

Light Λ -hypernuclei and CSB interaction

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(Hyper)nuclear charge symmetry breaking

Nuclear :

(Phys. Rev. C 63 (2001) 034005)

- most clearly seen in 1S_0 scattering lengths (corrected for electromagnetic effects)

$$a_{nn}^0 = -18.95 \pm 0.4 \text{ fm}$$

$$a_{pp}^0 = -17.3 \pm 0.4 \text{ fm}$$

- $B(^3\text{H}) - B(^3\text{He}) = 764 \text{ keV}$

→ CSB in ^3H and ^3He nuclei $\Delta_{^3\text{H}-^3\text{He}}^{\text{CSB}} = 68 \pm 9 \text{ keV}$

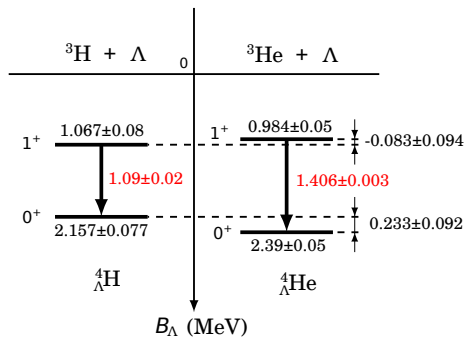
Hypernuclear :

(AIP Conf. Proc. 2130 (2019) 030003)

- Λp vs. Λn interaction
- strong CSB experimentally measured in ${}^4_{\Lambda}\text{H}$ - ${}^4_{\Lambda}\text{He}$ hypernuclei
- on the two-body level mostly Λp information
- neutron-rich hypernuclei Λnn , $\Lambda\Lambda n$, $\Lambda\Lambda nn$, ${}^6_{\Lambda}\text{H}$, ${}^6_{\Lambda}\text{He}$, ${}^7_{\Lambda}\text{He}$, ${}^8_{\Lambda}\text{He}$, ${}^9_{\Lambda}\text{He}$

Charge symmetry breaking in ${}^4_{\Lambda}\text{H}/{}^4_{\Lambda}\text{He}$

- $B_{\Lambda}({}^4_{\Lambda}\text{H}; 0^+)$ measurement at MAMI
(Nucl. Phys. A, 954 (2016) 149)
- $B_{\Lambda}({}^4_{\Lambda}\text{He}; 0^+)$ measurement (emulsion)
(Nucl. Phys. A 754 (2005) 3c)
- $E_{\gamma}({}^4_{\Lambda}\text{H}; 1^+ \rightarrow 0^+)$, $E_{\gamma}({}^4_{\Lambda}\text{He}; 1^+ \rightarrow 0^+)$
 γ -ray energies (J-PARC)
(Phys. Rev. Lett., 115 (2015) 222501)



Sizable CSB splitting in 0^+ ground states, while small in 1^+ excited states.

STAR collaboration (Phys. Lett. B 834 (2022) 137449):

$$\Delta B_{\Lambda}(0^+_{\text{g.s.}}) = -\Delta B_{\Lambda}(1^+_{\text{exc.}}) = 160 \pm 140(\text{stat}) \pm 100(\text{syst}) \text{ keV}$$

Theoretical works - DvH mechanism

- **R. H. Dalitz and F. von Hippel** (Phys. Lett. 10 (1964) 153)
 → CSB OPE contribution by allowing $\Lambda - \Sigma^0$ mixing in $SU(3)_f$

$$g_{\Lambda\Lambda\pi} = 2\mathcal{A}_{I=1}^{(0)} g_{\Lambda\Sigma\pi}; \quad \mathcal{A}_{I=1}^{(0)} = -\frac{\langle \Sigma^0 | \delta M | \Lambda \rangle}{M_{\Sigma^0} - M_{\Lambda}} = -0.0148(6)$$

- **A. Gal** (Phys. Let. B 744 (2015) 352)
 → generalization of DvH

$$\langle N\Lambda | V_{\Lambda N}^{CSB} | N\Lambda \rangle = -\frac{2}{\sqrt{3}} \mathcal{A}_{I=1}^{(0)} \tau_{Nz} \langle N\Lambda | V | N\Sigma \rangle$$

$$\rightarrow \Delta B_{\Lambda}(0_{g.s.}^+) \approx 240 \text{ keV} \quad \Delta B_{\Lambda}(1_{exc.}^+) \approx 35 \text{ keV}$$

- **D. Gazda and A. Gal**

(Phys. Rev. Lett. 116 (2016) 122501; Nucl. Phys. A 954 (2016) 161)

→ generalized DvH; LO χ EFT YN interaction; NSCM

$$\rightarrow \Delta B_{\Lambda}(0_{g.s.}^+) \approx 180 \pm 130 \text{ keV} \quad \Delta B_{\Lambda}(1_{exc.}^+) \approx -200 \pm 30 \text{ keV}$$

Theoretical works - χ EFT

J. Haidenbauer et al., Few-Body Syst. 62 (2021) 105

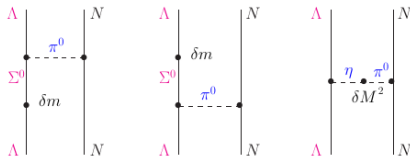


Fig. 1 CSB contributions involving pion exchange, according to Dalitz and von Hippel [1], due to $\Lambda - \Sigma^0$ mixing (left two diagrams) and $\pi^0 - \eta$ mixing (right diagram).

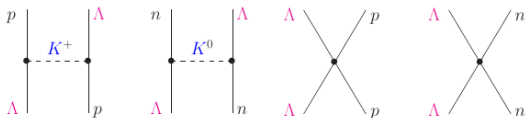


Fig. 2 CSB contributions from K^\pm/K^0 exchange (left) and from contact terms (right).

Contact terms fitted **perturbatively** to experimental CSB in ${}^4_\Lambda\text{He}$ and ${}^4_\Lambda\text{H}$.

$$\Delta B_\Lambda(0^+_{\text{g.s.}}) = 233 \pm 92 \text{ keV}$$

$$\Delta B_\Lambda(1^+_{\text{exc.}}) = -83 \pm 94 \text{ keV}$$

Theoretical works - χ EFT

J. Haidenbauer et al., *Few-Body Syst.* 62 (2021) 105

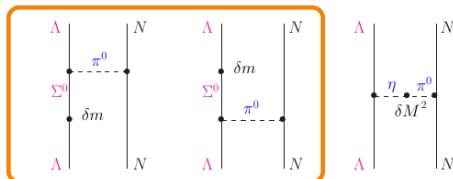


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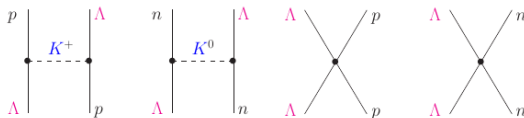


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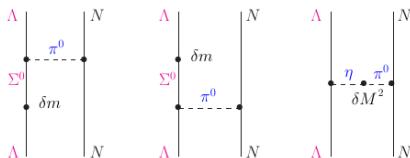


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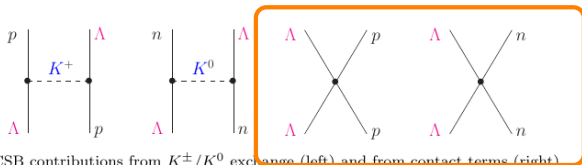


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Theoretical works - χ EFT

J. Haidenbauer et al., Few-Body Syst. 62 (2021) 105

Table 2 Comparison of different CSB scenarios, based on the YN interactions NLO13 and NLO19 with cutoff $\Lambda = 600$ MeV

	a_s^{Ap}	a_t^{Ap}	a_s^{An}	a_t^{An}	$\chi^2(\Lambda p)$	$\chi^2(\Sigma N)$	$\chi^2(\text{total})$	$\Delta E(0^+)$	$\Delta E(1^+)$
NLO13	-2.906	-1.541	-2.907	-1.517	4.47	12.34	16.81	58	24
CSB-OBE	-2.881	-1.547	-2.933	-1.513	4.39	12.43	16.83	57	20
CSB1	-2.588	-1.573	-3.291	-1.487	3.43	12.38	15.81	256	-53
CSB2	-3.983	-1.281	-2.814	-0.948	4.51	12.31	16.82	299	161
CSB3	-2.792	-1.666	-3.027	-1.407	9.52	12.41	21.93	370	56
NLO19	-2.906	-1.423	-2.907	-1.409	3.58	12.70	16.28	34	10
CSB-OBE	-2.877	-1.415	-2.937	-1.419	3.30	13.01	16.31	-6	-7
CSB1	-2.632	-1.473	-3.227	-1.362	3.45	12.68	16.13	243	-67
CSB2	-3.618	-1.339	-3.013	-1.117	4.02	12.09	16.12	218	129
CSB3	-2.758	-1.546	-3.066	-1.300	7.49	12.64	20.14	359	45

Results are shown for the original NLO interactions, with addition of OBE contribution to CSB, and for the scenarios CSB1, CSB2, CSB3 with added CSB contact terms. CSB1 corresponds to the present experimental status. Note that the χ^2 for the NLO interactions differs slightly from the ones given in Refs. [20,21] because there the small differences between Λp and Λn have not been taken into account. Small deviations of the CSB from values of the three scenarios are due to using perturbation theory for fitting and using a smaller number of partial waves for fitting

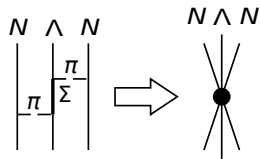
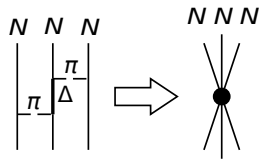
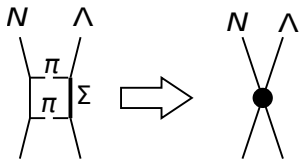
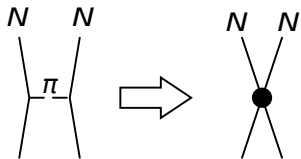
Theoretical works - χ EFT

H. Le et al., Phys. Rev. C 107 (2023) 024002

TABLE IV. Contributions to CSB in the $A = 7$ and 8 isospin multiplets, based on the YN potentials NLO13(500) and NLO19(500) (including $3N$ forces and SRG-induced YNN interactions). The results are for the original potentials (without CSB force) and for the scenario CSB1, see text. Results by Gal [37] and by Hiyama *et al.* [13] are included for the ease of comparison. All energies are in keV. The estimated uncertainties for $A = 7$ and 8 systems are 30 and 50 keV, respectively.

	ΔT	ΔV_{NN}	ΔV_{YN}			ΔB_A	
			1S_0	3S_1	Total		
${}^7_\Lambda\text{Be} - {}^7_\Lambda\text{Li}^*$	NLO13	7	-24	-1	0	0	-17
	NLO13-CSB	8	-24	-49	26	-24	-40
	NLO19	6	-40	-1	0	0	-34
	NLO19-CSB	6	-41	-43	42	9	-35
	Hiyama [13]		-70			200	150
	Gal [37]	3	-70			50	-17
	Experiment [6]						-100 ± 90
${}^7_\Lambda\text{Li}^* - {}^7_\Lambda\text{He}$	NLO13	8	-13	0	0	0	-5
	NLO13-CSB	7	-14	-49	26	-24	-31
	NLO19	5	-22	-43	42	0	-17
	NLO19-CSB	5	-21	-38	37	-1	-16
	Hiyama [13]		-80			200	130
	Gal [38]	2	-80			50	-28
	Experiment [6]						-20 ± 230^a -50 ± 190
${}^8_\Lambda\text{Be} - {}^8_\Lambda\text{Li}$	NLO13	12	8	-2	0	-4	16
	NLO13-CSB	12	7	100	56	159	178
	NLO19	7	-11	-1	0	-2	-6
	NLO19-CSB	6	-11	62	79	147	143
	Hiyama [13]		40				160
	Gal [37]	11	-81			119	49
	Experiment [4]						40 ± 60

^aThe difference between $B_\Lambda({}^7_\Lambda\text{Li}^*)$ and $B_\Lambda({}^7_\Lambda\text{He})$ is -20 ± 230 keV for the FINUDA and JLab results, but -50 ± 190 keV when the revised SKS and JLab results are used [6].



Hypernuclear CSB within $\not\chi$ EFT (Schäfer, Barnea, Gal, Phys. Rev. C 106 (2022) L031001)

Charge Symmetric (CS) LO $\not\chi$ EFT

Nuclear :

$$V_{NN} = \sum_S C_{NN}^S(\lambda) \mathcal{P}^S e^{-\frac{\lambda^2}{4} r_{12}^2}$$

$$V_{NNN} = D_\lambda^{1/2 \ 1/2} Q^{1/2 \ 1/2} \sum_{\text{cyc}} e^{-\frac{\lambda^2}{4} (r_{12}^2 + r_{23}^2)}$$

Hypernuclear :

$$V_{\Lambda N} = \sum_S C_{\Lambda N}^S(\lambda) \mathcal{P}^S e^{-\frac{\lambda^2}{4} r_{12}^2}$$

$$V_{\Lambda NN} = \sum_{IS} D_{\Lambda NN}^{IS}(\lambda) Q^{IS} \sum_{\text{cyc}} e^{-\frac{\lambda^2}{4} (r_{12}^2 + r_{23}^2)}$$

Fitted to explicit CS input :

$$\rightarrow B(^2\text{H})$$

$$\rightarrow a_{nn/pp}^0 = -18.13 \text{ fm}$$

$$\rightarrow \text{several sets of } (a_{\Lambda N}^0; a_{\Lambda N}^1)$$

$$\rightarrow B_\Lambda(^3\text{H})$$

$$\rightarrow \text{CS average } B(^3\text{H}/^3\text{He})$$

$$\rightarrow \text{CS average } B_\Lambda(^4\text{H}/^4\text{He})$$

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CSB in NN interaction (spin singlet)

$$C_{NN}^{S=0} \mathcal{P}^{S=0} \rightarrow (C_{pp}^{S=0} \mathcal{P}^{pp} + C_{nn}^{S=0} \mathcal{P}^{nn} + C_{pn}^{S=0} \mathcal{P}^{pn}) \mathcal{P}^{S=0}$$

$$C_{NN}^{S=0} = \frac{1}{2} (C_{pp}^{S=0} + C_{nn}^{S=0}), \quad \delta C_{NN}^{S=0} = \frac{1}{2} (C_{pp}^{S=0} - C_{nn}^{S=0})$$

$$V_{NN} = \underbrace{\sum_S C_{NN}^S(\lambda) \mathcal{P}^S e^{-\frac{\lambda^2}{4} r_{12}^2}}_{\text{part of LO CS } \not\chi\text{EFT}} + \underbrace{\delta C_{NN}^{S=0}(\lambda) \mathcal{P}^{S=0} (\mathcal{P}^{pp} - \mathcal{P}^{nn}) e^{-\frac{\lambda^2}{4} r_{12}^2}}_{\text{perturbative CSB}}$$

Nuclear CSB within $\not\pi$ EFT (Schäfer, Barnea, Gal, Phys. Rev. C 106 (2022) L031001)

$$\delta C_{NN}^{S=0}(\lambda)$$



Fit to $\Delta_{\text{H-}^3\text{He}}^{\text{CSB}} = 68 \pm 9 \text{ keV}$



CSB + DWBA
 $(a_{pp/nn}^0 = -18.13 \pm 0.3 \text{ fm})$

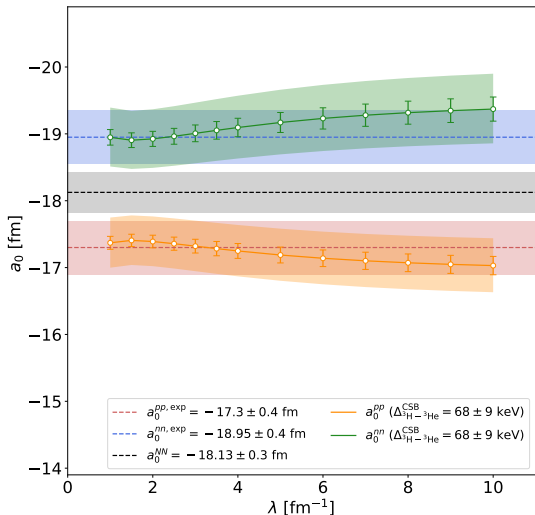


a_{pp}^0 and a_{nn}^0 scattering lengths



$$a_{nn}^0 = -18.95 \pm 0.4 \text{ fm}$$

$$a_{pp}^0 = -17.3 \pm 0.4 \text{ fm}$$



Hypernuclear CSB within $\not\equiv$ EFT (Schäfer, Barnea, Gal, Phys. Rev. C 106 (2022) L031001)

Charge Symmetric (CS) LO $\not\equiv$ EFT

Nuclear :

$$V_{NN} = \sum_S C_{NN}^S(\lambda) \mathcal{P}^S e^{-\frac{\lambda^2}{4} r_{12}^2}$$

$$V_{NNN} = D_\lambda^{1/2} Q^{1/2} \sum_{\text{cyc}} e^{-\frac{\lambda^2}{4} (r_{12}^2 + r_{23}^2)}$$

Hypernuclear :

$$V_{\Lambda N} = \sum_S C_{\Lambda N}^S(\lambda) \mathcal{P}^S e^{-\frac{\lambda^2}{4} r_{12}^2}$$

$$V_{\Lambda NN} = \sum_{IS} D_{\Lambda NN}^{IS}(\lambda) Q^{IS} \sum_{\text{cyc}} e^{-\frac{\lambda^2}{4} (r_{12}^2 + r_{23}^2)}$$

CSB in ΛN interaction

$$C_{\Lambda N}^S \mathcal{P}^S \rightarrow \left(C_{\Lambda p}^S \frac{1 + \tau_{Nz}}{2} + C_{\Lambda n}^S \frac{1 - \tau_{Nz}}{2} \right) \mathcal{P}^S$$

$$C_{\Lambda N}^S = \frac{1}{2} (C_{\Lambda p}^S + C_{\Lambda n}^S), \quad \delta C_{\Lambda N}^S = \frac{1}{2} (C_{\Lambda p}^S - C_{\Lambda n}^S)$$

$$V_{\Lambda N} = \overbrace{\sum_S C_{\Lambda N}^S(\lambda) \mathcal{P}^S e^{-\frac{\lambda^2}{4} r_{12}^2}}^{\text{part of LO CS } \not\equiv \text{EFT}} + \overbrace{\sum_S \delta C_{\Lambda N}^S(\lambda) \mathcal{P}^S \tau_{Nz} e^{-\frac{\lambda^2}{4} r_{12}^2}}^{\text{perturbative CSB}}$$

Fitting CSB LECs (Schäfer, Barnea, Gal, Phys. Rev. C 106 (2022) L031001)

→ perturbatively

→ two experimental constraints (${}^4_{\Lambda}\text{He}$ and ${}^4_{\Lambda}\text{H}$)

$$\Delta B_{\Lambda}(0_{\text{g.s.}}^+) = 233 \pm 92 \text{ keV} \quad \Delta B_{\Lambda}(1_{\text{exc.}}^+) = -83 \pm 94 \text{ keV}$$

System of two linear equation for $\delta C_{\Lambda N}^0$ and $\delta C_{\Lambda N}^1$:

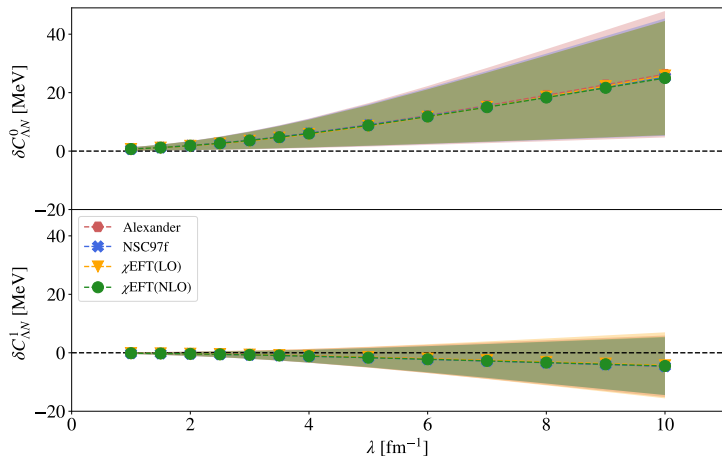
$$\begin{aligned} 2 \delta C_{\Lambda N}^0 \Delta V_{\Lambda N; 0^+}^0 + 2 \delta C_{\Lambda N}^1 \Delta V_{\Lambda N; 0^+}^1 &= \Delta B_{\Lambda}(0_{\text{g.s.}}^+) \\ 2 \delta C_{\Lambda N}^0 \Delta V_{\Lambda N; 1^+}^0 + 2 \delta C_{\Lambda N}^1 \Delta V_{\Lambda N; 1^+}^1 &= \Delta B_{\Lambda}(1_{\text{exc.}}^+) \end{aligned}$$

where

$$\Delta V_{\Lambda N; J^{\pi}}^S = \underbrace{\langle {}^4_{\Lambda}\text{H}; J^{\pi} | \tau_{Nz} \mathcal{P}_S \delta_{\lambda}(\Lambda N) | {}^4_{\Lambda}\text{H}; J^{\pi} \rangle}_{\text{CS LO } \not\epsilon\text{EFT wave function}}$$

Fitting CSB LECs

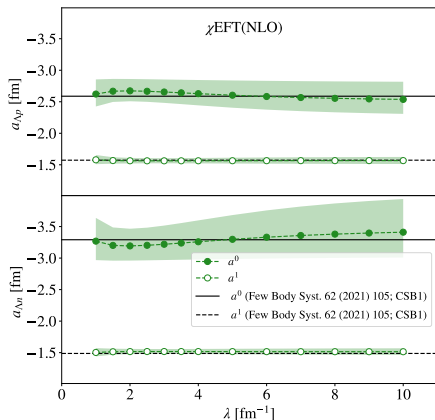
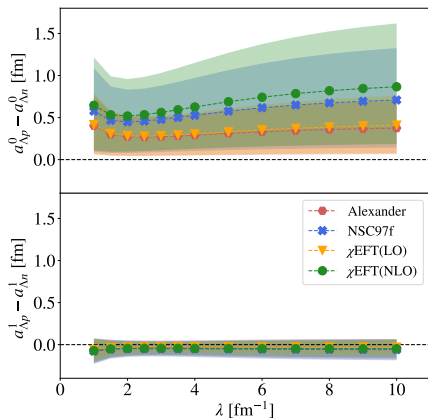
(Schäfer, Barnea, Gal, Phys. Rev. C 106 (2022) L031001)



$|\delta C_{\Lambda N}^1| < |\delta C_{\Lambda N}^0|$; predominantly opposite sign

Λp and Λn scattering lengths (Schäfer, Barnea, Gal, Phys. Rev. C 106 (2022) L031001)

→ CSB propagated into ΛN scattering length (perturbatively; DWBA)



$S = 0$: Stronger Λn and weaker Λp interaction

$S = 1$: Hardly affected; mostly stronger Λp and weaker Λn interaction

In-medium Λ isospin impurity (Schäfer, Barnea, Gal, Phys. Rev. C 106 (2022) L031001)

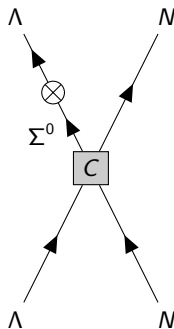
DvH ansatz :

(A. Gal, Phys. Lett. B 744 (2015) 352)

$$\langle \Lambda N | V_{\text{CSB}} | \Lambda N \rangle = -\frac{2}{\sqrt{3}} \mathcal{A}_{I=1}^{(0)} \langle \Sigma N | V_{\text{CS}} | \Lambda N \rangle \tau_{Nz}$$

↓

$$\delta C_{\Lambda N}^S = -\frac{2}{\sqrt{3}} \mathcal{A}_{I=1}^S C_{\Lambda N; \Sigma N}^S$$

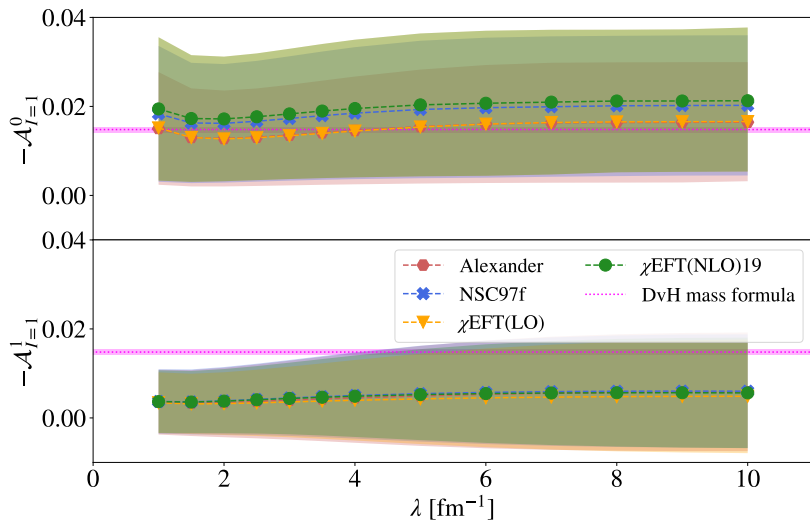


$SU(3)_f$ symmetry:

(C.B. Dover, H. Feshbach, Ann. Phys. (NY) 198 (1990) 321)

$$\left. \begin{aligned} C_{\Lambda N, \Sigma N}^0 &= -3(C_{NN}^0 - C_{\Lambda N}^0) \\ C_{\Lambda N, \Sigma N}^1 &= (C_{NN}^1 - C_{\Lambda N}^1) \end{aligned} \right\} \longrightarrow \begin{aligned} -\mathcal{A}_{I=1}^0 &= (\sqrt{3}/2)\delta C_{\Lambda N}^0 / [-3(C_{NN}^0 - C_{\Lambda N}^0)] \\ -\mathcal{A}_{I=1}^1 &= (\sqrt{3}/2)\delta C_{\Lambda N}^1 / [(C_{NN}^1 - C_{\Lambda N}^1)] \end{aligned}$$

In-medium Λ isospin impurity (Schäfer, Barnea, Gal, Phys. Rev. C 106 (2022) L031001)



In-medium Λ isospin impurity (Schäfer, Barnea, Gal, Phys. Rev. C 106 (2022) L031001)

→ considering more precise $\Delta E_\gamma = 316 \pm 20$ keV

Relation between CSB LECs and ΔE_γ :

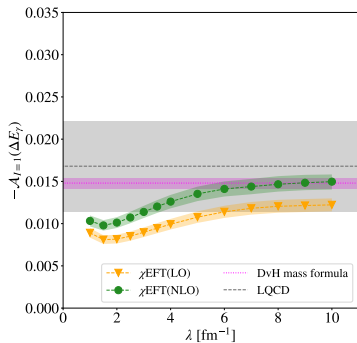
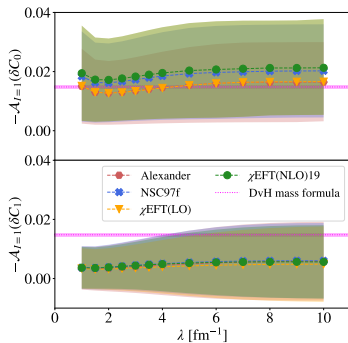
$$2 \delta C_{\Lambda N}^0 \left[\Delta V_{\Lambda N; 0^+}^0 - \Delta V_{\Lambda N; 1^+}^0 \right] + 2 \delta C_{\Lambda N}^1 \left[\Delta V_{\Lambda N; 0^+}^1 - \Delta V_{\Lambda N; 1^+}^1 \right] = \Delta E_\gamma$$

→ assuming DvH ansatz, $SU(3)_f$ symmetry, and $\mathcal{A}_{I=1}^0 = \mathcal{A}_{I=1}^1$

Relation between $I = 1$ admixture amplitude and ΔE_γ :

$$-\mathcal{A}_{I=1} = \frac{\sqrt{3}}{2} \Delta E_\gamma \left(-6(C_{NN}^0 - C_{\Lambda N}^0) [\Delta V_{\Lambda N; 0^+}^0 - \Delta V_{\Lambda N; 1^+}^0] + 2(C_{NN}^1 - C_{\Lambda N}^1) [\Delta V_{\Lambda N; 0^+}^1 - \Delta V_{\Lambda N; 1^+}^1] \right)^{-1}$$

In-medium Λ isospin impurity (Schäfer, Barnea, Gal, Phys. Rev. C 106 (2022) L031001)



Method/Input	B	$-\mathcal{A}_{I=1}$
SU(3) $_f$ (Phys. Lett 10 (1964) 153)	1	0.0148 ± 0.0006
LQCD (Phys. Rev. D 101 (2020) 034517)	1	0.0168 ± 0.0054
χ EFT (LO)/[χ EFT(LO); $\Lambda \rightarrow \infty$]	4	0.0139 ± 0.0013
χ EFT (LO)/[χ EFT(NLO); $\Lambda \rightarrow \infty$]	4	0.0168 ± 0.0014

Summary

- DvH mechanism; χ EFT approach using CSB contact terms
- perturbative inclusion of CSB in LO $\not\chi$ EFT (fitted to CSB in ${}^4_{\Lambda}\text{H}/{}^4_{\Lambda}\text{He}$)
- microscopic calculations of CSB in $A=4, 7, 8$ hypernuclei
- Spin-singlet : Stronger Λn and weaker Λp interaction
Spin-triplet : Hardly affected; mostly stronger Λp and weaker Λn

LO $\not\chi$ EFT - assumption of DvH ansatz and $\text{SU}(3)_f$ symmetry

- extraction of in-medium Λ isospin impurity $\mathcal{A}_{I=1}$; all cases in agreement with free-space LQCD prediction and in most cases with free-space DvH value
- using $\mathcal{A}_{I=1}^{(0)}$ DvH value the procedure can be applied in reverse thus predicting experimental CSB in ${}^4_{\Lambda}\text{H}/{}^4_{\Lambda}\text{He}$