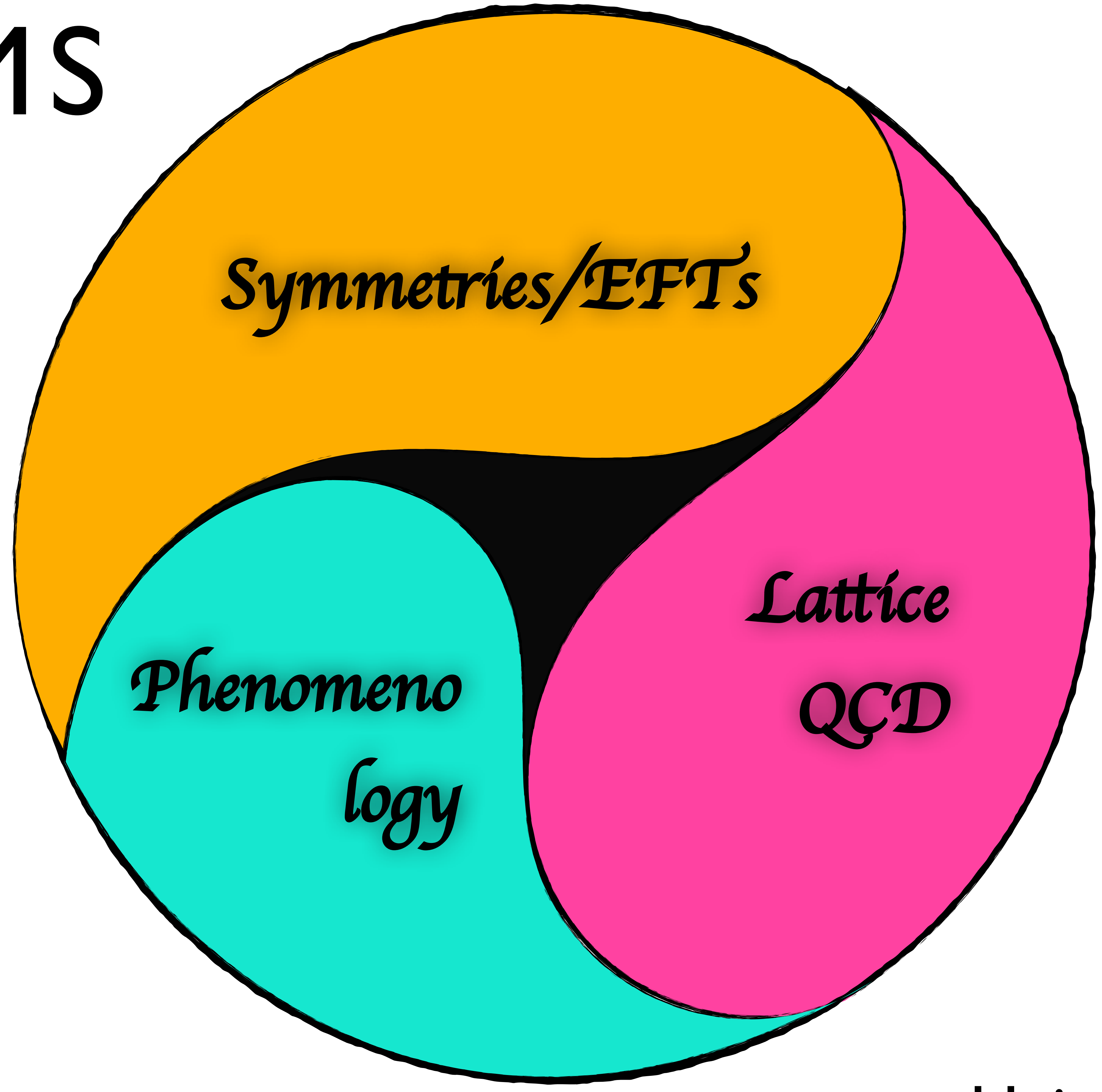
RESONANT HADRON SYSTEMS FROM:



Maxim Mai
University of Bonn
The George Washington University

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Deutsche Forschungsgemeinschaft
DFG

Effective field theory

Ulf-G Meißner (U. Bonn, FZ Jülich)

Bastian Kubis (U. Bonn)

Evgeny Epelbaum (U. Bochum)

. . .

Chiral Unitary approaches

Ulf-G Meißner (U. Bonn, FZ Jülich)

Daniel Sadasivan (Ave Maria U.)

LiSheng Geng (Beihang U.)

Aleš Cieply (Rež)

Michael Döring (GWU/JLAB)

. . .

Data driven tools

Justin Landay (GWU)

Dave Ireland (U. Glasgow)

. . .

Meson electroproduction [Jülich-Bonn-Washington Collaboration]

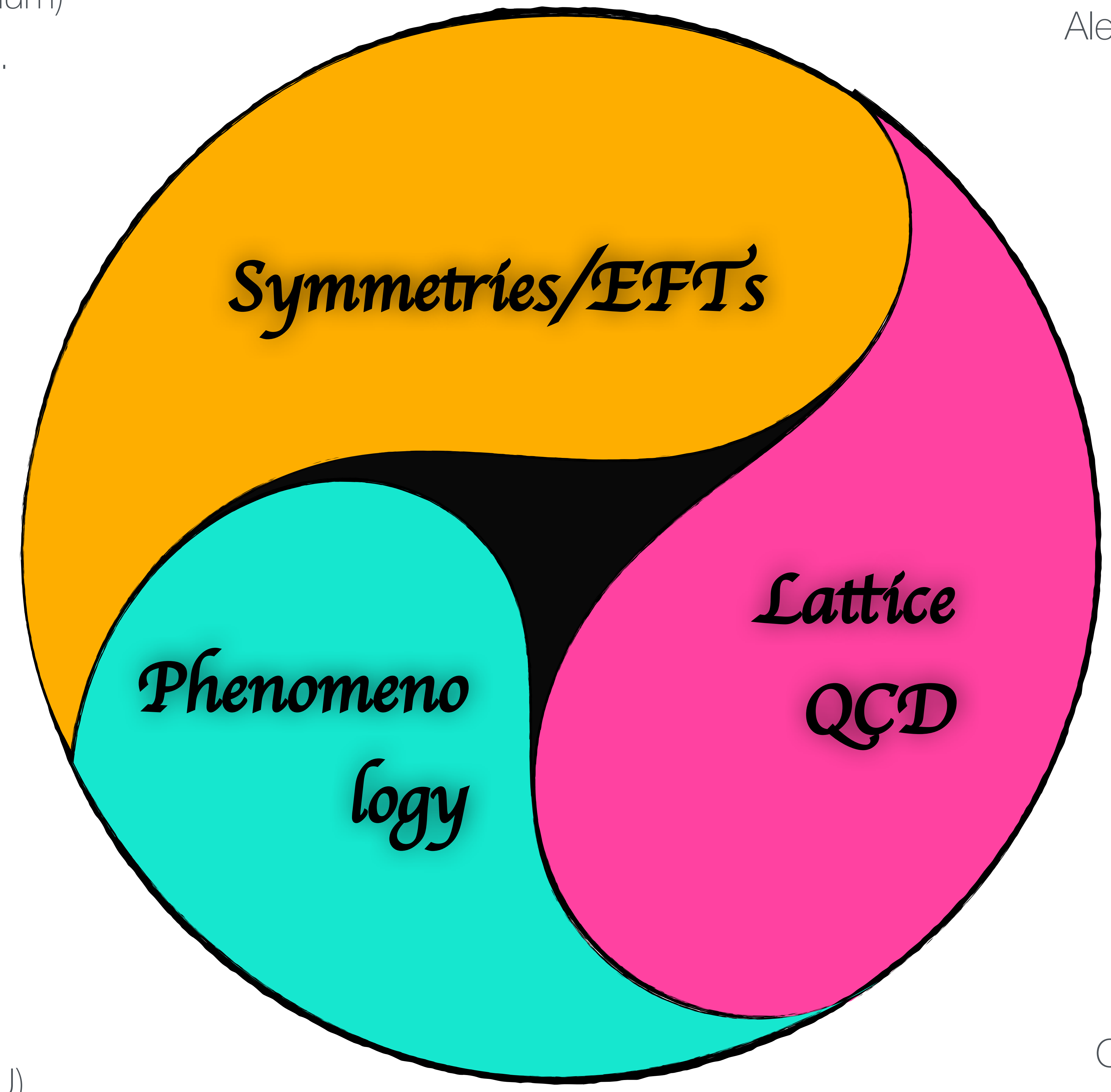
Ron Workman (JBW, GWU)

Michael Döring (GWU/JLAB)

Deborah Rönchen (FZ Jülich)

Helmut Haberzettl (GWU)

. . .



3-body scattering and quantisation condition [FVU]

Michael Döring (GWU/JLAB)

Adam P. Szczepaniak (U. Indiana/JLAB)

Alessandro Pilloni (U. Messina)

Finite-volume effects

Akaki Rusetsky (U. Bonn)

Ulf-G. Meißner (U. Bonn)

Michael Döring (GWU/JLAB)

Daniel Severt (U. Bonn)

2/3-body lattice spectroscopy [GWQCD collaboration]

Michael Döring (GWU/JLAB)

Chris Culver (U. Liverpool)

Ruairi Brett (GWU)

Andrei Alexandru (GWU)

Frank X. Lee (GWU)

[Extended Twisted Mass collaboration]

Carsten Urbach (U. Bonn)

Ferenc Pittler (U. Cyprus)

Marco Garofalo (U. Bonn)

. . .

HADRON SPECTROSCOPY

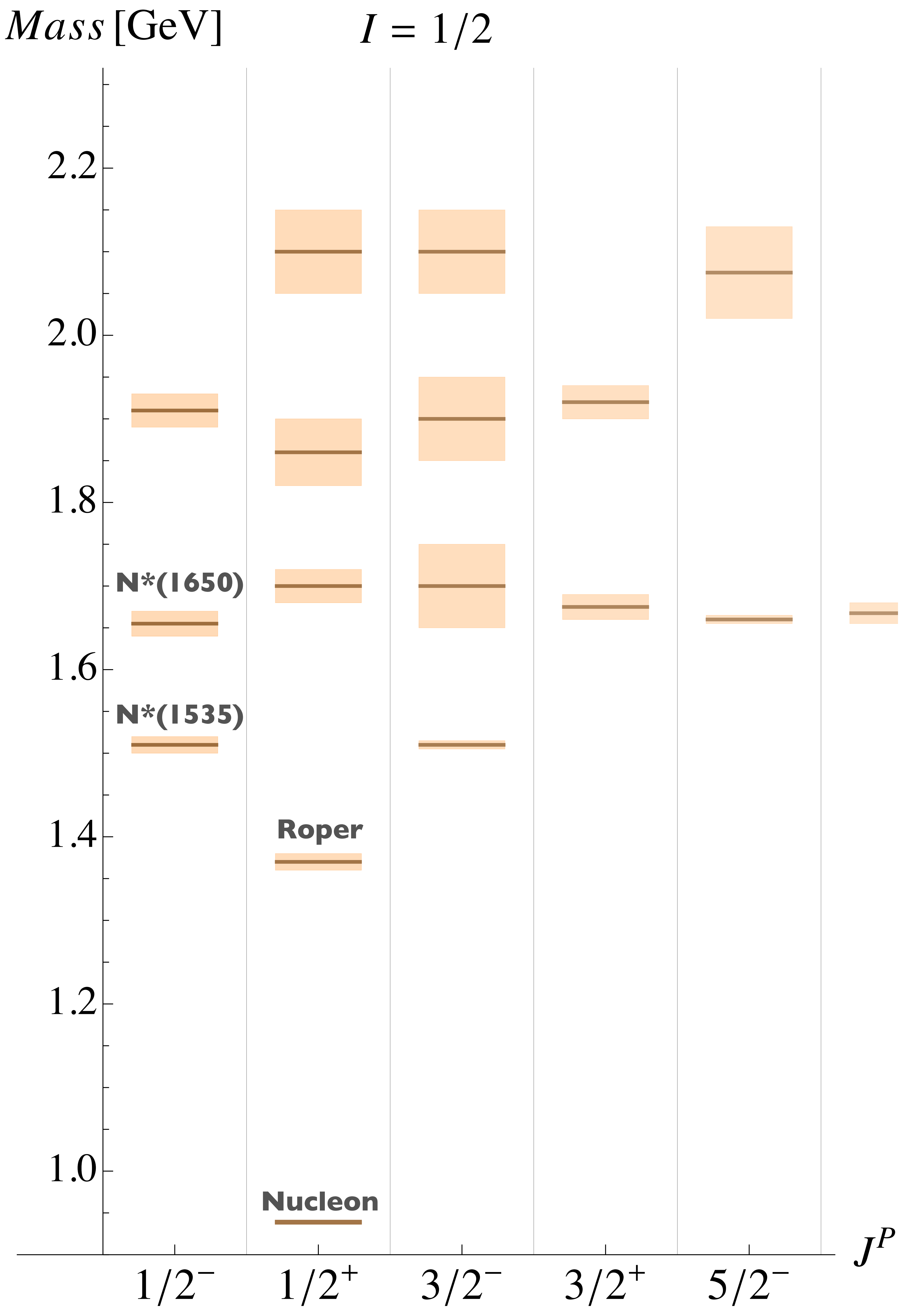
Mostly excited states

≈100 mesons & ≈50 baryons (****)

Key questions

"what is the pattern of these states?"

"how they are formed?"



 R.L. Workman et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2022, 083C01 (2022)

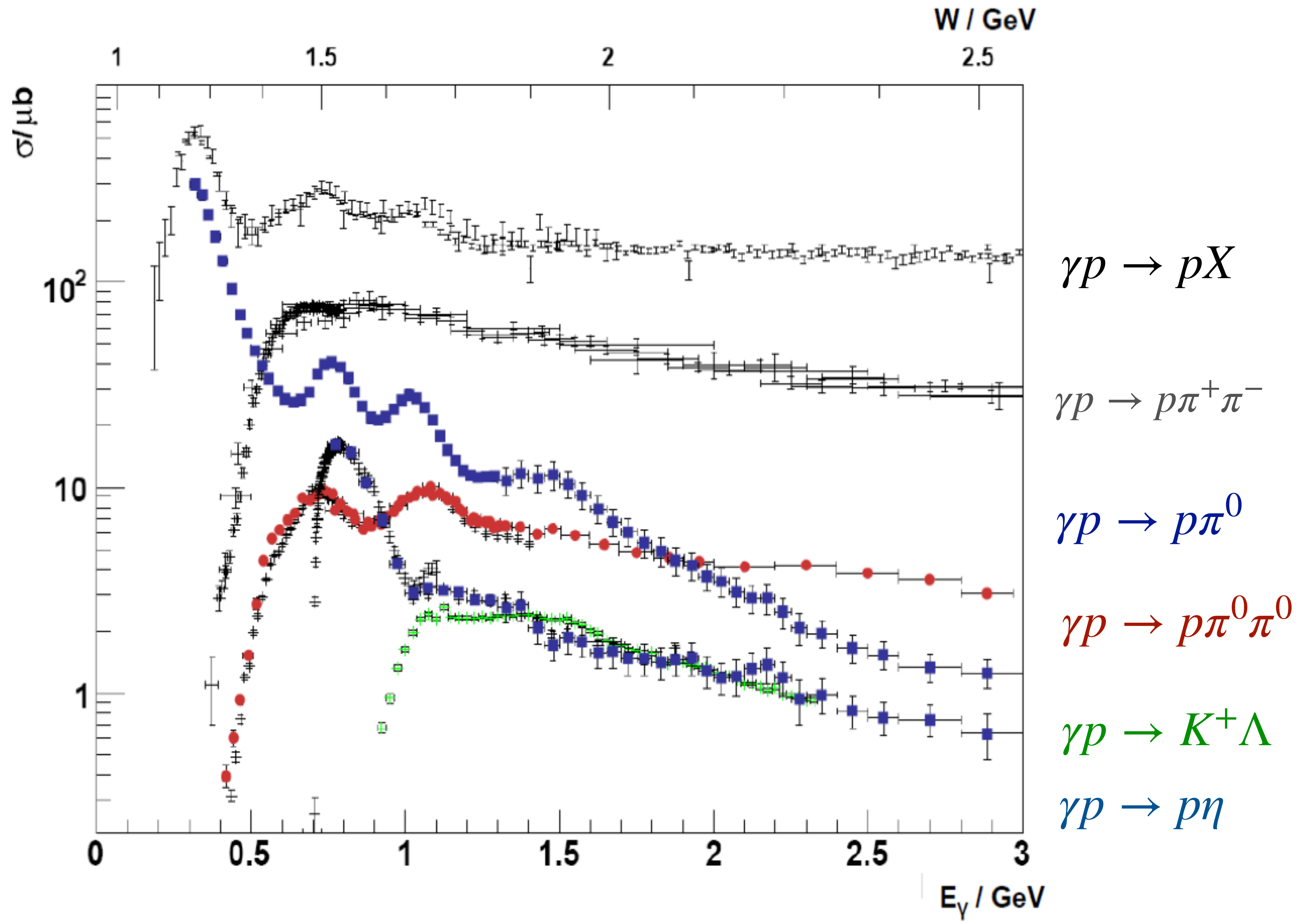
HADRON SPECTROSCOPY

Physical input

- many experimental data
- ongoing experiments[1]

Resonance parameter

"Not every bump is a resonance, not every resonance is a bump" - R. G. Moorhouse (1960s)

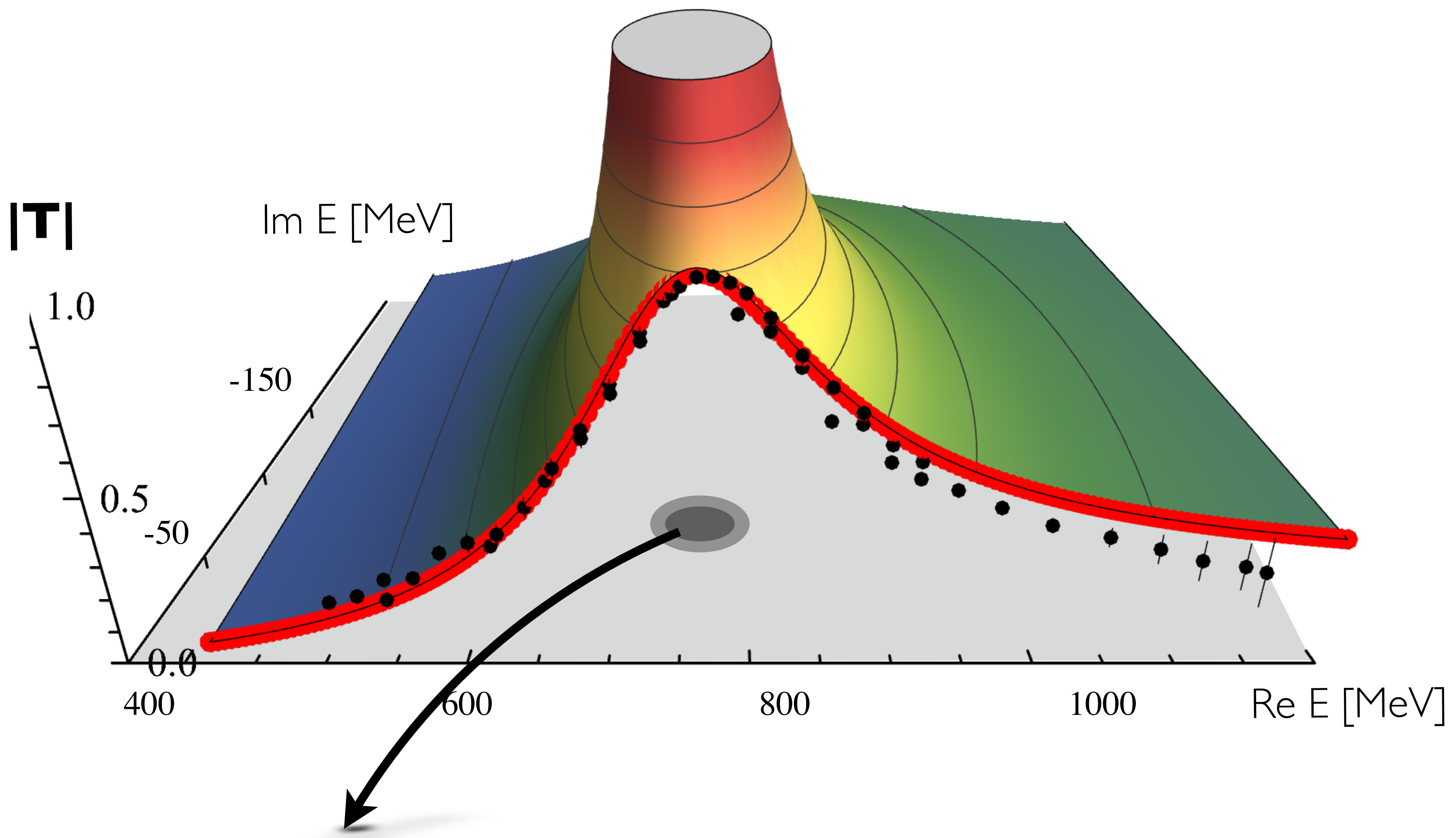


Data: Jefferson Laboratory, ELSA, MAMI

UNIVERSAL PARAMETERS

Reaction-independent parameters

- pole positions on unphysical Riemann Sheets
- central quantity: **transition amplitudes**



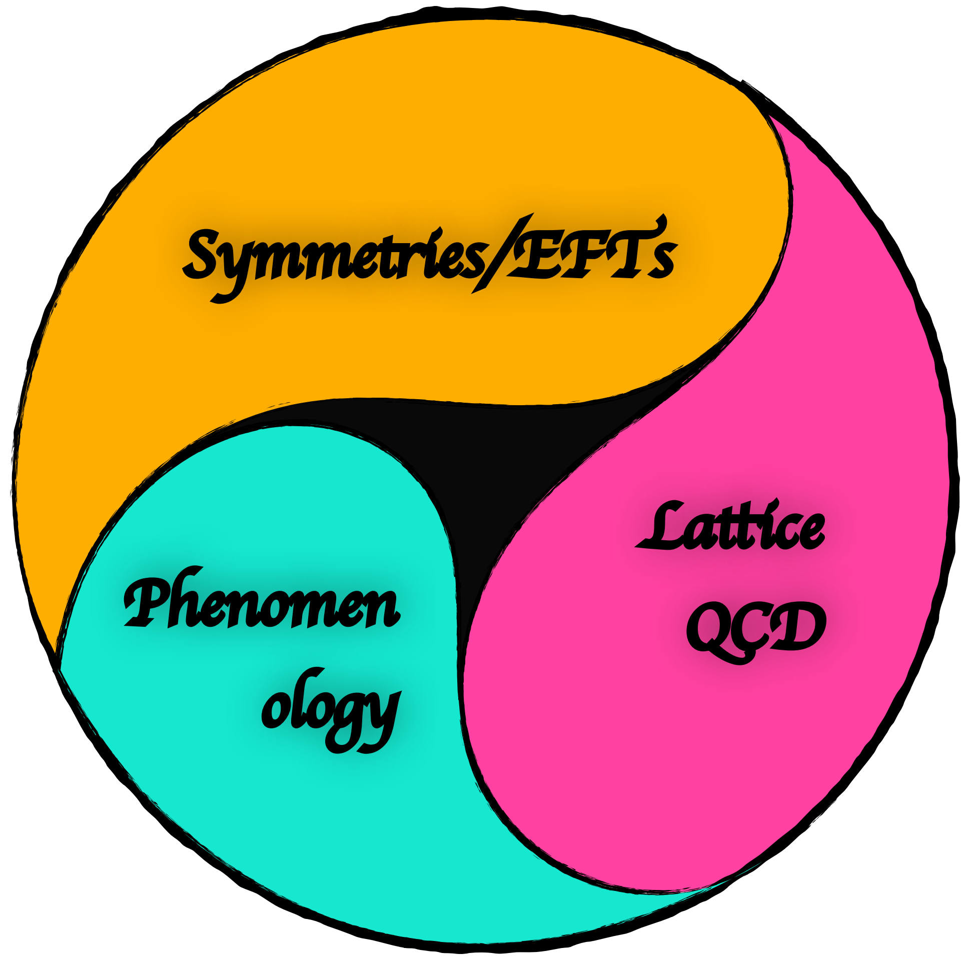
$M^* = (750 - i60)$ MeV
Universal property of the ρ – meson

Review: ``Towards a theory of hadron resonances'' Phys. Rept. 1001 (2023) – MM/Meißner/Urbach

Talks: Szczepaniak, Guo, Rodas, Pelaez, Döring, …

Data: Estabrooks et al. Nucl.Phys.B 79; Protopopescu et al. Phys.Rev.D 7;

UNIVERSAL PARAMETERS

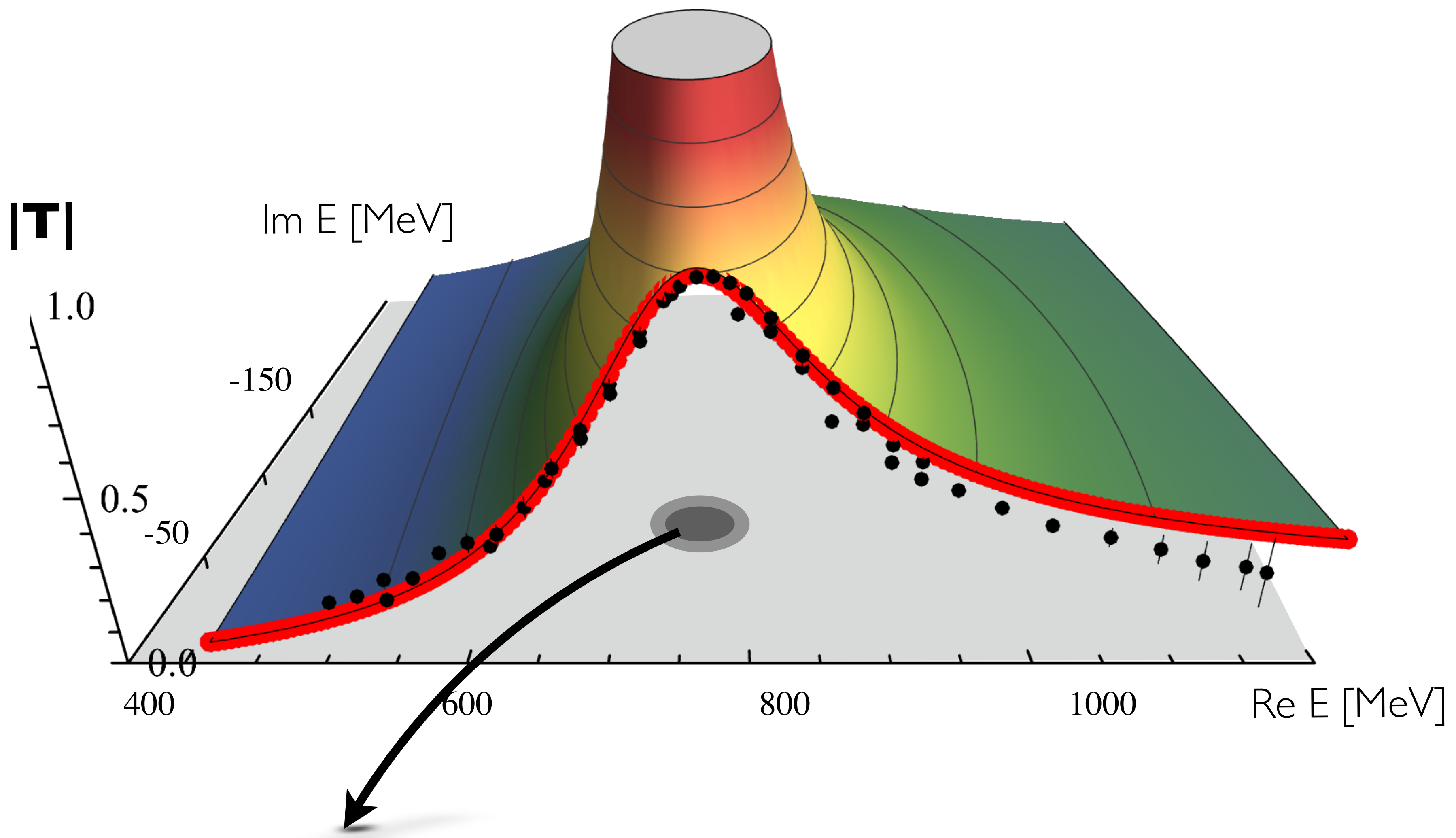


Reaction-independent parameters

- pole positions on unphysical Riemann Sheets
- central quantity: **transition amplitudes**

→ Constraints from S-matrix (Unitarity/Analyticity/Crossing)

→ Constrains from QCD (CHPT/LatticeQCD)



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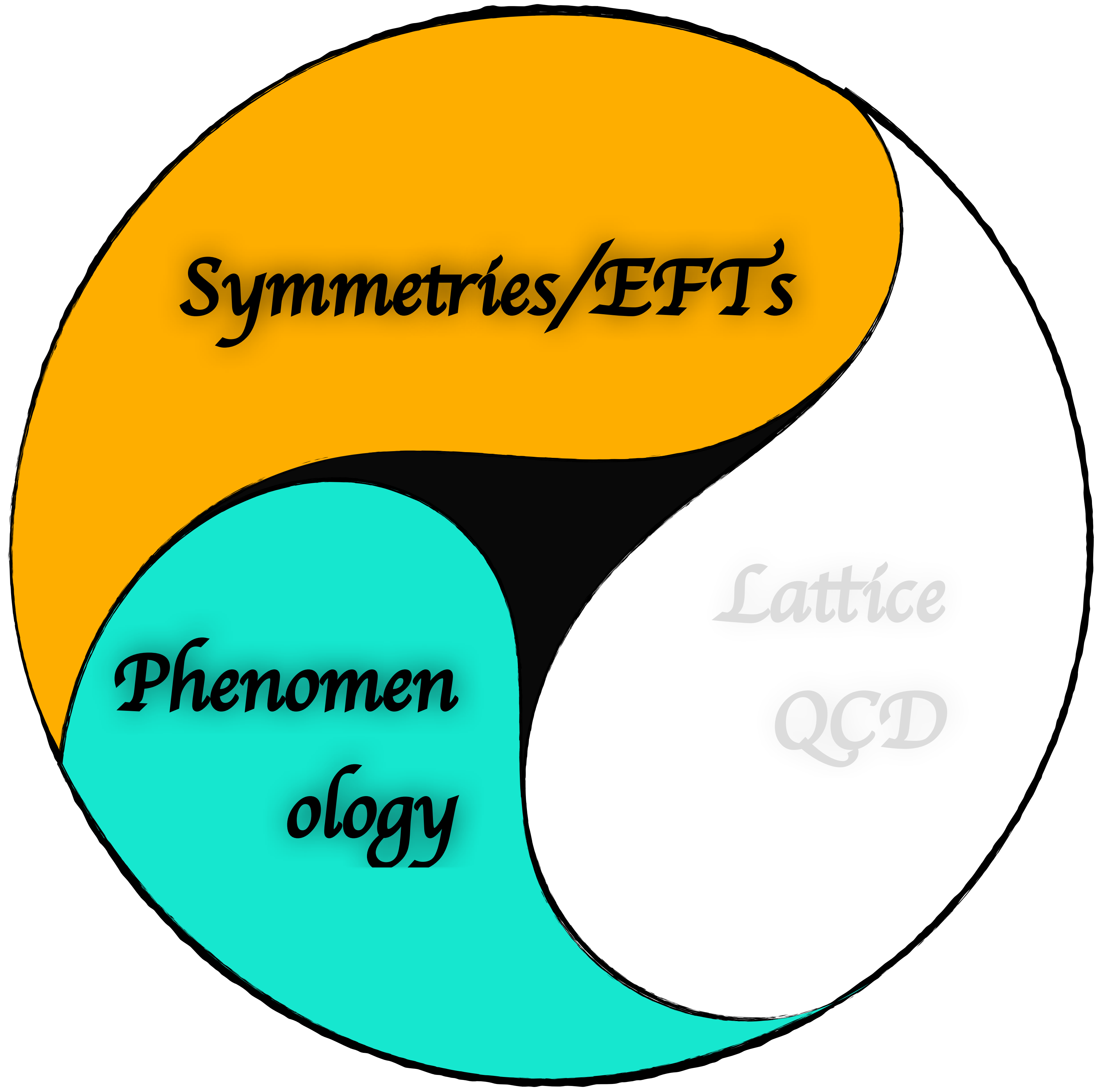
INDIVIDUAL STATES

MESON-BARYON RESONANCES FROM CHPT AND UNITARITY

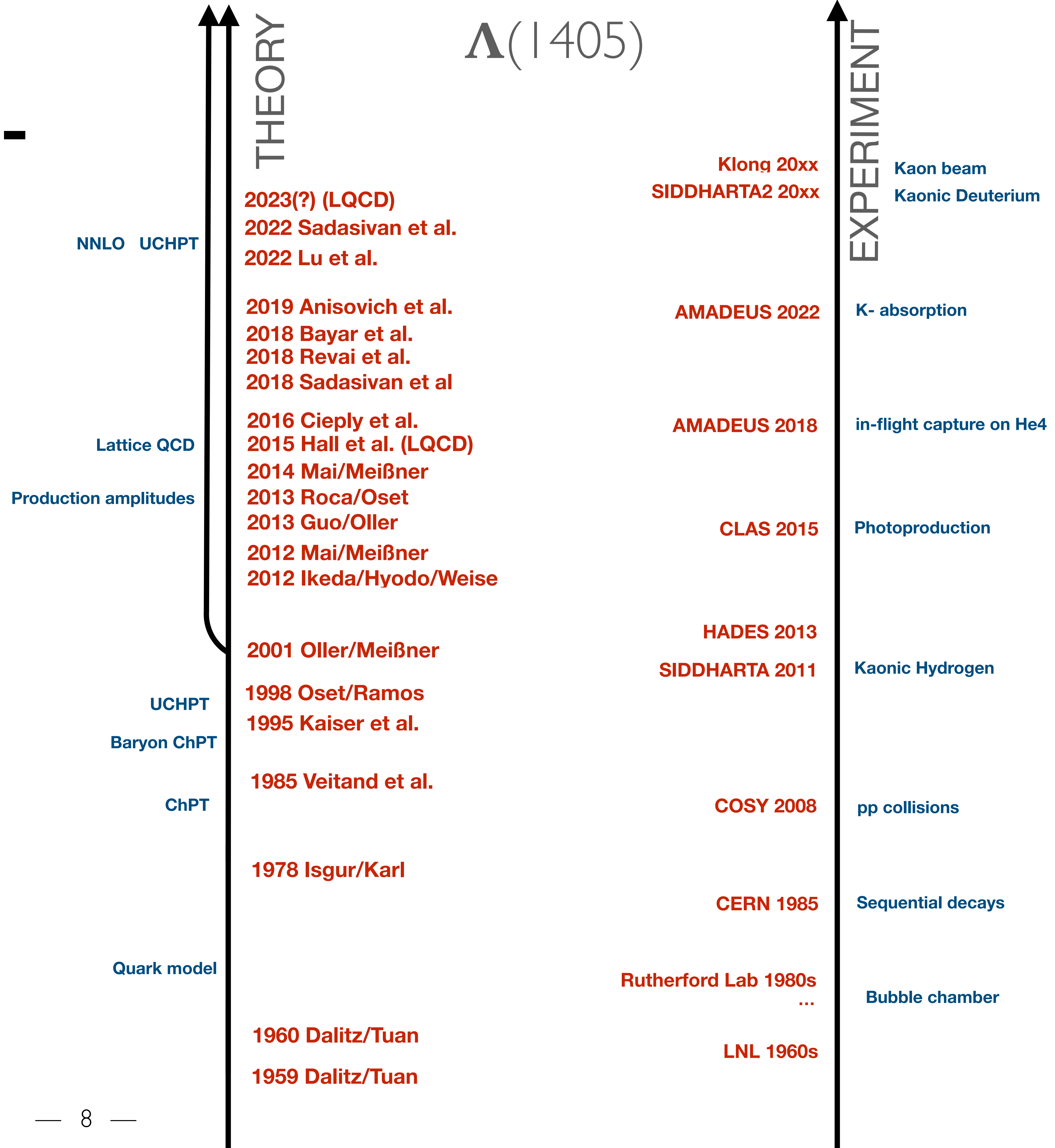


RESONANT MESON-BARYON SYSTEMS

Many examples:

- N*(1535), N*(1650), . . .
- $\mathbf{\Lambda}$(1405), $\mathbf{\Lambda}$(1380)

→ Long history of experimental and theoretical efforts[1]



[1] **Reviews**: MM Eur.Phys.J.ST 230 (2021); Hyodo/Niiyama Prog.Part.Nucl.Phys. 120 (2021)

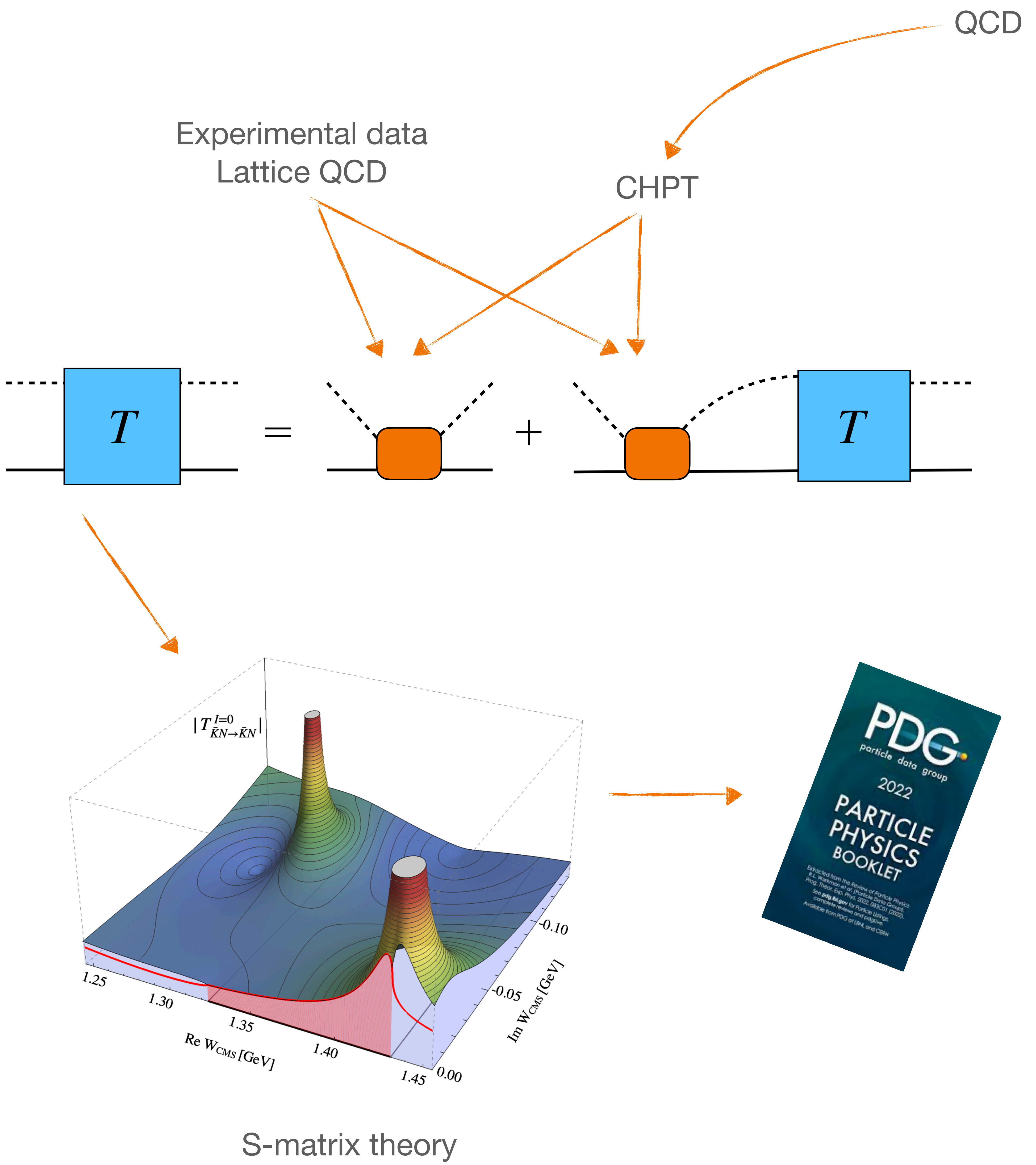
TRANSITION AMPLITUDE

One way:

- Chiral Perturbation Theory (#QCD#EFT) dictates the form of the interaction at low energies
- Unitary scattering amplitude from the Bethe-Salpeter equation

→ **Fit free parameters to experimental data / LQCD**

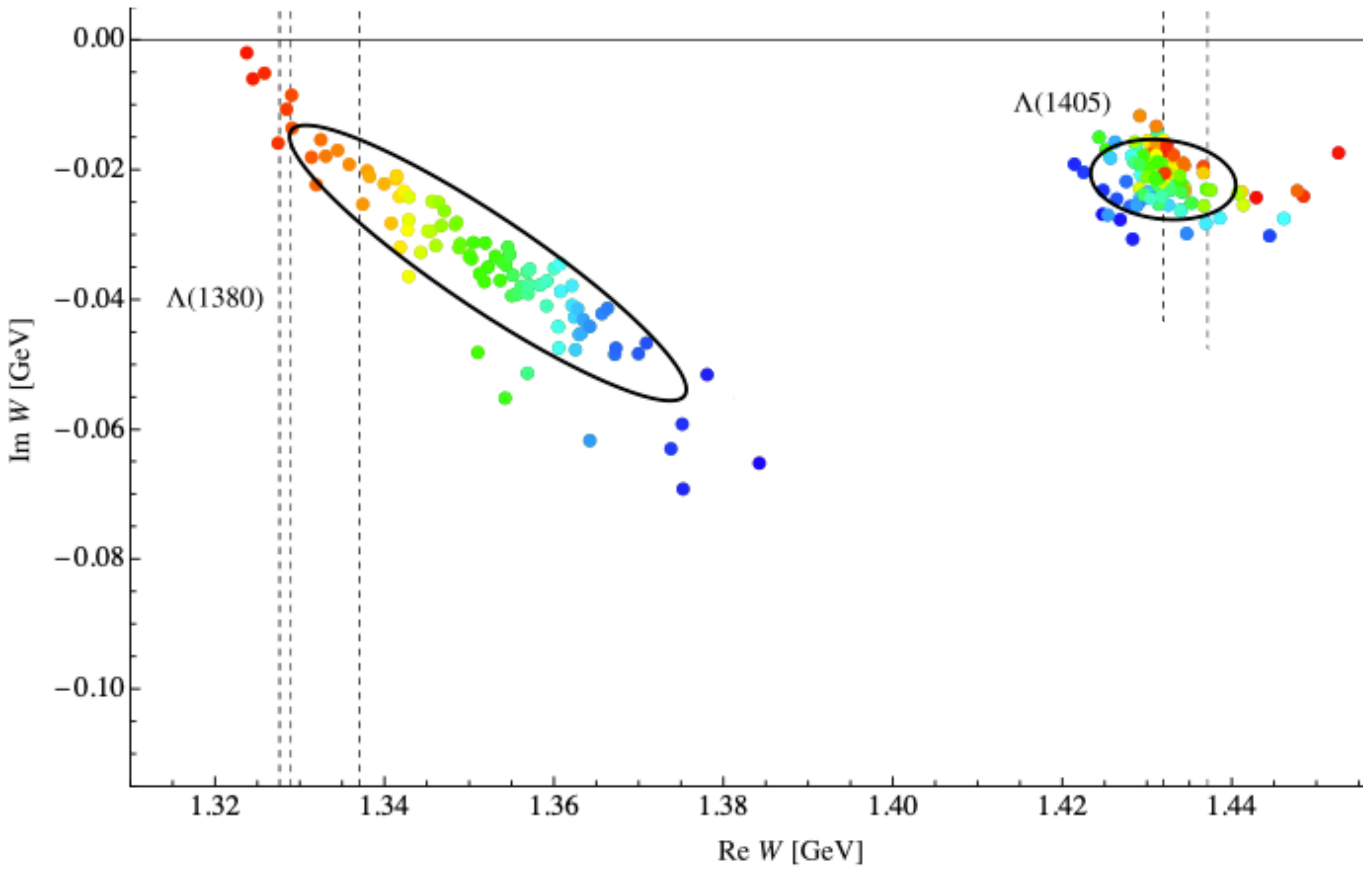
→ **Extract complex pole positions for complex energies**



CURRENT FRONTIER

Statistical tests and new data sources

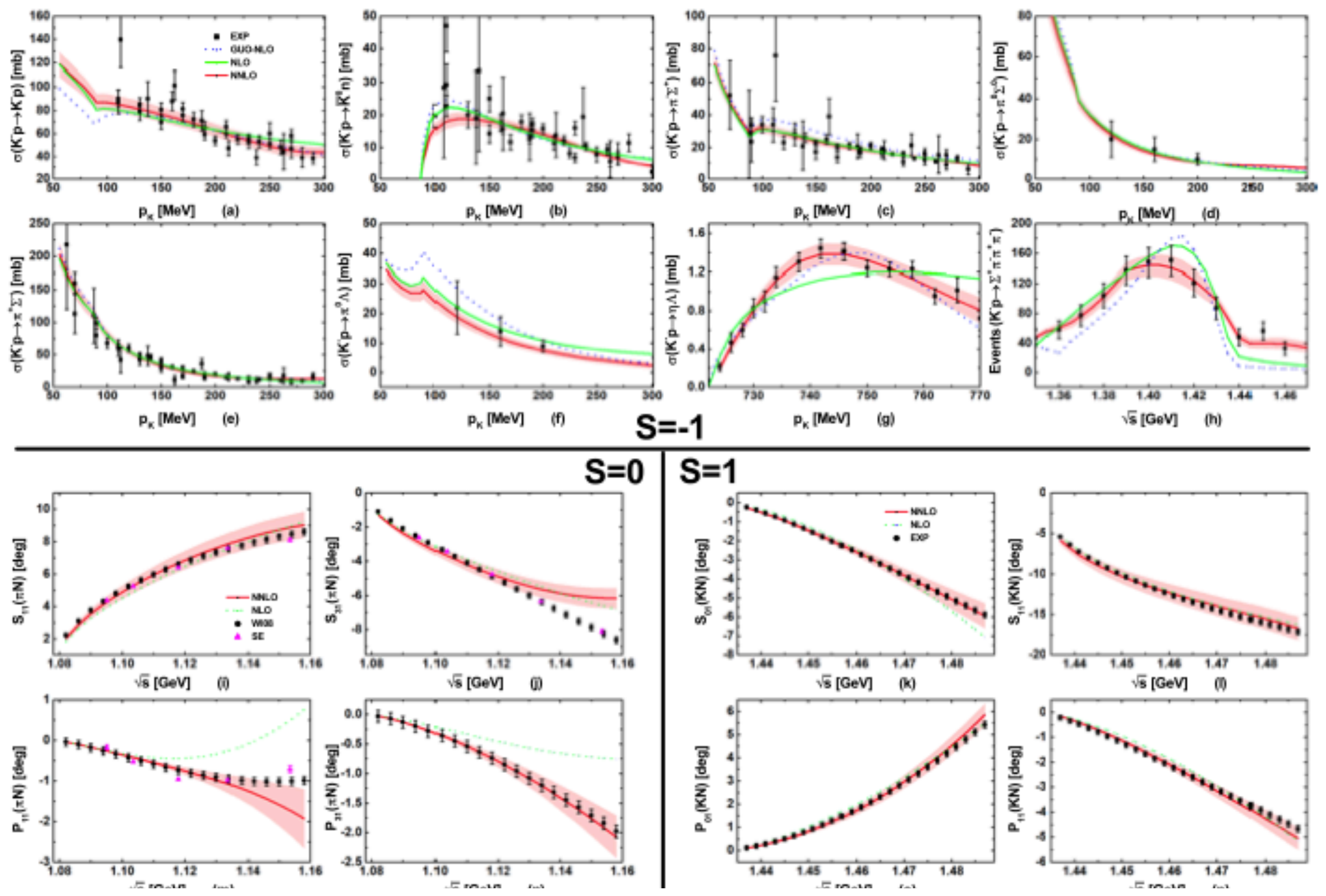
AMADEUS Phys. Lett. B 782 (2018)
Sadasivan et al Front.Phys. 11 (2023)



- New correlations $\mathbf{\Lambda}(1405)/\mathbf{\Lambda}(1380)$

Theory update: NNLO UCHPT

Lu/Geng/Döring/MM Phys.Rev.Lett. 130 (2023)

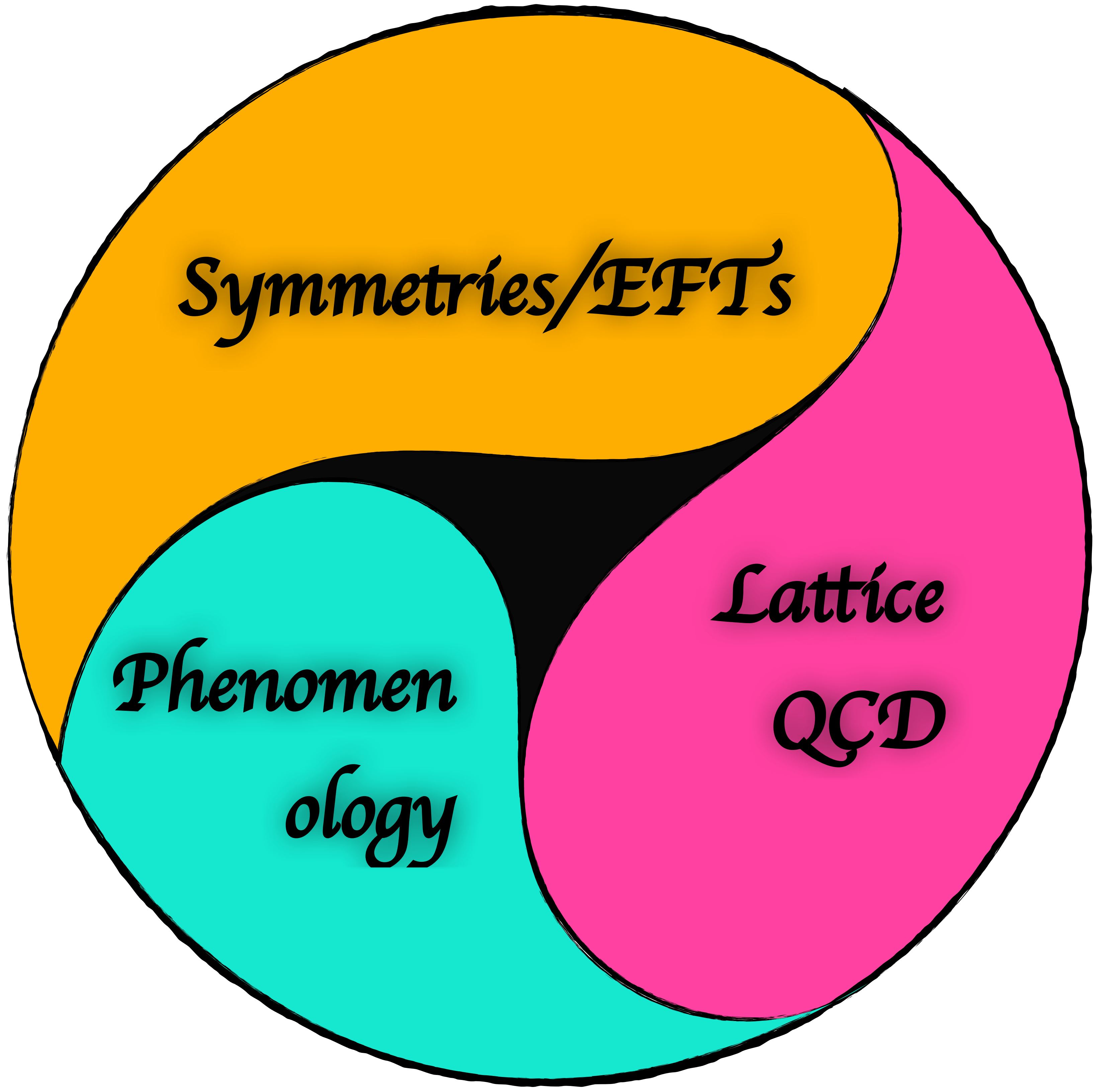


- Simultaneous description of $\pi N/KN/K$bar$N$ scattering
- Two-pole structure $\mathbf{\Lambda}(1405)/\mathbf{\Lambda}(1380)$

- **Lattice QCD [2 poles]** (Talks: Morningstar/Leinweber)
- **New experimental facilities** (Talks: Francesco Sgaramella/…)

INDIVIDUAL STATES

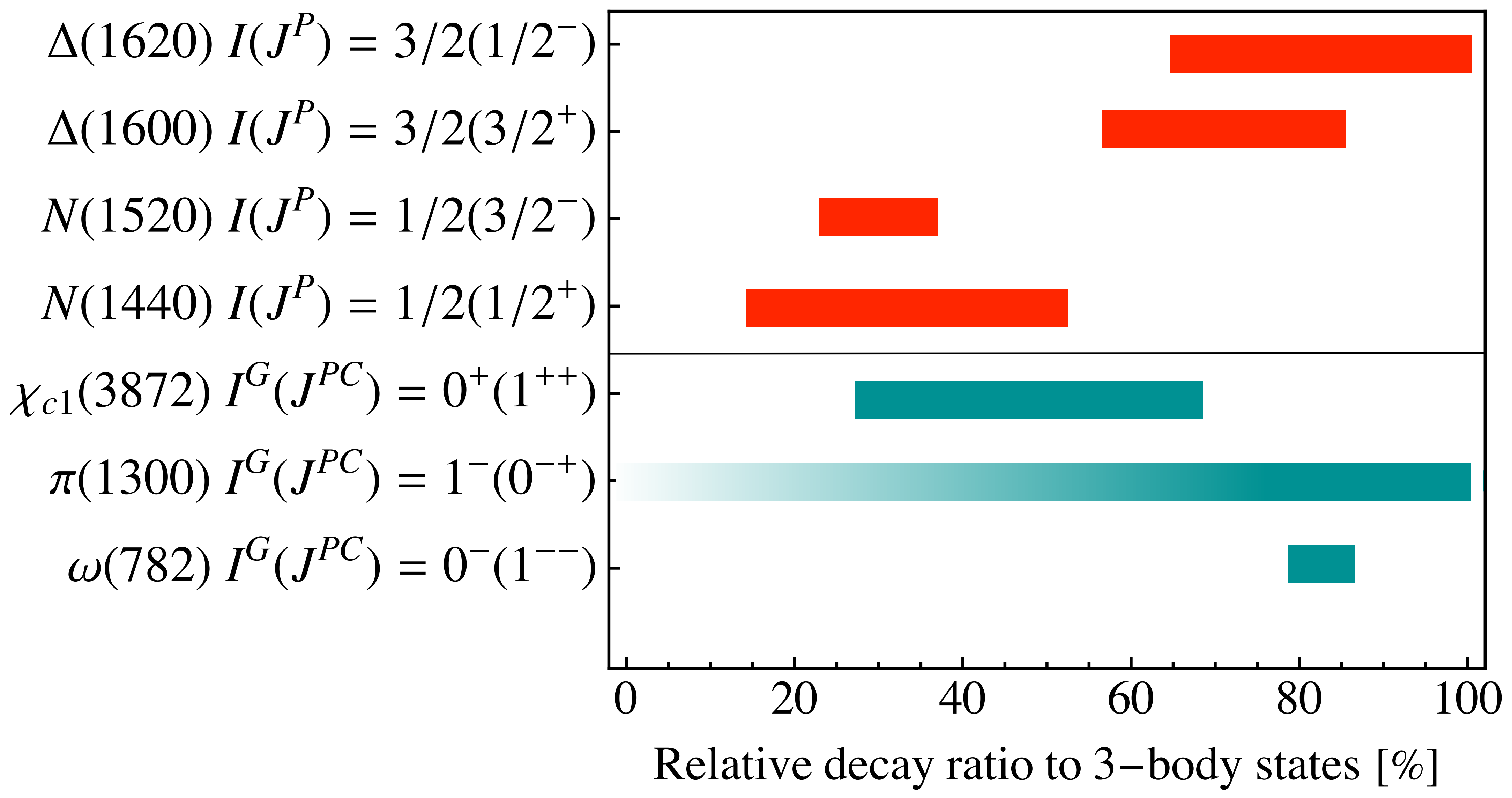
THREE-BODY SYSTEMS



HADRONIC 3-BODY PROBLEM

- Many known states have large 3-body content:

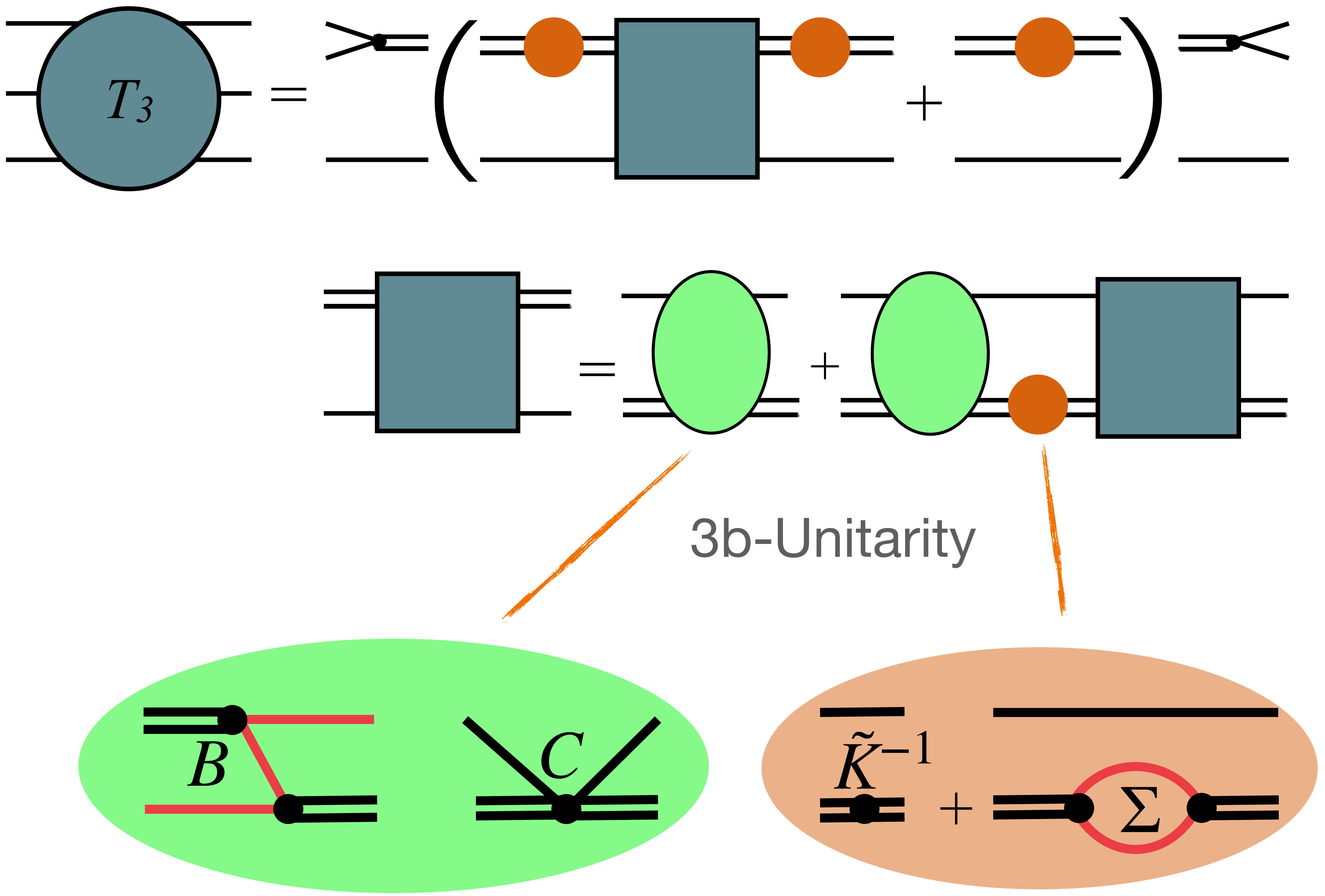
  → Roper(1440)/X(3872)/a_1(1260)/...
- Beyond Standard Model searches (τ-EDM/...)
- Exotic states of matter[1]



[1] Experimental programs: GlueX@JLAB; COMPASS@CERN;

R.L. Workman et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2022, 083C01 (2022)

TRANSITION AMPLITUDE



Three-body scattering amplitude[1][2]

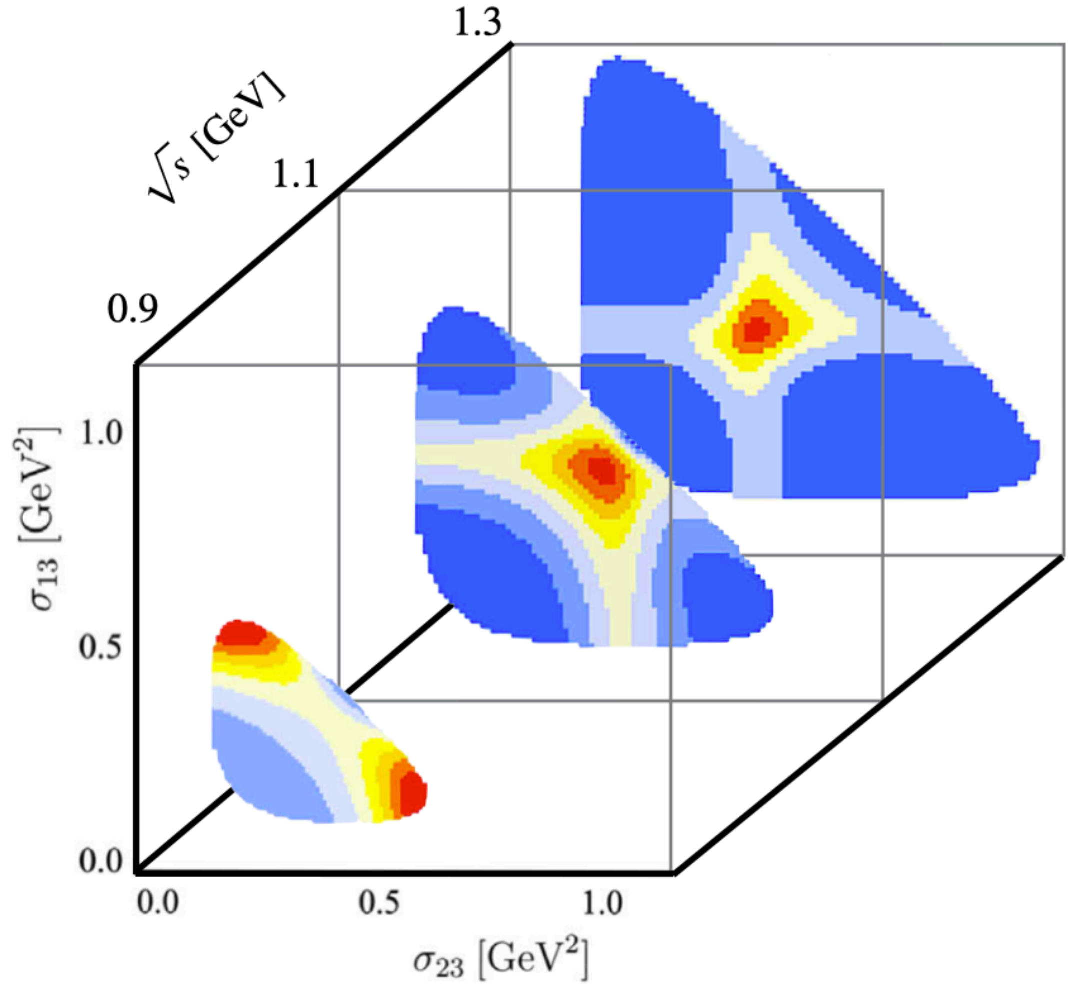
- constructed from unitarity
- novel result from the S-matrix theory

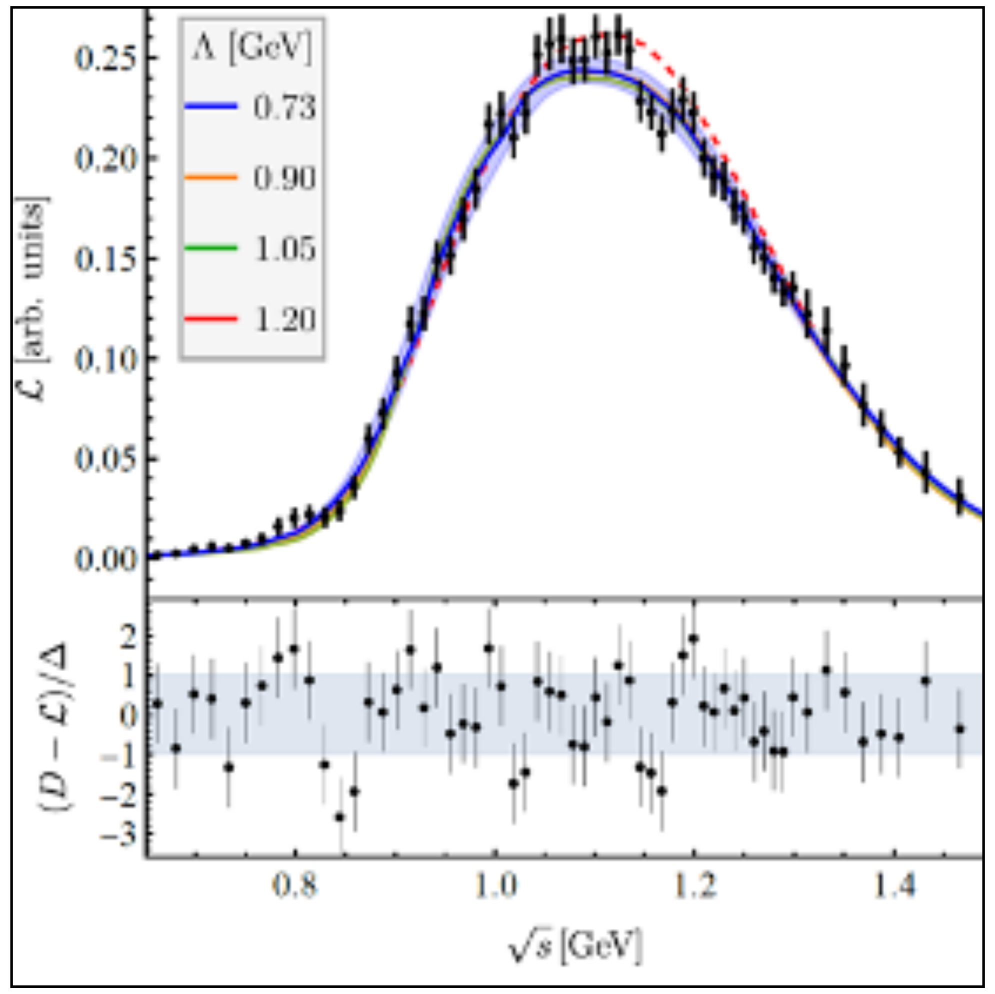
[1] MM/Hu/Döring/Pilloni/Szczepaniak Eur.Phys.J.A 53 (2017)

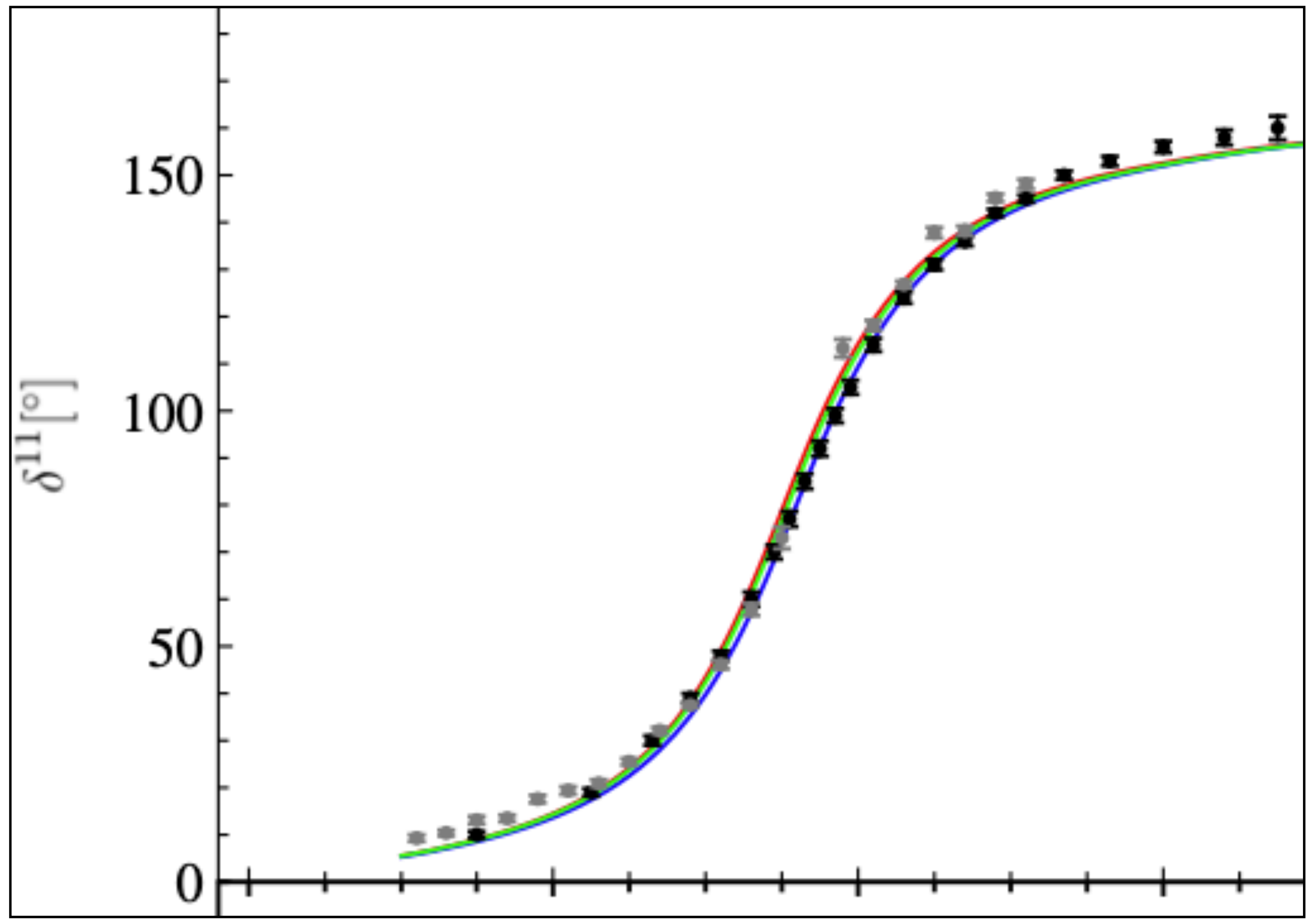
[2] Related approaches: Wunderlich et al. JHEP 08 (2019); Jackura et al. Eur.Phys.J.C 79 (2019);

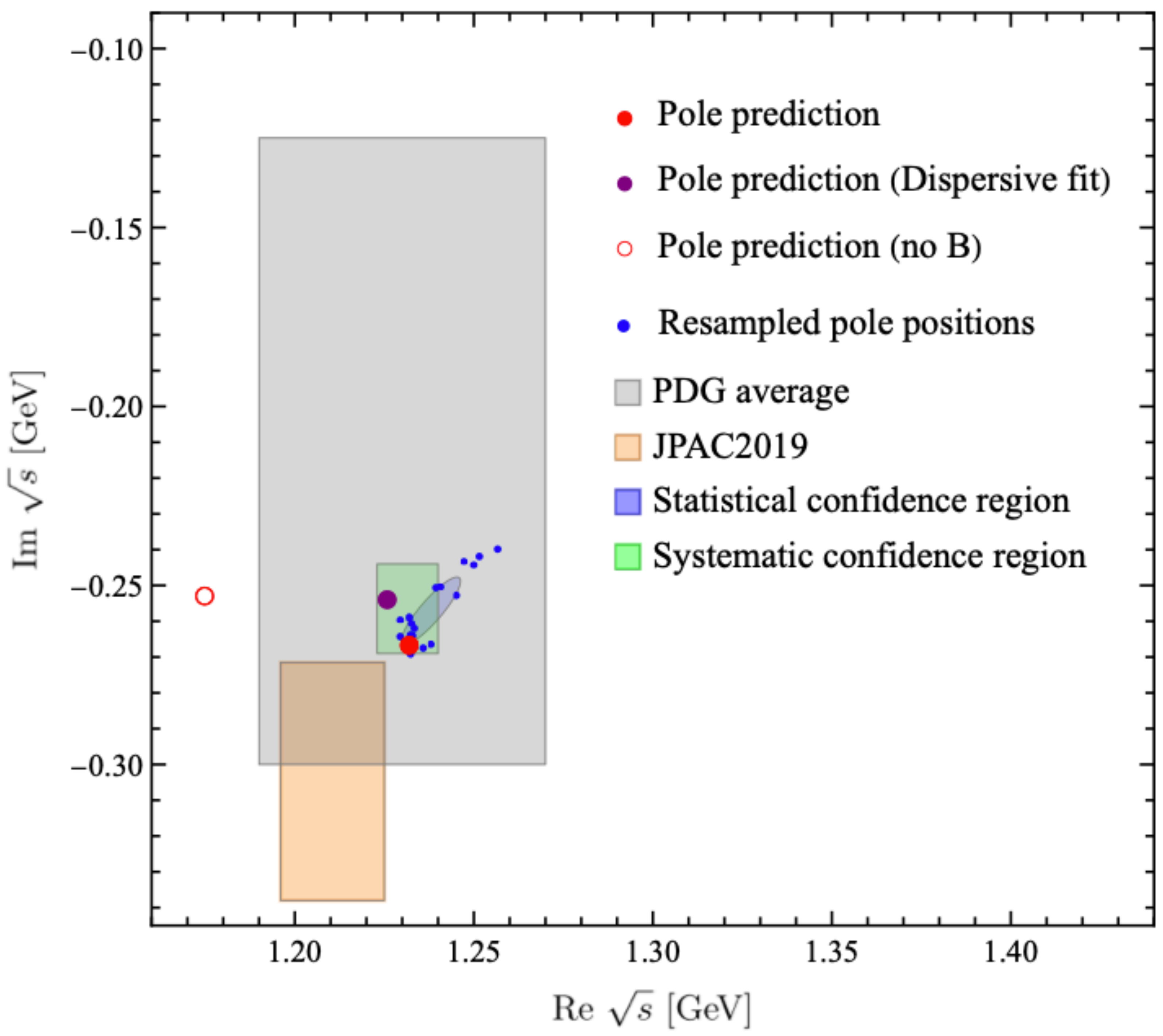
a1(1260)
Our Universe

$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3}\frac{(B+C)}{2E_l}\frac{1}{\tilde{K}_n^{-1}-\Sigma_n}T^c$$









Data: Schael [ALEPH] Phys.Rept. 421 (2005); Estabrooks et al. Nucl.Phys.B 79; Protopopescu et al. Phys.Rev.D 7;

Sadasivan/MM/Döring/Alexandru/Culver/Lee Phys.Rev.D 101 (2020)

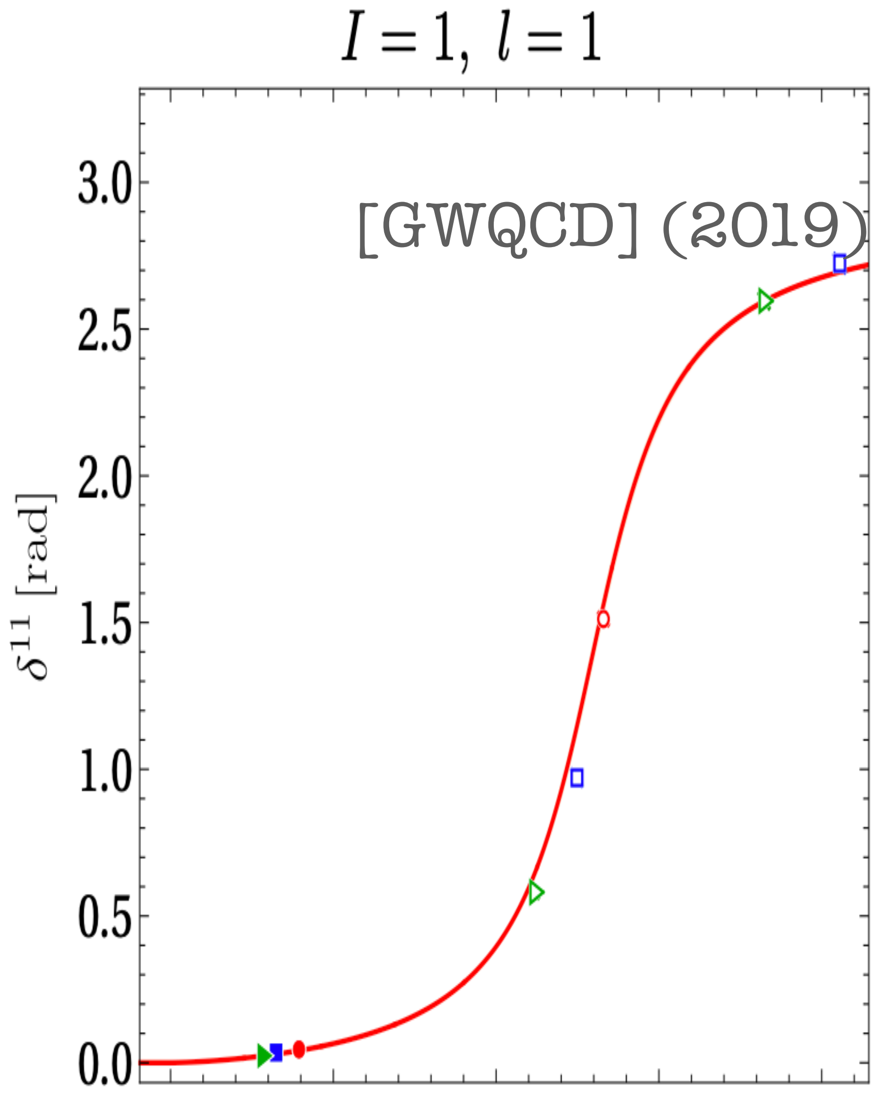
a1(1260)

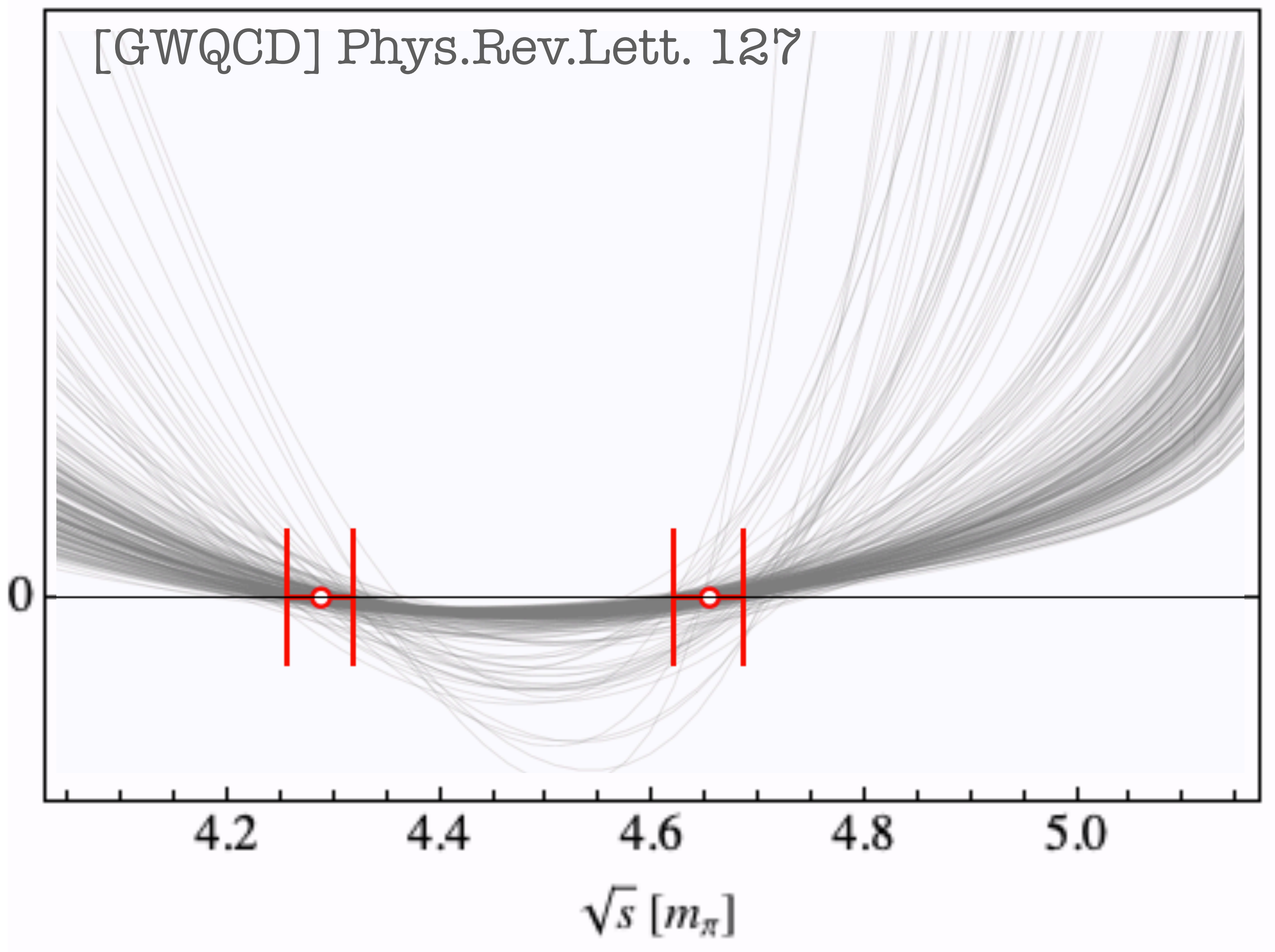
"Heavy Universe" (M_π : 138 MeV $\rightarrow$ 224 MeV)

3-body quantisation condition — FVU [1][2]

$$\det\left[2L^3E\left(\tilde{K}_n^{-1}-\Sigma\right)-B-C\right]_{\mathbf{p}'\mathbf{p}}=0$$

$$T^c=B+C+\int\frac{d^3\ell}{(2\pi)^3}\frac{(B+C)}{2E_l}\frac{1}{\tilde{K}_n^{-1}-\Sigma_n}T^c$$





[1] MM/Culver/Döring/Alexandru/Lee/Brett/Sadasivan [GWQCD] Phys.Rev.Lett. 127

[2] **Reviews:** Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Döring/Rusetsky Eur.Phys.J.ST 230 (2021);

More in
M. Döring's talk #1

a1(1260)
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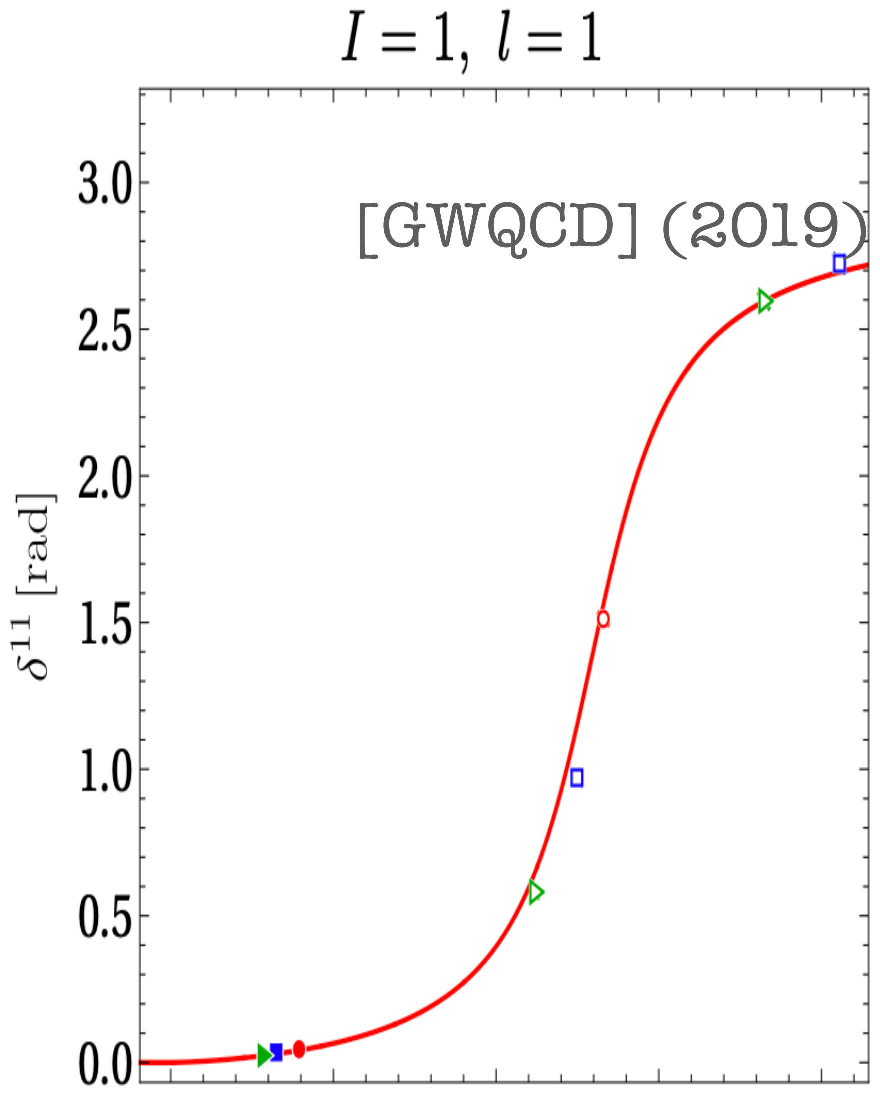
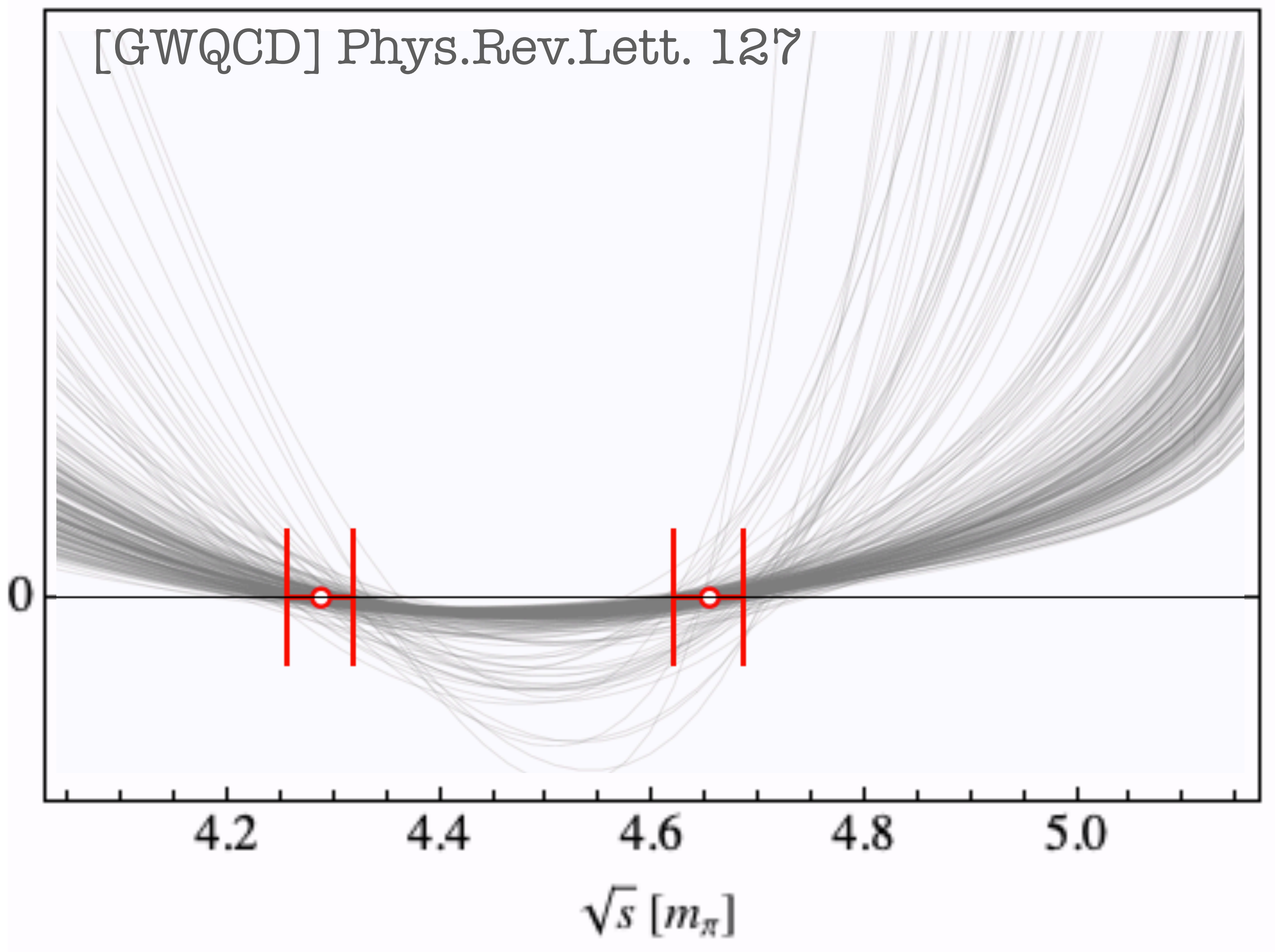
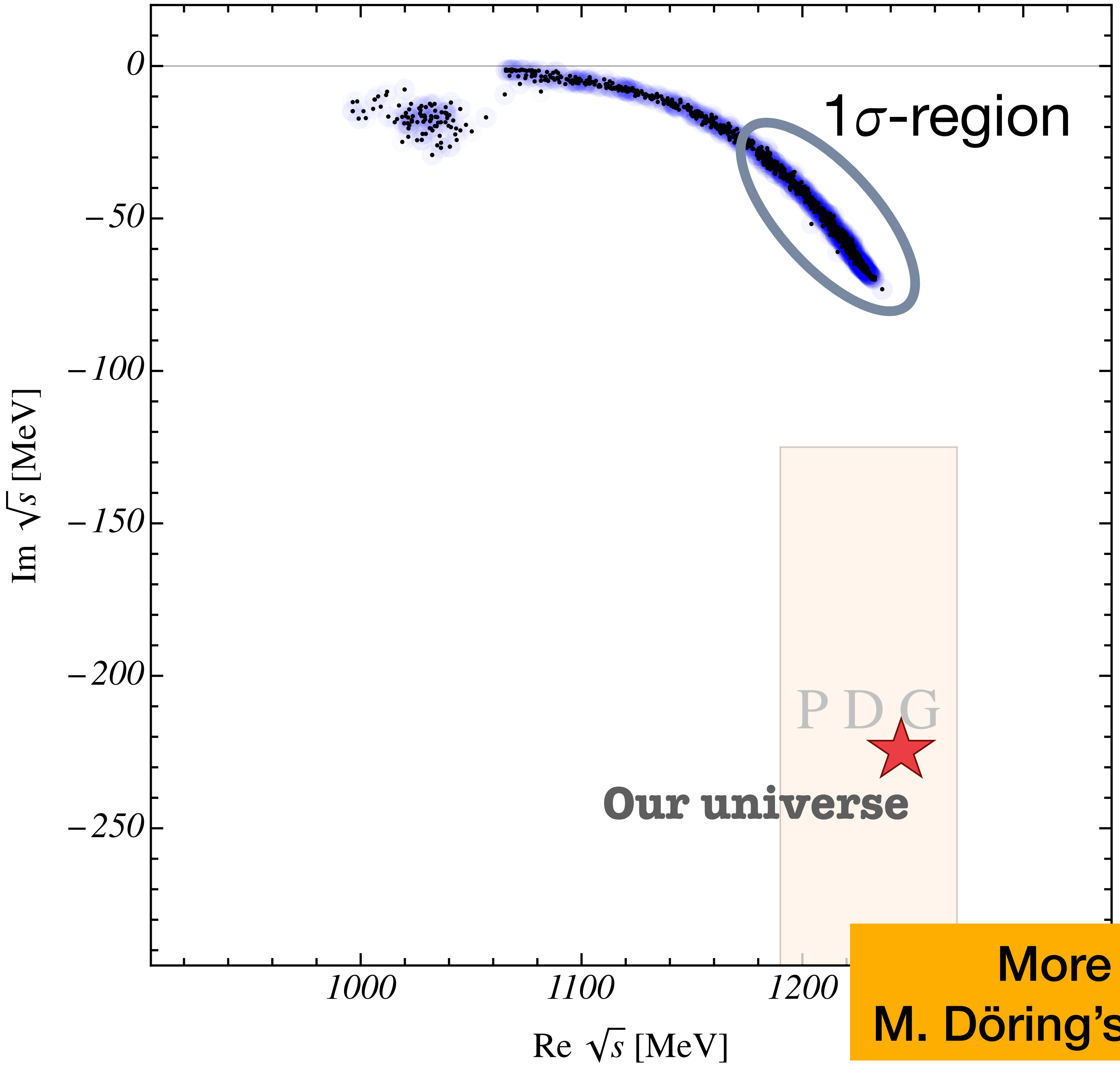






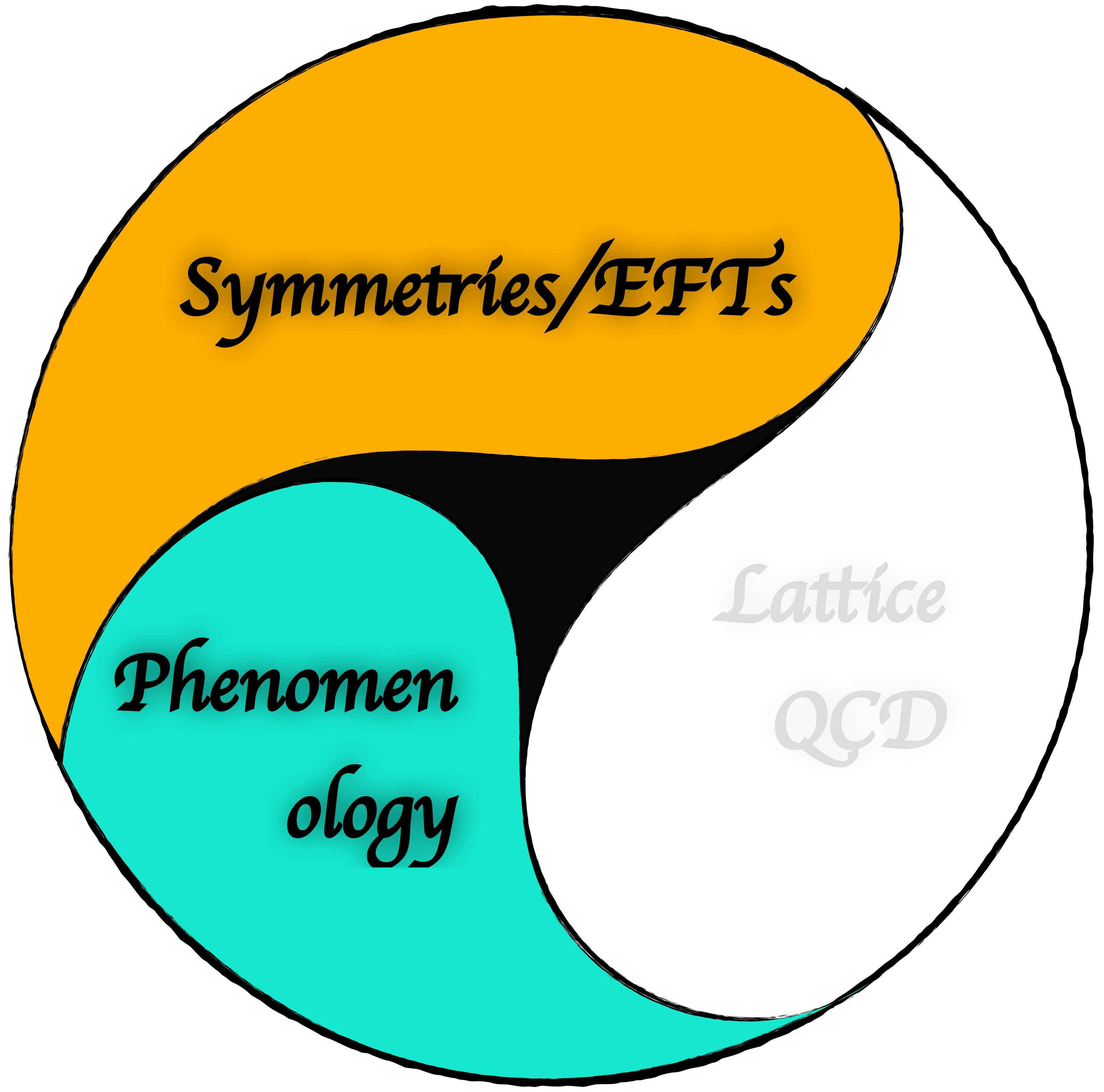
[1] MM/Culver/Döring/Alexandru/Lee/Brett/Sadasivan [GWQCD] Phys.Rev.Lett. 127

[2] **Reviews:** Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Döring/Rusetsky Eur.Phys.J.ST 230 (2021);

More in
M. Döring's talk #1

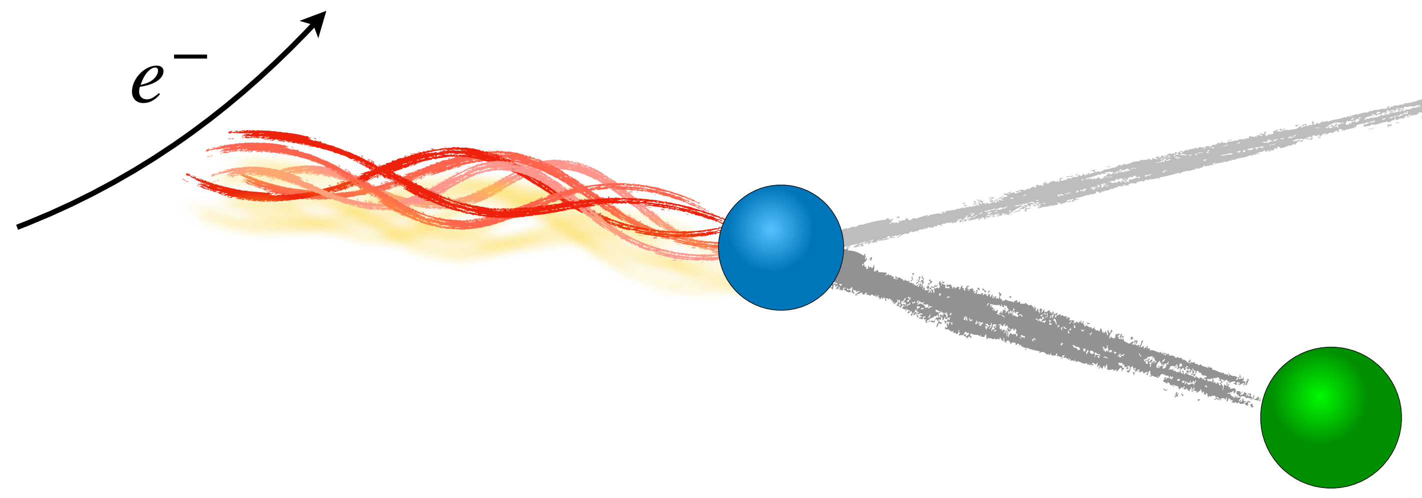
GLOBAL PROPERTIES

MESON-ELECTROPRODUCTION



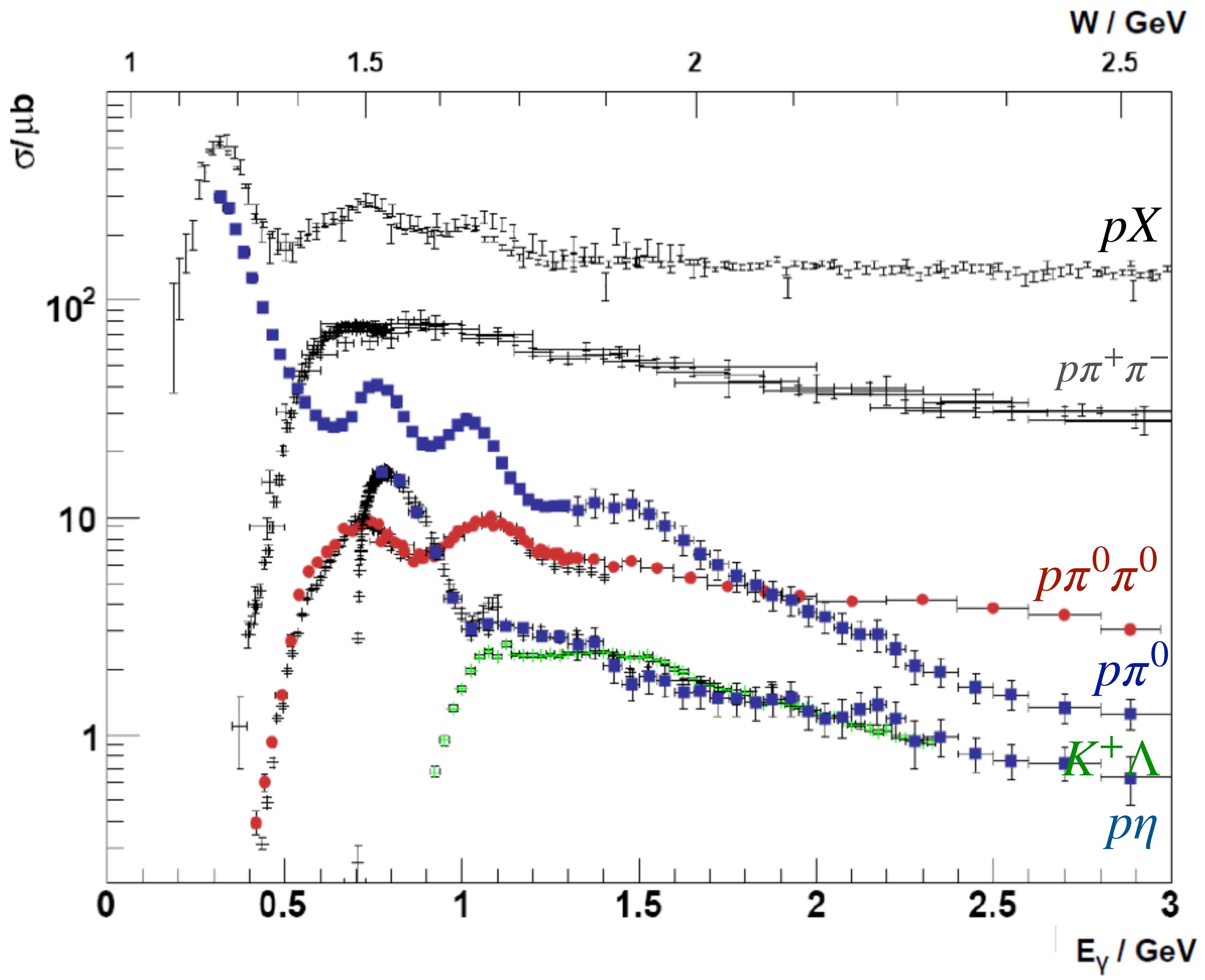
PHOTON-INDUCED EXCITATION

New probe



- 5 independent variables
- Momentum transfer dependence
- State compositeness[1] (?)

Experimental accessibility

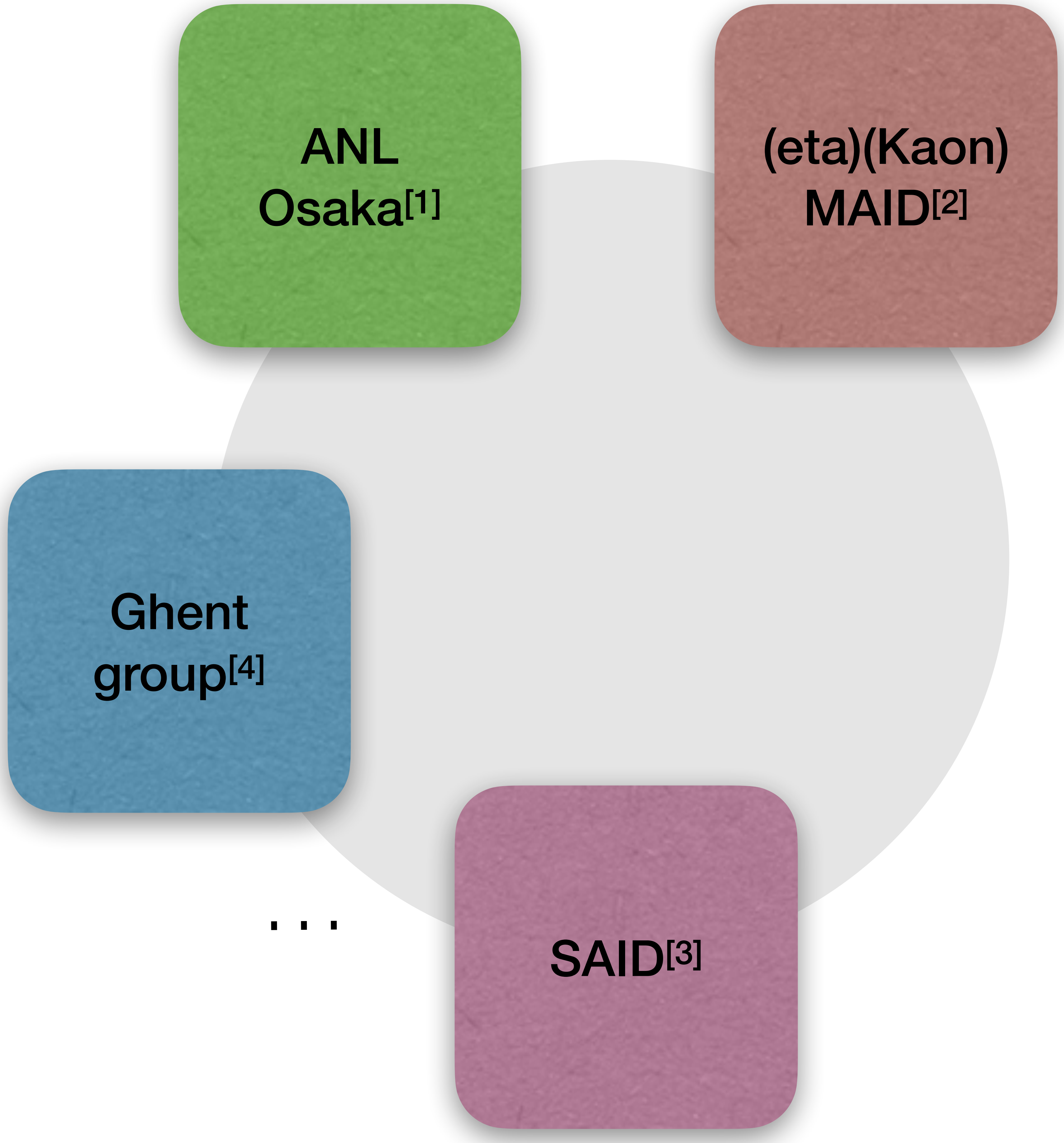


- Large amount of data (*10^5*)
- more data coming up[1] (*Q^2=5-12 GeV^2*)

[1] e.g., **Review:** Burkert, Roberts, Rev.Mod.Phys. 91 (2019)

[2] Carman, Joo, Mokeev, Few Body Syst. 61, 29 (2020) ... ; [CLAS] Phys.Rev.C 105 (2022) 065201; ...

TRANSITION AMPLITUDES: PREVIOUS APPROACHES



Some highlights

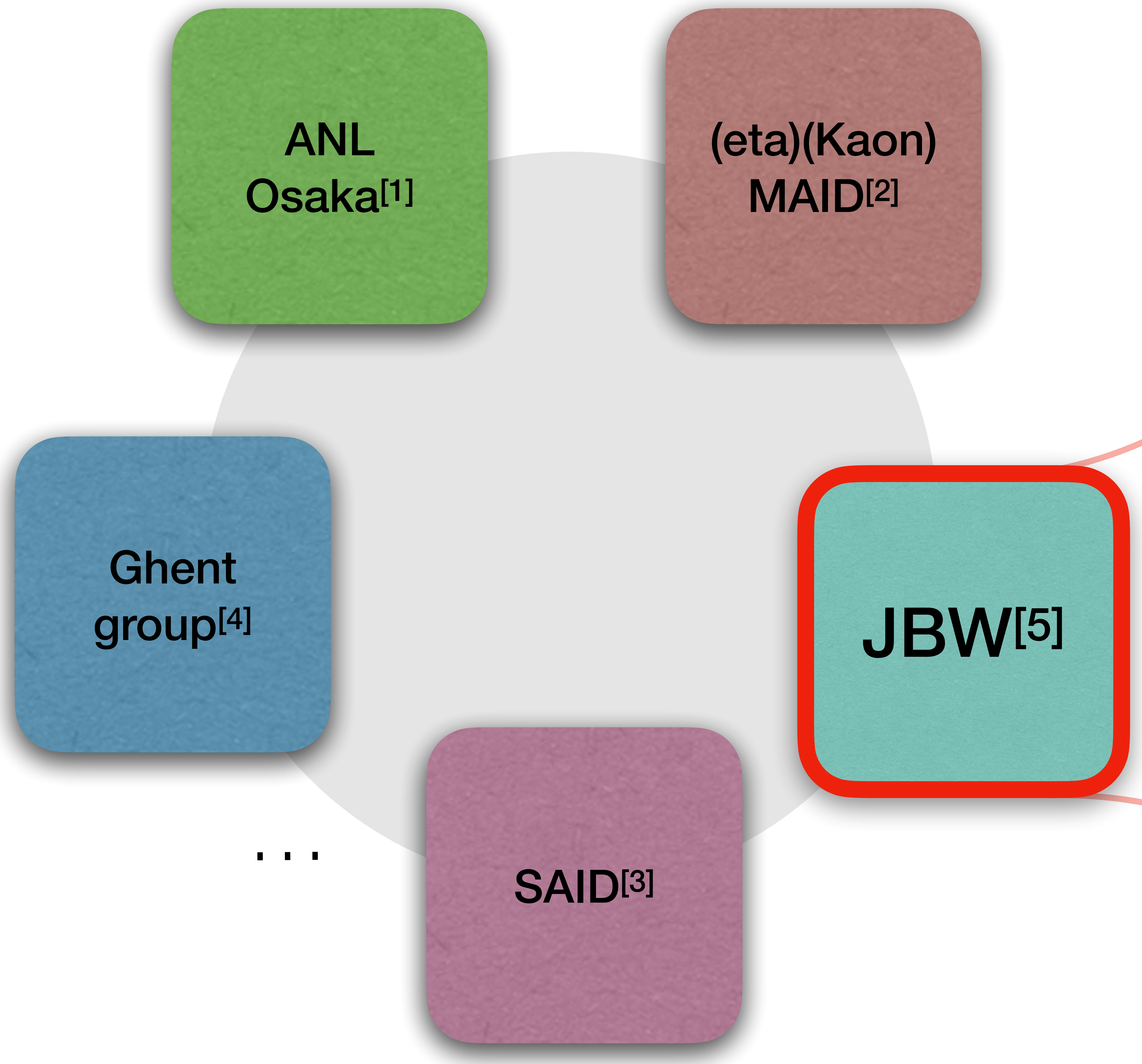
→ simultaneous description of pion photo-/ electroproduction (MAID)

→ low-energy constraints from CHPT (chiral MAID)

→ . . .

[1] ANL-Osaka PRC 80(2009), Few-Body Syst. 59(2018),... [2] MAID2007, EPJA 34(2007) EtaMAID2018, EPJA 54(2018) [3] SAID, PiN Newsletter 16(2002) [4] Gent group PRC 89(2014),... Aznauryan et al., PRC 80(2009), IJMP(2013),...

TRANSITION AMPLITUDES: NEW APPROACH



Jülich-Bonn-Washington

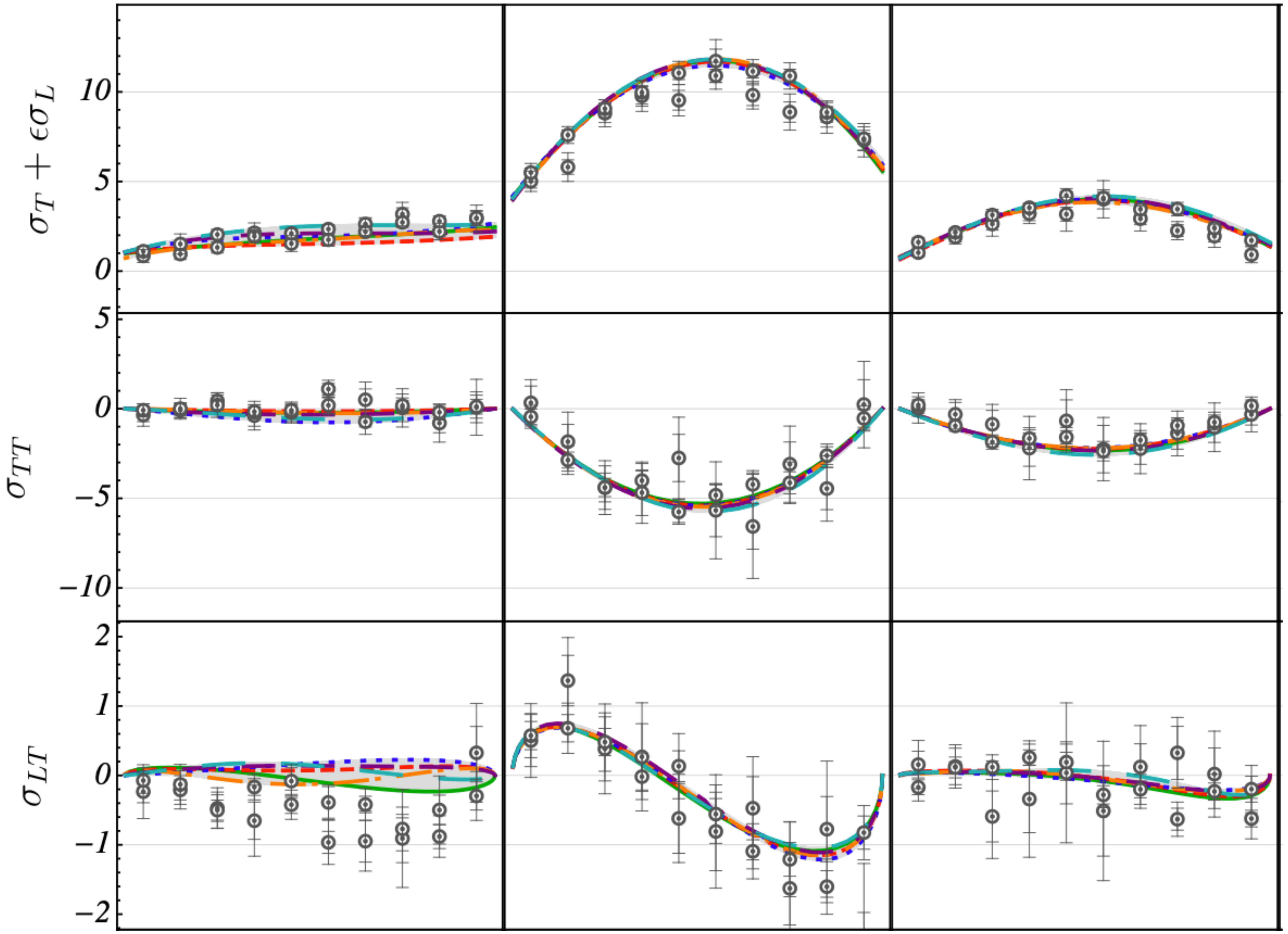
- coupled-channel approach (πN, ηN, $K\Lambda$, $K\Sigma$, ...)
- constraints from scattering data and theory:
  - → gauge invariance
  - →(pseudo)threshold behaviour
  - → Final-state unitarity
- **web-interface: https://jbw.phys.gwu.edu/**

More in
M. Döring's talk #2 [6 Jun 2023, 17:20 DAD/5L]

[1] ANL-Osaka PRC 80(2009), Few-Body Syst. 59(2018),... [2] MAID2007, EPJA 34(2007) EtaMAID2018, EPJA 54(2018) [3] SAID, PiN Newsletter 16(2002) [4] Gent group PRC 89(2014),... Aznauryan et al., PRC 80(2009), IJMP(2013),...

[5][JBW] MM et al. Phys.Rev.C 103 (2021) 6 / Phys.Rev.C 106 (2022) 015201

INTERPOLATIONS AND PREDICTIONS



Fits accomplished: π^0p/π^+n/ηp/(+$K\Lambda$ upcoming)[1]

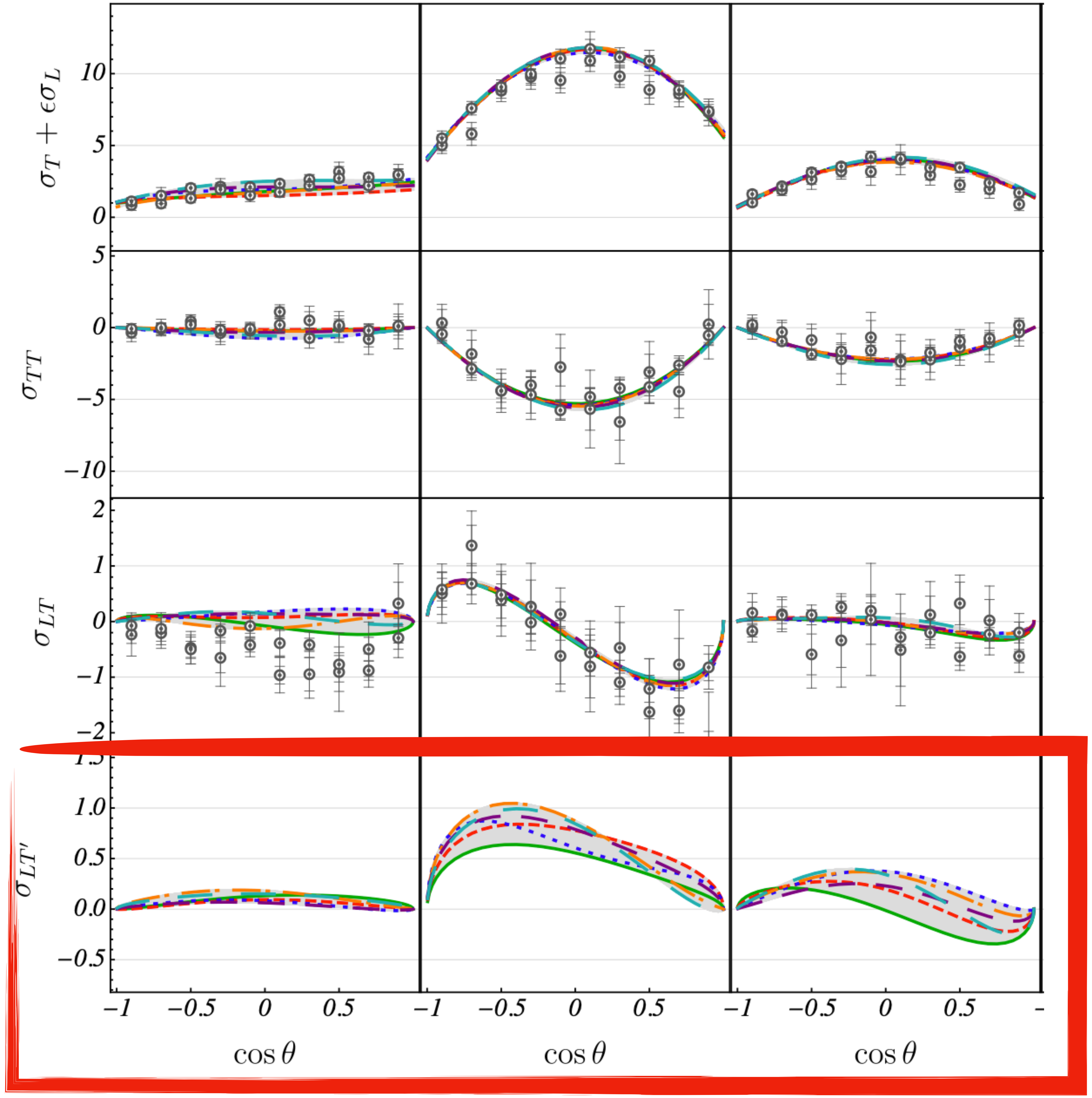
Example: Joo data[2]

- not measured quantities can be estimated
- **interpolator** over observable types and kinematics

[1] [JBW] MM et al. Phys.Rev.C 103 (2021) 6 / Phys.Rev.C 106 (2022) 015201

[2] Joo et al. [CLAS] PRC (2003), PRL (2002)

INTERPOLATIONS AND PREDICTIONS



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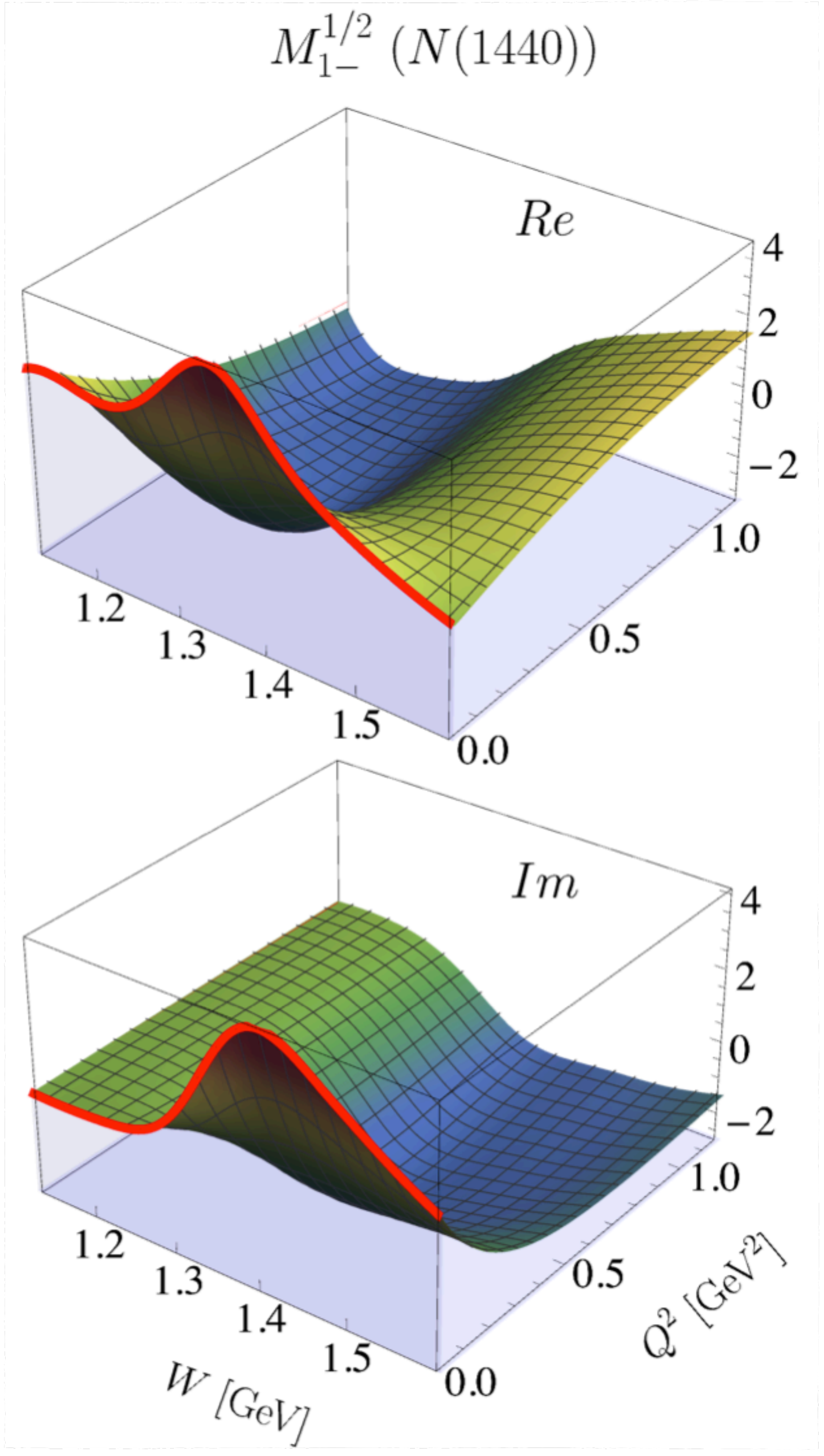
[1] [JBW] MM et al. Phys.Rev.C 103 (2021) 6 / Phys.Rev.C 106 (2022) 015201

[2] Joo et al. [CLAS] PRC (2003), PRL (2002)

INTERPOLATIONS AND PREDICTIONS

Example: Roper(1440)

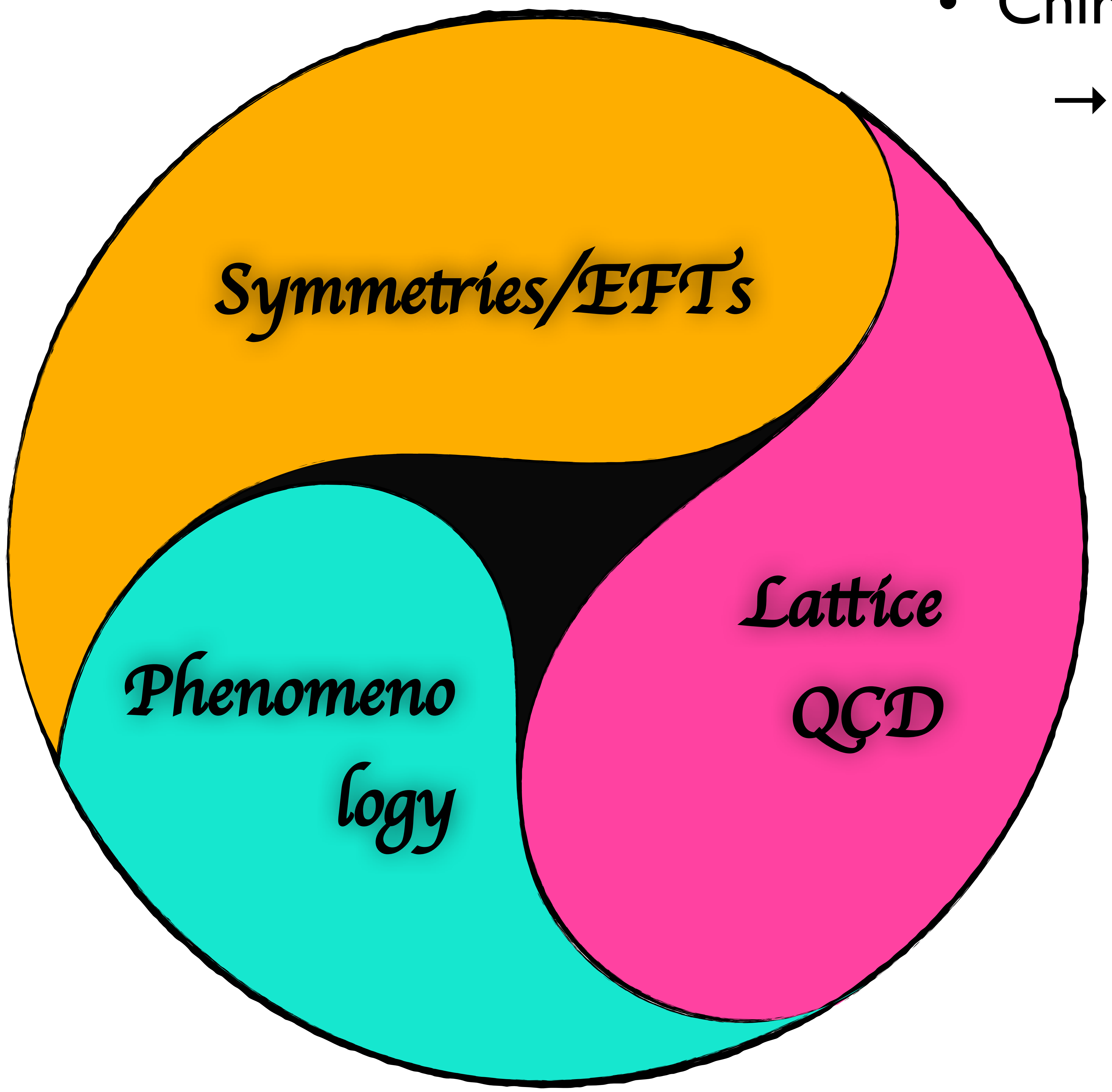
- Non-trivial Q^2 behaviour
  - → complex structure (**3-body effects, …)**
  - → Helicity couplings (upcoming)



[1] [JBW] MM et al. Phys.Rev.C 103 (2021) 6 / Phys.Rev.C 106 (2022) 015201

SUMMARY

New synergetic approaches to universal parameters of resonance become available



- **Chiral unitary models**
  - →QCD symmetries constraints to hadron-hadron dynamics
    - →Strong predictive power

- **Lattice hadron spectroscopy**
  - →Novel 3-body methodology has matured
  - →EFTs and S-matrix theory: bridge to real world physics

- **Phenomenological models**
  - →Link between observables and transition amplitudes

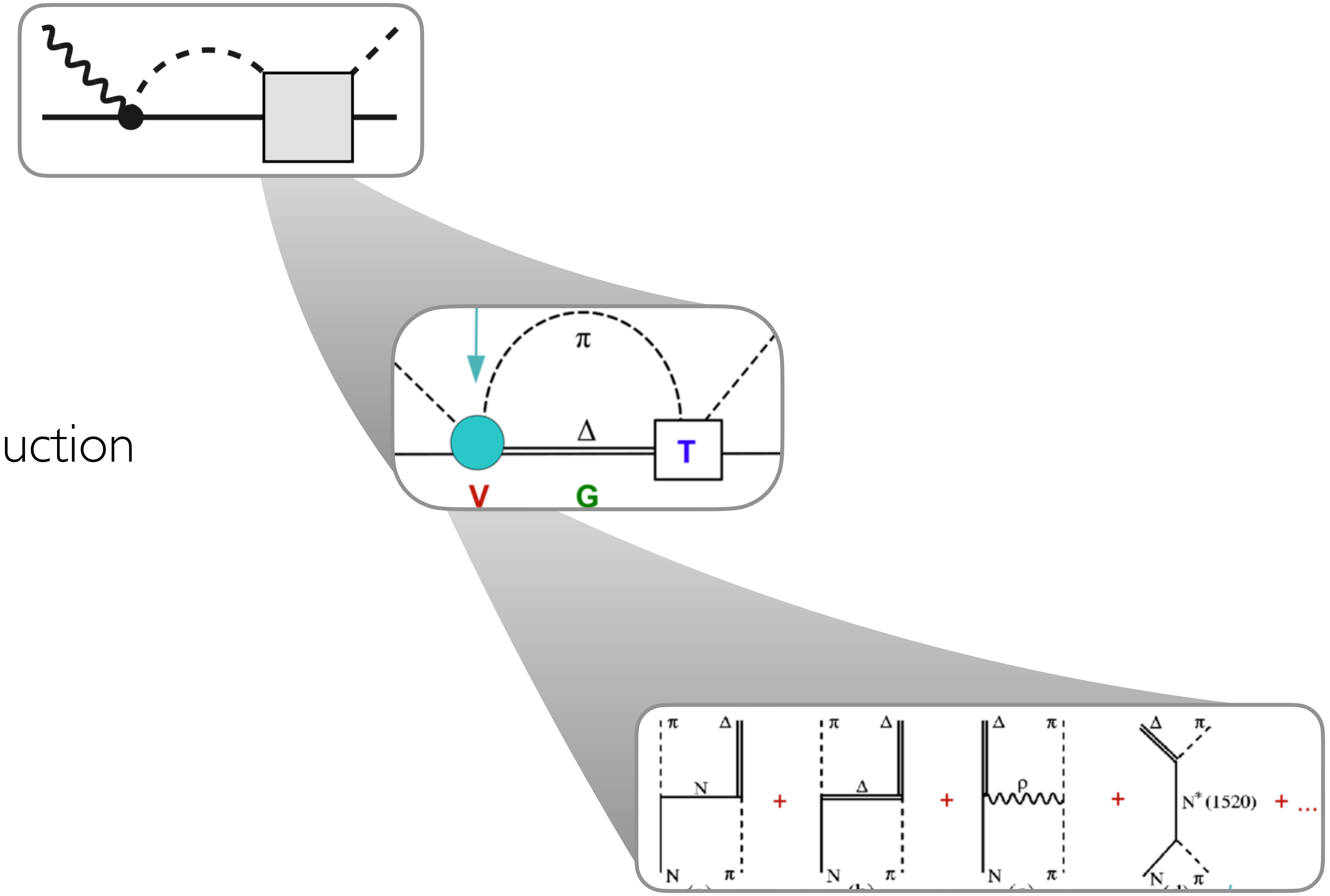
THANK YOU

TRANSITION AMPLITUDE

Final-state unitarity

- Jülich-Bonn dynamical coupled-channel model[1]
- Amplitudes fixed from scattering and photoproduction data

  πN→xX and γN→xX (~60k data)



[JBW] MM et al. Phys.Rev.C 103 (2021) 6 / Phys.Rev.C 106 (2022) 015201

SYMMETRIES

Five kinematical variables (3*(2+3)-10=5)

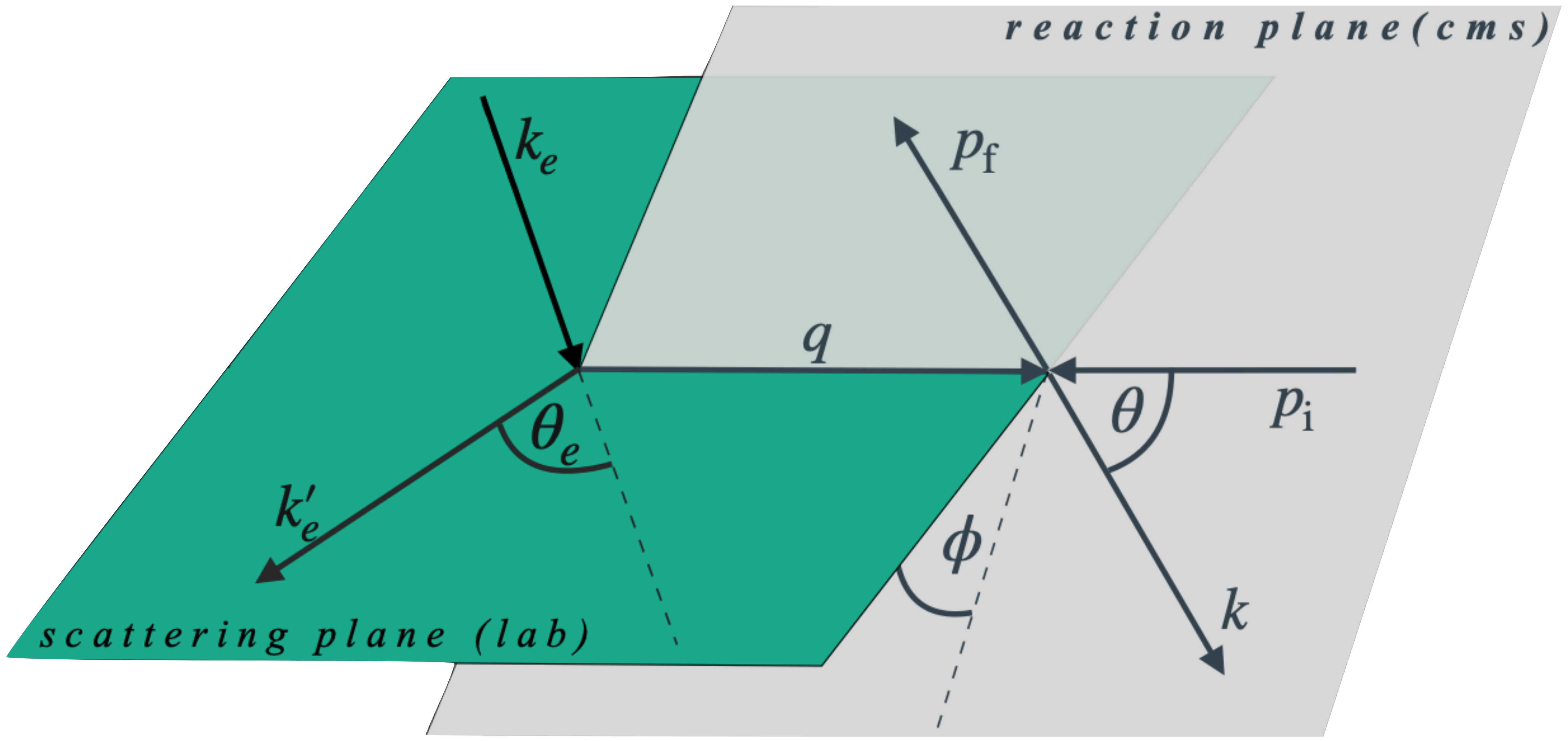
- total energy: **W**
- photon virtuality: $\mathbf{Q^2}$
- angles: θ_e, θ, φ

Underlying objects[1,2]:

- Helicity amplitudes:
- Multipoles:

$$\{H_i\left(W, Q^2, \theta\right) \mid i = 1..8\}$$

$$\mathcal{M}_{\ell\pm}(W, Q^2) \in \{E_{\ell\pm}(W, Q^2), L_{\ell\pm}(W, Q^2), M_{\ell\pm}(W, Q^2)\}$$



1) Chew et al. Phys.Rev. 106 (1957); Dennery Phys.Rev. 124 (1961); Berends et al. Nucl.Phys.B 4 (1967);
2) (for explicit formulas) MM et al. Phys.Rev.C 103 (2021)

THEORETICAL CONSTRAINTS

Gauge invariance

- manifest implementation[1] too costly
- instead Ward-Takahashi identity by construction

$$\longrightarrow \quad H_7 = \sum_{i=1}^{6} a_i H_i \quad H_8 = \sum_{i=1}^{6} b_i H_i$$

Pseudo/threshold constraints:

Siegert's theorem[2]

- Long-wavelength limit: electric and magnetic multipoles are related

... fewer parameters needed 👍

$$\longrightarrow \quad lim_{k\to 0} E_{\ell+} = k^{\ell} \ldots$$

$$\longrightarrow \quad L_{\ell\pm} \sim E_{\ell\pm} \quad \textbf{for} \quad q = 0$$

1) Afnan et al.(1995); Kvinikhidze et al.(1999); Haberzettl(19xx-2021); Borasoy et al. (2007); Ruic et al.(2011); MM et al. (2012); Bruns, Cieplý, MM 2206.08767 [nucl-th]

2) Siegert(1973) Amaldi et al.(1979) Tiator(2016)

MULTIPOLES

$$\mathscr{M}_{\mu\gamma^*}(W,Q^2) = R_{\ell'}(\lambda, q/q_\gamma)\left(V_{\mu\gamma^*}(W,Q^2) + \sum_\kappa \int_0^\infty dp\, p^2\, T^{\rm JUBO}_{\mu\kappa}(p,W) G_\kappa(p,W) V_{\kappa\gamma^*}(p,W,Q^2) \right)$$

$$V^{\rm JUBO}_{\mu\gamma}(W) e^{-\beta^0_\mu Q^2/m_p^2} \left(1 + Q^2/m_p^2 \beta^1_\mu + (Q^2/m_p^2)^2 \beta^2_\mu\right)$$

👍 Fulfils:

- Final state unitarity / Gauge invariance / Siegert's theorem / Threshold behaviour

👍 Describes:

- Scattering and photoproduction data -- parameters (λ, β) from fits to electroproduction data

🫣 Parametrization dependence due to incomplete data

... even for a truncated complete electroproduction experiment

... in future: Bias-variance tradeoff with statistical criteria

Tiator et al.(2017)

Landay et al., Phys.Rev.D (2019), 1810.00075 [nucl-th] (2019)

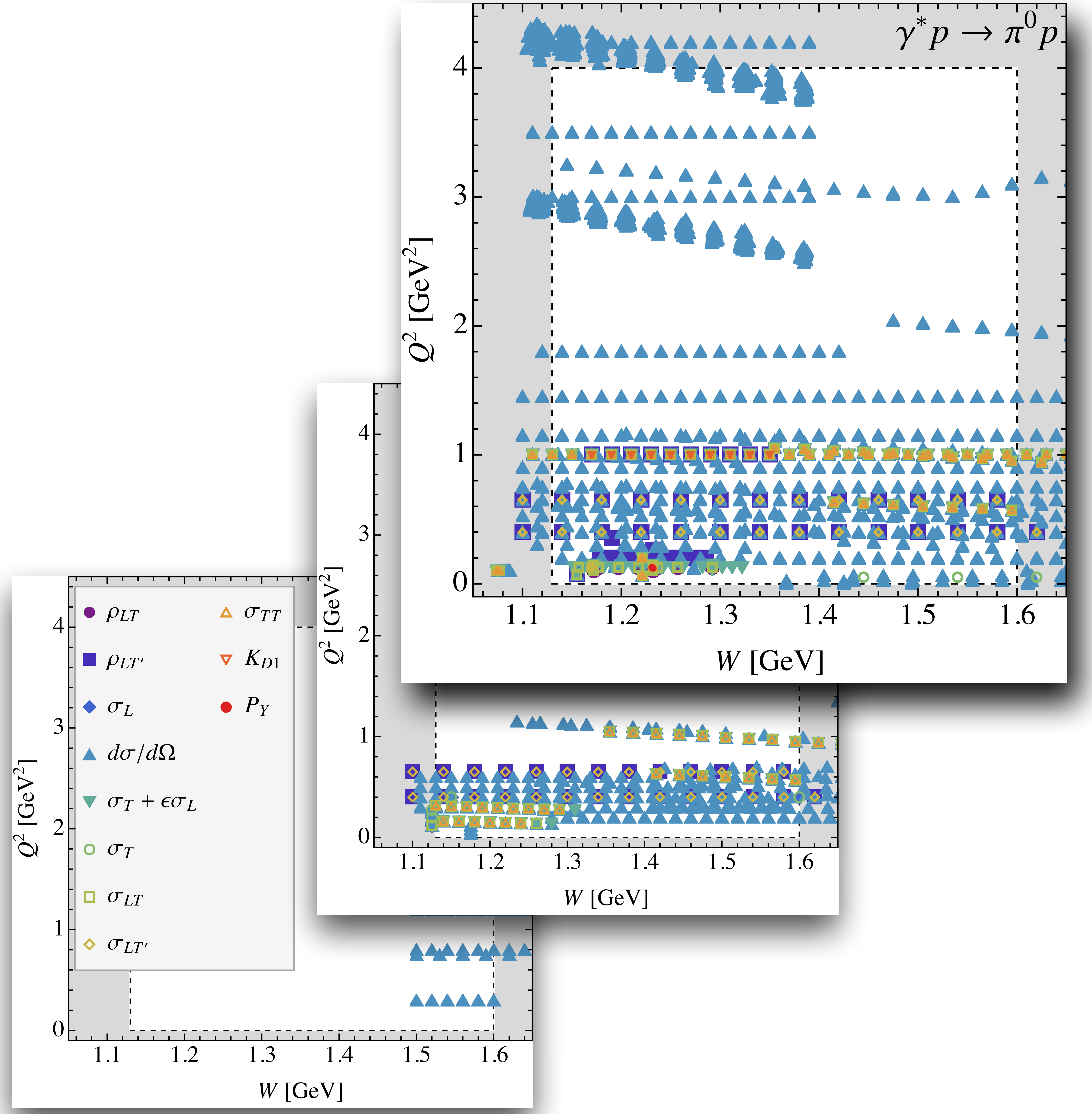
DATA AND PARAMETERS

Experimental data

- $45\text{k}(\pi^0 p) + 37\text{k}(\pi^+ n) + 2\text{k}(\eta p) = 84\text{k}$ data

Parameters $\{\lambda, \beta\}$

- S/P/D waves
- 26 multipoles × (10..13 pars) = 257 pars



1) [JBW] MM et al. Phys.Rev.C 103 (2021) 6; Phys.Rev.C 106 (2022) 015201

DATA AND PARAMETERS

Fits[1]:

- multiple solutions obtained
- systematic uncertainties studied

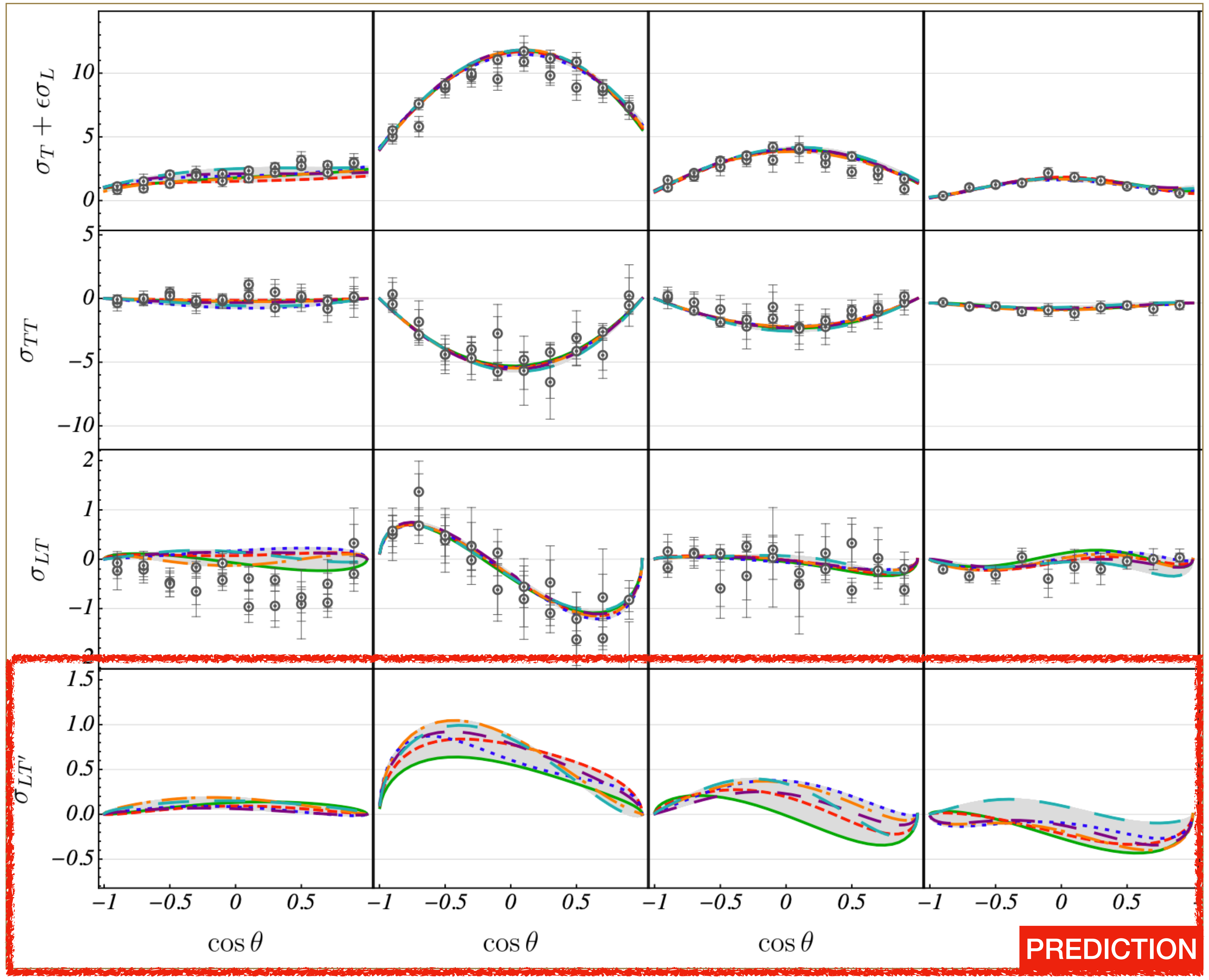
	χ^2/dof	$\chi^2_{\pi^0 p/\mathrm{data}}$	$\chi^2_{\pi^+ n/\mathrm{data}}$	$\chi^2_{\eta p/\mathrm{data}}$
$\mathfrak{F}_1^{\mathrm{reg}}$	1.66	1.68	1.61	1.77
$\mathfrak{F}_2^{\mathrm{reg}}$	1.73	1.71	1.71	2.29
$\mathfrak{F}_3^{\mathrm{reg}}$	1.69	1.69	1.66	1.89
$\mathfrak{F}_4^{\mathrm{reg}}$	1.69	1.7	1.64	2.05
$\mathfrak{F}_1^{\mathrm{wt}}$	1.54	1.74	1.63	1.25
$\mathfrak{F}_2^{\mathrm{wt}}$	1.63	1.82	1.79	1.27

1) [JBW] MM et al. Phys.Rev.C 103 (2021) 6; Phys.Rev.C 106 (2022) 015201

GLOBAL FEATURES

Predictive power

- example: Joo data[2]
- not measured quantities can be estimated:
  - **interpolator** over observable types and kinematics
  - already usable through web-interface



https://jbw.phys.gwu.edu/

1) [JBW] MM et al. Phys.Rev.C 103 (2021) 6; Phys.Rev.C 106 (2022) 015201

2) Joo et al. [CLAS] PRC (2003), PRL (2002)

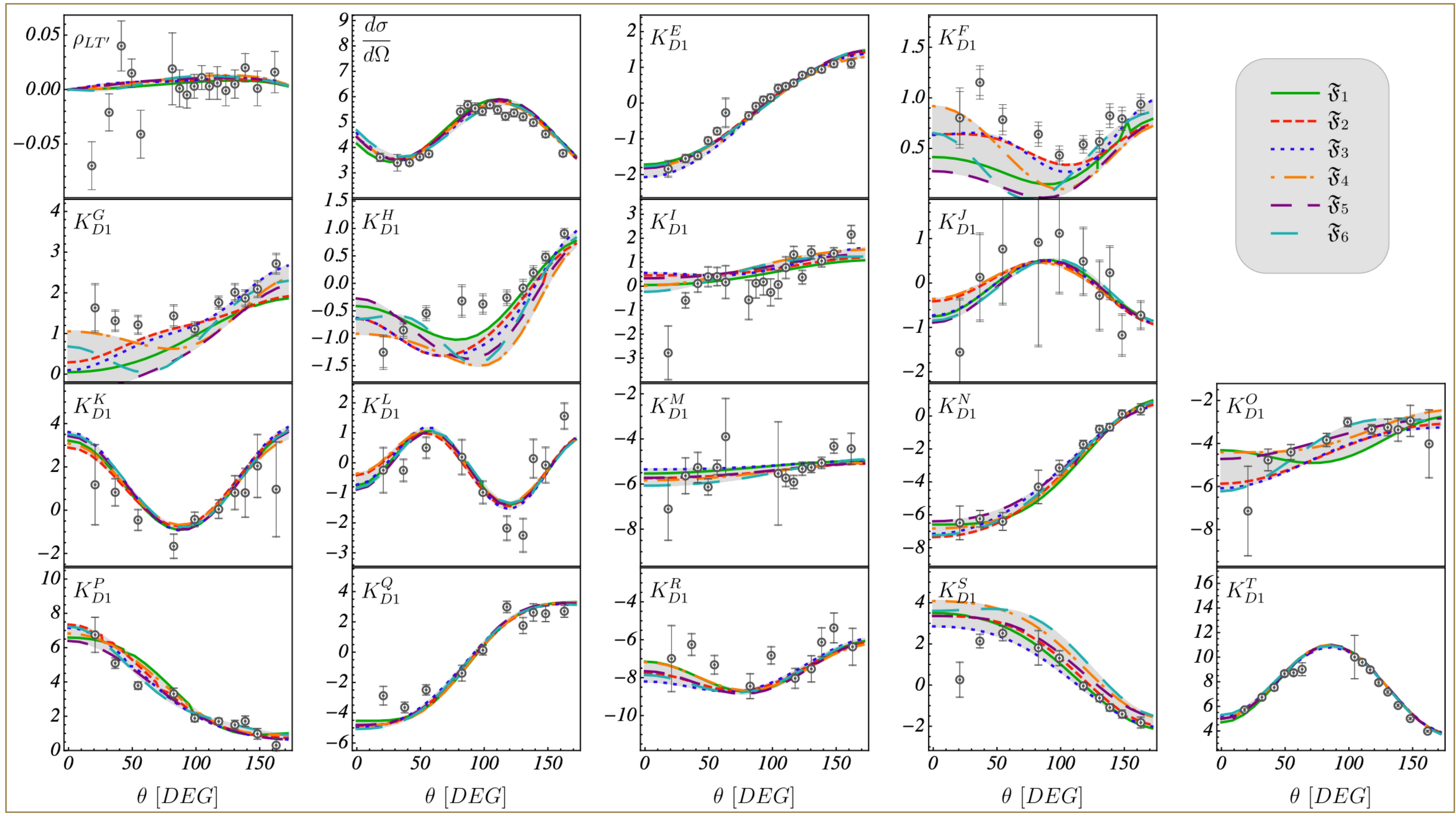
RESULTS

πN data fits[1]:

→all strategies converge

→different minima (systematic uncertainties)

→Kelly data[2]



1) [JBW] MM et al. Phys.Rev.C 103 (2021) 6; Phys.Rev.C 106 (2022) 015201

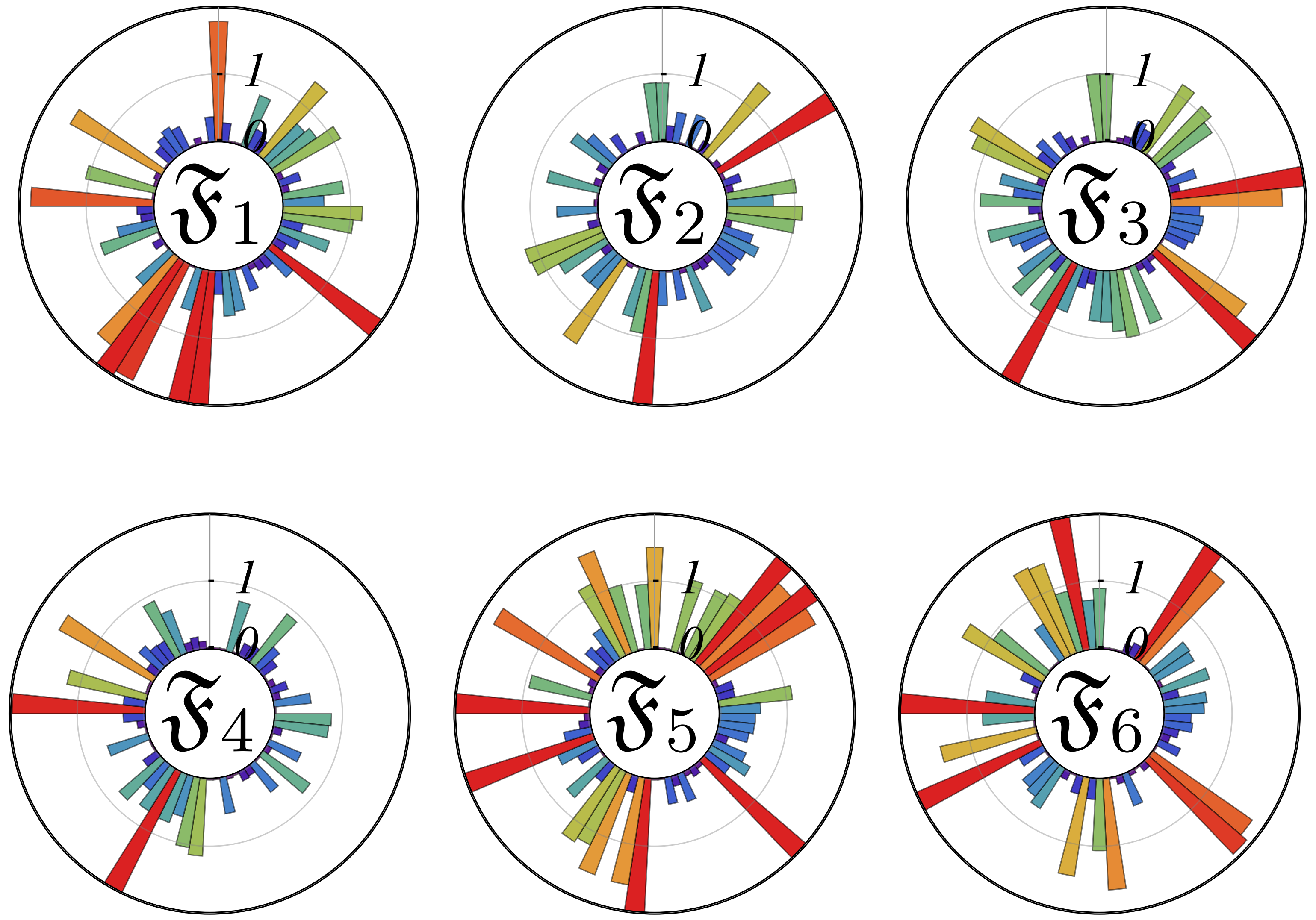
2) Jefferson Lab Hall A Collaboration Phys.Rev.Lett. 95 (2005) 102001

RESULTS

Fit	σ_L		$d\sigma/d\Omega$		$\sigma_T+\epsilon\sigma_L$		σ_T		σ_{LT}		$\sigma_{LT'}$		σ_{TT}		K_{D1}		P_Y		ρ_{LT}		$\rho_{LT'}$		$\chi^2_{\rm dof}$
	π^0p	π^+n	π^0p	π^+n	π^0p	π^+n	π^0p	π^+n	π^0p	π^+n	π^0p	π^+n	π^0p	π^+n	π^0p	π^+n	π^0p	π^+n	π^0p	π^+n	π^0p	π^+n	
$\mathfrak{F}_1$	–	9	65355	53229	870	418	87	88	1212	133	862	762	4400	251	4493	–	234	–	525	–	3300	10294	**1.77**
$\mathfrak{F}_2$	–	4	69472	55889	1081	619	65	78	1780	150	1225	822	4274	237	4518	–	325	–	590	–	3545	10629	**1.69**
$\mathfrak{F}_3$	–	8	66981	54979	568	388	84	95	1863	181	1201	437	3934	339	4296	–	686	–	687	–	3556	9377	**1.81**
$\mathfrak{F}_4$	–	22	63113	52616	562	378	153	107	1270	146	1198	1015	4385	218	5929	–	699	–	604	–	3548	11028	**1.78**
$\mathfrak{F}_5$	–	20	65724	53340	536	528	125	81	1507	219	1075	756	4134	230	5236	–	692	–	554	–	3580	11254	**1.81**
$\mathfrak{F}_6$	–	18	71982	58434	1075	501	29	68	1353	135	1600	1810	3935	291	5364	–	421	–	587	–	3932	11475	**1.78**

Fits[1]:

- →all strategies converge
- →different minima (systematic uncertainties)



1) [JBW] MM et al. Phys.Rev.C 103 (2021) 6; Phys.Rev.C 106 (2022) 015201

MULTIPOLES

Observable (e.g. cross section) $\dfrac{d\sigma^v}{d\Omega}\left(W, Q^2, \epsilon, \theta, \phi\right) = \sigma_T + \epsilon\sigma_L + \sqrt{2\epsilon(1+\epsilon)}\sigma_{LT}\cos\phi + \ldots$

Structure functions $\sigma_T\left(W, Q^2, \theta\right) = k/q_\gamma \left(|H_1|^2 + |H_2|^2 + |H_3|^2 + |H_4|^2 \right)/2, \ldots$

Helicity amplitudes $H_1\left(W, Q^2, \theta\right) = \sin\theta\cos\theta/2(-\mathscr{F}_3 - \mathscr{F}_4)/\sqrt{2}, \ldots$

CGLN amplitudes $\mathscr{F}_1\left(W, Q^2, \theta\right) = \sum_{\ell \geq 0} \ell M_{\ell+}(W, Q^2) P'_{\ell+1}(\cos\theta) + \ldots$

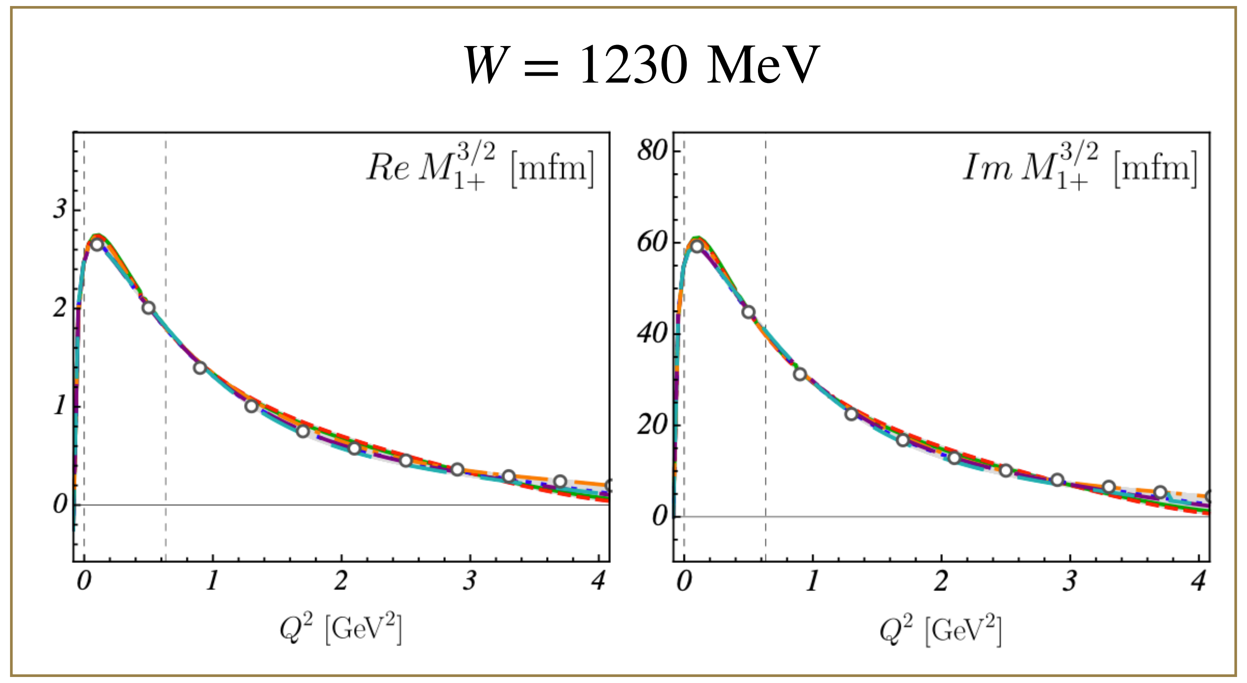
Multipoles $\{E_{\ell\pm}(W, Q^2), L_{\ell\pm}(W, Q^2), M_{\ell\pm}(W, Q^2)\}$

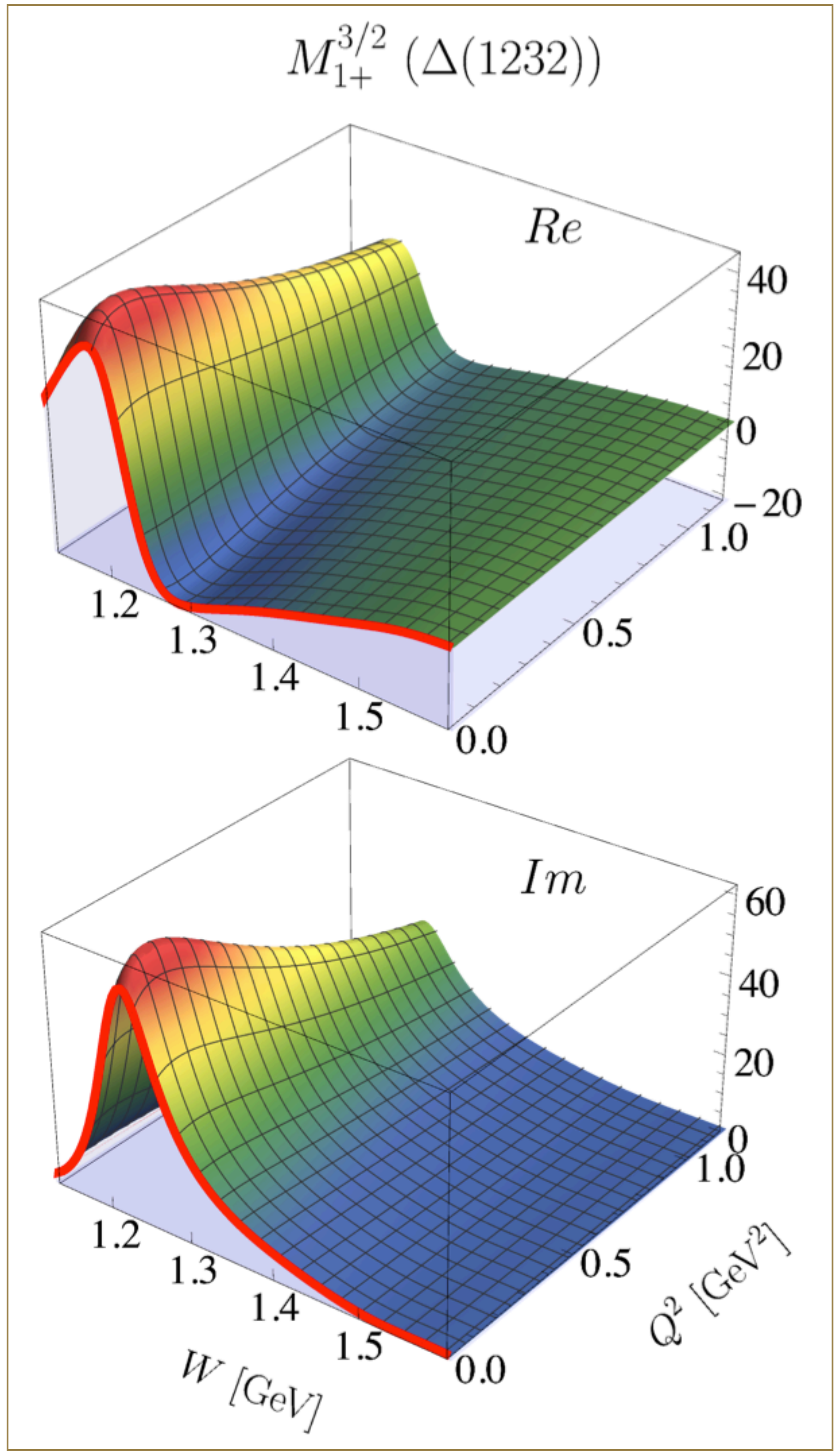
Chew et al. Phys.Rev. 106 (1957); Dennery Phys.Rev. 124 (1961); Berends et al. Nucl.Phys.B 4 (1967); ... (for explicit formulas) MM et al. Phys.Rev.C 103 (2021)

RESULTS

Delta(1232):

- Large multipoles well determined
- simple Q^2 dependence



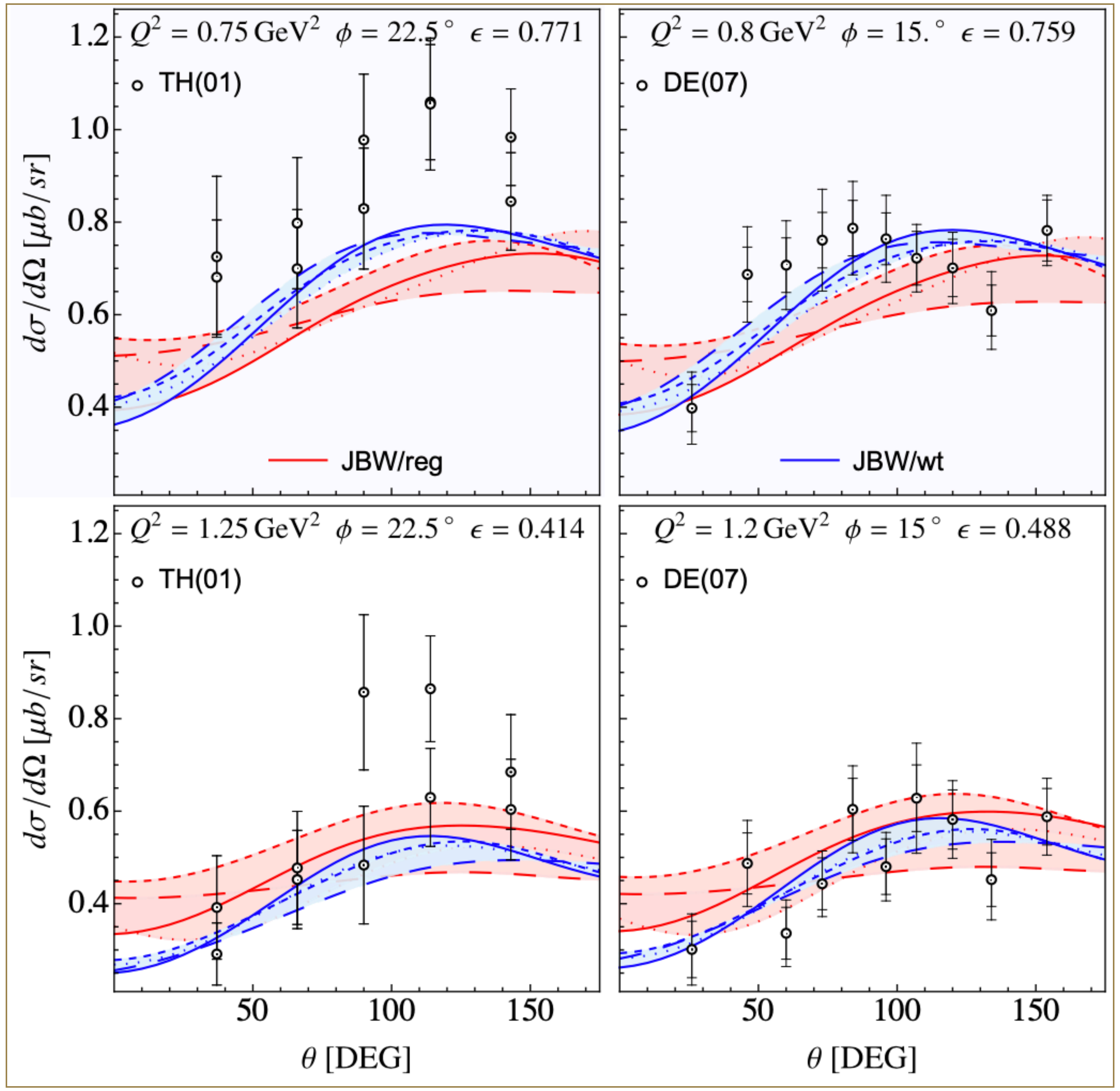


[JBW] MM et al. Phys.Rev.C 103 (2021) 6; Phys.Rev.C 106 (2022) 015201

RESULTS

Ambiguities in the data:

- example ηN data[2]



1) [JBW] MM et al. Phys.Rev.C 103 (2021) 6; Phys.Rev.C 106 (2022) 015201

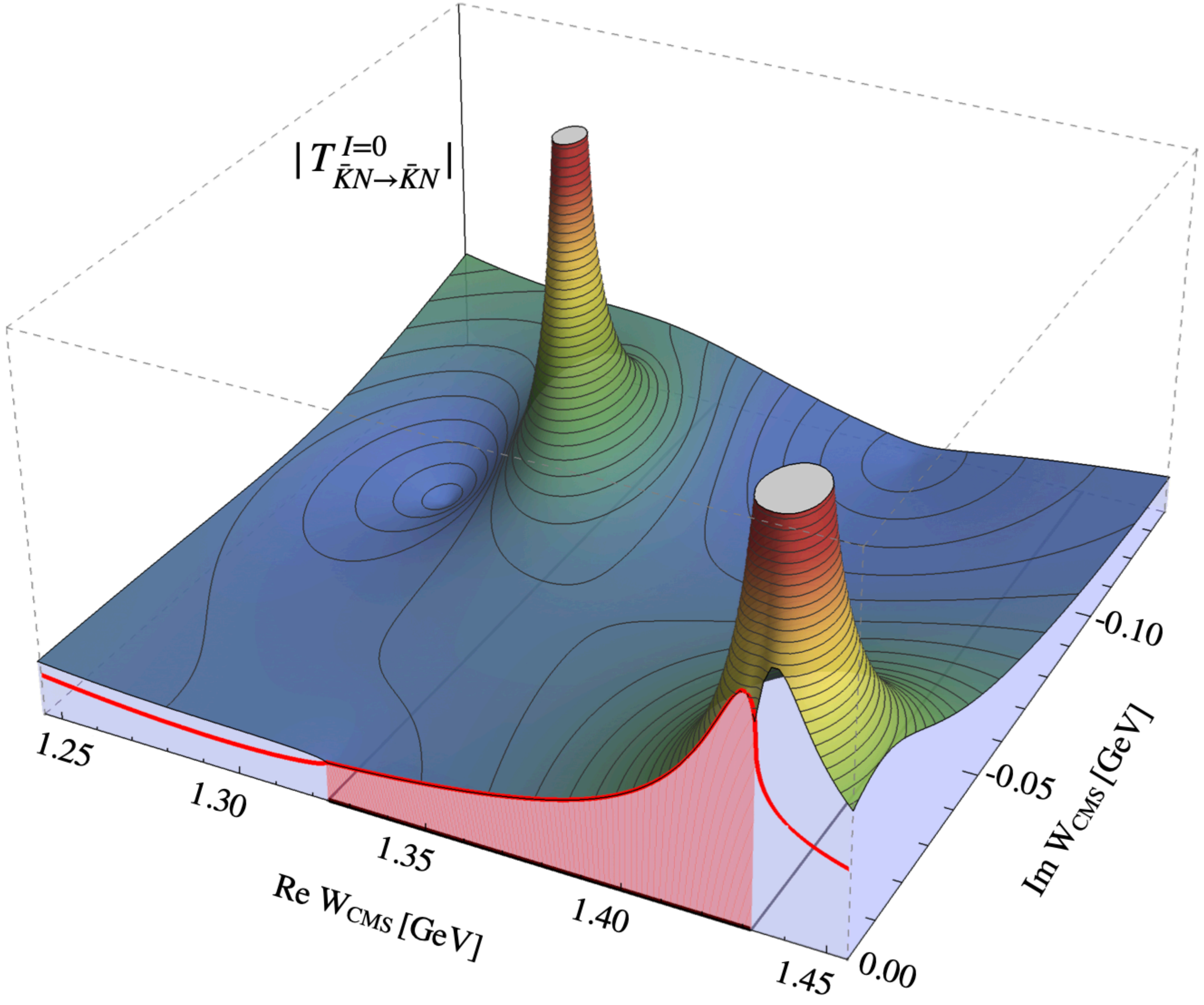
2) H. Denizli et al. (CLAS) PRC 76, 015204 (2007); Thompson et al. (CLAS), PRL86, 1702-1706 (2001); ...

RESONANCE POLE(S)

- Inclusion of chiral symmetry constants demands a second state[1]:

  W= (1325…1381)-i(56…114) MeV*

→ Common phenomenon in hadron physics[2]



1)Oller/Meißner (2001); Ikeda/Hyodo/Weise(2011); MM/Meißner(2012); Guo/Oller(2012),…

2)Meißner, Symmetry 12 (2020) 6, 981

HADRONIC 3-BODY PROBLEM: IMPACT

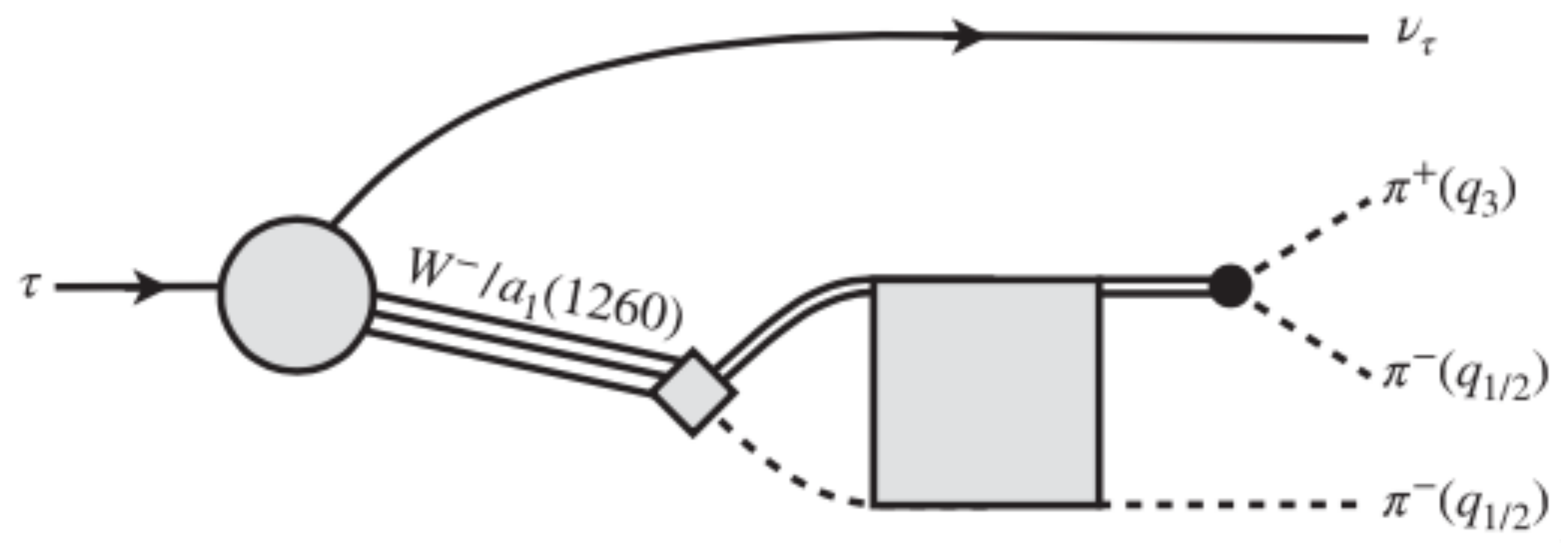
Intricate kinematics/dynamics

- 8 variables
- 2-body sub-channel dynamics

Hadron spectroscopy riddles

- Roper(1440) → **$\pi\pi$N** [first FV evaluations[1]]
- X(3872) → **DbarDπ**
- a_1(1260) → **$\pi\pi\pi$**
- …

- Beyond Standard Model: τ-EDM



- Precision physics: rare hadronic W-decays[2]
- Exotic states of matter[3]

1) Severt/MM/Meißner JHEP04(2023) >>> PHD talk on Friday
2) Sirunyan et al. [CMS@CERN] PRL122
3) Experimental programs: GlueX@JLAB; COMPASS@CERN;

BROADER IMPACT

Twice non-perturbative regime of QCD

- too low for perturbative QCD
- too high for low-energy EFT

KbarNN & KbarNNN bound states

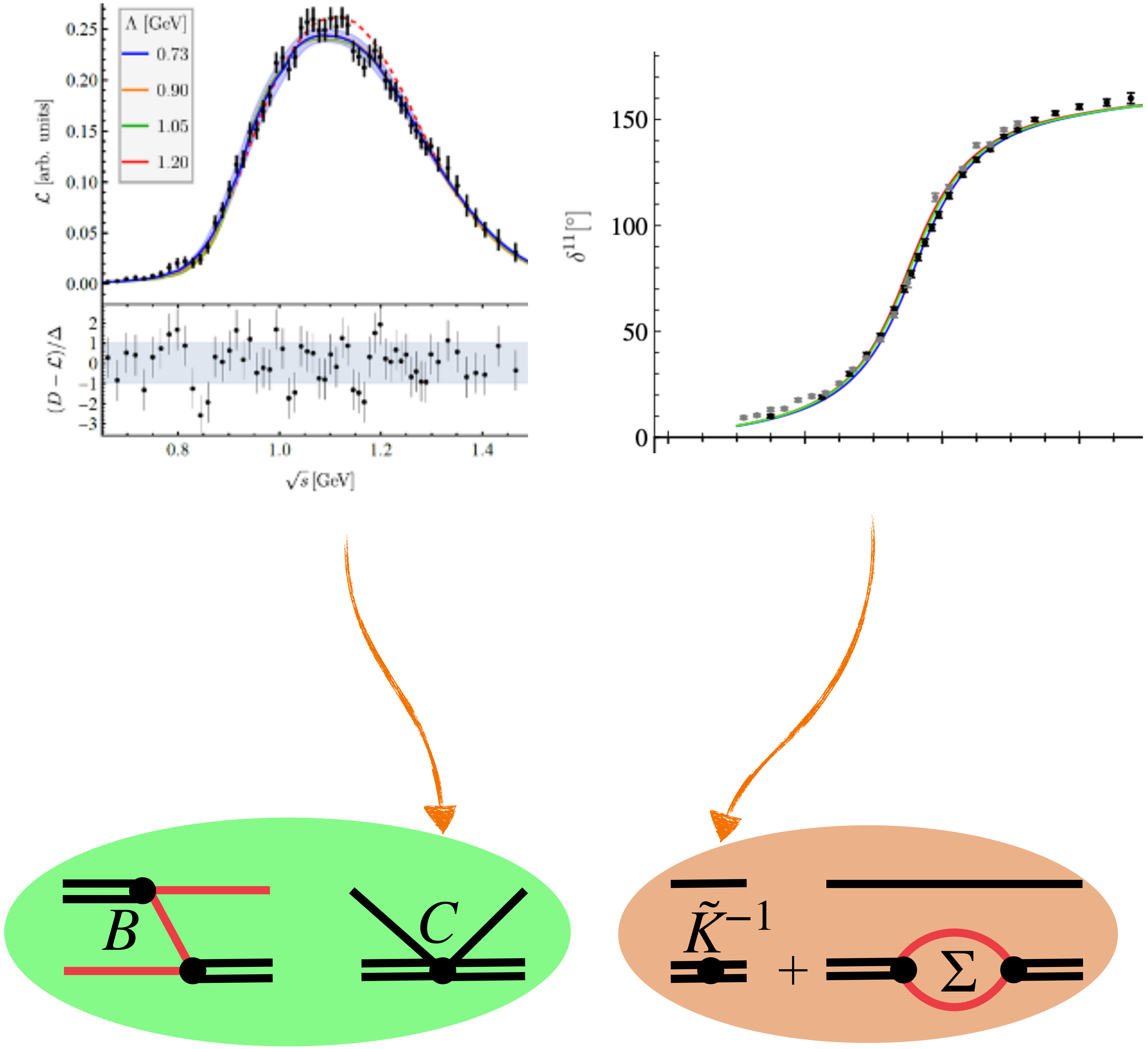
- dominated by KbarN interaction
- KbarN input is critical for interpretation

Antikaons in nuclear medium

- Strangeness in the EoS of neutron stars
- *K*-condensate can change EoS-stiffness

a_1(1260) PHENOMENOLOGY

- Fix quantum numbers to **a_1(1260) → $\pi\pi\pi$**
- solution via complex spectator momentum
- unknown parameter from fits[1] to data

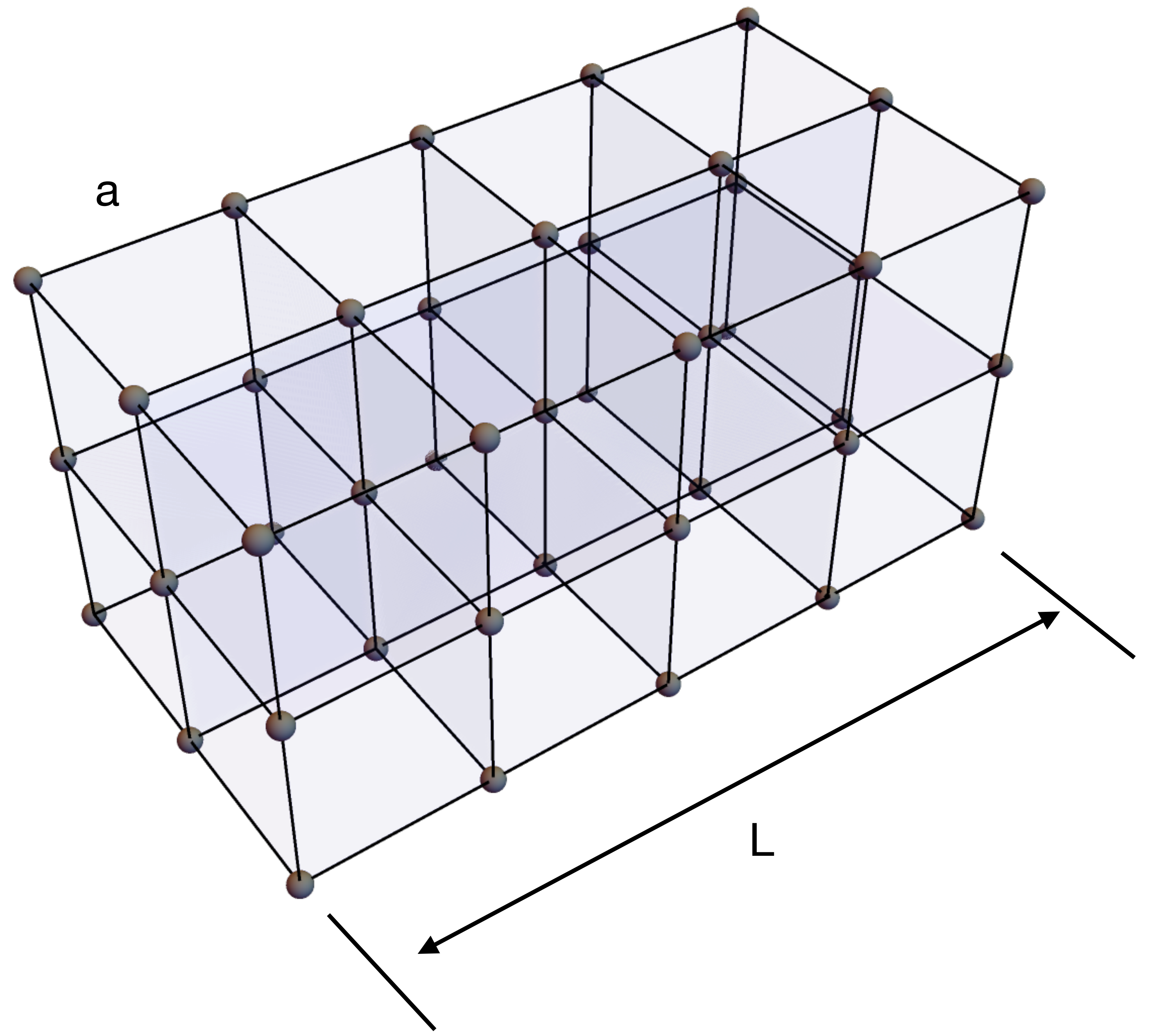


1) Sadasivan/MM/Döring/Alexandru/Culver/Lee Phys.Rev.D 101 (2020)

Data: Schael [ALEPH] Phys.Rept. 421 (2005); Estabrooks et al. Nucl.Phys.B 79; Protopopescu et al. Phys.Rev.D 7;

LATTICE HADRON SPECTROSCOPY

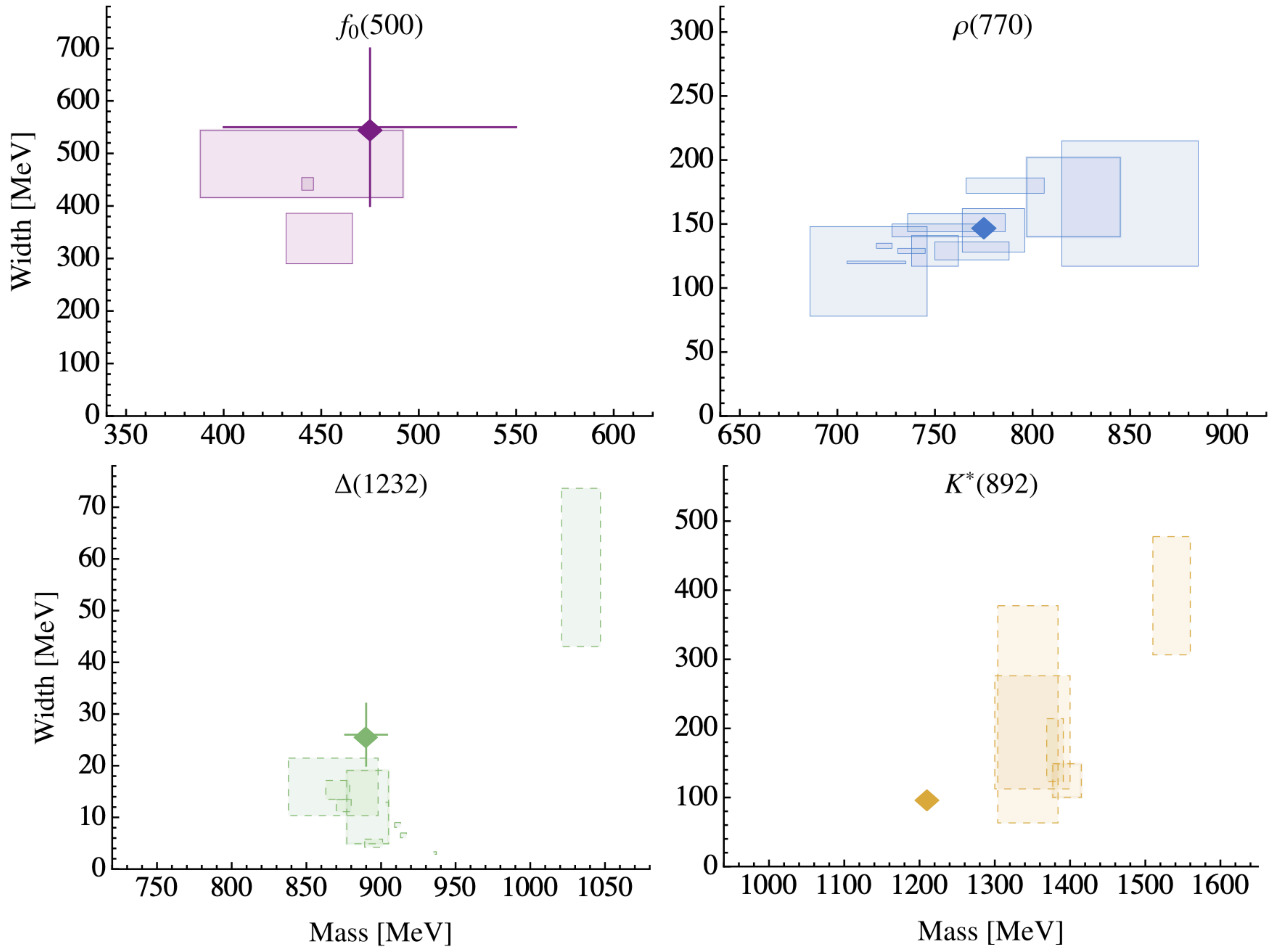
$$\mathcal{L}_{QCD} = \sum \bar{q}_f^a \left(i \not{D}_{ab} - m_f \delta_{ab} \right) q_f^b - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$$



- Numerical evaluation of QCD Green's functions

LATTICE HADRON SPECTROSCOPY

- Many studies of 2-body systems[1]



MM/Meißner/Urbach 2206.01477 Phys.Rept. 1001 (2023)

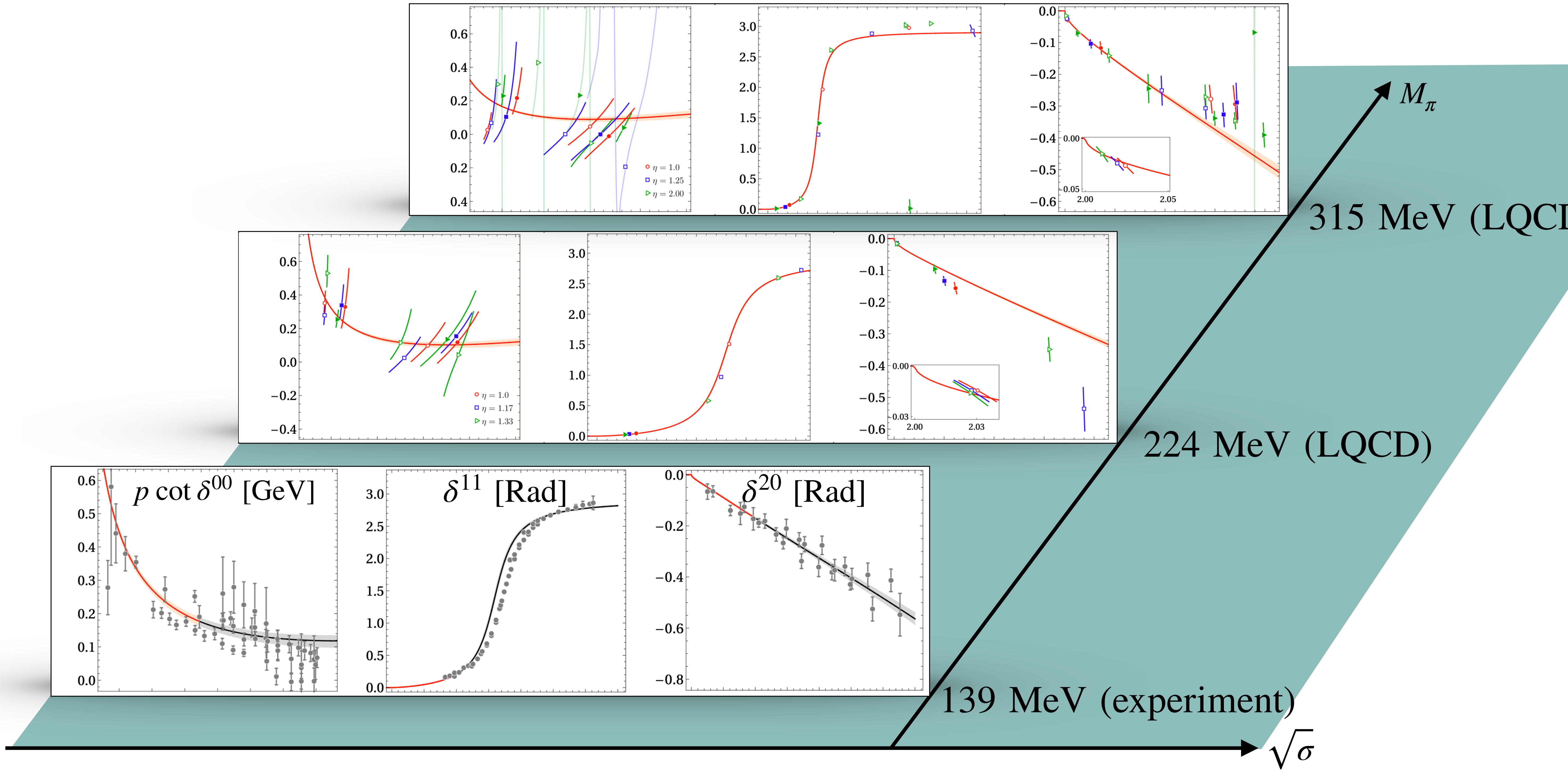
1)[NPLQCD], [RQCD], [ETMC], [HadSpec], ...

2)Reviews: Briceño/Dudek/Young Rev.Mod.Phys. 90 (2018); MM/Meißner/Urbach Phys.Rept. 1001 (2023)

LATTICE HADRON SPECTROSCOPY

- Experimentally inaccessible scenarios:
  - → Unconventional quantum numbers
  - → Three-body scattering
  - → Unphysical pion mass (chiral trajectories)

  …



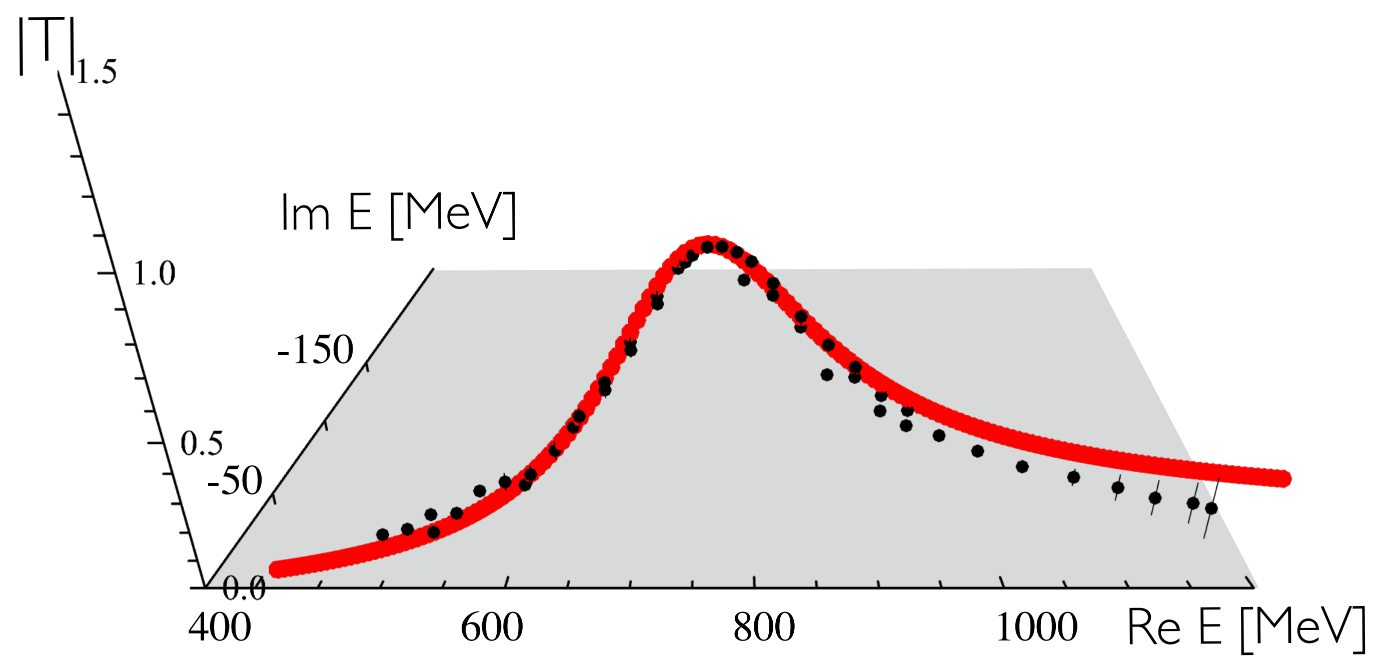
MM/Culver/Brett/Alexandru/Döring/Lee Phys.Rev.D

Review: MM/Döring/Rusetsky EPJ ST (2021)

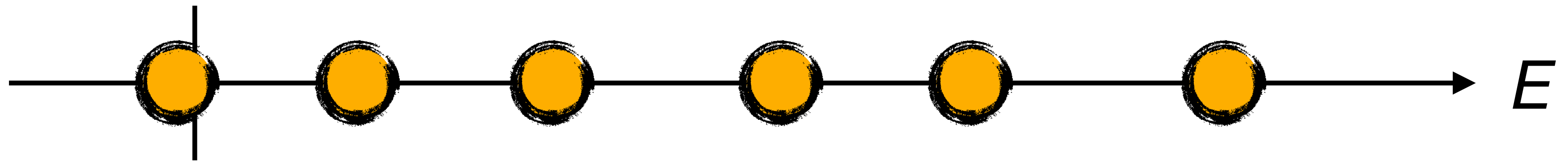
1)[NPLQCD], [RQCD], [ETMC], [HadSpec], …

2)Reviews: Briceño/Dudek/Young Rev.Mod.Phys. 90 (2018); MM/Meißner/Urbach Phys.Rept. 1001 (2023)

HADRONS IN A BOX



🤩 *A unitary "T-matrix" accounts for all O(mL) effects!*



🤗 Heavily simplified:

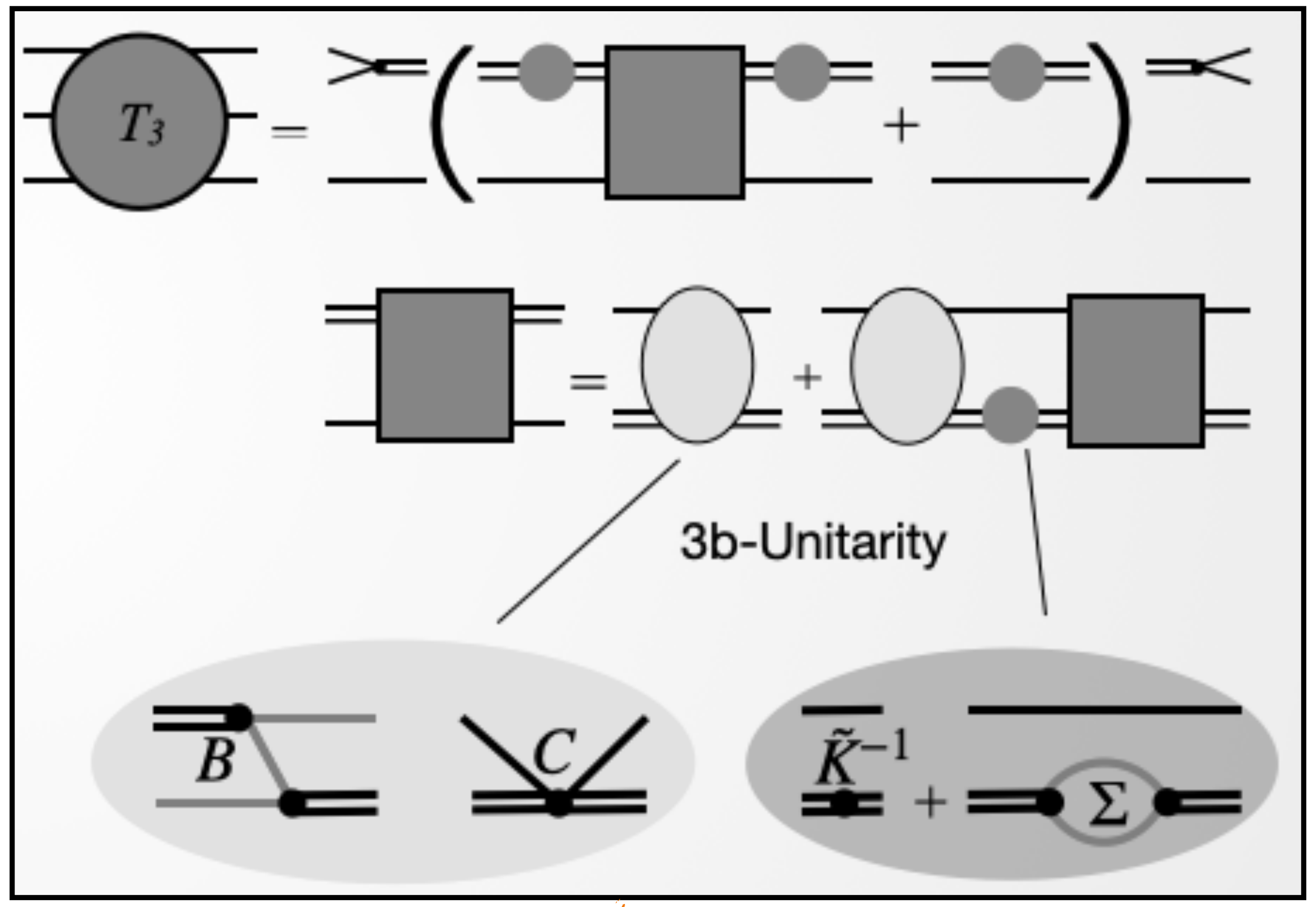
on-shell particle-configurations: $\boldsymbol{\Delta} E \sim mL$

off-shell particle-configurations: $\boldsymbol{\Delta} E \sim e^{-mL}$

1) Lüscher, Gottlieb, Rummukainen, Feng, Li, Döring, Briceño, Meißner, Rusetsky, Hansen, MM, Blanton, ...

2) Reviews: Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Döring/Rusetsky Eur.Phys.J.ST 230 (2021);

3-BODY QUANTISATION CONDITION



Reminder

Finite-volume unitarity (FVU)[1,2]

- separates volume dependent terms
- volume independent terms connect infinite/finite-volume spectra

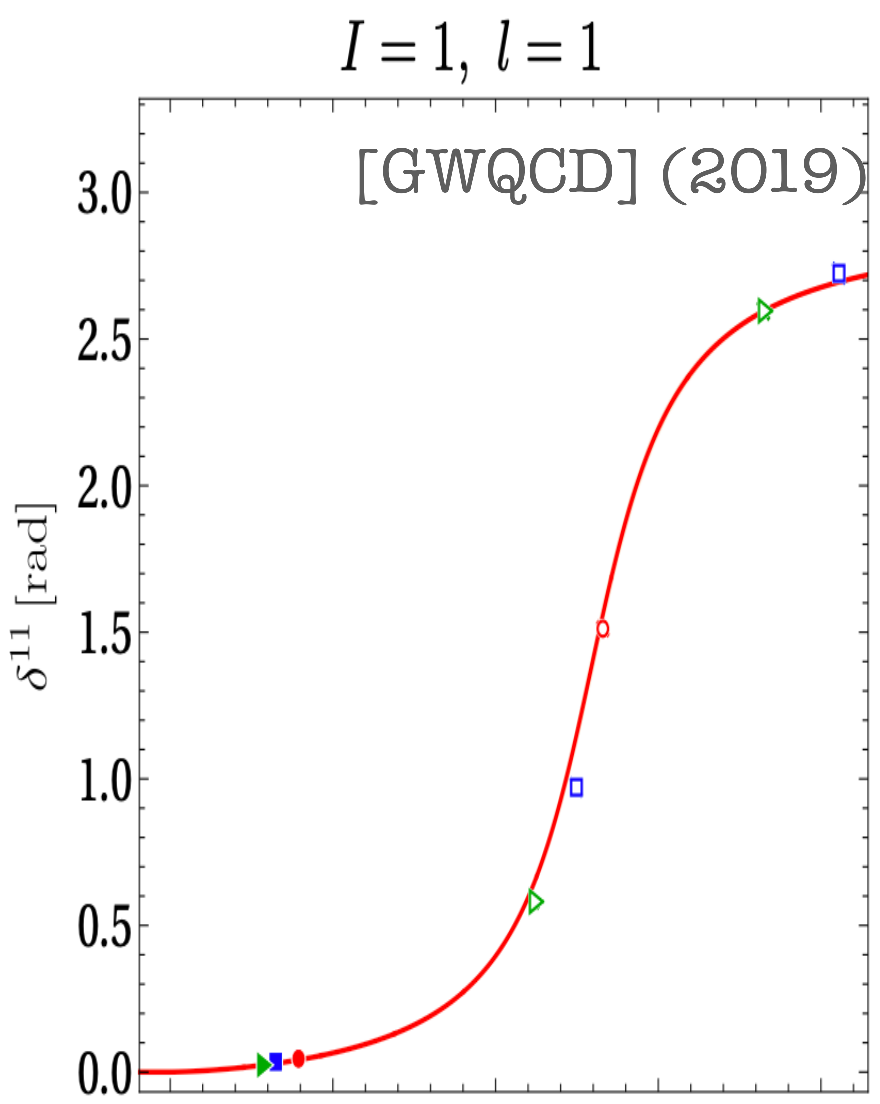
$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p}'\mathbf{p}}$$

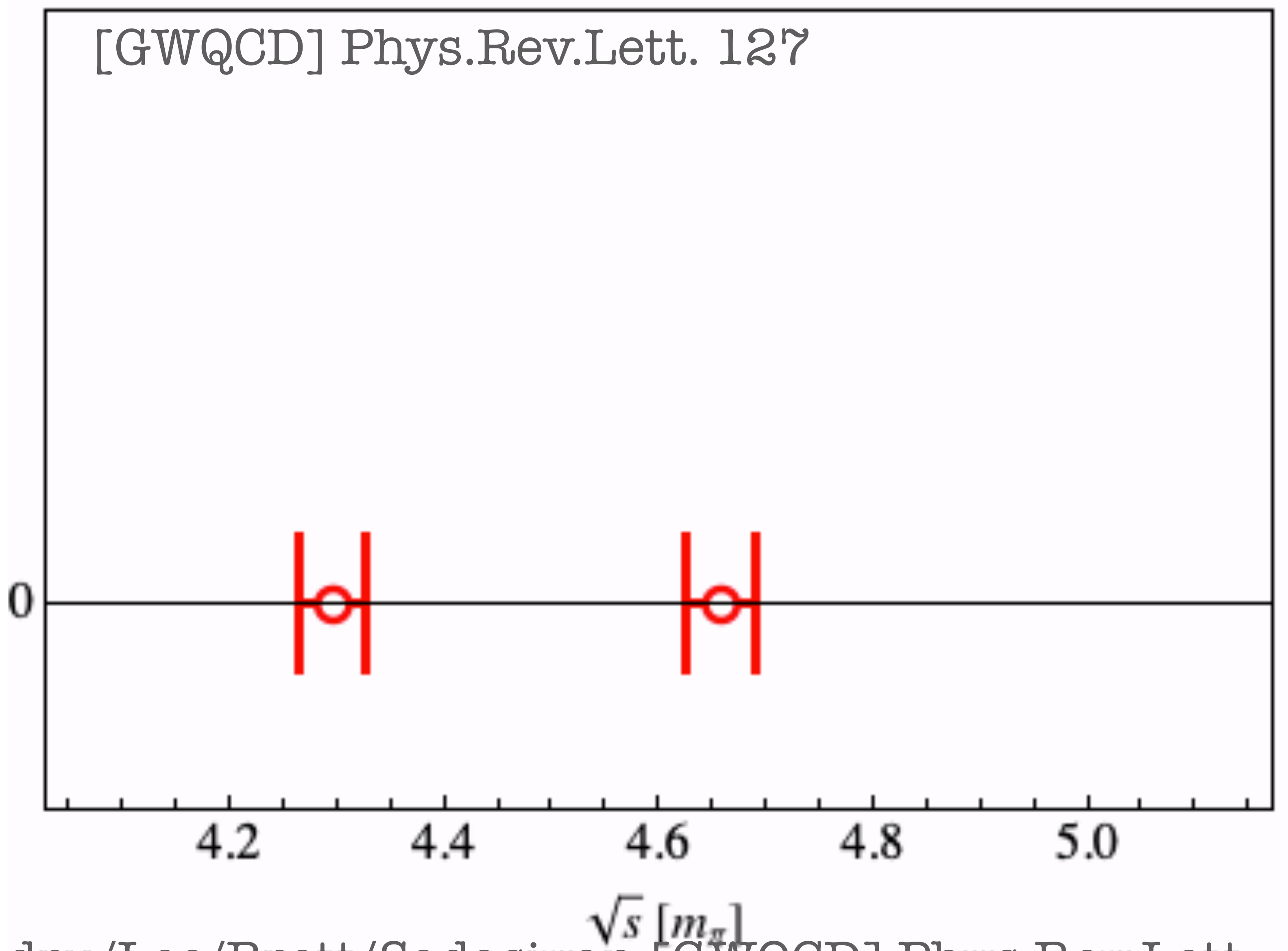
1) Lüscher, Gottlieb, Rummukainen, Feng, Li, Döring, Briceño, Meißner, Rusetsky, Hansen, MM, Blanton, ...

2) Reviews: Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Döring/Rusetsky Eur.Phys.J.ST 230 (2021);

Heavier Universe"

- First LQCD calculation[1] of a resonant 3b system
  - $N_f = 2$ dynamical fermions
  - LapH smearing
  - $\boldsymbol{P}=(0,0,0)$
  - $m_\pi=\mathbf{224}$ MeV, $m_\pi L=3.3$
  - *GEVP with one-/two-/three-meson operators*



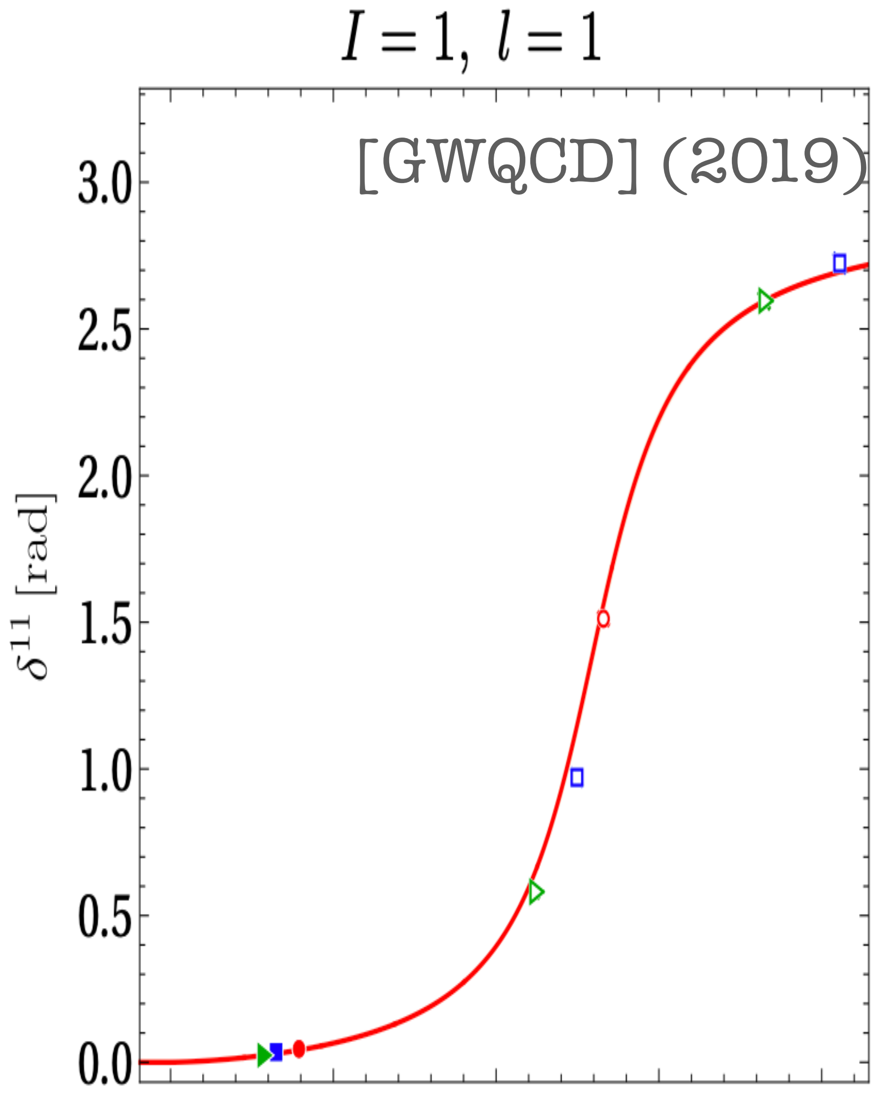


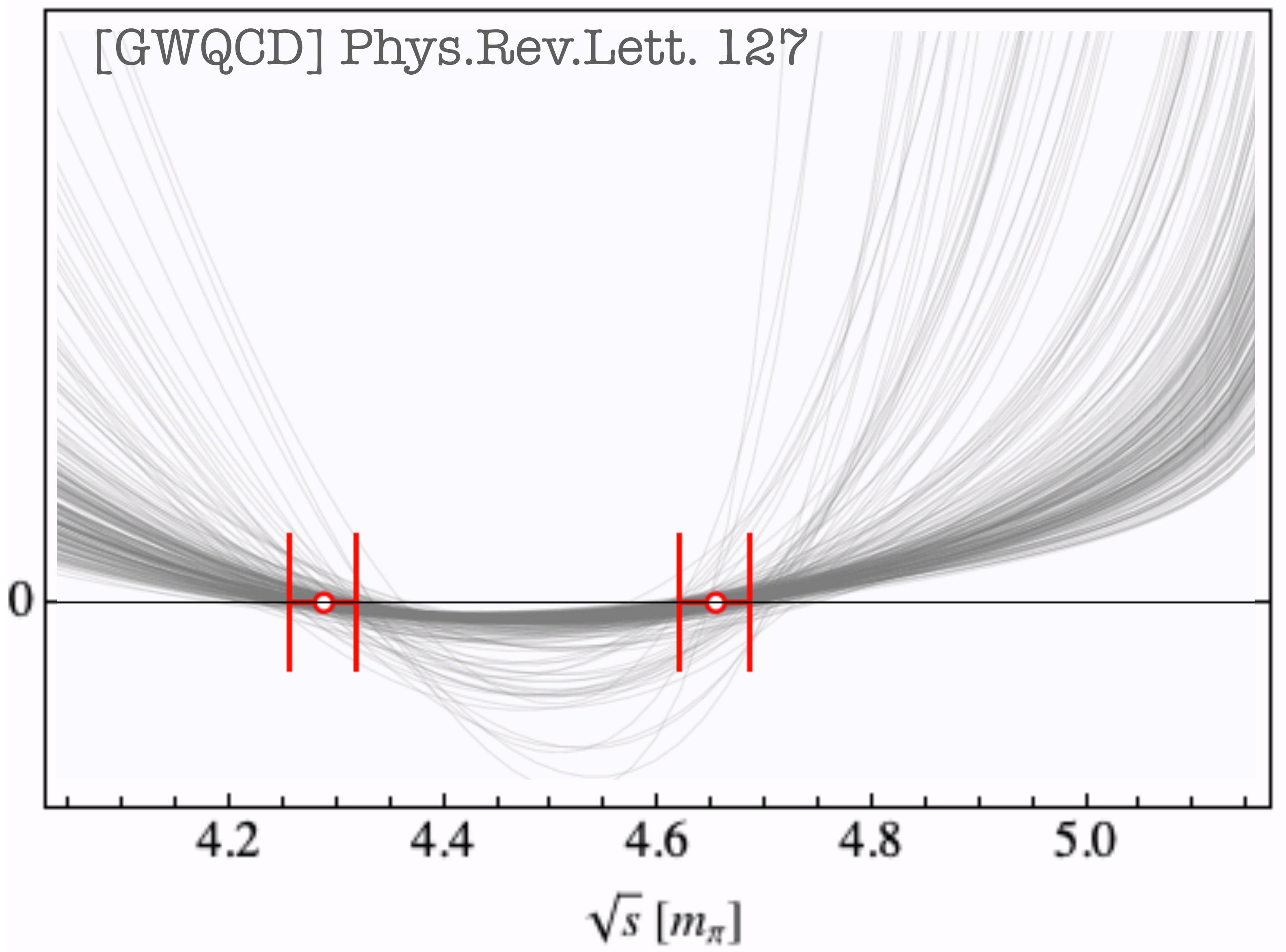
1) MM/Culver/Döring/Alexandru/Lee/Brett/Sadasivan [GWQCD] Phys.Rev.Lett. 127

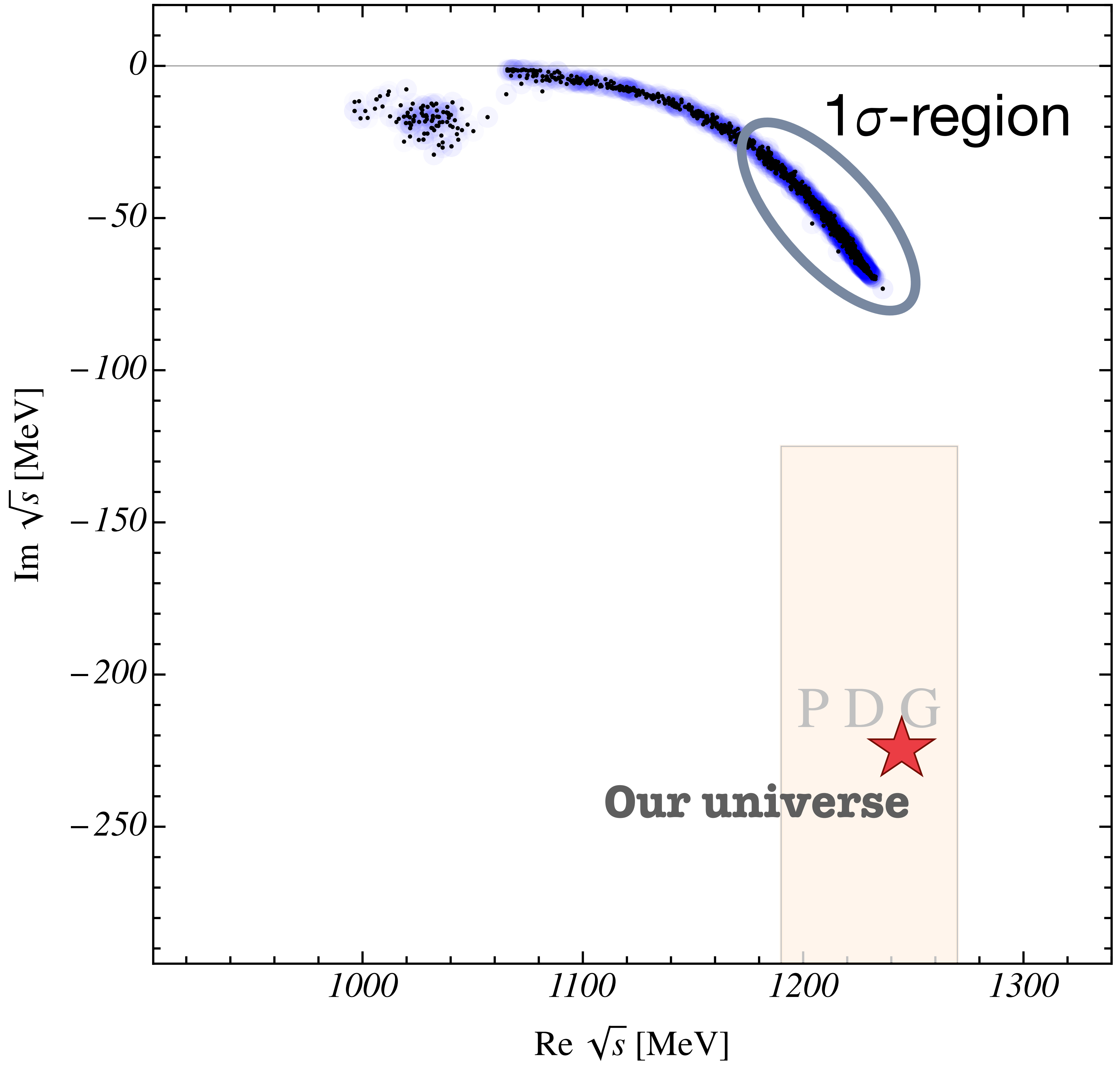
Heavier Universe"

$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p}'\mathbf{p}}$$

$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3}\frac{(B+C)}{2E_l}\frac{1}{\tilde{K}_n^{-1} - \Sigma_n}T^c$$







1) MM/Culver/Döring/Alexandru/Lee/Brett/Sadasivan [GWQCD] Phys.Rev.Lett. 127

HADRONS IN A BOX

Finite-volume spectrum is real and discrete!

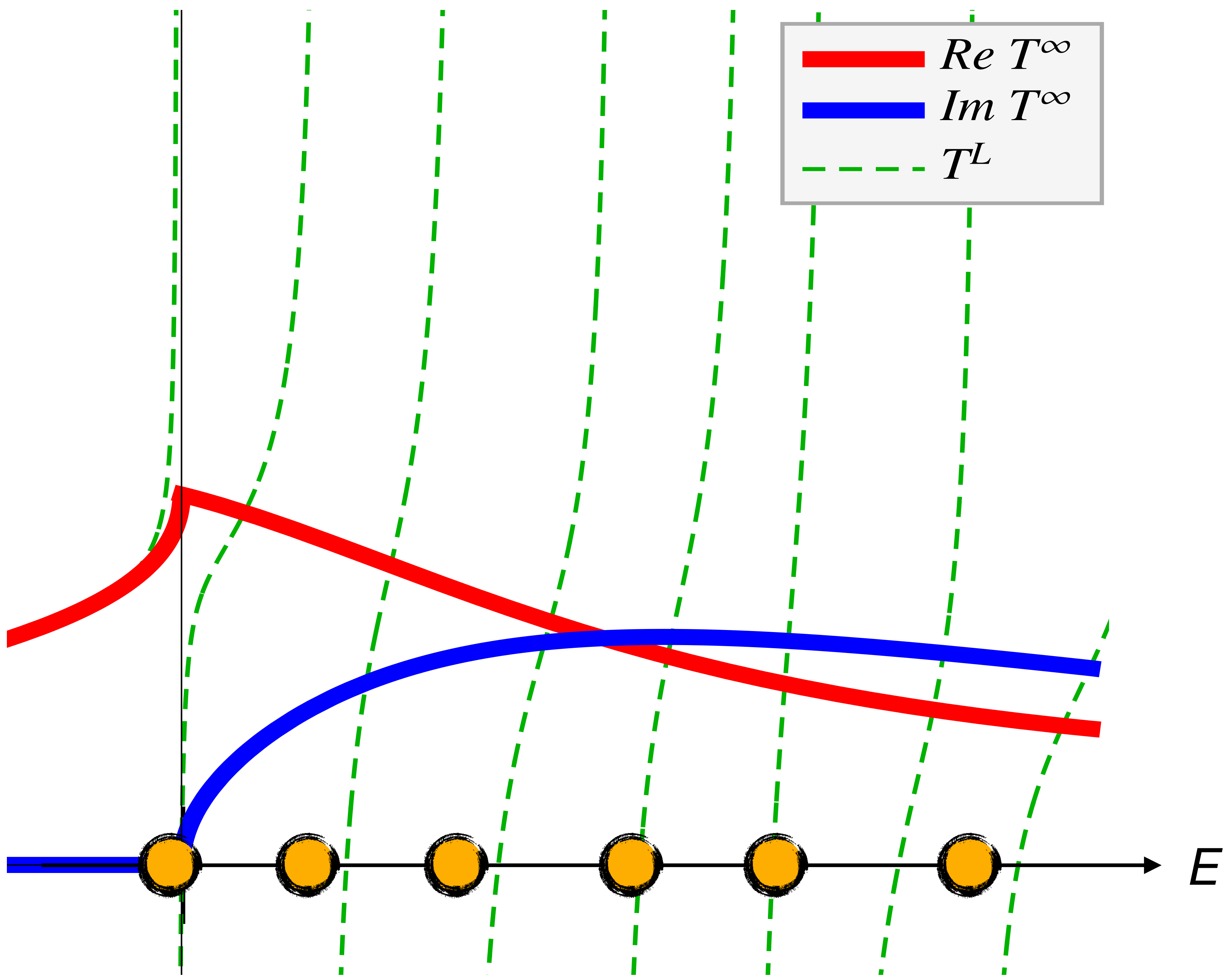
... requires mapping: Quantization condition[1,2]

🤗 Heavily simplified:

on-shell particle-configurations: $\mathbf{\Delta}E \sim mL$

off-shell particle-configurations: $\mathbf{\Delta}E \sim e^{-mL}$

🤩 *A unitary "T-matrix" accounts for all O(mL) effects!*

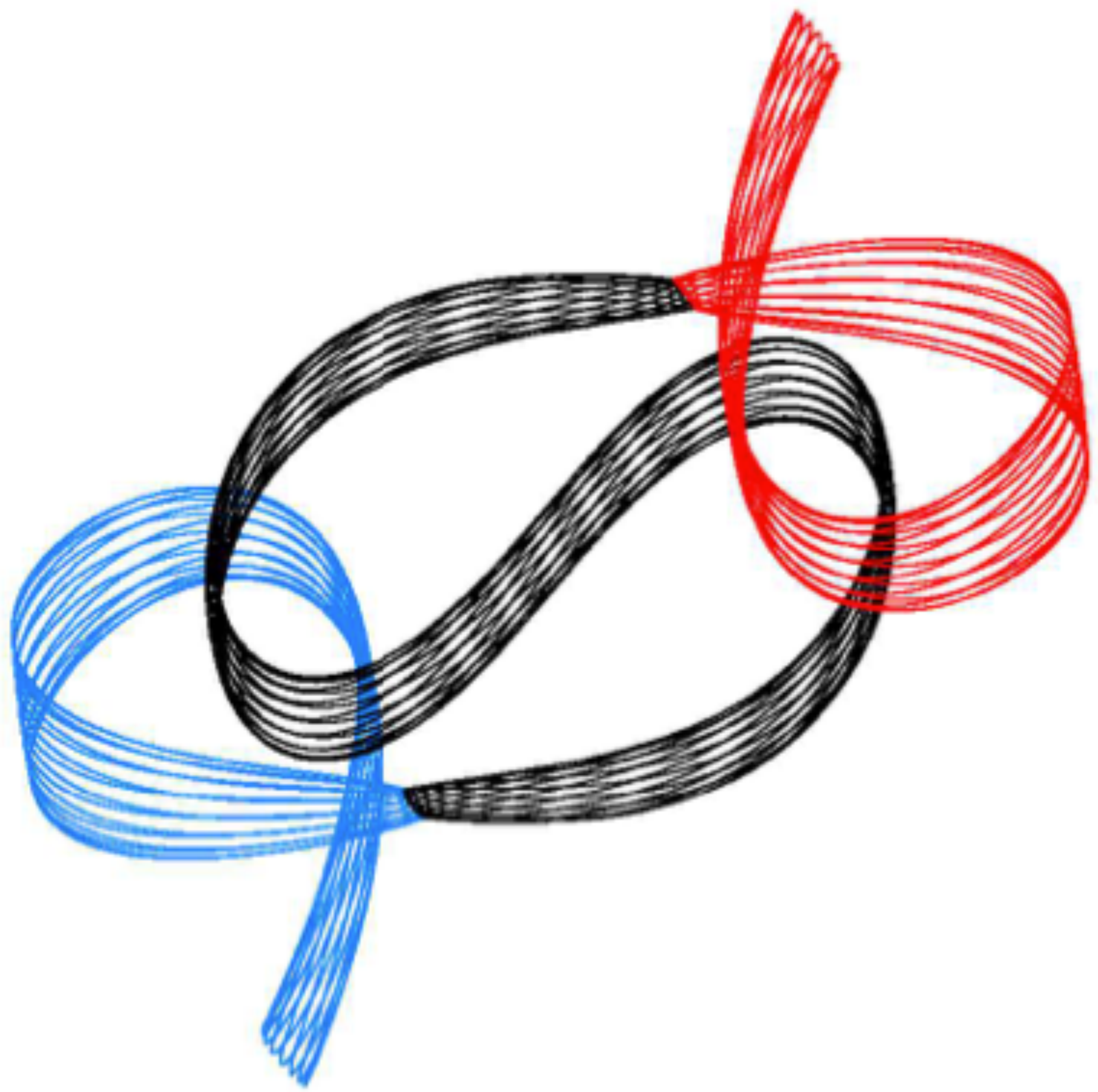


1) Lüscher, Gottlieb, Rummukainen, Feng, Li, Döring, Briceño, Meißner, Rusetsky, Hansen, MM, Blanton, ...

2) Reviews: Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Döring/

GRAVITATIONAL 3-BODY PROBLEM

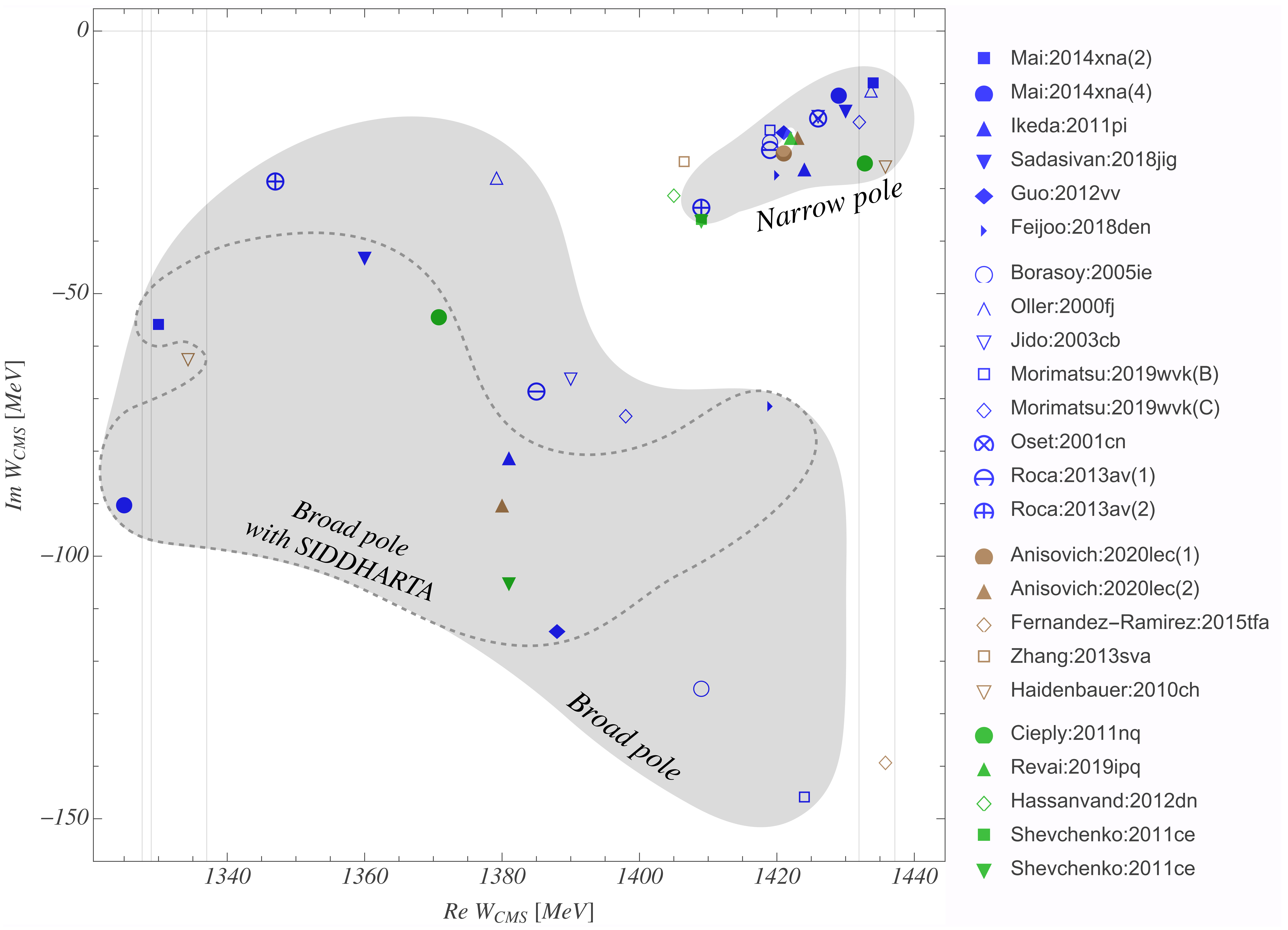
… not that we talk about



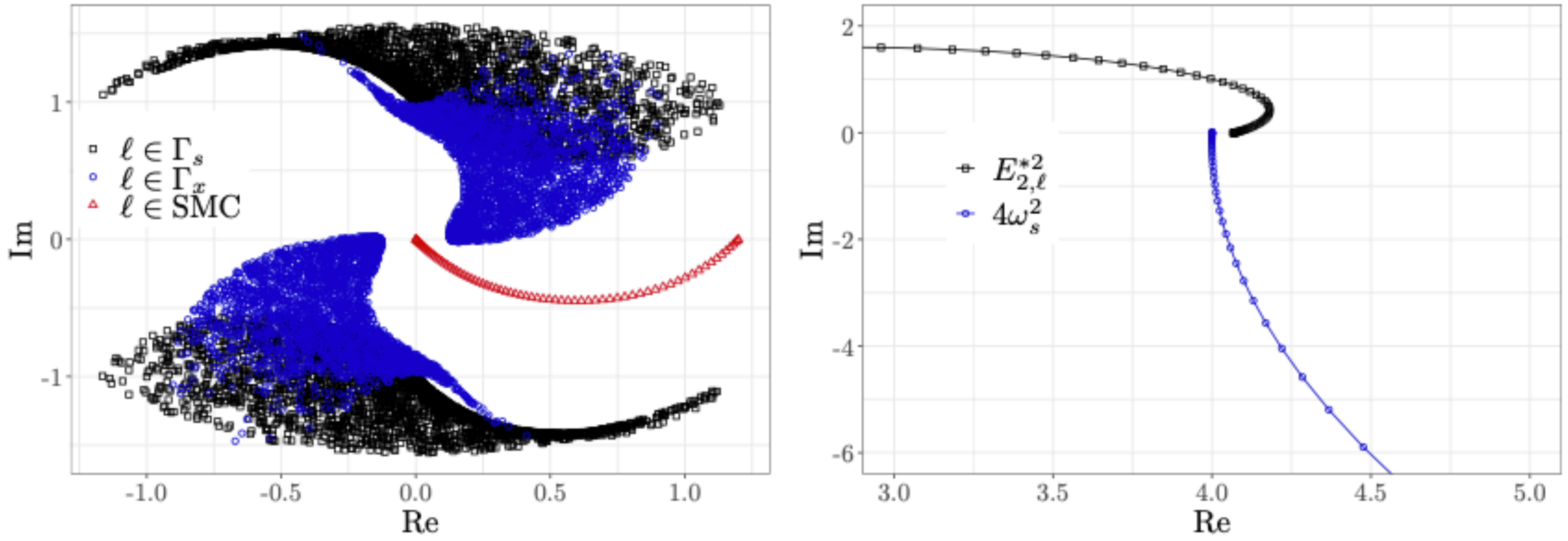
- birth of mathematical chaos[1]
- no closed solution
- in general non-repeating trajectories

1) H. Poincaré

Fig.: Šuvakov/Dmitrašinović PhysRevLett.110.114301 (2013)



0
−50
−100
−150
1340
1360
1380
1400
1420
1440
Re W_{CMS} [MeV]
Im W_{CMS} [MeV]
Narrow pole
Broad pole with SIDDHARTA
Broad pole
Mai:2014xna(2)
Mai:2014xna(4)
Ikeda:2011pi
Sadasivan:2018jig
Guo:2012vv
Feijoo:2018den
Borasoy:2005ie
Oller:2000fj
Jido:2003cb
Morimatsu:2019wvk(B)
Morimatsu:2019wvk(C)
Oset:2001cn
Roca:2013av(1)
Roca:2013av(2)
Anisovich:2020lec(1)
Anisovich:2020lec(2)
Fernandez−Ramirez:2015tfa
Zhang:2013sva
Haidenbauer:2010ch
Cieply:2011nq
Revai:2019ipq
Hassanvand:2012dn
Shevchenko:2011ce
Shevchenko:2011ce



$\ell \in \Gamma_s$
$\ell \in \Gamma_x$
$\ell \in$ SMC
Im
Re
1
0
-1
-1.0
-0.5
0.0
0.5
1.0
$E^{*2}_{2,\ell}$
$4\omega_s^2$
2
-2
-4
-6
3.0
3.5
4.0
4.5
5.0

Current frontier: 3-body dynamics from LQCD

➥ 3-body Quantization Conditions[1]

➥ RFT / FVU / NREFT

➥ many perturbatively interacting systems are studied[2]

$$0 = \det\left(L^3\left(\tilde{F}/3 - \tilde{F}(\tilde{K}_2^{-1} + \tilde{F} + \tilde{G})^{-1}\tilde{F}\right)^{-1} + K_{\mathrm{df},3}\right)$$ RFT

$$0 = \det\left(B_0 + C_0 - E_L\left(K^{-1}/(32\pi) + \Sigma_L\right)\right)$$ FVU

3-body force

2-body interaction

one-particle exchange

2-body self-energy

1) Rusetsky, Bedaque, Grießhammer, Sharpe, Meißner, Döring, Hansen, Davoudi, Guo....

Reviews:

Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019);

MM/Döring/Rusetsky Eur.Phys.J.ST 230 (2021);

2) MM/Döring PRL122(2019); Blanton et al. PRL 124 (2020); Hansen et al. PRL

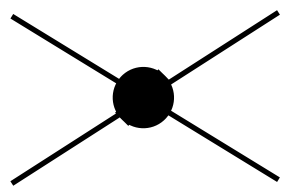
AVOIDED LEVEL CROSS

Variate g($\varphi_1 \to \varphi_0\varphi_0\varphi_0$) coupling:

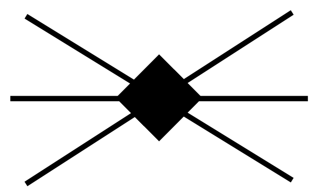
- avoided level crossing becomes wider
- RFT and FVU

g		a	m_1	c_0	c_1	m'_1	c'_0	c'_1	χ^2_{dof}
5	FVU	−0.1512(9)	3.0229(1)	−0.0188(35)	–	–	–	–	2.9
	RFT	−0.1522(12)	–	–	–	3.0232(2)	31.6(8.4)	–	2.5
	FVU	−0.1569(12)	3.0233(2)	−0.0297(57)	2.29(38)	–	–	–	1.5
	RFT	−0.1571(10)	–	–	–	3.0237(2)	37.6(9.0)	2789(540)	1.5
10	FVU	−0.1521(11)	3.0205(2)	−0.0475(66)	–	–	–	–	1.7
	RFT	−0.1531(13)	–	–	–	3.0212(3)	80(14)	–	1.6
	FVU	−0.1549(16)	3.0205(2)	−0.0595(99)	0.93(41)	–	–	–	1.5
	RFT	−0.1563(27)	–	–	–	3.0213(3)	97(16)	1773(980)	1.4
20	FVU	−0.1444(11)	3.0184(2)	−0.1136(77)	–	–	–	–	1.6
	RFT	−0.1450(17)	–	–	–	3.0199(2)	178(17)	–	1.6
	FVU	−0.1464(14)	3.0183(2)	−0.1363(148)	0.84(39)	–	–	–	1.3
	RFT	−0.1484(16)	–	–	–	3.0200(2)	210(23)	2227(600)	1.2

$$q^* \cot\delta = \frac{1}{aM_0}$$



$$C = \frac{c_0}{E_3^3 - m_1^2} + c_1$$



... same fit quality

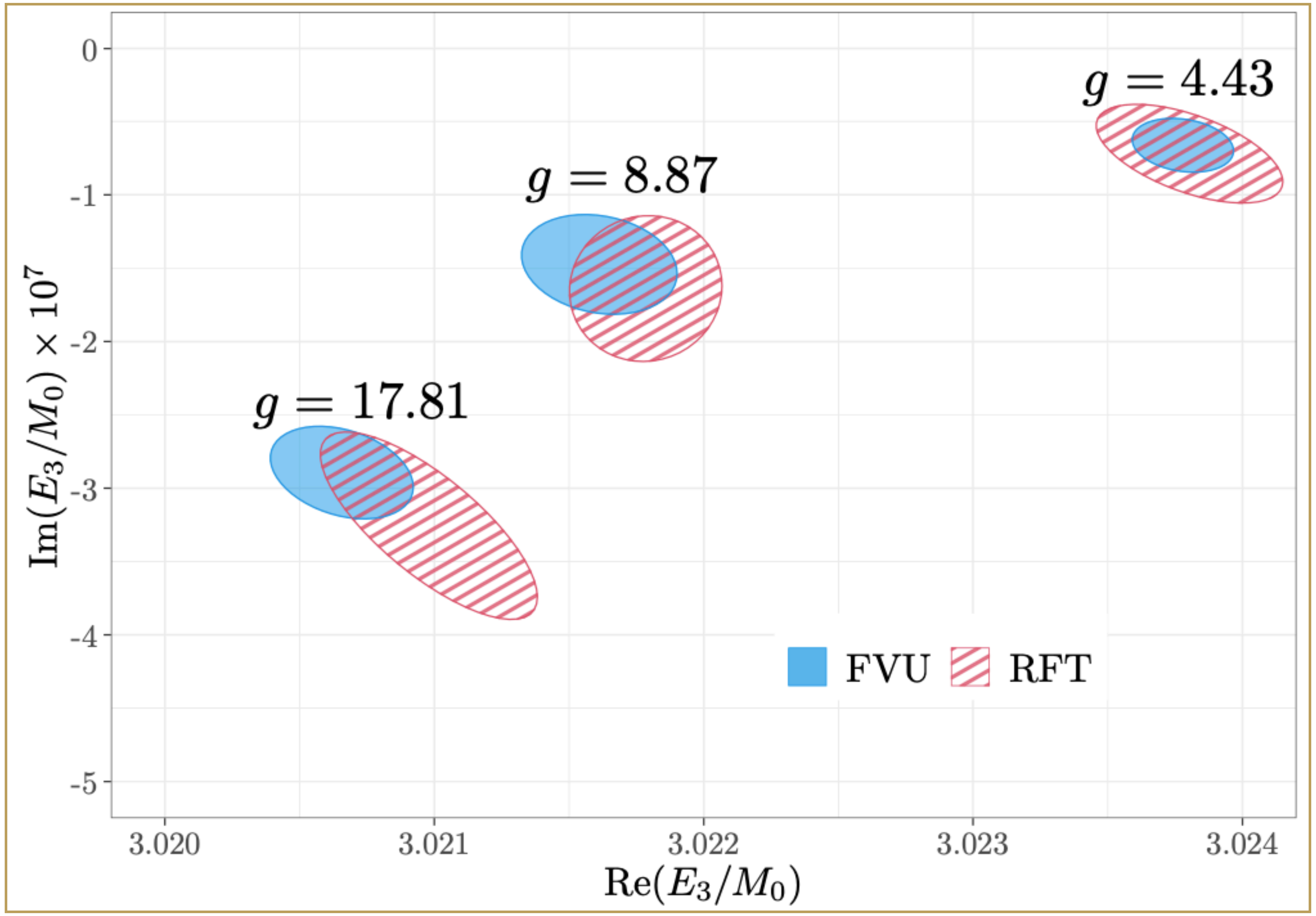
... observables determined consistently

Pole positions

- FVU: complex energy-plane analysis[1]

  -- resonance width grows ~ ***g²***

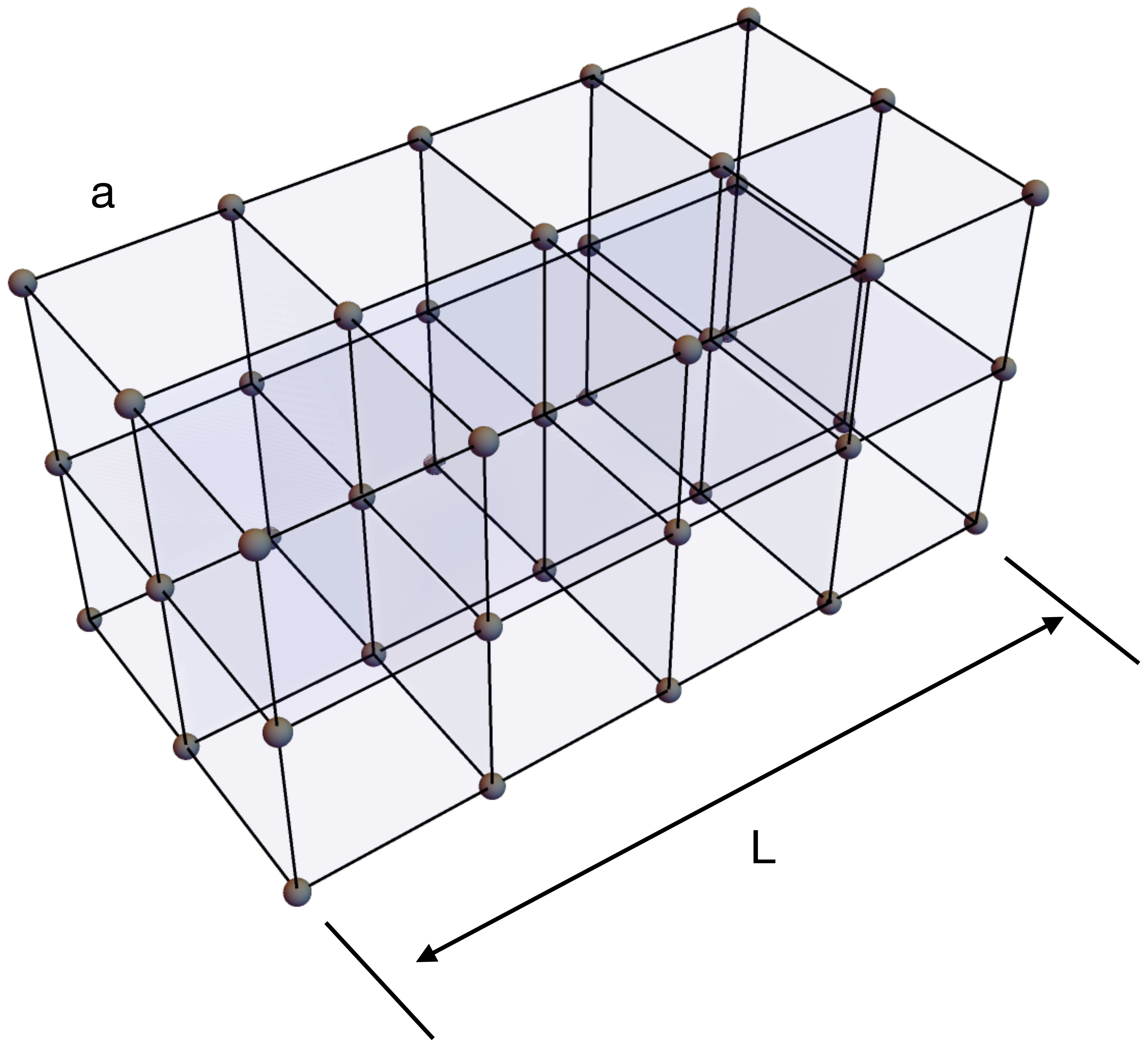
  -- avoided level crossing gap >> width
- Similarly from RFT with Breit-Wigner like approximation



1) Sadasivan/MM/.. Phys.Rev.D 101 (2020)

LATTICE QCD

$$\mathcal{L}_{\mathrm{QCD}} = \sum \bar{q}_f^a \left(i\not{D}_{ab} - m_f \delta_{ab} \right) q_f^b - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$$



Lattice QCD: numerical evaluation of QCD Green's functions. But…

- discretized Euclidean space time (***a>0***)
- in finite volume (***L<oo***)