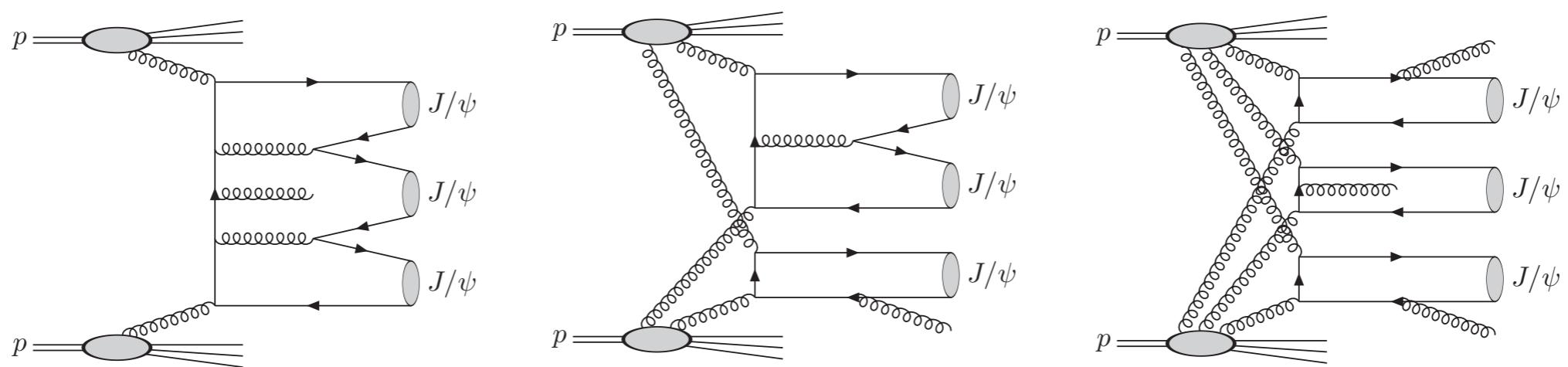




Multiple Parton Scattering from both theoretical and experimental point of views



Hua-Sheng Shao



HADRON 2023, Genova
05 June 2023

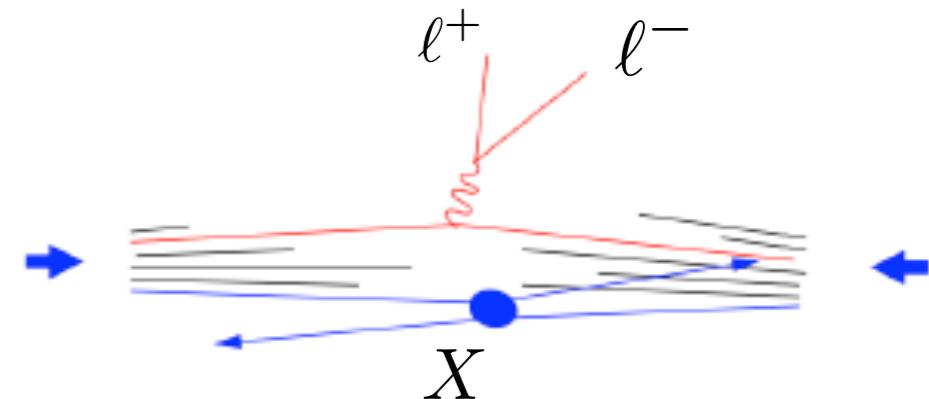
A Brief Introduction

- Cross section from factorization theorem (conjecture)

cross section = parton distribution \times partonic cross section

- Spectator-spectator interactions

- cancel in inclusive cross sections (unitarity)
- affect final state X



- Additional interaction (blue) will be sensitive if we probe X simultaneously
- If the second interaction is also hard \rightarrow **Double Parton Scattering**

e.g. $pp \rightarrow Z + H + X \rightarrow l\bar{l} + b\bar{b} + X$

- DPS contributes to signals and to backgrounds in many analyses at the LHC
- Inclusive cross section:

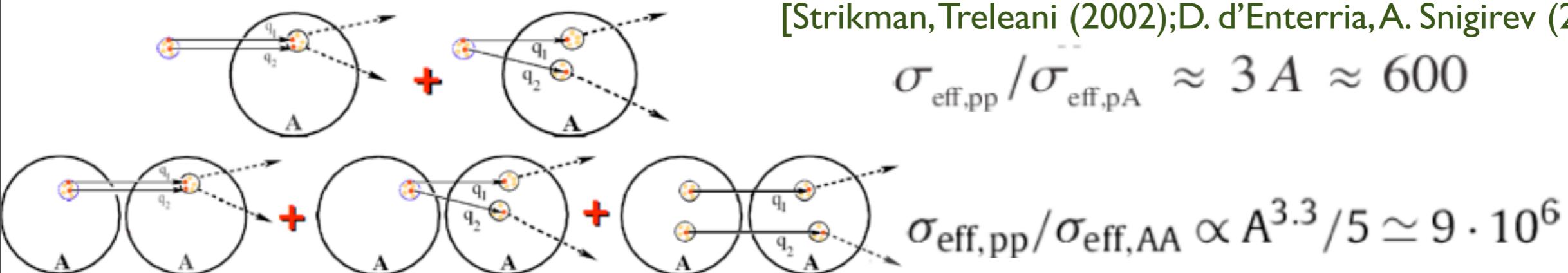
$$\sigma_{\text{SPS}} \sim \frac{1}{Q^2} \quad \text{v.s.} \quad \sigma_{\text{DPS}} \sim \frac{\Lambda_{\text{QCD}}^2}{Q^4}$$

- Higher energy \Rightarrow Larger parton density \Rightarrow enhance DPS

$$\sigma_{\text{SPS}} \propto (\text{parton density})^2 \quad \text{v.s.} \quad \sigma_{\text{DPS}} \propto (\text{parton density})^4$$

A Brief Introduction

- How to probe DPS at the LHC ?
- Processes of low hard scale Q (but still in the perturbative regime)
 - multiply hadron production, e.g. $J/\psi + J/\psi$
- Processes of large yields
 - multi-jet production
- Processes of precision measurements
 - multi-lepton production
- Enhancement of parton luminosity
 - higher energy [8 TeV to 14 TeV to 100 TeV (FCC)]
 - probe in proton-nucleus and nucleus-nucleus collisions



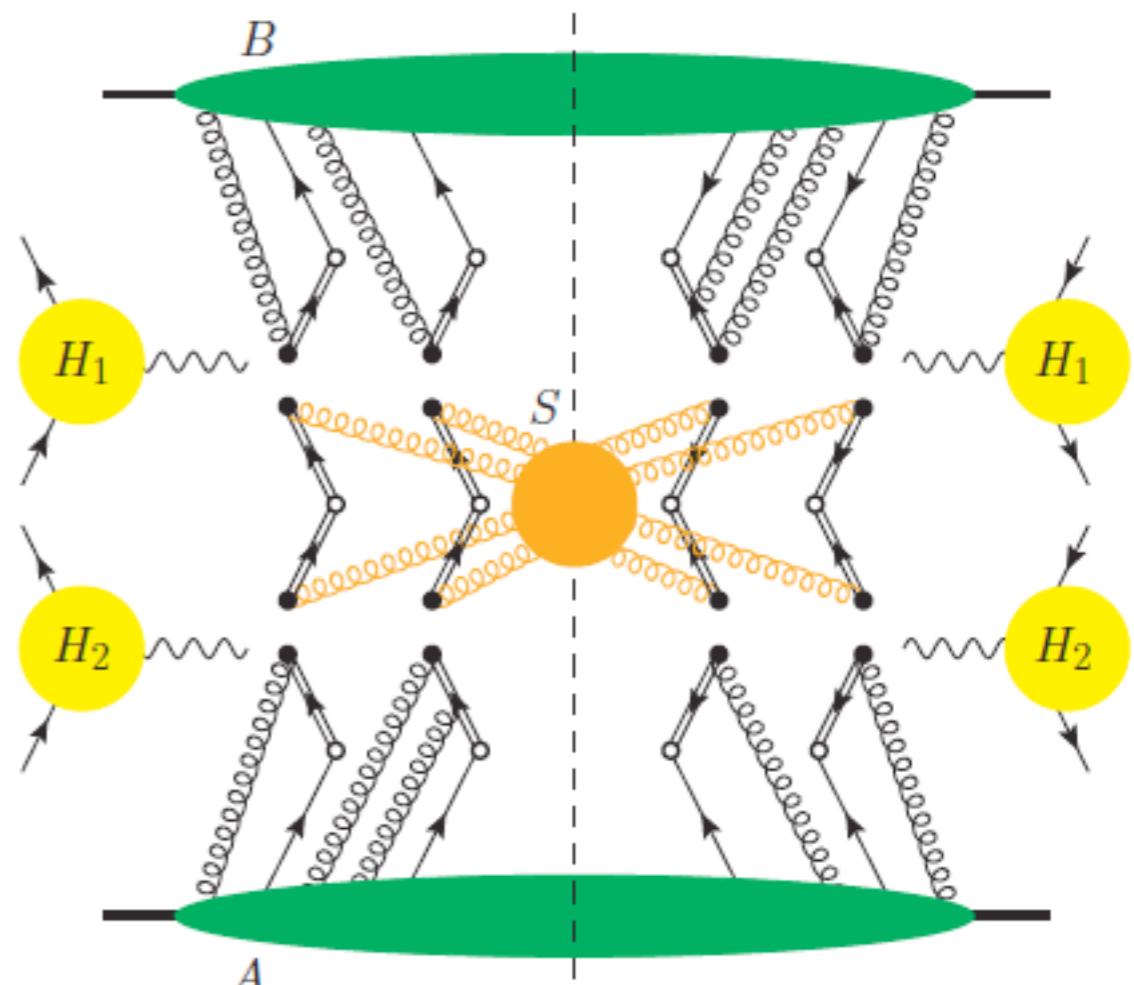
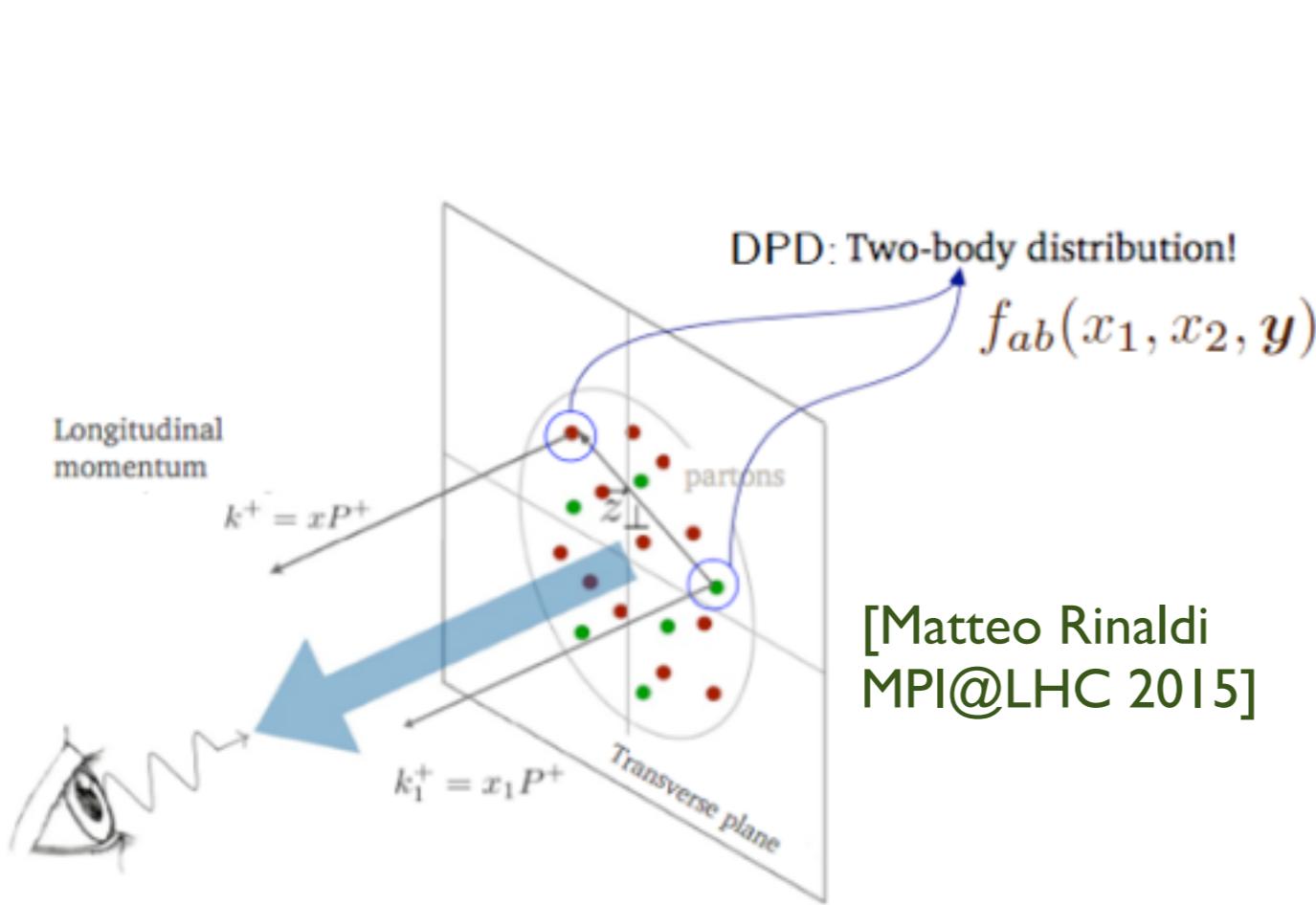
A DPS Theory Foundation

- Like SPS, we now have a first proven factorisation theorem for DPS (double Drell-Yan)

$$\sigma_{Q_1 Q_2} = \frac{1}{1 + \delta_{Q_1 Q_2}} \sum_{i,j,k,l} \int dx_1 dx_2 dx'_1 dx'_2 d^2 \mathbf{b}_1 d^2 \mathbf{b}_2 d^2 \mathbf{b}$$

$$\times \Gamma_{ij}(x_1, x_2, \mathbf{b}_1, \mathbf{b}_2) \hat{\sigma}_{ik}^{Q_1}(x_1, x'_1) \hat{\sigma}_{jl}^{Q_2}(x_2, x'_2) \Gamma_{kl}(x'_1, x'_2, \mathbf{b}_1 - \mathbf{b}, \mathbf{b}_2 - \mathbf{b}),$$

[Diehl, Gaunt, Ostermeier, Ploessl, Schafer (2015); Diehl, Nagar (2018)]



A NEW WAY TO ACCESS THE INFORMATION OF THE
NONPERTURBATIVE STRUCTURE OF NUCLEONS

A DPS Theory Foundation

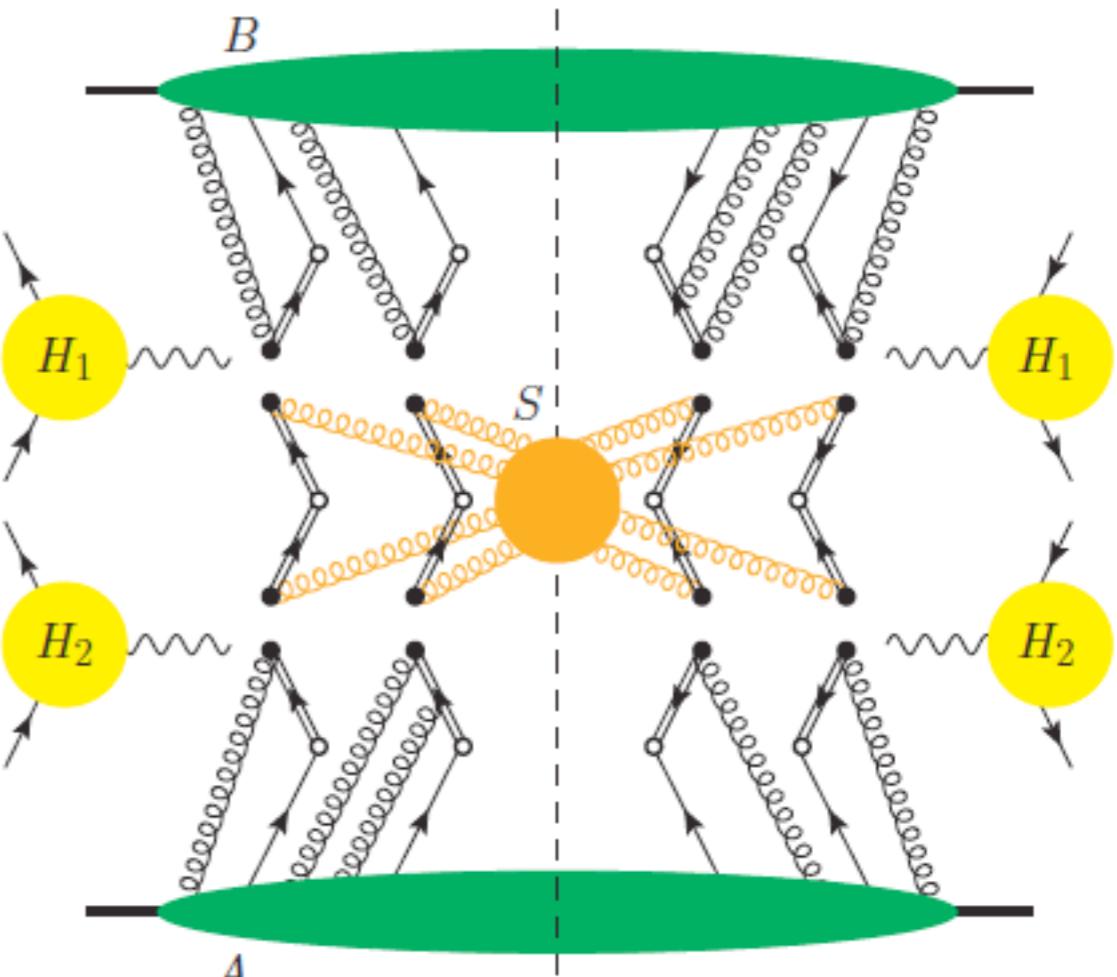
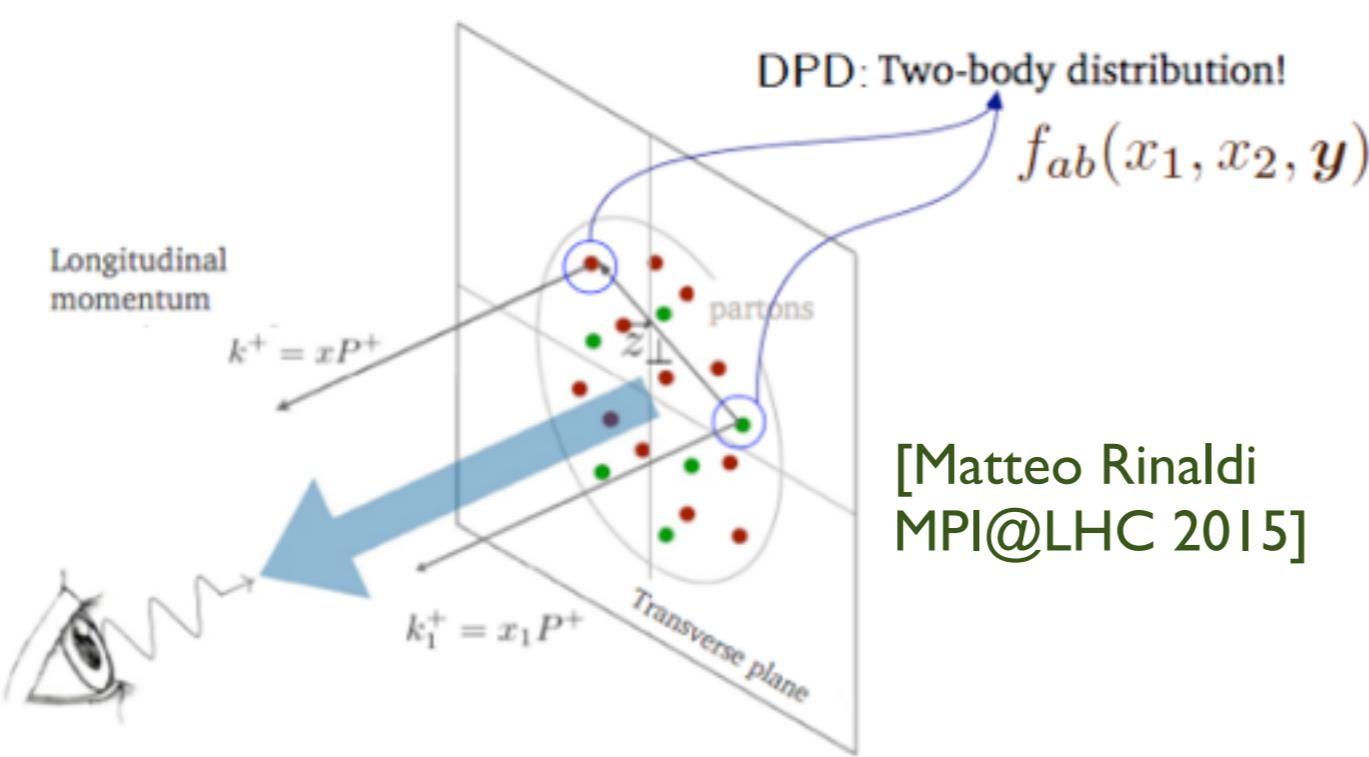
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[Diehl, Gaunt, Ostermeier, Ploessl, Schafer (2015); Diehl, Nagar (2018)]

Generalised double parton distribution



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- Widespread simplifications (most phenomenology relies on. Go beyond ?)

- factorization I $\Gamma_{ij}(x_1, x_2, \mathbf{b}_1, \mathbf{b}_2) = D_{ij}(x_1, x_2) T_{ij}(\mathbf{b}_1, \mathbf{b}_2)$, dPDF

- factorization II $D_{ij}(x_1, x_2) = f_i(x_1) f_j(x_2)$, PDF
 $T_{ij}(\mathbf{b}_1, \mathbf{b}_2) = T_i(\mathbf{b}_1) T_j(\mathbf{b}_2)$,

- assume flavor universality in T

$$\sigma_{\text{eff}} = \left[\int d^2 \mathbf{b} F(\mathbf{b})^2 \right]^{-1}.$$

$$\sigma_{Q_1 Q_2} = \frac{1}{1 + \delta_{Q_1 Q_2}} \frac{\sigma_{Q_1} \sigma_{Q_2}}{\sigma_{\text{eff}}}, \quad F(\mathbf{b}) = \int T(\mathbf{b}_i) T(\mathbf{b}_i - \mathbf{b}) d^2 \mathbf{b}_i,$$

Pocket Formula

A DPS Theory Foundation

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dPDF

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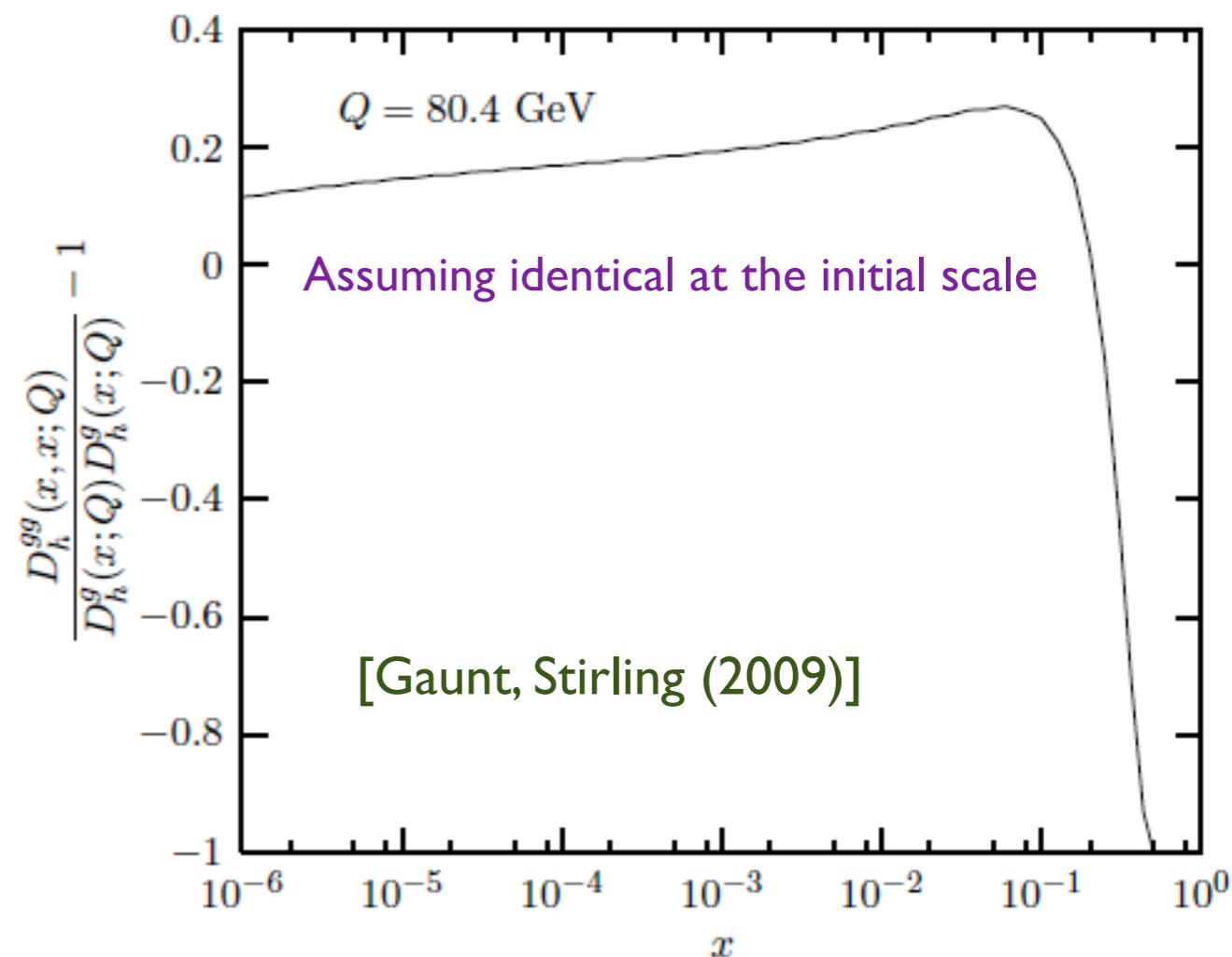
Pocket Formula

- Even these are complex objects to treat numerically

[Gaunt, Stirling; Elias, Golec-Biernat, Stasto; Diehl, Nagar, Tackmann]

DPS Theory Progress

- Let us start with the pocket formula and take any deviation wrt experiment as an indication of calling for a more rigorous treatment.
- Possible deviations (a few examples):
 - dDGLAP evolution (note high x !)



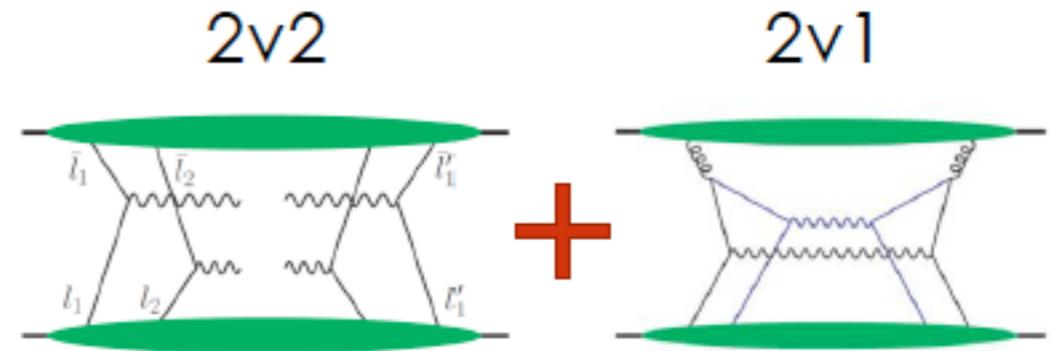
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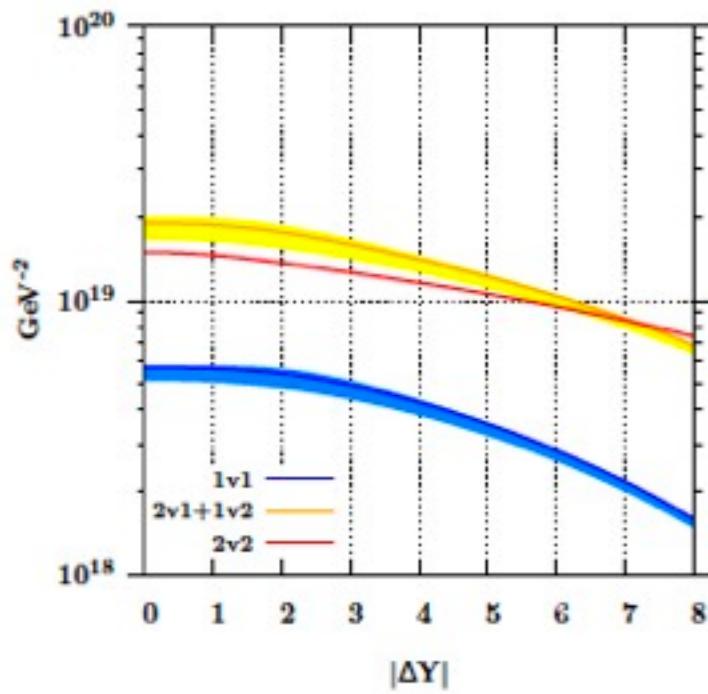
- dDGLAP evolution (note high x !)
- 1v2 (NLO ?) vs 2v2

parton luminosity is not suppressed !

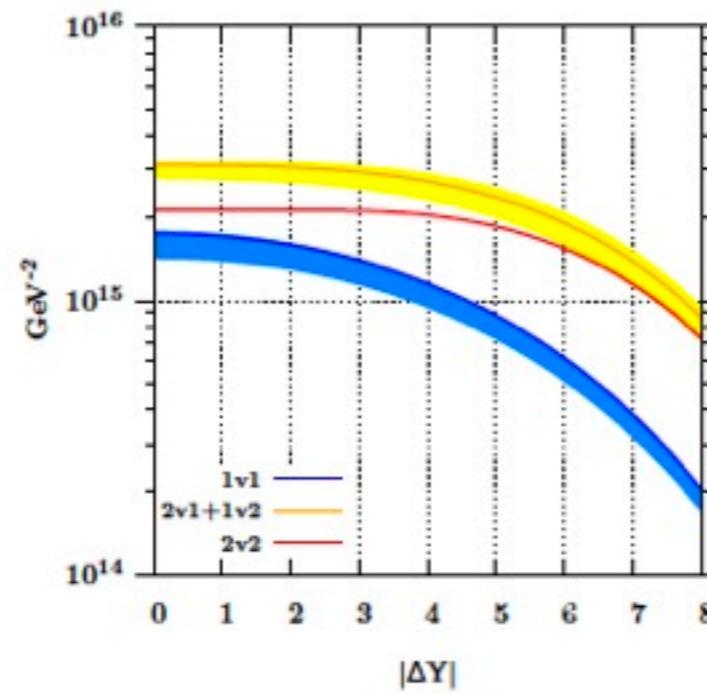
[Riccardo Nagar @ Quarkonia As Tools 2020]



[Gaunt, Stirling (2011); Block et al. (2012);
Manohar, Waalewijn (2012)]



$\mathcal{L}_{gggg} (m_{J/\Psi}, m_{J/\Psi})$

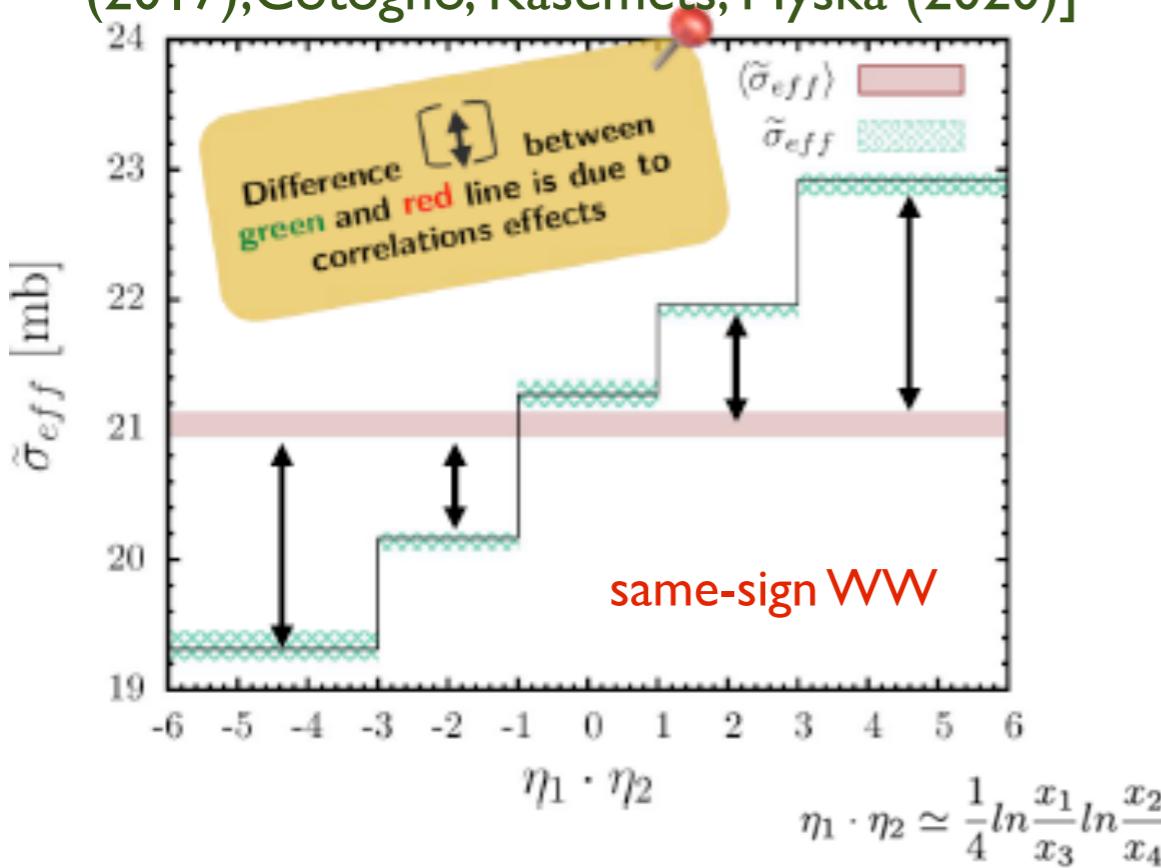


$\mathcal{L}_{gggg} (m_W, m_{J/\Psi})$

DPS Theory Progress

- Let us start with the pocket formula and take any deviation wrt experiment as an indication of calling for a more rigorous treatment.
- Possible deviations (a few examples):
 - dDGLAP evolution (note high x !)
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 - parton-parton correlations

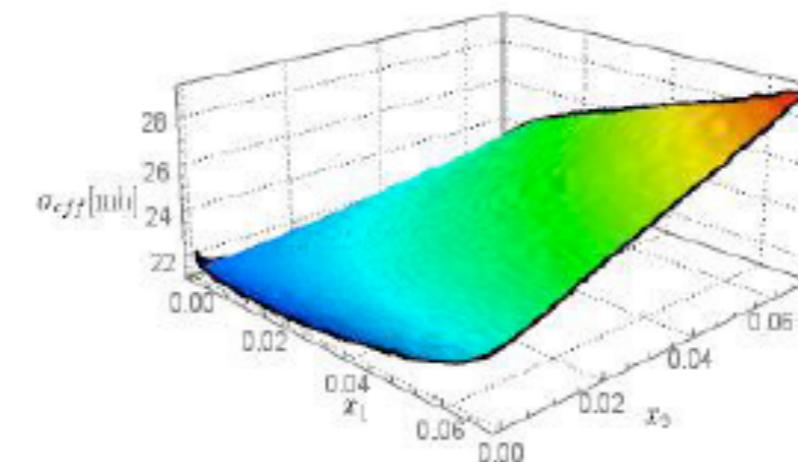
[Matteo Rinaldi @ Quarkonia As Tools
2020; Ceccopieri, Rinaldi, Scopetta
(2017); Cotogno, Kasemets, Myska (2020)]



the first and the last bins differ by 1 sigma,

$$\mathcal{L} = 1000 \text{ fb}^{-1}$$

is necessary to observe correlations



Gluons \otimes Gluons

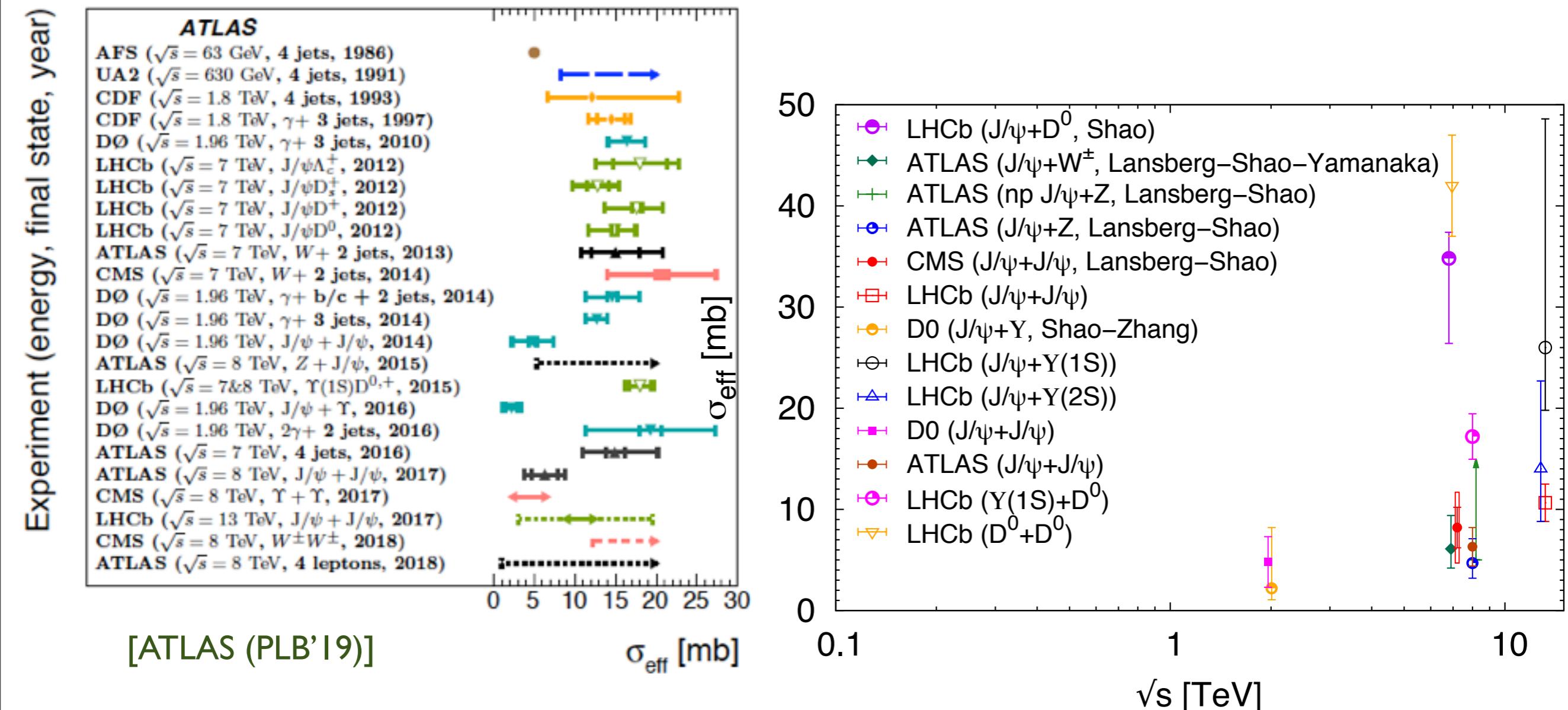
$$\sigma_{\text{eff}} \rightarrow \sigma_{\text{eff}}(x_1, x_2, \mu_F)$$

- Let us start with the pocket formula and take any deviation wrt experiment as an indication of calling for a more rigorous treatment.
- Possible deviations (a few examples):
 - dDGLAP evolution (note high x !)
 - 1v2 (NLO ?) vs 2v2
 - parton-parton correlations
- A few recent theoretical developments
 - DPS shower dShower [Cabouat, Gaunt, Ostrolenk (2019); Cabouat, Gaunt (2020)]
 - dDGLAP evolution beyond LO ChiliPDF [Diehl et al. (2023)]
 - Double parton distributions from lattice QCD [Bali et al. (2021); Zhang (2023); Jaarsma et al. (2023)]

Also see the section 7 in arXiv:2012.14161

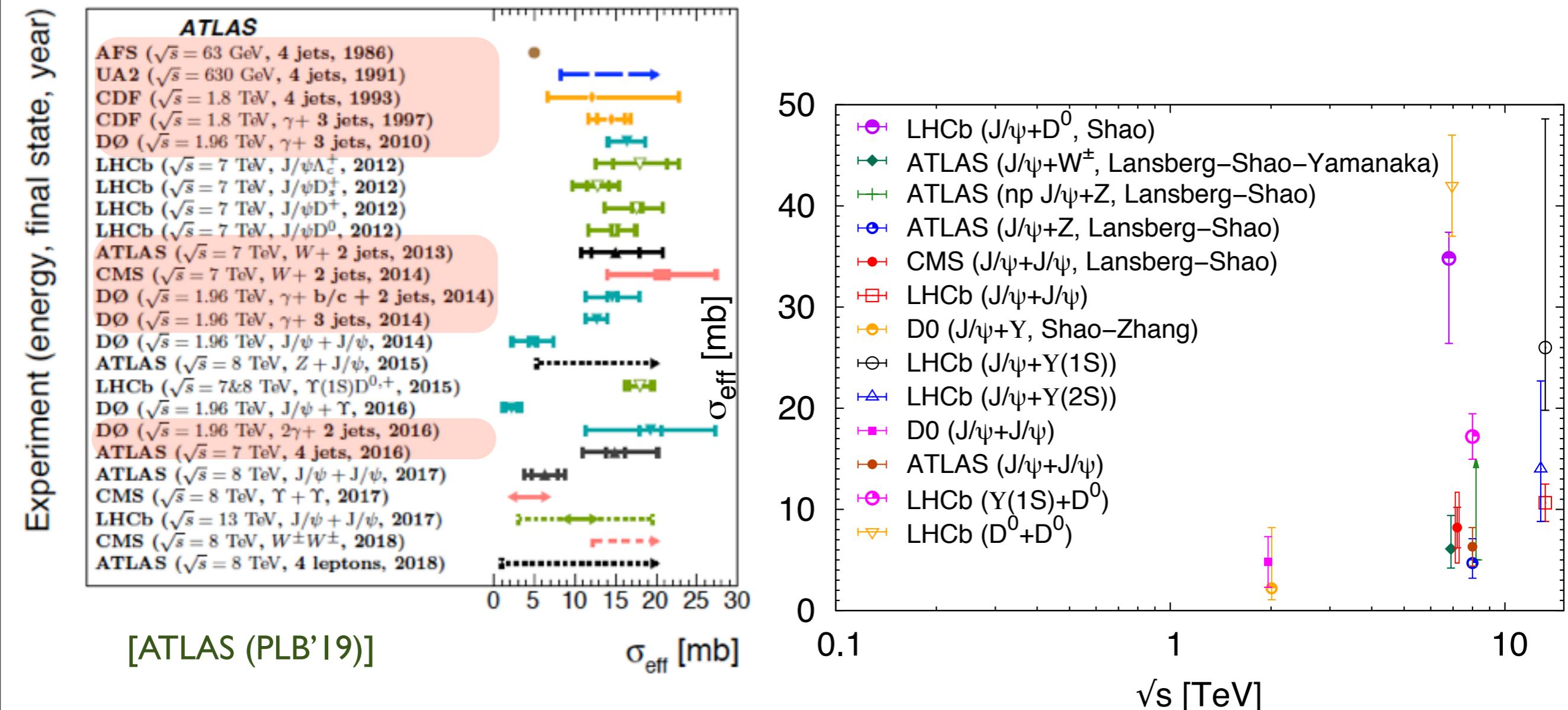
DPS Measurements

- Many DPS measurements at the LHC (Tevatron) in pp (ppbar)



DPS Measurements

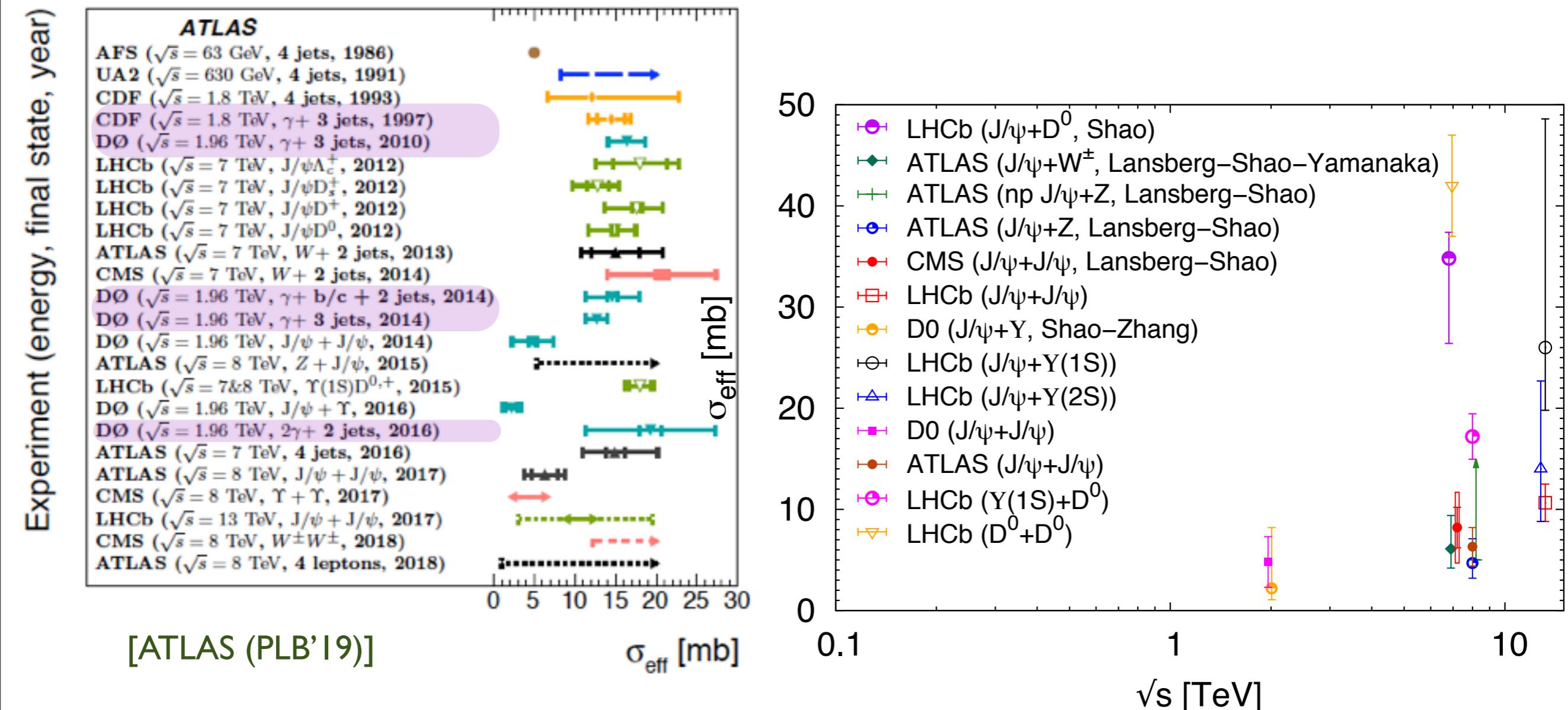
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jets

DPS Measurements

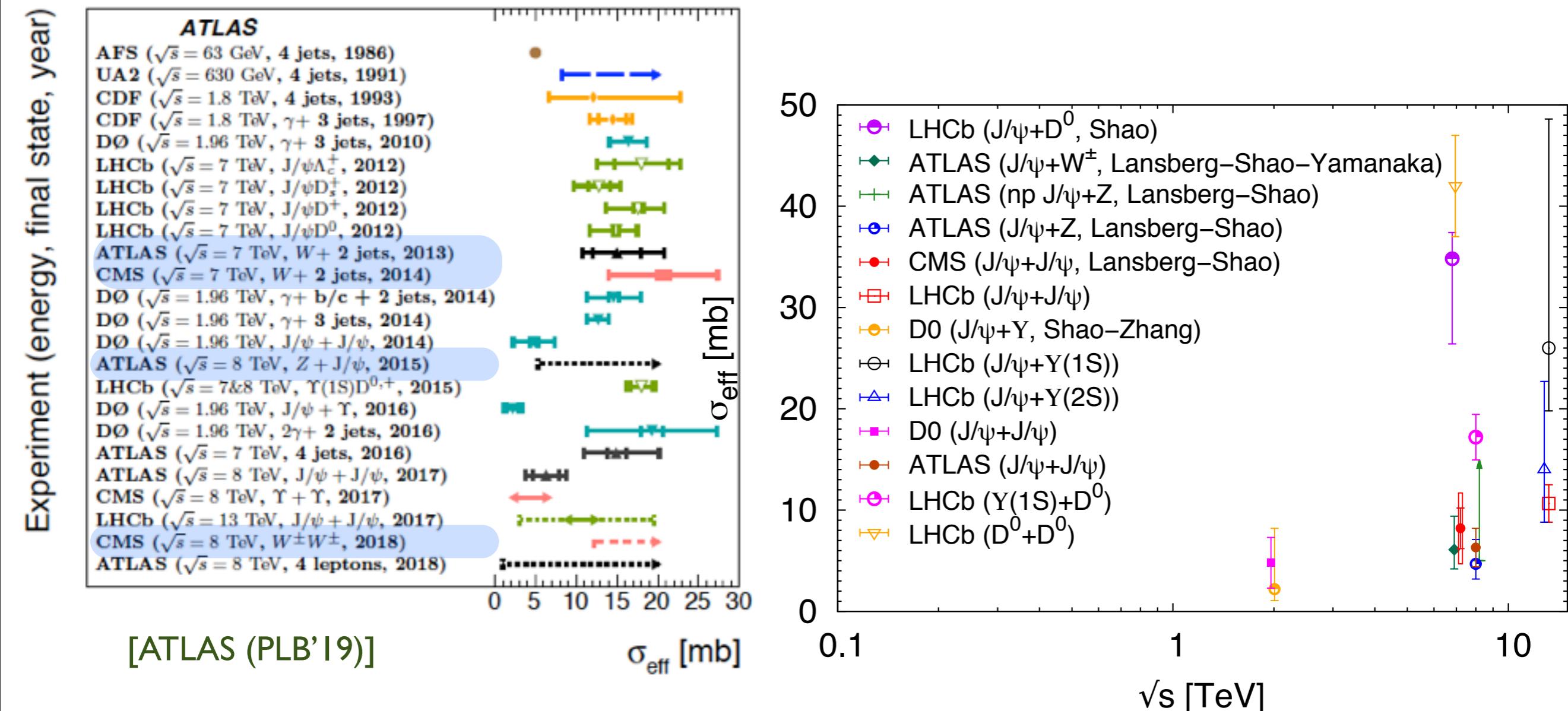
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jets photons

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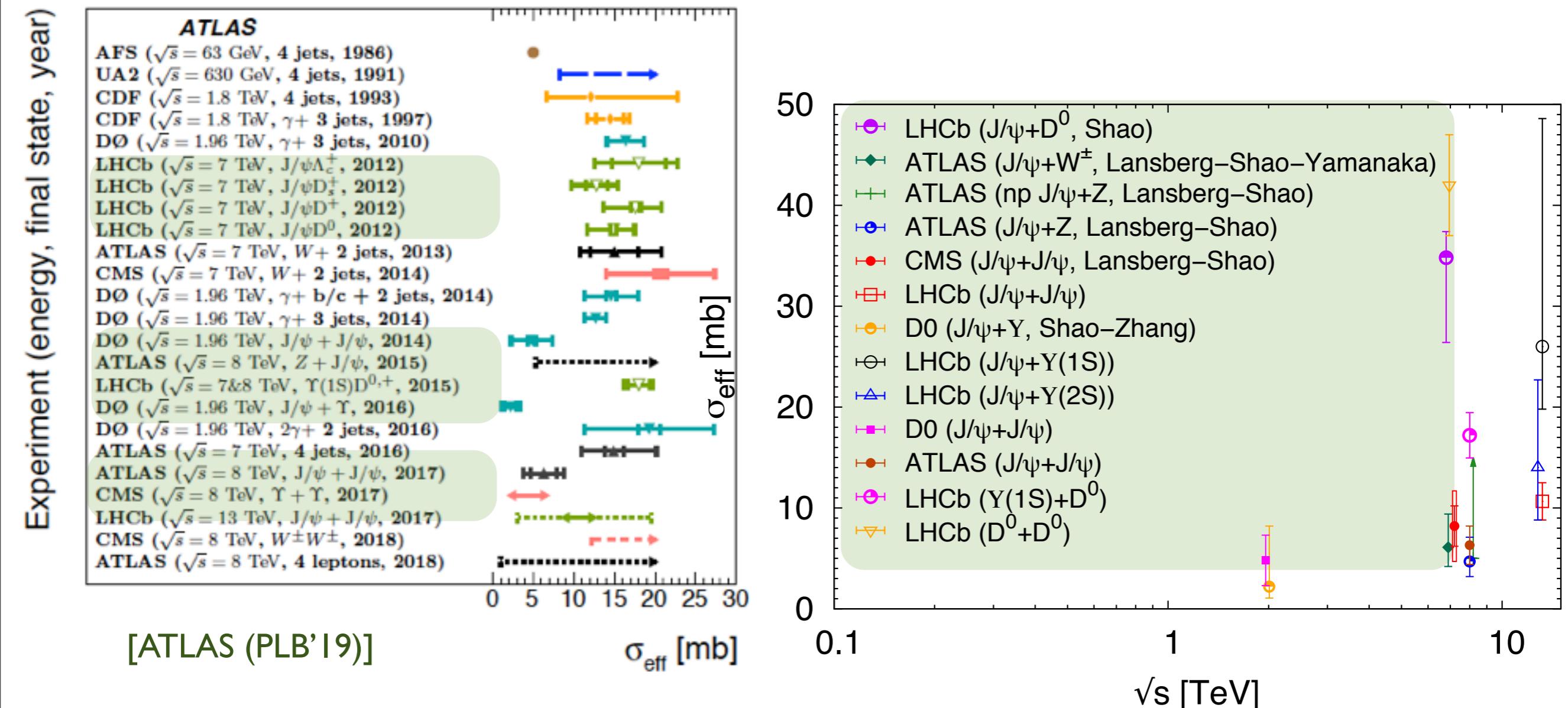


jets photons

W & Z

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jets

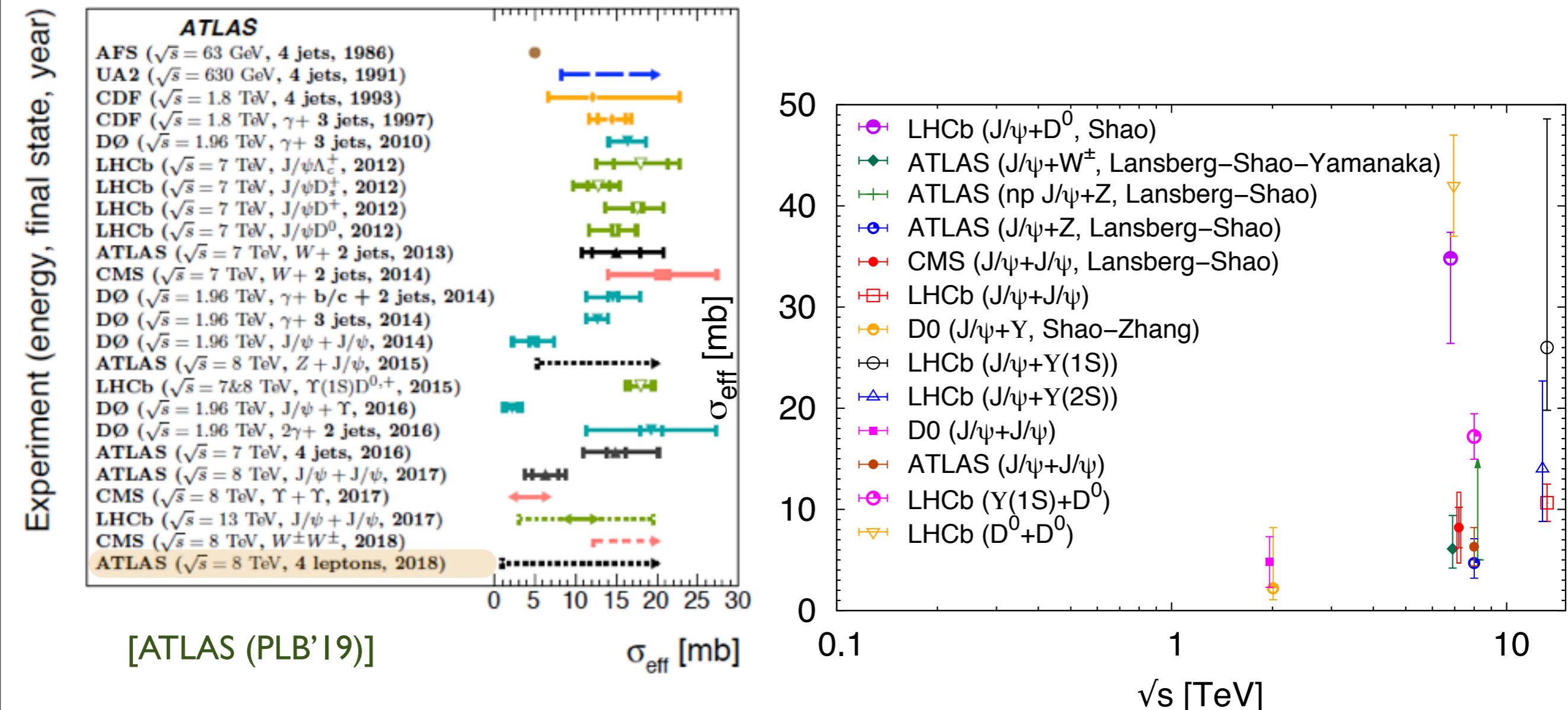
photons

W & Z

heavy flav. & onia

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jets

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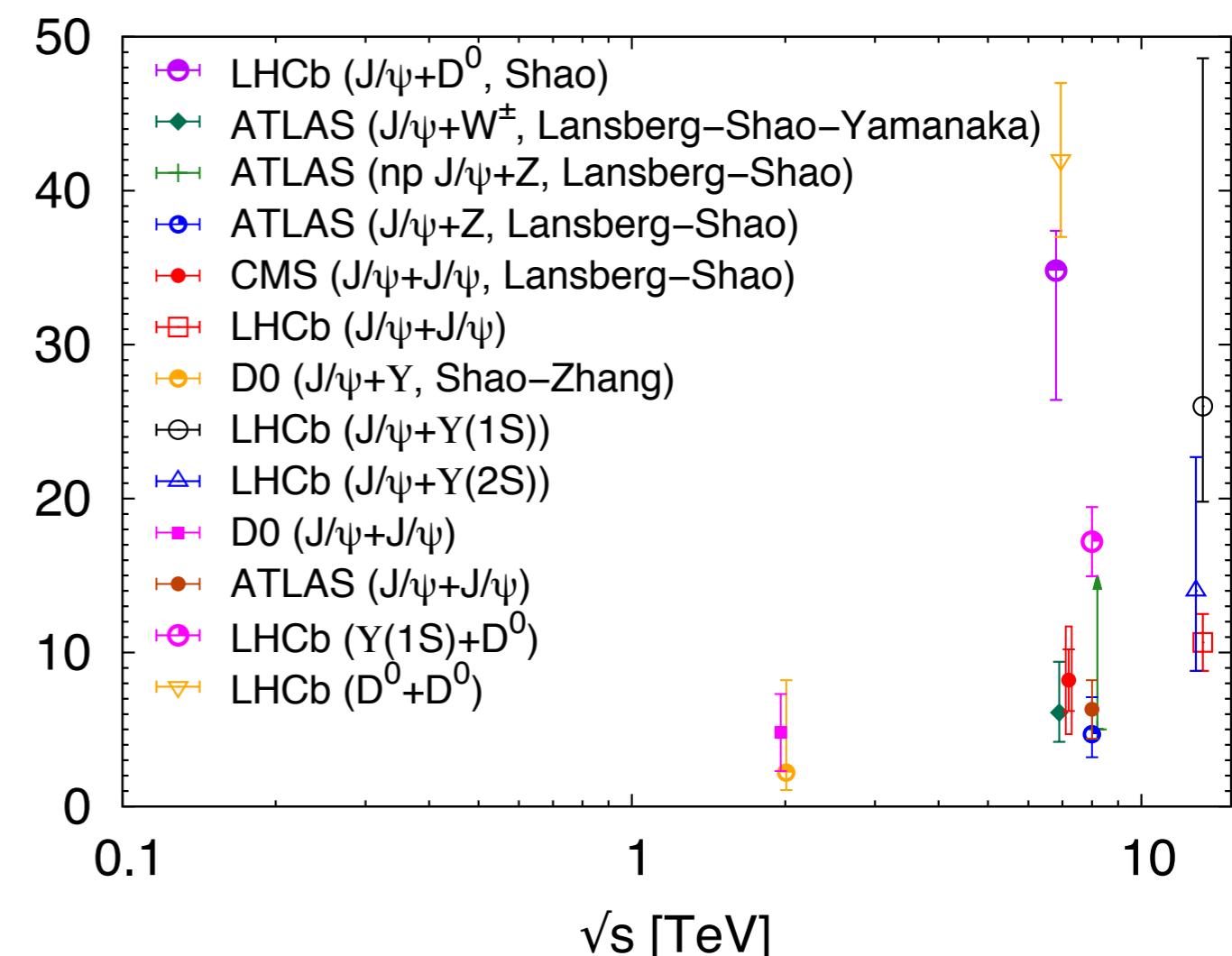
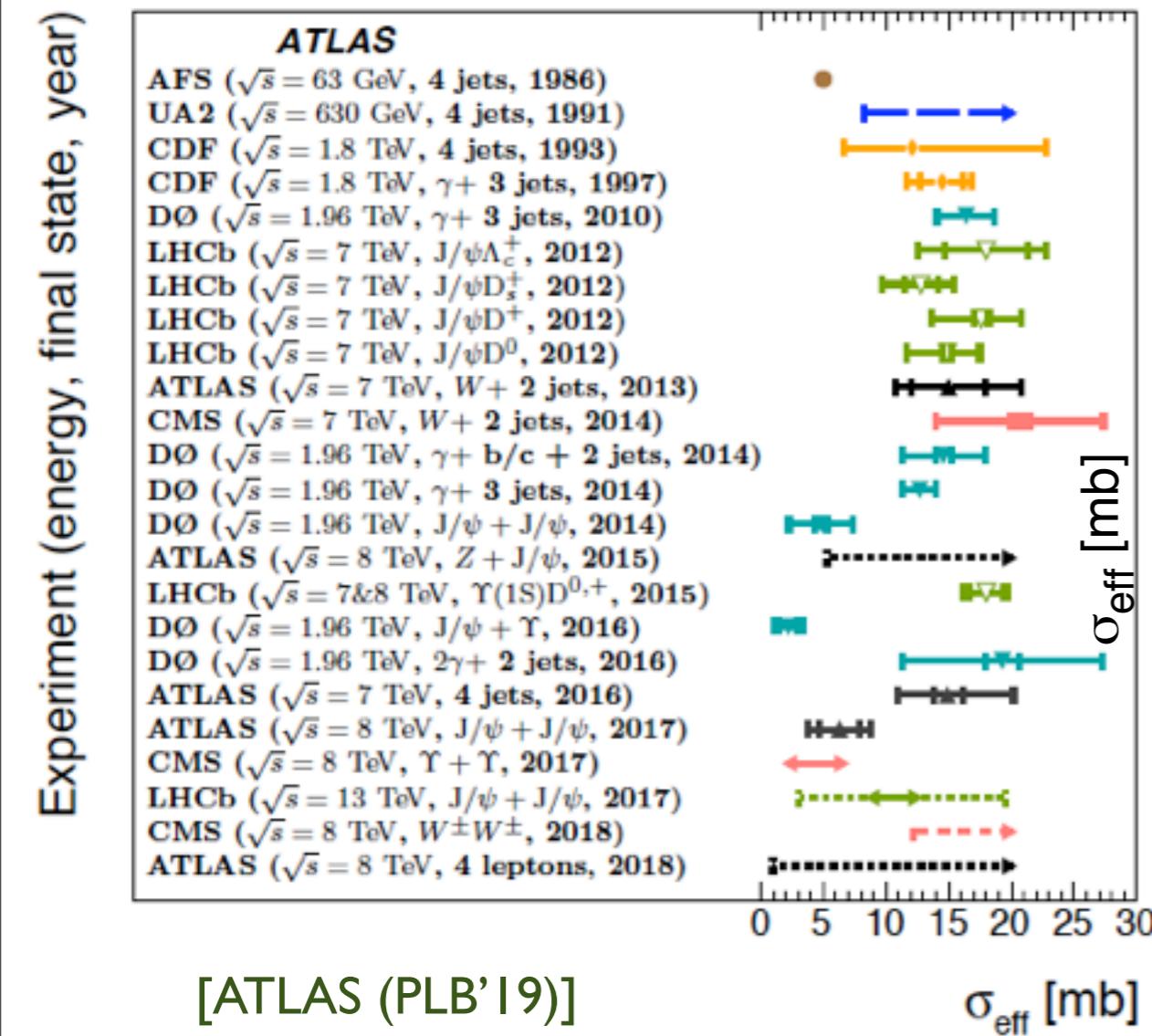
heavy flav. & onia

double DY

DPS Measurements

- Many DPS measurements at the LHC (Tevatron) in pp (ppbar)

- flavour dependent ?
- energy dependent ?
- kinematic dependent ?



jets

photons

W & Z

heavy flav. & onia

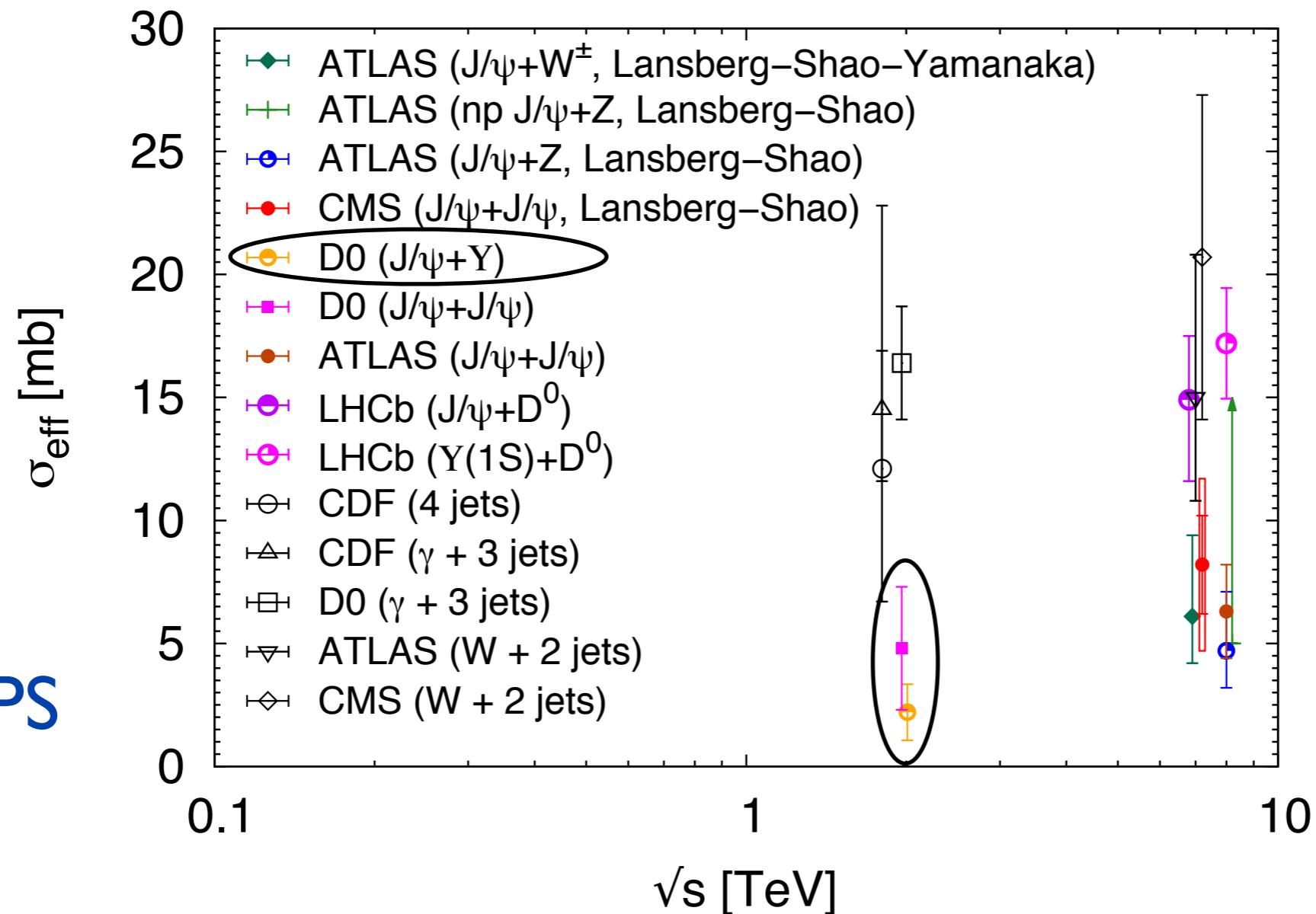
double DY

DPS Measurements

- Many DPS measurements at the LHC (Tevatron) in pp (ppbar)
 - Caveats with different extractions (challenging in differ. SPS & DPS)
 - How good are we understanding/controlling SPS ?

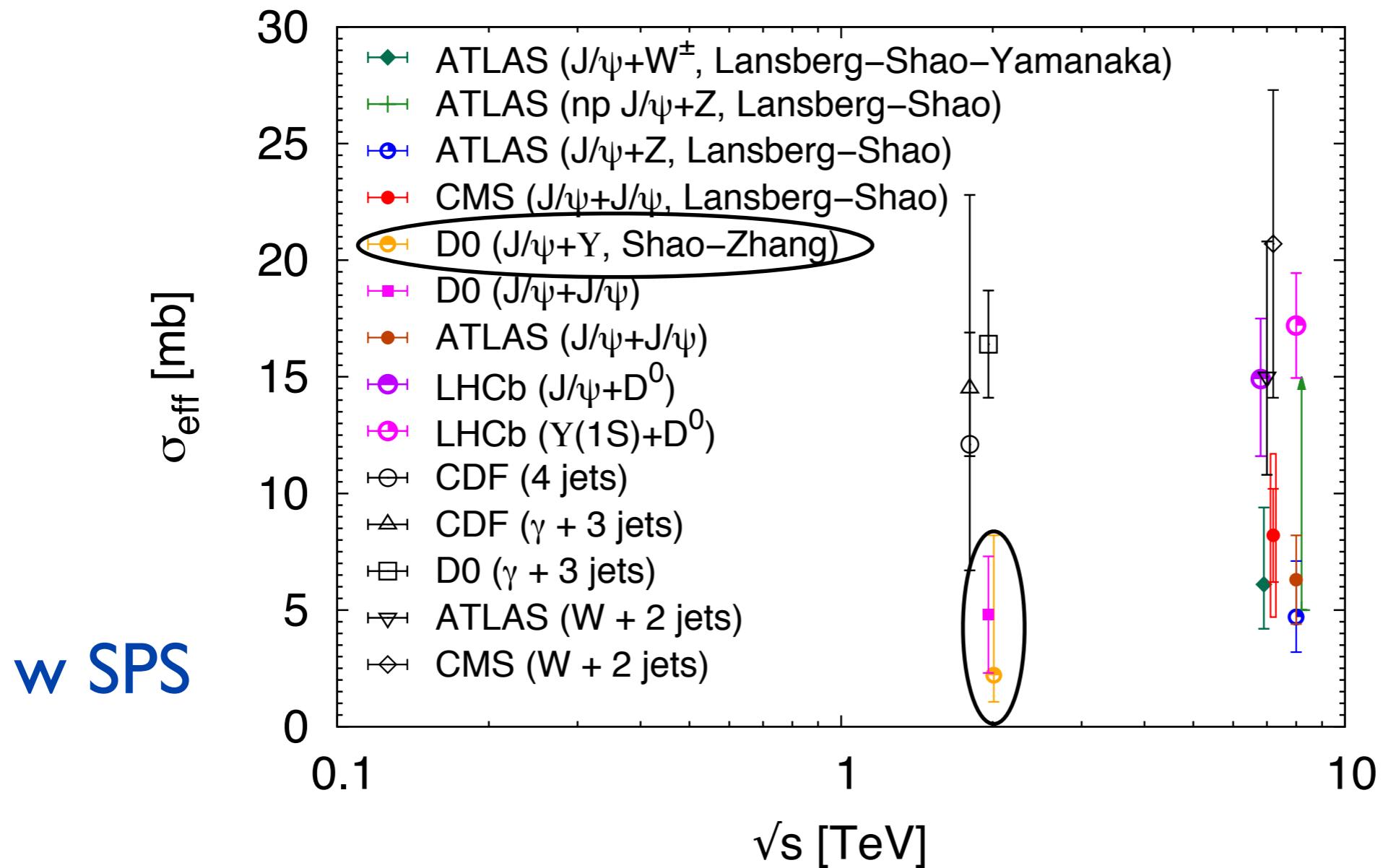
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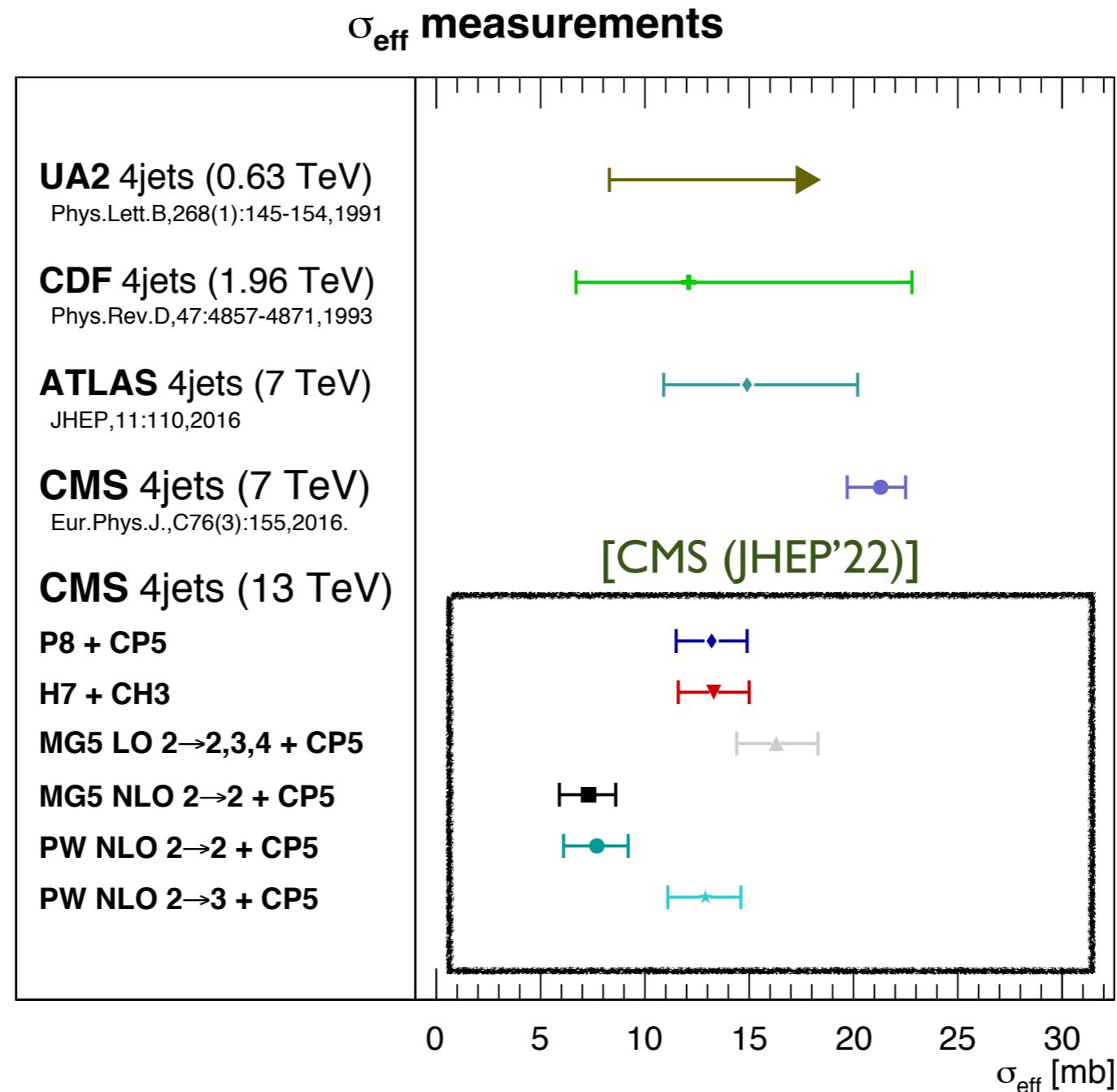
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Same observable but
different ME+MC

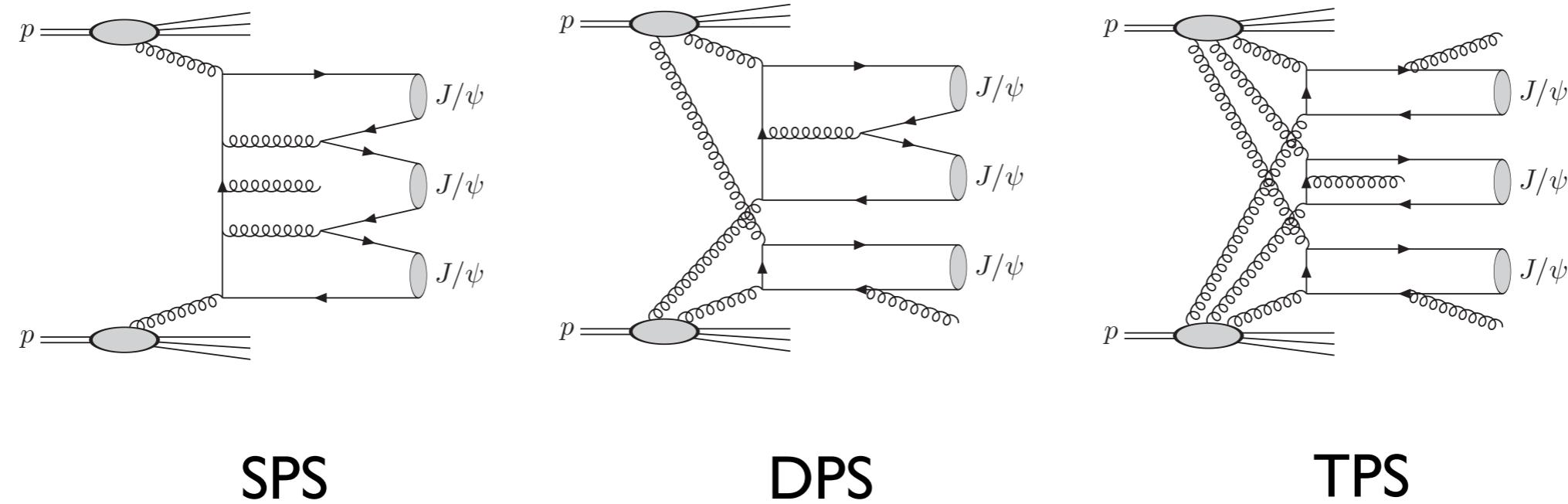
Two novel observables

- In the rest of the talk, I will focus on two novel observables that have been firstly measured by CMS and LHCb respectively

Triple Parton Scattering in pp

DPS in heavy-ion collisions

Triple Parton Scattering in pp



SPS

DPS

TPS

Triple Parton Scattering in pp

- Analogously, ignoring the parton correlations, the NPS pocket formula:

[D. d'Enterria, A. Snigirev (1708.07519)]

$$\sigma_{f_1 \dots f_N}^{\text{NPS}} = \frac{m}{N!} \frac{\prod_{i=1}^N \sigma_{f_i}^{\text{SPS}}}{(\sigma_{\text{eff},N})^{N-1}}$$

- A pure geometric consideration leads to

$$\sigma_{\text{eff},3} = (0.82 \pm 0.11) \times \sigma_{\text{eff},2}$$

[D. d'Enterria, A. Snigirev (PRL'17)]

- In general, the inclusive cross sections scale as

$$\sigma_{\text{SPS}} \sim \frac{1}{Q^2} \quad \text{v.s.} \quad \sigma_{\text{DPS}} \sim \frac{\Lambda_{\text{QCD}}^2}{Q^4} \quad \text{v.s.} \quad \sigma_{\text{TPS}} \sim \frac{\Lambda_{\text{QCD}}^4}{Q^6}$$

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- Any chance to see TPS at the LHC ?

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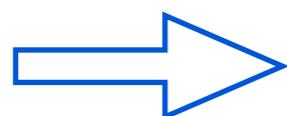
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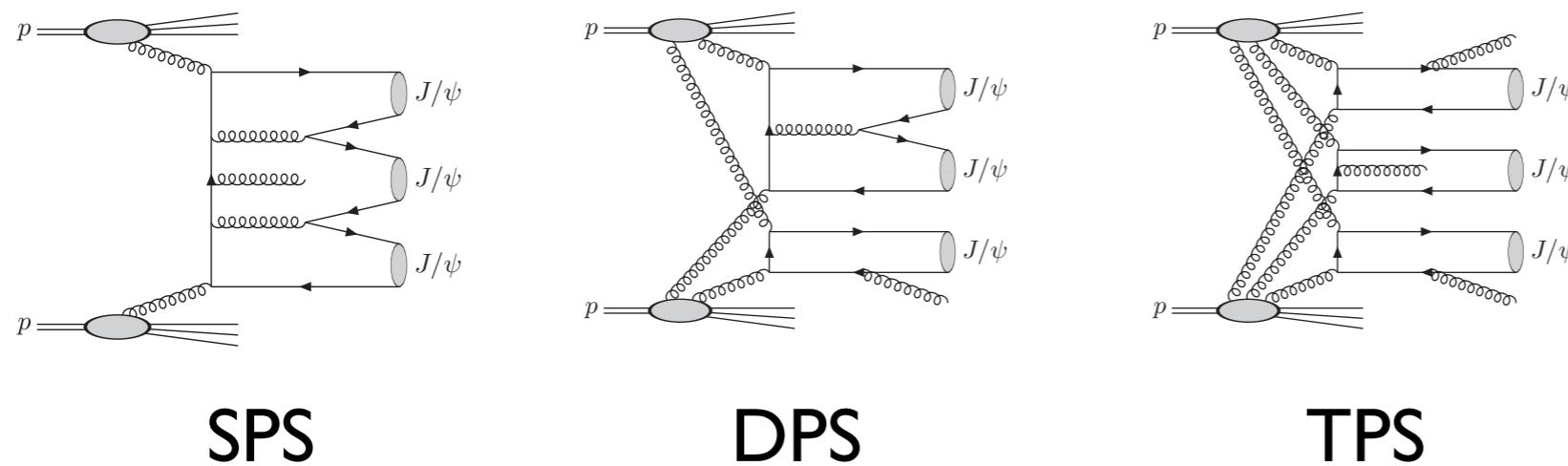
- Any chance to see TPS at the LHC ?



$$J/\psi J/\psi J/\psi \rightarrow \mu^+ \mu^- \mu^+ \mu^- \mu^+ \mu^-$$

Triple Parton Scattering in pp

- A first complete study of prompt triple J/psi as a probe of TPS



[HSS, Zhang (PRL'19)]

		inclusive	$2.0 < y_{J/\psi} < 4.5$	$ y_{J/\psi} < 2.4$
13 TeV	SPS	$0.41^{+2.4}_{-0.34} \pm 0.0083$	$(1.8^{+11}_{-1.5} \pm 0.18) \times 10^{-2}$	$(8.7^{+56}_{-7.5} \pm 0.098) \times 10^{-2}$
	DPS	$(190^{+501}_{-140}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(7.0^{+18}_{-5.1}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(50^{+140}_{-37}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$
	TPS	$130 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$1.3 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$18 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$
27 TeV	SPS	$0.46^{+2.9}_{-0.39} \pm 0.022$	$(3.2^{+22}_{-2.8} \pm 0.21) \times 10^{-2}$	$(5.8^{+39}_{-5.1} \pm 0.29) \times 10^{-2}$
	DPS	$(560^{+2900}_{-480}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(19^{+97}_{-16}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(120^{+630}_{-100}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$
	TPS	$570 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$5.0 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$57 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$
75 TeV	SPS	$0.59^{+4.4}_{-0.52} \pm 0.016$	$(3.0^{+25}_{-2.7} \pm 0.23) \times 10^{-2}$	$(7.2^{+63}_{-6.5} \pm 0.38) \times 10^{-2}$
	DPS	$(1900^{+11000}_{-1600}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(57^{+340}_{-50}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(310^{+2000}_{-270}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$
	TPS	$3900 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$27 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$260 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$
100 TeV	SPS	$1.1^{+8.4}_{-1.0} \pm 0.044$	$(4.5^{+33}_{-4.0} \pm 0.72) \times 10^{-2}$	$(36^{+290}_{-32} \pm 1.8) \times 10^{-2}$
	DPS	$(3400^{+19000}_{-2900}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(100^{+550}_{-86}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(490^{+3000}_{-430}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$
	TPS	$6500 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$45 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$380 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)^2$

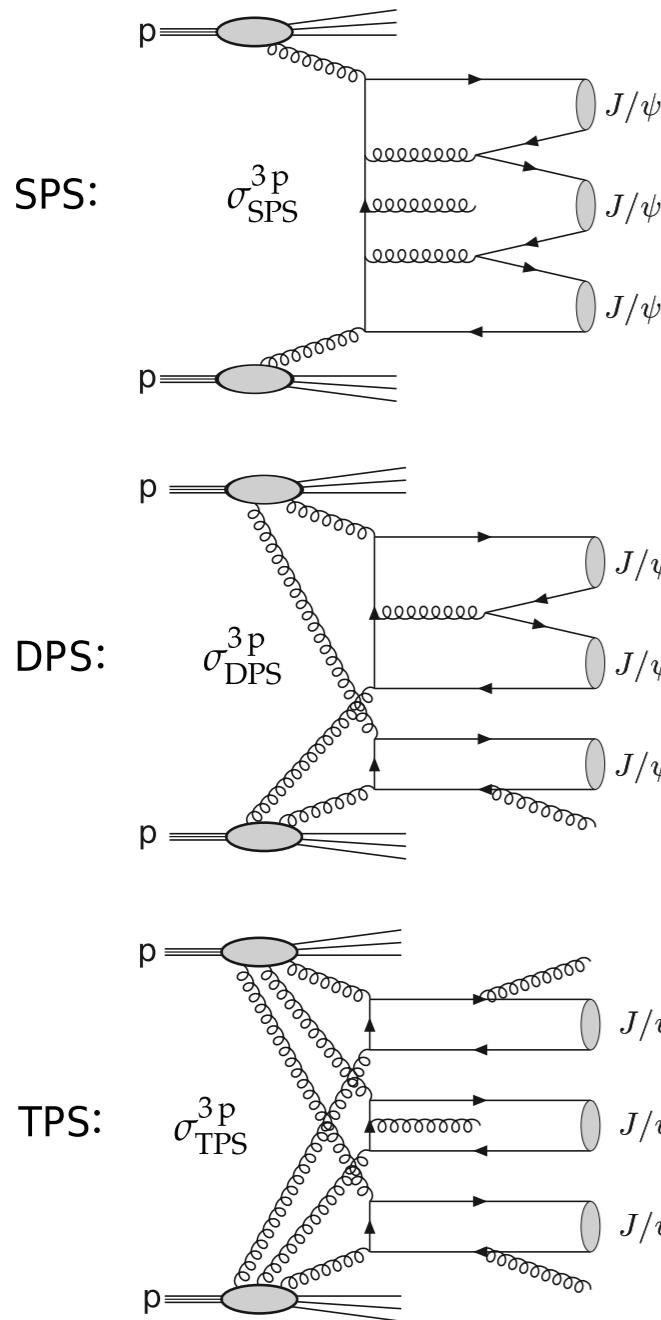
- With our knowledge of single J/psi and double J/psi, the process is predicted to be DPS and TPS dominant
- The number of events is large enough to be seen at the LHC unless $\sigma_{\text{eff},2}$ and $\sigma_{\text{eff},3}$ are significantly larger than 10 mb

Triple Parton Scattering in pp

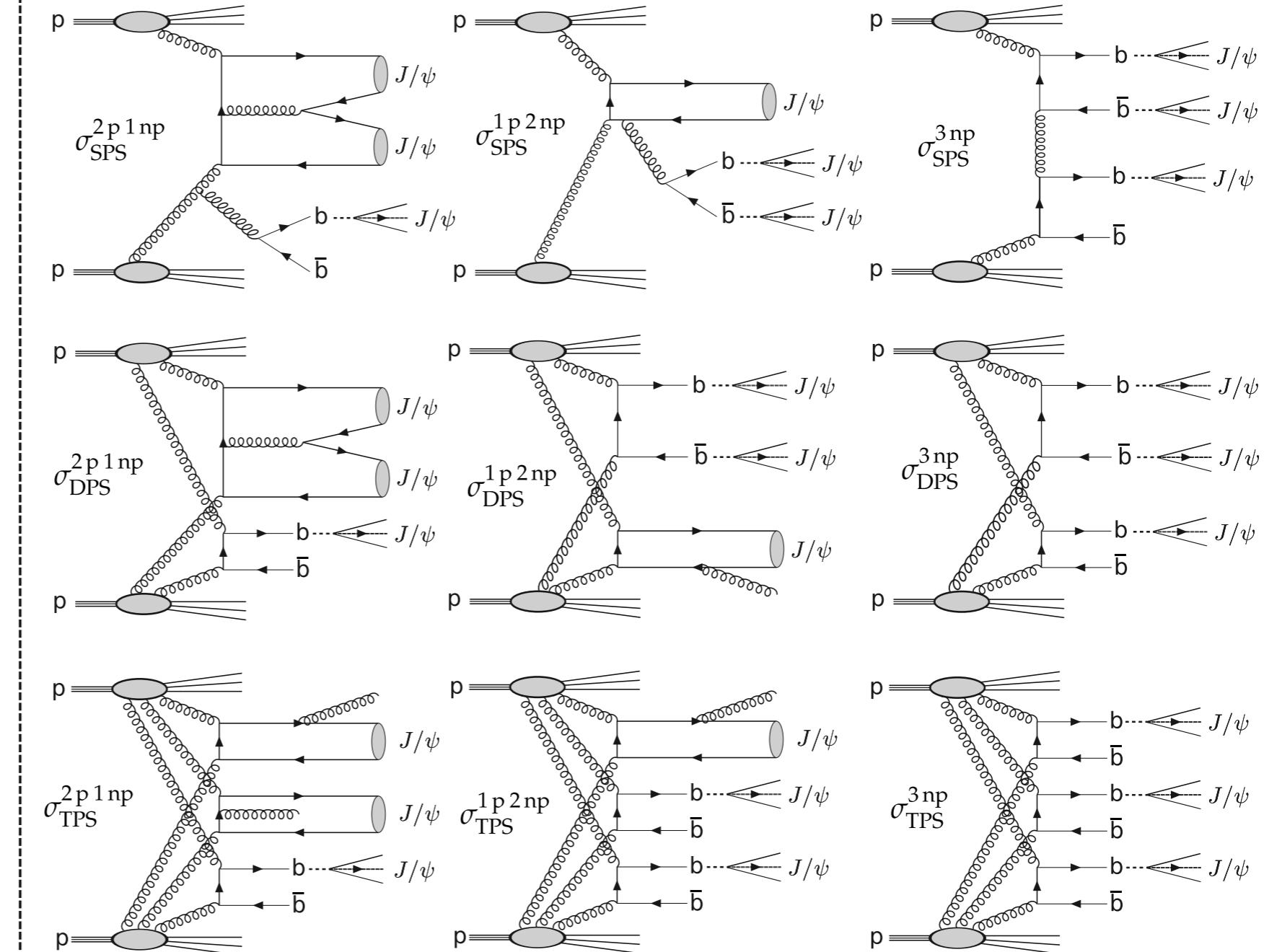
- First observation by CMS at 13 TeV in pp

[CMS (Nature Physics'23)]

Pure prompt production:

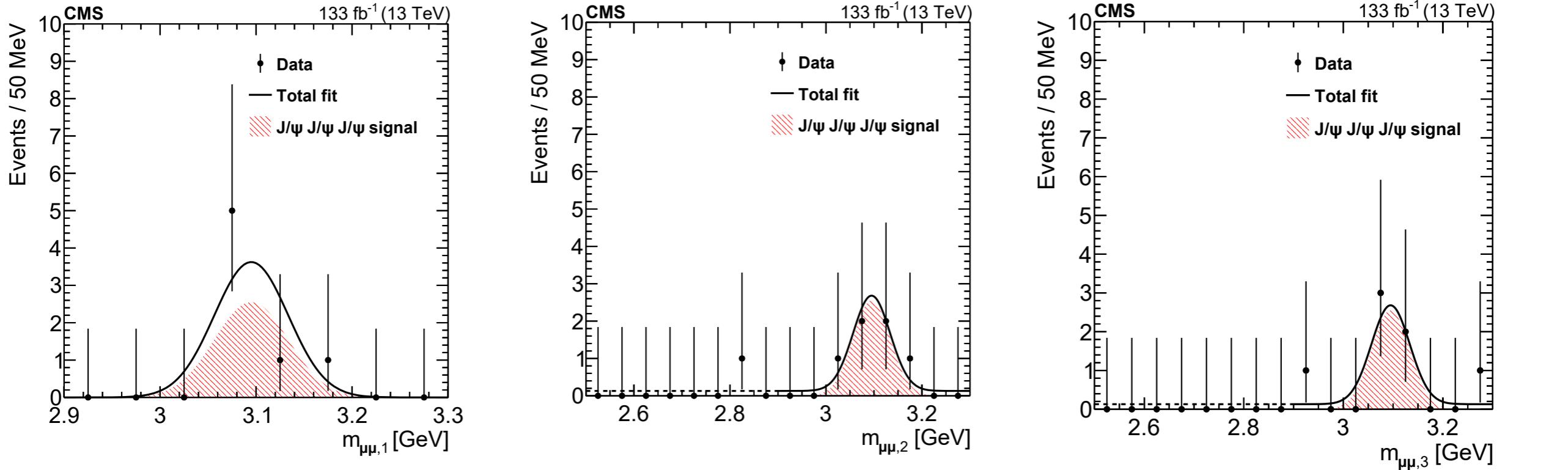


Nonprompt contributions:



Triple Parton Scattering in pp

- First observation by CMS at 13 TeV in pp



- Observation: 5 signal events + 1 background event
- The measurement of fiducial cross section

$$\sigma(pp \rightarrow J/\psi J/\psi J/\psi X) = 272^{+141}_{-104}(\text{stat}) \pm 17(\text{syst}) \text{ fb}$$

Triple Parton Scattering in pp

- **Theoretical interpretation of the CMS measurement** [CMS (Nature Physics'23)]
 - Using the pocket formula, we need to know the following theoretical inputs

SPS single-J/ ψ production		SPS double-J/ ψ production			SPS triple-J/ ψ production			
HO(DATA)	MG5NLO+PY8	HO(NLO*)	HO(LO)+PY8	MG5NLO+PY8	HO(LO)	HO(LO)+PY8	HO(LO)+PY8	MG5NLO+PY8
$\sigma_{\text{SPS}}^{1\text{p}}$	$\sigma_{\text{SPS}}^{1\text{np}}$	$\sigma_{\text{SPS}}^{2\text{p}}$	$\sigma_{\text{SPS}}^{1\text{p}1\text{np}}$	$\sigma_{\text{SPS}}^{2\text{np}}$	$\sigma_{\text{SPS}}^{3\text{p}}$	$\sigma_{\text{SPS}}^{2\text{p}1\text{np}}$	$\sigma_{\text{SPS}}^{1\text{p}2\text{np}}$	$\sigma_{\text{SPS}}^{3\text{np}}$
$570 \pm 57 \text{ nb}$	$600^{+130}_{-220} \text{ nb}$	$40^{+80}_{-26} \text{ pb}$	$24^{+35}_{-16} \text{ fb}$	$430^{+95}_{-130} \text{ pb}$	$< 5 \text{ ab}$	$5.2^{+9.6}_{-3.3} \text{ fb}$	14^{+17}_{-8} ab	$12 \pm 4 \text{ fb}$

HO: [HELAC-Onia](#)

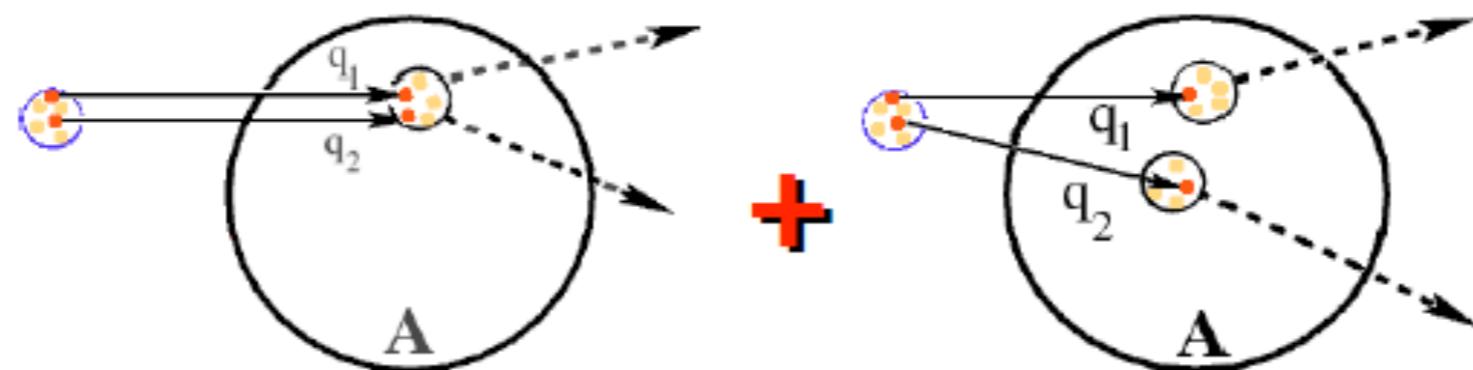
MG5NLO: [MadGraph5 aMC@NLO](#)

PY8: [Pythia8.2](#)

- Fixing $\sigma_{\text{eff},3} = (0.82 \pm 0.11) \times \sigma_{\text{eff},2}$ and fitting $\sigma_{\text{eff},2}$

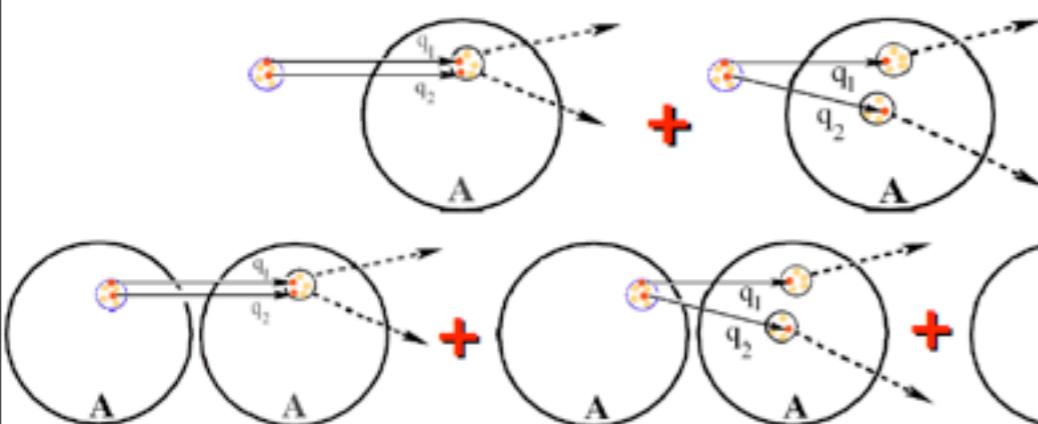
$$\sigma_{\text{eff},2} = 2.7^{+1.4}_{-1.0} (\text{exp})^{+1.5}_{-1.0} (\text{theo}) \text{ mb}$$
- Triple-J/psi fractions: ~6% SPS, ~74% DPS, ~20% TPS

DPS in heavy-ion collisions



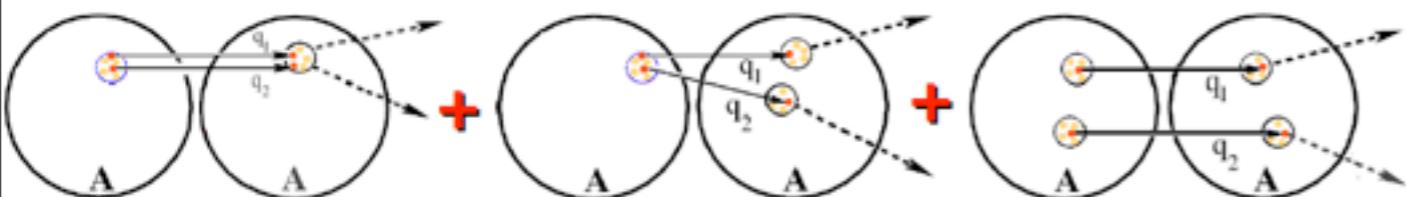
DPS in Heavy-Ion Collisions

- Geometrical enhancement because of several nucleons in a nucleus



[Strikman, Treleani (2002); D. d'Enterria, A. Snigirev (2013, 2014)]

$$\sigma_{pA}^{\text{DPS}} \approx 3A\sigma_{pp}^{\text{DPS}}, \sigma_{pA}^{\text{SPS}} \approx A\sigma_{pp}^{\text{SPS}}$$

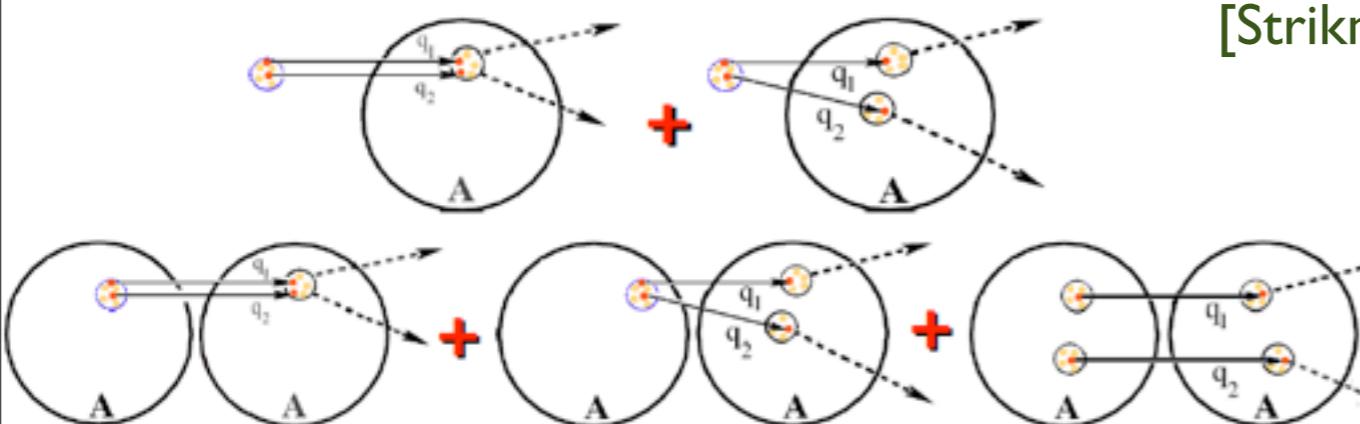


$$\sigma_{AA}^{\text{DPS}} \approx \frac{A^{3.3}}{5}\sigma_{pp}^{\text{DPS}}, \sigma_{AA}^{\text{SPS}} \approx A^2\sigma_{pp}^{\text{SPS}}$$

Assumptions: no nuclear modification and $\sigma_{\text{eff},pp} \simeq 15 \text{ mb}$

DPS in Heavy-Ion Collisions

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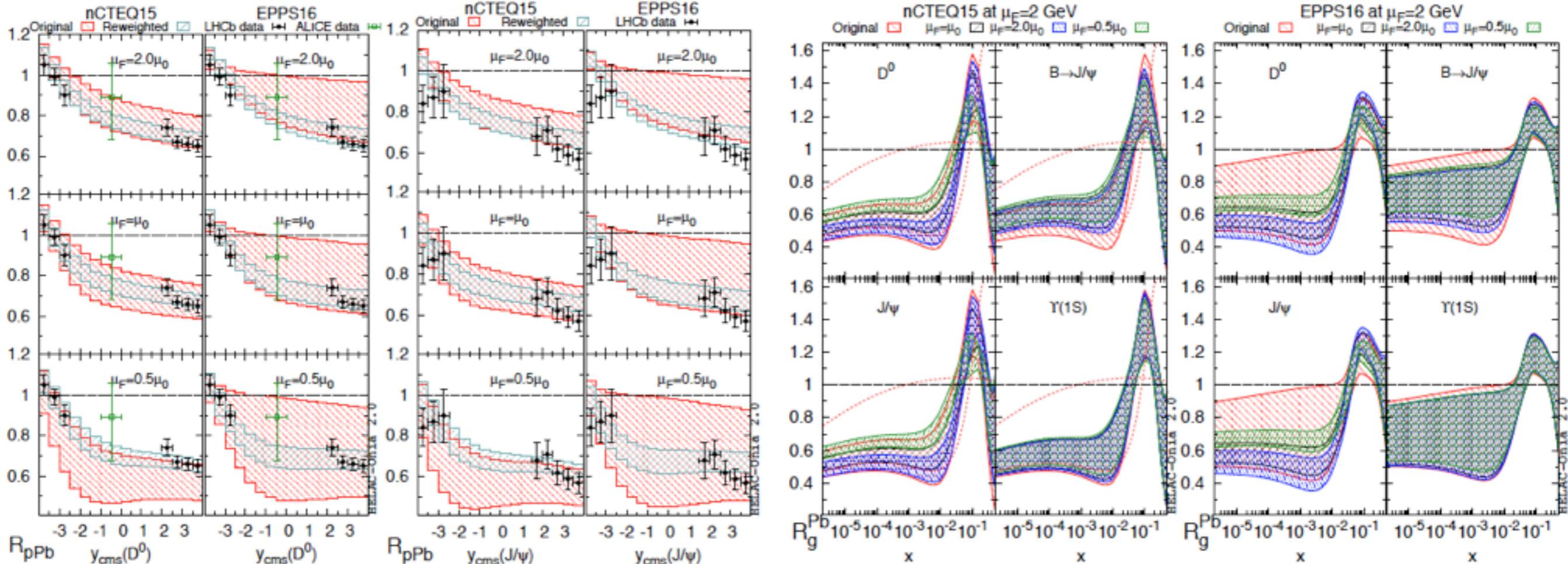


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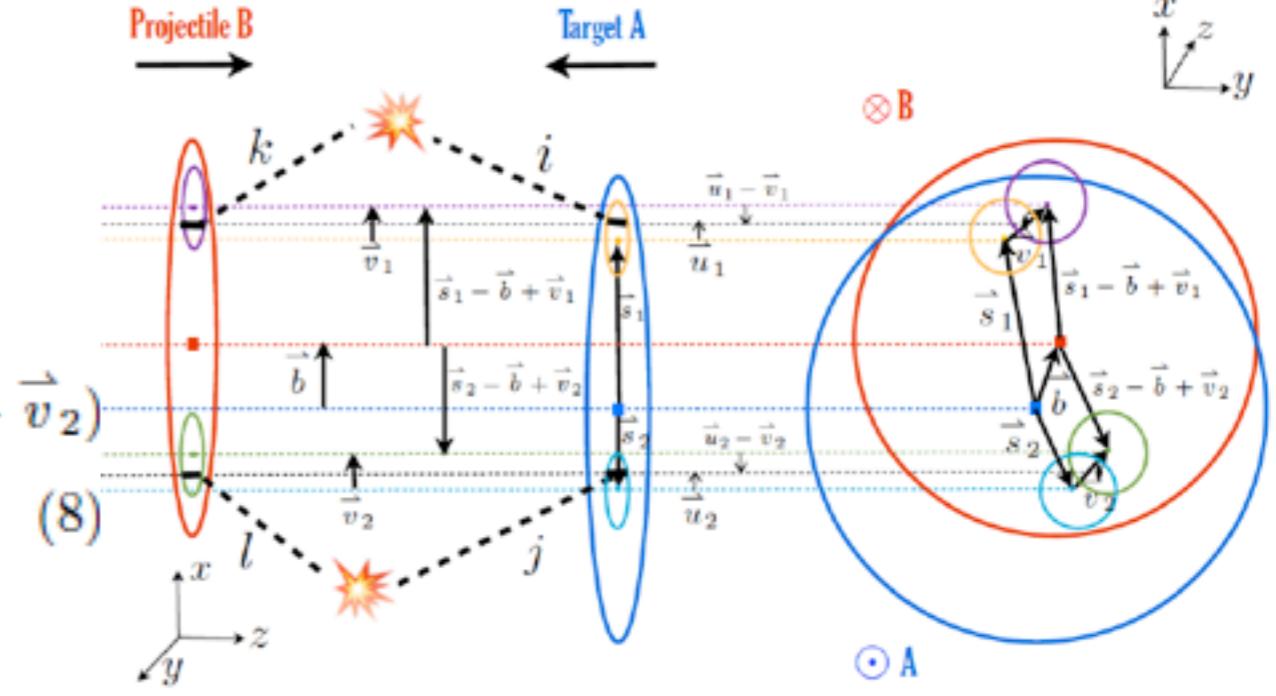
- Of course, we know we cannot neglect the nuclear modifications ...
 - E.g. gluon (anti)shadowing for heavy flavour and quarkonia [Kusina, et al. (PRL'18)]



DPS in Heavy-Ion Collisions

- Let us accommodate both nPDF and geometric effect [HSS (PRD'20)]

$$\begin{aligned}\sigma_{Q_1 Q_2} &= \frac{1}{1 + \delta_{Q_1 Q_2}} \sum_{i,j,k,l} \int dx_1 dx_2 dx'_1 dx'_2 d^2 \mathbf{b}_1 d^2 \mathbf{b}_2 d^2 \mathbf{b} \\ &\times \Gamma_{ij}(x_1, x_2, \mathbf{b}_1, \mathbf{b}_2) \hat{\sigma}_{ik}^{Q_1}(x_1, x'_1) \hat{\sigma}_{jl}^{Q_2}(x_2, x'_2) \Gamma_{kl}(x'_1, x'_2, \mathbf{b}_1 - \mathbf{b}, \mathbf{b}_2 - \mathbf{b}), \\ \sigma_{AB \rightarrow f_1 f_2}^{\text{DPS}} &= \frac{1}{1 + \delta_{f_1 f_2}} \sum_{i,j,k,l} \int dx_1 dx_2 dx'_1 dx'_2 \\ &\Gamma_A^{ij}(x_1, x_2, \vec{s}_1, \vec{s}_2, \vec{u}_1, \vec{u}_2) \hat{\sigma}_{ik}^{f_1}(x_1, x'_1) \hat{\sigma}_{jl}^{f_2}(x_2, x'_2) \times \\ &\Gamma_B^{kl}(x'_1, x'_2, \vec{s}_1 - \vec{b} + \vec{v}_1, \vec{s}_2 - \vec{b} + \vec{v}_2, \vec{u}_1 - \vec{v}_1, \vec{u}_2 - \vec{v}_2) \\ &d^2 \vec{u}_1 d^2 \vec{u}_2 d^2 \vec{v}_1 d^2 \vec{v}_2 d^2 \vec{s}_1 d^2 \vec{s}_2 d^2 \vec{b},\end{aligned}$$



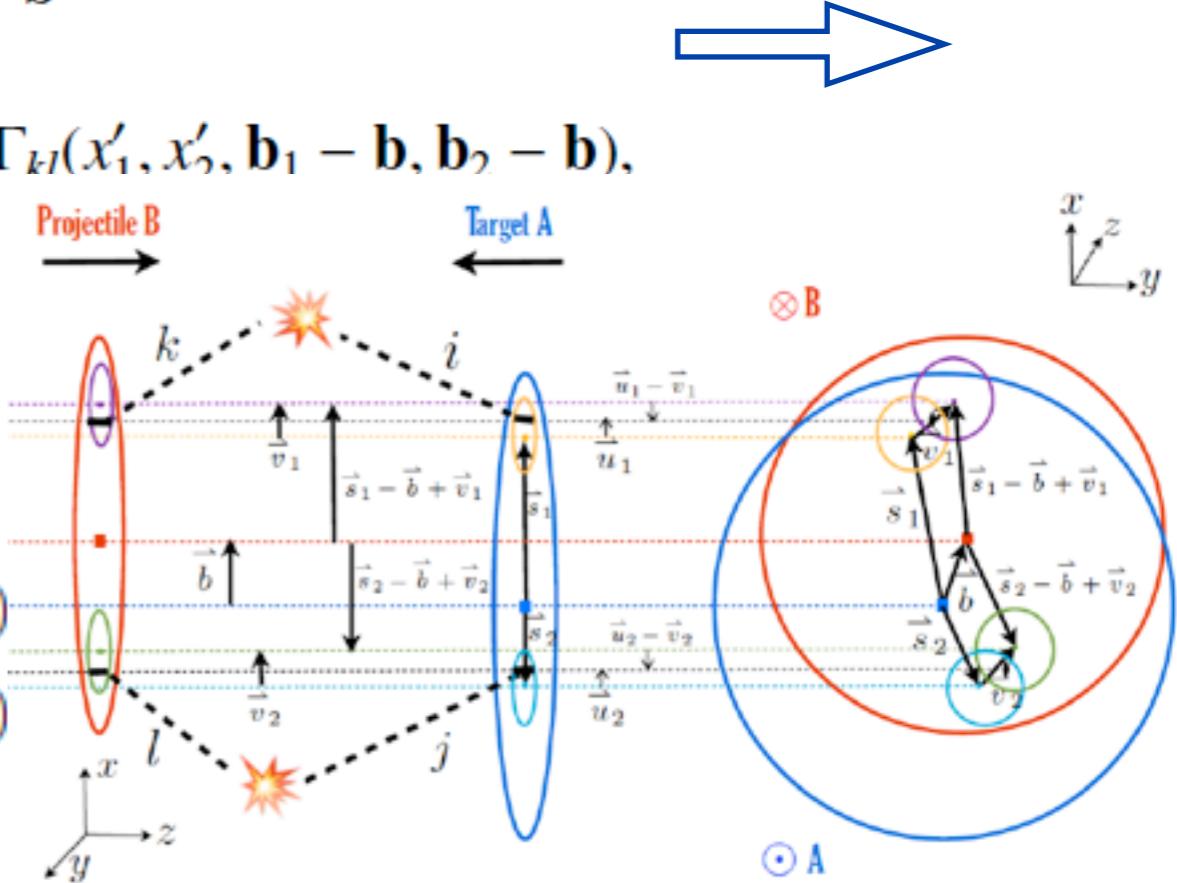
(a) Side view

(b) Beam view

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- We also need the knowledge of nuclear modification at different positions

$$R_k^A(x, \vec{b}) - 1 = (R_k^A(x) - 1) G \left(\frac{T_A(\vec{b})}{T_A(\vec{0})} \right) \quad k = g, q, \bar{q}$$

- Ubiquitous in centrality-dependent observables
- ...but most assume $G \left(\frac{T_A(\vec{s})}{T_A(\vec{0})} \right) = \frac{AT_A(\vec{s})}{T_{AA}(\vec{0})}$

DPS in Heavy-Ion Collisions

- For example, considering $p\text{Pb} \rightarrow D^0 D^0 X$ [HSS (PRD'20)]

$$\begin{aligned}
 R_{p\text{Pb} \rightarrow D^0 + D^0}^{\text{DPS}} = & R_{p\text{Pb}}^{D^0} R_{p\text{Pb}}^{D^0} \left[\frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{\text{eff},pp}}{\pi R_A^2} (A-1) \frac{9^{1-a}(a+3)^{2a}}{4(a+2)} \right] \\
 & + \left(R_{p\text{Pb}}^{D^0} + R_{p\text{Pb}}^{D^0} \right) \left[1 - \frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{\text{eff},pp}}{\pi R_A^2} (A-1) \left(\frac{3^{2-a}(a+3)^a}{2(a+4)} - \frac{9^{1-a}(a+3)^{2a}}{4(a+2)} \right) \right. \\
 & \left. + \left[-1 + \frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{\text{eff},pp}}{\pi R_A^2} (A-1) \left(\frac{9}{8} + \frac{9^{1-a}(a+3)^{2a}}{4(a+2)} - \frac{3^{2-a}(a+3)^a}{(a+4)} \right) \right] \right]
 \end{aligned}$$

$$G \left(\frac{T_A(\vec{b})}{T_A(\vec{0})} \right) \propto \left(\frac{T_A(\vec{b})}{T_A(\vec{0})} \right)^a$$

$$R_{pA}^f \equiv \frac{\sigma_{pA \rightarrow f}}{A \sigma_{pp \rightarrow f}}$$

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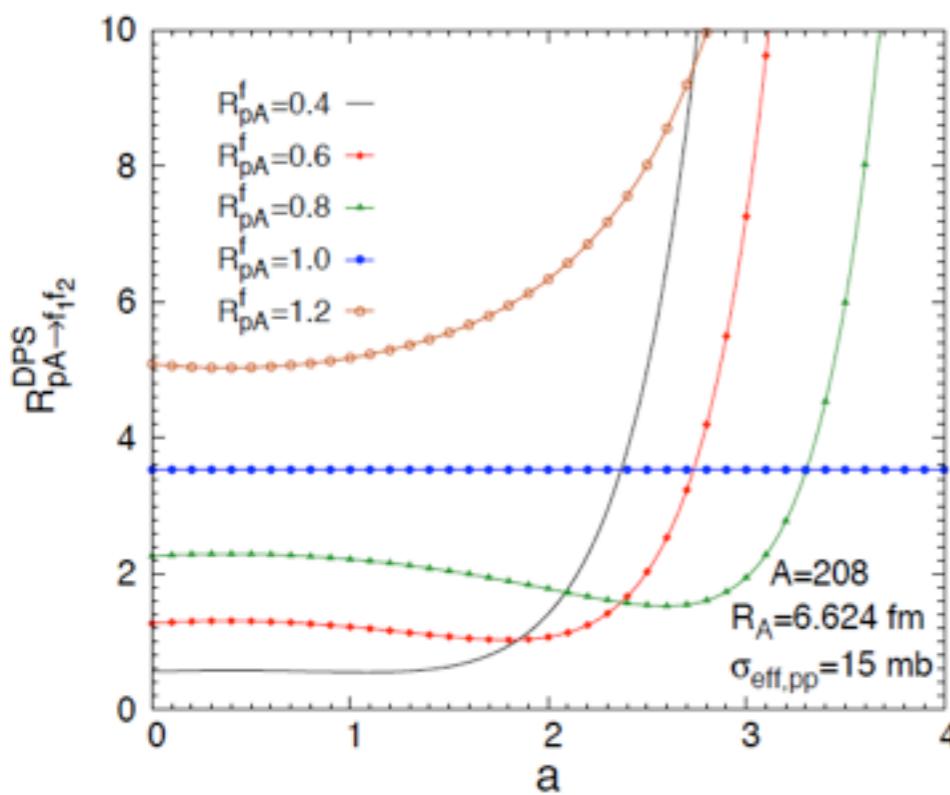
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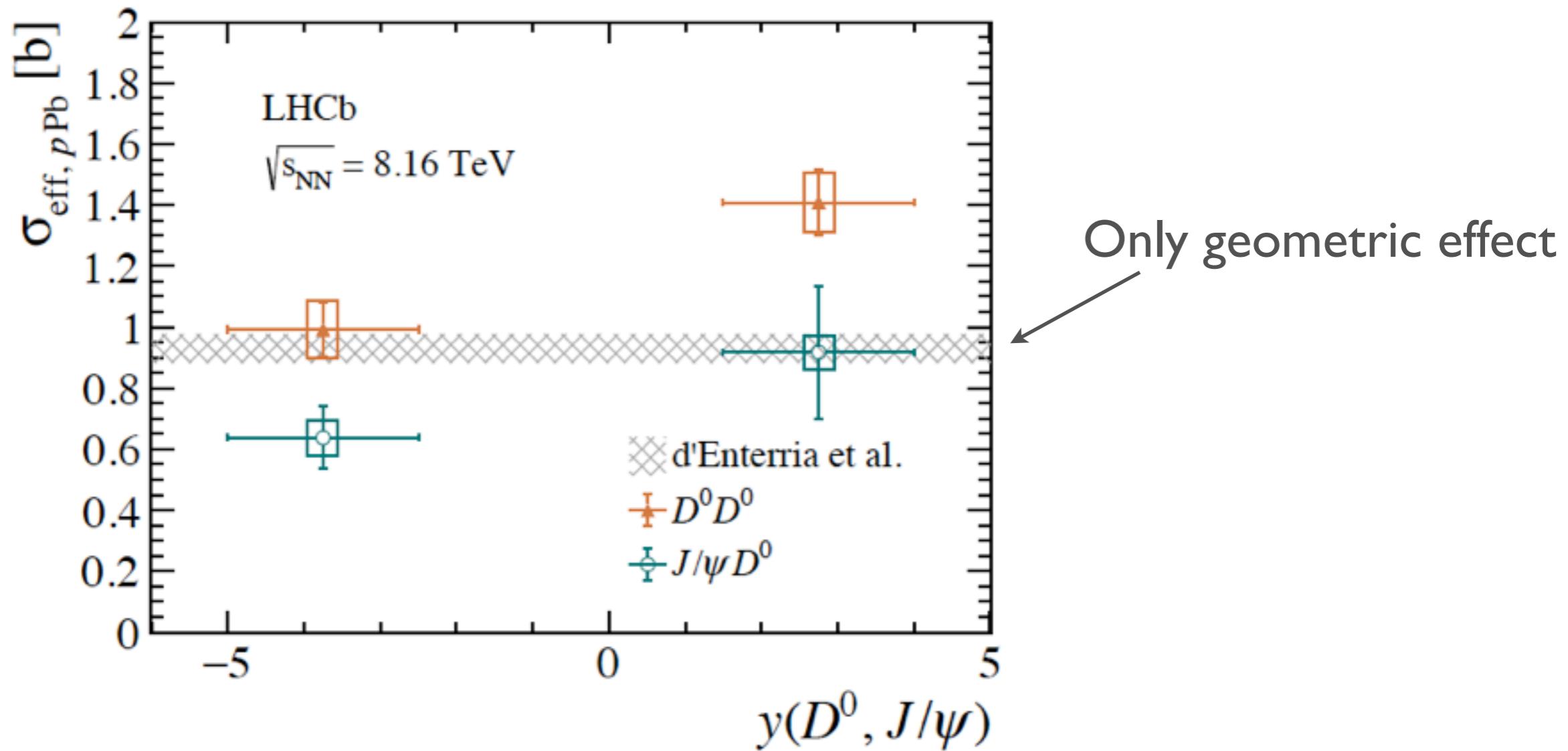
- DPS in heavy-ion potential to constrain $G()$!

DPS in Heavy-Ion Collisions

- First DPS measurement in heavy-ion collisions by LHCb [LHCb (PRL'20)]

$$\sigma_{\text{eff}} = \frac{1}{1 + \delta_{f_1 f_2}} \frac{\sigma_{p\text{Pb} \rightarrow f_1} \sigma_{p\text{Pb} \rightarrow f_2}}{\sigma_{p\text{Pb} \rightarrow f_1 f_2}}$$

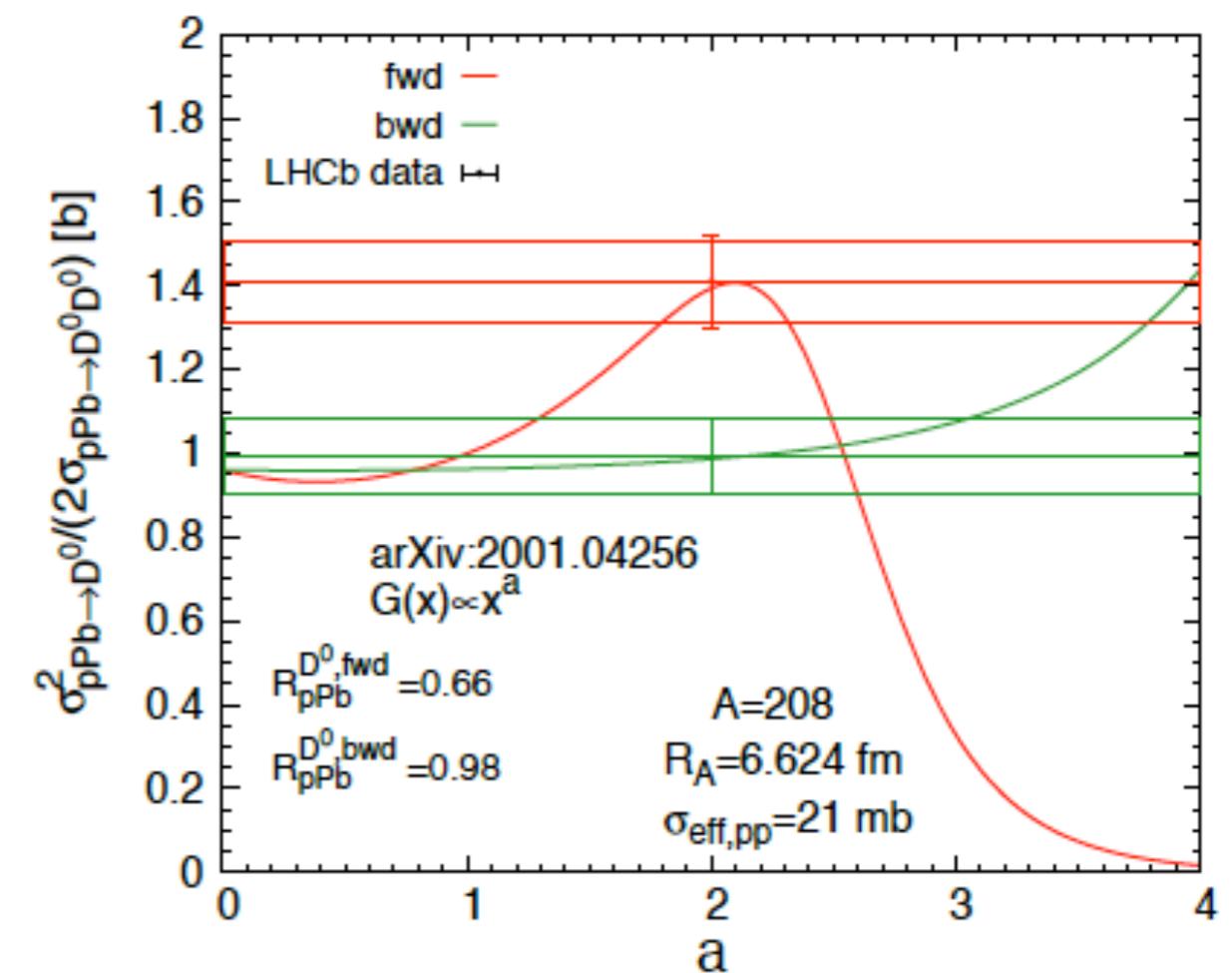
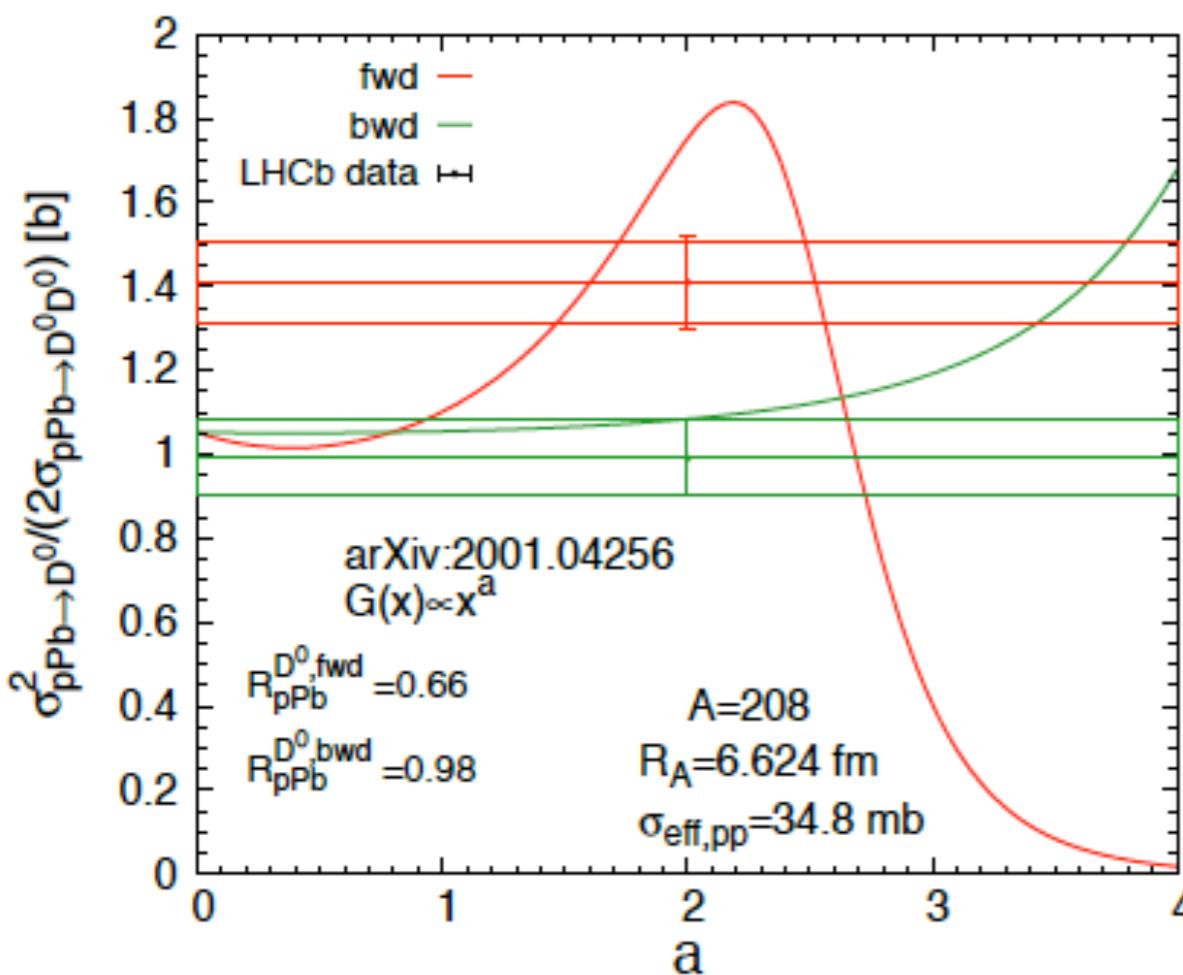
Caveats: $\sigma_{\text{eff}} \equiv \sigma_{\text{eff}, p\text{Pb}} \neq \sigma_{\text{eff}, pp}$



- Observe $\sim 3A$ enhancement in DPS wrt $\sim A$ enhancement in SPS by comparing pA vs pp xs
- The pure geometric effect cannot explain the rapidity dependence

DPS in Heavy-Ion Collisions

- Theoretical interpretation of the LHCb measurement
 - J/psi+D⁰ has the sizable SPS component [HSS (PRD'20)]
 - The SPS of D⁰+D⁰ is negligible in NLO pQCD calculations [Helenius, Paukkunen (PLB'20)]
 - The b-dependent gluon shadowing can explain the rapidity dependence [HSS (PRD'20)]



$$G \left(\frac{T_A(\vec{b})}{T_A(\vec{0})} \right) \propto \left(\frac{T_A(\vec{b})}{T_A(\vec{0})} \right)^a$$

favoring $a \sim 2$ and disfavoring $a \sim 1$

Conclusion

- LHC program offers an unprecedented avenue to study DPS & TPS.
- A lot of theoretical, phenomenological and experimental progress.
- NPS will reveal the first-ever multiple-body parton correlations in nucleon and nucleus
- Some novel observables can even tell us more (e.g. the impact parameter-dependent gluon shadowing)
- Don't be shy to attempt a 1st-ever measurement (e.g. TPS in pPb or DPS in PbPb ?)

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Thank you for your attention !

Backup Slides

DPS in Heavy-Ion Collisions

- **As a concrete example, let us take $p\text{Pb} \rightarrow J/\psi + D^0$ [HSS (PRD'20)]**

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 R_{p\text{Pb} \rightarrow J/\psi + D^0}^{\text{DPS}} = & R_{p\text{Pb}}^{J/\psi} R_{p\text{Pb}}^{D^0} \left[\frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{\text{eff},pp}}{\pi R_A^2} (A-1) \frac{9^{1-a}(a+3)^{2a}}{4(a+2)} \right] \\
 & + \left(R_{p\text{Pb}}^{J/\psi} + R_{p\text{Pb}}^{D^0} \right) \left[1 - \frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{\text{eff},pp}}{\pi R_A^2} (A-1) \left(\frac{3^{2-a}(a+3)^a}{2(a+4)} - \frac{9^{1-a}(a+3)^{2a}}{4(a+2)} \right) \right] \\
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