Model selection in kaon photoproduction

Dalibor Skoupil

(in collaboration with P. Bydžovský, A. Cieplý, D. Petrellis and N. Zachariou)

Phys. Rev. C **93**, 025204 (2016)
Phys. Rev. C **97**, 025202 (2018)
Phys. Rev. C **104**, 065202 (2021)
Phys. Rev. C **107**, 045206 (2023)

Department of Theoretical Physics
Nuclear Physics Institute of the Czech Academy of Sciences, Řež, Czech Republic

HADRON 2023, Genova, Italy
June 8, 2023

Motivation for the work on kaon photo/electroproduction

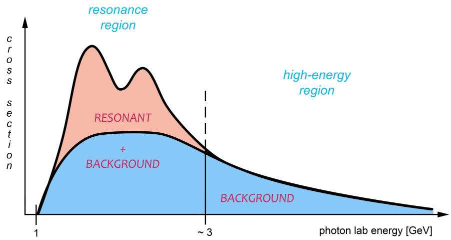
- We aim at understanding the baryon spectrum and production dynamics of particles with strangeness at low energies.
- Constituent Quark Model predicts a lot more N^* states than was observed in pion production experiments $\rightarrow$ “missing” resonance problem.
- Models for the description of elementary hyperon electroproduction are a suitable tool for hypernuclear physics calculations.
- New good-quality photoproduction data from LEPS, GRAAL, MAMI and (particularly) CLAS collaborations allow us to tune free parameters of the models.
- As the α_s increases with decreasing energy, we cannot use perturbative QCD at low energies $\rightarrow$ need for introducing effective theories and models.

Introduction

Photoproduction process

$$p(p) + \gamma(k) \rightarrow K^+(p_K) + \Lambda(p_\Lambda)$$

- Threshold: $E_\gamma^{lab} = 0.911\,\text{GeV}$, $W = 1.609\,\text{GeV}$
- In the lowest order, the reaction is described by the exchange of hadrons.
  - *The 3rd nucleon-resonance region:*
    many resonant states and no dominant one in the $K^+\Lambda$ production
    $\rightarrow$ need to assume a large number of nucleon resonances with mass $< 2.5\,\text{GeV}$



- Resonance region:
  resonance contributions dominate (N^*)
- Background:
  a plenty of nonresonant contributions
  (p, K, Λ; K^* and Y^*)

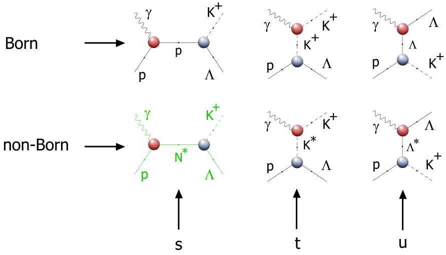
Isobar model

Single-channel approximation

- higher-order contributions (rescattering, FSI) included, to some extent, by means of effective values of the coupling constants

Use of effective hadron Lagrangian

- hadrons either in their ground or excited states
- amplitude constructed as a sum of tree-level Feynman diagrams
  - background and resonant part



- hadronic form factors account for the inner structure of hadrons

Free parameters (couplings, hff's cutoffs) adjusted to experimental data.

Satisfactory agreement with the data in the energy range $W = 1.6 - 2.5\,\text{GeV}$.

Isobar model

Novel features of our isobar model

Exchanges of high-spin resonant states

- non physical lower-spin components removed by appropriate choice of $\mathcal{L}_{int}$

$$V_S^{\mu}\,\mathcal{P}^{(1/2)}_{ij,\mu\nu}\,V_{EM}^{\nu}=0$$

Energy-dependent decay widths of nucleon resonances → restoration of unitarity

$$\Gamma(\vec{q})=\Gamma_{N^*}\,\frac{\sqrt{s}}{m_{N^*}}\sum_i x_i\left(\frac{|\vec{q}_i|}{|\vec{q}_i^{N^*}|}\right)^{2l+1}\frac{D(|\vec{q}_i|)}{D(|\vec{q}_i^{N^*}|)},$$

Extension from photoproduction to electroproduction

- Phenomenological form factors in the electromagnetic vertex
- Longitudinal couplings of N^*'s to γ^* (crucial at small Q^2)

$$V^{EM}(N^*_{1/2}p\gamma)=-i\frac{g_3^{EM}}{(m_R+m_p)^2}\Gamma_{\mp}\,\gamma_\beta\,\mathcal{F}^{\beta},$$

$$V^{EM}_{\mu}(N^*_{3/2}p\gamma)=-i\frac{g_3^{EM}}{m_R(m_R+m_p)^2}\gamma_5\Gamma_{\mp}\,(\not{q}\,g_{\mu\beta}-q_\beta\gamma_\mu)\,\mathcal{F}^{\beta},$$

$$V^{EM}_{\mu\nu}(N^*_{5/2}p\gamma)=-i\frac{g_3^{EM}}{(2m_p)^5}\Gamma_{\mp}(q_\alpha q_\beta g_{\mu\nu}+q^2 g_{\alpha\mu}g_{\beta\nu}-q_\alpha q_\nu g_{\beta\mu}-q_\beta q_\nu g_{\alpha\mu})\,p^{\alpha}\mathcal{F}^{\beta}.$$

New fits for $K^+\Sigma^-$ channel

χ^2 minimization and overfitting

Fitting procedure with MINUIT library: **minimizing the χ^2**

$$\chi^2 = \sum_{i=1}^{N} \frac{[d_i - p_i(c_1, \ldots, c_n)]^2}{\sigma_{d_i}^2},$$

$(c_1, \ldots, c_n)$ - set of free parameters, $(d_1, \ldots, d_N)$ - set of data points, p_i - theory, σ_{d_i} - error

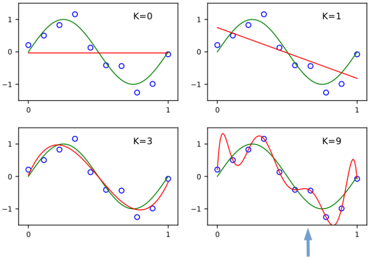
Problem: χ^2 minimization cannot prevent overfitting

Example: polynomial curve fitting

- $f(x, \mathbf{w}) = w_0 + w_1 x + w_2 x^2 + \cdots + w_k x^k$
- increasing order of polynomial k fits the data well...

  ...but gives only poor description of the function which generated them...

  ...and may fail to generalize to new data
- **Occam's razor** (law of parsimony): simpler models should be preferred



Model fits the noise in the sample

New fits of $K^+\Sigma^-$ channel

Least Absolute Shrinkage and Selection Operator (LASSO)

Remedy to the overfitting issue: regularization

- introduce a penalty term to the χ^2 $\rightarrow$ penalization of large parameter values
- penalized χ^2_T: $\chi^2_T = \chi^2 + P(\lambda)$
- penalty term: $P(\lambda) = \lambda^4 \sum_{i=1}^{N_{res}} |g_i|$

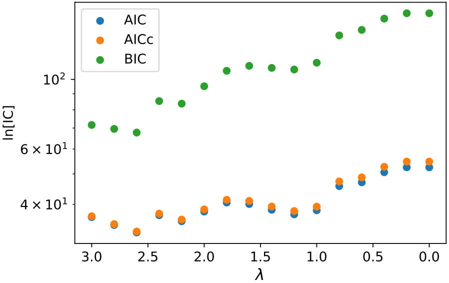
  λ - regularization parameter, g_i - resonances' couplings
- LASSO forces some of the parameters to zero
  $\rightarrow$ selection of a subset of the fit parameters
- λ controls the strength of the penalty and thus the complexity of the model
  $\rightarrow$ higher powers of λ allow fine sampling of the region of small λ

New fits of $K^+\Sigma^-$ channel

Information criteria:

- Akaike information criterion
  $\text{AIC} = 2n_i + \chi^2_T$
- Corrected Akaike information criterion
  $\text{AICc} = \text{AIC} + \frac{2n_i(n_i+1)}{N-n_i-1}$
- Bayesian information criterion
  $\text{BIC} = n_i \ln(N) + \chi^2_T$

n_i - no. of parameters corresponding to λ_i
N - number of data points



Applying the information criteria – forward selection

1. start with the full model: parameters initialized within $\langle -1; +1\rangle$; use $\lambda_{\max}$
2. perform LASSO χ^2_T minimization and compute IC
3. in each run reduce λ and run LASSO with the values of the previous run as starting values
4. repeat until $\lambda_{\min}$ is reached

Optimal λ occurs at the minimum of the IC

New fits of $K^+\Sigma^-$ channel

Fitting procedure

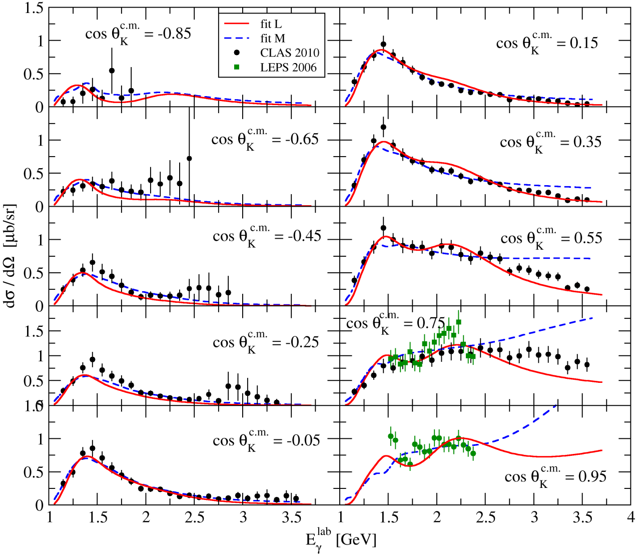
- non resonant part: Born terms and exchanges of K^* and K_1 and Σ^*'s
- resonant part: exchanges of N^*'s and Δ^*'s in the s channel
- around 600 data utilized to fit $\leq$ 25 parameters
- result with the smallest $\chi^2/\mathrm{ndf} = 2.3 \rightarrow$ **fit M** (25 parameters, 14 resonances)
- **LASSO method** used: $\chi^2_T/\mathrm{ndf} = 3.4 \rightarrow$ **fit L** (17 parameters, 9 resonances)

Characteristics of models

- only one Δ resonance introduced
- no hyperon resonances needed for reliable data description
- results in very good agreement with the cross-section and beam-asymmetry data
- fit L is very economical

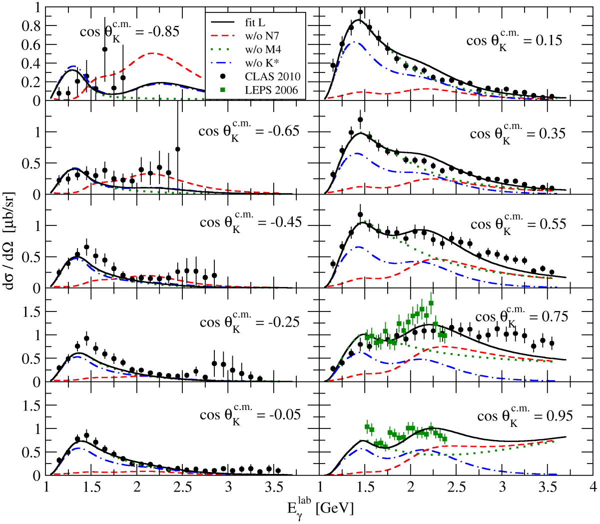
New fits of $K^+\Sigma^-$ channel

Differential cross section in dependence on the photon lab energy



New fits of $K^+\Sigma^-$ channel

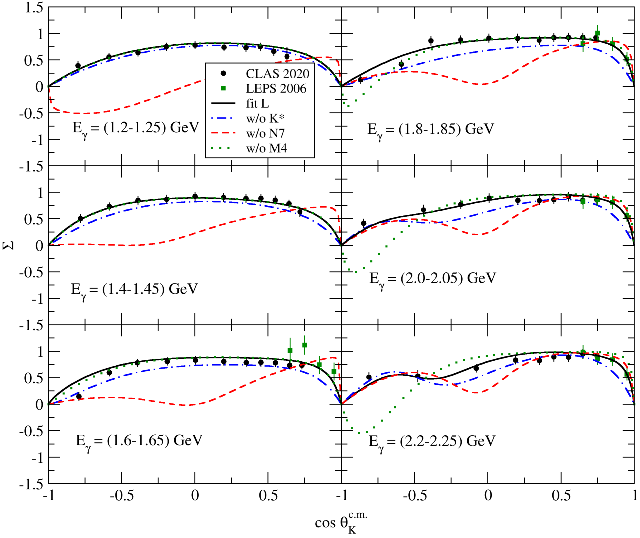
Differential cross section in dependence on the photon lab energy - fit L w/o individual resonances



N7: N(1720)$3/2^+$, M4: N(2060)$5/2^-$

New fits of $K^+\Sigma^-$ channel

Beam asymmetry in dependence on the kaon center-of-mass angle - fit L w/o individual resonances



Refitting the model's parameters in the $K^+\Lambda$ channel

Ridge regression and cross validation for suppressing hyperon couplings

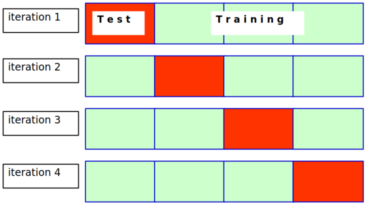
Why refit?

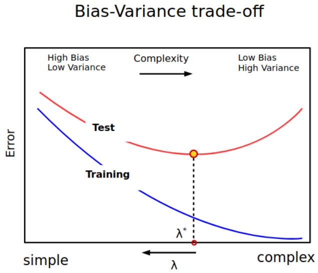
- include recent measurements of polarization observables
- need to investigate more the role of hyperon resonances in *KY* photoproduction
- large values of hyperon couplings: ridge regression to suppress them during the fitting procedure

Ridge regularization

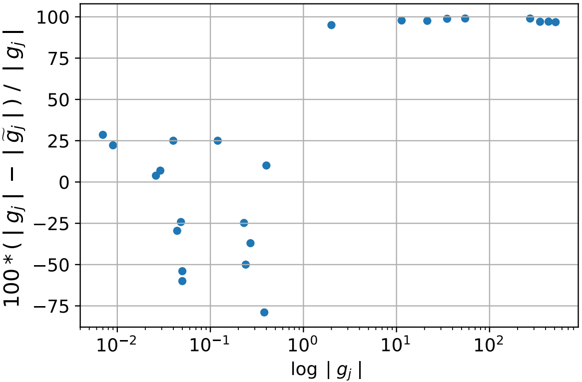
- penalized χ^2_T: $\chi^2_T = \chi^2 + \lambda^4 \sum_{i=1}^{n_\Lambda} g_i^2$, ($n_\Lambda$ = no. of Y couplings)
- parameter values reduced but they are *not* reduced to zero

Cross validation





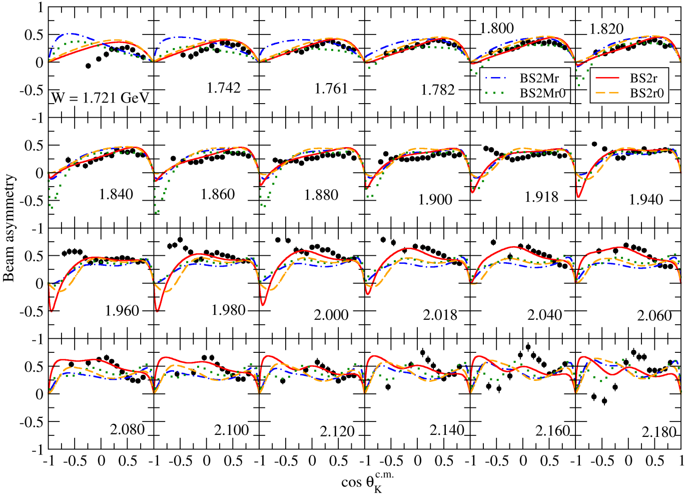
Relative percentage reduction of the resonance couplings



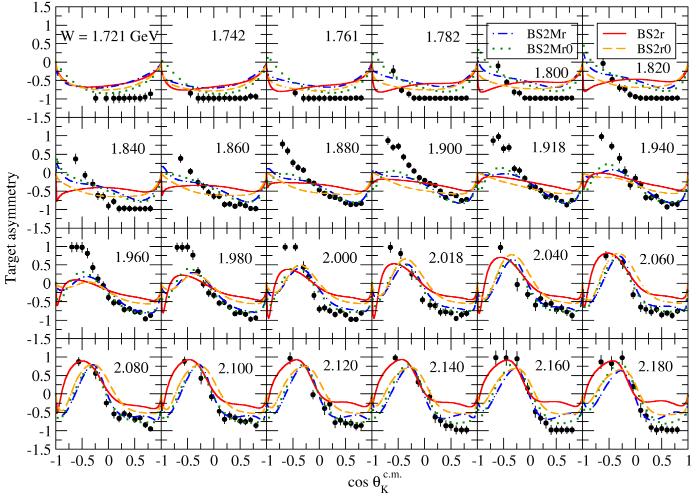
g_j - values from the unregularized fitting

$\tilde{g}_j$ - values after performing Ridge regularization

$K^+\Lambda$ channel: beam asymmetry Σ



$K^+\Lambda$ channel: target asymmetry T



Summary

New version of isobar modelfor the $K^+\Lambda$ channel

- available for calculations online at:

  http://www.ujf.cas.cz/en/departments/department-of-theoretical-physics/isobar-model.html

Description extended from the $K^+\Lambda$ channel to the $K^+\Sigma^-$ channel.

Regularization methods (LASSO, ridge) introduced as a remedy for overfitting.

Outlook

- testing the models in the DWIA calculations for hypernucleus production
- performing a multi-channel analysis of all Σ photoproduction channels
- extending the analysis of electroproduction beyond $Q^2 = 1\,\mathrm{GeV}^2$
- studying the production of Ξ hypernuclei

Thank you for your attention!