Thermal QCD axion production from the early universe

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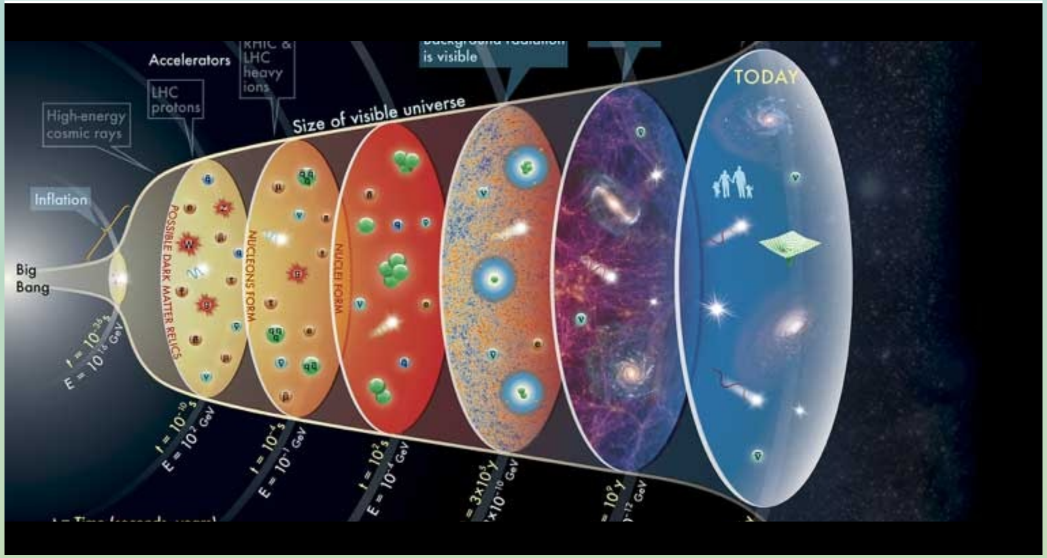
2211.03799 [hep-ph], to appear in PRL, with F. Rompineve and G.Villadoro

Relic particles in Cosmology

- Universe expanding with scale factor a,
- Primordial plasma made of g_* degrees of freedom and temperature T



- Approximately $T \propto 1/a$ (redshift)

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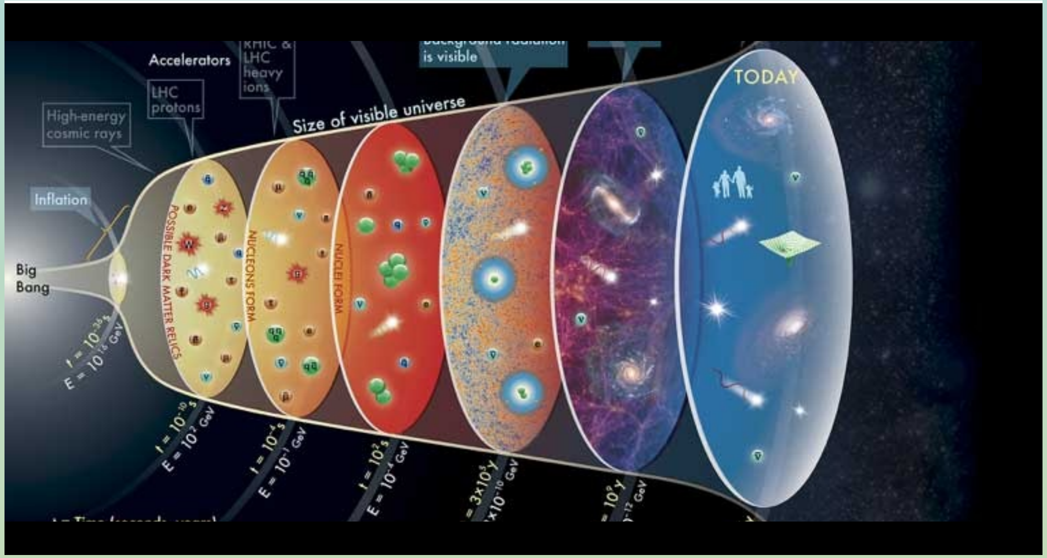

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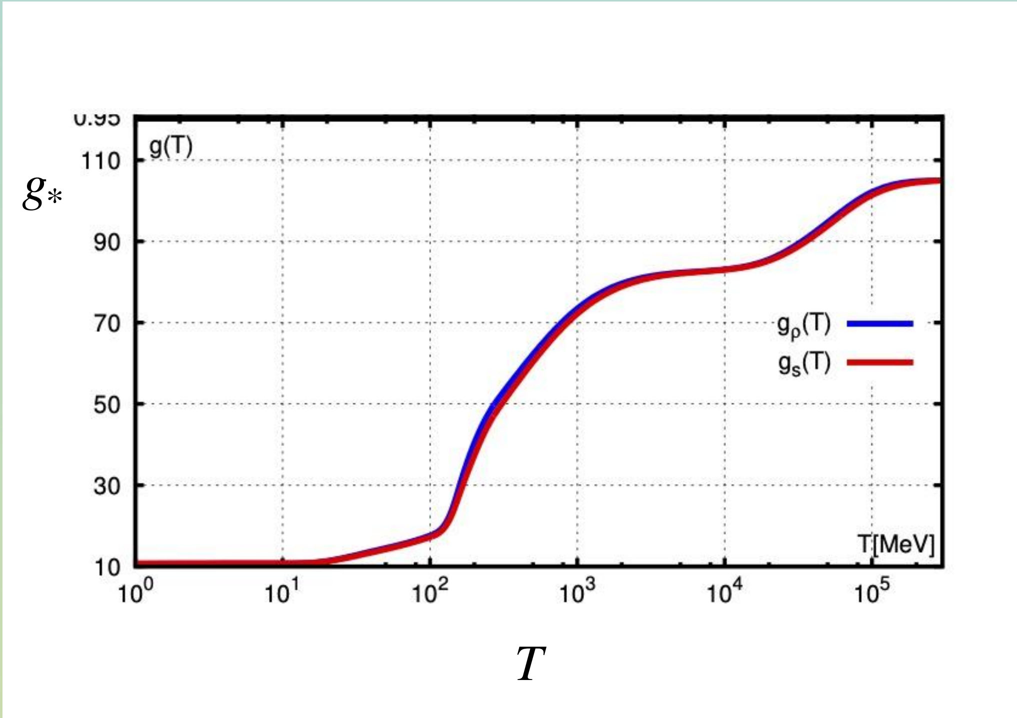


Total plasma energy density: $\rho_{\mathrm{TOT}} \propto g_* T^4$

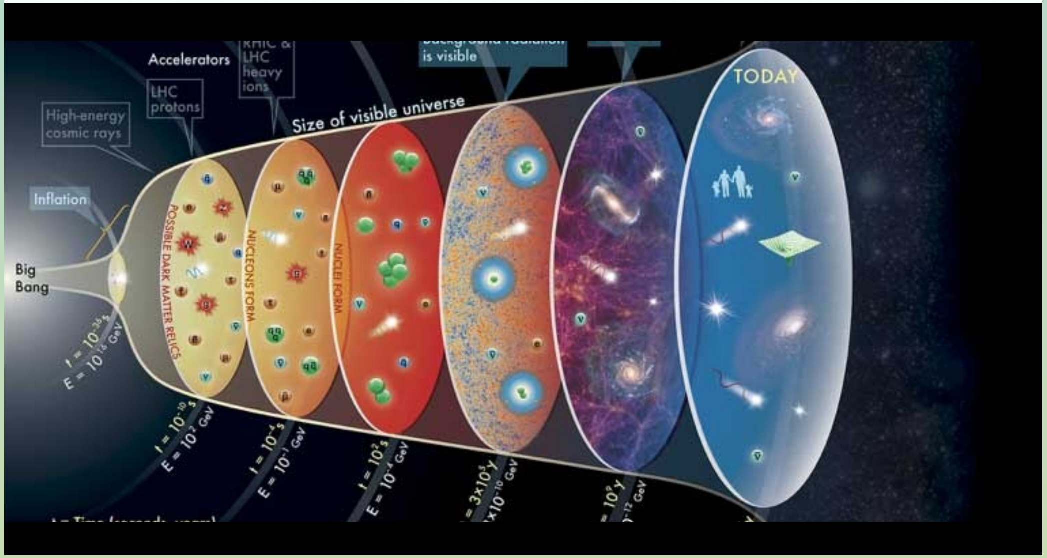
$$g_* \equiv \sum_{i=\mathrm{RELATIVISTIC\ BOSONS}} g_i + \frac{7}{8} \sum_{i=\mathrm{RELATIVISTIC\ FERMIONS}} g_i$$

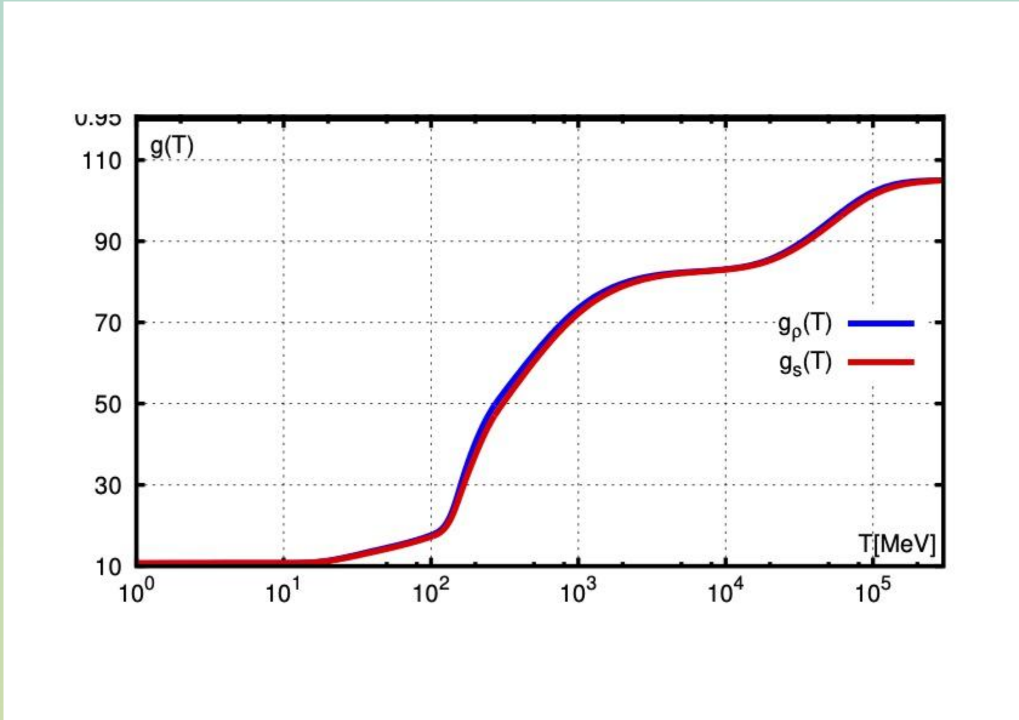
Early Universe Plasma

- More precisely $g_*^{1/3} T \propto 1/a$ (conservation of entropy)

- When a species becomes non-relativistic (e.g. $e^+ - e^-$ at $T \ll m_e$) ⇨ g_* decreases

⇨ T slightly "increases" (photons get slightly "heated")

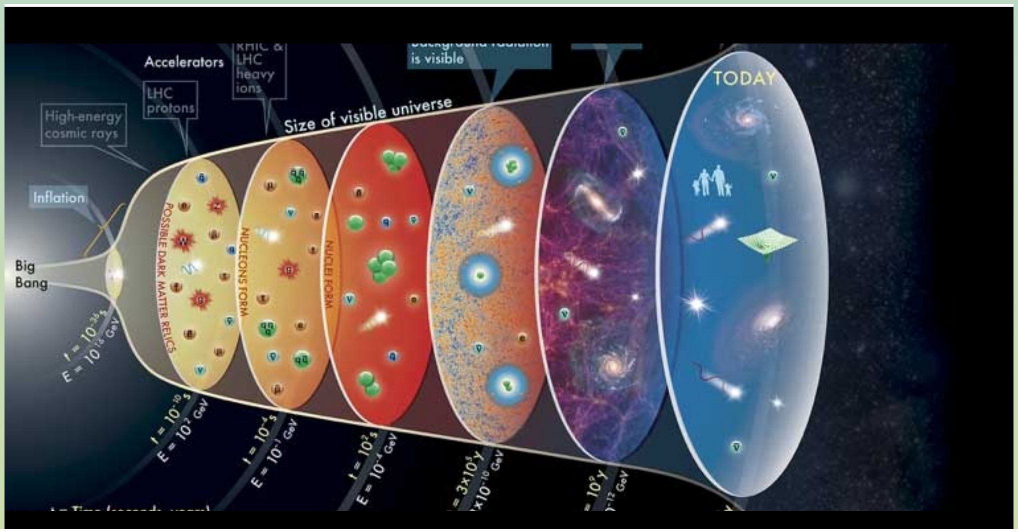




Relic light particles in Cosmology

- Light particles with small interaction (“thermalization rate” Γ, with time scale $1/\Gamma$) with primordial plasma (e.g. neutrinos, axions)

- Compare with timescale for the expansion of the universe $1/H$
  (Hubble rate $H \equiv \dot{a}/a$)



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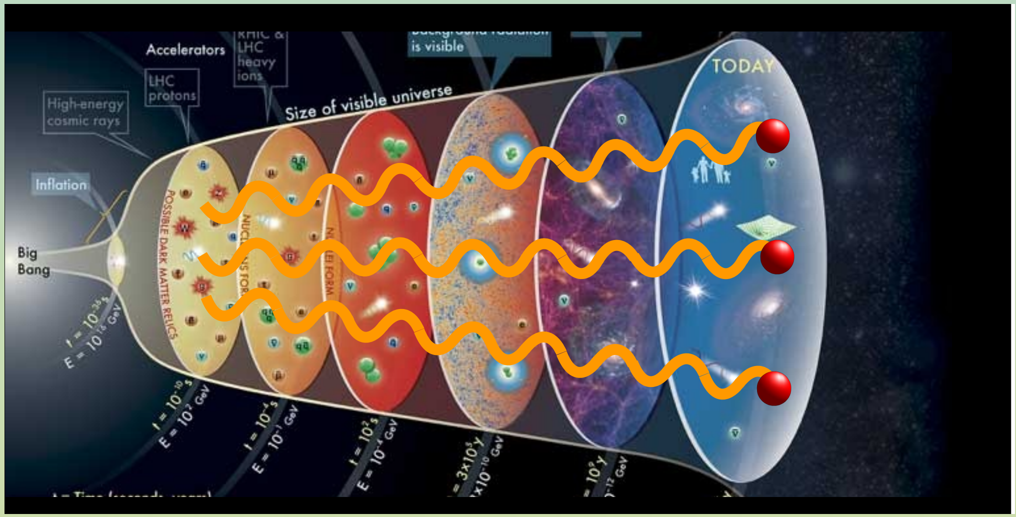
- If Particle Decouples below some Temperature $T_{\rm DEC}$, its distribution freezes at its "own temperature" and freely evolves, $\rho_P \propto T_P^4$, with $T_P = T_{\rm DEC}/a$



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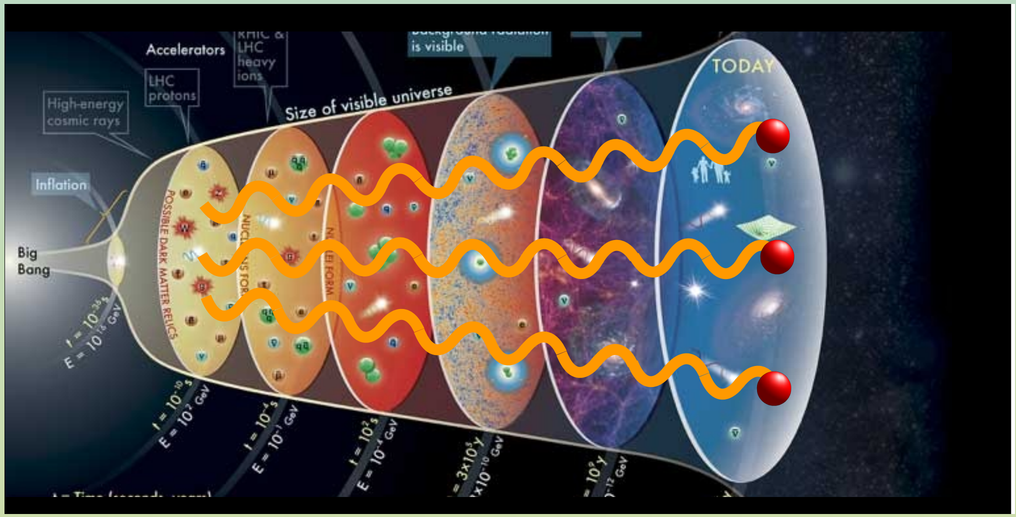
- Compared to photons it does
  not get heated after decoupling

- $\rho_P/\rho_\gamma \propto 1/g_{*DEC}^{4/3}$

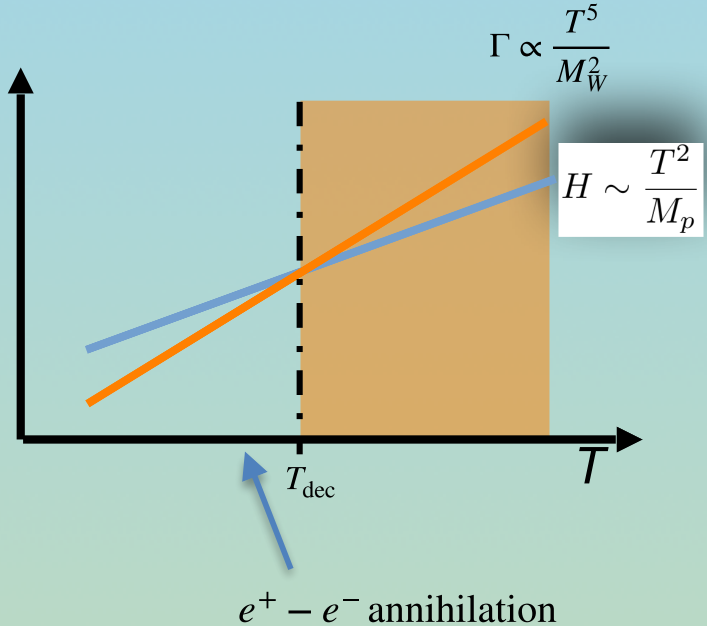


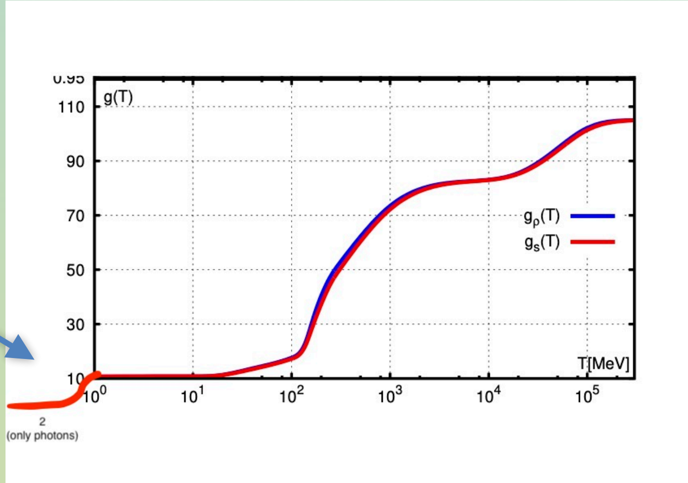
Example: Relic Neutrinos

- Thermalization rate $\Gamma \approx \frac{T^5}{M_W^4}$, vs. Hubble, $H \approx \frac{T^2}{M_{Pl}}$

- Neutrinos decouple at $T \approx MeV$

$$\frac{\rho_\nu}{\rho_\gamma} \propto \frac{1}{g_{*,DEC}^{4/3}} = \left(\frac{4}{11}\right)^{4/3}, \qquad T_\nu \approx 0.7\, T_\gamma \approx 1.96\,\mathrm{K}$$





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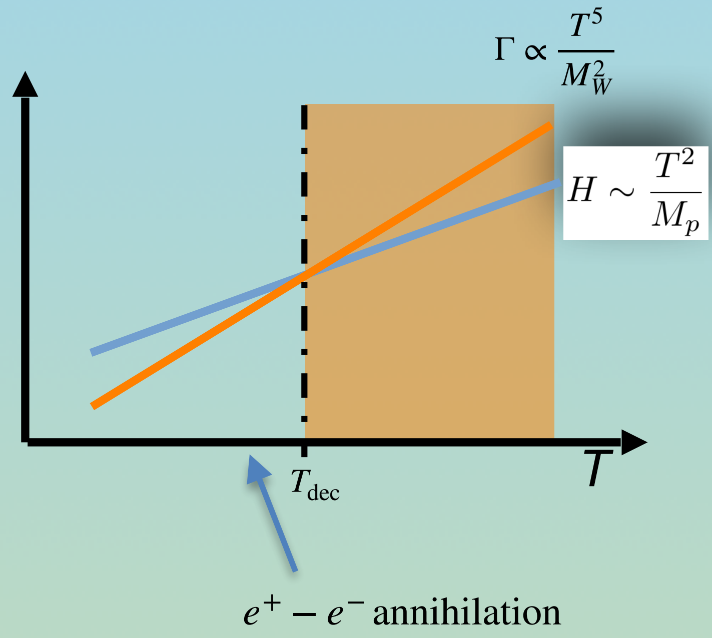
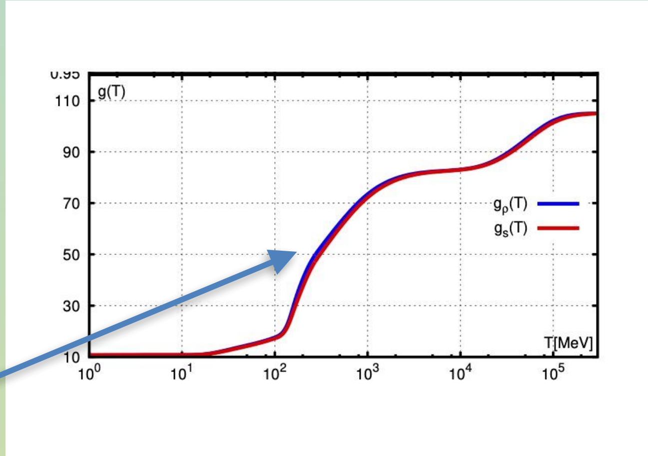


- Any light particle (axions,…) can do the same.
- Traditional parameterization as "extra neutrinos species":

$$\Delta N_{\rm eff} \equiv \left(\frac{8}{7}\right)\left(\frac{11}{4}\right)^{4/3} \frac{\rho_P}{\rho_\gamma}|_{\rm CMB}$$

- Relic abundance suppressed as: $\boxed{\Delta N_{\rm eff} \propto \frac{\rho_P}{\rho_\gamma}|_{\rm CMB} \propto \frac{1}{g_{*,DEC}^{4/3}}}$



Example: Relic Scalars

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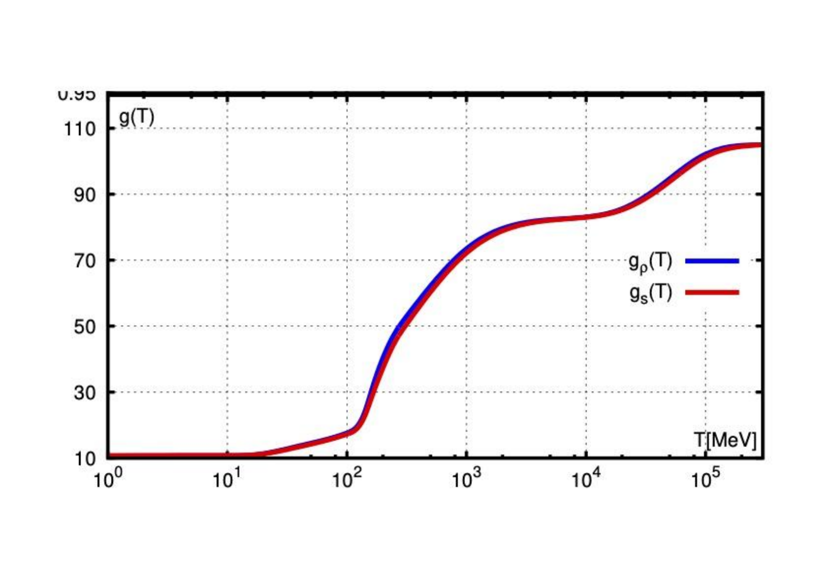

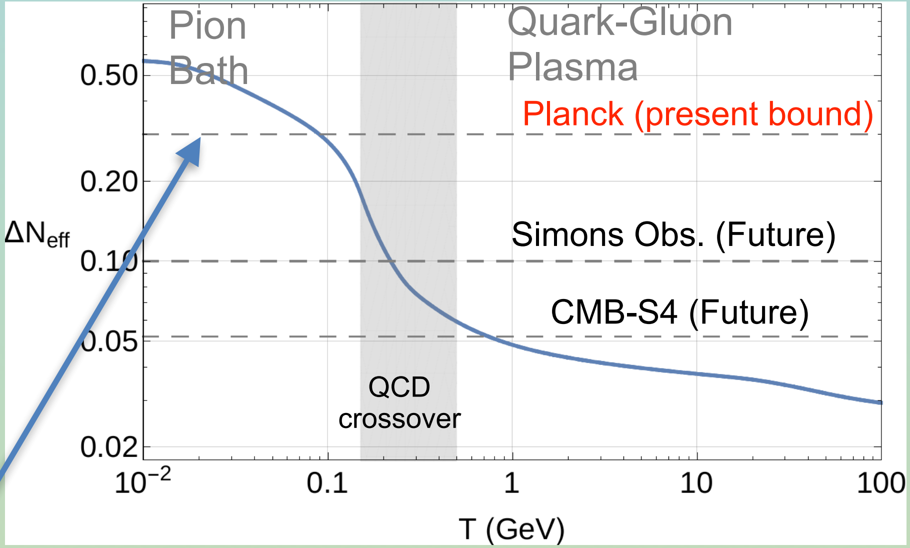

- Main effect: Extra “radiation” at CMB time ($T \approx 0.1 eV$) ⇨ $\Delta N_{\rm eff}$ affects angular spectra

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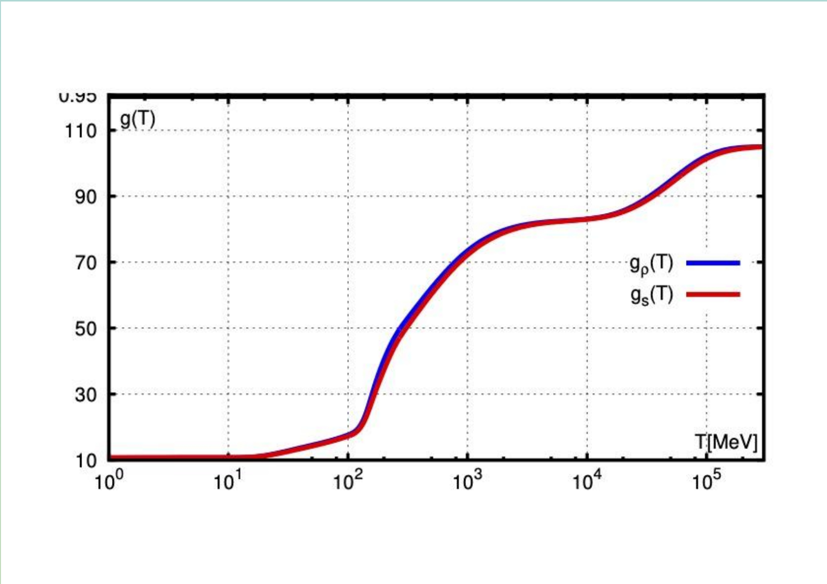

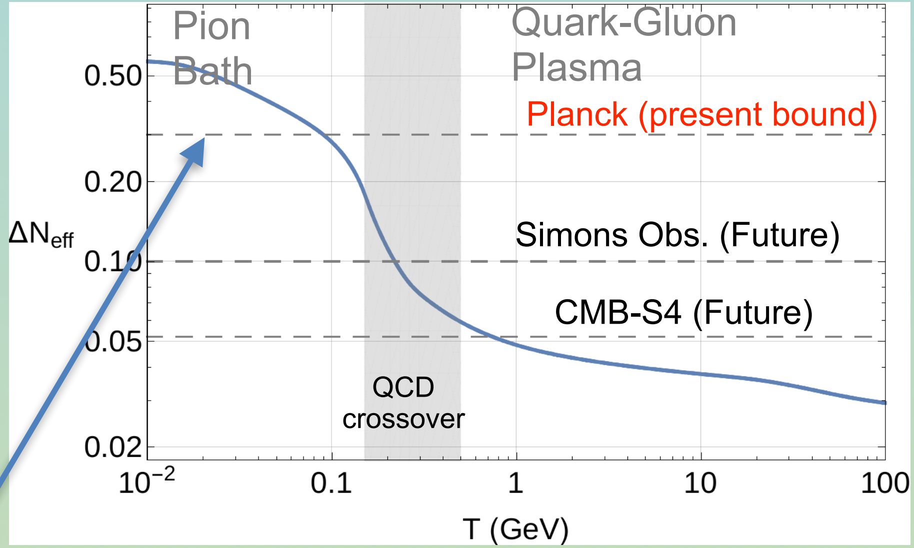

- Main effect: Extra "radiation" at CMB time ($T \approx 0.1eV$) ⇨ $\Delta N_{\rm eff}$ affects angular spectra
- If massive ($m \lesssim 0.1\text{eV}$) becomes non-relativistic after CMB time ⇨ adds to Dark Matter and affects its fluctuations (more constrained)

The (Minimal) QCD Axion

$$\mathscr{L}_{\mathrm{SM}} \supset \theta_{\mathrm{strong}} \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

- Why CP-violation is tiny ($\bar{\theta}_{\mathrm{strong}} \ll 1$)?

The (Minimal) QCD Axion

$$\mathscr{L}_{\rm SM} \supset \theta_{\rm strong} \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

- Why CP-violation is tiny ($\bar{\theta}_{\rm strong} \ll 1$)?
- QCD Axion solution: promote $\theta_{\rm strong}$ to a dynamical field $\rightarrow \dfrac{a}{f_a}$
- *Axion* potential minimized at $a = \bar{\theta}_{\rm strong} = 0$ (CP conserving)

The (Minimal) QCD Axion

$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \ldots$$

- Dynamical explanation of $\theta_{\text{strong}} \ll 1$

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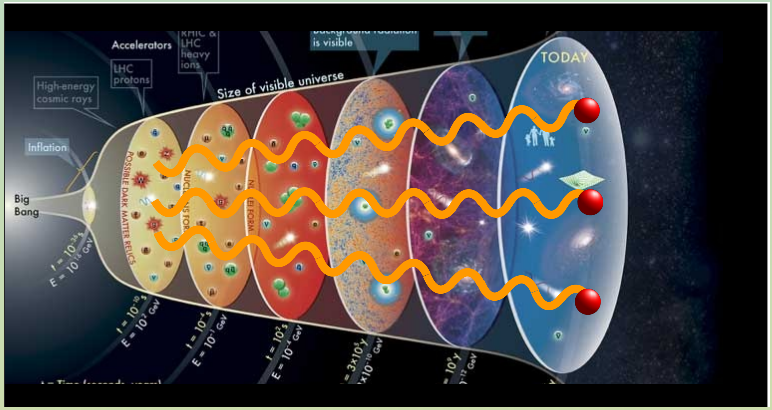
- Two populations of cosmological relic axions:

  - "Cold axions" contribute to part (or all) of Dark matter, not covered in this talk.

  - "Thermal axions": relativistic at production, May become non-relativistic later ⇨ small part of dark matter



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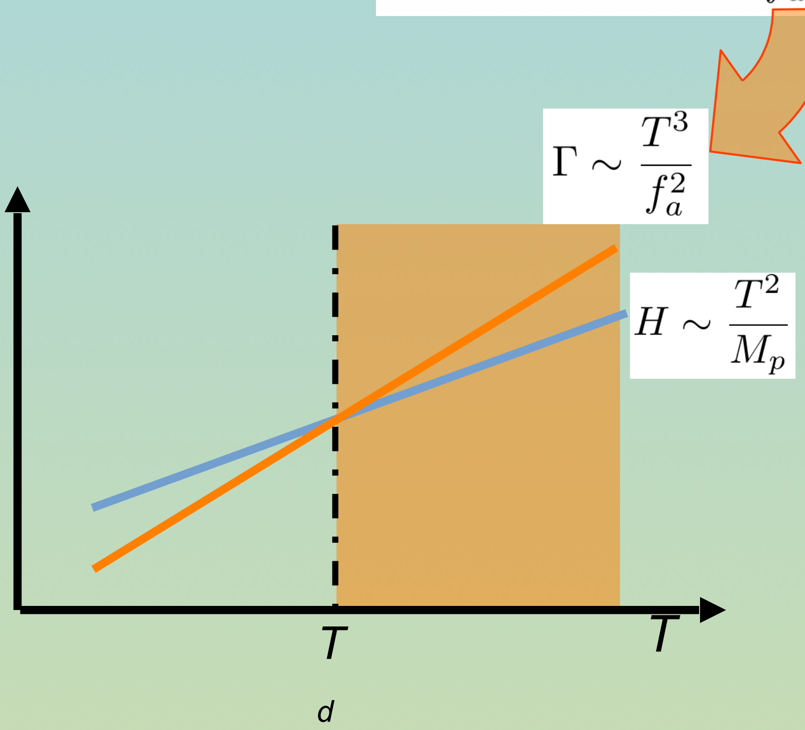

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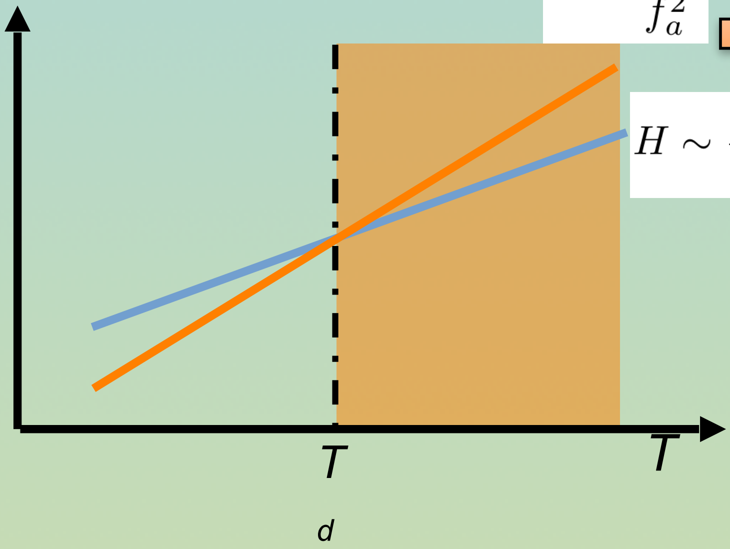
$$T_d \sim \frac{f_a^2}{M_p} \sim \Lambda_{QCD}\left(\frac{f_a}{10^8\ \mathrm{GeV}}\right)^2$$



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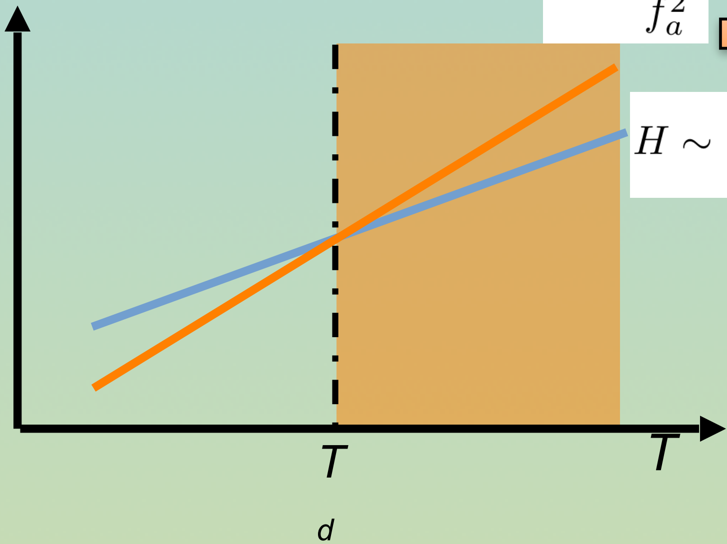

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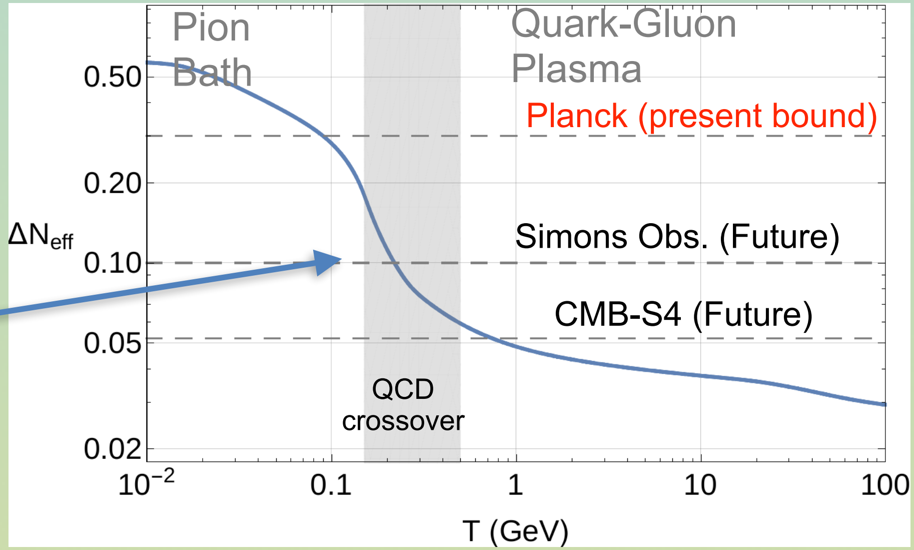
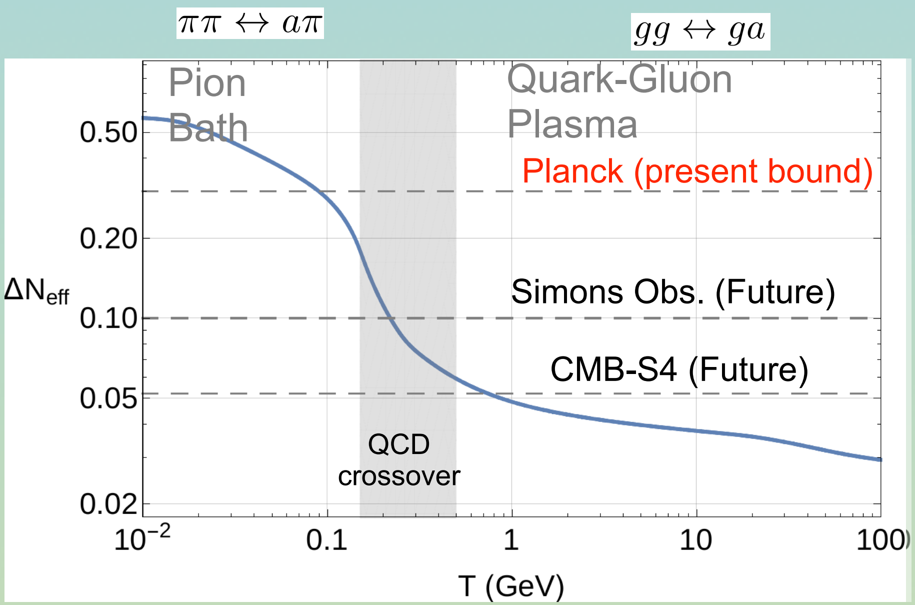


Axion $\Delta N_{\rm eff}$ has a long history:

Arias-Aragon, Baumann, Bernal, Berezhiani, Chang, Choi, D'Eramo, Di Luzio, Di Valentino, Dunsky, Ferreira, Giusarma, Graf, Green, Guo, Hall, Hajkarim , Hannestad, Harigaya, Khlopov, Lattanzi, Martinelli, Masso, Melchiorri, Mena, Merlo, Mirizzi, AN, Piazza, Raffelt, Rompineve, Rota, Salvio, Sakharov, Silk, Slosar, Steffen, Strumia, Wallisch, Wong, Yun, Zsembinszki, Xue, …



"Standard" treatments:

1. Instantaneous decoupling ($\Gamma = H$)
2. Single Boltzmann Eq.for abundance Y.

$$\frac{dY}{d\log x} = (Y^{\rm eq} - Y)\frac{\overline{\Gamma}}{H}\left(1 - \frac{1}{3}\frac{d\log g_{*,S}}{d\log x}\right)$$

$(x \equiv m/T)$

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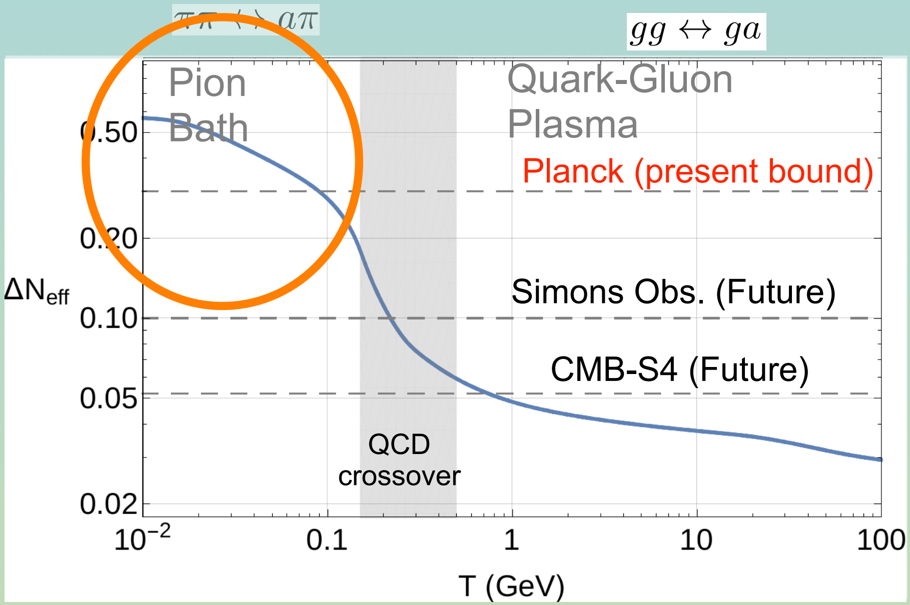
Our work: Improving present bounds



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Boltzmann Equation and Thermalization Rate Γ

$$\frac{df_{\mathbf{p}}}{dt} = (1 + f_{\mathbf{p}})\,\Gamma^{<} - f_{\mathbf{p}}\,\Gamma^{>}$$

$$\Gamma^{<} = e^{-\frac{E}{T}}\Gamma^{>}$$

(Detailed balance, plasma particles in equilibrium)

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Perturbatively, due to scatterings with pions:

$$\Gamma^{<} = \frac{1}{2E}\int\left(\prod_{i=1}^{3}\frac{d^3\mathbf{k}_i}{(2\pi)^3 2E_i}\right) f_1^{\mathrm{eq}} f_2^{\mathrm{eq}}(1 + f_3^{\mathrm{eq}})(2\pi)^4\delta^{(4)}(k_1^\mu + k_2^\mu - k_3^\mu - k^\mu)|\mathcal{M}|^2_{2\,\leftrightarrow\,2}$$

1. The Thermalization Rate Γ

$$\pi\pi \leftrightarrow a\pi$$

LO χPT rate
(Chang Choi '93)

NLO χPT rate
(Di Luzio, Martinelli, Piazza '21)

→ breaks down at $T \gtrsim 60$ MeV !

$$|\mathcal{M}^{\rm LO}|^2 = \theta_{a\pi}^2 \frac{s^2+t^2+u^2-3m_\pi^4}{f_\pi^4}$$

$$\boxed{\theta_{a\pi} = \frac{m_u - m_d}{m_u + m_d}\frac{f_\pi}{2f_a}}$$

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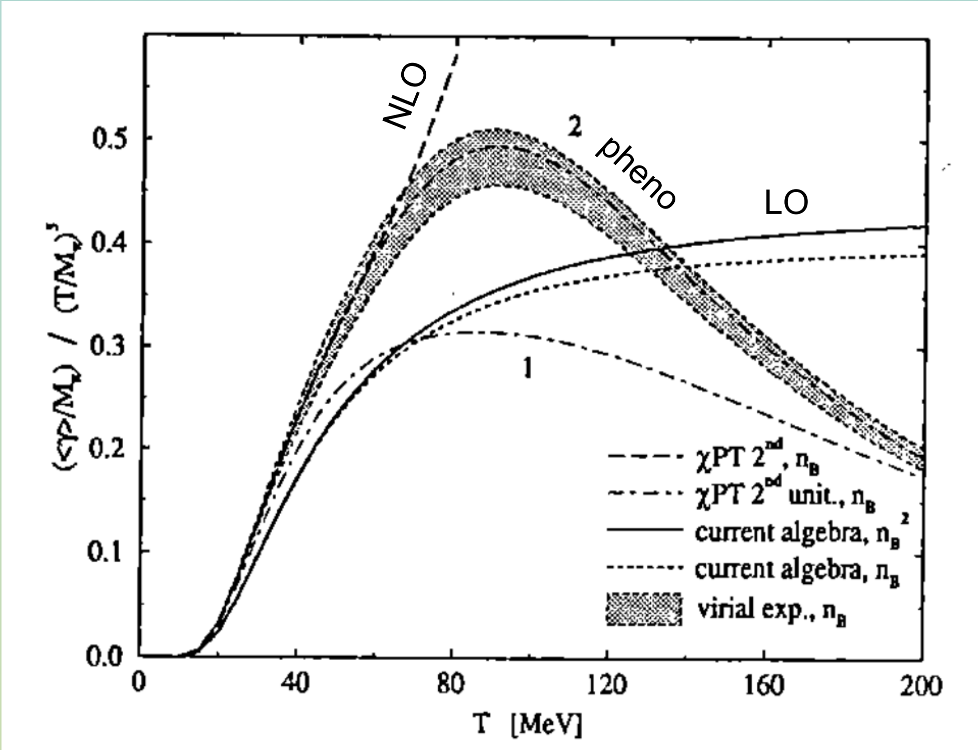

Schenk '94

1. The Thermalization Rate Γ

General form of minimal low energy axion QCD Lagrangian:

$$\mathcal{L} = \bar{q}\left(i\partial\!\!\!/ + \frac{c_0}{2f_a}\partial\!\!\!/ a\,\gamma_5\right) q - \bar{q}_L M_a q_R + h.c.\,, \qquad M_a \equiv \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} e^{i\frac{a}{2f_a}}$$

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$$\frac{\partial_\mu a}{2f_a} j_A^\mu \overset{\chi\mathrm{PT}}{=} \mathcal{O}(M_q)$$

$$\pi^0 = \cos(\theta_{a\pi})\,\pi^0_{\mathrm{phys}} + \sin(\theta_{a\pi})\,a_{\mathrm{phys}} \simeq \pi^0_{\mathrm{phys}} + \theta_{a\pi} a_{\mathrm{phys}}$$

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@ all orders in χPT

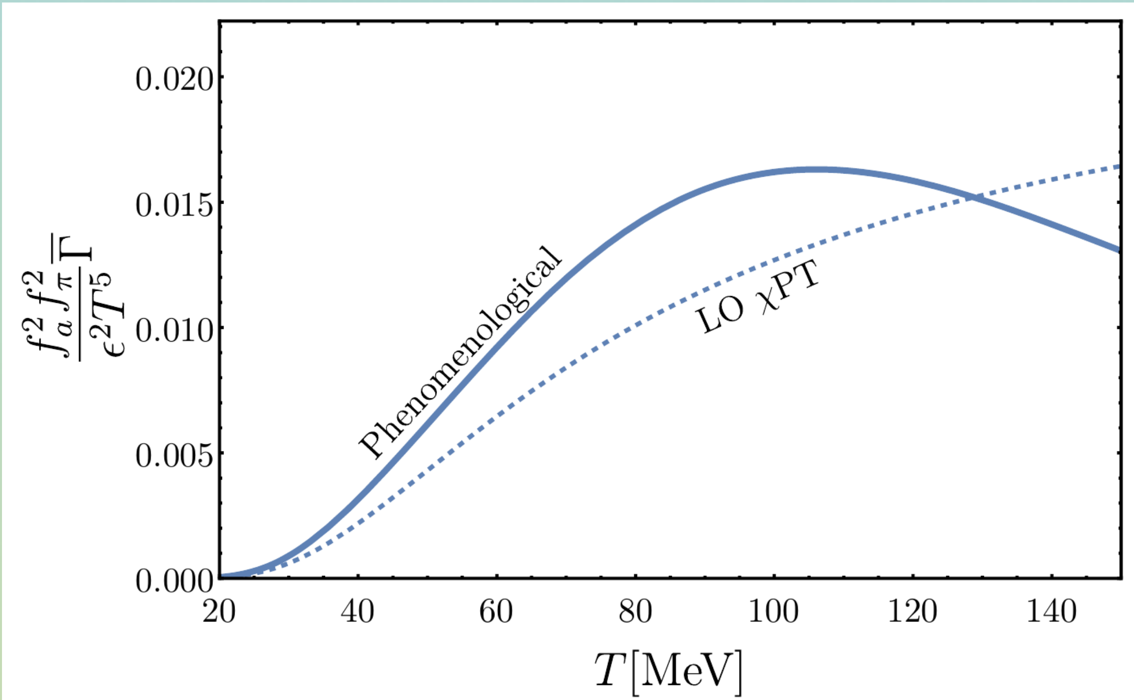
$$\mathcal{M}_{a\pi^i\to\pi^j\pi^k} = \theta_{a\pi}\cdot\mathcal{M}_{\pi^0\pi^i\to\pi^j\pi^k} + \mathcal{O}\left(\frac{m_\pi^2}{s}\right)$$

$\lesssim 10\%$

e.g. @ LO

$$|\mathcal{M}^{\rm LO}|^2 = \theta_{a\pi}^2\frac{s^2+t^2+u^2-3m_\pi^4}{f_\pi^4} \qquad |\mathcal{M}^{\rm LO}_{\pi\pi}|^2 = \frac{s^2+t^2+u^2-4m_\pi^4}{f_\pi^4}$$

1. The Axion Thermalization Rate Γ (from pions)



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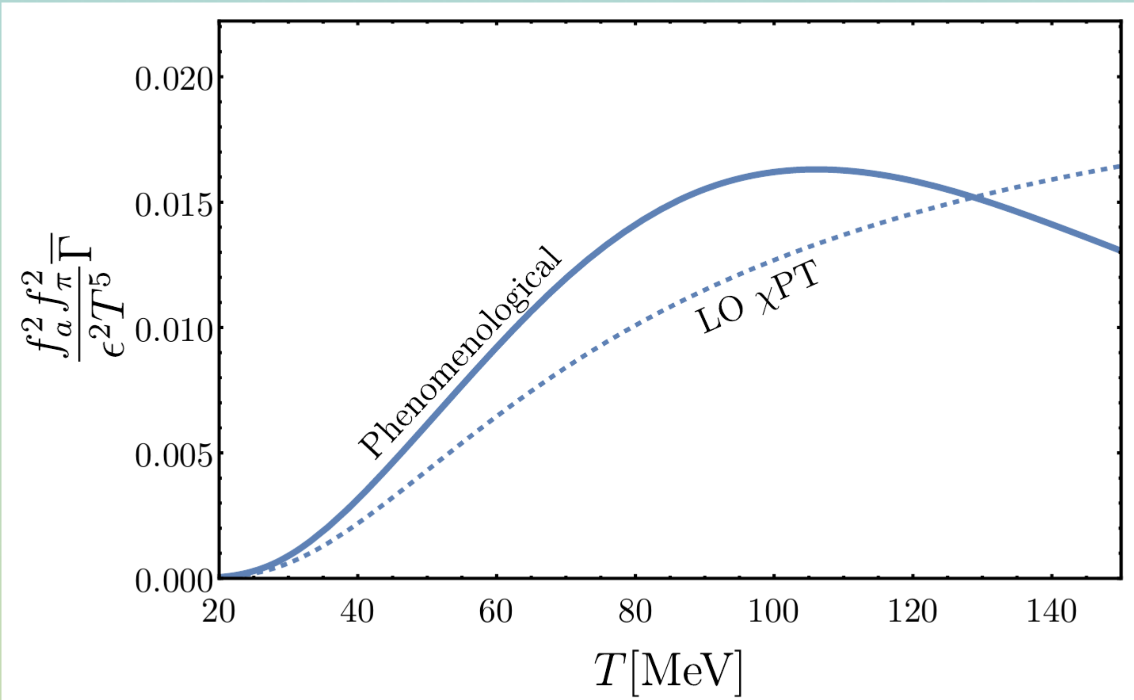

In reasonable agreement with: Di Luzio, Camalich, Martinelli, Oller, Piazza ‘22 (using NLO+unitarization)

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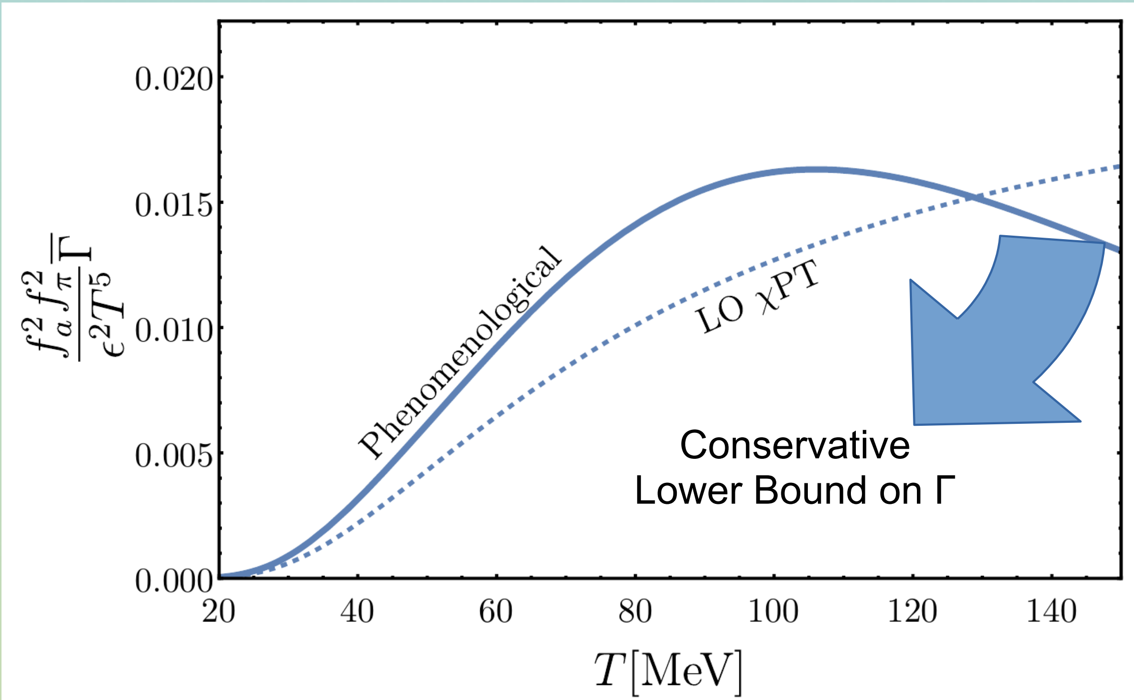

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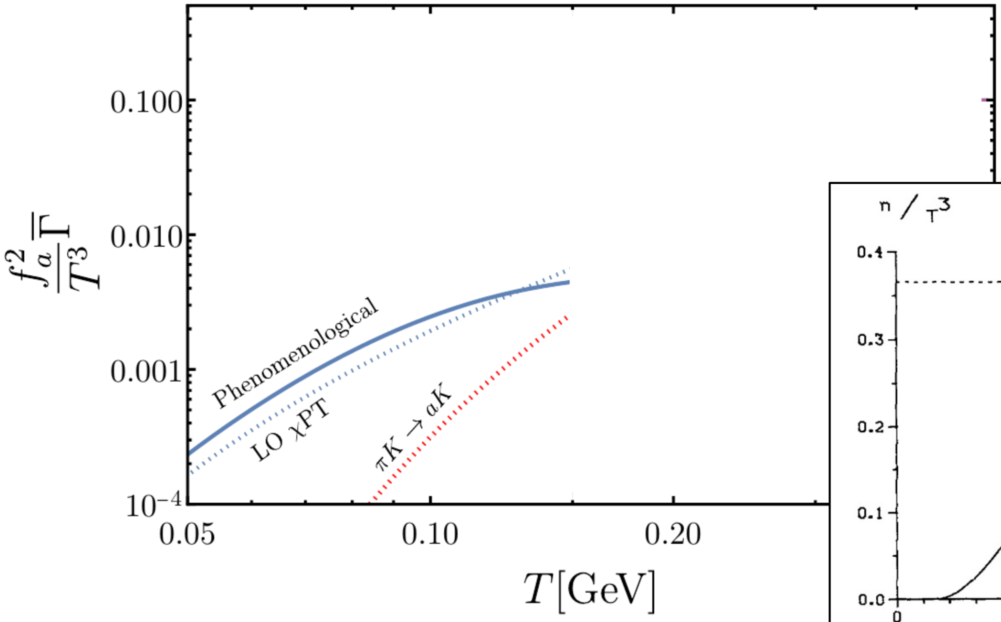
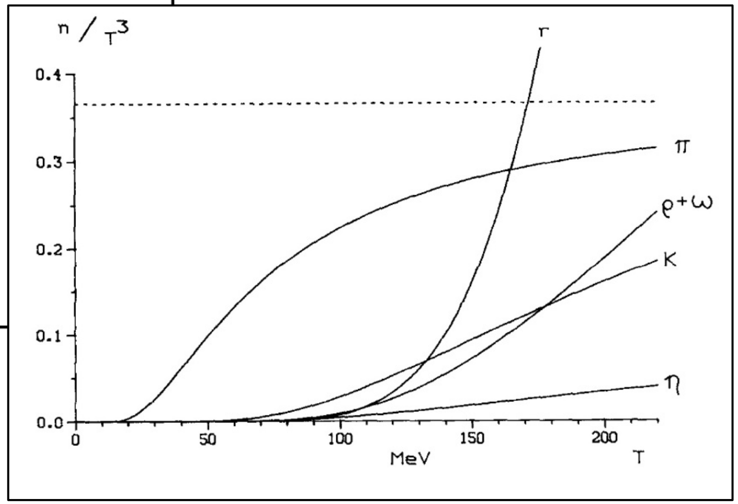
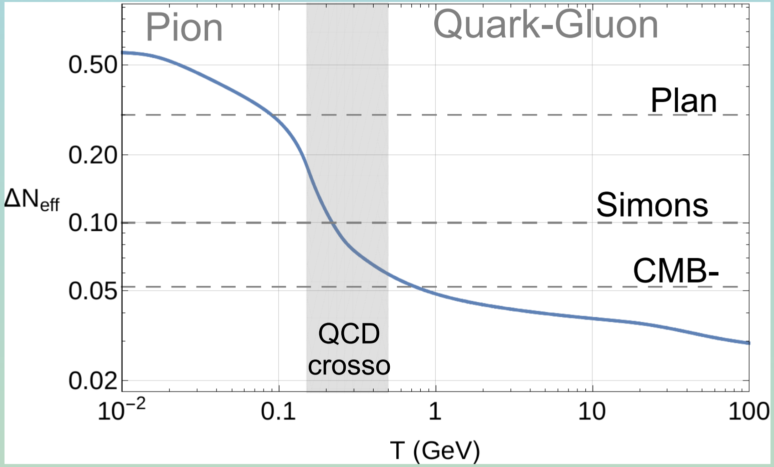
1. The Thermalization Rate Γ (Possible other channels)

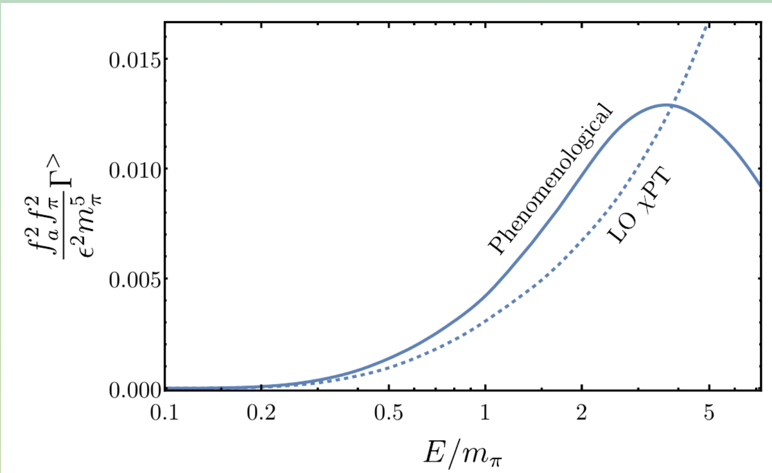




Gerber Leutwyler '89

2. Momentum Dependence





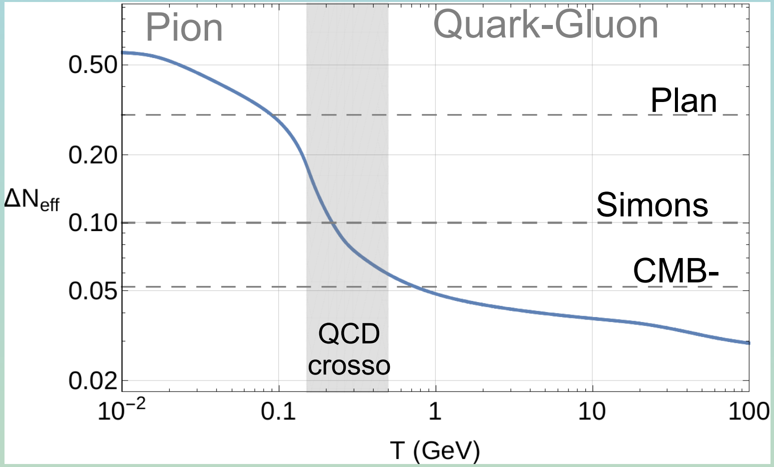
Boltzmann Eq.

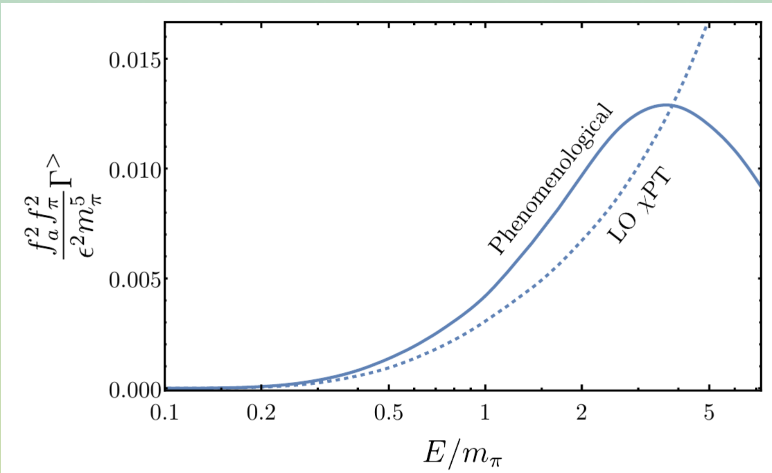
$$\frac{df_{\mathbf{p}}}{dt} = (1 + f_{\mathbf{p}})\,\Gamma^< - f_{\mathbf{p}}\,\Gamma^>$$

High momenta k decouple later than low k

They see a lower g_* → More abundant

2. Momentum Dependence





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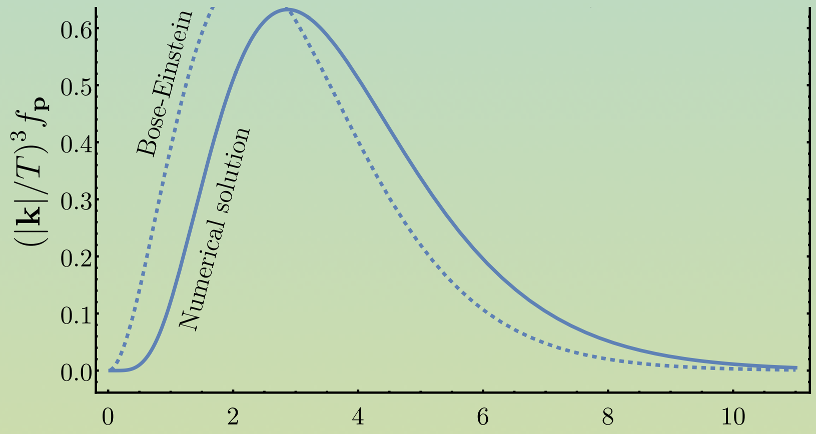
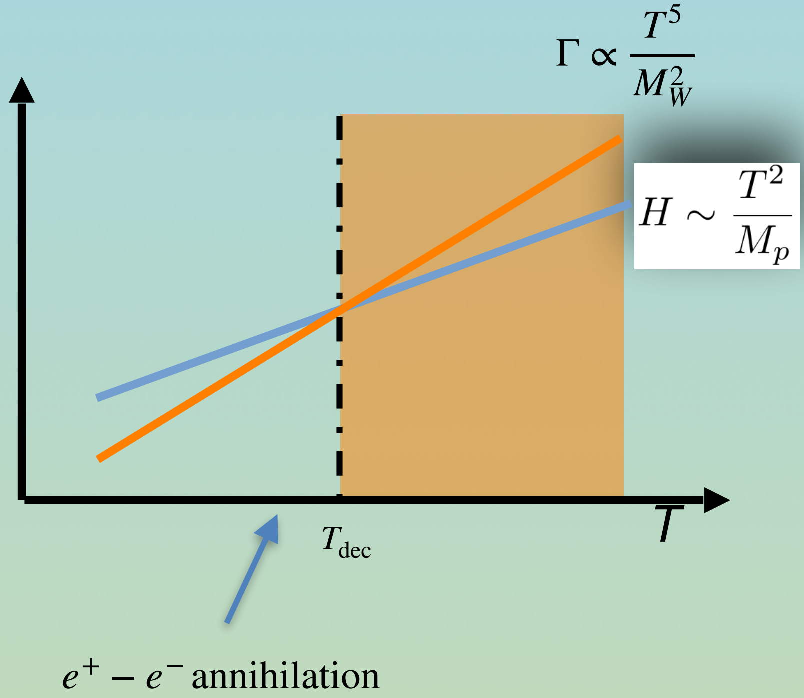
~ 40% enhanced total abundance



2. Momentum Dependence: Neutrinos



High momenta k decouple later than low k

They see a lower g_* ⇨ More abundant

Boltzmann Eq.

$$\frac{df_{\mathbf{p}}}{dt} = (1 + f_{\mathbf{p}})\,\Gamma^{<} - f_{\mathbf{p}}\,\Gamma^{>}$$

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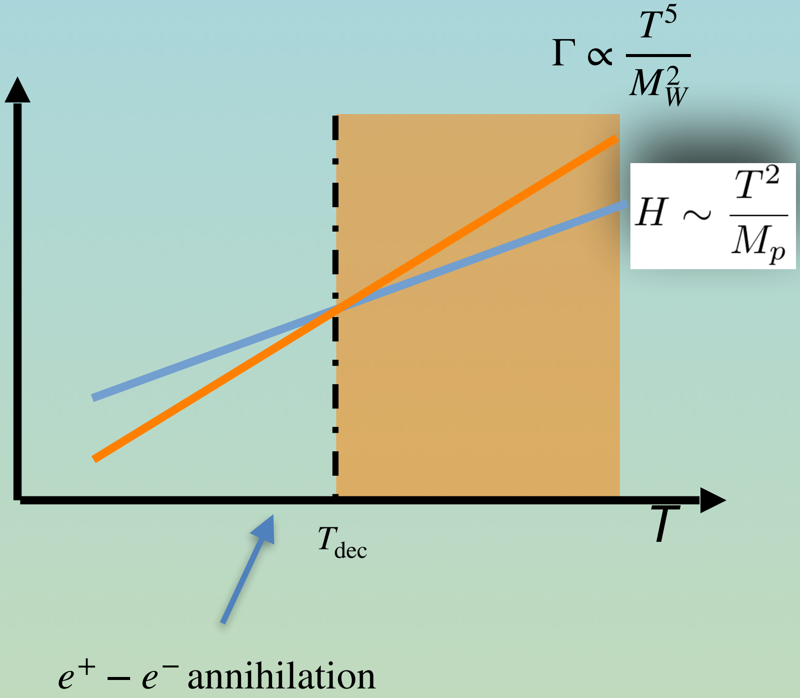

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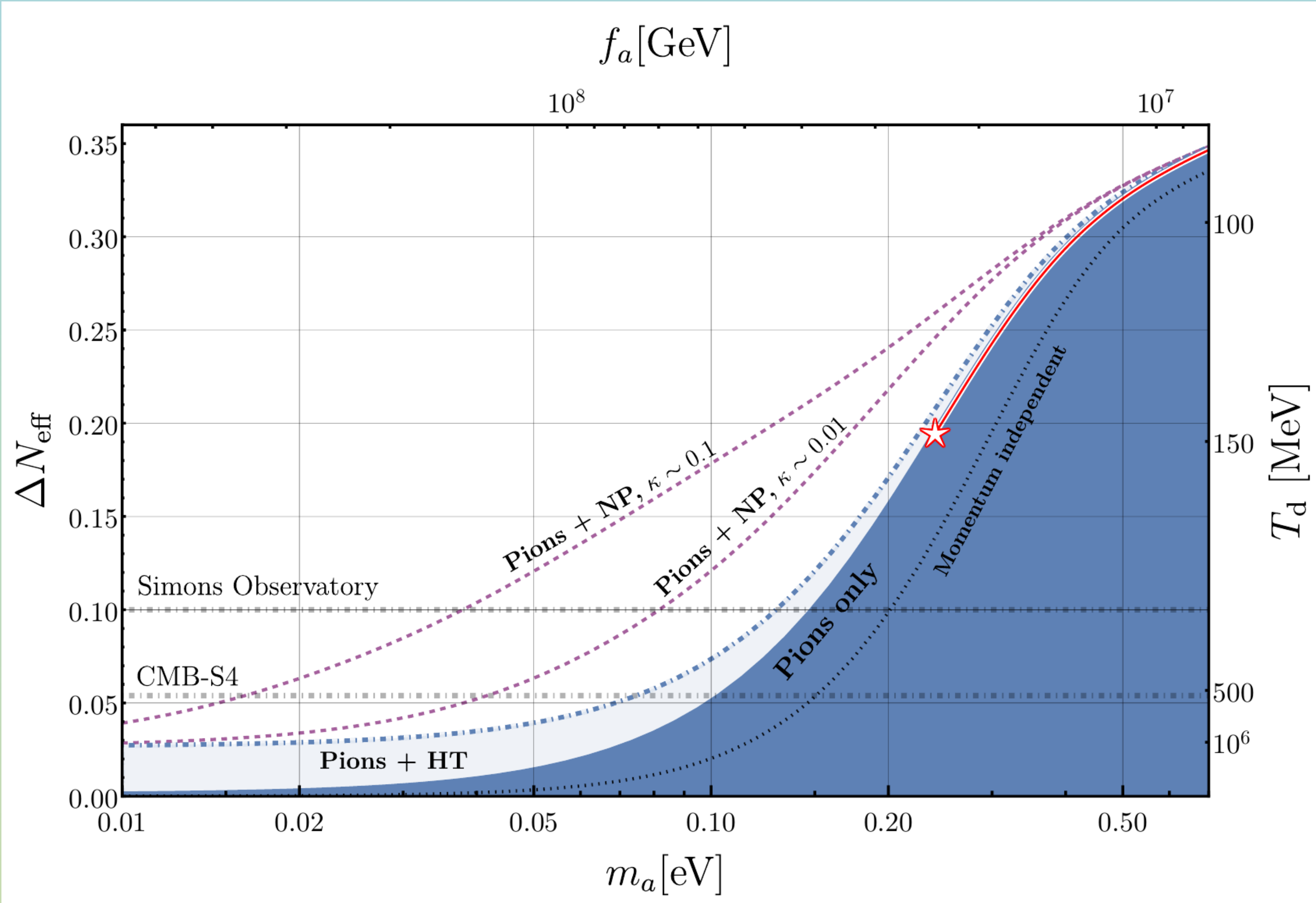
But m_e and $T_{\rm dec}$ are more separated ⇒ Only ~ 1% enhancement

$$N_{\rm eff} \approx 3.044$$

Present bound+Future Reach



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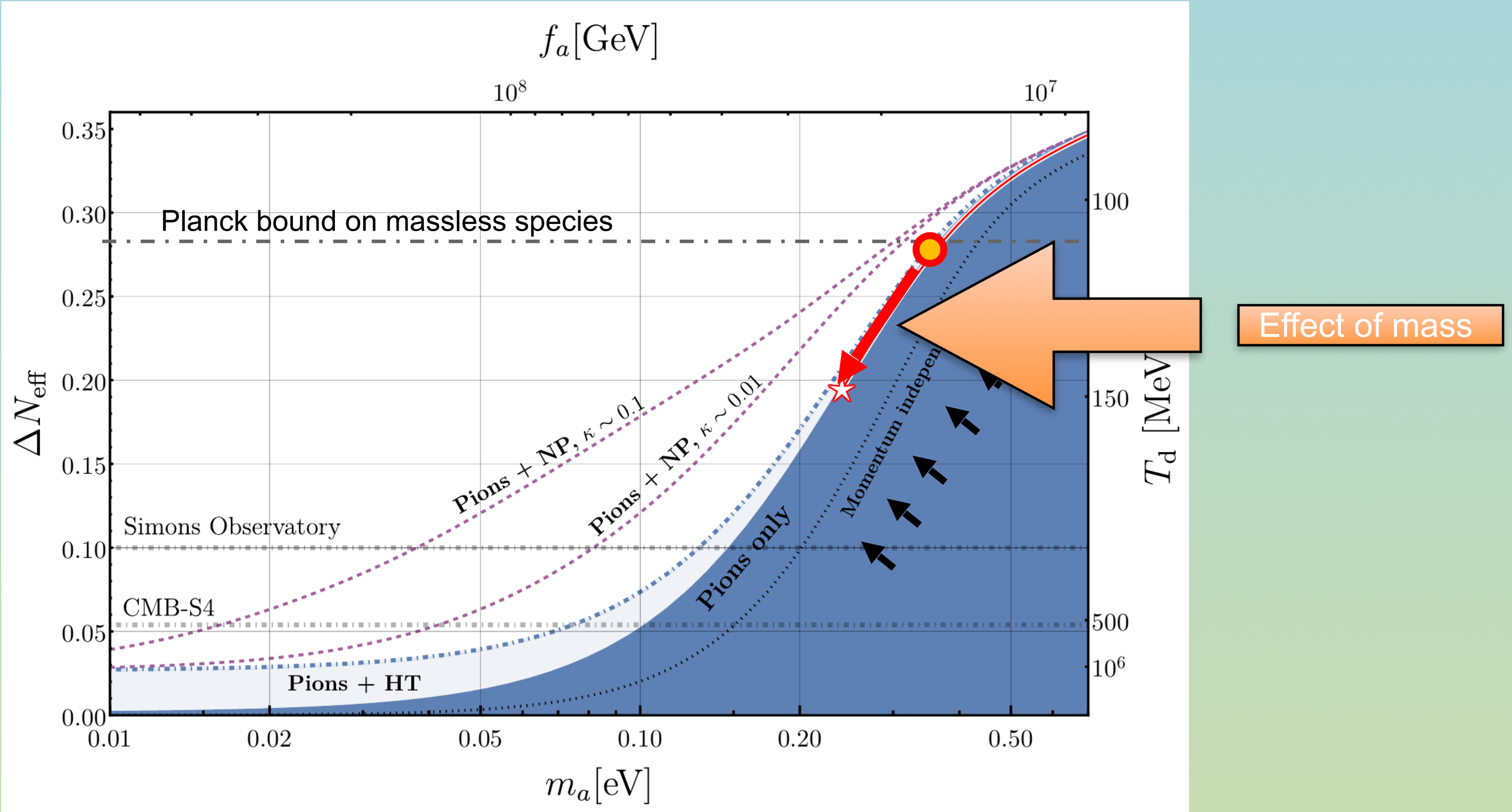
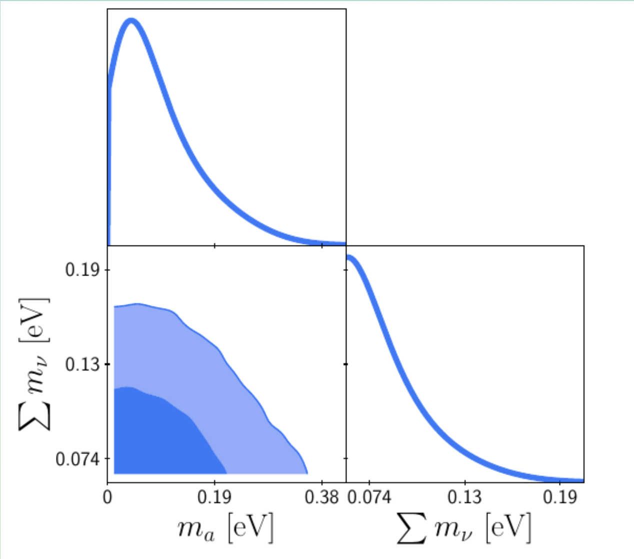


3. Combined cosmological Fit

(Λ_{CDM} + massive neutrinos + *axions*)



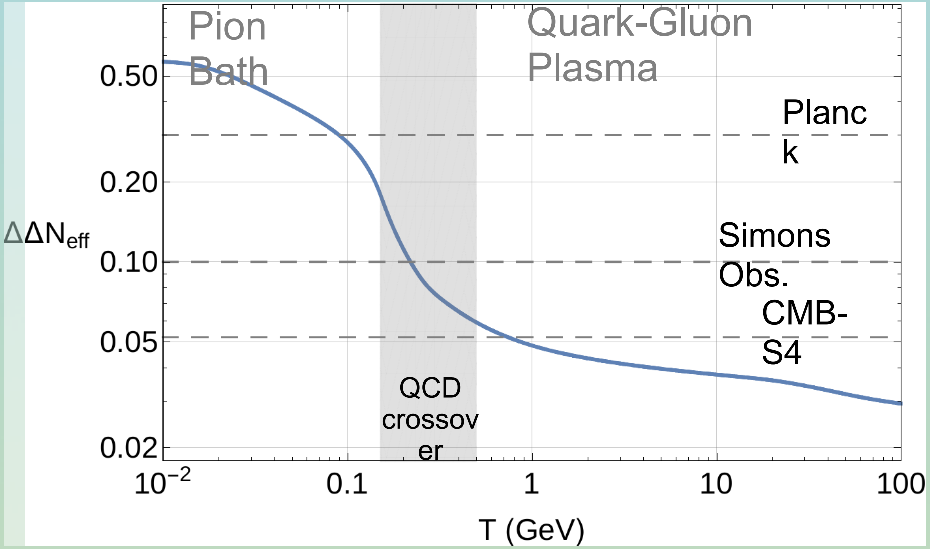
$$m_a \leq 0.24 \text{ eV}$$

$$\Leftrightarrow$$

$$\Delta N_{\text{eff}} \lesssim 0.19$$

$$f_a \geqslant 2.4 \cdot 10^7 \text{ GeV}$$

Future Reach



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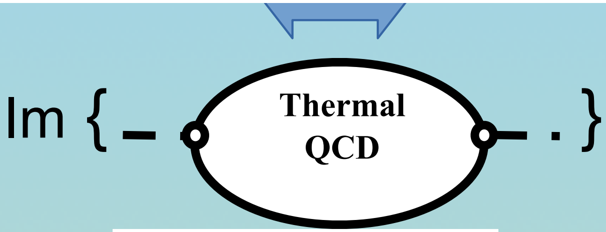
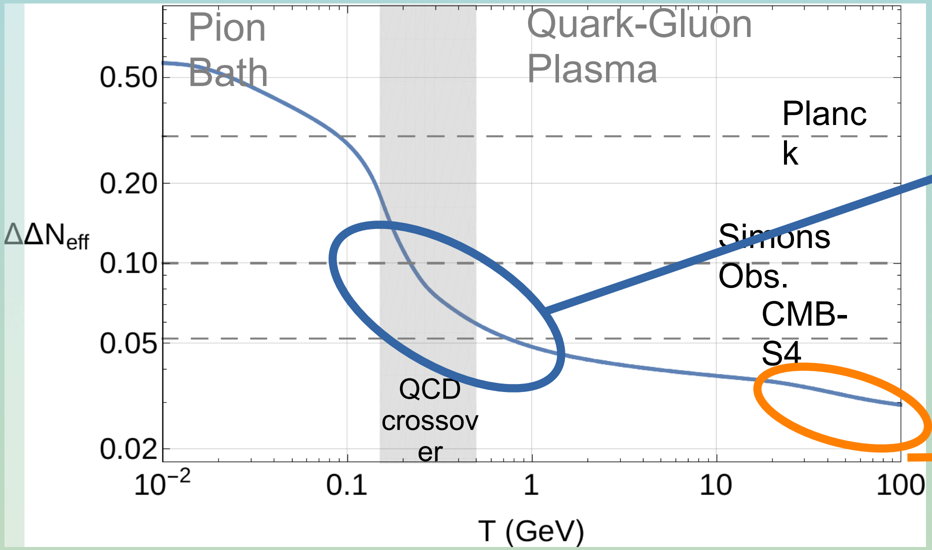


$$\Gamma^{>} = e^{\frac{E}{T}}\Gamma^{<} = \frac{\Gamma^{>}_{\rm top}}{2Ef_a^2}$$

Non-Perturbative

$$\Gamma^{>}_{\rm top} \equiv \int d^4x\, e^{ik^\mu x_\mu} \left\langle \frac{\alpha_s}{8\pi} G\tilde{G}(x^\mu)\, \frac{\alpha_s}{8\pi} G\tilde{G}(0) \right\rangle$$

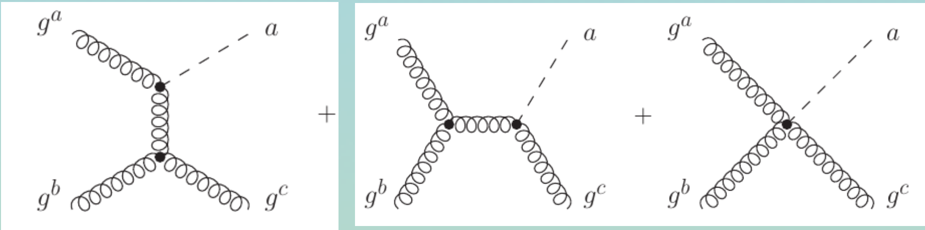


$gg \leftrightarrow ga$

Perturbative ?

$$\overline{\Gamma}_{\rm pert} = \frac{\alpha_s^2 T^3}{4\pi^3 f_a^2} F_3$$

High Temperatures Regime



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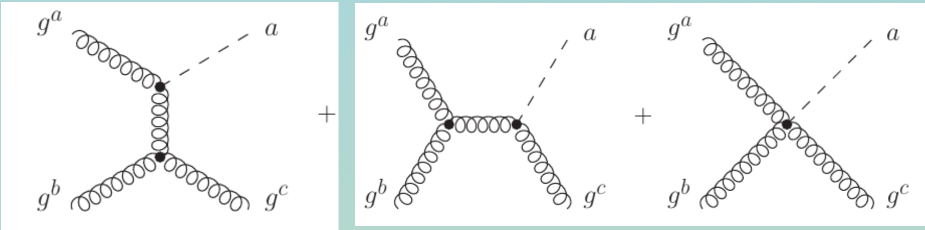

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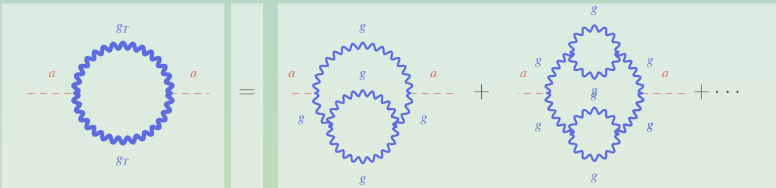
Masso, Rota, Zsembinszki ‘02
Graf, Steffen ‘10

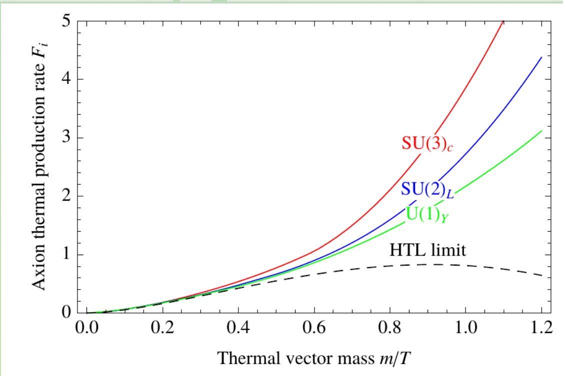
$$F_3 = g_s^2 \log\left(\frac{3T^2}{2m_g^2}\right) = g_s^2 \log\left(\frac{3}{2g_s}\right)^2$$

for $g_s \ll 1$

IR divergent

Salvio, Strumia, Xue ‘13





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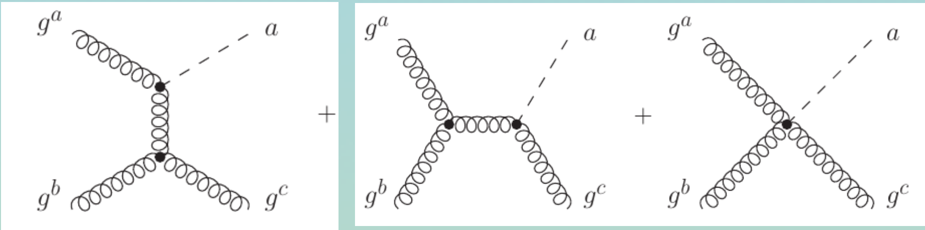

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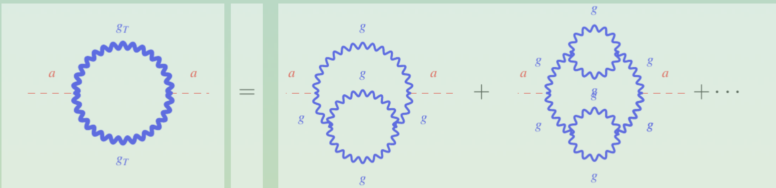
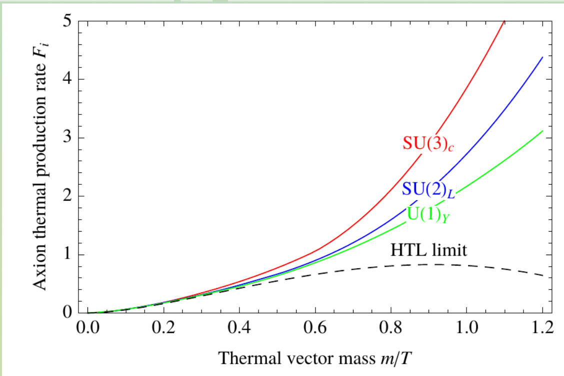
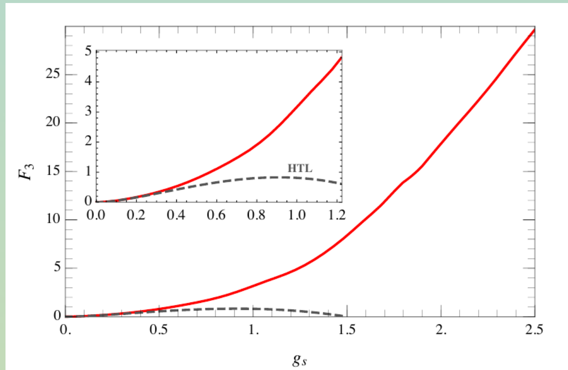


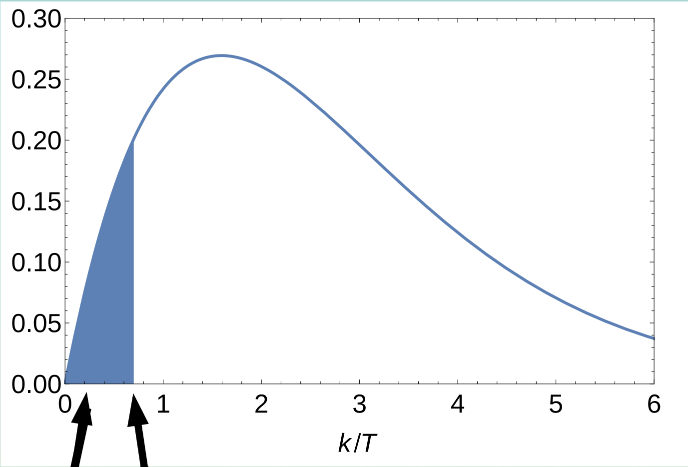


D'Eramo, Hajkarim, Yun ('21):
extrapolation of Salvio et al. to $g_s > 1$



*Matching gluon to pions through QCD crossover?
Pion-axion: suppressed by $\theta_{a\pi} \propto \dfrac{m_u - m_d}{m_u + m_d}$, gluon is not
Pion rates not monotonic with T
Rates could have sudden jumps as g_* does

High Temperatures Regime



$k \sim m_e \sim g_s T$

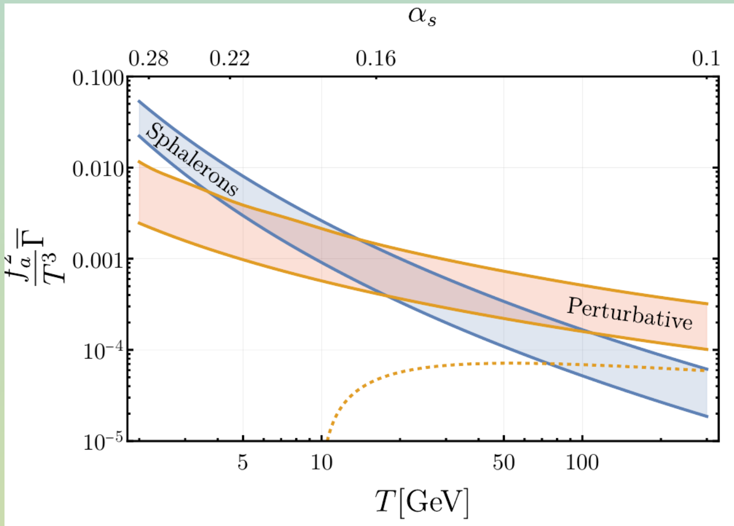
$k \sim m_m \sim g_s^2\, T$

$\# \sim 1 / g_s^2$

@ g_s << 1 :

large occupation numbers → dominated by semi-classical

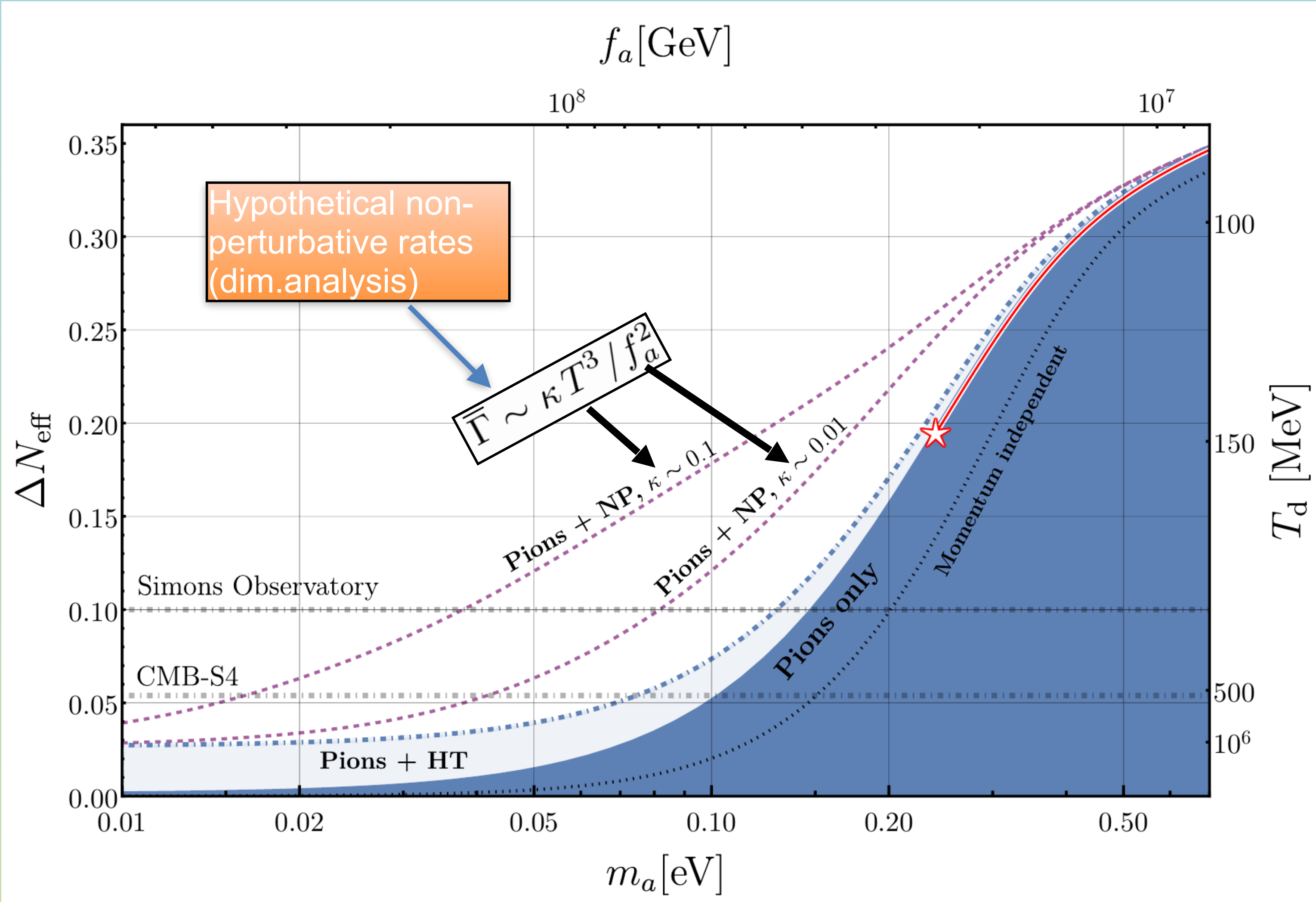
[non-linear YM equations - e.g. strong sphalerons]



$$\Gamma_{\mathrm{sphal}} \simeq \frac{(N_c \alpha_s)^5 T^3}{f_a^2}$$

(Adapted from: Moore, Tassler '10)

Future Reach



Conclusions:

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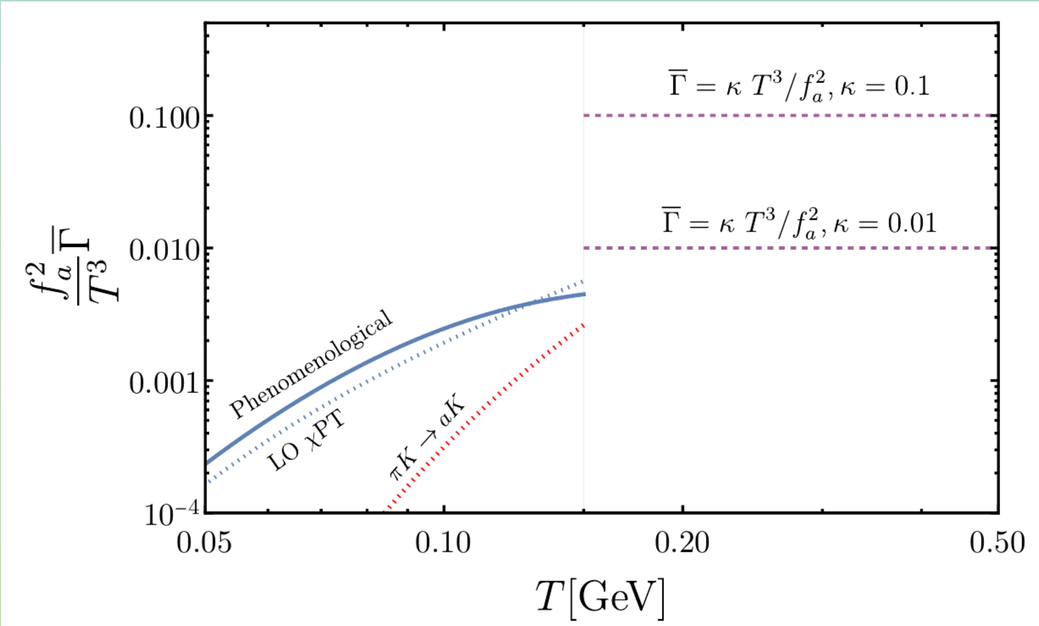
Thank you!

Back Up

Strong Sphaleron-like contribution to Axion rate

$$\overline{\Gamma}_{\rm sphal} = \frac{1}{n^{\rm eq}} \int^{|\mathbf{k}|<|\mathbf{k}_s|} \frac{d^3\mathbf{k}}{(2\pi)^3 2E} \frac{\Gamma_{\rm sphal}}{f_a^2} e^{-E/T} = \frac{(N_c\alpha_s)^5 T^3}{4\zeta_3 f_a^2} \left(1 - \left(1 + \frac{|\mathbf{k}_s|}{T}\right) e^{-|\mathbf{k}_s|/T}\right)$$



$$\Gamma^{>}_{\rm top}(E = |\mathbf{k}| < |\mathbf{k}_s|) \simeq \Gamma_{\rm sphal} \simeq (N_c\alpha_s)^5 T^4$$

$$|\mathbf{k}_s| \sim N_c \alpha_s T$$

The Thermal Width:

Challenge for Lattice QCD: <u>Compute Γ_k for $T > T_c$</u>

Existing Attempts (at k=0) e.g.

Moore, Tassler '10 : Classical SU(N) simulations

$$\Gamma_{\rm sphal} = 2T \lim_{\omega\to 0} \frac{\rho(\omega)}{\omega}$$

Kotov '18 ,

Altenkort et al. '20,

$$G(\tau) = \int \mathrm{d}^3x\, \langle q(\vec{0},0) q(\vec{x},\tau)\rangle = -\int_0^\infty \frac{\mathrm{d}\omega}{\pi}\, \rho(\omega) \frac{\cosh[\omega(1/2T-\tau)]}{\sinh(\omega/2T)}$$

Mancha, Moore '22 : Quantum Euclidean (plus modeling)

Important to exploit upcoming experiments!