

# Charm mesons in magnetized nuclear matter - effects of (inverse) magnetic catalysis

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# Motivation

- Heavy flavor mesons: (a) heavy quarkonia (b) open heavy flavor mesons
- Produced at the early times of ultra-relativistic heavy-ion collision experiments
- Strong magnetic fields are estimated during the early stages of peripheral heavy-ion collisions
- Produced magnetic field strength —  $|eB| \sim 10^{18} \text{ G} \approx m_\pi^2$  at RHIC in BNL,  $|eB| \sim 10^{19} \text{ G} \approx 15m_\pi^2$  at LHC in CERN
- Modifications of the properties of heavy flavor mesons due to such external effects may have experimental observable consequences

# The Charm Mesons

## Open charm mesons

- Pseudoscalar :  $D^\pm(\bar{d}c/\bar{c}d)$ ,  $D^0(\bar{u}c)$ ,  $\bar{D}^0(\bar{c}u)$
- Vector :  $D^{*\pm}$ ,  $D^{*0}$ ,  $\bar{D}^{*0}$
- Charmed strange :  $D_s^+(\bar{s}c)$ ,  $D_s^-(\bar{c}s)$ ,  $D_s^{*+}$ ,  $D_s^{*-}$

## Charmonium ( $\bar{c}c$ ) states

- $\Psi(3770)(1D) \rightarrow D\bar{D}$
- $\Psi(4040)(3S) \rightarrow D_s^+D_s^-$

# Hadronic chiral $SU(3)_L \times SU(3)_R$ Model

Effective hadronic chiral Lagrangian density contains the terms:

$$\mathcal{L} = \mathcal{L}_{kin} + \sum_{W=X, V_\mu, A_\mu, u} \mathcal{L}_{BW} + \mathcal{L}_{vec} + \mathcal{L}_0 + \mathcal{L}_{scale-break} + \mathcal{L}_{SB} + \mathcal{L}_{mag}$$

Under mean-field approximation :  $s(x) \rightarrow \langle s \rangle$ ;  $V^\mu(x) \rightarrow \langle V^\mu \rangle \equiv (V^0, 0)$

$\Rightarrow$  Scalar fields are treated to be classical in solving  $\partial_\mu \left[ \frac{\partial \mathcal{L}}{\partial (\partial_\mu x)} \right] - \frac{\partial \mathcal{L}}{\partial x} = 0 \Big|_{x=\sigma, \zeta, \delta, \chi}$

■  $\mathcal{L}_{BX} = - \sum_{i=p,n} \bar{\psi}_i m_i^* \psi_i \rightarrow$  mass of the  $i^{th}$  baryon  $m_i^* = -(g_{\sigma i} \sigma + g_{\zeta i} \zeta + g_{\delta i} \delta)$

■  $\mathcal{L}_{SB} = -\frac{1}{2} Tr[A_p(uXu + u^\dagger Xu^\dagger)] \equiv -Tr[diag(m_u \langle \bar{u}u \rangle, m_d \langle \bar{d}d \rangle, m_s \langle \bar{s}s \rangle)]$

With  $\langle X \rangle = diag \left[ \frac{(\sigma+\delta)}{\sqrt{2}}, \frac{(\sigma-\delta)}{\sqrt{2}}, \zeta \right]$ ,  $A_p = \frac{1}{\sqrt{2}} diag [m_\pi^2 f_\pi, m_\pi^2 f_\pi, (2m_K^2 f_K - m_\pi^2 f_\pi)]$

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P. Papazoglou, D. Zschesche, S. Schramm, J. Schaffner-Bielich, H. Stöcker, and W. Greiner, Phys. Rev. C **59**, 411 (1999).

# Chiral Effective Model

- Scalar fields are related to the light quark condensates,

$$\sigma \sim (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle), \quad \zeta \sim \langle \bar{s}s \rangle \quad \text{and} \quad \delta \sim (\langle \bar{u}u \rangle - \langle \bar{d}d \rangle).$$

- $\mathcal{L}_{scale-break} = -\frac{1}{4}\chi^4 \ln\left(\frac{\chi^4}{\chi_0^4}\right) + \frac{d}{3}\chi^4 \ln\left(\frac{(\sigma^2 - \delta^2)\zeta}{\sigma_0^2 \zeta_0} \frac{\chi^3}{\chi_0^3}\right) \rightarrow$

$$\theta_\mu^\mu = \sum_{i=u,d,s} m_i \langle \bar{q}_i q_i \rangle + \langle \frac{\beta_{QCD}}{2g} G^{a\mu\nu} G_{\mu\nu}^a \rangle \equiv -(1-d)\chi^4$$

- $\mathcal{L}_{mag} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - e_i \bar{\psi}_i \gamma_\mu A^\mu \psi_i - \frac{1}{4}\kappa_i \mu_N \bar{\psi}_i \sigma^{\mu\nu} F_{\mu\nu} \psi_i$

- Medium effects incorporated through number ( $\rho_{p,n}$ ) and scalar ( $\rho_{p,n}^s$ ) densities of nucleons.

$$\bar{\psi}_i \gamma^\mu \psi_j \rightarrow \delta_{ij} \delta_\mu^0 \langle \bar{\psi}_i \gamma^\mu \psi_i \rangle \equiv \delta_{ij} \rho_i, \quad \rightarrow \bar{\psi}_i \psi_j \rightarrow \delta_{ij} \langle \bar{\psi}_i \psi_i \rangle \equiv \delta_{ij} \rho_i^s,$$

# Dirac Sea contribution in presence of magnetic field

Free energy density of nucleons:

- $\Omega_N = \Omega_{N,sea} + \Omega_{N,mat}$

- $\Omega_{N,sea} = -\frac{|qB|}{2\pi} \sum_{\nu=0}^{\infty} \alpha_{\nu} \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} \epsilon_{k,\nu}$      $\alpha_{\nu} = (2 - \delta_{\nu 0}) \rightarrow$  for zero AMM  
 $= -\frac{|qB|}{2\pi} \sum_{\nu,s=\pm 1} \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} \epsilon_{k,\nu,s}$      $\alpha_{\nu} = 1 \rightarrow$  for non-zero AMM

- $\epsilon_{k,\nu} = \sqrt{k_z^2 + 2\nu|qB| + m_N^{*2}}$  — for zero AMM

$$\epsilon_{k,\nu,s} = \sqrt{k_z^2 + (\sqrt{2\nu|qB| + m_N^{*2}} + s\Delta_b)^2} \quad \text{— for non-zero AMM}$$

- For zero AMM,  $\frac{\partial \Omega}{\partial x}|_{x=\sigma,\zeta,\delta} \rightarrow \Delta \rho_s^N \approx \frac{|qB|m_N^*}{2\pi^2} \left[ y(1 - \ln y) + \frac{1}{2} \ln \frac{y}{2\pi} + \ln \Gamma(y) \right],$

$$y \equiv \frac{m_N^{*2}}{2|qB|}; \quad \Delta_b = -\frac{1}{2} \kappa_b \mu_N B$$

- Scalar fields ( $\sigma$ ,  $\zeta$ ,  $\delta$  and  $\chi$ ) solved at given  $\rho_B (= \rho_p + \rho_n)$ ,  $\eta = \frac{\rho_n - \rho_p}{2\rho_B}$  and  $|eB|$  incorporating this effect

# Generalized chiral effective model for $D$ meson

$$\mathcal{L}_{total}^D = \mathcal{L}_{free}^D + \mathcal{L}_{int}^D$$
$$\mathcal{L}_{int}^D = \mathcal{L}_{WT} + \mathcal{L}_{SME} + \mathcal{L}_{1^{st}range} + \mathcal{L}_{d_1} + \mathcal{L}_{d_2}$$

- $\mathcal{L}_{WT} = -\frac{i}{8f_D^2} [3(\bar{N}\gamma^\mu N)(\bar{D}(\partial_\mu D) - (\partial_\mu \bar{D})D) + (\bar{N}\gamma^\mu \tau^a N)(\bar{D}\tau^a(\partial_\mu D) - (\partial_\mu \bar{D})\tau^a D)]$
- $\mathcal{L}_{SME} = \frac{m_D^2}{2f_D} [(\sigma + \sqrt{2}\zeta)(\bar{D}D) + \delta^a(\bar{D}\tau^a D)]$
- $\mathcal{L}_{1^{st}range} = -\frac{1}{f_D} [(\sigma + \sqrt{2}\zeta)(\partial_\mu \bar{D})(\partial^\mu D) + (\partial_\mu \bar{D})\tau^a(\partial^\mu D)\delta^a]$
- $\mathcal{L}_{d_1} = \frac{d_1}{2f_D^2} (\bar{N}N)(\partial_\mu \bar{D})(\partial^\mu D)$
- $\mathcal{L}_{d_2} = \frac{d_2}{4f_D^2} (\bar{N}N)(\partial_\mu \bar{D})(\partial^\mu D) + (\bar{N}\tau^a N)[(\partial_\mu \bar{D})\tau^a(\partial_\mu D)]$

$$N \equiv (p, n), \quad D \equiv (D^0, D^+) \text{ and } \bar{D} \equiv (\bar{D}_0, D^-)$$

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Arvind Kumar, Amruta Mishra, Eur. Phys. J. A **47** 164 (2011); S. Reddy P., A. Jahan. C. S., N. Dhale, A. Mishra, J. S. Bielich, Phys. Rev. C **97** 065208 (2018).



# Dispersion Relation for Pseudoscalar $D$ mesons

Fourier transform of the e.o.m's

$$-\omega^2 + \vec{k}^2 + m_{D(\bar{D})}^2 - \Pi_{D(\bar{D})}(\omega, \vec{k}) = 0$$

$\Pi_{D(\bar{D})}$  : self energy of  $D$  ( $\bar{D}$ ) mesons in the medium.

$$\begin{aligned}\Pi(\omega, |\vec{k}|)_D &= \frac{1}{4f_D^2} \left[ 3(\rho_p + \rho_n) \pm (\rho_p - \rho_n) \right] \omega + \frac{m_D^2}{2f_D} (\sigma' + \sqrt{2}\zeta_c' \pm \delta') \\ &+ \left[ -\frac{1}{f_D} (\sigma' + \sqrt{2}\zeta_c' \pm \delta') + \frac{d_1}{2f_D^2} (\rho_p^s + \rho_n^s) \right. \\ &+ \left. \frac{d_2}{4f_D^2} \left( (\rho_p^s + \rho_n^s) \pm (\rho_p^s - \rho_n^s) \right) \right] (\omega^2 - \vec{k}^2),\end{aligned}$$

$\Rightarrow \Pi(\omega, |\vec{k}|)_{\bar{D}} = -1^{\text{st}} \text{ term} + (\dots) \rightarrow \text{for } \bar{D} \text{ mesons doublet } (\bar{D}^0, D^-)$

$\Rightarrow \pm \rightarrow (D^0, D^+)$  in  $D$  and  $(\bar{D}^0, D^-)$  in  $\bar{D}$ .

$\Rightarrow \sigma' = (\sigma - \sigma_0), \quad \zeta_c' = (\zeta_c - \zeta_{c0}) = 0, \quad \delta' = (\delta - \delta_0)$

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Arvind Kumar, Amruta Mishra, Eur. Phys. J. A **47** 164 (2011); S. Reddy P., A. Jahan. C. S., N. Dhale, A. Mishra, J. S. Bielich, Phys. Rev. C **97** 065208 (2018).

## In-medium mass of $D$ and $D^*$ mesons

- In-medium mass of pseudoscalar  $D$  ( $\bar{D}$ ) mesons,

$$m_{D/\bar{D}}^* \rightarrow \text{Solution for } \omega \text{ at } \vec{k} = 0$$

- In-medium mass of vector  $D^*$  ( $\bar{D}^*$ ) mesons,

$$m_{D^*}^* - m_{D^*}^{\text{vac}} = m_D^* - m_D^{\text{vac}}$$

- Lowest Landau level contribution for the charged mesons:

$$m_{D^\pm}^{\text{eff}} = \sqrt{m_{D^\pm}^{*2} + |eB|}, \quad m_{D^{*\pm}}^{\text{eff}} = \sqrt{m_{D^{*\pm}}^{*2} + (1 + gS_z)|eB|}$$

- $S_z$ : z-component of spin,  $\vec{B} = B\hat{z}$ .  
 $g$ : Landé g-factor  $\approx 2$ .

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Amruta Mishra, S. P. Misra, Int.J.Mod.Phys.E **30** 2150064 (2021);  
Sourodeep De, Amruta Mishra, arXiv: 2208.09820 (2022) [hep-ph] (To be published in  
Physical Review C).

# Effective PV Mixing Lagrangian

- Effective hadronic Lagrangian describing  $PV$  interaction

$$\mathcal{L}_{PV\gamma} = \frac{g_{PV}}{m_{av}} e \tilde{F}_{\mu\nu} (\partial^\mu P) V^\nu$$

$$\mathcal{L} = -\frac{1}{2} \partial_\mu P \partial^\mu P + \frac{1}{2} m_P^2 P^2 - \frac{1}{2} \partial_\mu V_\nu \partial^\mu V^\nu + \frac{1}{2} m_V^2 V^2 + \mathcal{L}_{PV\gamma}$$

- $\mathbf{B} = B \hat{z}$ ,  $\tilde{F}_{03} = -\tilde{F}_{30} = B$   $V^\mu = (V_0, \mathbf{V}_\perp, V_{||})$

- $m_{av} = (m_V + m_P)/2$ ;  $\Gamma(V \rightarrow P\gamma) = \frac{e^2}{12\pi} \frac{g_{PV}^2 p_{cm}^3}{m_{av}^2}$

- Eq. of motions  $\rightarrow$

$$P : (\partial^2 + m_P^2)P - \frac{g_{PV}}{m_0} e \tilde{F}_{\alpha\beta} \partial^\alpha V^\beta = 0$$

$$V : (\partial^2 + m_V^2)V_\mu + \frac{g_{PV}}{m_0} e \tilde{F}_{\alpha\mu} \partial^\alpha P = 0$$

- For vanishing spatial momentum  $q^\mu = (\omega, 0, 0, 0)$ ,
- $m_P$  decreases and  $m_V^{||}$  increases with  $|eB|$

# Effective model Lagrangian for $D_s$ mesons

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{int}$$

- $\mathcal{L}_{free} = (\partial^\mu D_s^+)(\partial_\mu D_s^-) - m_{D_s}^2 D_s^+ D_s^-$
- $\mathcal{L}_{int} = \frac{m_{D_s}^2}{\sqrt{2}f_{D_s}} \left[ (\zeta + \zeta_c)(D_s^+ D_s^-) \right] - \frac{\sqrt{2}}{f_{D_s}} \left[ (\zeta + \zeta_c)(\partial_\mu D_s^+)(\partial^\mu D_s^-) \right] + \frac{d_3}{2f_{D_s}^2} \left[ (\bar{p}p + \bar{n}n)((\partial^\mu D_s^+)(\partial_\mu D_s^-)) \right]$
- Scalar fields,  $\zeta \sim \langle \bar{s}s \rangle$  and  $\zeta_c \sim \langle \bar{c}c \rangle$

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Divakar Pathak and Amruta Mishra, Adv.High Energy Phys. **2015** (2015) 697514;  
Sourodeep De, Amruta Mishra, arXiv: 2208.09820 (2022) [hep-ph] (To be published in Physical Review C)..

# In-medium masses of $D_s$ , $D_s^*$ mesons

Dispersion relation for the  $D_s^\pm$  mesons

$$-\omega^2 + \vec{k}^2 + m_{D_s}^2 - \Pi_{D_s}(\omega, |\vec{k}|) = 0$$

$$\Pi(\omega, |\vec{k}|) = \left[ \frac{d_1}{2f_{D_s}^2} (\rho_p^s + \rho_n^s) - \frac{\sqrt{2}}{f_{D_s}} (\zeta' + \zeta'_c) \right] (\omega^2 - \vec{k}^2) + \frac{m_{D_s}^2}{\sqrt{2}f_{D_s}} (\zeta' + \zeta'_c)$$

scalar fields fluctuations,  $\zeta' = \zeta - \zeta_0$ , and  $\zeta'_c = \zeta_c - \zeta_{c0} = 0$

- In-medium mass  $m_{D_s^*}^*$  of vector  $D_s^*$  meson:

$$m_{D_s^*}^* - m_{D_s^*}^{vac} = m_{D_s^*}^* - m_{D_s^*}^{vac}$$

- Lowest Landau level contribution for the charged mesons:

$$m_{D_s^\pm}^{eff} = \sqrt{m_{D_s^\pm}^{*2} + |eB|}, \quad m_{D_s^*}^{eff} = \sqrt{m_{D_s^*}^{*2} + (1 + gS_z)|eB|}$$

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Sourodeep De, Amruta Mishra, arXiv: 2208.09820 (2022) [hep-ph] (To be published in Physical Review C).

# Spin-Mixing Effect

The spin-magnetic field interaction Hamiltonian

$$H = - \sum_i \vec{\mu}_i \cdot \mathbf{B}$$

$$\mathbf{B} = B\hat{z}, \quad \vec{\mu}_i = g' q_i \vec{S}_i / 2m_i$$

The effective masses of the pseudoscalar and longitudinal component of vector meson due to spin-magnetic field interaction

$$m_{V||/P}^{\text{eff}} = m_{V/P}^* \pm \Delta m;$$

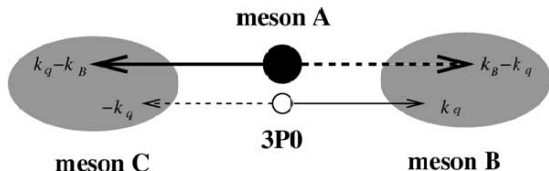
$$\Delta m = \frac{\Delta M}{2} \left( (1 + \chi_{sB}^2)^{1/2} - 1 \right); \quad \chi_{sB} = \frac{2}{\Delta M} \frac{(-g|eB|)}{4} \left( \frac{q_1}{m_1} - \frac{q_2}{m_2} \right);$$

$$\Delta M = m_V^* - m_P^*; \quad g = 2;$$

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J. Alford and M. Strickland, Phys. Rev. D **88**, 105017 (2013); Amruta Mishra, S.P. Misra, Int.J.Mod.Phys.E **31** (2022) 06, 2250060; S. De, P. Parui, A. Mishra, Int.J.Mod.Phys.E **31** 2250106 (2022).

# The $^3P_0$ model



- $M_{A \rightarrow BC} = \langle A | \gamma [\bar{q}_s q_s]^3 P_0 | BC \rangle$ ;  $\gamma$  is the coupling strength, probability for creating quark-antiquark pair.
- Matrix element:  $M = \frac{\gamma}{\pi^{1/4} \beta^{1/4}} \mathcal{P}(x, r) e^{-\frac{x^2}{4(1+2r^2)}}$
- Scaled momentum:  $x = \frac{1}{\beta} \sqrt{m_A^2/4 - m_B^2}$
- The decay rate:  $\Gamma(A \rightarrow BC) = 2\pi \frac{p_B E_B E_C}{M_A} |M|^2$

B. Friman, S.H. Lee, T. Song, Phys. Lett. B **548**, 153 (2002); E.S. Ackleh, T. Barnes, E.S. Swanson, Phys. Rev. D, **54** 11 (1996); B. Friman, S.H. Lee, T. Song, Phys. Lett. B **548**, 153 (2002); S. De, P. Parui, A. Mishra, arXiv: 2208.14953 (2022) (To be published in Physical Review C); A. Mishra, et. al, Eur.Phys.J.A **55** 99 (2019).

# In-medium hadron decay widths

- The decay width for charmonium states  $\Psi(3770) \rightarrow D\bar{D} \rightarrow$

$$\Gamma_{\Psi(3770) \rightarrow D\bar{D}} = \frac{\gamma_{\Psi}^2 \sqrt{\pi} E_D E_{\bar{D}}}{2m_{\Psi(3770)}} \frac{2^{11} 5}{3^2} \left( \frac{r_{\Psi}}{1 + 2r_{\Psi}^2} \right)^7 \\ \times x_{\Psi}^3 \left( 1 - \frac{1 + r_{\Psi}^2}{5(1 + 2r_{\Psi}^2)} x_{\Psi}^2 \right)^2 \exp \left( -\frac{x_{\Psi}^2}{2(1 + 2r_{\Psi}^2)} \right)$$

- The decay width of  $\psi(4040) \rightarrow D_s^+ D_s^- \rightarrow$

$$\Gamma_{\psi(4040) \rightarrow D_s^+ D_s^-} = \frac{\sqrt{\pi} E^2 x^3 \gamma^2 2^{12}}{M \times 3^5 \times 5} \left[ \frac{15}{8} \frac{1 + r^2}{1 + 2r^2} - \frac{5r^2(4 + r^2)}{(1 + 2r^2)^3} + \frac{r^2(5 - 9r^2 - 10r^4)x^2}{2(1 + 2r^2)^4} \right. \\ \left. + \frac{r^4(1 + 2r^2)x^4}{2(1 + 2r^2)^5} \right] e^{-\frac{x^2}{2(1+2r^2)}}$$

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E.S. Ackleh, T. Barnes, E.S. Swanson, Phys. Rev. D, **54** 11 (1996); B. Friman, S.H. Lee, T. Song, Phys. Lett. B **548**, 153 (2002); S. De, P. Parui, A. Mishra, arXiv: 2208.14953 (2022) (To be published in Physical Review C); A. Mishra, et. al, Eur.Phys.J.A **55** 99 (2019).



# Mass shifts of charmonia

Mass shifts in terms of medium modifications of scalar gluon condensate —

$$\Delta m_\psi = \frac{4}{81}(1-d) \int dk^2 \left\langle \left| \frac{\partial \psi(\vec{k})}{\partial \vec{k}} \right|^2 \right\rangle \frac{k}{k^2/m_c + \epsilon} \underbrace{(\chi^4 - \chi_0^4)}_{\sim \Delta \langle \frac{\alpha_s}{\pi} G^{a\mu\nu} G_{\mu\nu}^a \rangle}$$

$$\psi_{N,l} = P \times Y_l^m(\theta, \phi)(\beta^2, r^2)^{l/2} e^{-\frac{\beta^2 r^2}{2}} L_{N-1}^{l+1/2}(\beta^2 r^2)$$

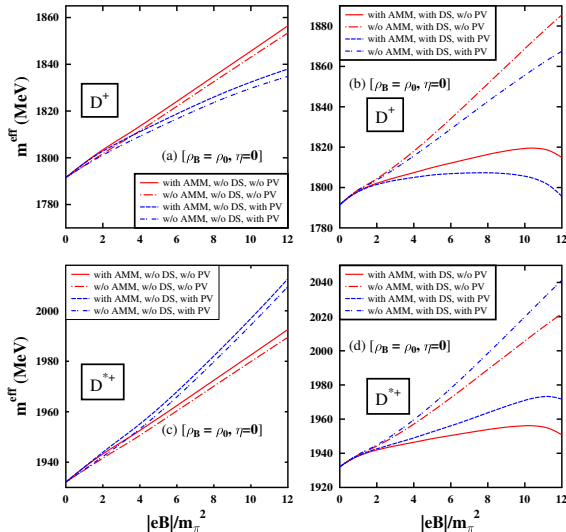
with

- $P \rightarrow \int \frac{d^3k}{(2\pi)^3} |\psi(k)|^2 = 1$
- $\beta^2 = M\omega/\hbar$ ;  $M = \frac{m_c}{2}$
- $\epsilon = 2m_c - m_\psi$

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Arvind Kumar, Amruta Mishra, Eur. Phys. J. A **47**, 164 (2011); A. Jahan C.S, N. Dhale, S. Reddy P, S. Kesarwani, A. Mishra, Phys.Rev.C **98** 065202 (2018); B. Friman, S. H. Lee, T. Song, Phys. Lett. B **548** 153 (2002).

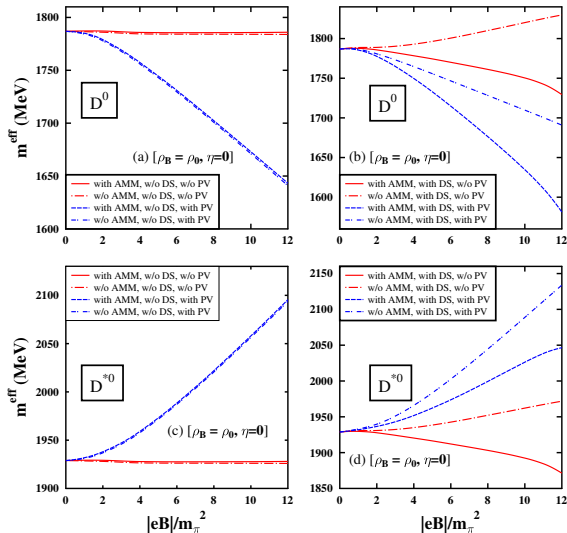
# $D^+$ and $D^{*+}$ meson masses at $\rho_B = \rho_0$



The masses (in MeV) of  $D^+$  and  $D^{*+}$  mesons plotted at  $\rho_B = \rho_0$  for symmetric matter ( $\eta = 0$ ) with  $eB/m_\pi^2$ , including the Dirac sea effects (b,d), without the Dirac sea effects (a,c). PV ( $D^+ - D^{*+}$ ) mixing effect and the AMM of the nucleons along with LLL contribution are included.

S. De, A. Mishra, arXiv: 2208.09820 [hep-ph].

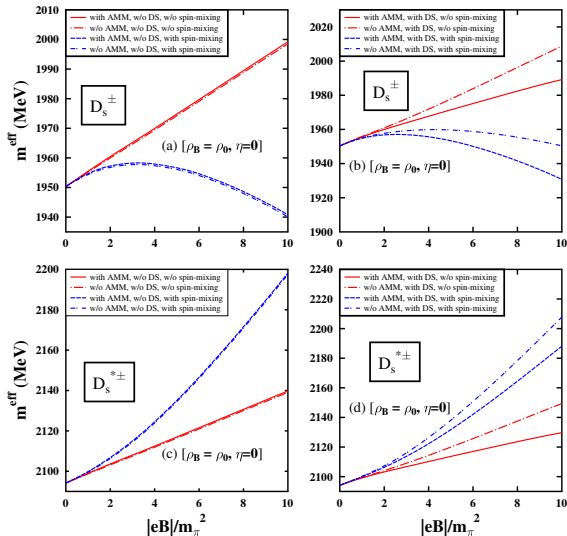
# $D^0$ and $D^{*0}$ masses at $\rho_0$



The masses (in MeV) of  $D^0$  and  $D^{*0}$  mesons are plotted at  $\rho_B = \rho_0$  in the symmetric matter, as functions of  $|eB|/m_\pi^2$ , including the Dirac sea (DS) effects (in (b) and (d)). It is compared to the case when the DS effect is absent (in (a) and (c)). The masses are plotted with and without PV mixing ( $D^0 - D^{*0}|$ ), considering the AMMs of the nucleons.

S. De, A. Mishra, arXiv: 2208.09820 [hep-ph];

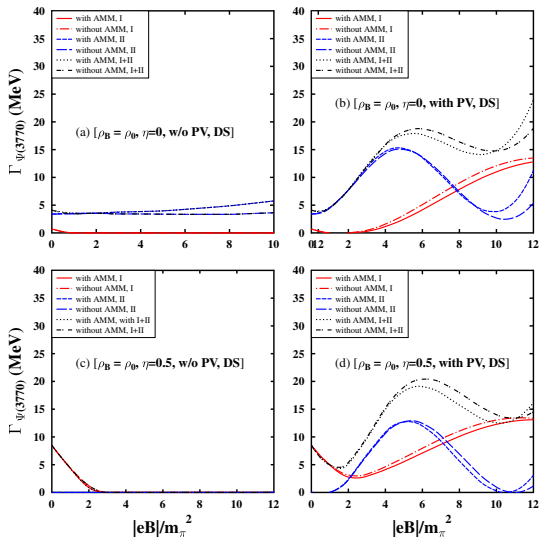
# $D_s^\pm$ and $D_s^{*\pm}$ meson masses $\rho_B = \rho_0$



In-medium masses of  $D_s^+$  [in (a) and (b)] and  $D_s^{*+}$  [in (c) and (d)] are plotted as functions of  $|eB|/m_\pi^2$ , at  $\rho_B = \rho_0$ , for  $\eta = 0$ , accounting for the Dirac sea (DS) effects. The spin mixing between  $D_s^+ - D_s^{*+}$  is considered, along with the LLL contribution for the charged  $D_s$  mesons.

S. De, A. Mishra, arXiv: 2208.09820 [hep-ph];

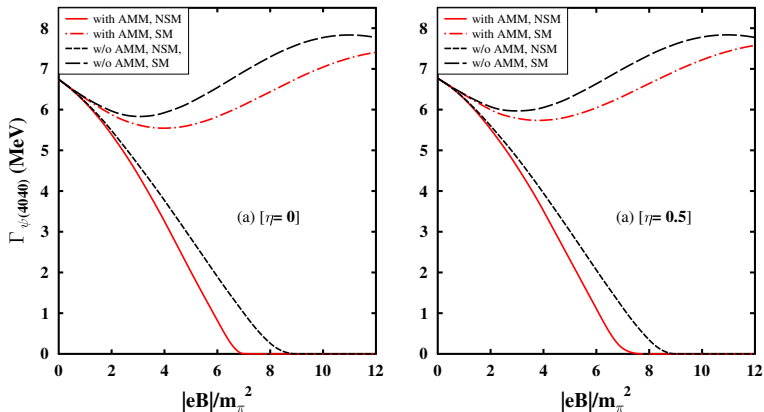
# Decay widths of $\Psi(3770) \rightarrow D\bar{D}$ at $\rho_0$



The decay widths (in MeV) of  $\Psi(3770) \rightarrow D^+ D^-$  (I) and  $\Psi(3770) \rightarrow D^0 \bar{D}^0$  (II) and the total (I+II), are plotted as functions of  $|eB|/m_\pi^2$ , at  $\rho_0, \eta = 0$ , incorporating the Dirac sea effects. Effects of Dirac sea in (a) and (c) are compared to the cases when the DS effect is not considered, in (b) and (d).

S. De, P. Parui, A. Mishra, arXiv: 2208.14953 [hep-ph].

# Decay widths of $\Psi(4040) \rightarrow D_s^+ D_s^-$ at $\rho_0$



The decay widths (in MeV) of  $\Psi(4040) \rightarrow D_s^+ D_s^-$  is plotted as functions of  $|eB|/m_\pi^2$ , at  $\rho_0$ ,  $\eta = 0, 0.5$ , incorporating the Dirac sea effects. The effects of spin-magnetic field interactions are compared.

# Conclusion

- Magnetized Dirac sea at nuclear matter saturation density results in (inverse) magnetic catalysis, leading to change in the masses and decay widths of open heavy flavor mesons with magnetic field.
- The pseudoscalar meson receives negative mass contribution due to PV mixing ( $D$  mesons), with positive contribution to the vector mesons ( $D^*$ ).
- The decay widths of the charmonia to  $D\bar{D}$  shows center-of-mass momentum dependence of the polynomial. The decay widths have dipped at certain values of the magnetic field.
- Major aspect of the medium modification of these heavy flavor mesons is **the effect they have on the production of heavy quarkonia and open heavy flavor mesons** produced at the early stages of the non-central heavy ion collision experiments e.g., at RHIC, LHC, where produced magnetic fields are estimated to be large.

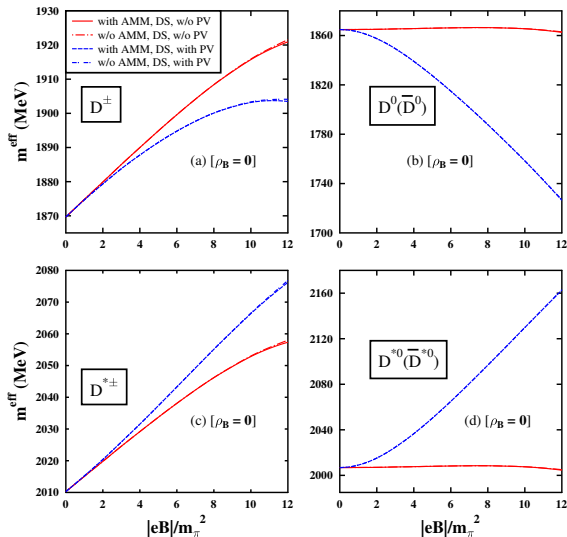


THANK  
YOU





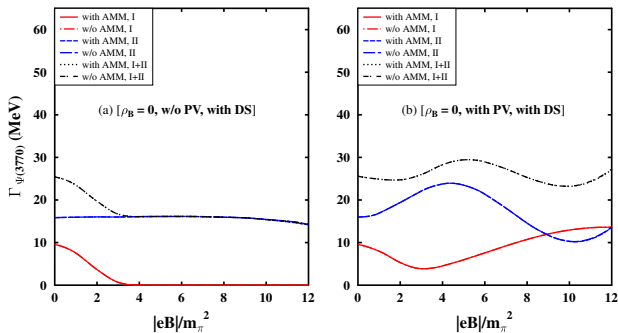
# Results : $D$ and $D^*$ meson masses $\rho_B = 0$



The masses (in MeV) of  $D^\pm$ ,  $D^0$  ( $\bar{D}^0$ ),  $D^{*\pm}$  and  $D^{*0}$  ( $\bar{D}^{*0}$ ) mesons plotted with  $|eB|/m_\pi^2$ , at  $\rho_B=0$ , including Dirac sea effects, PV mixing effects.

S. De, A. Mishra, arXiv: 2208.09820 [hep-ph].

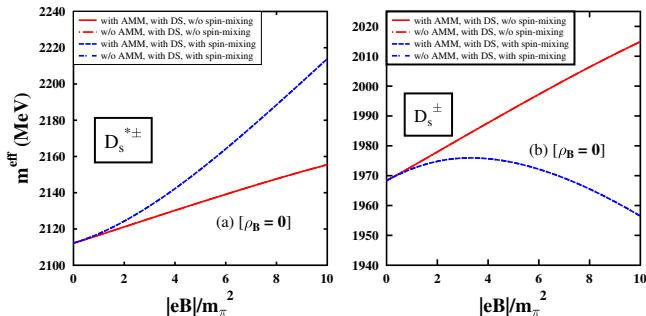
# In-medium decay widths of $\Psi(3770) \rightarrow D\bar{D}$ at $\rho_B = 0$



**Figure:** The decay widths (in MeV) of  $\Psi(3770) \rightarrow D^+ D^-$  (I) and  $\Psi(3770) \rightarrow D^0 \bar{D}^0$  (II) and the total (I+II), are plotted as functions of  $eB/m_\pi^2$ , at  $\rho_B = 0$ , incorporating the Dirac sea effects.

S. De, P. Parui, A. Mishra, arXiv: 2208.14953 [hep-ph].

# $D_s^+$ and $D_s^{*+}$ meson masses $\rho_B = 0$



**Figure:** In-medium masses of  $D_s^+$  [(b)] and  $D_s^{*+}$  [(a)] are plotted as functions of  $|eB|/m_\pi^2$ , at  $\rho_0, \eta = 0$ , accounting for the Dirac sea (DS) effects. The spin mixing between  $D_s^+ - D_s^{||*+}$  are considered, along with the LLL contribution for the charged  $D_s$  mesons.

# Nuclear matter in magnetic field

Transverse momenta of a particle charged  $q$ , in presence of uniform magnetic field (z-direction), is restricted to discrete Landau levels,  $k_{\perp}^2 = 2\nu|q|B$ ,  $\nu > 0$

$$\int_k \rightarrow \frac{|q|B}{(2\pi)^2} \sum_n \int_{-\infty}^{\infty} dk_z$$

For spin 1/2 particle:  $\nu = n + \frac{1}{2} - \frac{s}{2} \frac{q}{|q|}$   $s = \pm 1$  is the spin projection of the particle along  $B$

Total energy of the charged particle gets quantized

$$E = \sqrt{k_z^2 + ((m^2 + 2\nu|q|B)^{1/2} - s\kappa B)^2}$$

Momentum of the charged particle along the direction of the magnetic field,

$$k_{z,F} = \sqrt{E^2 - ((m^2 + 2\nu|q|B)^{1/2} - s\kappa B)^2}; \quad \nu_{max} = \left\lfloor \frac{(E + s\kappa B)^2 - m^2}{2|q|B} \right\rfloor$$

For uncharged particles,  $\int_k \rightarrow \int \frac{d^3k}{(2\pi)^3}$  and

$$m^{*2} = \left( \sqrt{m^2 + k_{\perp}^2} - s\kappa B \right)^2$$

# Contribution of matter part in external magnetic field

- $\rho_p = \frac{eB}{4\pi^2} \left[ \sum_{\nu=0}^{\nu_{max}} k_{f,\nu,1} + \sum_{\nu=1}^{\nu_{max}} k_{f,\nu,-1} \right]$
- $\rho_p^s = \frac{|eB|m_p^*}{2\pi^2} \left[ \sum_{\nu=0, s=1} \frac{\sqrt{m_p^{*2} + 2eB\nu + \Delta_p}}{\sqrt{m_p^{*2} + 2eB\nu}} \times \ln \left| \frac{k_{f,\nu,1}^p + E_f^p}{\sqrt{m_p^{*2} + 2eB\nu + \Delta_p}} \right| + \sum_{\nu=1, s=-1} \frac{\sqrt{m_p^{*2} + 2eB\nu - \Delta_p}}{\sqrt{m_p^{*2} + 2eB\nu}} \times \ln \left| \frac{k_{f,\nu,-1}^p + E_f^p}{\sqrt{m_p^{*2} + 2eB\nu - \Delta_p}} \right| \right]$
- $k_{f,\nu,S}^p = \sqrt{E_f^{(p)2} - (\sqrt{m_p^{*2} + 2eB\nu} + S\Delta_p)^2}$
- $\rho_n = \frac{1}{4\pi^2} \sum_{s=\pm 1} \left[ \frac{2}{3} k_{f,S}^{(n)3} + S\Delta_n [(m_n^* + S\Delta_n) k_{f,S}^n + E_f^{n2} (\arcsin(\frac{m_n^* + S\Delta_n}{E_f^{(n)}}) - \frac{\pi}{2})] \right]$
- $\rho_n^s = \frac{m_n^*}{4\pi^2} \sum_{s=\pm 1} \left[ k_{f,S}^{(n)} E_f^n - (m_n^* + S\Delta_n)^2 \ln \left| \frac{k_{f,S} + E_f^n}{m_n^* + S\Delta_n} \right| \right]$
- $k_{f,S}^n = \sqrt{E_f^{(n)2} - (m_n^* + S\Delta_n)^2}$

A.E. Broderick, M. Prakash and J. M. Lattimer, Phys. Lett. B **531**, 167 (2002) ; M. Strickland, V. Dexheimer, and D. P. Menezes, Phys. Rev. D **86**, 125032 (2012); Sushruth Reddy P, Amal Jahan CS, Nikhil Dhale, Amruta Mishra, J. Schaffner-Bielich, Phys. Rev. C **97**, 065208 (2018).

# Form of Harmonic Oscillator wave function

$$\Psi = N \times Y_l^m(\theta, \phi)(\beta^2, r^2)^{l/2} e^{-\frac{\beta^2 r^2}{2}} L_{N-1}^{l+1/2}(\beta^2 r^2)$$

- $L_p^k(z)$  is a Laguerre polynomial.
- $\beta^2 = M\omega/\hbar$
- $E_n = \hbar\omega(n + 1/2)$ ,  $n = 2k + l + 1 = 2(N - 1) + l + 1$

Wave function of  $\Psi(3770)$

$$\Psi(3770) = \frac{1}{\sqrt{3}} \frac{\beta^{3/2}}{\pi^{3/4}} \beta^2 r^2 (3\cos^2\theta - 1) \exp(-\beta^2 r^2/2)$$

Wave function of  $\Psi(4040)$

$$\Psi(4040) = \sqrt{\frac{8}{15}} \frac{\beta^{3/2}}{\pi^{3/4}} \left( \frac{15}{8} - \frac{5}{2} \beta^2 r^2 + \frac{\beta^4 r^4}{2} \right) \exp(-\beta^2 r^2/2)$$

# PV mixing

$$m_{P,V||}^2 (PV) = \frac{1}{2} \left( M_+^2 + \frac{C_{PV}^2}{m_{av}^2} \mp \sqrt{M_-^4 + \frac{2C_{PV}^2 M_+^2}{m_{av}^2} + \frac{C_{PV}^4}{m_{av}^4}} \right)$$

# Back up slide

- $h[\pi_a(x)] \in SU(3)_V$
- $u(\pi_a) = \exp(-\frac{i}{\sqrt{2}} \frac{\pi \gamma_5}{\sigma_0}); \pi = \frac{1}{\sqrt{2}} \sum_{a=1}^8 \lambda_a \pi_a$
- $X' = hXh^\dagger, V'_\mu = hV_\mu h^\dagger, A'_\mu = hA_\mu h^\dagger, B' = hBh^\dagger$

$$\mathcal{L} = \mathcal{L}_{kin} + \sum_{W=X, V_\mu, A_\mu, u} \mathcal{L}_{BW} + \mathcal{L}_{vec} + \mathcal{L}_0 + \mathcal{L}_{scale-break} + \mathcal{L}_{SB} + \mathcal{L}_{mag}$$

Kinetic energy term of baryons & mesons multiplets:

■  $\mathcal{L}_{kin} = iTr \bar{B} \gamma_\mu D^\mu B + Tr(u_\mu X u^\mu X + X u_\mu u^\mu X) + (\text{K.E. of other mesons})$

- $D_\mu B = \partial_\mu B + i[\Gamma_\mu, B]$
- $\Gamma_\mu = -\frac{i}{2}[u^\dagger \partial_\mu u + u \partial_\mu u^\dagger]$
- $u_\mu = -\frac{i}{2}[u^\dagger \partial_\mu u - u \partial_\mu u^\dagger]$

■ For baryons :  $iTr(\bar{B} \gamma_\mu D^\mu B) \rightarrow$  vectorial Weinberg-Tomozawa term

$$\mathcal{L}_{WT} = -\frac{1}{2} \sum_{i,j,k,l} \bar{B}_{i,j,k} \gamma^\mu \left( (\Gamma_\mu)_l^k B^{ijl} + 2(\Gamma_\mu)_l^j B^{ilk} \right); \quad B^{121} = p, B^{122} = n$$



# Linear $SU(2) \times SU(2)$ $\sigma$ model

$$\mathcal{L}_\sigma = \bar{N}[i\gamma_\mu \partial^\mu - g(\sigma + i\vec{\tau} \cdot \vec{\pi} \gamma_5)]N + \frac{1}{2}(\partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \partial_\mu \sigma \partial^\mu \sigma) - \frac{\lambda}{4}((\pi^2 + \sigma^2) - f_\pi^2)^2$$

- Vector and axial-vector transformations :

$$\checkmark SU(2)_V \equiv e^{-i\frac{\vec{\tau}}{2} \cdot \vec{\theta}} : \sigma \rightarrow \sigma, \quad \pi_i \rightarrow \pi_i + \epsilon_{ijk} \theta_j \pi_k$$

$$\checkmark SU(2)_A \equiv e^{-i\gamma_5 \frac{\vec{\tau}}{2} \cdot \vec{\theta}} : \sigma \rightarrow \sigma - \theta_i \pi_i; \quad \pi_i \rightarrow \pi_i + \theta_i \sigma$$

$$(\sigma^2 + \vec{\pi}^2) \xrightarrow{SU(2)_V \times SU(2)_A} (\sigma^2 + \vec{\pi}^2)$$

- Nucleon mass  $\rightarrow g\langle\sigma\rangle = g\sigma_0$
- In vacuum limit,  $\langle\vec{\pi}\rangle = 0$ , due to negative parity state
- $-m\bar{q}q \equiv \epsilon\sigma$  leads to non-zero mass for pions  $\rightarrow m_\pi^2 = \frac{\partial^2 V}{\partial \pi^2} \frac{\epsilon}{f_\pi}$
- $\langle\epsilon\sigma\rangle = -\langle m\bar{q}q\rangle \rightarrow m_\pi^2 f_\pi^2 = -\frac{(m_u + m_d)}{2}(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle)$  — GOR relation

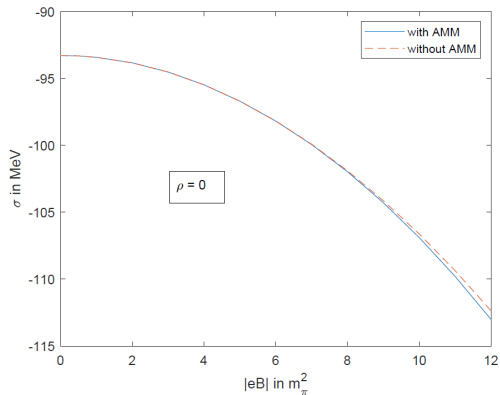
# Kaon nucleon scattering length

$$a_{KN}^{(0)} = \frac{m_K}{4\pi f_K^2(1 + m_K/m_N)} \times \left[ -\frac{m_K f_K}{2} \left( \frac{g_{\sigma N}}{m_\sigma^2} + \sqrt{2} \frac{g_{\zeta N}}{m_\zeta^2} - 3 \frac{g_{\delta N}}{m_\delta^2} \right) + \frac{(d_1 - d_2)m_K}{2} \right] \quad (1)$$

$$a_{KN}^{(1)} = \frac{m_K}{4\pi f_K^2(1 + m_K/m_N)} \times \left[ -1 - \frac{m_K f_K}{2} \left( \frac{g_{\sigma N}}{m_\sigma^2} + \sqrt{2} \frac{g_{\zeta N}}{m_\zeta^2} + \frac{g_{\delta N}}{m_\delta^2} \right) + \frac{(d_1 + d_2)m_K}{2} \right] \quad (2)$$

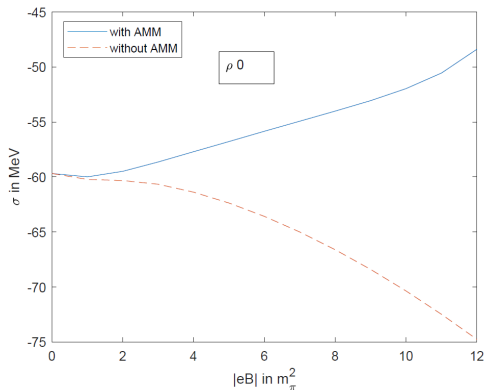
$$\bar{a}_{KN} = \frac{1}{4} a_{KN}^{(0)} + \frac{3}{4} a_{KN}^{(1)} \quad (3)$$

# Solution of scalar-isoscalar field $\sigma$ with $|eB|$ at $\rho_B = 0$



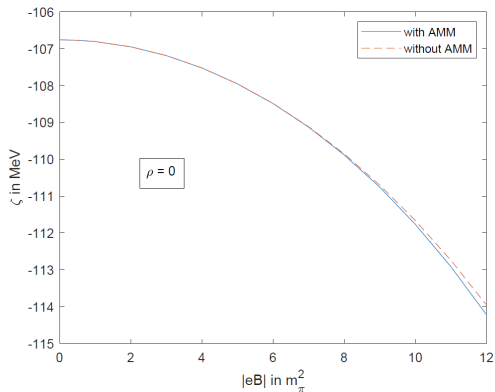
**Figure:** Scalar isoscalar field (non-strange)  $\sigma$  in MeV is plotted as a function of magnetic field  $|eB|$  in units of  $m_\pi^2$  at vacuum  $\rho_B = 0$ .

# Solution of scalar-isoscalar field $\sigma$ with $|eB|$ at $\rho_0$



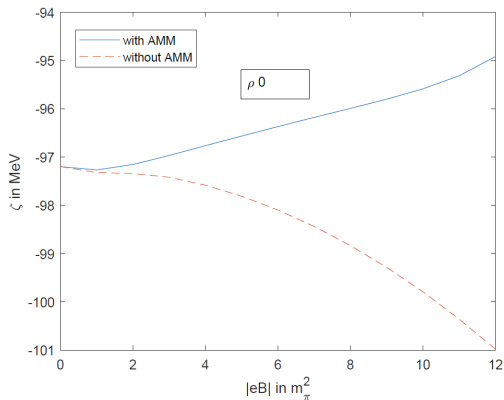
**Figure:** Scalar isoscalar field (non-strange)  $\sigma$  in MeV is plotted as a function of magnetic field  $|eB|$  in units of  $m_\pi^2$  at  $\rho_0$  and  $\eta = 0$ .

# Solution of scalar-isoscalar field $\zeta$ with $|eB|$ at $\rho_B = 0$



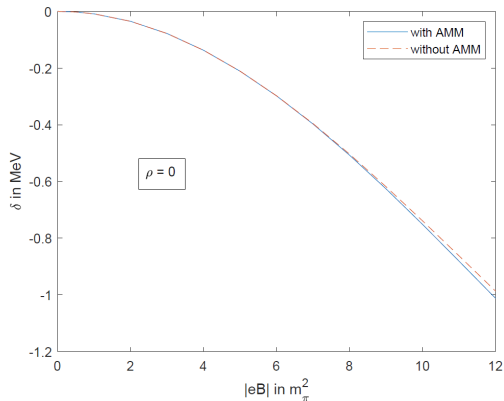
**Figure:** Scalar isoscalar field (strange)  $\zeta$  in MeV is plotted as a function of magnetic field  $|eB|$  in units of  $m_\pi^2$  at  $\rho_B = 0$ .

# Solution of scalar-isoscalar field $\zeta$ with $|eB|$ at $\rho_0$



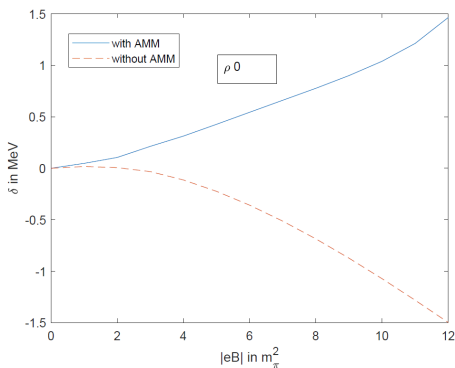
**Figure:** Scalar isoscalar field (strange)  $\zeta$  in MeV is plotted as a function of magnetic field  $|eB|$  in units of  $m_\pi^2$  at  $\rho_0$  and  $\eta = 0$ .

# Solution of scalar-isovector field $\delta$ with $|eB|$ at $\rho_B = 0$



**Figure:** Scalar isovector field  $\delta$  in MeV is plotted as a function of magnetic field  $|eB|$  in units of  $m_\pi^2$  at  $\rho_B = 0$ .

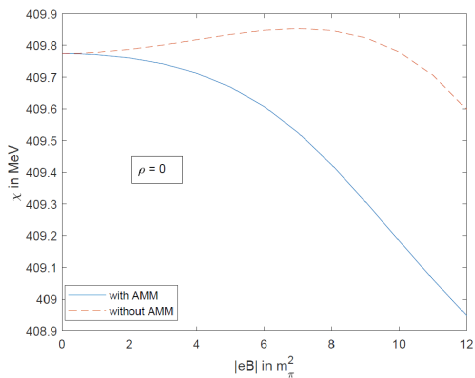
# Solution of scalar-isovector field $\delta$ with $|eB|$ at $\rho_0$



**Figure:** Scalar isovector field  $\delta$  in MeV is plotted as a function of magnetic field  $|eB|$  in units of  $m_\pi^2$  at  $\rho_0$  and  $\eta = 0$ .

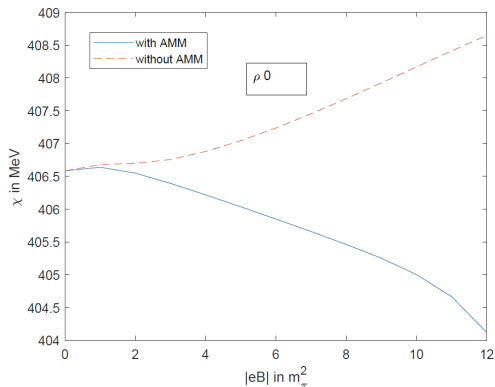


# Solution of scalar dilaton field $\chi$ with $|eB|$ at $\rho_B = 0$



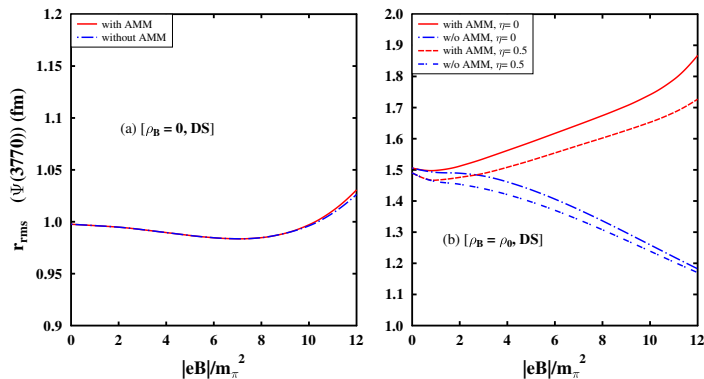
**Figure:** Scalar dilaton field  $\chi$  in MeV is plotted as a function of magnetic field  $|eB|$  in units of  $m_\pi^2$  at  $\rho_B = 0$ .

# Solution of scalar dilaton field $\chi$ with $|eB|$ at $\rho_0$



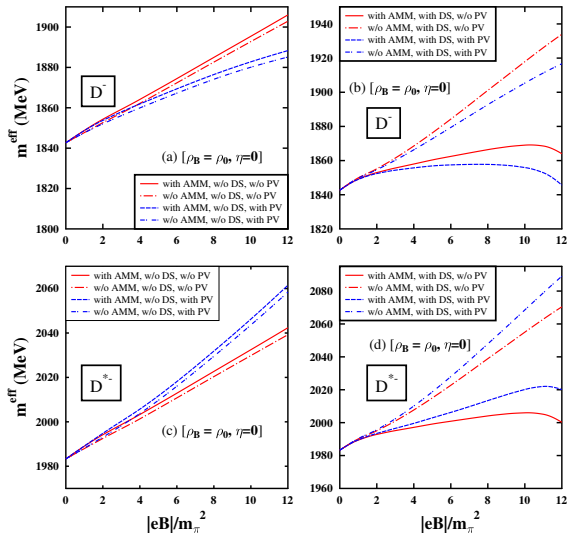
**Figure:** Scalar dilaton field  $\chi$  in MeV is plotted as a function of magnetic field  $|eB|$  in units of  $m_\pi^2$  at  $\rho_0$  and  $\eta_0$ .

# Variation of rms size of charmonium state $\psi(1D)$



**Figure:** The rms radius (in fm) of  $\Psi(3770)$  state as a function of  $|eB|/m_\pi^2$ , at  $\rho_B = 0$ ,  $\rho_0$  and  $\eta = 0, 0.5$ , due to the medium modifications of the masses of the state.

# $D^-$ and $D^{*-}$ meson masses at $\rho_B = \rho_0$

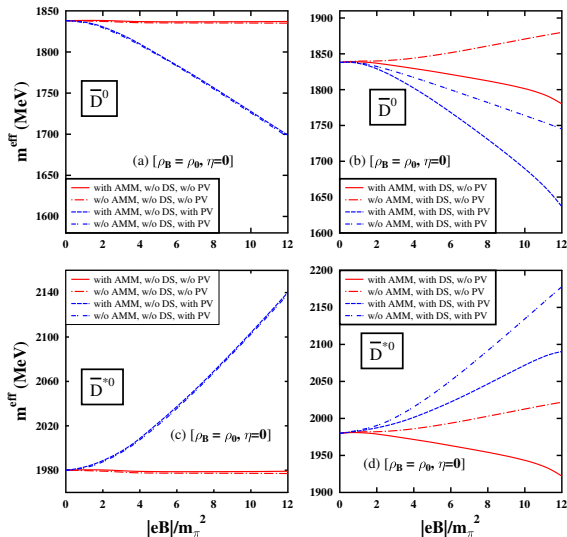


The masses (in MeV) of  $D^-$  and  $D^{*-}$  mesons are plotted at  $\rho_B = \rho_0$  ( $\eta=0$ ), as functions of  $|eB|/m_\pi^2$ , including the Dirac sea effects (in (b) and (d)). The masses are compared to the case when the Dirac sea effects are not considered (in (a) and (c)). The masses are plotted with and without PV ( $D^- - D^{*-}$  mixing effect, accounting for the AMMs of the nucleons and the LLL contribution).

S. De, A. Mishra, arXiv: 2208.09820

[hep-ph];

# $\bar{D}^0$ and $\bar{D}^{*0}$ masses at $\rho_0$



Mass (in MeV) of  $\bar{D}^0$  and  $\bar{D}^{*0}$  mesons are plotted at  $\rho_B = \rho_0$  in the symmetric matter ( $\eta=0$ ) as functions of  $eB/m_\pi^2$ , including the Dirac sea (DS) effects (in (b) and (d)). It is compared to the case when the DS effect is absent (in (a) and (c)). The masses are plotted with and without PV mixing ( $\bar{D}^0 - \bar{D}^{*0}$ ).

S. De, A. Mishra, arXiv: 2208.09820 [hep-ph];

# Chiral Symmetry and Lagrangian of QCD

$$\mathcal{L}_{QCD} = \bar{\psi}(i\gamma_{\mu}D^{\mu} - m)\psi - \frac{1}{4}Tr[G^{\mu\nu}G_{\mu\nu}]$$

- $\psi_{i1} = u, d, s$  for  $i = 1, 2, 3$ ,  $m_{ij} = \delta_{ij}m_i$  for  $i, j = 1, 2, 3$
- $D_{\mu} = \partial_{\mu} - igA_{\mu}$ ;  $A_{\mu} = \sum_{a=1}^8 A_{\mu}^a \frac{\lambda^a}{2}$
- $G_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} - ig[A_{\mu}, A_{\nu}]$ ;
- $\psi_{R,L} = P_{R,L}\psi$ ;  $P_{R,L} = \frac{1 \pm \gamma_5}{2}$
- $m \rightarrow 0$ ,  $\psi_R \rightarrow R\psi_R = \exp(i \sum_{a=1}^8 \theta_{R,a} \frac{\lambda_a}{2})\psi_R$ ,  $\psi_L \rightarrow L\psi_L = \exp(i \sum_{a=1}^8 \theta_{L,a} \frac{\lambda_a}{2})\psi_L$
- $J_{R,a}^{\mu} = \bar{\psi}_R \gamma^{\mu} \frac{\lambda_a}{2} \psi_R$ ,  $J_{L,a}^{\mu} = \bar{\psi}_L \gamma^{\mu} \frac{\lambda_a}{2} \psi_L \Rightarrow \partial_{\mu} J_{R,L}^{\mu,a} = 0$ ;  $a = 1, \dots, 8$
- $V_a^{\mu} = J_{R,a}^{\mu} + J_{L,a}^{\mu} = \bar{\psi} \gamma^{\mu} \frac{\lambda_a}{2} \psi$ ,  $A_a^{\mu} = J_{R,a}^{\mu} - J_{L,a}^{\mu} = \bar{\psi} \gamma^{\mu} \gamma_5 \frac{\lambda_a}{2} \psi$
- $G = SU(3)_L \times SU(3)_R \equiv SU(3)_V \times SU(3)_A \rightarrow$  *Spontaneously broken to*  $SU(3)_V$
- $Q_a^V |0\rangle = 0$ ,  $Q_a^A |0\rangle \neq 0 \rightarrow$  Goldstone boson fields

# Chiral Symmetry and Symmetry Breaking Effects

- $SU(3)_A$  symmetry breaks  $\rightarrow$  Eight massless modes (Goldstone's Theorem)
- Quark condensates occupy the vacuum  $\langle \bar{\psi}\psi \rangle = \langle \bar{u}u \rangle + \langle \bar{d}d \rangle + \langle \bar{s}s \rangle$
- Chiral  $SU(3)_L \times SU(3)_R$  is explicitly broken by non-zero light quark masses
- $\partial_\mu A^\mu = i\bar{\psi}\{m, \frac{\lambda_a}{2}\}\gamma_5\psi$ , Partially conserved Axial current (**PCAC**)
- GOR (Gell-Mann, Oakes, Renner) relation:  $m_\pi^2 f_\pi^2 \approx -\frac{m_u+m_d}{2} \langle \bar{u}u + \bar{d}d \rangle$
- Chiral symmetry broken : (1) spontaneously  $\rightarrow \langle 0|\bar{\psi}\psi|0\rangle \neq 0$   
(2) explicitly  $\rightarrow m_u, m_d, m_s \neq 0$
- Scale-invariance breaking effect  $\rightarrow$  trace anomaly

$$\theta_\mu^\mu = \langle \frac{\beta_{QCD}}{2g} G_{\mu\nu}^a G^{\mu\nu,a} \rangle + \sum_i m_i \langle \bar{q}_i q_i \rangle$$

$$\beta_{QCD} \approx -\frac{11N_c g^3}{48\pi^2} \left(1 - \frac{2N_f}{11N_c}\right) \text{ with } N_c = 3, \quad N_f = 3 \text{ (at the leading order)}$$