Open Strange Mesons in (magnetized) Nuclear Matter

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under the supervision

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9th June 2023

Presented at

HADRON 2023

UNIVERSITÀ DEGLI STUDI DI GENOVA

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Motivation and Relevance

Objective: To study in-medium properties* of **open strange mesons** under extreme conditions.

- **High baryon density :** several times the nuclear matter saturation density ($\rho_0 = 0.15 \text{ fm}^{-3}$).
- **Isospin asymmetry:** unequal proton and neutron number density.
- **High magnetic field** : upto **15** $\boldsymbol{m_\pi^2} \sim \mathbf{10^{19} Gauss}$[1,2].

➢ produced in ultrarelativistic heavy ion collision (HIC) experiments as well as in certain astronomical objects.

➢ * In-medium masses from the medium-modified scalar quark and gluon condensates.
  * In-medium decay widths from medium modified masses of mesons.

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[1] V. Skokov, A. Y. Illarionov and V. Toneev, Int. J. Mod. Phys. A 24, 5925 (2009).
[2] D. Kharzeev, L. McLerran and H. Warringa, Nucl. Phys. A 803, 227 (2008); K. Fukushima, D. Kharzeev and H. Warringa, Phys. Rev. D 78, 074033 (2008).

Effective field theories (EFTs):

- In low energy regime: α_s is large, Perturbative techniques can't be applied.

- Construction of EFTs in terms of hadronic degrees of freedom

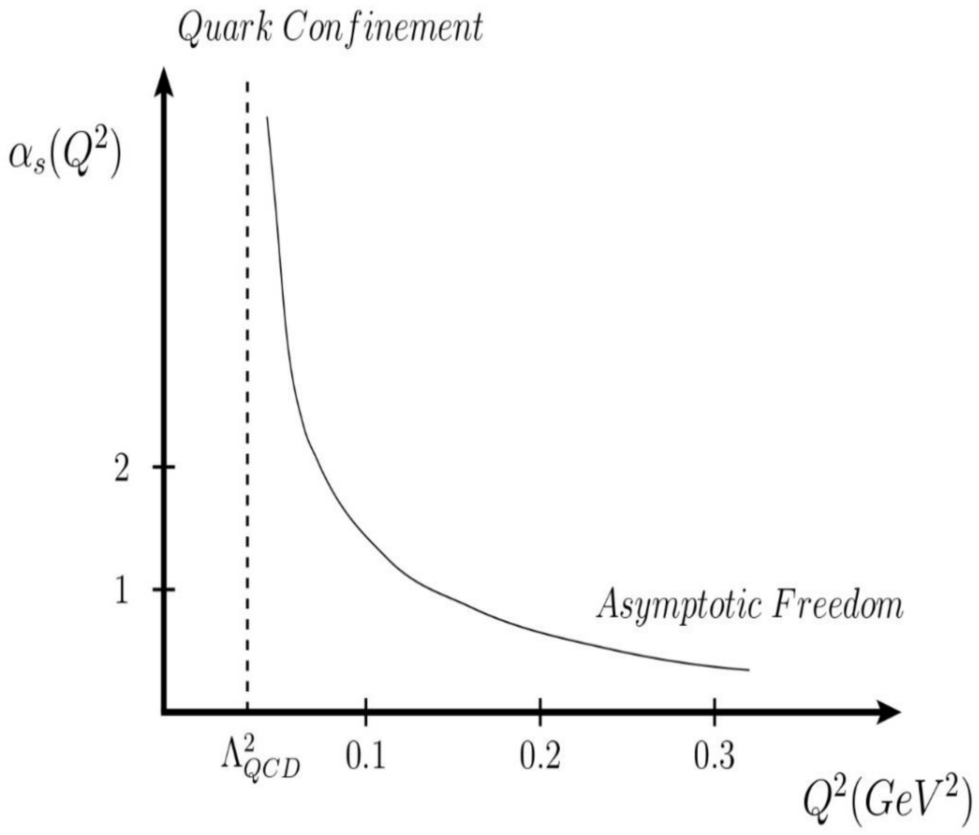
- Based on symmetries and symmetry breaking patterns of low energy QCD.

(1) Spontaneous Breaking of Chiral Symmetry (SCSB):

The QCD Lagrangian is symmetric under chiral transformations, but the ground state is not.

- **SCSB** → 8 massless Goldstone Bosons: **Pseudoscalar mesons.**
- SCSB leads to QCD vacuum being populated by **scalar chiral condensates**

$$\langle 0|\bar{\psi}\psi|0\rangle = \langle\bar{\psi}\psi\rangle = \langle\bar{u}u\rangle + \langle\bar{d}d\rangle + \langle\bar{s}s\rangle$$



(2) Scale invariance Breaking: In massless quarks limit, under scale transformation $x \to \lambda x$,

Action, $S = \int \mathcal{L}\, d^4x$ is invariant

$\Rightarrow$ conserved dilaton current, $j^{\mu}_{\text{dilaton}} = x_\nu \theta^{\mu\nu},\ and\ \partial_\mu\left(j^{\mu}_{\text{dilaton}}\right) = \theta^{\nu}_{\nu} = 0$

- However, trace is non-vanishing as gluon condensates contribute,

$\theta^{\mu}_{\mu} = \frac{\beta_{\text{QCD}}}{2g} G^{a}_{\mu\nu}\, G^{a\mu\nu} + \sum_i m_i \bar{q}_i q_i,$ where at one loop level,

This is **Trace Anomaly** of QCD.

$$\beta_{\text{QCD}}(g) = \mu\frac{\partial g}{\partial \mu} = -\frac{11N_c g^3}{48\pi^2}\left(1 - \frac{2N_f}{11N_c}\right)$$

(3) **Explicit Chiral Symmetry Breaking (ECSB)**:

The quark mass term breaks the chiral invariance,

$$\mathcal{L}_m = -\left(\bar{\psi}_L m\, \psi_R + \bar{\psi}_R m\, \psi_L\right)$$

$$\mathcal{L}^{\text{QCD}}_{SB} = -\text{Tr}\left[\text{diag}\left(m_u \bar{u} u, m_d \bar{d} d, m_s \bar{s} s\right)\right]$$

- Due to **ECSB,** pseudoscalar mesons get mass.

Chiral $SU(3)_L \times SU(3)_R$ Model

The effective hadronic chiral Lagrangian density[1,2] :

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \sum_W \mathcal{L}_{BW} + \mathcal{L}_{\rm vec} + \mathcal{L}_0 + \mathcal{L}_{\rm scale\ break} + \mathcal{L}_{SB} + \mathcal{L}_{\rm mag}$$

$B = \text{baryons}, W = \text{mesons.}$

$\Longrightarrow$ nonlinear coupled quantum field equations with large couplings.

Mean-field approximation:

- All the meson fields are treated as classical fields.
- Further, $\langle \bar{\psi}_i \psi_j \rangle = \delta_{ij} \rho_i^s$; $\langle \bar{\psi}_i \gamma^\mu \psi_j \rangle = \delta_{ij} \delta^{\mu 0} \rho_i$; $\rho_i^s (\rho_i) \equiv$ scalar (number) density of *i*th baryon.
- $\mathcal{L}_{BS} \rightarrow$ baryon mass generation, $m_i^* = -\left(g_{\sigma i}\, \sigma + g_{\zeta i} \zeta + g_{\delta i} \delta\right)$

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[1] P. Papazoglou, D. Zschiesche, S. Schramm, J. Schaffner-Bielich, H. Stöcker, and W. Greiner, Phys. Rev. C 59, 411 (1999). [2] A. Mishra, K. Balazs, D. Zschiesche, S. Schramm, H. Stöcker, and W. Greiner, Phys. Rev. C 69, 024903 (2004).

- $\mathcal{L}_0$ incorporates SCSB through non-zero VEVs of scalar fields (σ, ζ, δ)

- **Explicitly chiral symmetry breaking** term, $\mathcal{L}_{SB} = \mathrm{Tr}\left[\mathrm{diag}\left(-\frac{m_\pi^2 f_\pi(\sigma+\delta)}{2}, -\frac{m_\pi^2 f_\pi(\sigma-\delta)}{2}, -(\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}} m_\pi^2 f_\pi)\zeta\ \right)\right]$

  Quark condensates are related with the scalar fields as,

$$\boldsymbol{m_u \langle \bar{u}u \rangle = \frac{m_\pi^2 f_\pi(\sigma+\delta)}{2}, \qquad m_d \langle \bar{d}d \rangle = \frac{m_\pi^2 f_\pi(\sigma-\delta)}{2}, \quad m_s \langle \bar{s}s \rangle = (\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}} m_\pi^2 f_\pi)\zeta}$$

- **Scale invariance breaking** term, $\mathcal{L}_{\text{scale break}} = -\frac{1}{4}\chi^4 \ln\left(\frac{\chi^4}{\chi_0^4}\right) + \frac{d}{3}\chi^4 \ln\left(\left(\frac{(\sigma^2-\delta^2)\zeta}{\sigma_0^2\zeta_0}\right)\left(\frac{\chi}{\chi_0}\right)^3\right)$

$$\boldsymbol{\left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu}\, G^{a\mu\nu} \right\rangle = \frac{8}{9}\left\{(1-d)\chi^4 + \left[m_\pi^2 f_\pi \sigma + (\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}} m_\pi^2 f_\pi)\zeta\right]\right\}}$$

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[1] A. E. Broderick, M. Prakash, and J. M. Lattimer, Phys. Lett. B 531, 167 (2002).

QCD Sum Rule (QCDSR) approach

The current-current correlation function[1,2],

$$\Pi_{\mu\nu}(q) = i\int d^4x\, e^{iq.x}\,\langle 0\left|T j_\mu^V(x) j_\nu^V(0)\right|0\rangle\,; \qquad j_\mu^V \text{ is the vector meson current}$$

The scalar correlator function, $\Pi^V(q^2) = \frac{1}{3} g^{\mu\nu}\Pi_{\mu\nu}^V(q)$

(a) For large space-like region $Q^2 = -q^2 \gg 1\ GeV^2$, using operator product expansion (OPE)

$$12\pi^2\widetilde{\Pi}^V(q^2 = -Q^2) = d_V\left[-c_0^V \ln\left(\frac{Q^2}{\mu^2}\right) + \frac{C_1^V}{Q^2} + \frac{C_2^V}{Q^4} + \frac{C_3^V}{Q^6}\right]; \qquad \text{with } \widetilde{\Pi}^V(q^2 = -Q^2) = \frac{\Pi^V(q^2=-Q^2)}{Q^2}$$

The scale $\mu = 1\ GeV$, c_0^V term for perturbative QCD; $\mathrm{c}_2^{\mathrm{V}}$, $\mathrm{c}_3^{\mathrm{V}}$ for non-perturbative effects of QCD.

(b) On phenomenological side, $12\pi^2\widetilde{\Pi}_{\text{phen}}^V(Q^2) = \int_0^\infty ds\,\frac{R_{\text{phen}}^V(s)}{s+Q^2}$; spectral density, $R_{\text{phen}}^V(s) = 12\pi\,\mathrm{Im}\Pi_{\text{phen}}^V(s)$

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[1] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B 147, 385 (1979); ibid, Nucl. Phys. B 147, 448 (1979).
[2] F. Klingl, N. Kaiser, and W. Weise, Nucl. Phys. A 624, 527 (1997); ibid, Z. Phys. A 356, 193 (1996).

The spectral density separates to resonance part and a perturbative continuum,

$$R^V_{\text{phen}}(s) = R^{V(\text{res})}_{\text{phen}}\theta(s^V_0 - s) + d_V c^V_0 \theta(s - s^V_0)$$

Borel transformation: $\hat{f}(M^2) = \lim_{\substack{Q^2\to\infty, n\to\infty \\ \frac{Q^2}{n}=M^2=\text{constant}}} \frac{1}{(n-1)!}(Q^2)^n \left(-\frac{d}{dQ^2}\right)^n f(Q^2)$

$$12\pi^2 \tilde{\Pi}^V_{\text{phen}}(M^2) = \int_0^\infty ds\, e^{-s/M^2}\, R^V_{\text{phen}}(s),$$ … phenomenological side

$$12\pi^2 \tilde{\Pi}^V(M^2) = d_V\left[c^V_0 M^2 + c^V_1 + \frac{c^V_2}{M^2} + \frac{c^V_3}{2M^4}\right];$$ … OPE side

For $M > \sqrt{s^V_0}$, exponential term in the integral is expanded in powers of $\frac{s}{M^2}$ for $s < s^V_0$.

The finite energy sum rules (FESRs) are[1],

$$F^*_V = d_V(c^V_0 s^{*V}_0 + c^V_1)$$

$$F^*_V m^{*2}_V = d_V\left(\frac{(s^{*V}_0)^2 c^V_0}{2} - c^{*V}_2\right)$$

$$F^*_V m^{*4}_V = d_V\left(\frac{(s^{*V}_0)^3 c^V_0}{3} + c^{*V}_3\right)$$

F^*_V is the overlap strength; s^{*V}_0 is the delineation scale.

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[1] A. Mishra, Phys. Rev. C 91, 035201 (2015).

In-medium properties of open strange mesons

Strange Vector mesons : $K^{*+}(\bar{s}\gamma^{\mu}u)$; $K^{*0}(\bar{s}\gamma^{\mu}d)$.

Strange Axial Vector Meson : $K_1^{+}(\bar{s}\gamma^{\mu}\gamma^{5}u)$; $K_1^{0}(\bar{s}\gamma^{\mu}\gamma^{5}d)$.

The coefficients $c_i's$ for the charged mesons are given by[1,2]

$$c_0^{K^*,K_1} = 1 + \frac{\alpha_s(Q^2)}{\pi}; \text{ and } \quad c_1^{K^*,K_1} = -3(m_u^2 + m_s^2);$$

$$c_2^{K^*,K_1} = \frac{\pi^2}{3}\langle\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}\rangle \pm \frac{16\pi^2}{d_{K^*}}\langle m_u\bar{s}s + m_s\bar{u}u\rangle \mp \frac{4\pi^2}{d_{K^*}}\langle m_s\bar{s}s + m_u\bar{u}u\rangle$$

$$c_3^{K^*,K_1} = -8\alpha_s\pi^3\kappa_q\left(\pm\frac{32}{9}(\langle\bar{s}s\rangle\langle\bar{u}u\rangle) + \frac{32}{81}(\langle\bar{u}u\rangle^2 + \langle\bar{s}s\rangle^2)\right)$$

For axial vector meson, $R_{\text{phen}}^{K_1}(s) = f_K^2\delta(s - m_K^2) + F_{K_1}\delta\left(s - m_{K_1}^2\right) + d_{K_1}c_0^{K_1}\theta(s - s_0^{K_1})$

The FESRs are:

$$F_{K_1}^* = d_{K_1}\left(c_0^{K_1}s_0^{*K_1} + c_1^{K_1}\right) - f_K^2$$

$$F_{K_1}^* m_{K_1}^{*2} = d_{K_1}\left(\frac{\left(s_0^{*K_1}\right)^2 c_0^{K_1}}{2} - c_2^{*K_1}\right) - f_K^2 m_K^2$$

$$F_{K_1}^* m_{K_1}^{*4} = d_{K_1}\left(\frac{\left(s_0^{*K_1}\right)^3 c_0^{K_1}}{3} + c_3^{*K_1}\right) - f_K^2 m_K^4$$

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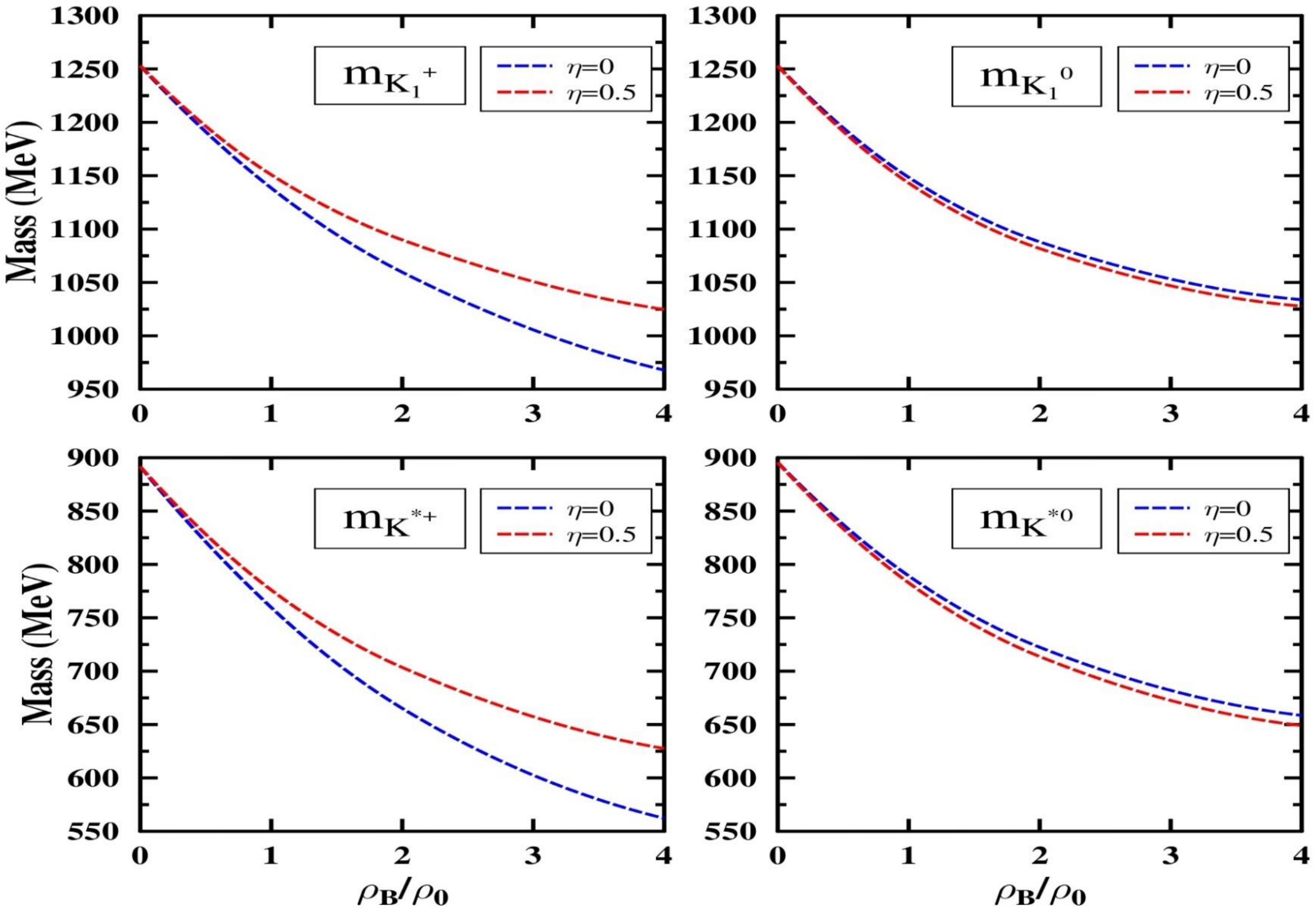
[1] T. Song, T. Hatsuda and S. H. Lee, Phys. Lett. B 792, 160 (2019).
[2] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B 147, 385 (1979); ibid, Nucl. Phys. B 147, 448 (1979).

Vacuum masses[1]:
$K^{*+} = 891.67$ MeV;
$K^{*0} = 895.55$ MeV;
$K_1^{+} = 1253$ MeV;
$K_1^{0} = 1253$ MeV;

[1] R.L. Workman et al.(Particle Data Group), Prog. Theor. Exp. Phys. 2022, 083C01 (2022).



Masses of K^* and K_1 mesons plotted as a function of ρ_B/ρ_0 for various values of isospin asymmetry parameter (η), within QCD sum rule approach.

*Ankit Kumar and Amruta Mishra, arXiv:2302.14493 [hep-ph].
*Ankit Kumar and Amruta Mishra, Proceedings of the DAE Symp. on Nucl. Phys. **65**, 617 (2021).

Effects of Strong Magnetic fields

Landau Quantization: The charged particles undergo Landau quantization[1],

$$m_P(eB) = \sqrt{[m_P^2 + (2n+1)eB + p_z^2]};$$

$$m_V(eB) = \sqrt{[m_V^2 + (2n+1)eB + p_z^2 + gS_z eB]};$$

Spin Magnetic Field Interaction: Due to the spin-magnetic field interaction $(-\mu_i . B)$ term[2],

$$(-\mu_i . B)|00\rangle = \frac{gB}{4}\left(\frac{q_1}{m_1} - \frac{q_2}{m_2}\right)|10\rangle$$

The transverse polarized states do not get mixed

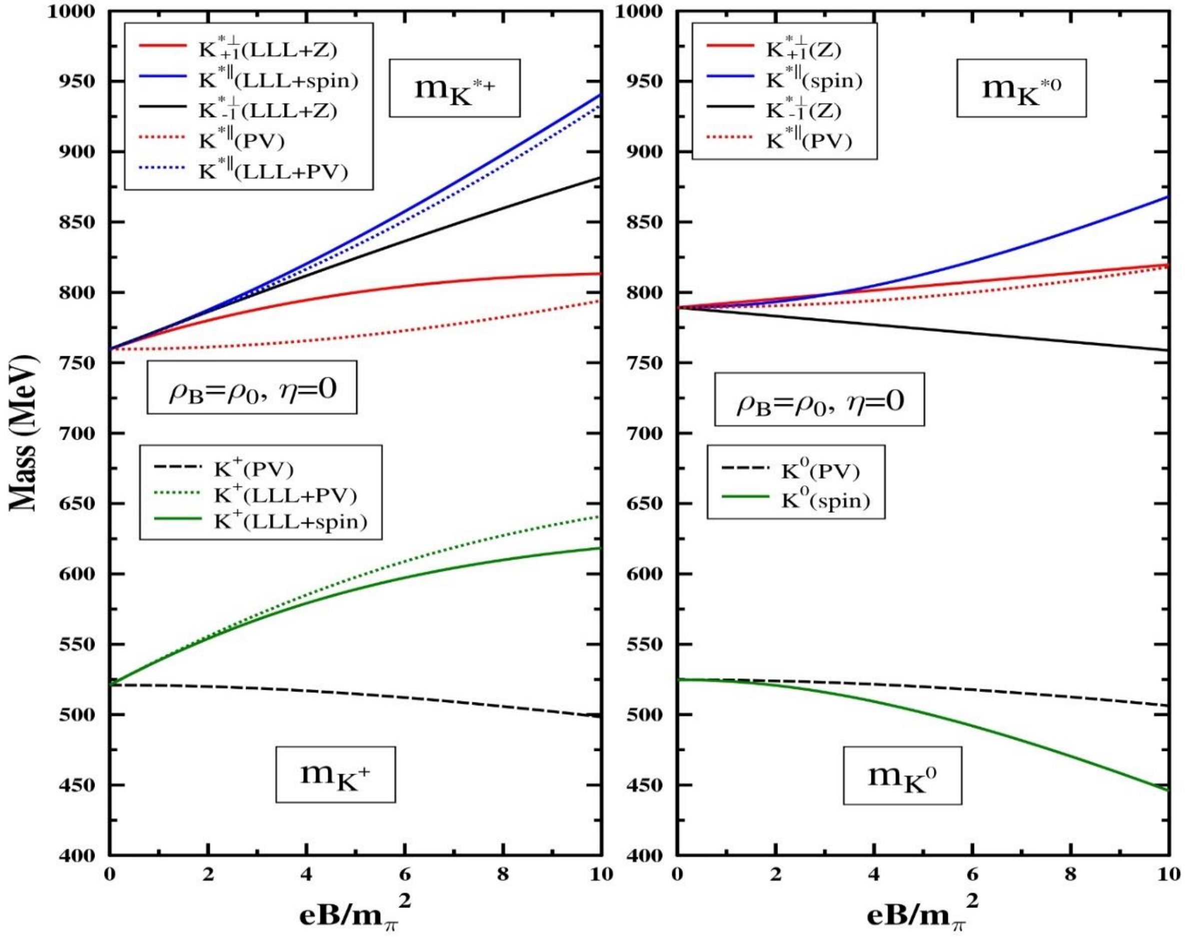
$$(-\mu_i . B)|1 \pm 1\rangle = \mp\frac{gB}{4}\left(\frac{q_1}{m_1} + \frac{q_2}{m_2}\right)|1 \pm 1\rangle$$

where μ_i is the magnetic moment, S_z is the spin projection, p_z is continuous momentum.

PV mixing: The mixing can also be introduced through the interaction Lagrangian

$$\mathcal{L}_{PV\gamma} = \frac{g_{PV}}{m_{\text{avg}}} e\, \tilde{F}_{\mu\nu}(\partial^\mu P)V^\nu$$

[1] P. Gubler, K. Hattori, S. H. Lee, M. Oka, S. Ozaki and K. Suzuki, Phys. Rev. D 93, 054026 (2016).
[2] J. Alford and M. Strickland, Phys. Rev. D 88, 105017 (2013).



Masses of vector K^* and pseudoscalar K mesons plotted as a function of eB/m_π^2 at $\rho_B = \rho_0$. The three polarization states are written as $K^{*\perp}_{+1}$, $K^{*||}$, and $K^{*\perp}_{-1}$.

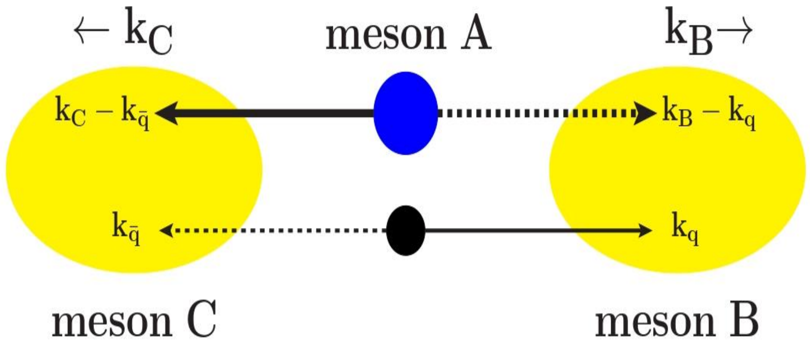
*Ankit Kumar and Amruta Mishra, arXiv:2302.14493 [hep-ph].
*Ankit Kumar and Amruta Mishra, Proceedings of the DAE Symp. on Nucl. Phys. **66**, 841 (2022).

Decay width of $K^* \to K\pi$ in ${}^3P_0$ model

- A light $\bar{q}q$ pair is assumed to be produced with 3_{P_0} quantum numbers.

The matrix element for the general decay $A \to BC$,

$$M_{\mathrm{A \to BC}} = \langle A | \gamma [\bar{q}_s q_s]^{3P_0} | BC \rangle$$



The decay width is then given by[1]

$$\Gamma(K^* \to K\pi) = 2\pi \frac{p_K \, E_K E_\pi}{M_{K^*}} \sum_{LS} |M_{LS}|^2$$

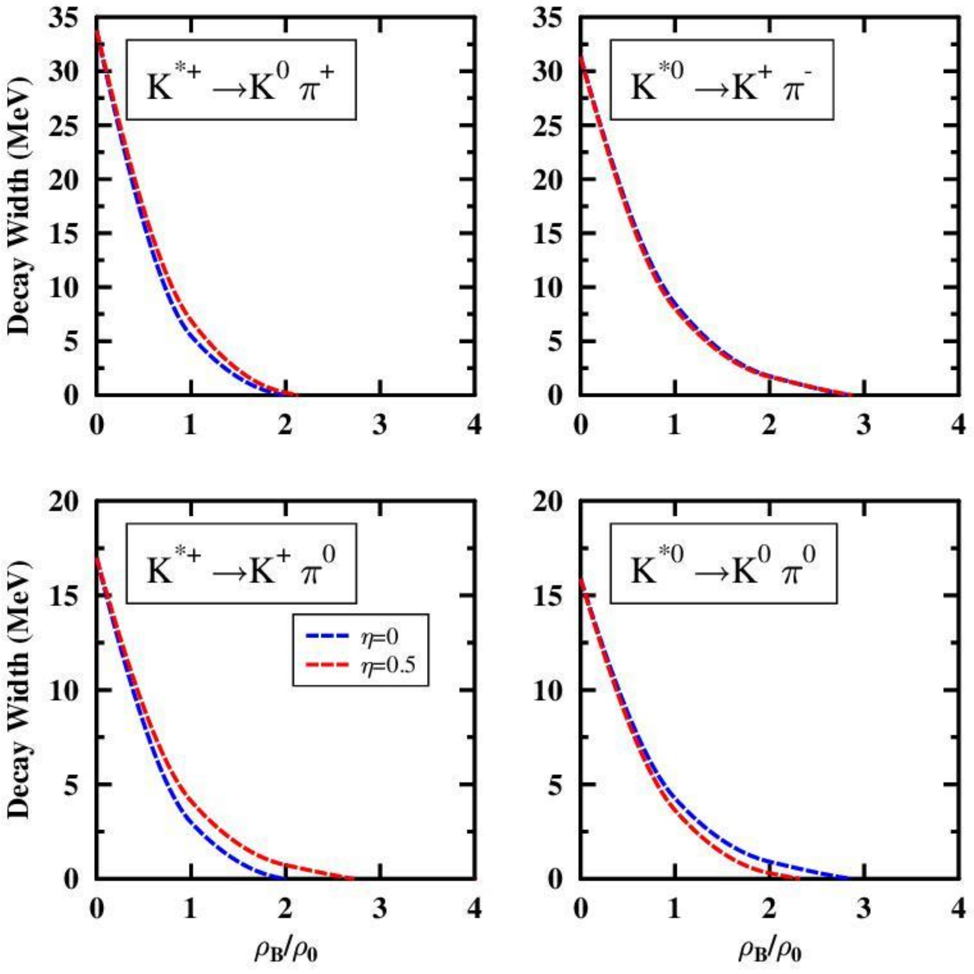
The coupling strength γ is related to strength of 3_{P_0} vertex.

- In vacuum[2]: $\Gamma(K^{*+} \to K^+\pi^0,\ K^0\pi^+) = (16.98, 33.77)$ MeV; $\gamma = (0.1243, 0.1782)$
  $\Gamma(K^{*0} \to K^0\pi^0,\ K^+\pi^-) = (15.87, 31.31)$ MeV; $\gamma = (0.1198, 0.1673)$

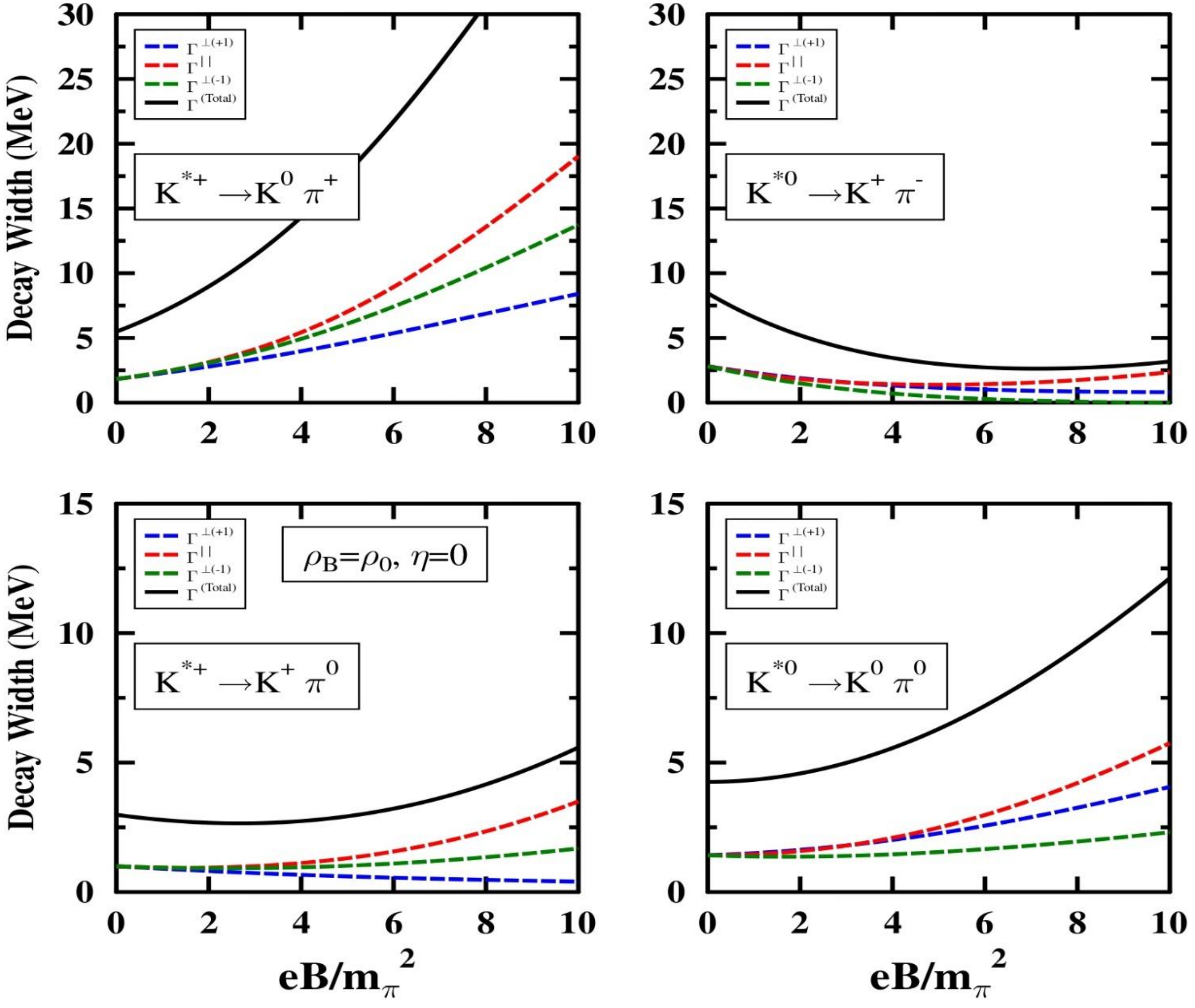
- Within the chiral model[3], m_{K^+} *and* m_{K^0} are observed to increase as a function of ρ_B.

- The pions are assumed to retain their vacuum properties during this work.

[1] T. Barnes, F.E. Close, P.R. Page, and E.S. Swanson, Phys. Rev. D 55, 4157 (1997); B. Friman, S.H. Lee, and T. Song, Phys. Lett. B 548, 153 (2002).
[2] P. A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020) and 2021 update.
[3] A. Mishra, A. K. Singh, N. S. Rawat, and P. Aman, Eur. Phys. J. A 55, 107 (2019).



Partial decay widths of vector K^* meson are plotted as a function of ρ_B/ρ_0 at zero magnetic field (eB), as calculated within 3_{P_0} model.



Partial decay widths of vector K^* meson with polarization states (+1,0,-1) plotted as a function of eB/m_π^2 at $\rho_B = \rho_0$ with $\eta = 0$, within 3_{P_0} model.

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*Ankit Kumar and Amruta Mishra, arXiv:2302.14493 [hep-ph].

*Ankit Kumar and Amruta Mishra, Proceedings of the DAE Symp. on Nucl. Phys. **66**, 841 (2022).

Decay Width of Axial Vector Meson $\boldsymbol{K_1} \rightarrow \boldsymbol{K^*\pi}$

The $\boldsymbol{K_1}$(1270) and $\boldsymbol{K_1}$(1400) mesons are mixture of the two strange axial vector nonets (1^3P_1 and 1^1P_1).

$$|K_1(1270)\rangle = \sin\theta_{K_1}|K_{1A}\rangle + \cos\theta_{K_1}|K_{1B}\rangle$$

$$|K_1(1400)\rangle = \cos\theta_{K_1}|K_{1A}\rangle - \sin\theta_{K_1}|K_{1B}\rangle; \qquad \theta_{K_1} \sim 60°$$

The decay width is given by[1]

$$\Gamma(K_1 \rightarrow K^*\pi) = 2\pi \frac{p_{K^*}\, E_{K^*} E_\pi}{M_{K_1}} \sum_{LS} |M_{LS}|^2 \; ;$$

The matrix element gets contribution from each decay channel accordingly.

- In vacuum[2] : $\Gamma(K_1^+ \rightarrow K^{*+}\pi^0,\ K^{*0}\pi^+) = (6.28, 12.32)$ MeV;
  $\Gamma(K_1^0 \rightarrow K^{*0}\pi^0,\ K^{*+}\pi^-) = (6.20, 12.48)$MeV;

- The in-medium decay width is calculated from in-medium masses of K^* and K_1 mesons calculated within QCDSR approach.

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[1] Tayduganov, E. Kou, and A. Le Yaouanc, Phys. Rev. D 85, 074011 (2012).
[2] L. Roca, J. E. Palomar, and E. Oset, Phys. Rev. D 70, 094006 (2004).

- The effects of baryon density on the decay widths reflects the in-medium changes of K_1 and K^* meson masses.

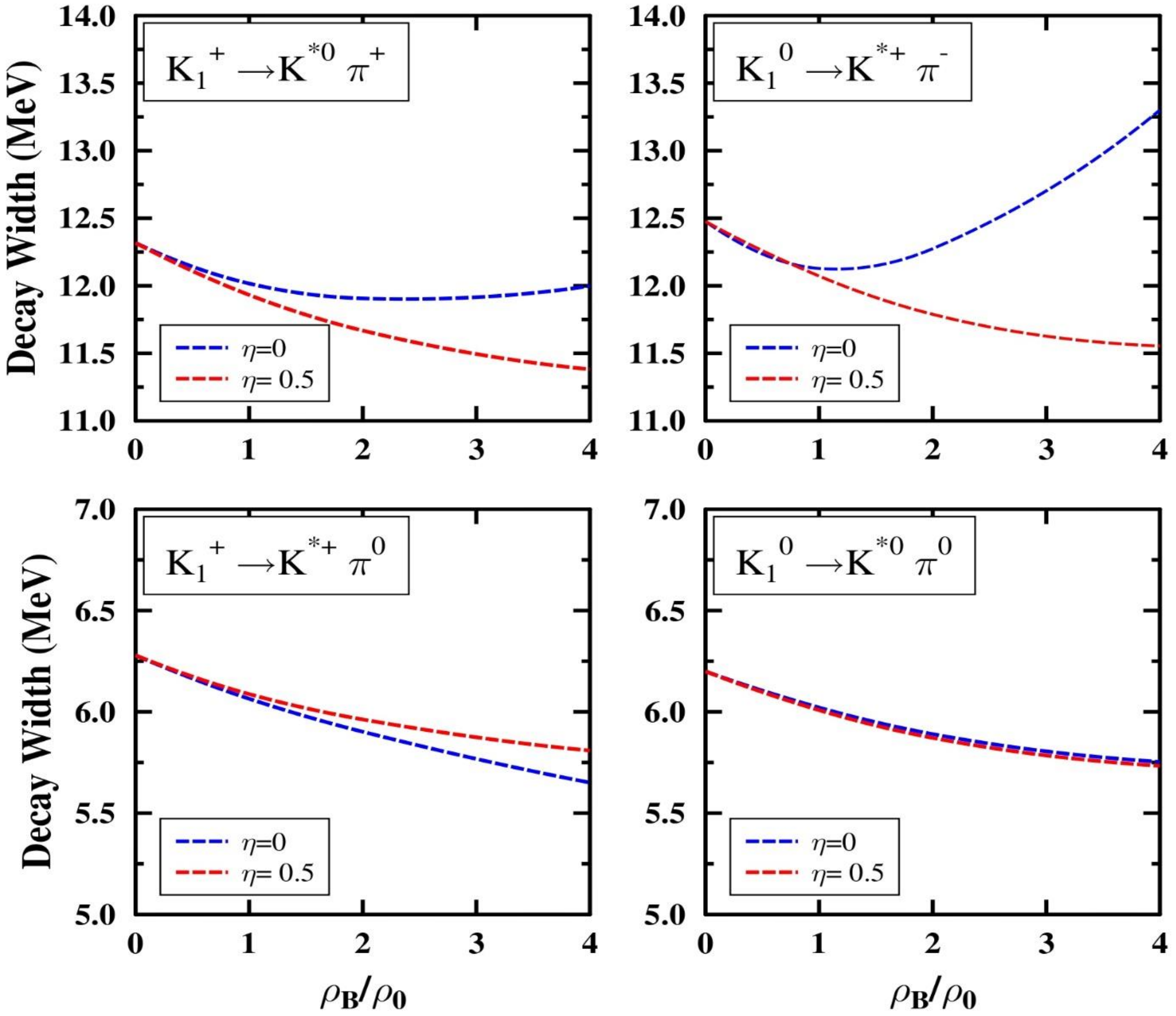
- Experimental observables like yield and spectra of these mesons in HICs (Strangeness production).

Relevance:

- Intermediate energy central collisions, where produced medium has high density.

- High energy peripheral collision experiments where large magnetic fields are produced but the produced medium has low density.

- In nuclear astrophysical objects like (proto) neutron stars where the strange mesons are speculated to be present.



Decay width of Axial vector K_1 meson, plotted as a function of ρ_B/ρ_0 at zero magnetic field, calculated within the 3_{P_0} model.

Ankit Kumar and Amruta Mishra, arXiv:2302.14493 [hep-ph].

Thank You

Back-up Slides

Chiral $SU(3)_L \times SU(3)_R$ symmetry

In low-energy regime, the Dirac Lagrangian: $\mathcal{L}_{Dirac} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$; $\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$

$\Longrightarrow \mathcal{L}_{Dirac} = i(\bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R \gamma^\mu \partial_\mu \psi_R) - (\bar{\psi}_L m\, \psi_R + \bar{\psi}_R m\, \psi_L)$; $P_{R,L}\psi = \frac{1 \pm \gamma_5}{2}\psi = \psi_{R,L}$;

In massless quarks limit, $\mathcal{L}_{Dirac}$ is invariant under $\psi_{R,L} \to \exp\left(-i\theta^a_{R,L}\frac{\lambda^a}{2}\right)\psi_{R,L}$;

$J^\mu_{R,a} = \bar{\psi}_R \gamma^\mu \frac{\lambda_a}{2} \psi_R$; $J^\mu_{L,a} = \bar{\psi}_L \gamma^\mu \frac{\lambda_a}{2} \psi_L$ ……… **Noether's Theorem**

Vector and axial vector currents: $V^\mu_a = J^\mu_{R,a} + J^\mu_{L,a} = \bar{\psi}\gamma^\mu \frac{\lambda_a}{2}\psi$; $A^\mu_a = J^\mu_{R,a} - J^\mu_{L,a} = \bar{\psi}\gamma^\mu \gamma_5 \frac{\lambda_a}{2}\psi$

With transformations, $\psi \to \exp(-i\theta^a_V \lambda^a)\,\psi$; $\psi \to \exp(-i\gamma_5 \theta^a_A \lambda^a)\,\psi$, with $\theta_{V,A} = (\theta_L \pm \theta_R)/2$

$$\Longrightarrow \quad SU(3)_L \times SU(3)_R \equiv SU(3)_V \times SU(3)_A$$

Spontaneous Chiral Symmetry Breaking (SCSB)

- For any symmetric charge Q based on **Noether's theorem**

$$e^{i\alpha Q}|0\rangle = |0\rangle, \forall\, \alpha \qquad \Rightarrow Q|0\rangle = 0$$

We expect $Q_V|0\rangle = 0 = Q_A|0\rangle$

But from the observed spectra, $Q_V|0\rangle = 0,\ Q_A|0\rangle \neq 0$

$$\Rightarrow SU(3)_V \times\ SU(3)_A \xrightarrow{\text{spontaneous breaking}} SU(3)_V.$$

In the massless quarks limit, Lagrangian is symmetric under chiral transformations, but the ground state is not.

SCSB → 8 massless Goldstone Bosons: **Pseudoscalar mesons.**

- SCSB leads to QCD vacuum being populated by **chiral condensates**

$$\langle 0|\bar{\psi}\psi|0\rangle = \langle\bar{\psi}\psi\rangle = \langle\bar{u}u\rangle + \langle\bar{d}d\rangle + \langle\bar{s}s\rangle$$

- Mass term ($\bar{\psi}m\psi$) causes **explicit chiral symmetry breaking (ESCB)** and pseudoscalar mesons get mass.

Scale Symmetry and its Breaking:

- The QCD Lagrangian, $\boldsymbol{\mathcal{L}}_{QCD} = \sum_i \bar{\psi}_i \left(i\gamma^{\mu} D_{\mu} - m_{\mathrm{i}} \right) \psi_i \, - \frac{1}{4} \, G^{\mu\nu a} G^a_{\mu\nu}$

  In massless quarks limit, under the scale transformations:

  $$x \to \lambda x, \qquad \psi(x) \to \psi'(x) = \lambda^{-\frac{3}{2}} \psi(\lambda x), \quad A^a_{\mu}(x) \to {A'_{\mu}}^{a}(x) = \lambda^{-1} A^a_{\mu}(\lambda x)$$

  Action, $S = \int \boldsymbol{\mathcal{L}} \, d^4x$, is invariant.

  The conserved dilaton current, $j^{\mu}_{dilat} = x_{\nu} \theta^{\mu\nu}, \; and \; \partial_{\mu} \left(j^{\mu}_{dilat} \right) = \theta^{\nu}_{\nu} = 0$

- However, trace is non-vanishing as gluon condensates contribute,

  $\theta^{\mu}_{\mu} = \frac{\beta_{QCD}}{2g} G^a_{\mu\nu} \, G^{a\mu\nu} + \sum_i m_i \bar{q}_i q_i,$ where at one loop level,

  This is **Trace Anomaly** of QCD. $\beta_{QCD}(g) = \mu \frac{\partial g}{\partial \mu} = -\frac{11 N_c g^3}{48\pi^2} \left(1 - \frac{2N_f}{11 N_c} \right)$

- Any scale invariant theory is characterized by the vanishing of its β-function.

Chapter II: Chiral $SU(3)_L \times SU(3)_R$ Model

The effective hadronic chiral lagrangian density[1,2] :

$$\mathcal{L} = \mathcal{L}_{kin} + \sum_W \mathcal{L}_{BW} + \mathcal{L}_{vec} + \mathcal{L}_0 + \mathcal{L}_{scale\ break} + \mathcal{L}_{SB} + \mathcal{L}_{mag}$$

$\mathcal{L}_{kin}$ is kinetic energy term for baryons and mesons
$\mathcal{L}_{BW}$ is the baryon-meson interaction term
$\mathcal{L}_{vec}$ is the dynamical mass generation term for vector mesons
$\mathcal{L}_0$ contains the meson-meson interaction term
$\mathcal{L}_{scale\ break}$ is the scale-invariance breaking logarithmic potential term
$\mathcal{L}_{SB}$ describes the explicit chiral symmetry breaking term.
$\mathcal{L}_{mag}$ is the magnetic field contribution

$\Longrightarrow$ nonlinear coupled quantum field equations with large couplings.

[1] P. Papazoglou, D. Zschiesche, S. Schramm, J. Schaffner-Bielich, H. Stöcker, and W. Greiner, Phys. Rev. C 59, 411 (1999).
[2] A. Mishra, K. Balazs, D. Zschiesche, S. Schramm, H. Stöcker, and W. Greiner, Phys. Rev. C 69, 024903 (2004).

Chiral SU(3) Model

(1) Kinetic energy term, $\boldsymbol{\mathcal{L}}_{kin} = i\, tr\left(\bar{B}\gamma_\mu D^\mu B\right) + \frac{1}{2} tr\left(\left(D_\mu X\right)(D^\mu X)\right) + \frac{1}{2} tr\left(\left(D_\mu Y\right)(D^\mu Y)\right) + \frac{1}{2}\left(D_\mu \chi\right)(D^\mu \chi) +$

$$tr\left(\left(u_\mu X u^\mu X\right) + \left(X u_\mu u^\mu X\right)\right) - \frac{1}{4} tr(V_{\mu\nu}V^{\mu\nu}) - \frac{1}{4} tr(A_{\mu\nu}A^{\mu\nu})$$

$D_\mu B = \partial_\mu B + i[\Gamma_\mu, B]$

The composite vector-type field, $\Gamma_\mu = -\frac{i}{4}[u^\dagger\left(\partial_\mu u\right) - \left(\partial_\mu u^\dagger\right)u + u\left(\partial_\mu u^\dagger\right) - \left(\partial_\mu u\right)u^\dagger]$

With $u = \exp(\frac{i}{\sqrt{2}\sigma_0} M\gamma_5)$

Tensor fields for vector and axial vector mesons, $V_{\mu\nu} = D_\mu V_\nu - D_\nu V_\mu \; and \; A_{\mu\nu} = D_\mu A_\nu - D_\nu A_\mu$

(2) Baryon-meson interaction term, $\boldsymbol{\mathcal{L}}_{BW} = -\sqrt{2} g_8^W (\alpha_W [\bar{B}\boldsymbol{\mathcal{O}}BW]_F + (1-\alpha_W)[\bar{B}\boldsymbol{\mathcal{O}}BW]_D) - \frac{g_1^W}{\sqrt{3}} tr(\bar{B}\boldsymbol{\mathcal{O}}B) tr(W)$

Symmetric coupling, $[\bar{B}\boldsymbol{\mathcal{O}}BW]_D = tr(\bar{B}\boldsymbol{\mathcal{O}}WB + \bar{B}\boldsymbol{\mathcal{O}}BW) - \frac{2}{3} tr(\bar{B}\boldsymbol{\mathcal{O}}B) tr(W)$

Antisymmetric coupling, $[\bar{B}\boldsymbol{\mathcal{O}}BW]_F = tr(\bar{B}\boldsymbol{\mathcal{O}}WB - \bar{B}\boldsymbol{\mathcal{O}}BW)$

(3) Scalar meson interaction term, $\boldsymbol{\mathcal{L}}_0 = -\frac{1}{2} k_0 \chi^2 I_2 + k_1 (I_2)^2 + k_2 I_4 + 2k_3 \chi I_3$

Where , $I_1 = tr(X), I_2 = tr(X^2), I_3 = \det(X)$

(4) Scale breaking term, $\boldsymbol{\mathcal{L}}_{scale\ break} = -\frac{1}{4}\chi^4 \ln\left(\frac{\chi^4}{\chi_0^4}\right) + \frac{d}{3}\chi^4 \ln\left(\left(\frac{I_3}{det\langle X\rangle_0}\right)\left(\frac{\chi}{\chi_0}\right)^3\right)$

Lagrangian term required to generate $\chi_0$is given as, $\boldsymbol{\mathcal{L}}'_\chi = -k_4\chi^4$

The energy-momentum tensor, $\theta_{\mu\nu} = (\partial_\mu \chi)\left(\frac{\partial \boldsymbol{\mathcal{L}}_\chi}{\partial(\partial^\nu \chi)}\right) - g_{\mu\nu}\boldsymbol{\mathcal{L}}_\chi$

Trace, $\theta_\mu^\mu = \chi\left(\frac{\partial \boldsymbol{\mathcal{L}}_\chi}{\partial \chi}\right) - 4\boldsymbol{\mathcal{L}}_\chi = -(1-d)\chi^4$

Equating, $\theta_\mu^\mu = \left\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a\, G^{a\mu\nu} \right\rangle + \sum m_i \langle \bar{q}_i q_i \rangle = -(1-d)\chi^4$

(5) Explicit chiral breaking term, $\boldsymbol{\mathcal{L}}_{SB} = -\frac{1}{2} m_{\eta_0}^2 tr(Y^2) - \frac{1}{2} tr\left(A_p\left(uXu + u^\dagger X u^\dagger\right)\right) - tr((A_s - A_p)X)$

$$A_p = \frac{m_\pi^2 f_\pi}{2} diag\left(1,1, \frac{2m_K^2 f_K}{m_\pi^2 f_\pi} - 1\right)$$

(6) Vector meson interaction term, $\boldsymbol{\mathcal{L}}_{vec} = \frac{1}{2} m_V^2 \left(\frac{\chi^2}{\chi_0^2}\right) tr\left(\tilde{V}_\mu \widetilde{V^\mu}\right) + \frac{\mu}{4} tr\left(\tilde{V}_{\mu\nu} \tilde{V}^{\mu\nu} X^2\right) + \frac{\lambda_V}{12} tr\left(\left[\tilde{V}_{\mu\nu}\right]\right)^2$

Where , V_μ are the re-normalised vector meson fields.

$$B=\frac{1}{\sqrt{2}}\psi^a\lambda_a=\begin{pmatrix}\frac{\Sigma^0}{\sqrt{2}}+\frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p\\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}}+\frac{\Lambda^0}{\sqrt{6}} & n\\ \Xi^- & \Xi^0 & -2\frac{\Lambda^0}{\sqrt{6}}\end{pmatrix};\ X=\frac{1}{\sqrt{2}}\sigma^a\lambda_a=\begin{pmatrix}\frac{\sigma^0+\delta^0}{\sqrt{2}} & \delta^+ & \kappa^+\\ \delta^- & \frac{\sigma^0-\delta^0}{\sqrt{2}} & \kappa^0\\ \kappa^- & \bar{\kappa}^0 & \zeta\end{pmatrix}$$

$$V_\mu=\frac{1}{\sqrt{2}}\sum_a v_\mu^a\lambda_a=\begin{pmatrix}\frac{\rho_\mu^0+\omega_\mu}{\sqrt{2}} & \rho_\mu^+ & K_\mu^{*+}\\ \rho_\mu^- & \frac{-\rho_\mu^0+\omega_\mu}{\sqrt{2}} & K_\mu^{*0}\\ K_\mu^{*-} & \bar{K}_\mu^{*0} & \phi_\mu\end{pmatrix};\ \mathrm{M}=\frac{1}{\sqrt{2}}\pi^a\lambda_a=\begin{pmatrix}\frac{\pi^0}{\sqrt{2}}+\frac{\eta^8}{\sqrt{2(1+2w^2)}} & \pi^+ & \left(\frac{2}{w+1}\right)K^+\\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{\eta^8}{\sqrt{2(1+2w^2)}} & \left(\frac{2}{w+1}\right)K^0\\ \left(\frac{2}{w+1}\right)K^- & \left(\frac{2}{w+1}\right)\bar{K}^0 & -\frac{2\eta^8}{\sqrt{2(1+2w^2)}}\end{pmatrix}$$

Conditions to apply MFT:
Large baryon density
Uniform and stationary matter
Rotationally invariant

The meson field operators are replaced by their classical expectation values.
Here the fluctuations of the field operators around their constant vacuum expectation values are neglected.

Mean-field approximation : All the meson fields are treated as classical fields.
Under this approximation, meson field operators are replaced by their vacuum expectation values.

$$\phi \rightarrow \langle \phi \rangle \equiv \phi_0, \qquad V^{\mu}\{\equiv (V_0, \vec{V})\} \rightarrow \langle V^{\mu} \rangle \equiv (V_0, 0)$$

Further, $\langle \bar{\psi}_i \psi_j \rangle = \delta_{ij} \langle \bar{\psi}_i \psi_i \rangle = \delta_{ij} \rho_i^s$; $\langle \bar{\psi}_i \gamma^{\mu} \psi_j \rangle = \delta_{ij} \delta^{\mu 0} \langle \bar{\psi}_i \gamma^0 \psi_i \rangle = \delta_{ij} \delta^{\mu 0} \rho_i$;
$\rho_i^s (\rho_i) \equiv$ scalar (number) density of *i*th baryon.

- Only the scalar and vector fields contribute,

$$\mathcal{L}_{BS} + \mathcal{L}_{BV} = -\sum_i \bar{\psi}_i \left[m_i^* + g_{\omega i} \gamma_0 \omega + g_{\rho i} \gamma_0 \rho + g_{\phi i} \gamma_0 \phi \right] \psi_i \text{ , where } m_i^* = -\left(g_{\sigma i}\, \sigma + g_{\zeta i} \zeta + g_{\delta i} \delta \right)$$

- Interaction term for mesons,

$$\mathcal{L}_{vec} = \frac{1}{2} \left(m_{\omega}^2 \omega^2 + m_{\rho}^2 \rho^2 + m_{\phi}^2 \phi^2 \right) \left(\frac{\chi^2}{\chi_0^2} \right) + g_4 (\omega^4 + 6\rho^2 \omega^2 + \rho^4 + 2\phi^4)$$

$$\mathcal{L}_0 = -\frac{1}{2} k_0 \chi^2 (\sigma^2 + \zeta^2 + \delta^2) + k_1 (\sigma^2 + \zeta^2 + \delta^2)^2 + \mathrm{k}_2 \left(\frac{\sigma^4}{2} + \frac{\delta^4}{2} + 3\sigma^2 \delta^2 + \zeta^4 \right) + k_3 \chi (\sigma^2 - \delta^2) \zeta$$

- $\mathcal{L}_{BS} + \mathcal{L}_{BV} = -\sum_i \bar{\psi}_i\left[m_i^* + g_{\omega i}\gamma_0\omega + g_{\rho i}\gamma_0\rho + g_{\phi i}\gamma_0\phi\right]\psi_i$, where $m_i^* = -\left(g_{\sigma i}\,\sigma + g_{\zeta i}\zeta + g_{\delta i}\delta\right)$

- $\mathcal{L}_0 = -\frac{1}{2}k_0\chi^2(\sigma^2+\zeta^2+\delta^2) + k_1(\sigma^2+\zeta^2+\delta^2)^2 + \mathrm{k}_2\left(\frac{\sigma^4}{2} + \frac{\delta^4}{2} + 3\sigma^2\delta^2 + \zeta^4\right) + k_3\chi(\sigma^2-\delta^2)\zeta$

- **Scale invariance breaking** term, $\mathcal{L}_{scale\ break} = -\frac{1}{4}\chi^4\ln\left(\frac{\chi^4}{\chi_0^4}\right) + \frac{d}{3}\chi^4\ln\left(\left(\frac{(\sigma^2-\delta^2)\zeta}{\sigma_0^2\zeta_0}\right)\left(\frac{\chi}{\chi_0}\right)^3\right)$

- **Explicitly chiral symmetry breaking** term, $\mathcal{L}_{SB} = Tr\left[diag\left(-\frac{m_\pi^2 f_\pi(\sigma+\delta)}{2}, -\frac{m_\pi^2 f_\pi(\sigma-\delta)}{2}, -(\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi)\zeta\right)\right]$

- Quark and gluon condensates are related with the scalar fields as,

$$m_u\langle\bar{u}u\rangle = \frac{m_\pi^2 f_\pi(\sigma+\delta)}{2}, \qquad m_d\langle\bar{d}d\rangle = \frac{m_\pi^2 f_\pi(\sigma-\delta)}{2}, \quad m_s\langle\bar{s}s\rangle = (\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi)\zeta$$

$$\left\langle\frac{\alpha_s}{\pi}G_{\mu\nu}^a\, G^{a\mu\nu}\right\rangle = \frac{8}{9}\left\{(1-d)\chi^4 + \left[m_\pi^2 f_\pi\sigma + (\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi)\zeta\right]\right\}$$

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[1] A. E. Broderick, M. Prakash, and J. M. Lattimer, Phys. Lett. B 531, 167 (2002).

The coupled equations of motion derived from chiral SU(3) model for the fields $\sigma, \zeta, \delta,$ and χ are given as:

$$k_0\chi^2\sigma - 4k_1(\sigma^2+\zeta^2+\delta^2)\sigma - 2k_2(\sigma^3+3\sigma\delta^2) - 2k_3\chi\sigma\zeta - \frac{d}{3}\chi^4\left(\frac{2\sigma}{\sigma^2-\delta^2}\right) + \left(\frac{\chi}{\chi_0}\right)^2 m_\pi^2 f_\pi - \sum g_{\sigma i}\rho_i^s = 0 \quad ...(1)$$

$$k_0\chi^2\zeta - 4k_1(\sigma^2+\zeta^2+\delta^2)\zeta - 4k_2\zeta^3 - k_3\chi(\sigma^2-\delta^2) - \frac{d}{3}\frac{\chi^4}{\zeta} + \left(\frac{\chi}{\chi_0}\right)^2 [\sqrt{2}m_k^2 f_k - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi] - \sum g_{\zeta i}\rho_i^s = 0 \quad ...(2)$$

$$k_0\chi^2\delta - 4k_1(\sigma^2+\zeta^2+\delta^2)\delta - 2k_2(\delta^3+3\sigma^2\delta) + 2k_3\chi\sigma\delta + \frac{2d}{3}\chi^4\left(\frac{\delta}{\sigma^2-\delta^2}\right) - \sum g_{\delta i}\rho_i^s = 0 \quad ...(3)$$

$$k_0\chi(\sigma^2+\zeta^2+\delta^2) - k_3(\sigma^2-\delta^2)\zeta + \chi^3\left[1+ln\left(\frac{\chi^4}{\chi_0^4}\right)\right] + (4k_4-d)\chi^3 - \frac{4}{3}d^3\ln\left[\left(\frac{(\sigma^2-\delta^2)\zeta}{\sigma_0^2\zeta_0}\right)\left(\frac{\chi}{\chi_0}\right)^3\right] + \frac{2\chi}{\chi_0}\left[m_\pi^2 f_\pi\sigma + \left(\sqrt{2}m_k^2 f_k - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi\right)\zeta\right] = 0 \quad ...(4)$$

where, scalar density for protons i, $\rho_p^s = \frac{eBm_p^*}{2\pi^2}\sum_{\nu=0}^{\nu_{max}^{S=\pm1}}\left[\frac{\sqrt{m_p^*+2eB\nu}+S\Delta_p}{\sqrt{m_p^{*2}+2eB\nu}} \times ln\left|\frac{k_{f,\nu,S}^p+E_f^p}{\sqrt{m_p^{*2}+2eB\nu}+S\Delta_p}\right|\right]$

neutrons, $\rho_n^s = \frac{m_n^*}{4\pi^2}\sum_{S=\pm1}\left\{k_{f,S}^{(n)}E_f^{(n)} - (m_n^*+S\Delta_n)^2 ln\left|\frac{k_{f,S}^{(n)}+E_f^{(n)}}{m_n^*+S\Delta_p}\right|\right\}$; $\bar{\psi}_i\psi_j = \delta_{ij}\rho_i^s$, $\bar{\psi}_i\gamma^\mu\psi_j = \delta_{ij}\delta^{\mu 0}\rho_i^s$

Fermi momentum, $k_{f,\nu,S}^p = \sqrt{E_f^{(p)^2} - \left(\sqrt{m_p^{*^2}+2eB\nu}+S\Delta_p\right)^2}$; $k_{f,S}^{(n)} = \sqrt{E_f^{(n)^2} - (m_n^*+S\Delta_n)^2}$; Δ_i refers to AMMs

The effect of **magnetic field** is introduced via the term :

$$\mathcal{L}_{mag} = -\frac{1}{4} F^{\mu\nu}F_{\mu\nu} - q_i\bar{\psi}_i\gamma_\mu A^\mu\psi_i - \frac{1}{4}\kappa_i\mu_N\bar{\psi}_i\sigma^{\mu\nu}F_{\mu\nu}\psi_i$$

where, ψ_i is the baryon field operator

A^μ is the magnetic vector potential

κ_i corresponds to anomalous magnetic moment (AMMs) of the baryons

μ_N is the nuclear Bohr magneton

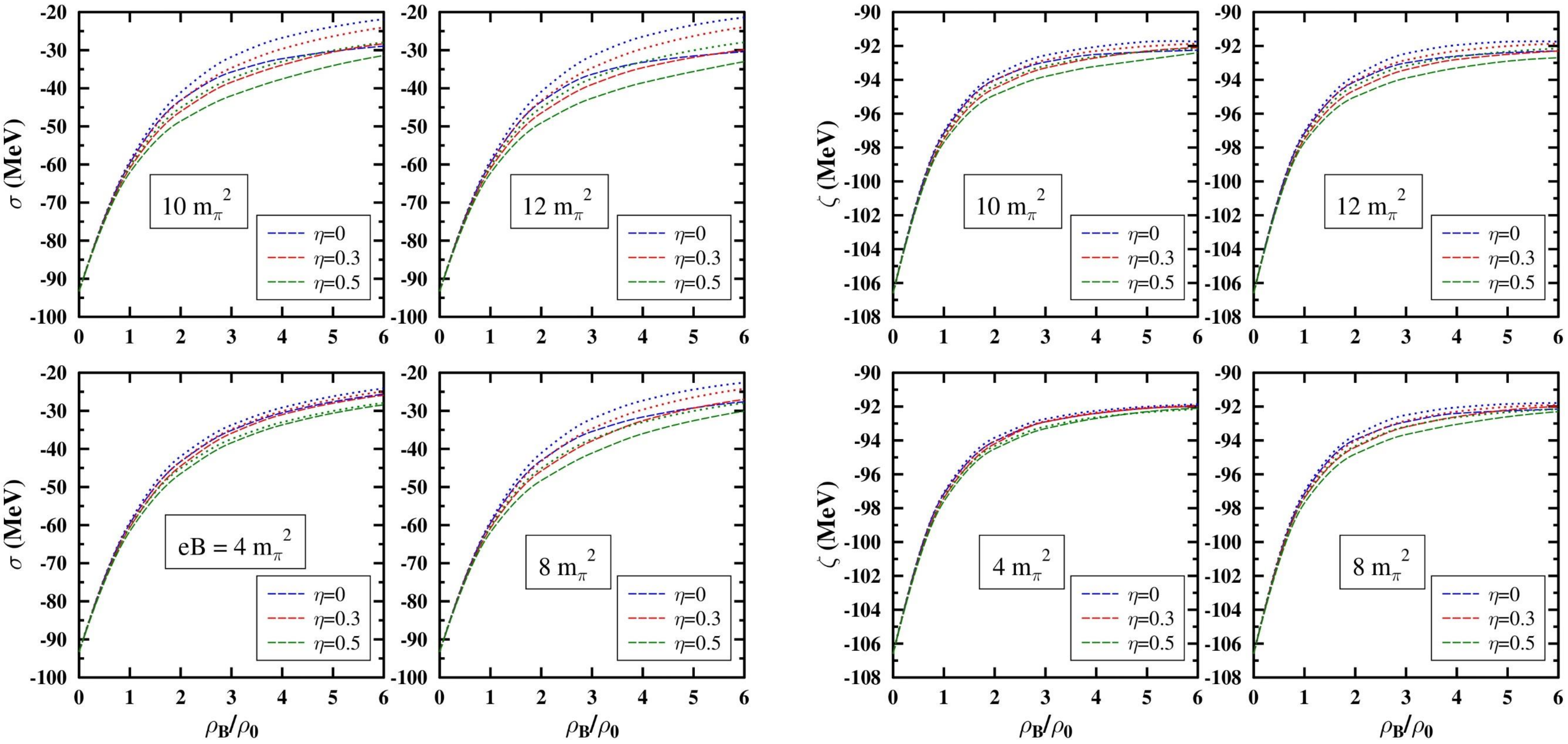
$F_{\mu\nu}$ is the electromagnetic field strength tensor

Where, scalar density for baryons i, $\rho_i^s = \gamma_i \int \frac{d^3k}{(2\pi)^3}\frac{m_i^*}{E_i^*(k)}\left(\frac{1}{e^{\left[E_i^*(k)-\mu_i^*\right]/T}+1} + \frac{1}{e^{\left[E_i^*(k)+\mu_i^*\right]/T}+1}\right)$

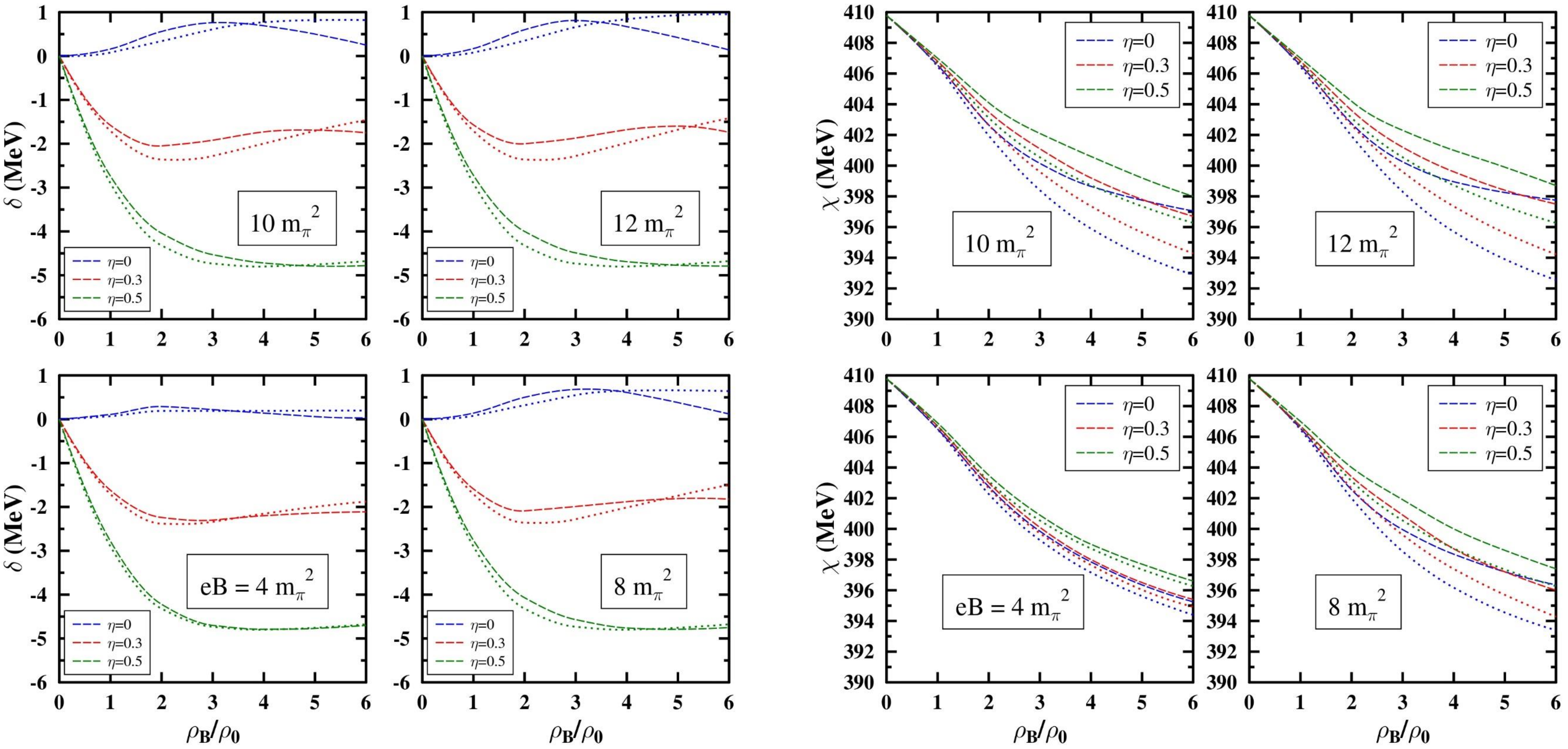
Where $E_i^*(k) = \left(k^2 + {m_i^*}^2\right)^{\frac{1}{2}}$ and $\mu_i^* = \mu_i - g_{\omega i}\omega - g_{\rho i}\rho - g_{\phi i}\phi$; spin degeneracy factor $\gamma_i = 2$

$$\rho_n = \frac{1}{4\pi^2}\sum_{S=\pm1}\left\{\frac{2}{3}{k_{f,S}^{(n)}}^3 + S\Delta_n\left[(m_n^* + S\Delta_n){k_{f,S}^{(n)}}^3 + {E_f^{(n)}}^2\left(arcsin\left(\frac{m_n^*+S\Delta_n}{E_f^{(n)}}\right) - \frac{\pi}{2}\right)\right]\right\}$$

$$\rho_p = \frac{\mathrm{eB}}{4\pi^2}\left[\sum_{\nu=0}^{\nu_{max}^{S=1}} k_{f,\nu,1}^{(p)} + \sum_{\nu=1}^{\nu_{max}^{S=-1}} k_{f,\nu,-1}^{(p)}\right]$$



σ (MeV)
ζ (MeV)
ρ_B/ρ_0
10 m_π^2
12 m_π^2
eB = 4 m_π^2
8 m_π^2
4 m_π^2
η=0
η=0.3
η=0.5



δ (MeV)
χ (MeV)
ρ_B/ρ_0
η=0
η=0.3
η=0.5
10 m_π^2
12 m_π^2
eB = 4 m_π^2
8 m_π^2

Constants, $f_\pi = 93.3\ MeV$; $f_K = 122.143\ MeV$; $f_\eta = 93 \pm 9\ MeV$; $f_\eta' = 83 \pm 7\ MeV$; $m_\pi = 139$; $m_K = 498$; $\sigma_0 = -93.3$; $\zeta_0 = -106.7$; $g_{\sigma p} = g_{\sigma n} = 10.567$; $g_{\zeta p} = g_{\zeta n} = -0.46707$; $g_{\delta p} = 2.487$; $g_{\delta n} = -2.487$;

$k_0 \rightarrow \frac{\partial\Omega}{\partial\sigma}|_{vacuum} = 0$ fixes $k_0 = 2.5365899$;
$k_2 \rightarrow \frac{\partial\Omega}{\partial\zeta}|_{vacuum} = 0$ fixes $k_2 = -4.775199$,
$k_1 \rightarrow m_\sigma \approx 500\ MeV$ fixes $k_1 = 1.35436$,
$k_3 \rightarrow \eta - \eta'$ splitting fixes $k_3 = -2.77257$;
$k_4 \rightarrow \frac{\partial\Omega}{\partial\chi}|_{vacuum} = 0$ fixes $k_4 = -0.2348811$
$\chi_0 \rightarrow p|_{\rho_B=\rho_0} = 0$ fixes$\chi_0 = 409.77$; ,
$d \rightarrow \beta_{QCD}$ as $d = 0.064018$;

Borel Transform:

$$\hat{f}(M^2) = \lim_{\substack{Q^2\to\infty, n\to\infty \\ \frac{Q^2}{n}=M^2=constant}} \frac{1}{(n-1)!}(Q^2)^n\left(-\frac{d}{dQ^2}\right)^n f(Q^2)$$

- enhanced contribution from the resonance part.
- the continuum in high-energy perturbative region is suppressed exponentially.
- Detailed description of the crossover and continuum are no longer significant.
- Better convergence for the OPE side.

Comparing the explicit symmetry breaking terms from QCD and mean field approximation,

$$\mathcal{L}_{SB}^{QCD} = -Tr\left[diag\left(m_u\bar{u}u, m_d\bar{d}d, m_s\bar{s}s\right)\right]$$

and $\mathcal{L}_{SB} = Tr\left[diag\left(-\frac{m_\pi^2 f_\pi(\sigma+\delta)}{2}, -\frac{m_\pi^2 f_\pi(\sigma-\delta)}{2}, -(\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi)\zeta\right)\right]$

scattering term[1] $\Pi^V(0) = \frac{1}{4}\sum_i\left(\frac{g_{Vi}}{g_{VN}}\right)^2\frac{\rho_i}{M_i}$, and ρ_i= number density of ith baryon

running coupling parameter $\alpha_s = 4\pi/[b\ln(\frac{Q^2}{\Lambda_{QCD}^2})] = 0.3551$ at $\mathrm{Q}^2 = 1\mathrm{GeV}^2$ and $d_\rho = \frac{3}{2}, d_\omega = \frac{1}{6}, d_\phi = \frac{1}{3}$

Effects of Strong Magnetic fields

Spin Magnetic Field Interaction: Also, due to the spin-magnetic field interaction ($-\mu_i.B$) term.
The effective physical mass eigenvalues are given by

$$m_{V||,P}^{eff} = m_{V||,P} \pm \frac{\Delta \mathrm{E}}{2}\left(\sqrt{[1+\chi_{SB}^2]^{1/2} - 1}\right)$$

Where $\Delta E = m_{V||} - m_P$ and $\chi_{SB} = \frac{2}{\Delta E}\left(\frac{gq_1 B}{4m_1} - \frac{gq_2 B}{4m_2}\right)$

*$\mu_i = \frac{gq_i\sigma_i}{4m_i}$; $and\ \sigma_1.B(|\uparrow\downarrow> \pm|\downarrow\uparrow>) = B(|\uparrow\downarrow> \mp|\downarrow\uparrow>); \sigma_2.B(|\uparrow\downarrow> \pm|\downarrow\uparrow>) = -B(|\uparrow\downarrow> \mp|\downarrow\uparrow>)$

PV mixing:

$$\mathcal{L}_{PV\gamma} = \frac{g_{PV}}{m_{avg}} e\, \tilde{F}_{\mu\nu}(\partial^\mu P)V^\nu$$

The masses after PV mixing are given by,

$$m_{V||,P}^2 = \frac{1}{2}\left(M_+^2 + \frac{c_{PV}^2}{m_{avg}^2} \pm \sqrt{M_-^4 + \frac{2c_{PV}^2 M_+^2}{m_{avg}^2} + \frac{c_{PV}^4}{m_{avg}^4}}\right),$$

with $c_{PV} = g_{PV}eB;\ \ M_+^2 = m_V^2 + m_P^2;\ \ \ M_-^2 = m_V^2 - m_P^2$

The radiative decay width is given by

$$\Gamma(V \to P\gamma) = \frac{e^2}{12}\frac{g_{PV}^2 p_{cm}^3}{\pi m_{avg}^2}$$

p_{cm} is the centre of mass-momentum in final state.

Decay width of $K^* \rightarrow K\pi$ in $^3P_0$ model

$$\Gamma(K^* \rightarrow K\pi) = \frac{\sqrt{\pi}\, E_K E_\pi \gamma^2}{4\, M_{K^*}} \left(\frac{2^8 r^3 (1+r^2)^2}{3(1+2r^2)^5}\right) x^3 \exp\left(\frac{-x^2}{2(1+2r^2)}\right)$$

Where $E_K = \sqrt{p^2 + m_K^2}$; $E_\pi = \sqrt{p^2 + m_\pi^2}$; The 3-momentum $p (= p_K = p_\pi)$ is given by

$$p = \sqrt{\left[\frac{M_{K^*}^2}{4} - \frac{m_K^2 + m_\pi^2}{2} + \frac{(m_K^2 - m_\pi^2)^2}{4\, M_{K^*}^2}\right]}$$

The quantity $x = p/\beta_{avg}$ is the scaled momentum and $r = \beta_{K^*}/\beta_{avg}$, with β_{K^*} as the strength of harmonic oscillator potential for parent particle and β_{avg} as average of two daughter particles.

$$\Gamma(K_1 \rightarrow K^*\pi) = 2\pi \frac{p_{K^*}\, E_{K^*} E_\pi}{M_{K_1}} \sum_{LS} |M_{LS}|^2 \;;$$

$$M_{LS} = \frac{\gamma_{K_1}}{\pi^{1/4} \beta_{avg}^{1/2}} P_{LS}(x, r) \exp\left(-\frac{x^2}{4(1+2r^2)}\right) I_f$$

Where the polynomials are given by[2]

$$P_{01}^{(1^3P_1 \rightarrow 1^3S_1 + 1^1S_0)} = \left(2^5 \left(\frac{r}{1+2r^2}\right)^{5/2} \left(1 - \frac{1+r^2}{3(1+2r^2)} x^2\right)\right); \quad P_{01}^{(1^1P_1 \rightarrow 1^3S_1 + 1^1S_0)} = -\frac{1}{\sqrt{2}} P_{01}^{(1^3P_1 \rightarrow 1^3S_1 + 1^1S_0)}$$

$$P_{21}^{(1^3P_1 \rightarrow 1^3S_1 + 1^1S_0)} = -\sqrt{\frac{5}{6}} \left(\frac{1}{\sqrt{15}} \frac{r^{5/2} 2^5 (1+r^2)}{(1+2r^2)^{7/2}} x^2\right); \quad P_{21}^{(1^1P_1 \rightarrow 1^3S_1 + 1^1S_0)} = \sqrt{2}\, P_{21}^{(1^3P_1 \rightarrow 1^3S_1 + 1^1S_0)}$$

The Harmonic oscillator potential strength[3]: $\beta_{K_1} = 136.563$ MeV; $\beta_{K^*} = 184.84$ MeV;
$\beta_K = 238.3$ MeV; $\beta_\pi = 211$ MeV.

Extra Back-up Slides

Introduction

In **Standard Model (SM)** of particle physics**,** the fundamental constituents of matter are **Quarks and Leptons**.

Light quarks[1]: up, down, strange quarks: $m_{u,d} \approx 4 - 8\ MeV$; $m_s \approx 150\ MeV$
Heavy quarks: charm, bottom, top quarks : $m_c \approx 1.28\ GeV$; $m_b \approx 4.18\ GeV$; $m_t \approx 173.1\ GeV$

Leptons: (1) electron (e^-), muon (μ^-), tau (τ^-)
(2) electron neutrino (ν_e), muon neutrino (ν_μ), tau neutrino (ν_τ).

Interactions: strong, electromagnetic**,** and weak interactions.

Vector Bosons: gluons (g), photon (γ), W bosons ($W^\pm$), Z boson.

Scalar Boson: Higgs boson (H).

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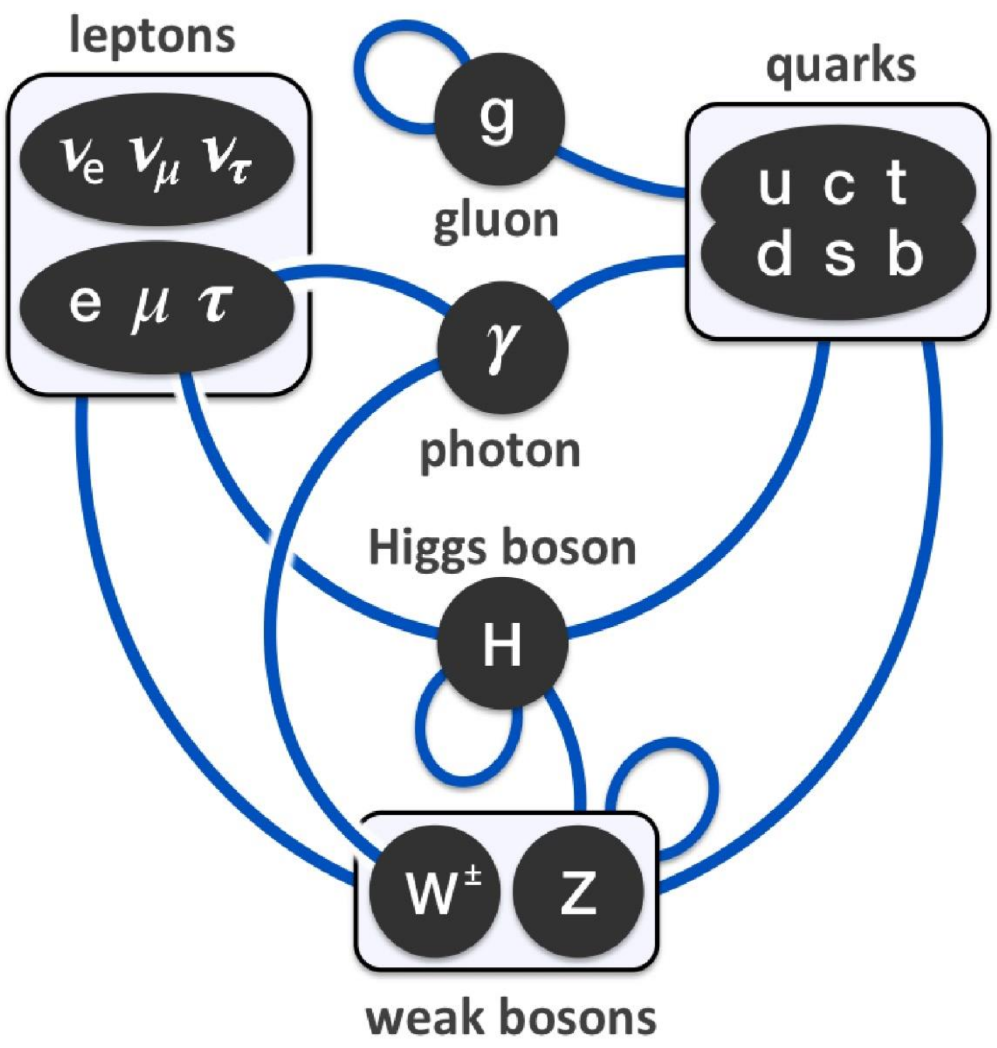
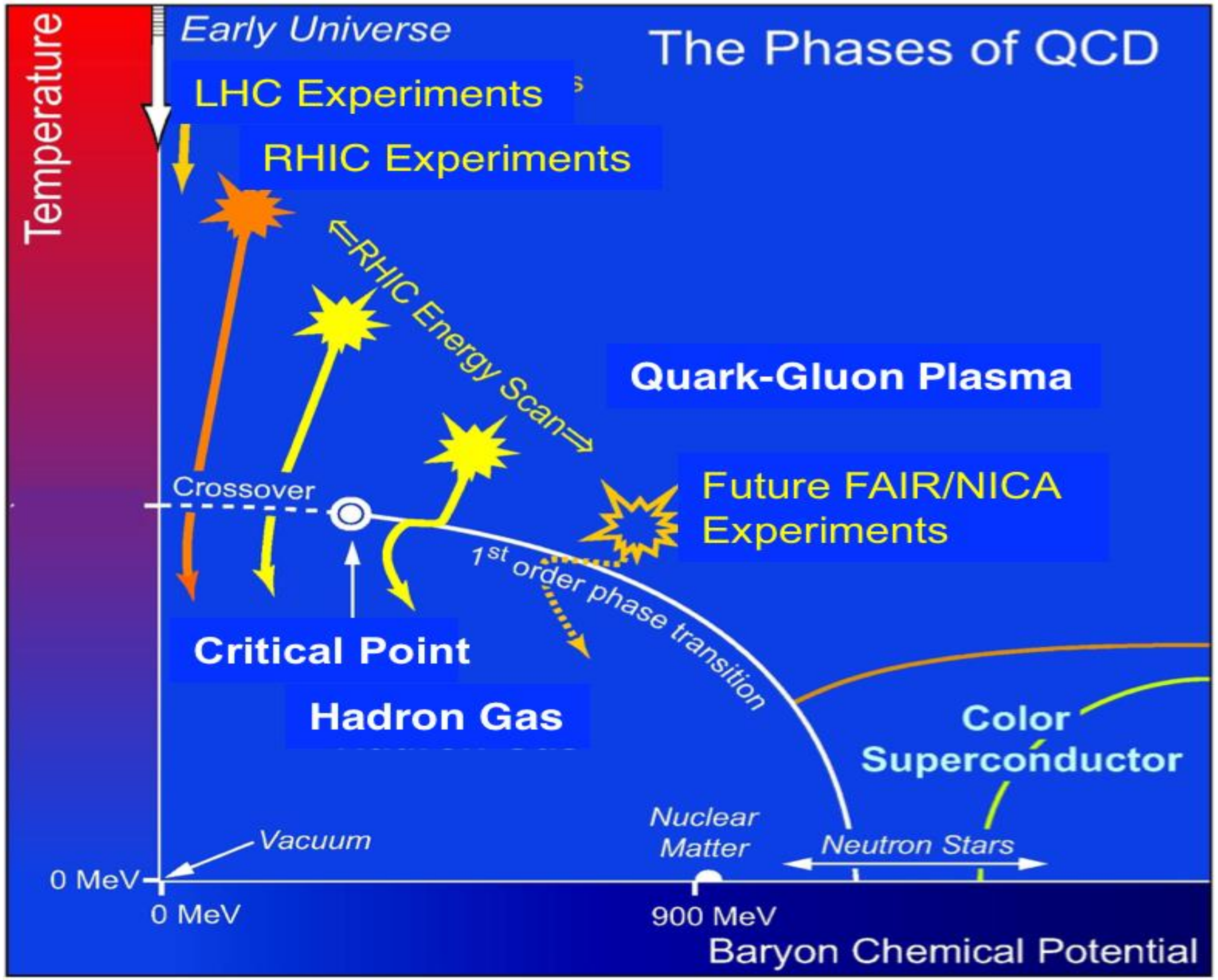
[1] P. A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020) and 2021 update.

In Particle Physics, we study the fundamental particles of nature and their interactions.



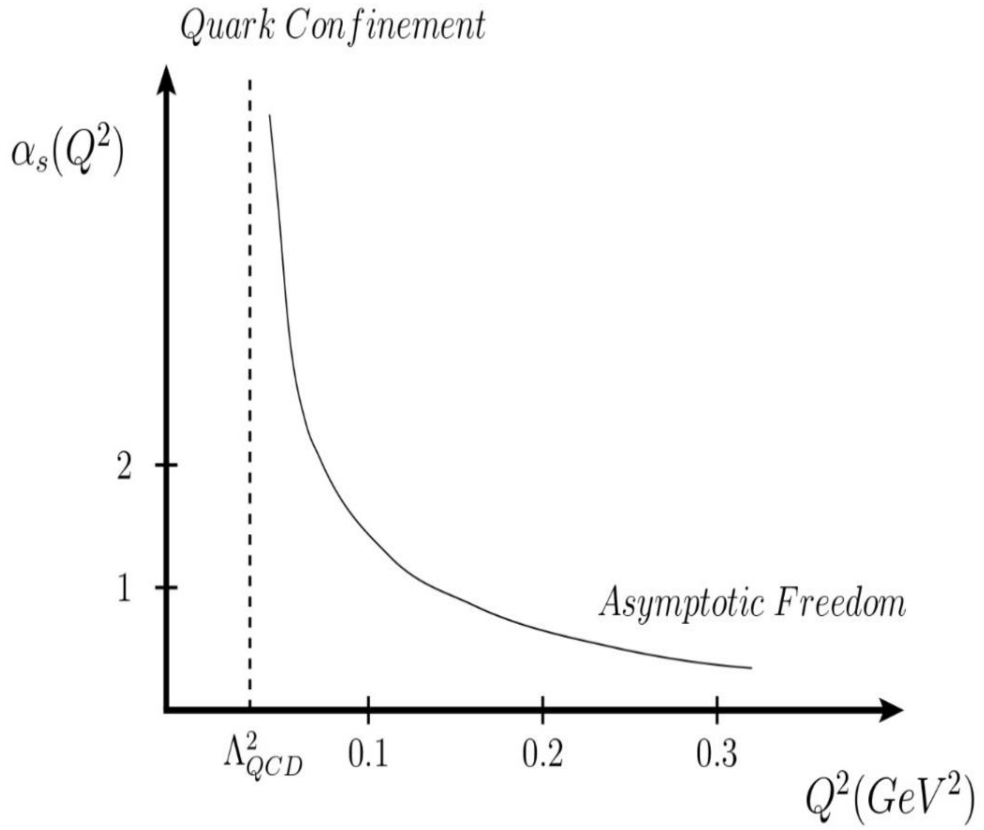


Effective field theories:

- Running coupling parameter of Quantum Chromodynamics (QCD):

$$\alpha_s(Q^2) = \frac{4\pi}{\left(11 - 2N_f/3\right)\ln\left(\frac{Q^2}{\Lambda_{QCD}^2}\right)}$$

N_f is the no. of flavors of quarks.
QCD scale parameter $\Lambda_{QCD} \sim 200\ \mathrm{MeV}$



- In low energy regime: α_s is large, Perturbative techniques can't be applied.
- Construction of effective field theories in terms of hadronic degrees of freedom based on symmetries and symmetry breaking patterns of low energy QCD.

Quantum Chromodynamics (QCD)

- The QCD Lagrangian density is written as

$$\mathcal{L}_{QCD} = \sum_i \bar{\psi}_i \left(i\gamma^{\mu} D_{\mu} - m_{\mathrm{i}}\right) \psi_i - \frac{1}{4}\, G^{\mu\nu a} G^{a}_{\mu\nu}$$

$$\psi_i = (u, d, s);\, D_{\mu} \equiv \partial_{\mu} + igA^{a}_{\mu}\lambda^{a}/2$$

$$\mathrm{G}^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} - g f_{bca} A^{b}_{\mu} A^{c}_{\nu}$$

A^{a}_{μ} are the gluon fields; λ^{a} are the Gell-Mann matrices
a= 1, 2, …. 8.

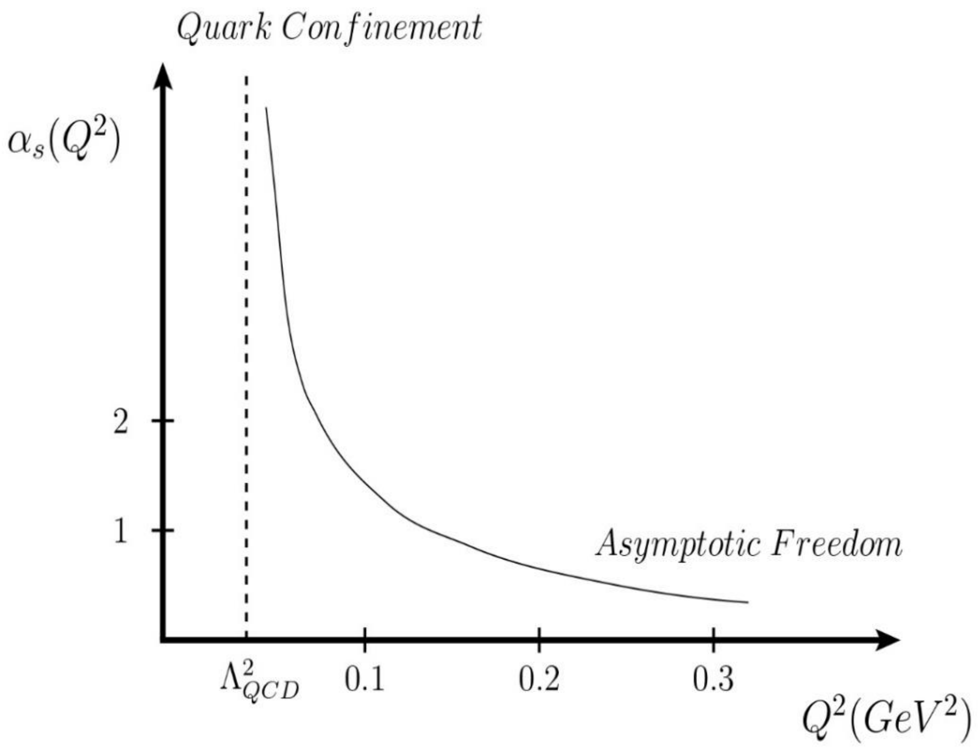
- **Hadrons:** bound states of quarks;
  (i) Baryons: bound state of three quarks
  (ii) Mesons: bound state of quark-antiquark.

- In low energy regime: α_s is large, Perturbative techniques can't be applied.
- Construction of effective field theories in terms of Hadronic degrees of freedom based on symmetries and symmetry breaking patterns of low energy QCD.

- Running coupling parameter of QCD:

$$\alpha_s(Q^2) = \frac{4\pi}{\left(11 - 2N_f/3\right)\ln\left(\frac{Q^2}{\Lambda^2_{QCD}}\right)}$$

N_f is the no. of flavors of quarks.
QCD scale parameter $\Lambda_{QCD} \sim 200$ MeV



Symmetries and Symmetry Breaking effects

(1) Chiral Symmetry: The QCD Lagrangian,

$$\mathcal{L}_{QCD} = \sum_{i=u,d,s} \bar{\psi}_i \left(\mathrm{i}\gamma^\mu D_\mu - m_i \right) \psi_i - \frac{1}{4} G^{\mu\nu a} G^a_{\mu\nu}$$

in massless quark limit, is invariant under $\psi_{R,L} \rightarrow \exp\left(-\mathrm{i}\theta^a_{R,L} \frac{\lambda^a}{2}\right) \psi_{R,L}$, with $\psi = \psi_R + \psi_L$

Vector and axial vector currents: $V^\mu_a = J^\mu_{R,a} + J^\mu_{L,a} = \bar{\psi}\gamma^\mu \frac{\lambda_a}{2} \psi$; $A^\mu_a = J^\mu_{R,a} - J^\mu_{L,a} = \bar{\psi}\gamma^\mu \gamma_5 \frac{\lambda_a}{2} \psi$

$$\Longrightarrow SU(3)_L \times SU(3)_R \equiv SU(3)_V \times SU(3)_A$$

(2) Spontaneous Breaking of Chiral Symmetry (SCSB) : Lagrangian is symmetric under chiral transformations, but the ground state is not.

$$\Rightarrow SU(3)_V \times SU(3)_A \xrightarrow{\text{spontaneous breaking}} SU(3)_V$$

- **SCSB** → 8 massless Goldstone Bosons: **Pseudoscalar mesons.**
- SCSB leads to QCD vacuum being populated by **scalar chiral condensates**

$$\langle 0 | \bar{\psi}\psi | 0 \rangle = \langle \bar{\psi}\psi \rangle = \langle \bar{u}u \rangle + \langle \bar{d}d \rangle + \langle \bar{s}s \rangle$$

Spontaneous chiral symmetry breaking:

- $Q_V|0\rangle = 0 = Q_A|0\rangle$ … **Wigner-Weyl realization** of chiral symmetry
  (Ground state properties reflect the underlying symmetry of the theory.

  We expect that the spectra of **vector** ($J^P = 1^-$) and **axial-vector** ($J^P = 1^+$) should coincide.
  But $\rho(770)$ meson have much smaller mass than the lightest axial vector meson $a_1(1230)$.
  Lightest **pseudoscalar** $\pi(\sim 140\ MeV)$ and **scalar** mesons $a_0 = 980\ MeV$.

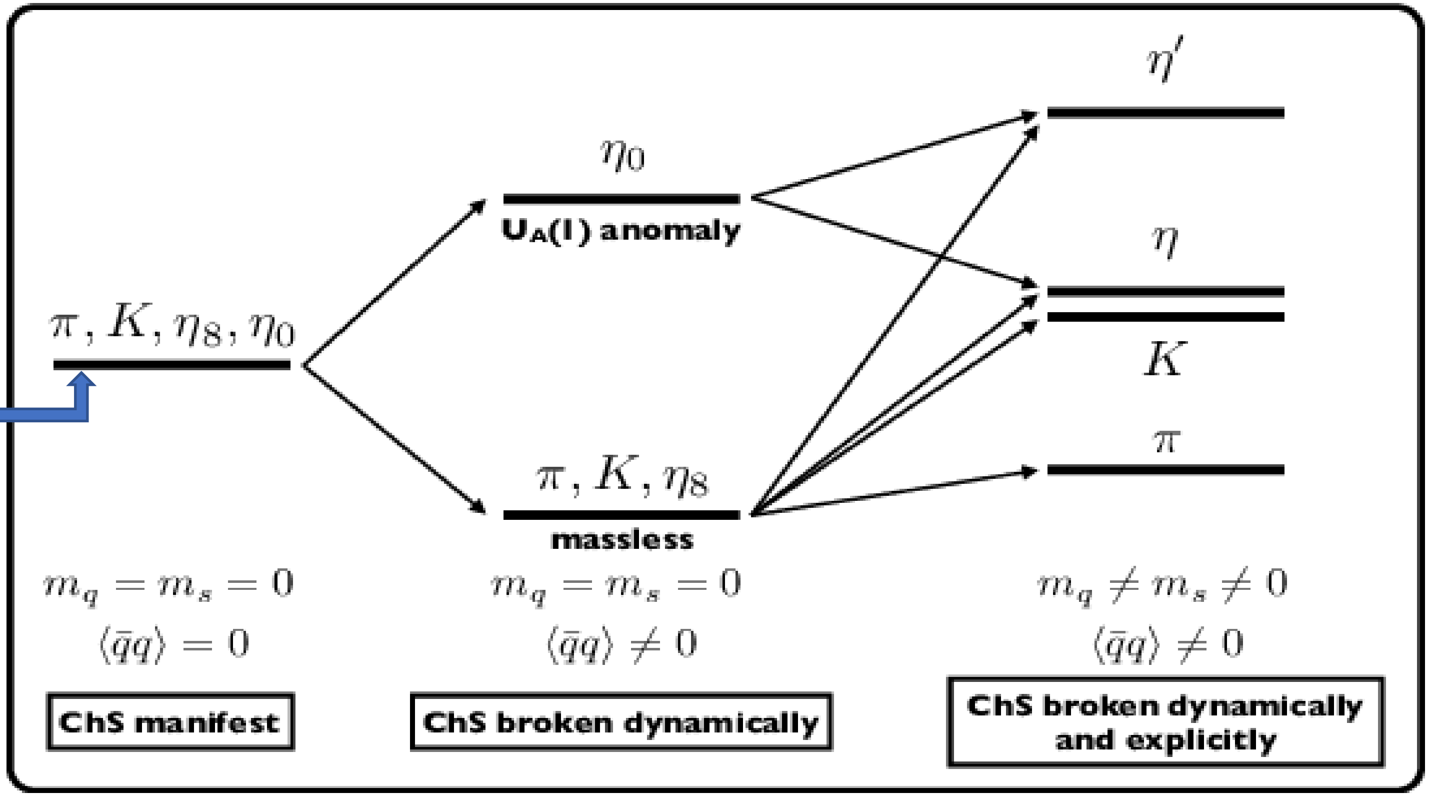
  $Q_V|0\rangle = 0$, but $Q_A|0\rangle \neq 0$ … **Nambu-Goldstone realization** of chiral symmetry.
- The chiral condensates are defined in terms of full quark propagator as
  $\left\langle 0\middle|\bar{\psi}\psi\middle|0\right\rangle = -i \lim_{y\to x_+} tr\, S_F(x,y)$; with $S_F(x,y) = -i\langle 0\middle|T\psi(x)\,\bar{\psi}(y)\middle|0\rangle$

- A non-zero VEV of $\langle\bar{\psi}\psi\rangle$ implies that the vacuum have lost the chiral symmetry of the Lagrangian.

- Likewise, the energy density of QCD vacuum is lowered by color electric and magnetic fields , so QCD vacuum also features a non-vanishing gluon condensates.
  $\langle\bar{u}u\rangle = \langle\bar{d}d\rangle = \langle\bar{s}s\rangle \sim -(225\ MeV)^3$
  And $\left\langle\frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{\mu\nu a}\right\rangle \sim (350\ MeV)^4$
- Axial current has a non vanishing divergence.
  $\partial_\mu A^\mu_a = \frac{i}{2}\ \bar{\psi}\{m, \lambda_a\}\gamma_5\psi$, **Partial Conservation of Axial Current (PCAC).**

Various symmetry breakings and meson spectra

Spontaneous breaking of $U(3)_L \times U(3)_R$ leads to 9 massless pseudoscalar Goldstone bosons



- **Scale invariance breaking** term, $\mathcal{L}_{scale\ break} = -\frac{1}{4}\chi^4 \ln\left(\frac{\chi^4}{\chi_0^4}\right) + \frac{d}{3}\chi^4 \ln\left(\left(\frac{(\sigma^2-\delta^2)\zeta}{\sigma_0^2\zeta_0}\right)\left(\frac{\chi}{\chi_0}\right)^3\right)$

- **Explicitly chiral symmetry breaking** term, $\mathcal{L}_{SB} = -\left(\frac{\chi}{\chi_0}\right)^2 \left[m_\pi^2 f_\pi \sigma + \left(\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi\right)\zeta\right]$

- **Magnetic field interaction** term[1], $\mathcal{L}_{mag} = -\frac{1}{4} F^{\mu\nu}F_{\mu\nu} - q_i\bar{\psi}_i\gamma_\mu A^\mu\psi_i - \frac{1}{4}\kappa_i\mu_N\bar{\psi}_i\sigma^{\mu\nu}F_{\mu\nu}\psi_i$

- Quark and gluon condensates are related with the scalar fields as,

$$m_u\langle\bar{u}u\rangle = \frac{m_\pi^2 f_\pi(\sigma+\delta)}{2}, \qquad m_d\langle\bar{d}d\rangle = \frac{m_\pi^2 f_\pi(\sigma-\delta)}{2}, \quad m_s\langle\bar{s}s\rangle = (\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi)\zeta$$

$$\left\langle\frac{\alpha_s}{\pi}G_{\mu\nu}^a\, G^{a\mu\nu}\right\rangle = \frac{8}{9}\left\{(1-d)\chi^4 + \left[m_\pi^2 f_\pi \sigma + (\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi)\zeta\right]\right\}$$

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[1] A. E. Broderick, M. Prakash, and J. M. Lattimer, Phys. Lett. B 531, 167 (2002).

$U(1)_A$ anomaly

Noether's theorem:

If $\mathcal{L}'(\psi', \partial_\mu \psi') \equiv \mathcal{L}(\psi, \partial_\mu \psi)$ under a global symmetry transformation defined as

$$\psi(x) \to \psi'(x) = \exp(i\Gamma_a \theta^a)\, \psi(x)$$

there will be a four-vector current $J_a^\mu(x) = \bar{\psi}(x)\gamma^\mu \Gamma_a \psi(x)$ for each Γ_a such that $\partial_\mu J_a^\mu(x) = 0$.

Here θ^a = parameter of transformation and generators Γ_a of the symmetry group.

The corresponding conserved charge $\mathrm{Q_a} = \int d^3x\, J_a^0(x)$

The $U(1)_L$ and $U(1)_R$ symmetries leads to conserved singlet left- and right-handed currents,

$$J_L^\mu = \bar{\psi}_L \gamma^\mu \psi_L;\; J_R^\mu = \bar{\psi}_R \gamma^\mu \psi_R$$

The vector and axial vector currents are formed as

$V^\mu = J_R^\mu + J_L^\mu = \bar{\psi}\gamma^\mu\psi;\; A^\mu = J_R^\mu - J_L^\mu = \bar{\psi}\gamma^\mu\gamma_5\psi$

(a) The vector current can be obtained independently through the $U(1)_V$ transformation $\psi(x) = e^{i\theta}\psi(x)$.

The corresponding conserved charge for vector current, after normalization is the baryon number given as

$$B = \frac{1}{3}\int d^3x\, \bar{\psi}\gamma^0\psi = \frac{1}{3}\int d^3x\, \psi^\dagger\psi$$

(b) The axial vector current can be obtained independently through the $U(1)_A$ transformation $\psi(x) = e^{i\theta\gamma_5}\psi(x)$.

This would lead to globally conserved axial vector current as $\partial_\mu A^\mu = 0$ and classically conserved axial current, the flavor singlet current, $A_\mu^0 = \bar{\psi}\gamma_\mu\gamma^5\psi = \bar{u}\gamma_\mu\gamma^5 u + \bar{d}\gamma_\mu\gamma^5 d + \bar{s}\gamma_\mu\gamma^5 s$

$U(1)_A$ anomaly ….. Continued

But this symmetry is no longer valid at the quantum level when we take quantum loop corrections into account.
The gluon fields contribute to the divergence of axial current and is known as $U(1)_A$ anomaly or axial anomaly

$$\partial_\mu A^\mu = \frac{\alpha_s}{4\pi} N_f G_{\mu\nu} \tilde{G}^{\mu\nu}$$

Dual field strength tensor, $\tilde{G}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}/2$ and $G_{\mu\nu}\tilde{G}^{\mu\nu} = 8\, \boldsymbol{E}.\boldsymbol{B}$
The anomaly also results in larger mass of η′ mesons (958 MeV) compared to η meson (548 MeV).

- The axial U $(1)_A$ breaking is connected to another important non-perturbative feature of QCD known as 'Instanton' which can couple to quarks through the divergence of the singlet axial-current.

- The GOR relations to leading order in the current quark masses,

$$m_\pi^2 f_\pi^2 = -2\bar{m}\langle \bar{q} q\rangle + \boldsymbol{\mathcal{O}}(m_q^2)$$
$$m_K^2 f_K^2 = -(\bar{m} + m_s)\langle \bar{q} q\rangle + \boldsymbol{\mathcal{O}}(m_q^2)$$
$$m_\eta^2 f_\eta^2 = -\frac{2}{3}(\bar{m} + 2m_s)\langle \bar{q} q\rangle + \boldsymbol{\mathcal{O}}\left(m_q^2\right); \quad \text{with } \bar{m} = \frac{m_u + m_d}{2}$$

In the chiral limit, the decay constants f_π, f_K, f_η are all equal. The η' and η decay constants are linked through $\eta - \eta'$ mixing.

$$f_\pi = (92.4 \pm 0.3) MeV$$
$$f_K = (113 \pm 1.3) MeV$$
$$f_\eta = (93 \pm 9) MeV$$
$$f_{\eta'} = (83 \pm 7) MeV$$

Origin of magnetic fields in HICs:

Huge magnetic fields are produced in non-central relativistic heavy ion colliders at various particle colliders like The Large Hadron Collider (LHC) at CERN near Geneva, Switzerland and The Relativistic Heavy Ion Collider (RHIC) at BNL, New York.

If two ions having charge 'Ze' and radius 'R' collide with an impact parameter 'b', then the magnetic field produced is given by Biot –Savart law $\boldsymbol{B \approx \gamma \frac{(Ze)b}{R^3}}$,

where lorentz factor, $\gamma = \frac{\sqrt{s_{NN}}}{2m_N}$, $\sqrt{s_{NN}}$ is the centre of mass energy per nucleon and $\mathrm{m_N}$ is the mass of nucleon

Estimation for RHIC: Heavy gold ions (Z=79) are collided with energy 200 GeV/nucleon, taking $b \approx R_{Au} = 7\ fm$ magnetic fields, **eB** $\approx \mathbf{6 \times 10^{18}\ Gauss = 2\ m_\pi^2}$ are produced.

At LHC energies, the amount of magnetic field produced are even higher , **eB** $\approx \mathbf{10^{19}\ Gauss \approx 15\ \boldsymbol{m_\pi^2}}$

- **Time evolution**: These produced magnetic fields decreases rapidly (by a factor of more than 3 orders of magnitude in first 3 fm/c time) as the ion remnants moves away from the collision zone.
- Induced currents are produced in accordance with Faraday's law, which slows down the decrease in magnetic field by inducing magnetic field according to Lenz's law.
- Time evolution of magnetic field also depends crucially on electrical conductivity of the medium.
- Lattice calculation studies have shown that external magnetic fields is a slowly varying function of time during the entire Quark-Gluon Plasma lifetime (5-10 fm/c).

Linear Sigma Model:

$$\mathcal{L}_{LS} = \frac{1}{2}\left(\partial_\mu \vec{\pi}.\partial^\mu \vec{\pi} + \partial_\mu \sigma \partial^\mu \sigma\right) - \frac{\lambda}{4}\left((\vec{\pi}^2 + \sigma^2) - f_\pi^2\right)^2 + i\bar{\psi}\gamma^\mu \partial_\mu \psi - g(\bar{\psi}\psi\sigma + (i\bar{\psi}\gamma_5 \vec{\tau}\psi).\vec{\pi})$$

The pion field: $\vec{\pi} = \bar{\psi}_q \vec{\tau}\gamma_5 \psi_q$; $\sigma = \bar{\psi}_q \psi_q$; $\psi = \begin{pmatrix} p \\ n \end{pmatrix}$

Under **vector transformation**, $\Lambda_V = e^{-i\frac{\vec{\tau}}{2}.\vec{\theta}} \Longrightarrow \pi_i \rightarrow \pi_i + \epsilon_{ijk}\theta_j \pi_k$; and $\sigma \rightarrow \sigma$ (rotation in flavor (isospin) space).

Under **axial vector transformation**, $\Lambda_A = e^{-i\gamma_5\frac{\vec{\tau}}{2}.\vec{\theta}} \Longrightarrow \pi_i \rightarrow \pi_i + \theta_i \sigma$; and $\sigma \rightarrow \sigma - \theta_i \pi_i$ (transforms the states)

But $\Lambda_V : \vec{\pi}^2 \rightarrow \vec{\pi}^2$; and $\sigma^2 \rightarrow \sigma^2$

$\Lambda_A : \vec{\pi}^2 \rightarrow \vec{\pi}^2 + 2\sigma\theta_i\pi_i$; and $\sigma^2 \rightarrow \sigma^2 - 2\sigma\theta_i\pi_i$

So, the combination $\vec{\pi}^2 + \sigma^2$ is invariant. This term is chiral invariant (under V and A) as well as Lorentz scalar.

Mass term for fermions: $M_N\bar{\psi}\psi \equiv g(\bar{\psi}\psi)\sigma_0$, $\text{if } \langle\sigma\rangle \neq 0, \text{but some } \sigma_0$

Thus, fermions acquire a non-zero mass arising due to a non-vanishing VEV of the scalar field.

This non-zero VEV is introduced by a specific potential term having minima at $\sigma = f_\pi$,

$V \equiv V(\vec{\pi}^2 + \sigma^2) = \frac{\lambda}{4}\left((\vec{\pi}^2 + \sigma^2) - f_\pi^2\right)^2$, Mexican hat potential

This ensures a minima for locus of all particles satisfying, $\left((\vec{\pi}^2 + \sigma^2) - f_\pi^2\right)^2 = 0$ referred as the chiral circle.

Since $\langle\vec{\pi}\rangle = 0$, we are concerned in $\vec{\pi} = 0$ direction only.

<u>Significance of Mexican hat potential</u>: We have ensured that V (0; 0) ≠ 0 but rather, has a hump, which shifts the ground state from the origin, to anywhere on the chiral circle.

The Lagrangian can be written in terms of $\Sigma \equiv \sigma - i\boldsymbol{\tau}.\boldsymbol{\pi}$ as,

$$\mathcal{L}_{LS} = \frac{1}{4} Tr\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right) - \frac{\lambda}{16}\left[Tr\Sigma^\dagger\Sigma\right]^2 + i\bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + i\bar{\psi}_R \gamma^\mu \partial_\mu \psi_R - g\left[\bar{\psi}_L \Sigma^\dagger \psi_R + \bar{\psi}_R \Sigma \psi_L\right]$$

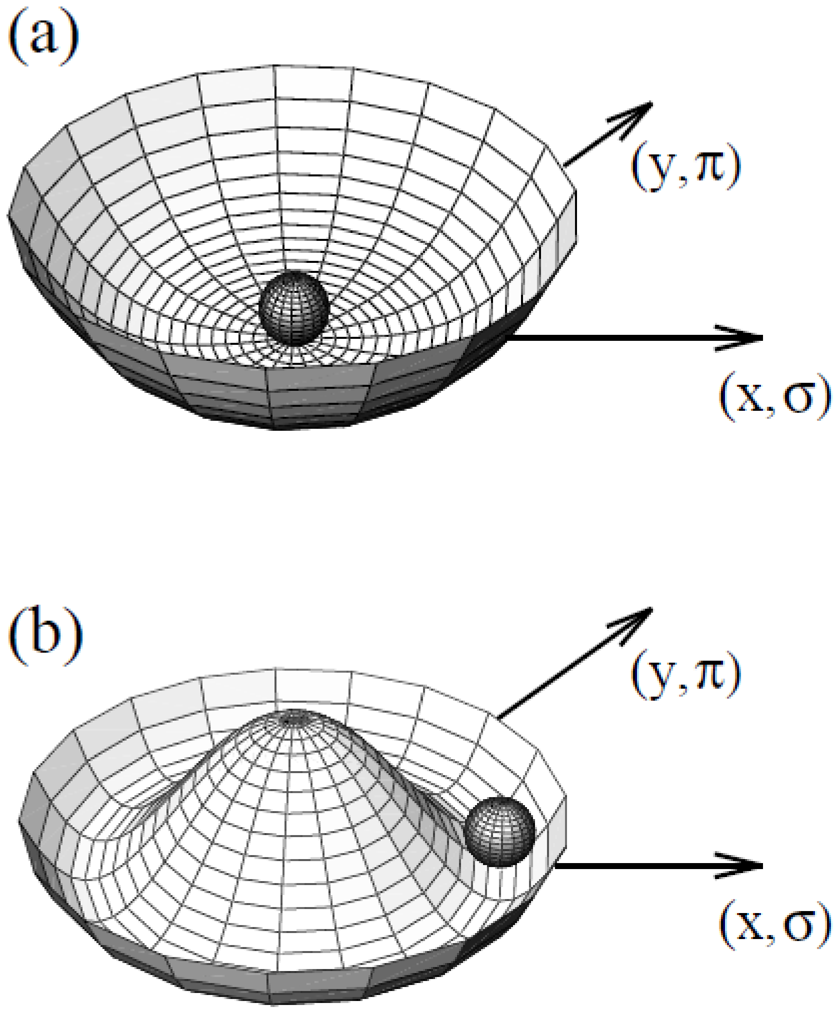
By introducing left- and right-handed fermion fields, the coupling term can be written as;

$$\mathcal{L}_c = g\bar{\psi}(\sigma + i\gamma_5 \boldsymbol{\tau}.\boldsymbol{\pi})\,\psi = g\left[\bar{\psi}_L \Sigma^\dagger \psi_R + \bar{\psi}_R \Sigma \psi_L\right];$$

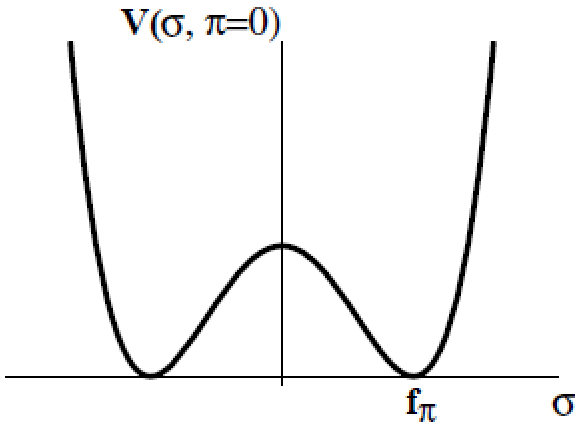
Where , $\Sigma \equiv \sigma - i\boldsymbol{\tau}.\boldsymbol{\pi}$ transforms like, $\Sigma \rightarrow R\Sigma L^\dagger$ under chiral $SU(2)_L \times SU(2)_R$ under chiral transformation,

$$\psi_L \rightarrow L\psi_L; \;\; and \;\; \psi_R \rightarrow R\psi_R$$

The fields σ *and* π transforms together (through the transformations of Σ) to maintain the chiral $SU(2)_L \times SU(2)_R$ invariance, hence the coupling constant is taken to be same.



Considering small perturbation; $\sigma = \langle\sigma\rangle + \delta\sigma$; $\vec{\pi} = \langle\vec{\pi}\rangle + \delta\vec{\pi} = 0 + \delta\vec{\pi}$
Expanding the Mexican hat potential → $V = \lambda f_\pi^2(\delta\sigma)^2 + O(\delta^3)$
We get $m_\sigma^2 = 2\lambda f_\pi^2 \neq 0$; $m_\pi^2 = 0$
Pion is massless as expected due to Goldstone theorem.

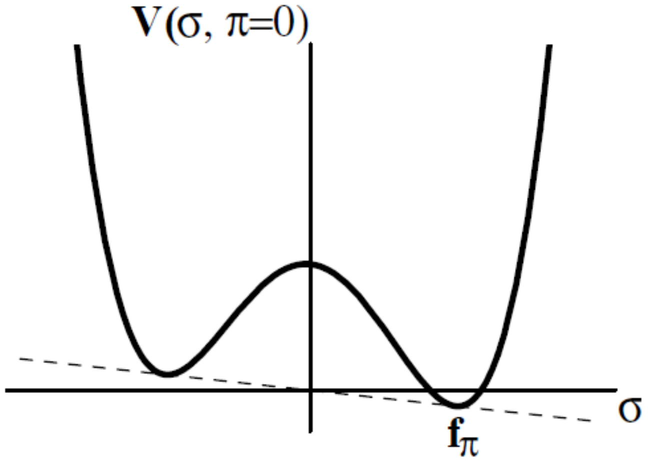


Now, if we add explicit symmetry breaking term;
$\mathcal{L}_{ESB} = -m\bar{q}q \equiv \epsilon\sigma$, Where ε is the symmetry breaking parameter

This term satisfies the requirement of respecting Λ_V but not Λ_A, as expected from a symmetry breaking term in this context.

The potential becomes; $V(\sigma,\vec{\pi}) = \frac{\lambda}{4}[(\vec{\pi}^2 + \sigma^2) - v_0^2]^2 - \varepsilon\sigma$
The symmetry breaking term ($\varepsilon\sigma$) is linear, potential gets tilted to one side along σ-axis
To preserve $\sigma_0 = f_\pi$ we adjust $v_0 = f_\pi - \varepsilon/2\lambda f_\pi^2$
Again expanding the vacuum around fluctuations,
We get, $m_\pi^2 = \frac{\partial^2 V}{\partial_i\pi\partial^i\pi} = \frac{\varepsilon}{f_\pi} \neq 0$; $m_\sigma^2 = 2\lambda f_\pi^2 + \frac{\varepsilon}{f_\pi}$
Pions get mass due to ECSB.
Using $-m\bar{q}q = \varepsilon\sigma$ *and* $m_\pi^2 f_\pi = \varepsilon$, $\langle\sigma_0\rangle = f_\pi$
We arrive at **Gell-Mann-Oakes-Renner relation:**

$$m_\pi^2 f_\pi^2 = -\frac{m_u + m_d}{2}\langle\bar{u}u + \bar{d}d\rangle$$



Kaons and antikaons in asymmetric nuclear matter

Kaons and antikaons: bound states of a strange quark (or antiquark) and an up or down antiquark (or quark).

$$\text{kaons } (K^+ \equiv u\bar{s}, K^0 \equiv s\bar{s}) \text{ and antikaons } \left(K^- \equiv \bar{u}s, \bar{K}^0 \equiv \bar{d}s\right)$$

Importance of study:

- Kaon-Nucleon scattering study is important in interpretation of strangeness fraction produced in high energy collisions.
- Kaon beams can be used as probes for nuclear structures as kaon-nucleon cross section is small.
- Due to relevance in neutron star phenomenology and relativistic heavy ion collisions (HICs).

We study the medium modifications of the energies of kaons (antikaons) using **chiral SU(3) model**.
These modifications arises due **to interaction** of kaons (antikaons) with scalar fields of the medium and vectorial interaction with nucleons (Weinberg-Tomozawa interaction).
The **interaction Lagrangian** consists of:

(1) Vectorial interaction term is attractive for antikaons $(\bar{K}^0, K^-)$ and repulsive for kaon (K^0, K^+) energy.
(2) The scalar meson exchange term, which arises from explicit symmetry breaking terms , is attractive for both kaon and antikaons.
(3) There are three range terms: First range term (which arises from kinetic terms of pseudoscalar mesons) is repulsive for both kaons and antikons, while last two range terms are attractive for both kaons and antikaons.

The last two range terms are:

$$\mathcal{L}_{d1} = \frac{d_1}{2\, f_K^2}[(\bar{N}N)\left(\partial_\mu \bar{K}\right)(\partial^\mu K)], \text{where } N \text{ represents nucleons and } K \text{ represents kaon doublet.}$$

$$\mathcal{L}_{d_2} = \frac{d_2}{2\, f_K^2}[(\bar{p}p)\left(\partial_\mu K^+\right)(\partial^\mu K^-) + (\bar{n}n)\left(\partial_\mu K^0\right)(\partial^\mu \bar{K}^0) + (\bar{p}n)\left(\partial_\mu K^+\right)(\partial^\mu \bar{K}^0) + (\bar{n}p)\left(\partial_\mu K^0\right)(\partial^\mu K^-)],$$

The parameters d_1 and d_2 are calculated from the empirical values of KN scattering lengths which gives $d_1 = \frac{5.5}{m_K}$ $and\ d_2 = \frac{0.66}{m_K}$; where m_K is the mass of kaon.

The total Lagrangian density is written $as\ \mathcal{L} = (\partial_\mu \bar{K})(\partial^\mu K) - m^2_{K(\bar{K})}\bar{K}K + \mathcal{L}_K^{int}$

Then the Fourier transformation of the equation of motion will give us the following dispersion relation:

$-\omega^2 + \vec{k} + m_K^{*2} - \Pi(\omega, |\vec{k}|, \rho) = 0$, where $\Pi_{K(\bar{K})}$ is the kaon self energy.

Interaction with magnetic field: Charged kaons (antikaons) $K^\pm$ have additional positive arising from direct interaction of these mesons with the magnetic field. $m_{K^\pm}^{eff} = \sqrt{(m_{K^\pm}^*)^2 + eB}$; while the neutral kaons (antikaons) have no such direct interaction with the magnetic field, $m_{K^0(\bar{K}^0)}^{eff} = m_{K^0(\bar{K}^0)}^*$.

Results: A. Mishra, A. K. Singh, N. S. Rawat, and P. Aman, Eur. Phys. J. A 55, 107 (2019).

		K^+	K^0
Vacuum		493.677	497.611
ρ_0	$\eta = 0$	521	524.8
	$\eta = 0.5$	518	529
$4\rho_0$	$\eta = 0$	544.6	547.1
	$\eta = 0.5$	528.9	591.5