$d_N^\theta \propto \left\langle N_\uparrow \middle| \int d^3x \, G^a_{\mu\nu} \widetilde{G}^a_{\mu\nu} \middle| N_\uparrow \right\rangle_{\mathcal{E}_z}$$

Nonzero in **CP-even** vacuum only if both ***spin***- and ***charge***-polarized

(permanent) EDM $\Longleftrightarrow$

- in CP-broken vacuum : **correlation** of *spin* and *charge*
  OR
- in CP-even vacuum : *"topological" polarization of glue* in **S** & **d** polarized nucleon

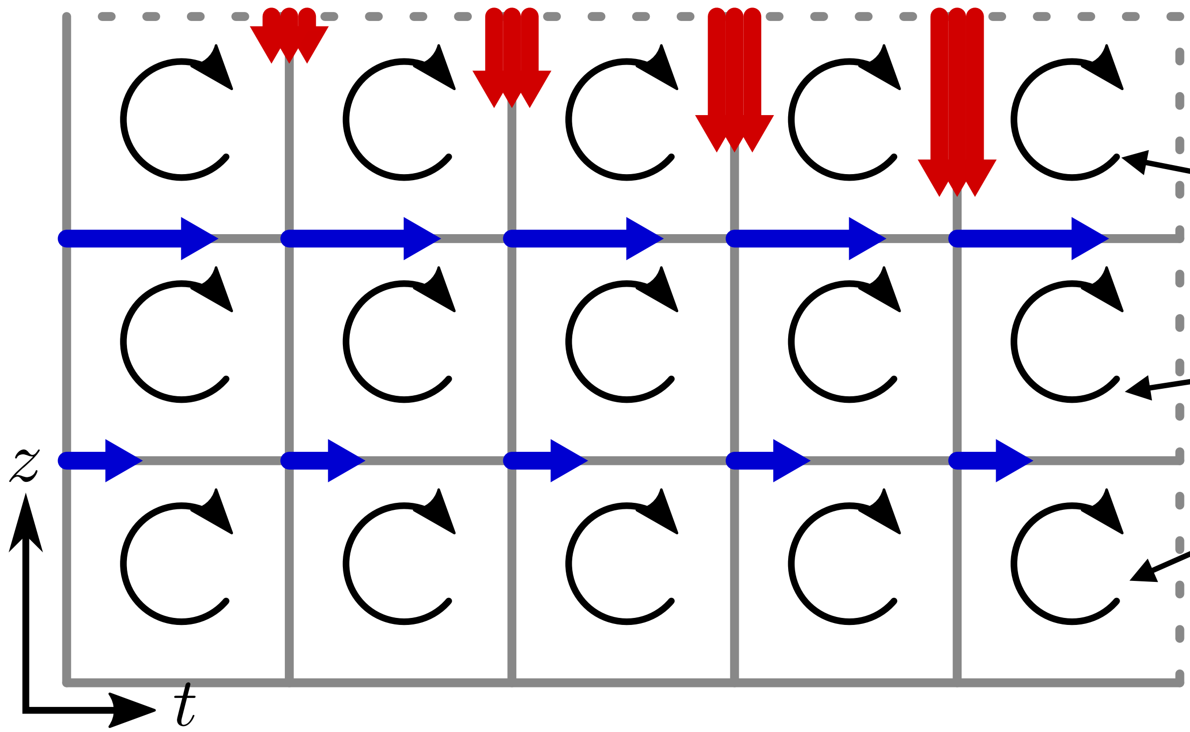
$\langle N|\, \mathbf{d} \,|N\rangle_{CPv} \sim \mathbf{S}$,
$\langle N|\, \mathbf{\Sigma} \,|N\rangle_{CPv} \sim \mathbf{E}$

$\langle N|\, G\tilde{G} \,|N\rangle_{CP\text{-}even} \sim \mathbf{S}\cdot\mathbf{E}$

Background Electric Field on a Lattice

[W.Detmold et al (2009)] :
Magnetic & electric moments at Q^2=0 ; hadron polarizabilities

Electric field: Real in Euclidean ≡ Imaginary in Minkowski space



Uniform Electric field <u>is quantized</u> in Finite volume with PBC:

$$|q_d|\mathcal{E}_{\rm min} = \frac{2\pi}{L_x L_t}$$

(Flux through the entire *z*–*t* "side" of the PBC box $q\Phi = 2\pi \cdot n$)

$$U_\mu \to e^{iqA_\mu} U_\mu$$

$$A_z(z,t) = n\,\mathcal{E}_{\rm min} \cdot t$$

$$A_t(z, t = L_t - 1) = -n\,\mathcal{E}_{\rm min} \cdot L_t z$$

Electric field on a 24^3x64 lattice

$$\mathcal{E} = \frac{6\pi}{L_x L_t} \approx 0.037\ \mathrm{GeV}^2$$
$$\approx 186\ \mathrm{MV/fm}$$

Unambiguous determination of EDM from the energy shift:
Most straightforward for neutron (Q_{el}=0); possible for proton

Topological Charge with Gradient Flow

[M.Luscher, JHEP08:071; 1006.4518]

Gradient flow: covariant *4D-diffusion* of quantum fields with "G.F." time t_{GF}:

$$\frac{d}{dt_{\mathrm{GF}}} B_\mu(t_{\mathrm{GF}}) = D_\mu G_{\mu\nu}(t_{\mathrm{GF}})\,, \quad B_\mu(0) = A_\mu$$

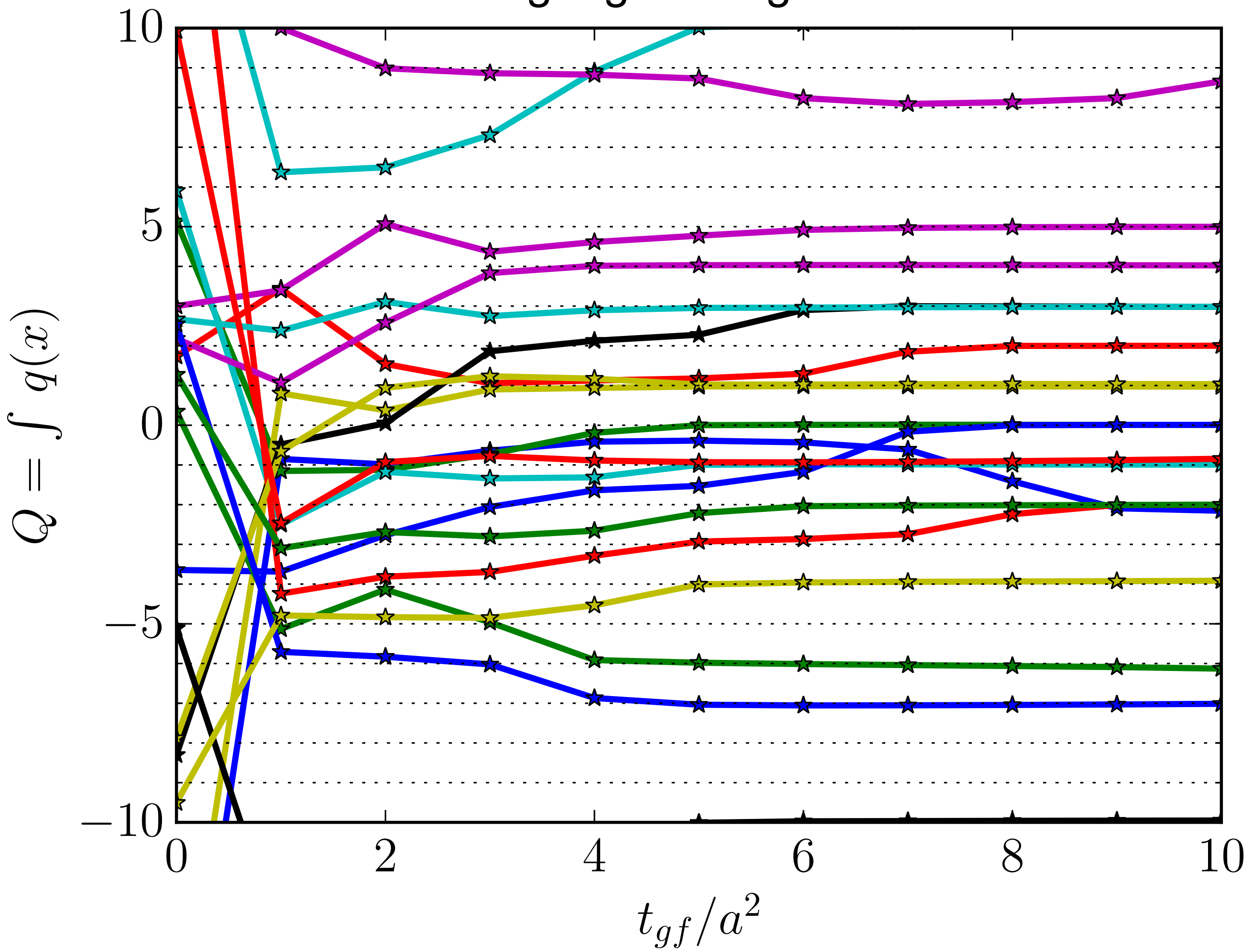
Tree-level:

$$B_\mu(x, t_{\mathrm{GF}}) \propto \int d^4y \, \exp\Big[-\frac{(x-y)^2}{4t_{\mathrm{GF}}}\Big] A_\mu(y)$$

Gradient-flowed topological charge:

$$\tilde{Q}(t_{\mathrm{GF}}) = \int d^4x \, \frac{g^2}{32\pi^2} \Big[G_{\mu\nu}\tilde{G}_{\mu\nu}\Big]\Big|_{t_{\mathrm{GF}}}$$

total top. charge on 20 randomly chosen gauge configurations



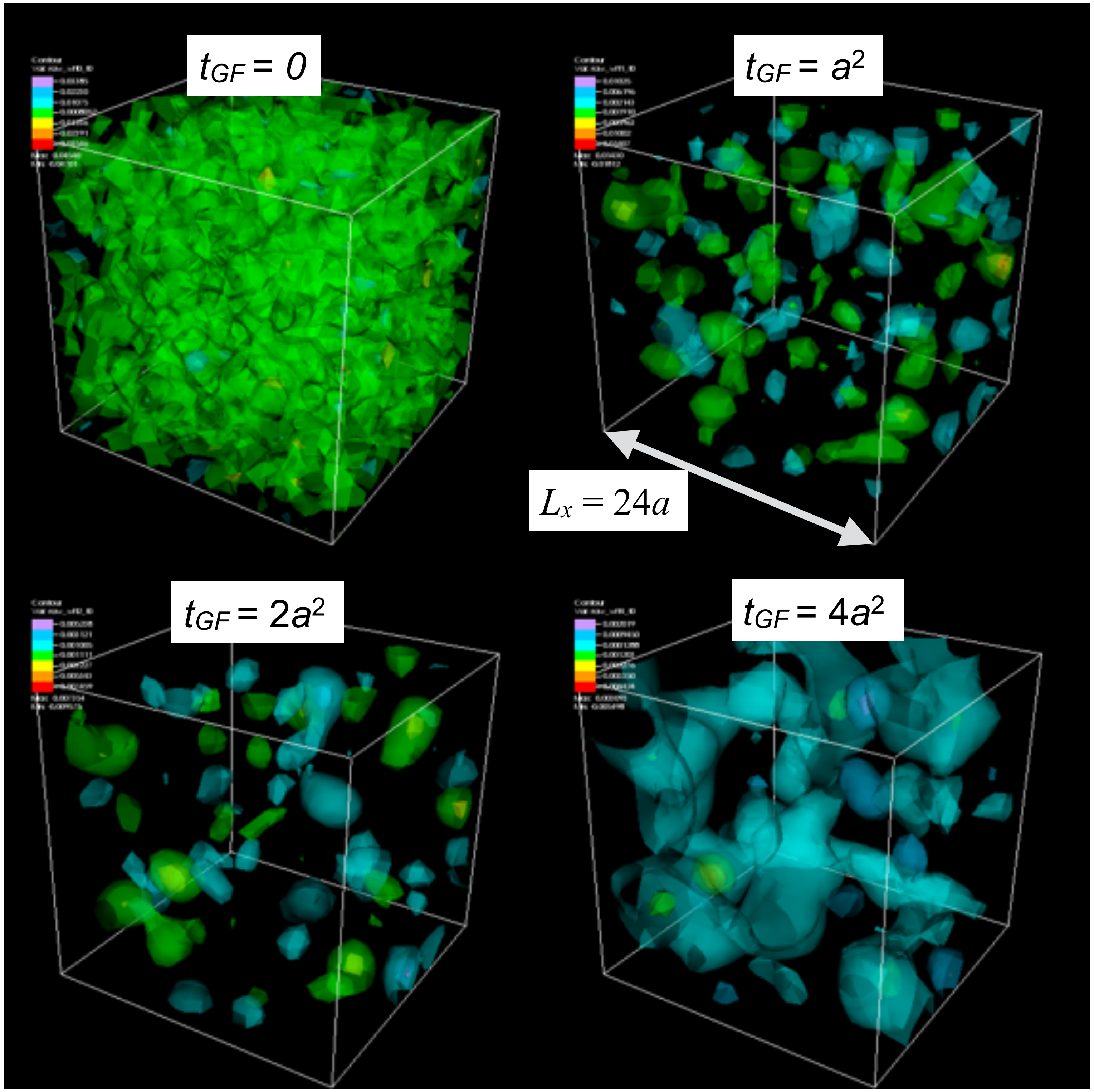
- effective scale $\Lambda_{\mathrm{UV}} \to (t_{\mathrm{GF}})^{-1/2}$
- smooth fields (reduce $|G_{\mu\nu}|$)

  $\Longleftrightarrow$ continuous "cooling"
- remove $G_{\mu\nu}$ dislocations

  ⇒dynamical separation of top. sectors

  [M.Luscher, JHEP08:071; 1006.4518]
- diffusion of top.charge density

Topological Charge Density



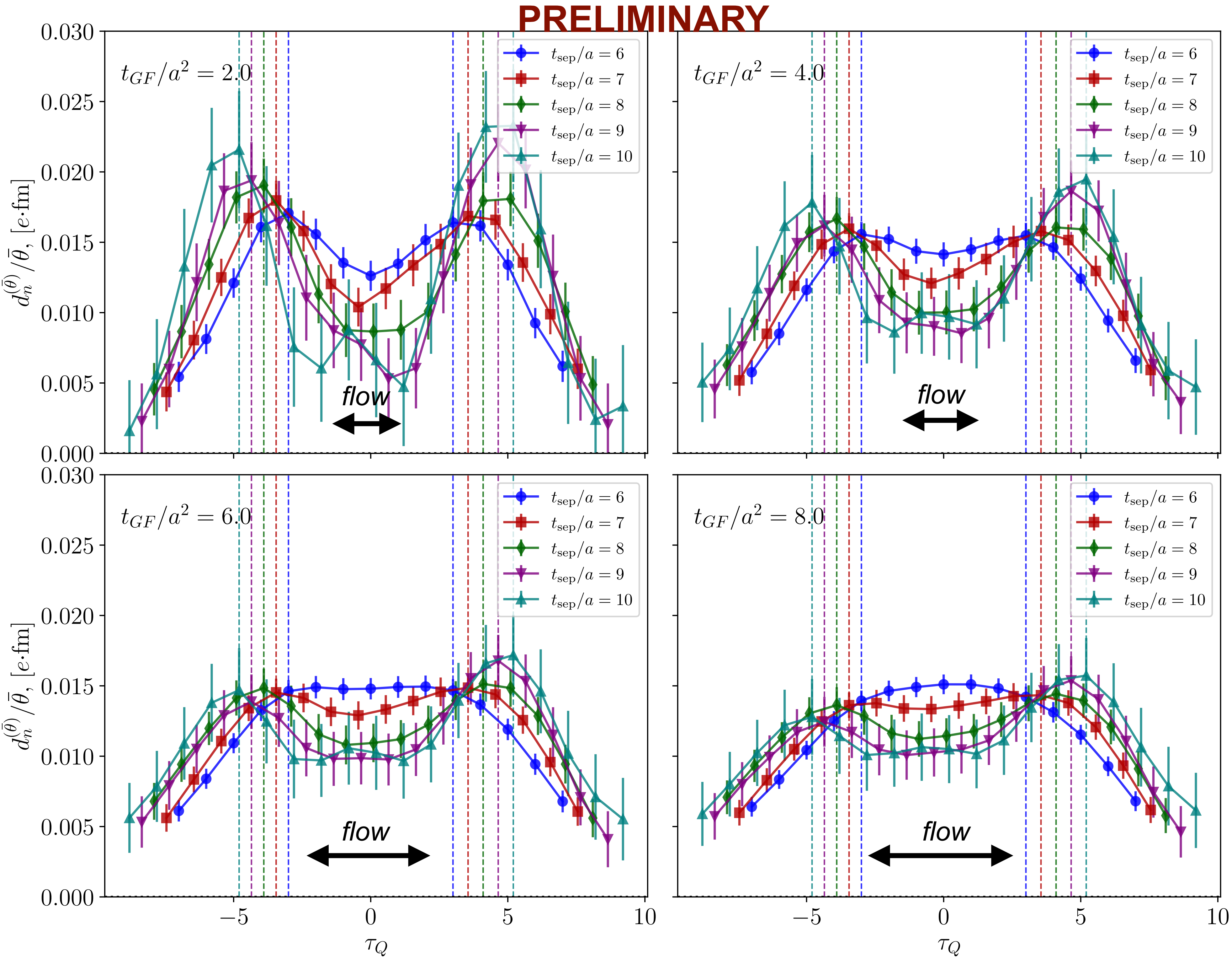
$24^3 \times 64$ lattice, $m\pi \approx 340\ MeV$

$$q(x) = \frac{g^2}{32\pi^2} G^a_{\mu\nu} \widetilde{G}^a_{\mu\nu}$$
$$\approx \frac{1}{16\pi^2}\frac{1}{a^4}\mathrm{Tr}\left[G^{\rm lat}_{\mu\nu}\widetilde{G}^{\rm lat}_{\mu\nu}\right]$$
$$\propto (\mathbf{E}\cdot\mathbf{H})_{\rm color}$$

Gradient flow:

- effective scale $(\Lambda_{UV})^{-1} \to (t_{GF})^{1/2}$
- make fields smooth (reduce $|G_{\mu\nu}|$)
- remove dislocations⇒dynamical separation of topological sectors [M.Luscher, JHEP08:071; 1006.4518]
- 4D-diffusion (including time) of $q(x)$
  $\langle q(x)q(0) \rangle \sim exp[-(x-y)^2 / 8t_{GF}]$

Top.Charge–Nucleon Correlation Functions



Two effects observed:

1. Convergence to ground state matrix el.
2. Diffusion of top.charge for $t_{sep} \lesssim 7a$

PRELIMINARY estimates
$2md_n = F_3(0) \approx 0.11 .. 0.13$
agree with form factor

Analysis of (τ_Q, t_{GF}) required to detangle

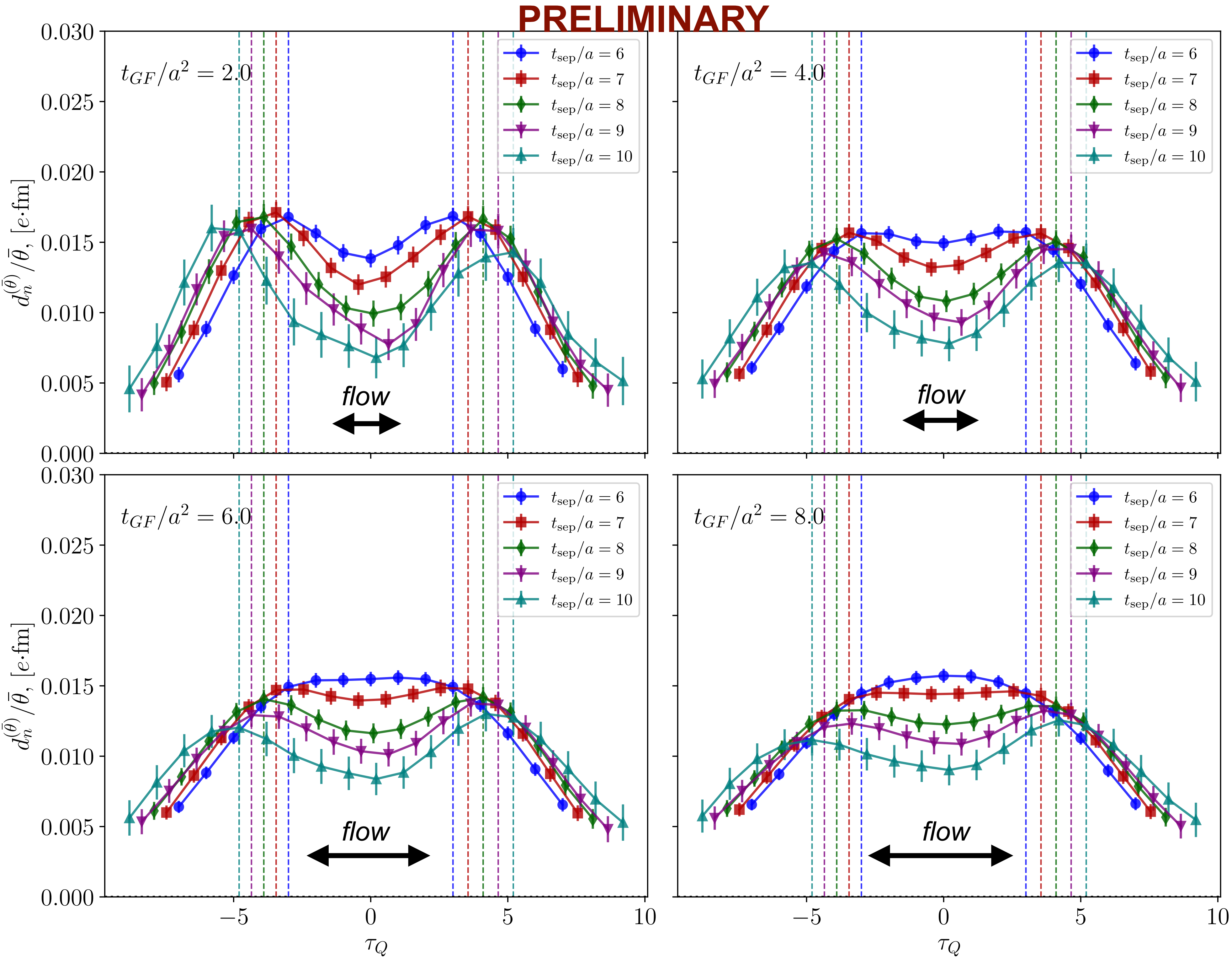
$$\langle N|G\widetilde{G}|N\rangle\,,$$

$$\langle N|G\widetilde{G}|N\rangle_{\rm exc}\,,$$

$$\langle {\rm vac}|G\widetilde{G}|N\bar{N}\rangle\,,$$

$$\ldots$$

Top.Charge–Nucleon C.F. (Low-mode Improved)



Two effects observed:

1. Convergence to ground state matrix el.
2. Diffusion of top.charge for $t_{sep} \lesssim 7a$

PRELIMINARY estimates
$2md_n = F_3(0) \approx 0.11 .. 0.13$
agree with form factor

Analysis of (τ_Q, t_{GF}) required to detangle

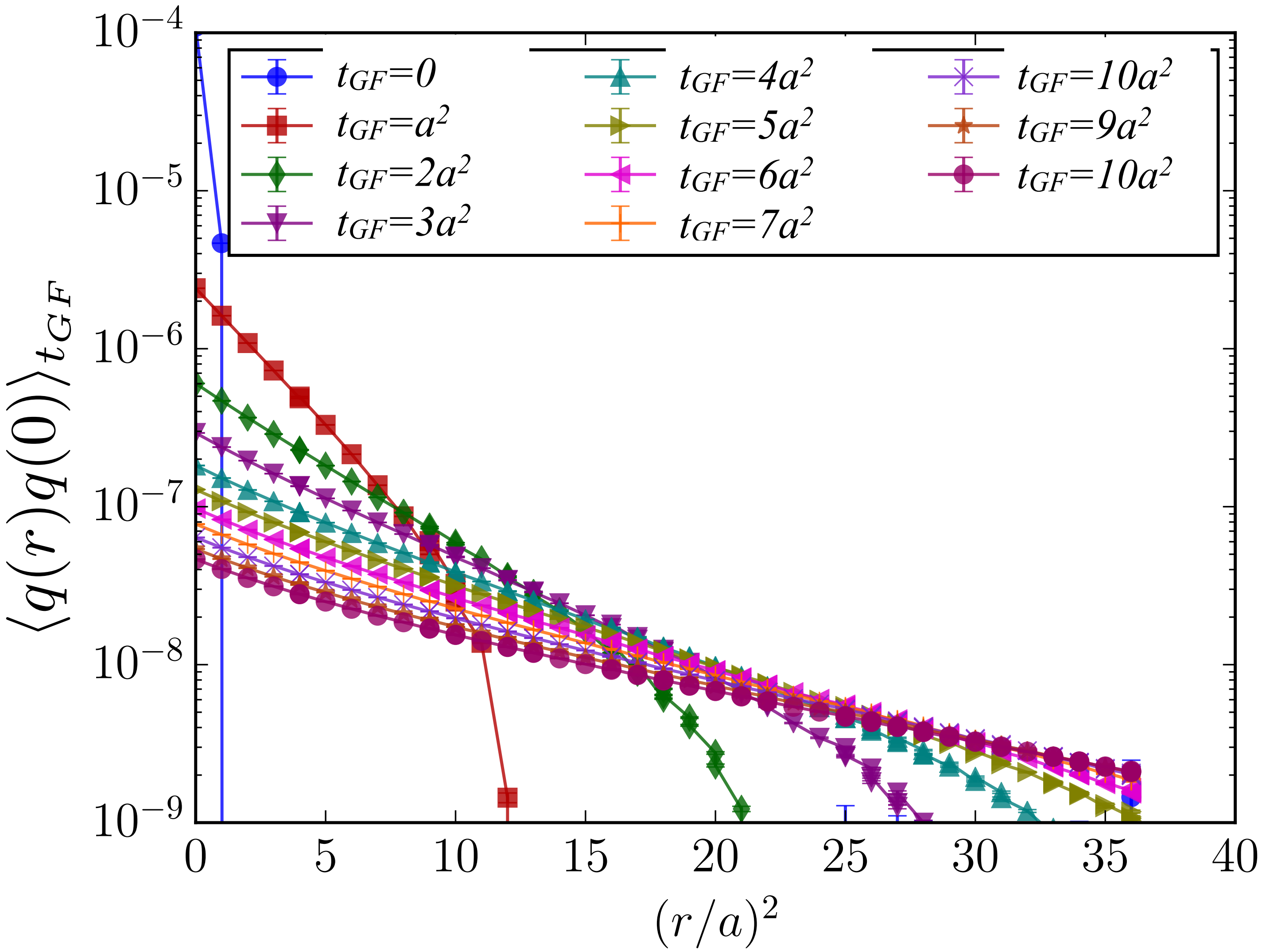
$\langle N|G\widetilde{G}|N\rangle$,

$\langle N|G\widetilde{G}|N\rangle_{\rm exc}$,

$\langle {\rm vac}|G\widetilde{G}|N\bar{N}\rangle$,

$\ldots$

"Diffusion" of Top.Charge under Gradient Flow



Empirically for $r, \sqrt{t_{\rm GF}} \gg m_{\eta'}^{-1}$

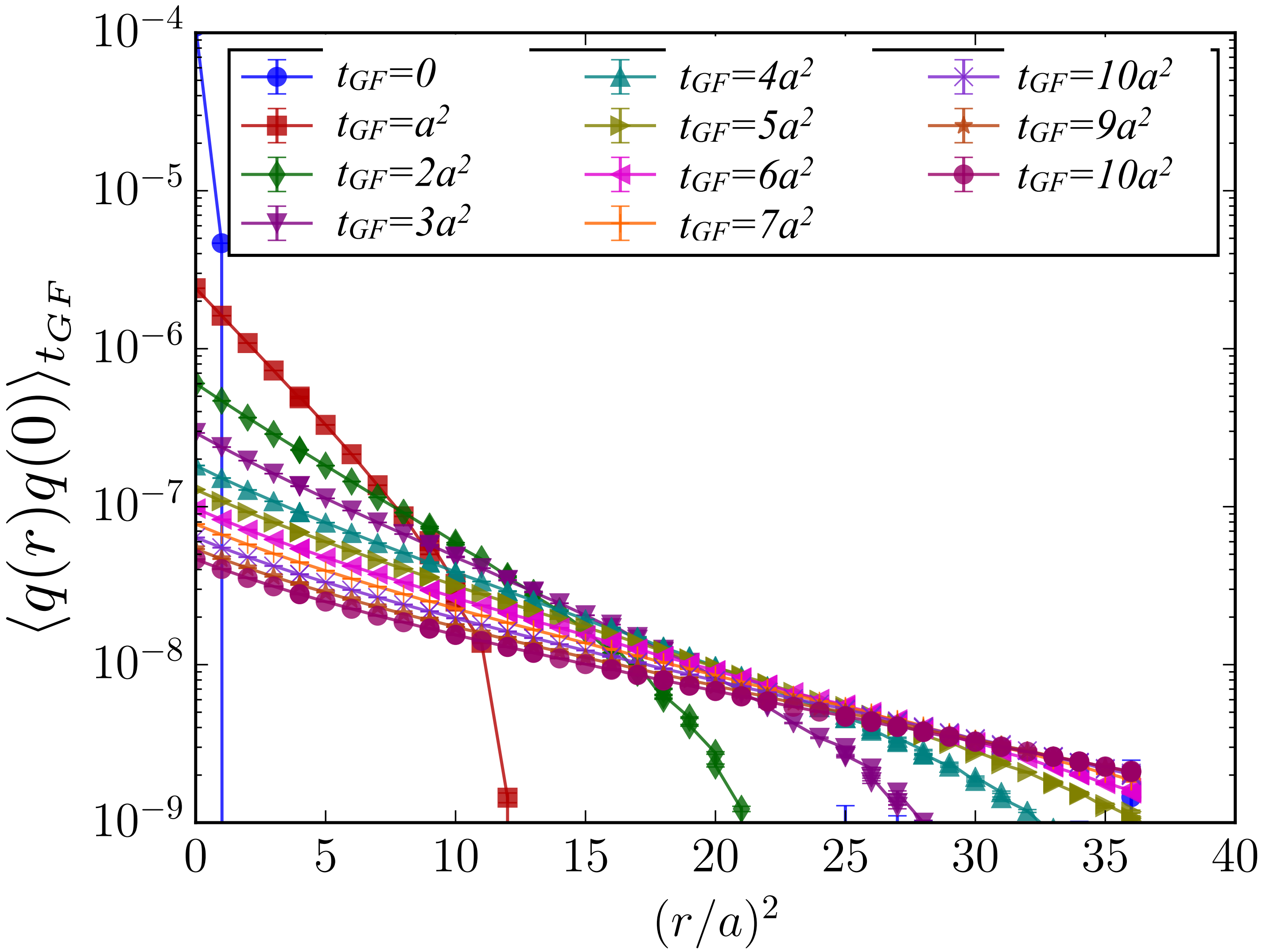
$$\langle \widetilde{q}(r)\widetilde{q}(0)\rangle \propto \exp\left[-\frac{r^2}{4r_Q^2(t_{\rm GF})}\right]$$

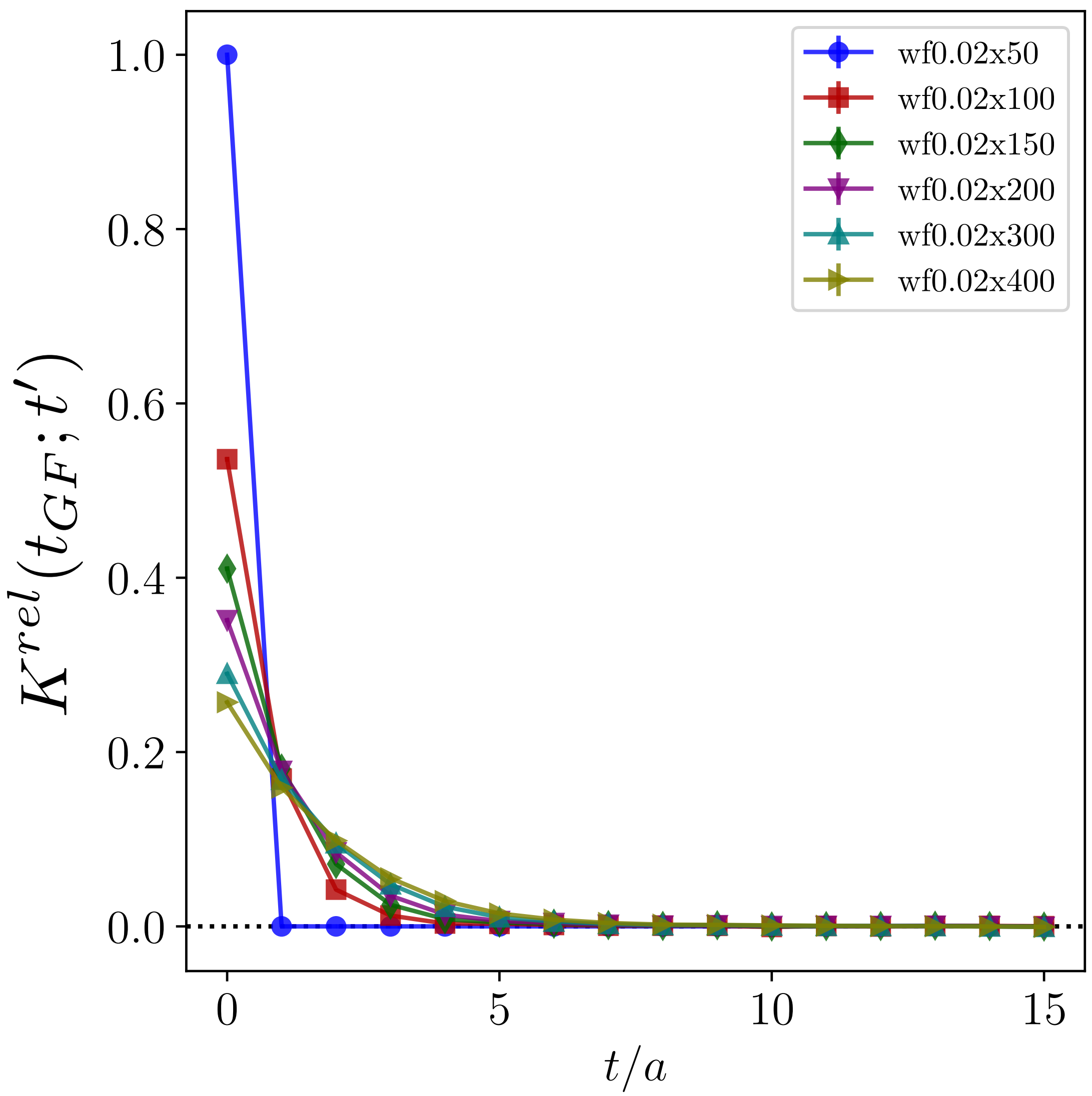
Diffusion of q(x) in Euclidean (lattice) time:

$$q(t_{GF};t) = \sum_{t'} K(t_{GF};t-t')q(t')$$

complications for matrix element analysis

"Diffusion" of Top.Charge under Gradient Flow





Diffusion of q(x) in Euclidean (lattice) time:

$$q(t_{GF};t) = \sum_{t'} K(t_{GF};t-t')q(t')$$

complications for matrix element analysis

Extract kernel K from lattice data

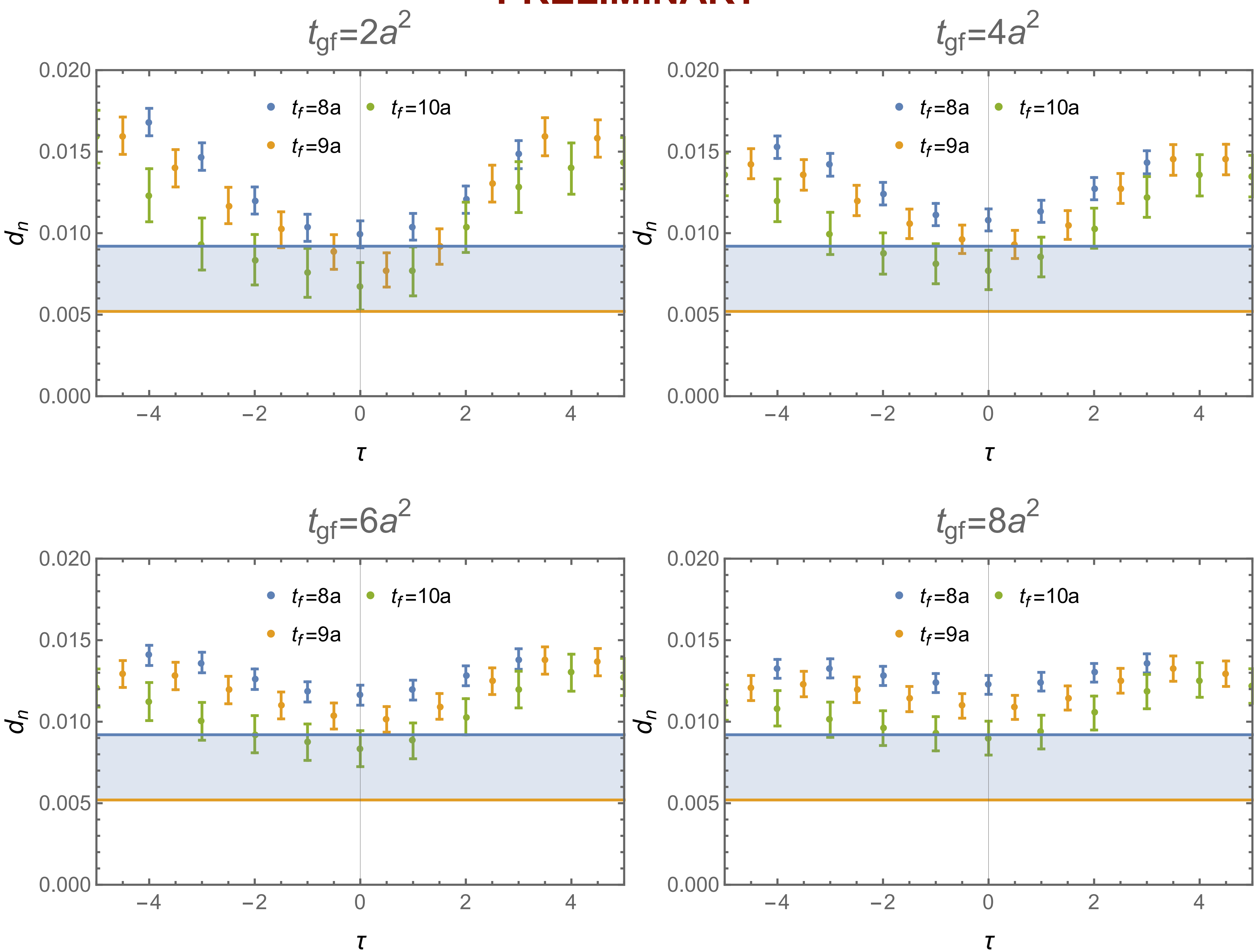
$$\tilde{K}^{rel}(t_{GF};\omega) = \sqrt{\frac{\tilde{\chi}(t_{GF};\omega)}{\tilde{\chi}(t_{GF0};\omega)}}$$

where

$$\chi(t_{GF};t_2-t_1) = \langle q(t_{GF};t_2)q(t_{GF};t_1)\rangle$$

Combined Fit: Euclidean Time & Gradient Flow

PRELIMINARY

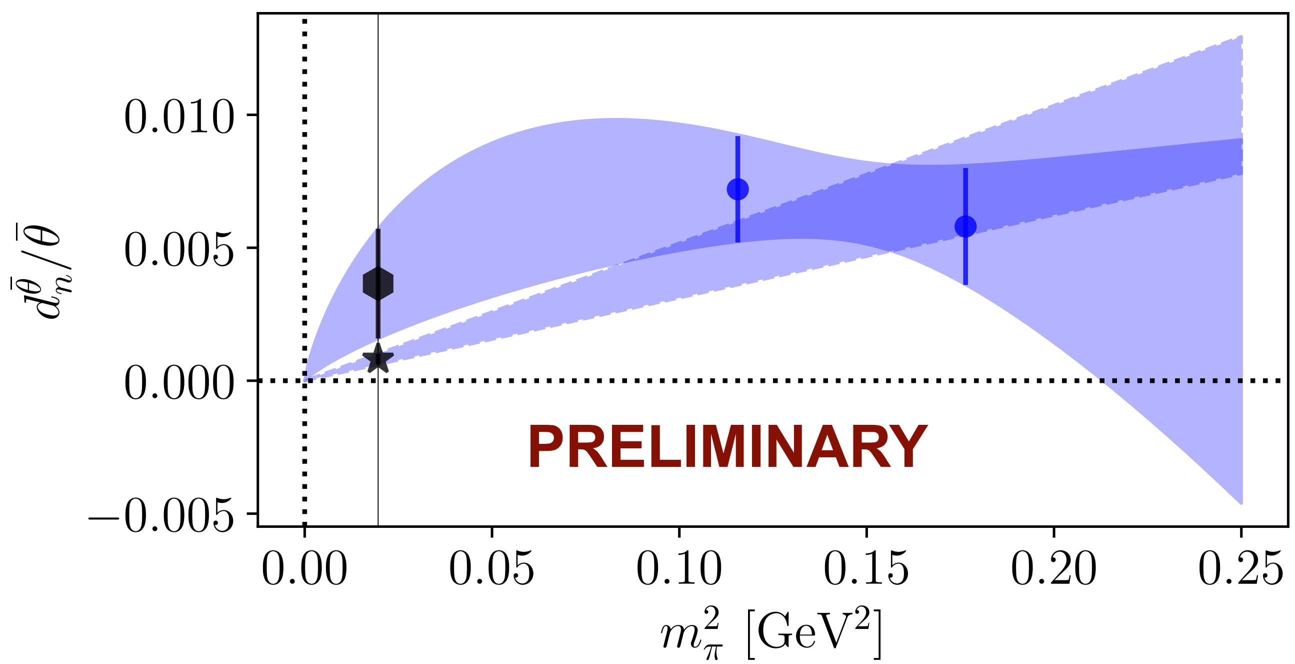


Combined Analysis of $(\tau_Q,\, t_{GF})$ dependence:

- ground state $\langle N|G\widetilde{G}|N\rangle$
- excited state(s) $\langle N|G\widetilde{G}|N\rangle_{\rm exc}$
- "contact" amplitudes $\langle N\,(G\widetilde{G})|N\rangle$
- $N\bar{N}$ annihilation by $G\tilde{G}$ $\langle \mathrm{vac}|G\widetilde{G}|N\bar{N}\rangle$
- grey band: "summation analysis"

Fit $\quad \langle N(t_{sep})\; q(t_{GF},\tau_Q)\; \bar{N}(0)\rangle \quad \sim \quad K(t_{GF}, |\tau_Q - \tau_Q'|) \otimes \langle N(t_{sep})\; q(\tau_Q)\; \bar{N}(0)\rangle$

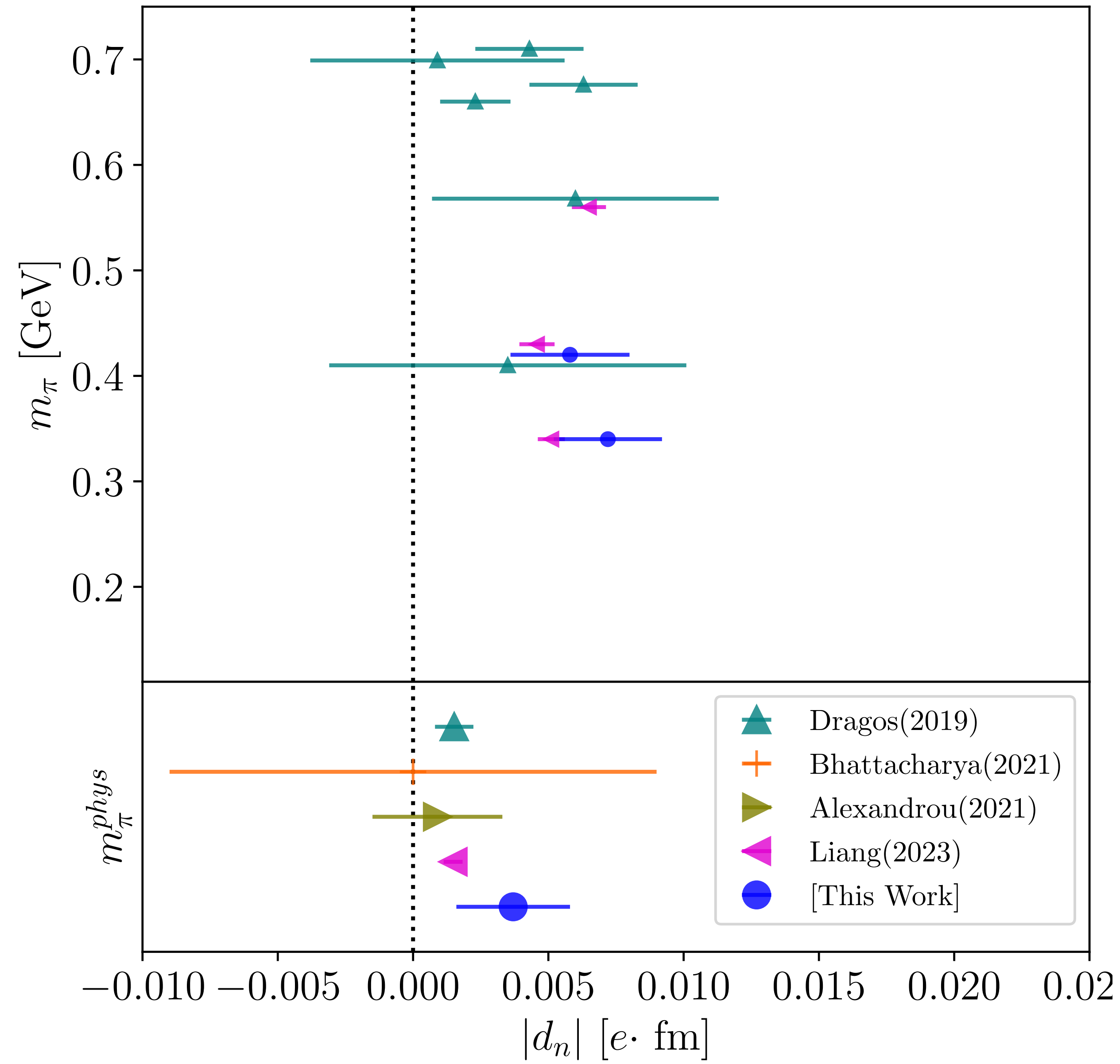
Extrapolation to the Physical Point



Chiral extrapolation
[Hockings, van Kolck (2005)]

$$d_n(m_\pi) = C_1 m_\pi^2 + C_2 m_\pi^2 \log \frac{m_\pi^2}{m_N^2}$$

(Only multiplicative O(a^2) corrections with chiral-symmetric lattice fermions)



Summary of neutron θ-EDM from Lattice QCD

Summary

- Novel method to compute nEDM from local topological charge
  Results consistent with earlier works (and also with zero)
  Potential method of choice for physical-point calculations with large V_4
- Important cross-check for E.D. form-factor calculations
  Controllable space cut-off of "disconnected" CPv interaction
- Current results within 2σ of zero;
  more statistics, additional pion-mass point needed
  Potential method of choice for physical-point calculations with large V_4

Outlook: nEDM from other CPv Operators

- EDM of the Proton : background field requires only energy shift (acc. cancels out)
- Background field method may reduce errors in calculations of nEDM from other "disconnected" CPv interactions: Weinberg, isoscalar (strange quark cEDM?)
- Simplified contractions for 4-quark CPv operators (L-R, SUSY)

$$\mathcal{O}^{(1)}_{\varphi ud} = \frac{1}{3}(\bar{u}u)\,(\bar{d}\gamma_5 d) + (\bar{u}T^A u)\,(\bar{d}\gamma_5 T^A d) - [u \leftrightarrow d]$$

$$\mathcal{O}^{(1)}_{quqd} = (\bar{u}\gamma_5 u)\,(\bar{d}d) + (\bar{u}u)\,(\bar{d}\gamma_5 d) - [(\bar{u}u)(\bar{d}d) \leftrightarrow (\bar{u}d)(\bar{d}u)]$$

$$\mathcal{O}^{(8)}_{quqd} = (\bar{u}\gamma_5 T^A u)\,(\bar{d}T^A d) + (\bar{u}T^A u)\,(\bar{d}\gamma_5 T^A d) - [(\bar{u}u)(\bar{d}d) \leftrightarrow (\bar{u}d)(\bar{d}u)]$$