Hadronic contributions to the muon $g-2$ from lattice QCD

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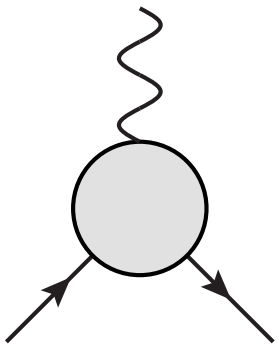
talk/material in collab. with Christoph Lehner

Hadron2023
20th International Conference on Hadron Spectroscopy and Structure
Genova, Italy, June 5th

Muon $(g-2)$

A quick recap

Anomalous magnetic moment



scattering of particle mass m off external photon (μ, q)

$$-ie\left[\gamma_\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q^\nu}{2m}F_2(q^2)\right], g = 2(F_1(0) + F_2(0))$$

$$F_1(0) = 1 \rightarrow F_2(0) = a = (g-2)/2$$

A rich history

electron a_e measured in experiment [Kusch, Foley '48]

confirms radiative corrections [Schwinger '48] $\rightarrow$ success of QFT

muon a_μ measured in experiement [Columbia exp. '59]

"muon is heavy electron" $\rightarrow$ families of leptons

Back to the future

new physics contribution to a: $(a - a^{\mathrm{SM}}) \propto m^2/\Lambda^2_{\mathrm{NP}}$

a_τ experimentally inaccessible, a_μ most promising

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Muon $(g-2)$

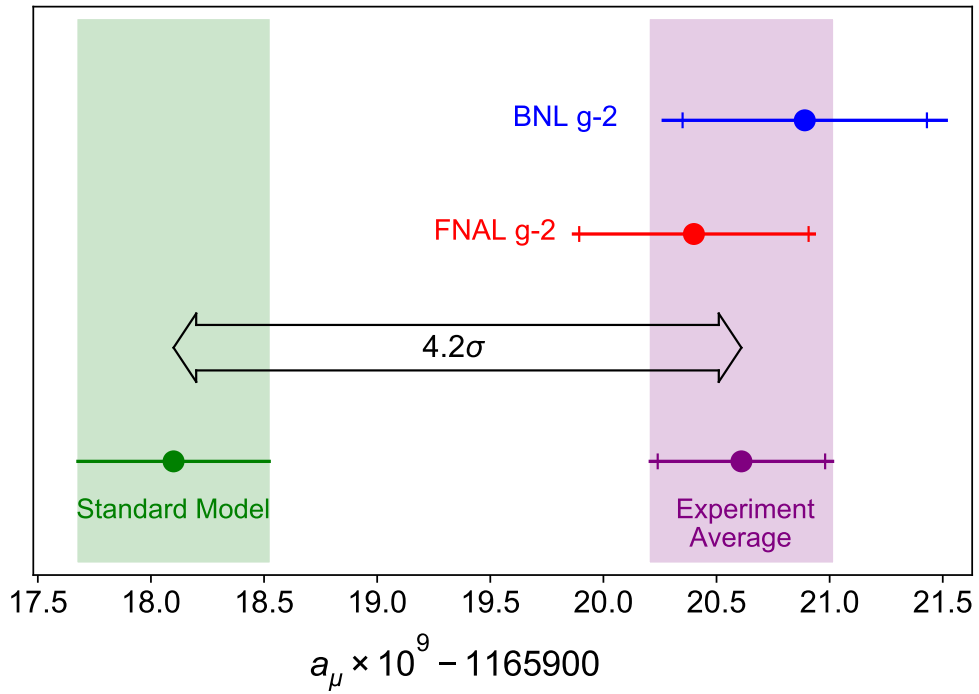
Status

$(g-2)$ theory initiative

[White Paper '20]

SM contributions to $a_\mu[\times 10^{10}]$

5-loop QED	11 658 471.8853(36)
2-loop EW	15.36(10)
HVP LO	693.1(4.0)
HVP NLO	-9.83(7)
HVP NNLO	1.24(1)
HLbL	9.2(1.9)



Theory error dominated by hadronic physics HVP and HLbL

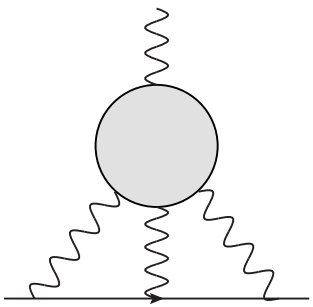
Hadronic Vacuum-Polarization and Light-by-Light

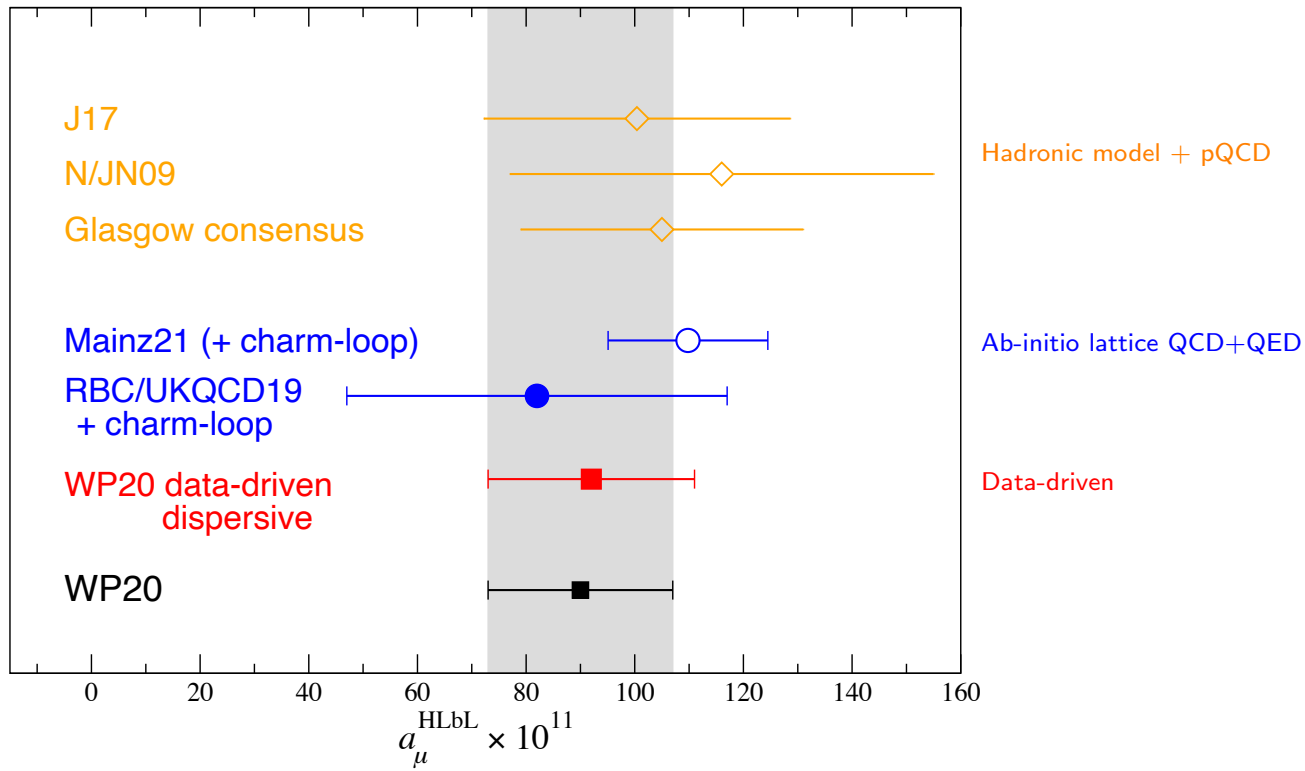
Precision goal for Fermilab $\times 4$ better

implies knowing HVP at 0.2-0.3 % accuracy

Hadronic Light-by-Light

Status

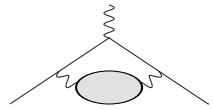




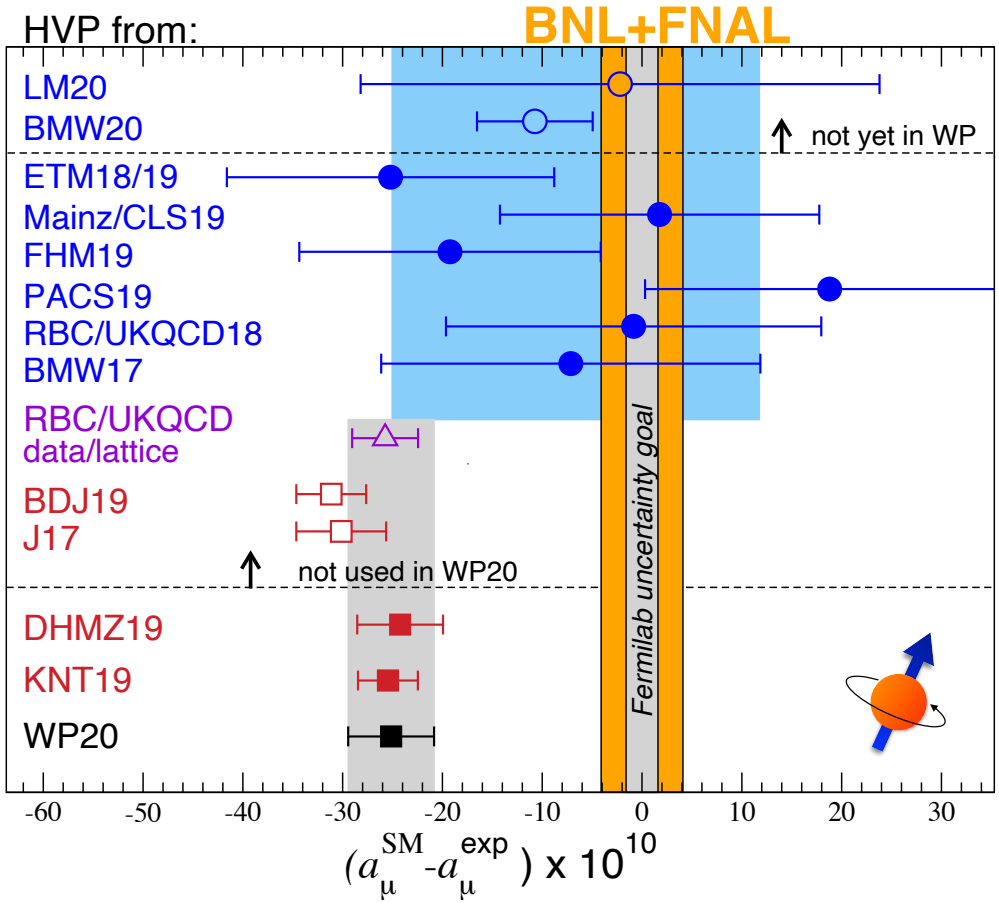
Consistency between lattice QCD+QED and dispersive

novel update $124.7(11.5)(9.9) \cdot 10^{11}$ [RBC/UKQCD '23]

Hadronic Vacuum Polarization



[Snowmass '21]



BMW20 first complete Lattice QCD+QED calculation below 1%

Lattice QCD+QED

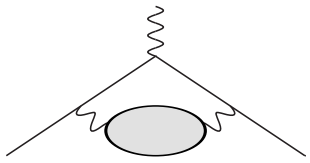
data-driven/dispersive

WP20: $g-2$ theory initiative community White Paper

$\rightarrow$ only data-driven/dispersive used in current best estimate

Dispersive approach

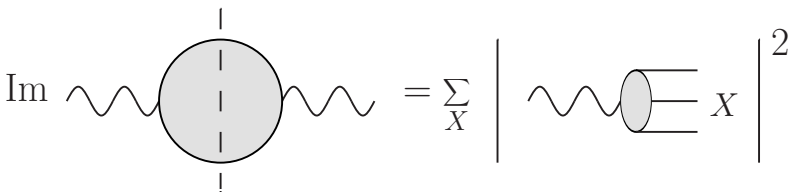
Method



$$a_\mu = \frac{\alpha}{\pi}\int \frac{ds}{s} K(s,m_\mu)\,\frac{\mathrm{Im}\Pi(s)}{\pi}$$ [Brodsky, de Rafael '68]

analyticity $\hat{\Pi}(s) = \Pi(s) - \Pi(0) = \dfrac{s}{\pi}\displaystyle\int_{4m_\pi^2}^{\infty} dx \frac{\mathrm{Im}\Pi(x)}{x(x-s-i\varepsilon)}$

unitarity



$$\frac{4\pi^2\alpha}{s}\frac{\mathrm{Im}\Pi(s)}{\pi} = \sigma_{e^+e^-\to\gamma^\star\to\mathrm{had}}$$

At present $O(30)$ channels: $\pi^0\gamma, \pi^+\pi^-, 3\pi, 4\pi, K^+K^-, \cdots$

$K(s,m_\mu) \to \pi^+\pi^-$ dominates due to ρ resonance

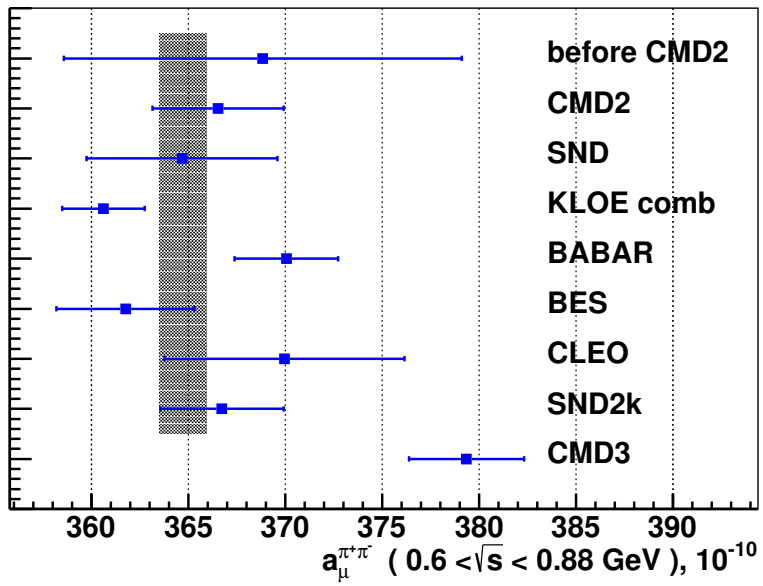
$\pi\pi$ channel is $\sim 70\%$ of signal and $\sim 70\%$ of error

Dispersive approach

Tensions in $\pi^+\pi^-$ channel

Large tensions among experiments: BaBar, KLOE, now CMD3

[CMD3 2302.08834]



very difficult to combine different experiments

what is the error of $\pi\pi$ contribution to a_μ?

motivates even more first-principles Lattice QCD calculations

Lattice field theories

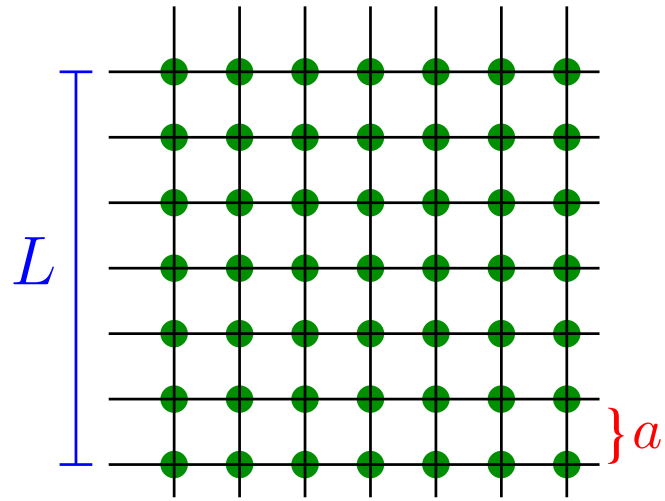
Non-perturbative predictions

lattice spacing $a \to$ regulate UV divergences

finite size $L \to$ infrared regulator

Continuum theory $a \to 0$, $L \to \infty$

Euclidean metric $\to$ Boltzman interpretation of path integral



$$\langle O \rangle = \mathcal{Z}^{-1} \int [DU] e^{-S[U]} O(U) \approx \frac{1}{N} \sum_{i=1}^{N} O[U_i]$$

Very high dimensional integral $\to$ Monte-Carlo methods

Markov Chain of gauge field configs $U_0 \to U_1 \to \cdots \to U_N$

HVP from Lattice

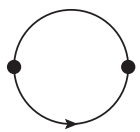
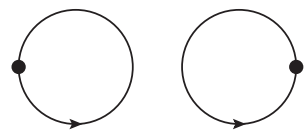
Method

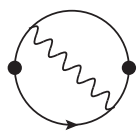
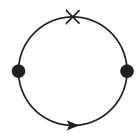
Vector electro-magnetic current $j^\gamma_\mu(x) = i \sum_{\rm f} Q_{\rm f} \overline{\psi}(x) \gamma_\mu \psi(x)$

Time-momentum representation [Bernecker, Meyer, '11]

$$G(t) = \frac{1}{3} \sum_k \int d\vec{x} \, \langle j^\gamma_k(x) j^\gamma_k(0) \rangle \qquad \langle \cdot \rangle = \text{QCD+QED exp. value}$$

$$a_\mu = 4\alpha^2 \int_0^\infty dt \, w(t) \, G(t) \,, \quad w(t) \text{ muon kernel (weights)}$$

Isospin limit: quark-conn  $\mathrm{ud, s, c}$ quark-disc  $\mathrm{ud, s}$

Isospin-breaking: $O(\alpha)$  $+ \ldots$ $O(m_u - m_d)$  $+ \ldots$

Dominant contribution (signal+noise): up-down quark-connected

HVP from Lattice

Theoretical advances

Formulation isospin-breaking schemes, isosymmetric points

[RM123][RBC/UKQCD 18][BMW 20][WP20][Portelli Lat22][Tantalo Lat22][...]

Analytic control of finite-volume effects

[Hansen, Patella '19 '20][Lehner, Meyer '20][Bijnens et al '19]

Improved understanding of scaling violations

[Mainz 20][Husung, Marquard, Sommer '22][Husung '23][Sommer Lat22]

HVP from Lattice

Roadmap

Accuracy goal ≤ 5 $[\times 10^{-10}]$

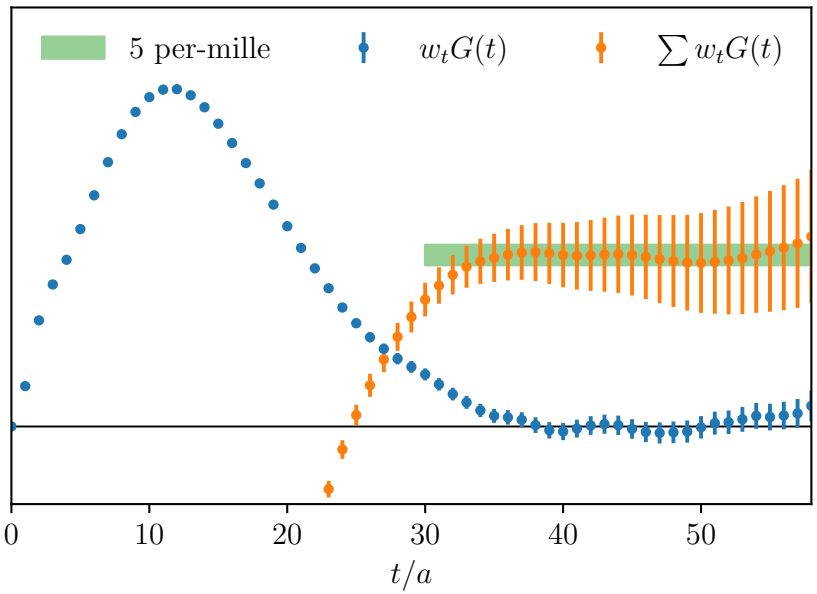
	conn-ud	conn-s	conn-c	disc	QED	SIB
$\approx$	650	53	14	-11	<10	<10
err	0.5%	5%	10%	10%	10%	10%

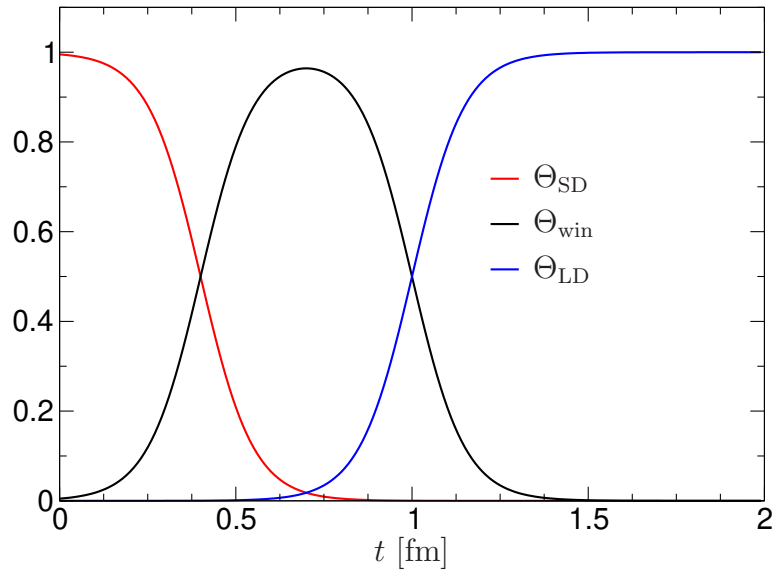
1. light-quark per-mille prediction from QCD in isospin limit
   up-down degenerate (and no QED)
   tune up, down, strange masses to physical values
   include charm dynamical effects, take $a \to 0$ and $L \to \infty$
   disconneted HVP, strange, charm, bottom

2. per-mille prediction from Standard Model
   isospin-breaking must be included at $O(\alpha), O(m_u - m_d)$

Euclidean windows

A novel paradigm





Smoothly divide integral in several parts

$$a_\mu = 4\alpha^2 \sum_t w_t \Big[\Theta_{\rm SD}(t) + \Theta_{\rm W}(t) + \Theta_{\rm LD}(t)\Big] G(t)$$ [RBC/UKQCD '18]

short-distance $\rightarrow$ cutoff effects

long-distance $\rightarrow$ Monte-Carlo noise

intermediate window: accessible today w/ current resources

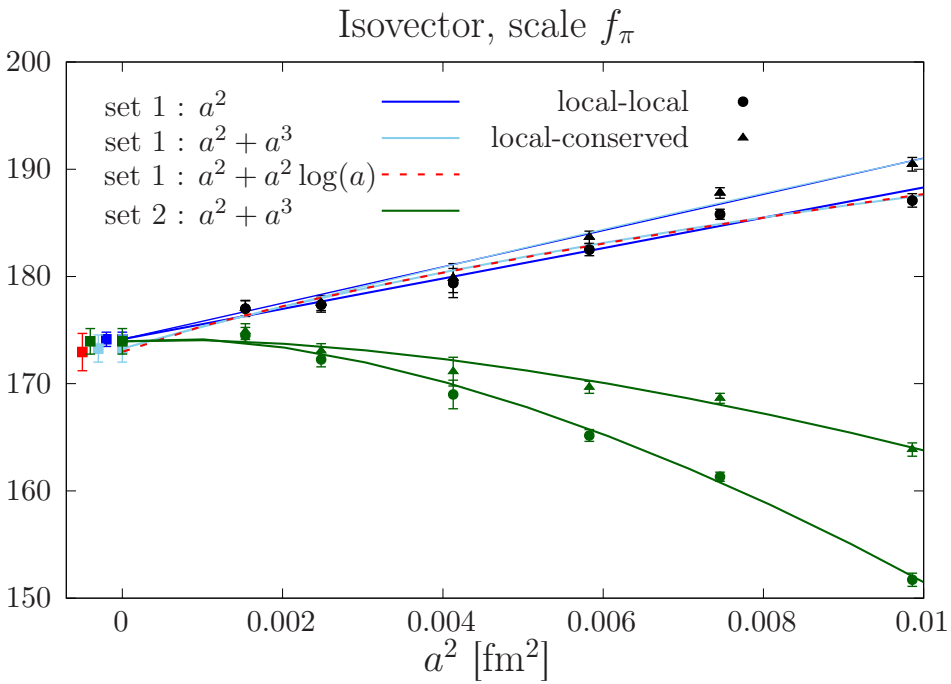
most collaborations precision of 0.4 - 0.6 %

Continuum limit

Intermediate window examples

[Mainz '22]

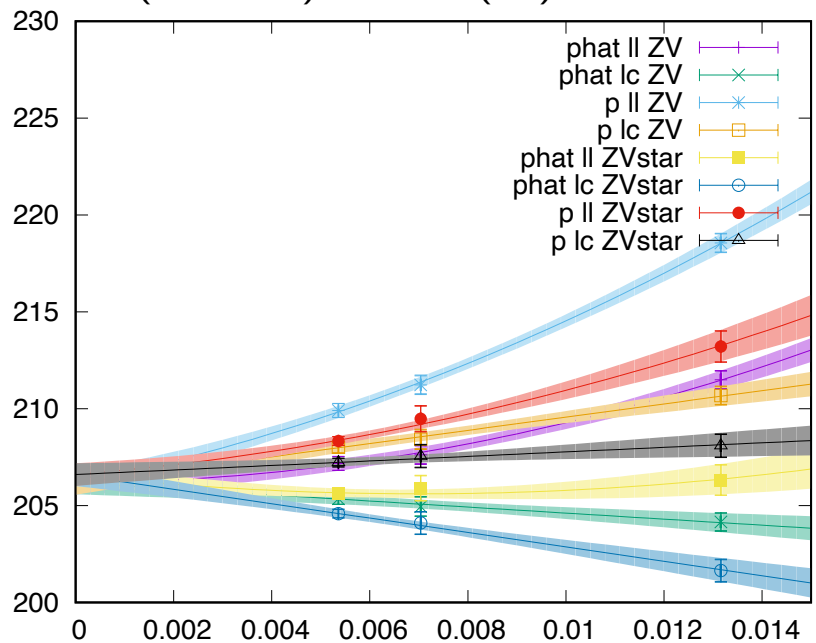
$m_\pi = m_K \simeq 420$ MeV



[RBC/UKQCD '22]

m_π, m_K physical

Fits ll ($a^2 + a^4$) and lc (a^2):



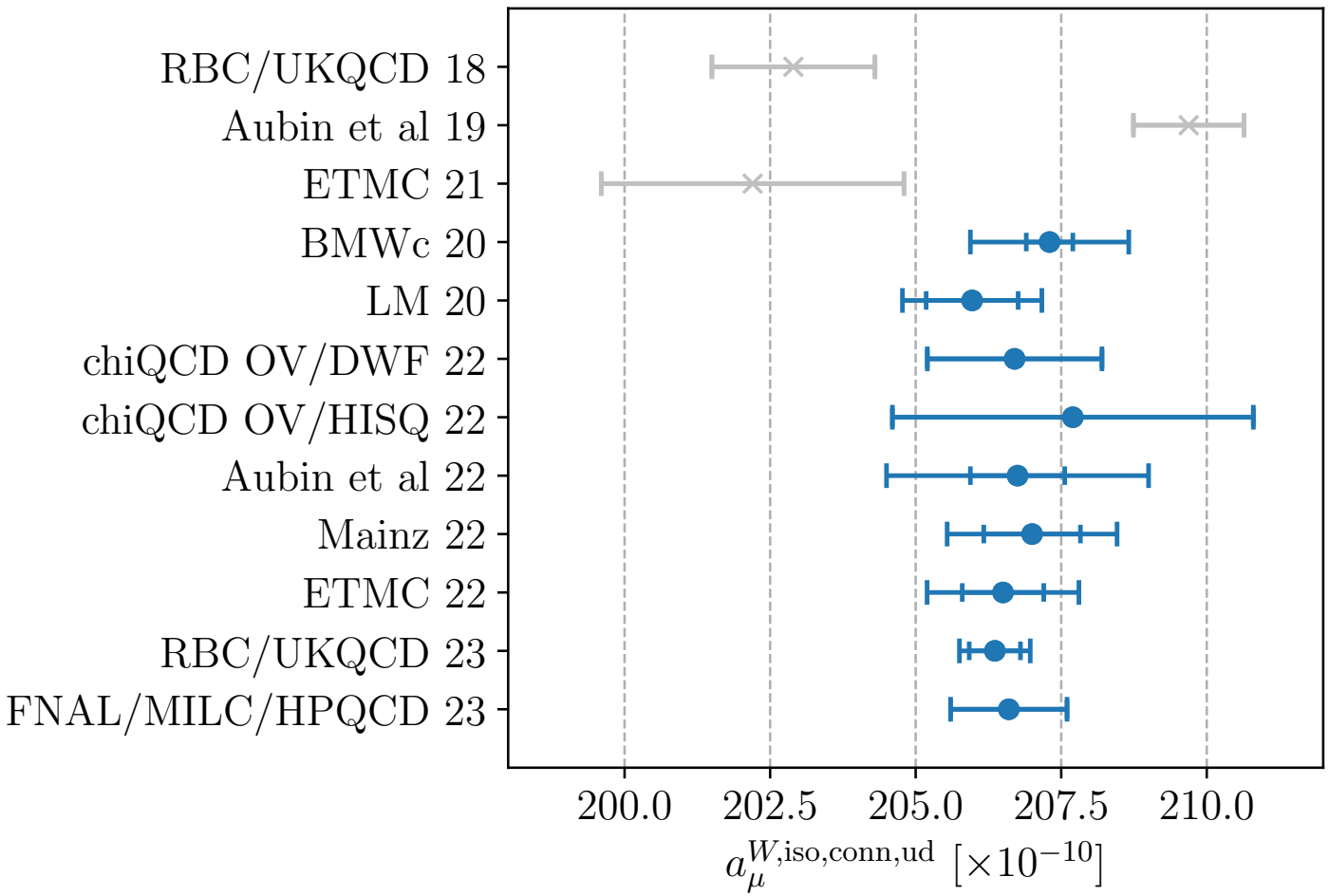
Different lattice collaborations = different systematic errors

unique answer in continuum limit → excellent consistency

Intermediate window

Status

isosymmetric intermediate window: internal lattice cross-checks



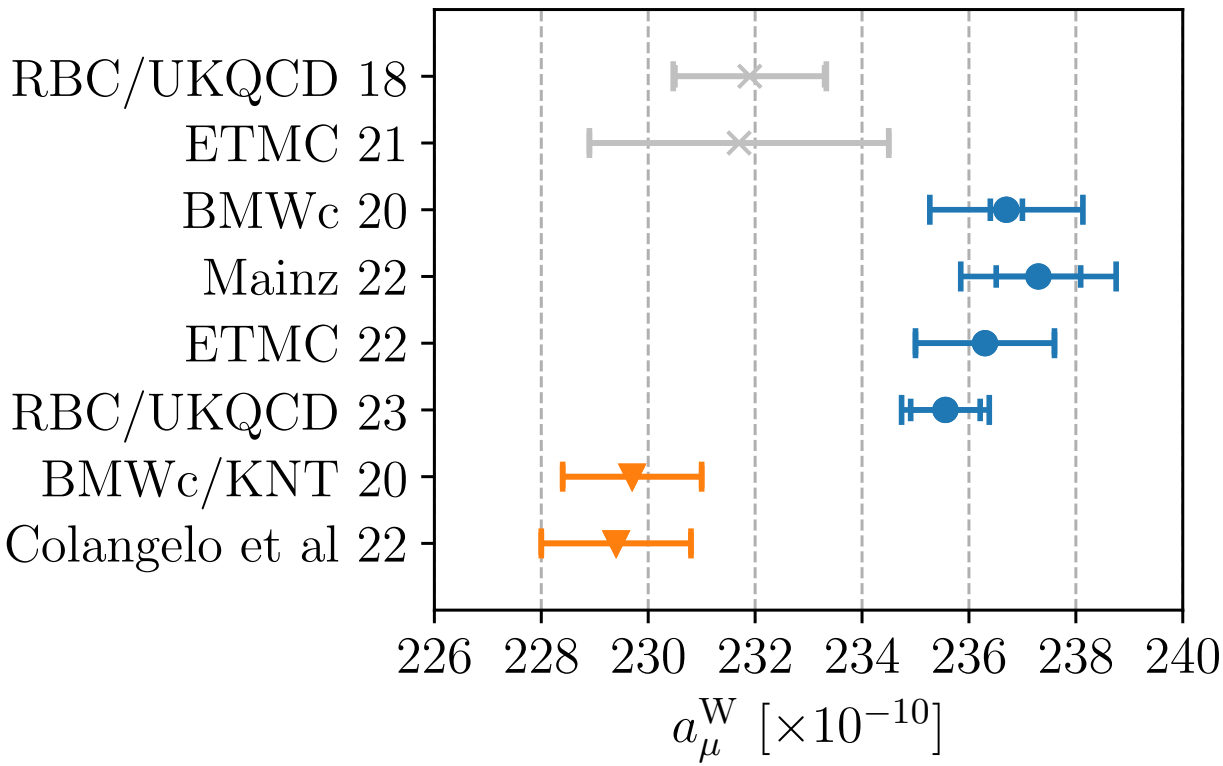
New puzzles forming

Comparison with data

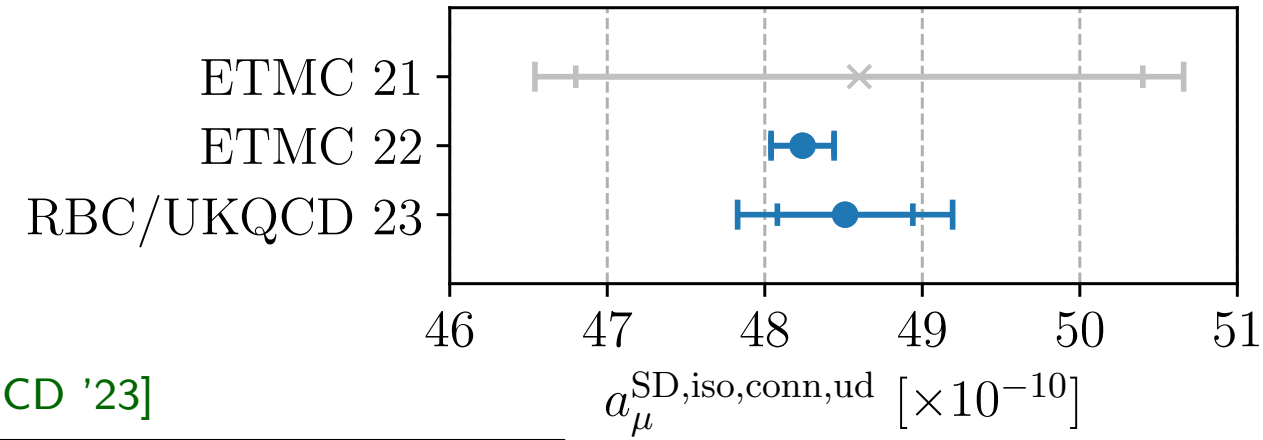
Windows calculable starting from $R(s)$: compare w/ Lattice QCD+QED

add isospin-breaking + strange + charm + disconnected $(206 \to 236)$

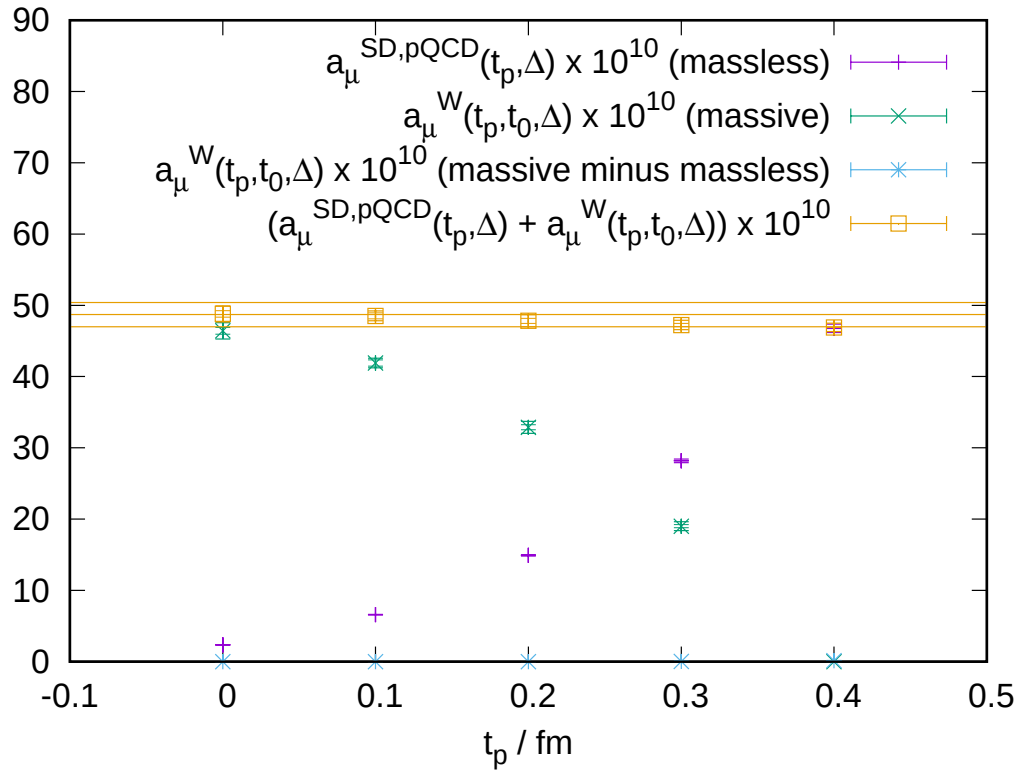
Situation before CMD3 (see also [Aubin et al/CL/KNT 19])



Short-Distance window



[plot from RBC/UKQCD '23]



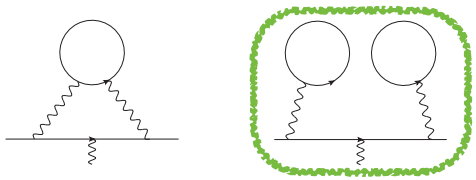
isospin limit

dominated by pert. theory

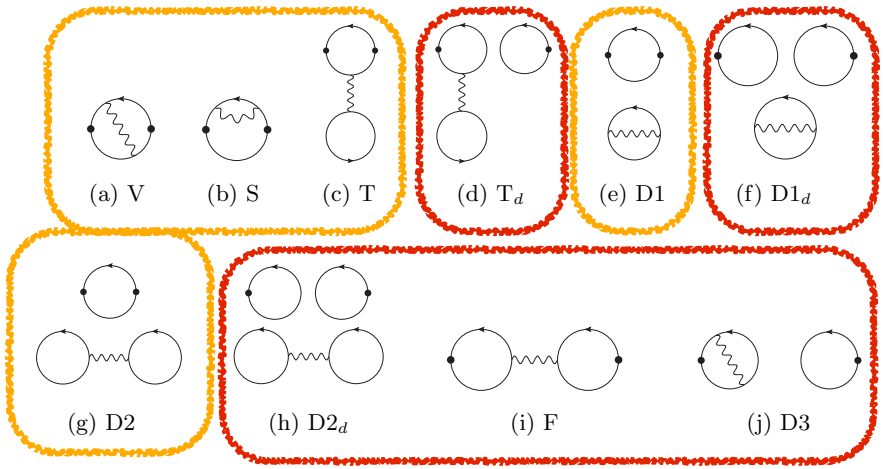
more results expected soon

Summary

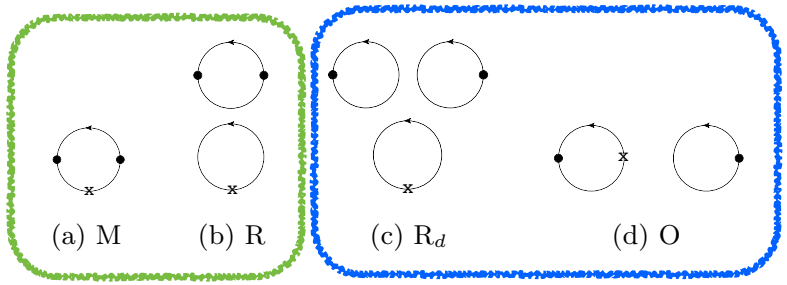
Isospin limit



QED corrections



Strong isospin breaking



Light-quark connected:
a_μ^W, a_μ^{SD}, a_μ^{LD}

Strange-quark connected

Charm-quark connected

Legend:
strong agreement, only BMW20, attention needed, agreement forming

Conclusions

Achievements

1. HLbL remarkable work from Lattice and Dispersive
2. a_μ^W (isosymmetric) remarkable agreement Lattice collaborations
3. a_μ^{SD} (isosymmetric) agreement of two Lattice collaborations
4. disconnected, strange, charm remarkable agreement Lattice collaborations

Outlooks Lattice community:

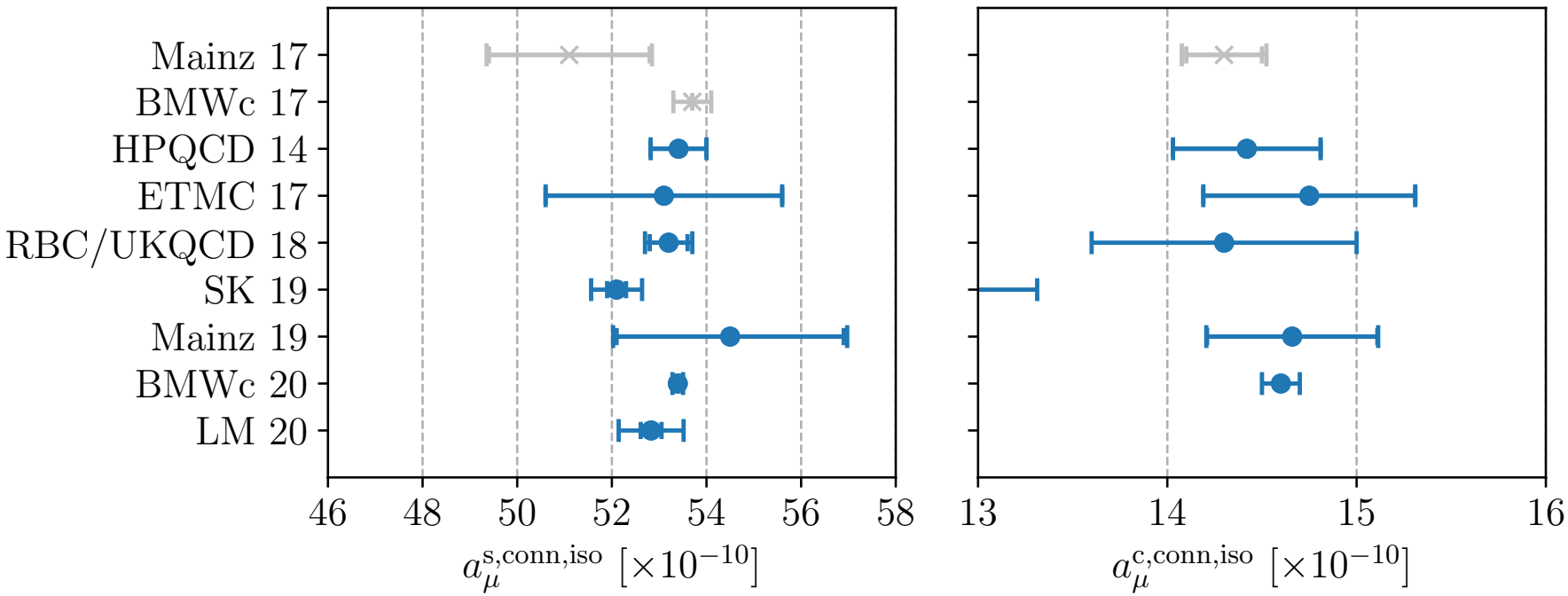
1. a_μ^{LD} (isosymmetric) high-priority, several results soon
2. isospin-breaking effects high-priority, several results soon

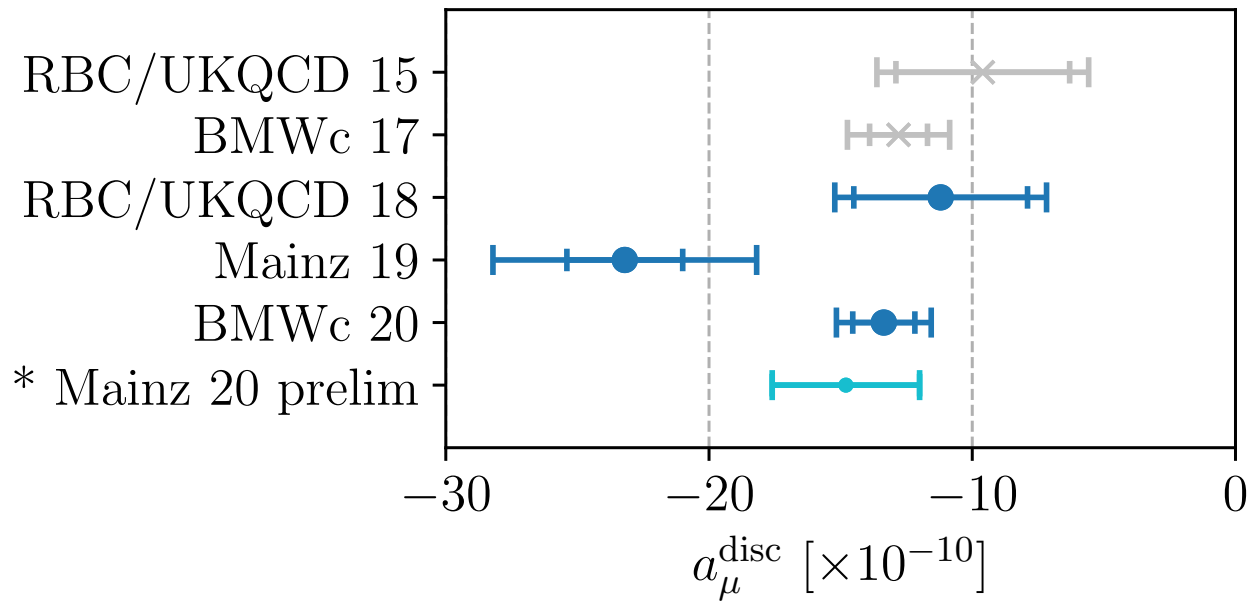
Outlook experimental community:

clarify tensions in $\pi^+\pi^-$ BaBar, KLOE, CMD3, high(er)-priority

Thanks for the attention!

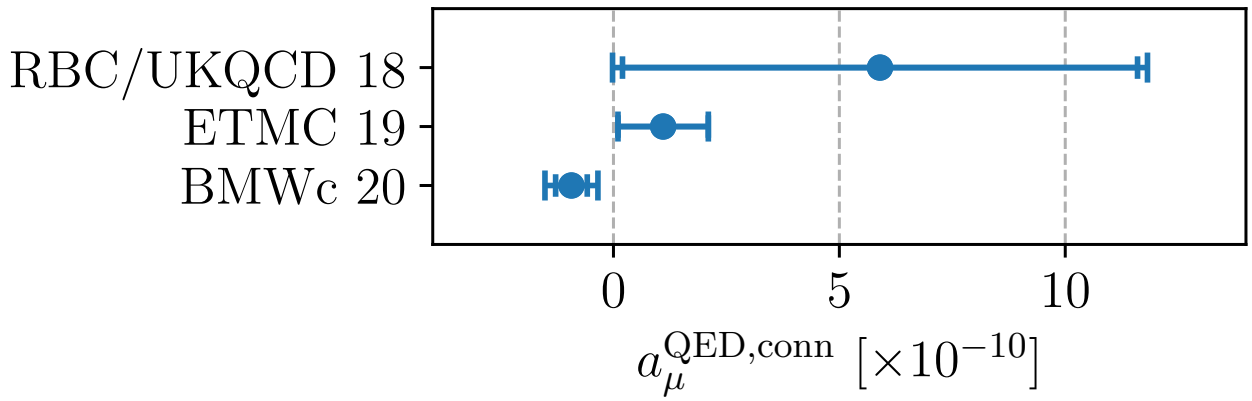
Strange, charm

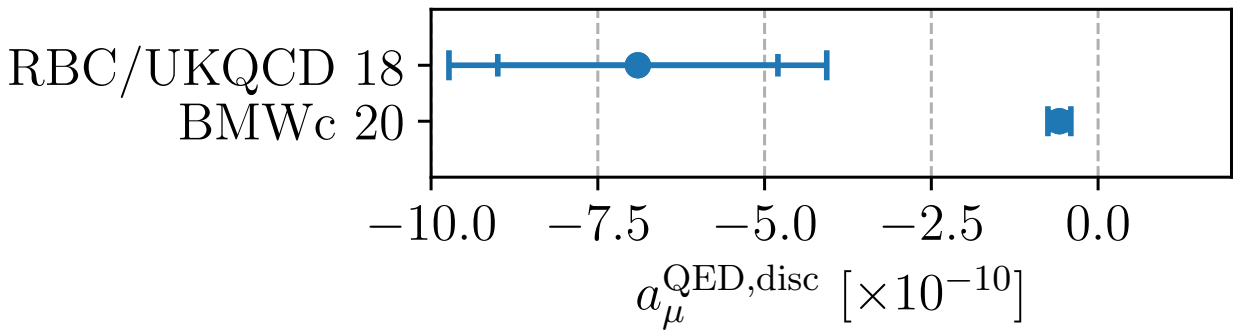




* = Workshop "The hadronic vacuum polarization from lattice QCD at high precision" Nov 2020
also preliminary results from FNAL/MILC/HPQCD

QED corrections





attention needed for QED,disc

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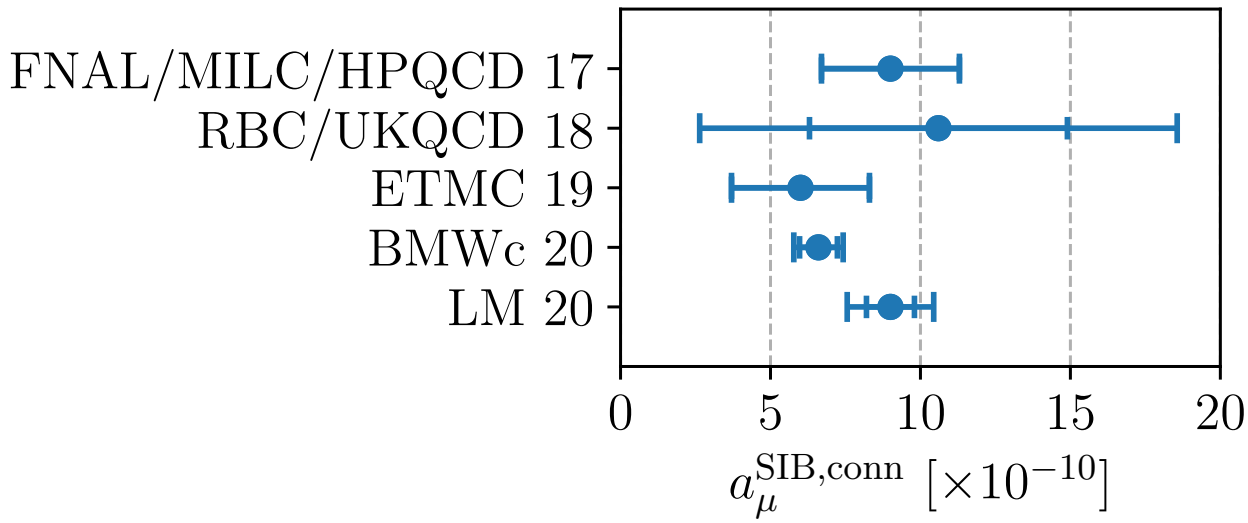
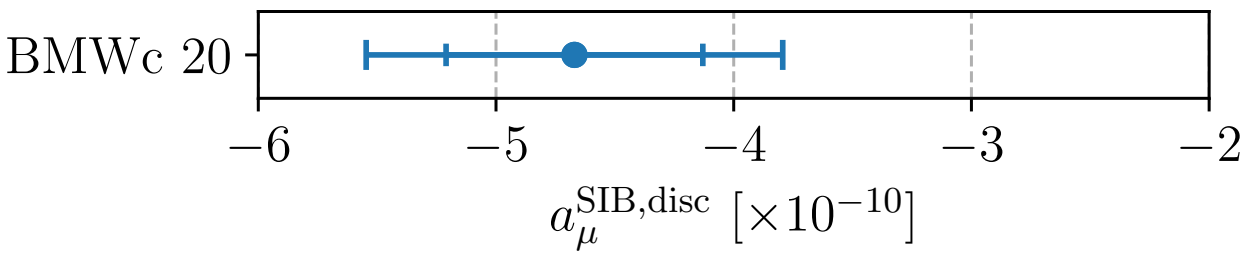
SIB corrections

Separation conn vs disc potentially large finite-vol errors [Lehner, Meyer '19]

pqChPT: $a_\mu^{\rm SIB,conn}(L = 5.5\ {\rm fm}) - a_\mu^{\rm SIB,conn}(L = \infty) \simeq 4\ [\times 10^{-10}]$





another result available from PQChPT [Lehner, Meyer '20]

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