Progress in the Partial-Wave Analysis Methods at COMPASS

Julien Beckers and Florian Kaspar for the COMPASS Collaboration

HADRON 2023: Analysis tools

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julien.beckers@tum.de florian.kaspar@tum.de

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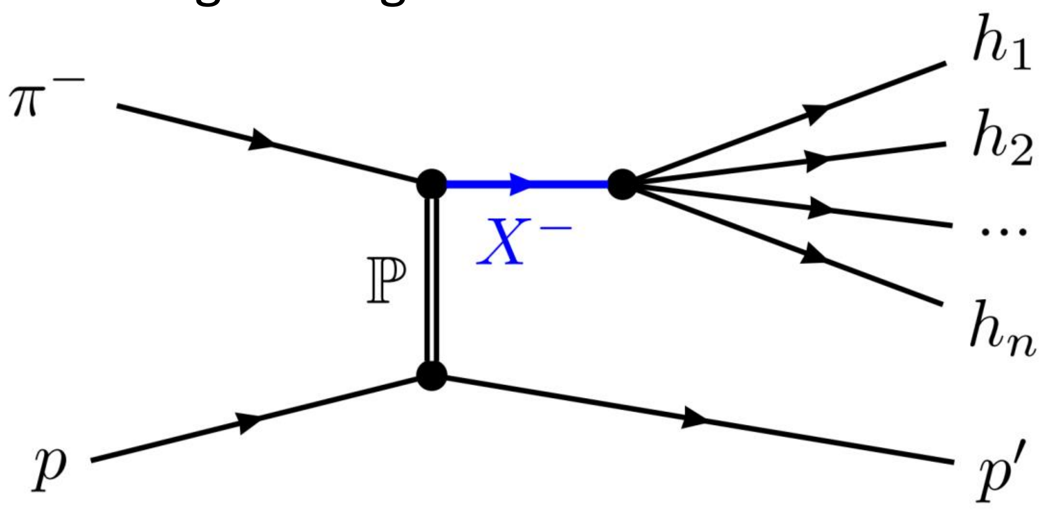
Excited Light Mesons at COMPASS

- Inelastic scattering reactions of high-energetic meson beam



- Strong interaction (Pomeron exchange) between beam meson and target proton
- Intermediate hadronic resonances X^- are created, then decay into n-body final state

  → wide range of allowed (spin) quantum numbers

- Final-state particles measured

The COMPASS Experiment

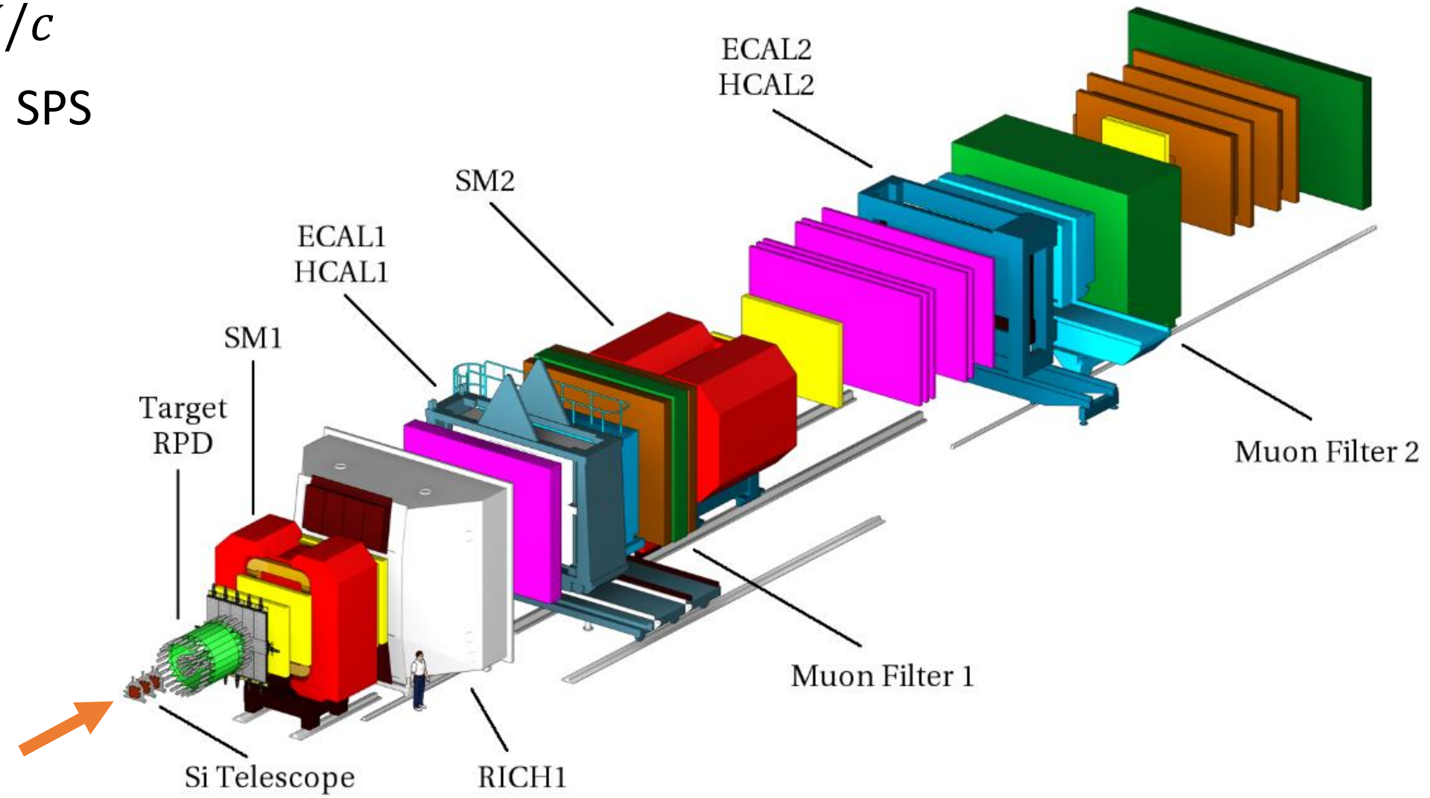
Large-acceptance magnetic spectrometer @ CERN-SPS

Beam:

- Secondary hadrons (π^-, K^-) at $190\ \mathrm{GeV}/c$
- produced via primary proton beam from SPS

Spectrometer:

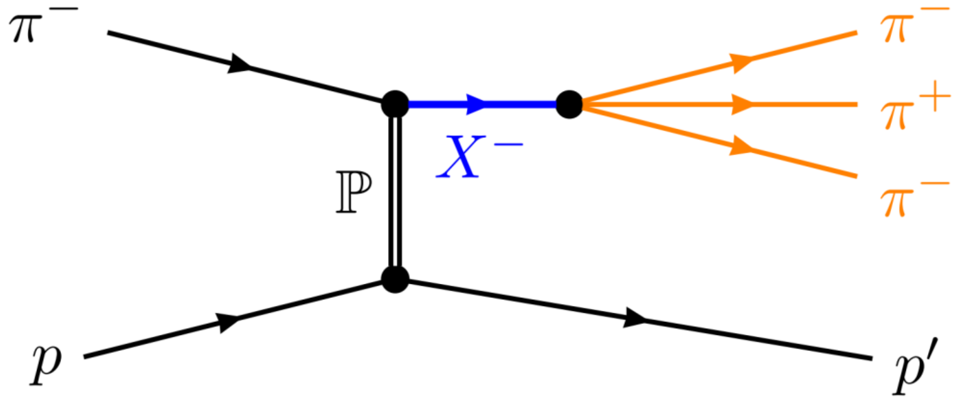
- Liquid-hydrogen target
- Two-stage spectrometer setup around two dipole magnets SM1/2



From COMPASS Collab., The COMPASS Setup for Physics with Hadron Beams (Nucl. Instrum. Methods Phys. Res. A 779 (2014), pp. 69–115)

Excited Light Mesons at COMPASS

$\boldsymbol{\pi^- + p \rightarrow \pi^-\pi^-\pi^+ + p}$



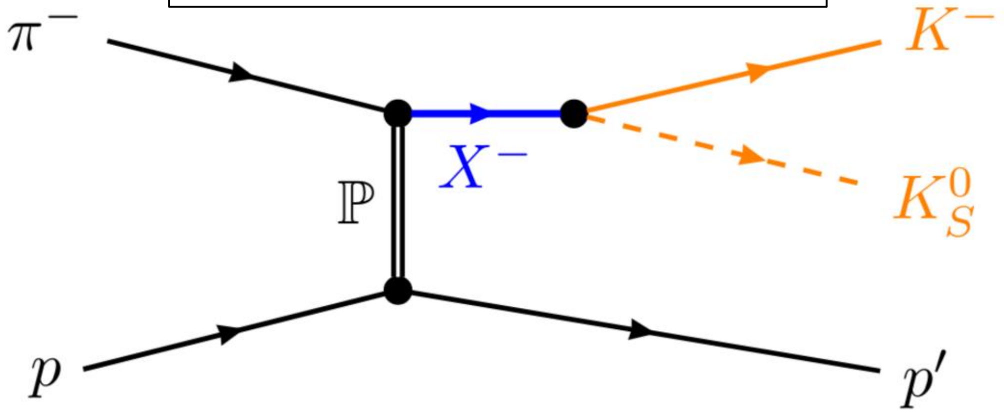
- Allowed quantum numbers:

$$J^{PC} = 0^{-+}, 1^{-+}, 1^{++}, \ldots$$

$\rightarrow$ $\boldsymbol{\pi_J}$ and $\boldsymbol{a_J}$ resonances

- COMPASS flagship channel: 115×10^6 evts

$\boldsymbol{\pi^- + p \rightarrow K_S^0 K^- + p}$



- Allowed quantum numbers:

$J^{PC} = 1^{--}, 2^{++}, 3^{--}, \ldots$ from decay

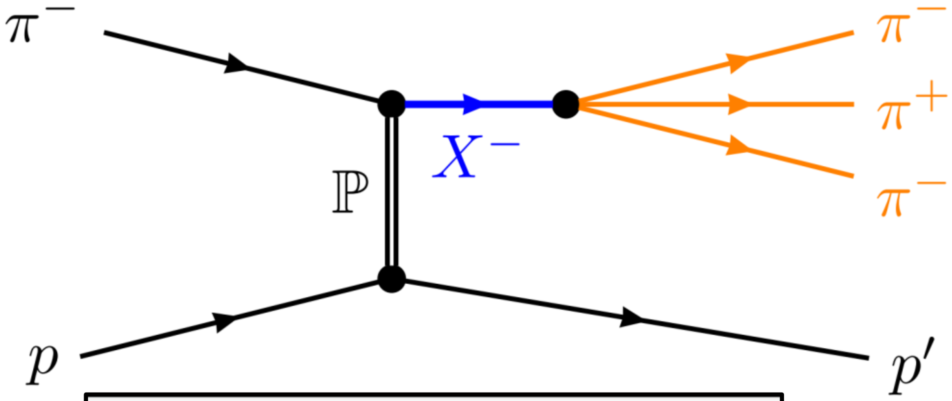
Dominated by Pomeron exchange

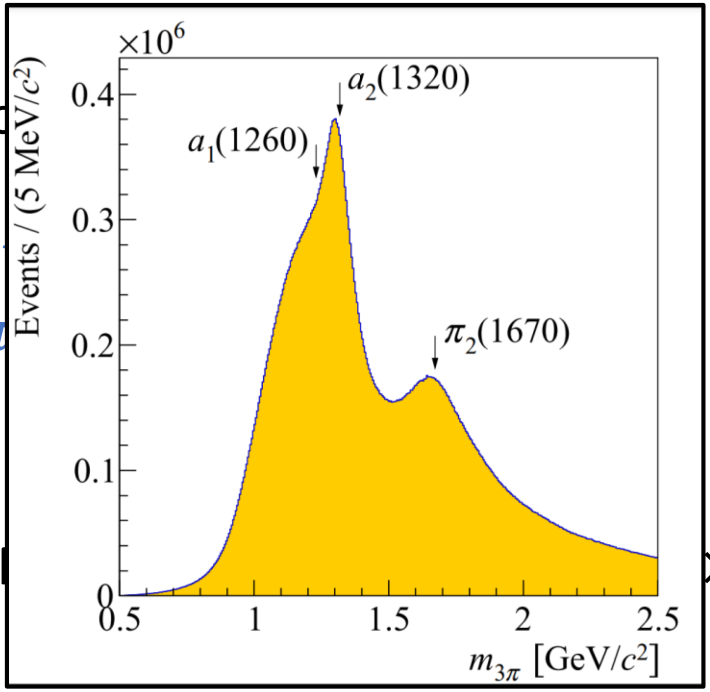
$\rightarrow$ $\boldsymbol{a_J}$ for even J

- Highly selective $\rightarrow$ search for a_4', a_6

Excited Light Mesons at COMPASS

$\pi^- + p \to \pi^-\pi^-\pi^+ + p$



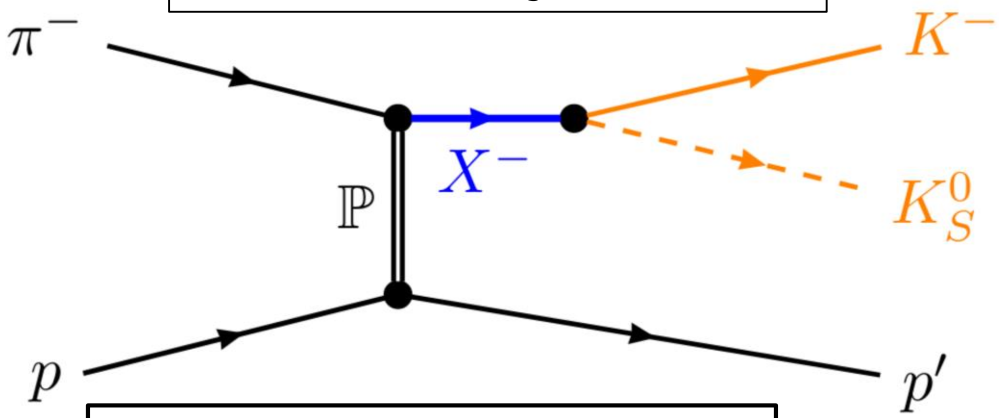


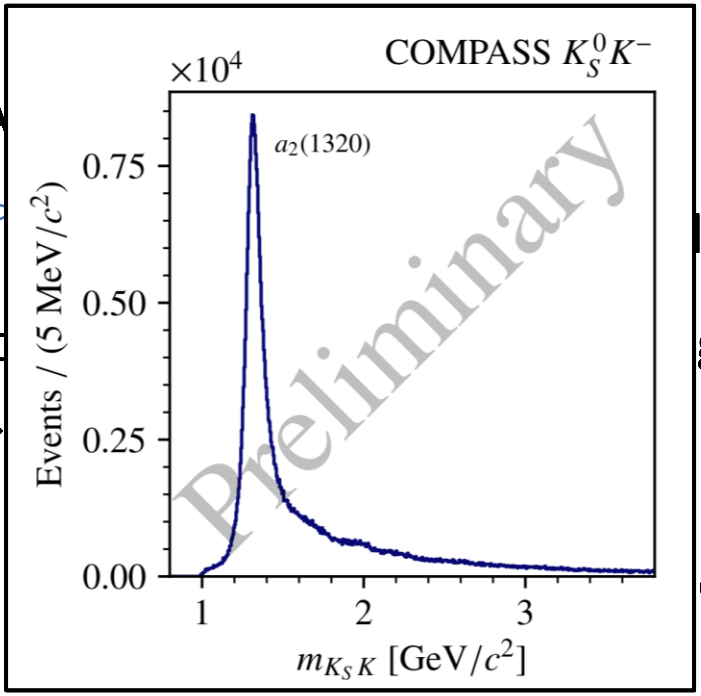
- Allo

→ π

- CO $\times\ 10^6$ evts

$\pi^- + p \to K_S^0 K^- + p$





- A

J^P lecay

Do ge

→

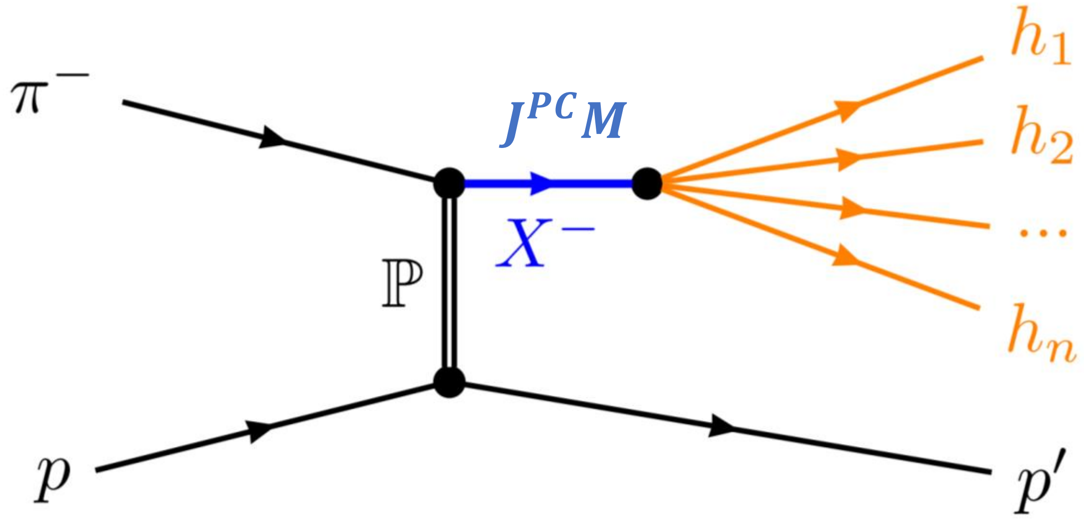
- Highl 6

Partial-Wave Decomposition

$$I(m_X, t'; \tau_n) = \left|M_{fi}\right|^2$$

- Separate process amplitude into **partial waves**
  - Spin $\boldsymbol{J}$ and spin-projection $\boldsymbol{M}$
  - Parity $\boldsymbol{P}$, charge conjugation $\boldsymbol{C}$
  - …

  → Partial wave index a



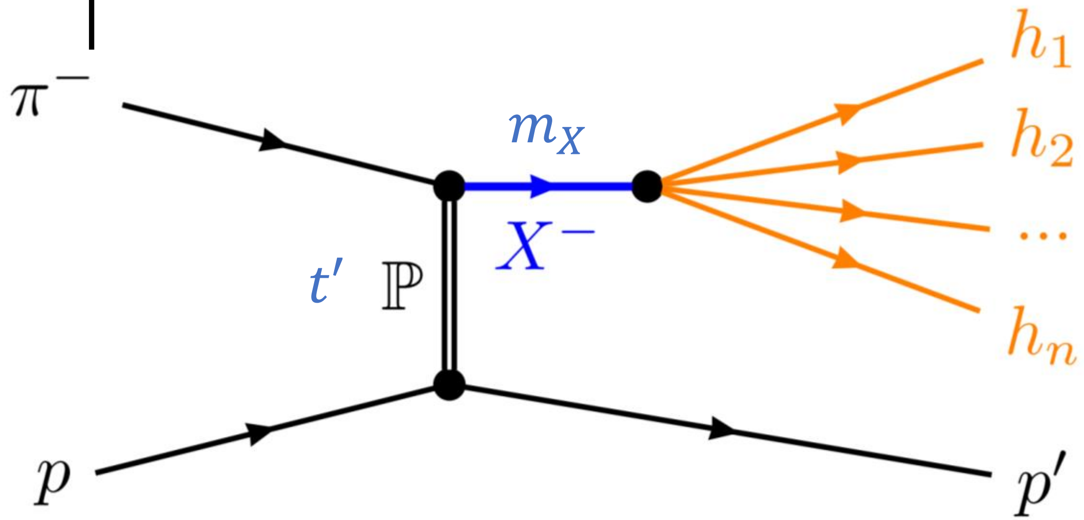
Partial wave $\boldsymbol{a}$:
specific $(\boldsymbol{J^{PC}M})$

Partial-Wave Decomposition

$$I(m_X, t'; \tau_n) = \left| \sum_a T_a(m_X, t')\, \psi_a(m_X, \tau_n) \right|^2$$

- Separate process amplitude into **partial waves**
  $\rightarrow$ Partial wave index a

- Production, propagation of X^-: $\boldsymbol{T_a(m_X, t')}$



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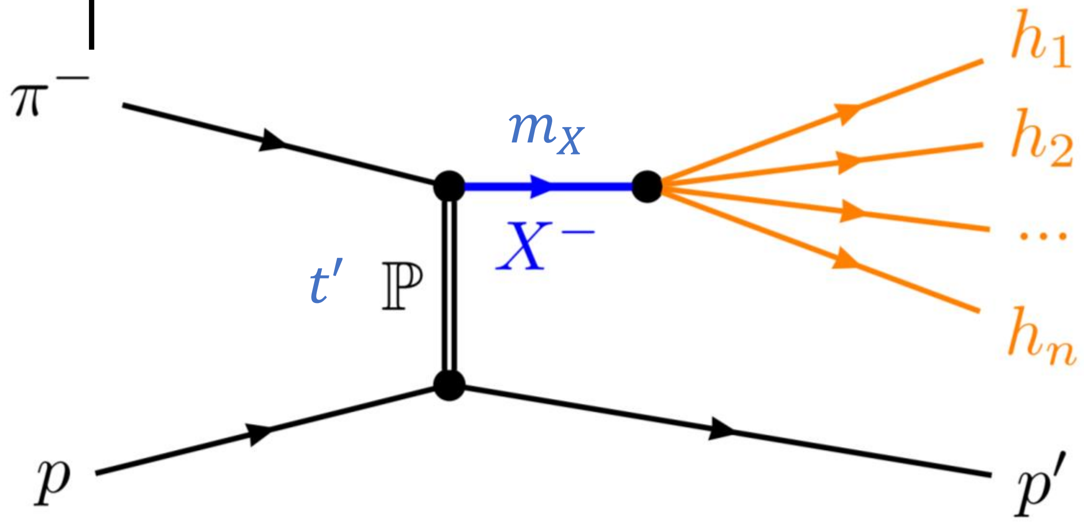
- Separate process amplitude into **partial waves**
  $\rightarrow$ Partial wave index a

- Production, propagation of X^-: $\boldsymbol{T_a(m_X, t')}$

- Decay of X^-: $\boldsymbol{\psi_a(m_X, \tau_n)}$



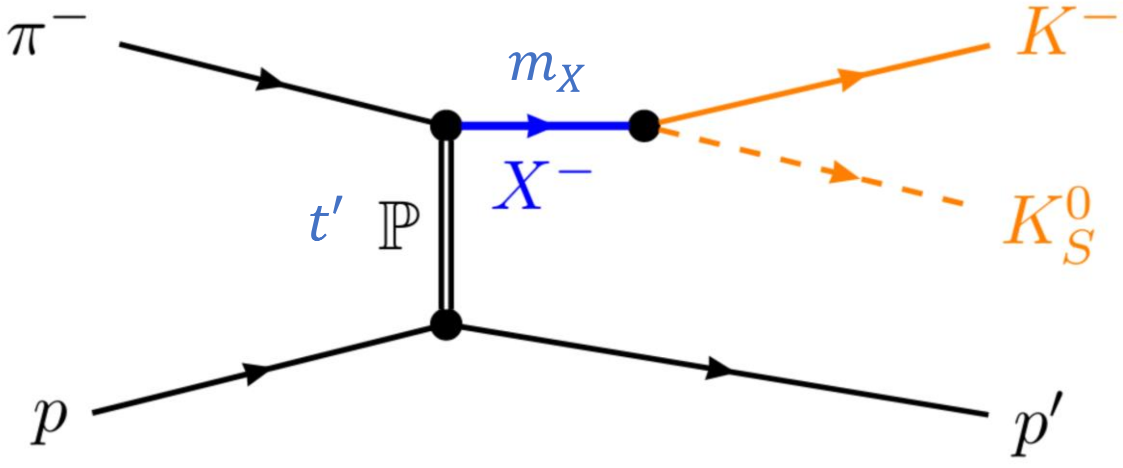
Partial wave $\boldsymbol{a}$:
specific $(\boldsymbol{J^{PC}M})$

Partial-Wave Decomposition

$$I(m_X, t'; \tau_n) = \left| \sum_a T_a(m_X, t')\, \psi_a(m_X, \tau_n) \right|^2$$

- Separate process amplitude into **partial waves**
  - $\rightarrow$ Partial wave index a
- Production, propagation of X^-: $\boldsymbol{T_a(m_X, t')}$
- Decay of X^-: $\boldsymbol{\psi_a(\tau_n) = Y_J^M(\theta, \phi)}$

$$\boldsymbol{\tau_n = (\theta, \phi)}$$



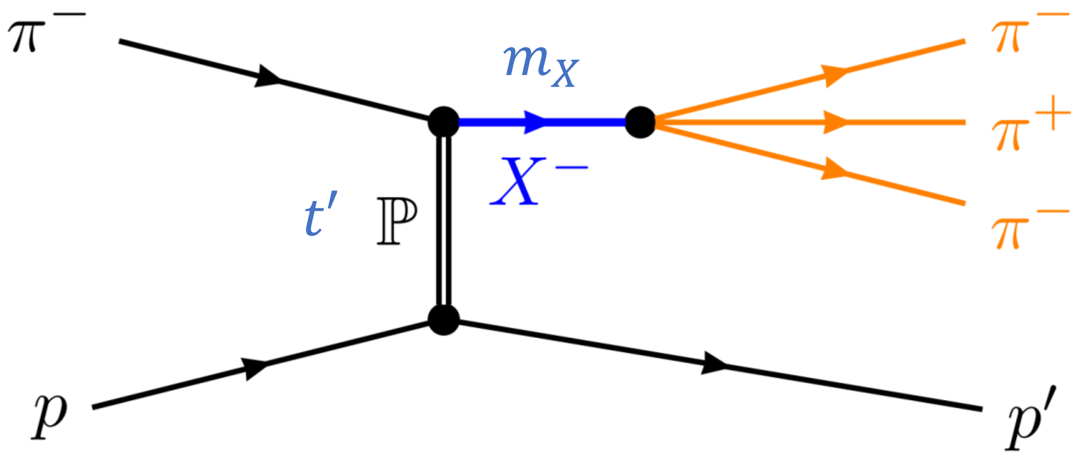
Partial wave a:
specific $(J^{PC}M)$

$\boldsymbol{K_S^0 K^-}$

Partial-Wave Decomposition

$$I(m_X, t'; \tau_n) = \left| \sum_a T_a(m_X, t')\, \psi_a(m_X, \tau_n) \right|^2$$

- Separate process amplitude into **partial waves**
  - → Partial wave index a
- Production, propagation of X^-: $\boldsymbol{T_a(m_X, t')}$
- Decay of X^-: via **isobar model**



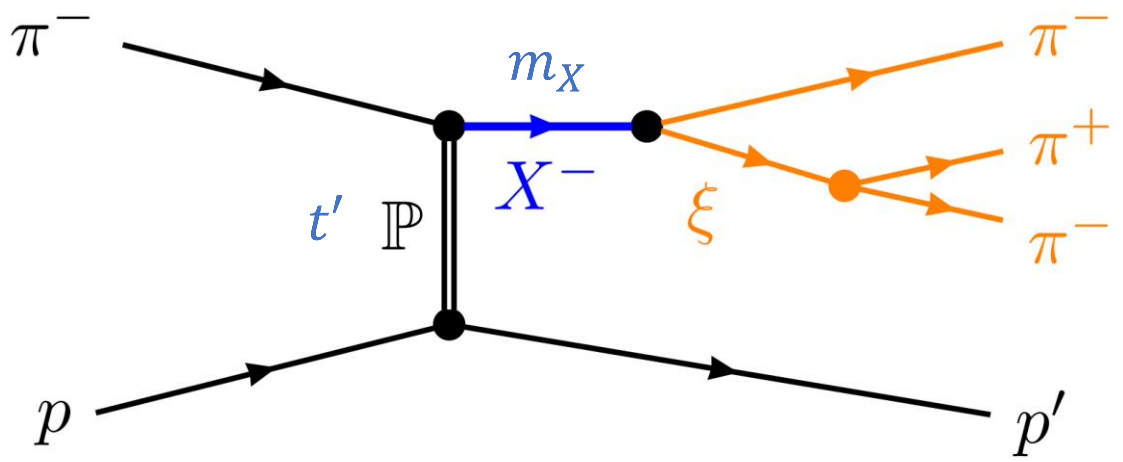
Partial wave $\boldsymbol{a}$:
specific $\boldsymbol{(J^{PC}M)}$

$\boldsymbol{\pi^-\pi^-\pi^+}$

Partial-Wave Decomposition

$$I(m_X, t'; \tau_n) = \left| \sum_a T_a(m_X, t')\, \psi_a(m_X, \tau_n) \right|^2$$

- Separate process amplitude into **partial waves**
  - → Partial wave index a



- Production, propagation of X^-

- Decay of X^-: via isobar model

$$\boldsymbol{\tau_n = (\theta_{GJ}, \phi_{GJ}, m_\xi, \theta_{HF}, \phi_{HF})}$$

$$\boldsymbol{\psi_a = \psi_X(m_X, \theta_{GJ}, \phi_{GJ}) \cdot \psi_\xi(m_\xi, \theta_{HF}, \phi_{HF})}$$

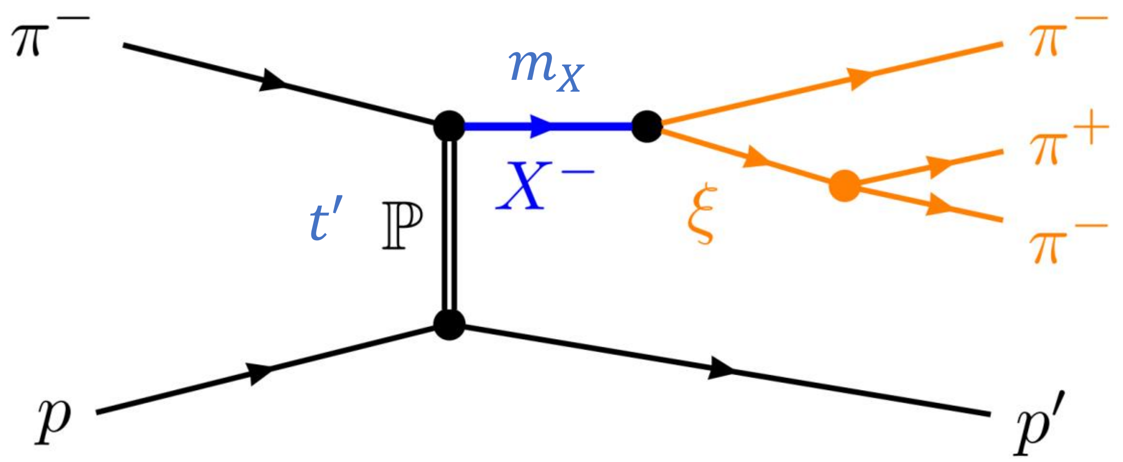
Partial wave $\boldsymbol{a}$:
specific $(\boldsymbol{J^{PC}} + \text{decay})$

$\boldsymbol{\pi^-\pi^-\pi^+}$

Partial-Wave Decomposition

$$I(m_X, t'; \tau_n) = \left| \sum_a T_a(m_X, t')\, \psi_a(m_X, \tau_n) \right|^2$$

- Separate process amplitude into **partial waves**
  → Partial wave index a
- Production, propagation of X^-
- Decay of X^-: via isobar model
- Fit $I(m_X, t'; \tau_n)$ to data **in $(\boldsymbol{m_X}, \boldsymbol{t'})$ bins**
  → parametrize T_a as step-wise functions
  → extract constant T_a in each bin



Partial wave $\boldsymbol{a}$:
specific $(J^{PC} + \text{decay})$

Resonance-Model Fit

Second step: **extract resonance parameters**

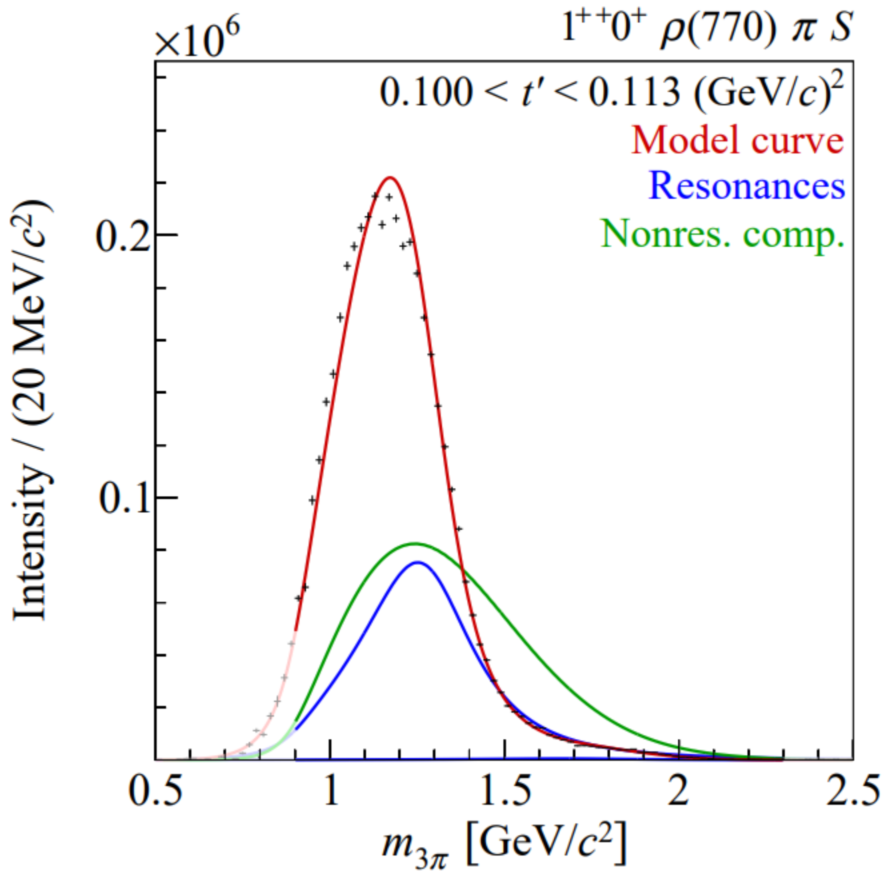
- Build model for **mass dep. of partial-wave amplitudes**:

  resonant (e.g. Breit-Wigner distribution)

  \+ **non-resonant background** components

- χ^2 fit to output of partial-wave decomposition

→ get masses and widths of parameterized resonances



COMPASS PRD 98 (2018) 092003

Understanding the Ambiguities in the Partial-Wave Decomposition of the $K_S^0 K^-$ Final State

julien.beckers@tum.de

Ambiguities in the Partial-Wave Decomposition

For any final state with **two spinless** particles ($\pi\pi, KK, \eta\pi, \ldots$):

- Decomposition of intensity into $\{T_J\}$ is not **unique** (see derivation later)

→ Several sets of $\{T_J\}$ lead to the **same** $\boldsymbol{I(\theta,\phi)}$ in each (m_X, t') bin

$$I(\theta,\phi) = \left|\sum_{JM} T_{JM}^{(1)}\,\psi_{JM}(\theta,\phi)\right|^2 = \left|\sum_{JM} T_{JM}^{(2)}\psi_{JM}(\theta,\phi)\right|^2$$

- The fit cannot distinguish between the **mathematically equivalent** solutions!

Ambiguities in the Partial-Wave Decomposition

$$I(\theta,\phi) = \left|\sum_{JM} T_{JM}\,\psi_{JM}(\theta,\phi)\right|^2$$

Assume strong dominance of $|M| = 1$ *

- Pomeron exchange dominant $\rightarrow M \neq 0$
- Higher $|M|$ suppressed

*using reflectivity basis for ψ_{JM} :
doi.org/10.1103/PhysRevD.11.633

Ambiguities in the Partial-Wave Decomposition

$$I(\theta, \phi) = \left| \sum_J T_J \, \psi_J(\theta, \phi) \right|^2$$

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Ambiguities in the Partial-Wave Decomposition

$$I(\theta,\phi) = \left|\sum_J T_J\, \psi_J(\theta,\phi)\right|^2 = \underbrace{\left|\sum_J T_J\, Y_J^1(\theta,0)\right|^2}_{\boldsymbol{a(\theta)}} \;|\sin\phi|^2$$

$$\boxed{Y_J^1(\theta,0) = \sum_{j=0}^{J-1} y_j \tan^{2j}\theta}$$

Polynomial in $\tan^2\theta$

$$a(\theta) = \sum_{j=0}^{J_{\max}-1} c_j(\{T_J\}) \tan^{2j}(\theta) = c(\{T_J\}) \prod_{k=1}^{J_{\max-1}} \left(\tan^2(\theta) - \boldsymbol{r_k}(\{T_J\})\right)$$

root decomposition

$a(\tan^2\theta = r_k) = 0$

“Barrelet zeros”

Chung, PRD 56 7299–7316 (1997)

Barrelet, *Nuov Cim A* **8**, 331–371 (1972)

Ambiguities in the Partial-Wave Decomposition

$$a(\theta) = c(\{T_J\}) \prod_{k=1}^{J_{\max-1}} \left(\tan^2(\theta) - \boldsymbol{r}_k(\{T_J\})\right)$$

$$I(\theta,\phi) = \Bigg|\underbrace{\sum_J T_J\, Y_J^1(\theta,0)}_{a(\theta)}\Bigg|^2 \;|\sin\phi|^2$$

$$= \left|\sum_{j=0}^{J_{\max}-1} c_j(\{T_J\})\, \tan^{2j}(\theta)\right|^2 |\sin\phi|^2$$

$$= c^2 \prod_{k=1}^{J_{\max-1}} |\tan^2(\theta) - \boldsymbol{r}_k|^2 \;|\sin\phi|^2 \quad = c^2 \prod_{k=1}^{J_{\max-1}} |\tan^2(\theta) - \boldsymbol{r}_k^*|^2 \;|\sin\phi|^2$$

$$\{c_j{}'\}$$

$$\{T_J{}'\} \neq \{T_J\}$$

Study of the Ambiguities

- How do the ambiguous solutions look like (**continuity, signals, …**)?
- What are the effects of the **partial-wave decomposition fit** on **finite data** on the ambiguities?

I. Continuous intensity model

- create an amplitude model for selected partial waves
- calculate exact ambiguities

II. Finite pseudo-data

- generate pseudo-data according to model
- perform partial-wave decomposition

Continuous Amplitude Model

I. Continuous intensity model

- create an amplitude model for four selected partial waves
- In $1.0 < m_X < 2.5\ \mathrm{GeV}/c^2$
- m_X-dependence by Breit-Wigner amplitudes (PDG parameters)

$$T(m_X) = \underbrace{\sqrt{m_X}\,\sqrt{\rho_2(m_X)}}_{\text{phase-space factor}} \cdot \underbrace{C e^{i\phi}}_{\text{complex scale}} \cdot \underbrace{D_{BW}(m_X; M_0, \Gamma_0)}_{\frac{M_0\,\Gamma_0}{M_0^2 - m_X^2 - iM_0\Gamma_0}}$$

J^{PC}	Resonances
$\mathbf{1}^{--}$	$\rho(1450)$
$\mathbf{2}^{++}$	$a_2(1320), a_2'(1700)$
$\mathbf{3}^{--}$	None
$\mathbf{4}^{++}$	$a_4(1970)$

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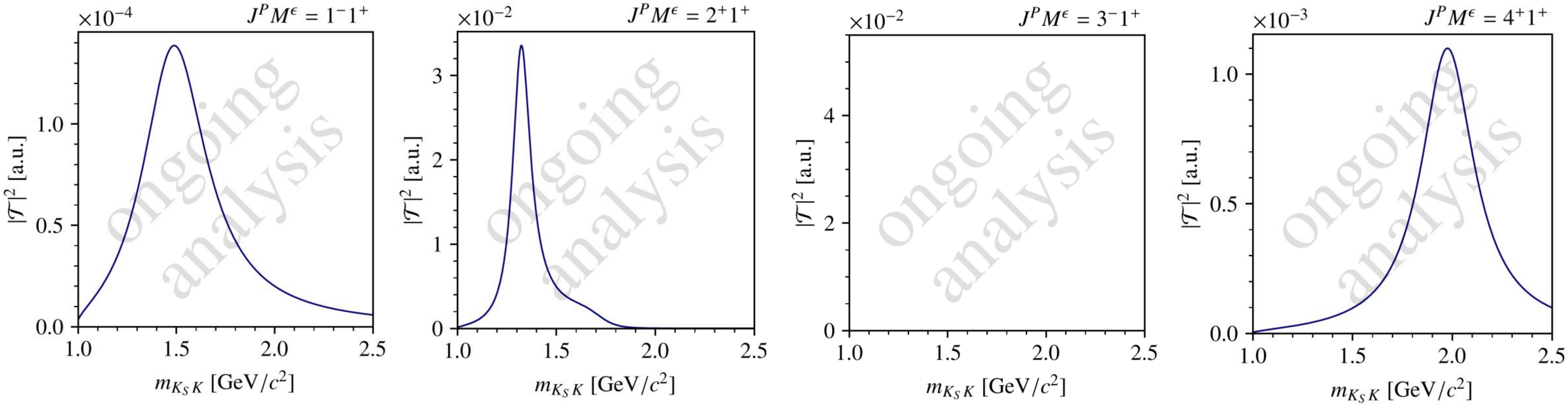
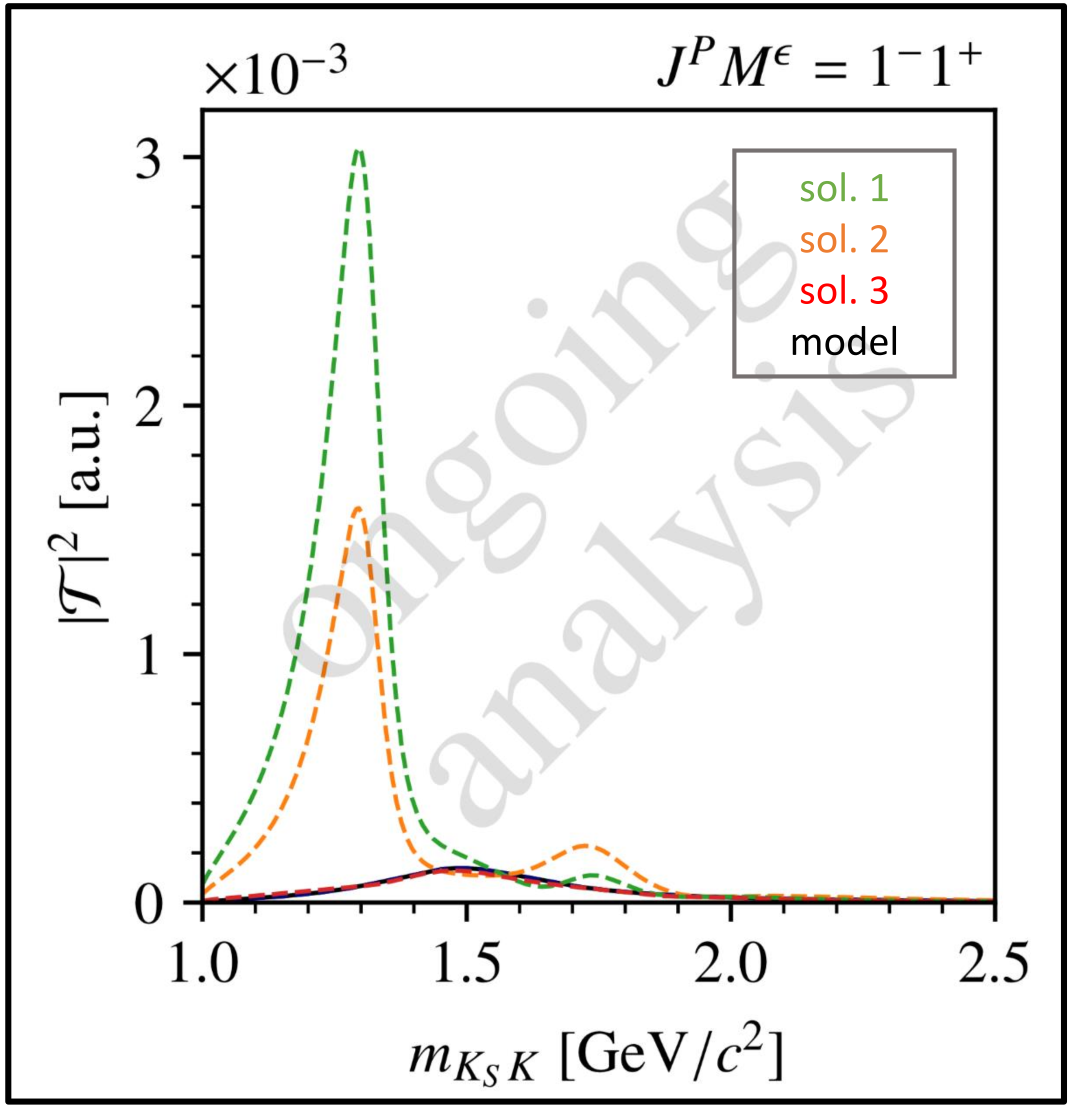


Continuous Amplitude Model

I. Continuous intensity model $\boxed{N_a = 3}$

- Sample points in m_X and calculate ambiguous solutions
- Ambiguous intensities are also continuous
- Not all solutions are different from each other!
- Highest-spin (4^{++}) intensity is invariant!



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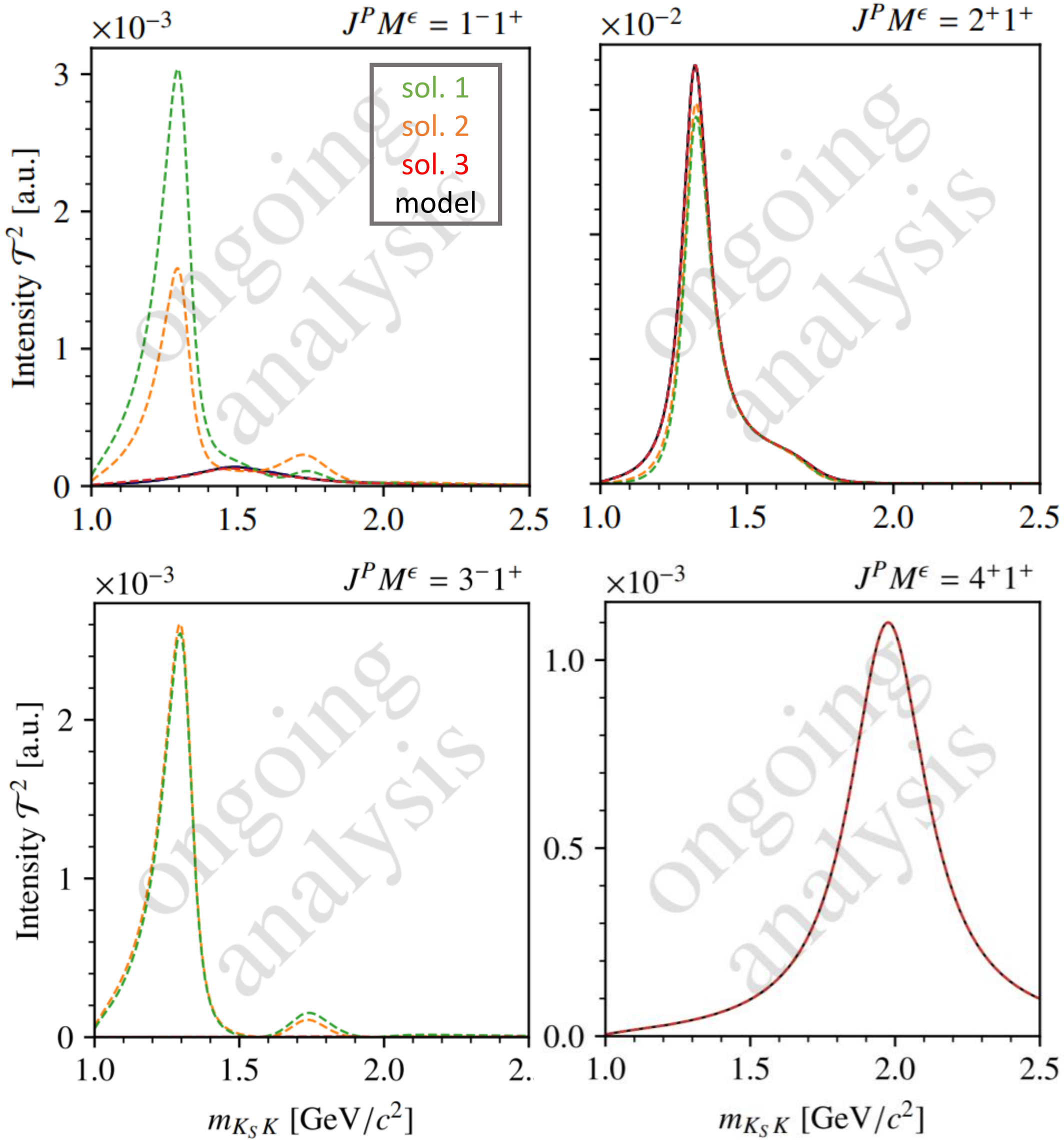

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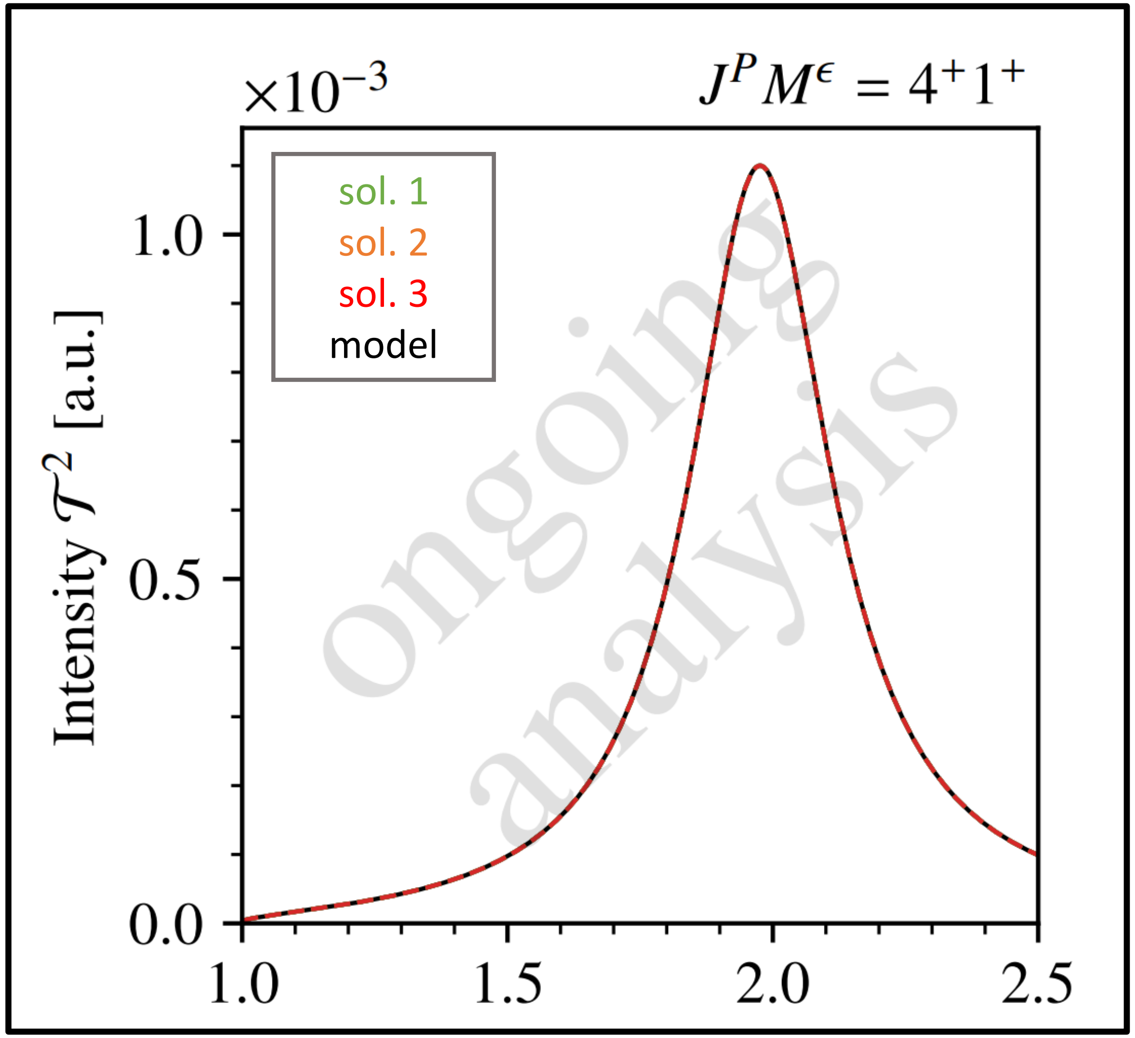

Study of the Ambiguities

II. Finite pseudo-data

- reality: **finite data** and **amplitudes unknown**
- generate pseudo-data according to model (10^5 events)
- perform a partial-wave decomposition fit

→ 3000 attempts with random start values

J^{PC}	Resonances
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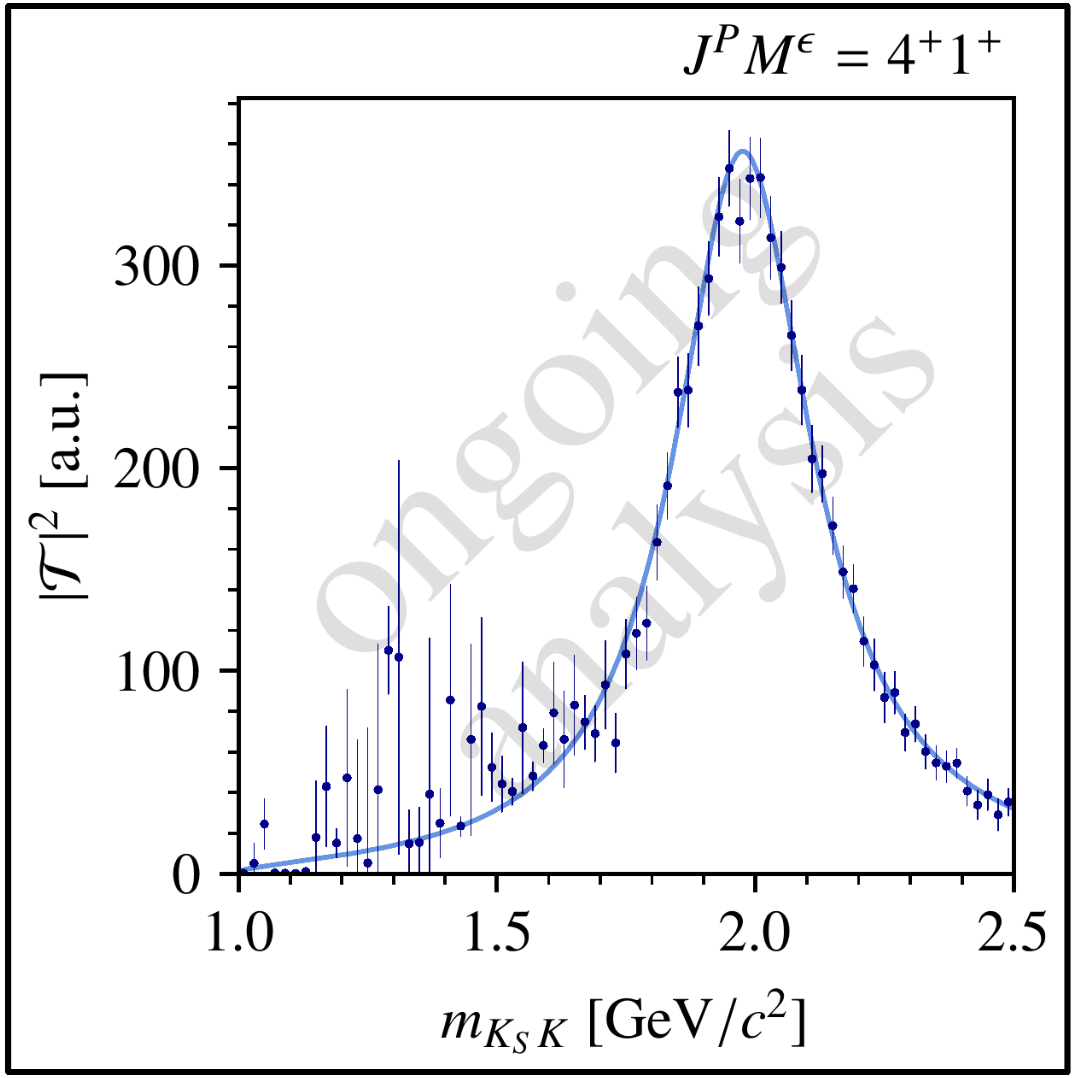
Partial-Wave Decomposition Fits on Pseudodata

II. Finite pseudo-data

- 4^{++} intensity is still invariant!
- Overall, amplitude values found by the fit follow the calculated distributions
- Not all solutions are found in each m_X bin

→ PWD fit distorts the intensity distribution!



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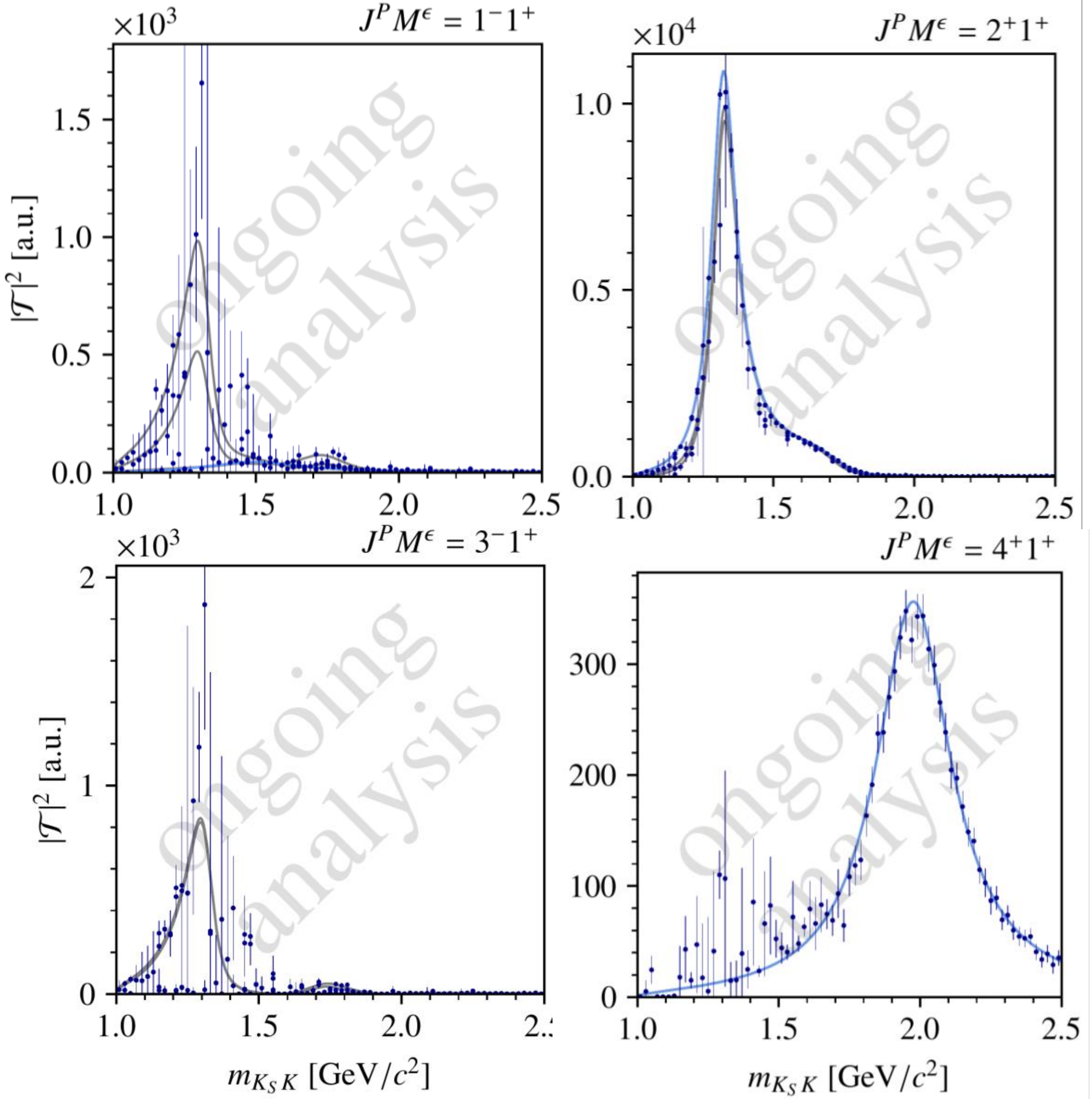

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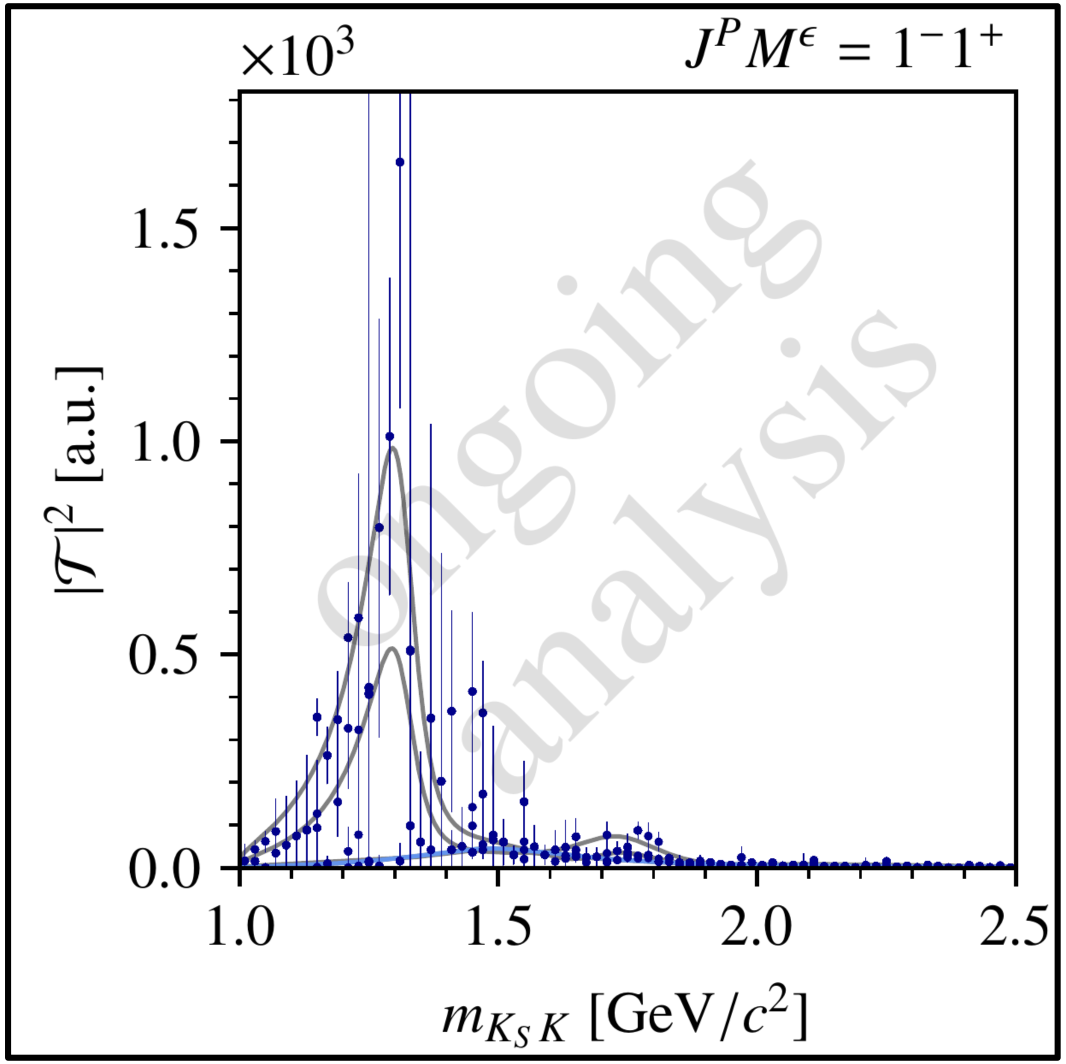
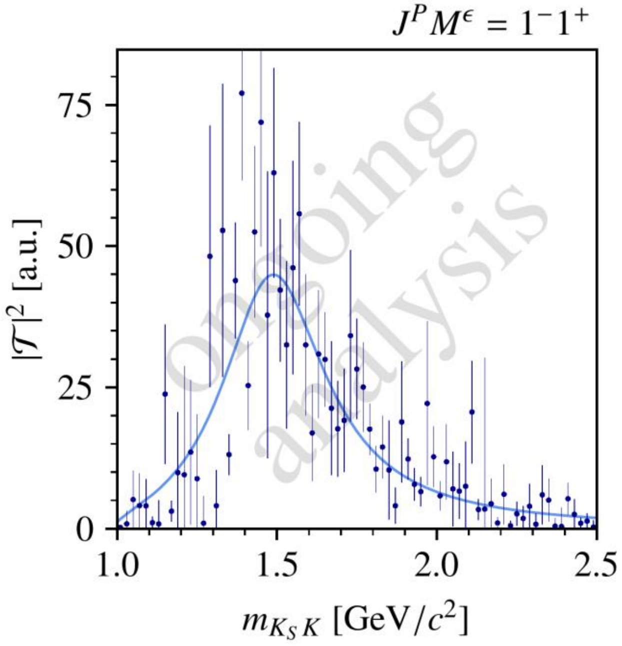
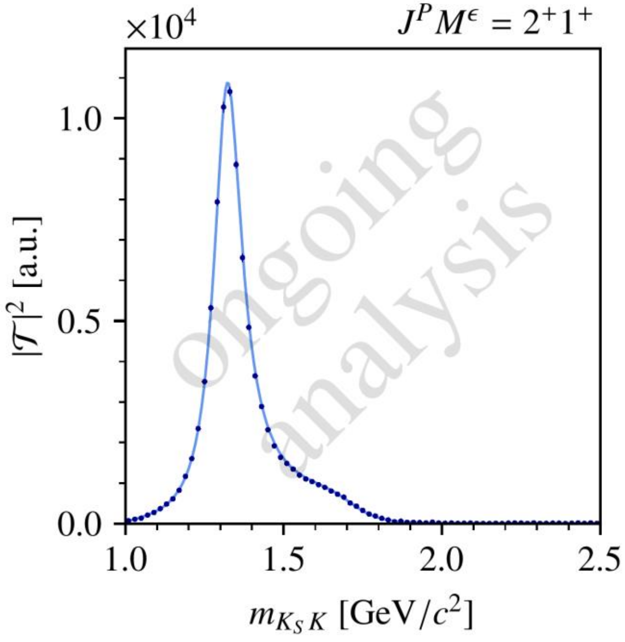
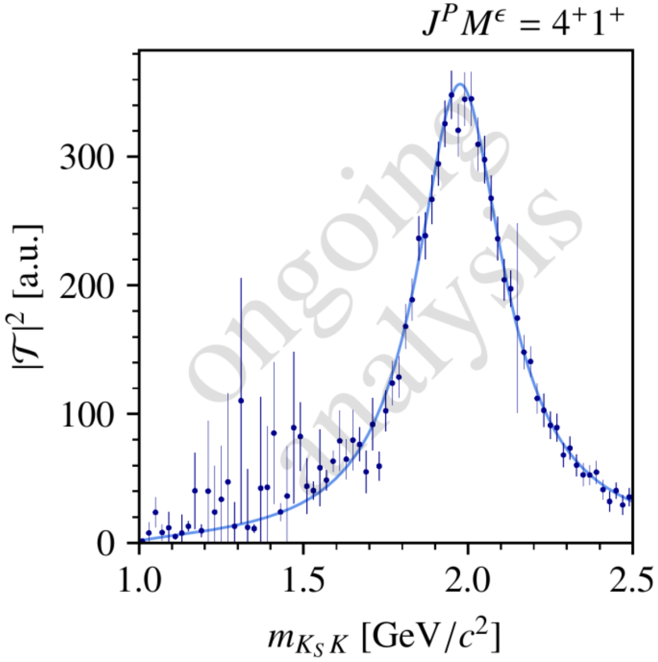


Reducing the Ambiguities

- Intensity of highest-spin wave is unaffected by ambiguities
- Including $M \geq 2$ $\rightarrow$ allows for additional angular structure $\rightarrow$ **resolves ambiguities**
- Remove one wave with $J < J_{\max}$ $\rightarrow$ **resolves ambiguities**







Continuity Constraints for Partial-Wave Analyses

Conventional Partial-Wave Analysis

We have some knowledge about the partial-wave amplitudes $T(m_X, t')$:

- Physics should be (mostly) **continuous** in m_X and t'

→ Solutions in neighboring bins should be similar (→ correlations between bins)

- Amplitudes should follow **phase-space** and **production kinematics**

Limitations of conventional PWA:

$$I(m_X, t'; \tau_n) = \left| \sum_{\text{waves}} T_i(m_X, t')\, \psi_i(m_X, \tau_n) \right|^2$$

- Binned analysis limits statistics, especially for small signals
- Continuity information is not imposed in the model
- We need to **select ("small") subset of partial waves** to include in the model

→ important source of systematic uncertainty

Constraints for Partial-Wave Analyses

Make use of this information to stabilize partial-wave decomposition fit:

→ Replace discrete amplitudes with **smooth, non-parametric curves**

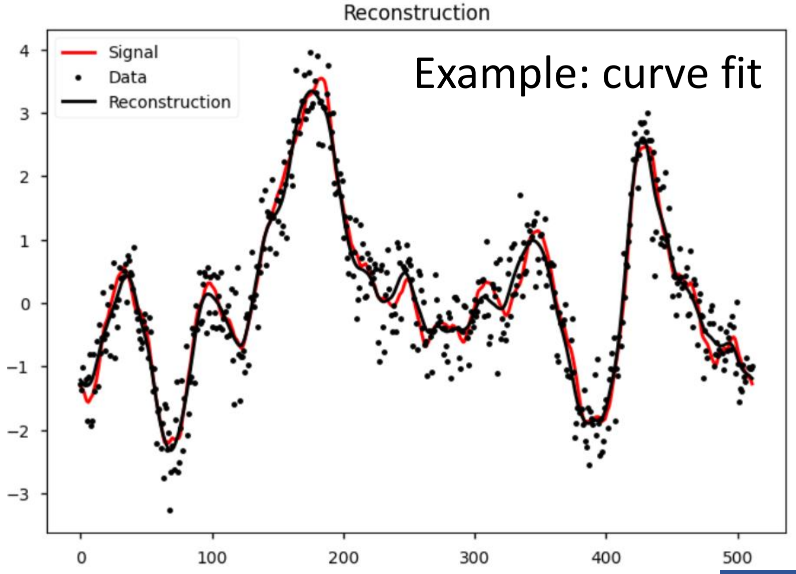
→ Incorporate **kinematic factors**

→ Include **regularization** for small amplitudes

Framework by team from the Max-Planck Institute for Astrophysics:

NIFTY: “**N**umerical **I**nformation **F**ield **T**heory”

https://ift.pages.mpcdf.de/nifty/



- Provides continuous non-parametric models
- Adapt to partial-wave analysis model
- **Learns smoothness and shape** of the amplitude curves

This work is done in collaboration with Jakob Knollmueller
(TUM / ORIGINS Excellence Cluster)

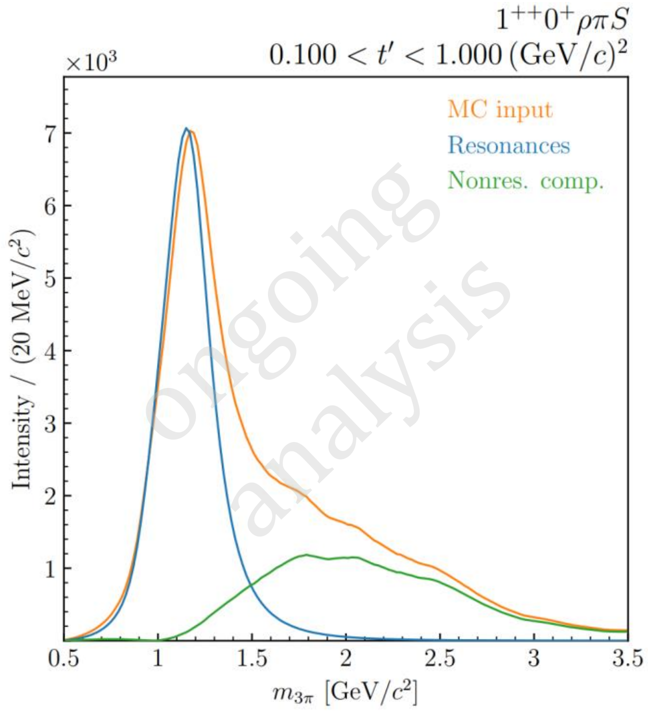
Input-Output Study

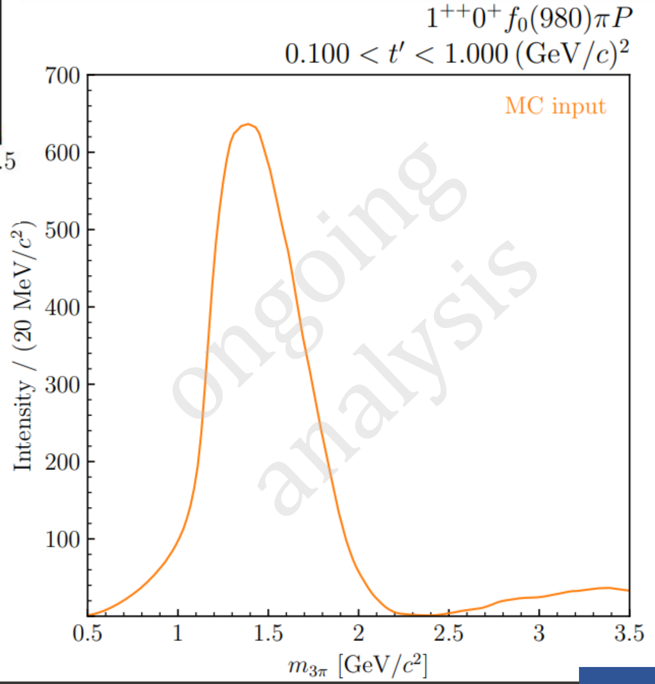
Verification on pseudodata:

- Generate pseudodata according to:
  - smooth model in mass
  - 81 partial waves
  - 5 resonances in selected waves

- resonance(s) (Breit-Wigner)
- nonres. component (broad curve)
- Combined signal → **input model**

- Perform PWA fit with NIFTy model on generated dataset
  - Same set of partial waves





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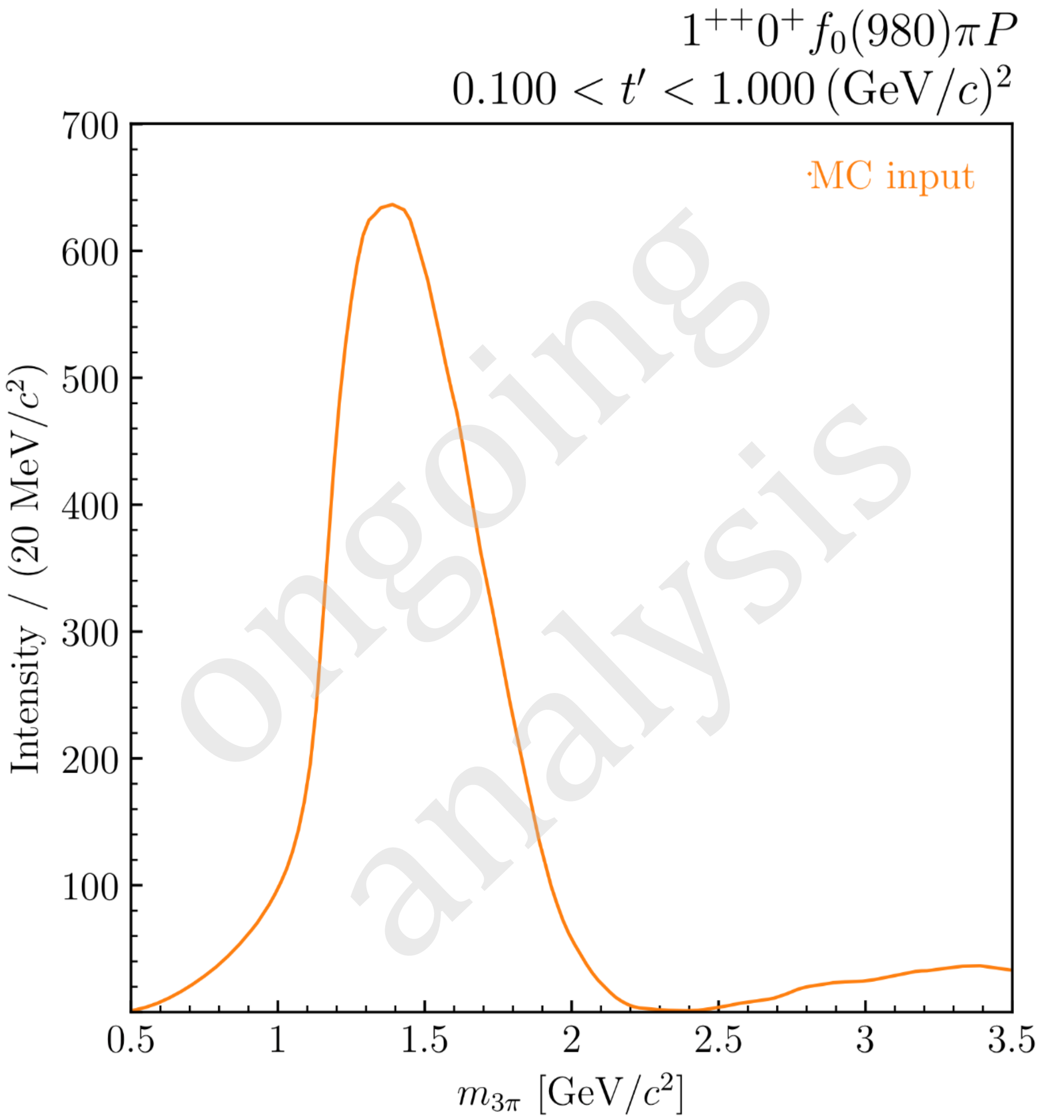

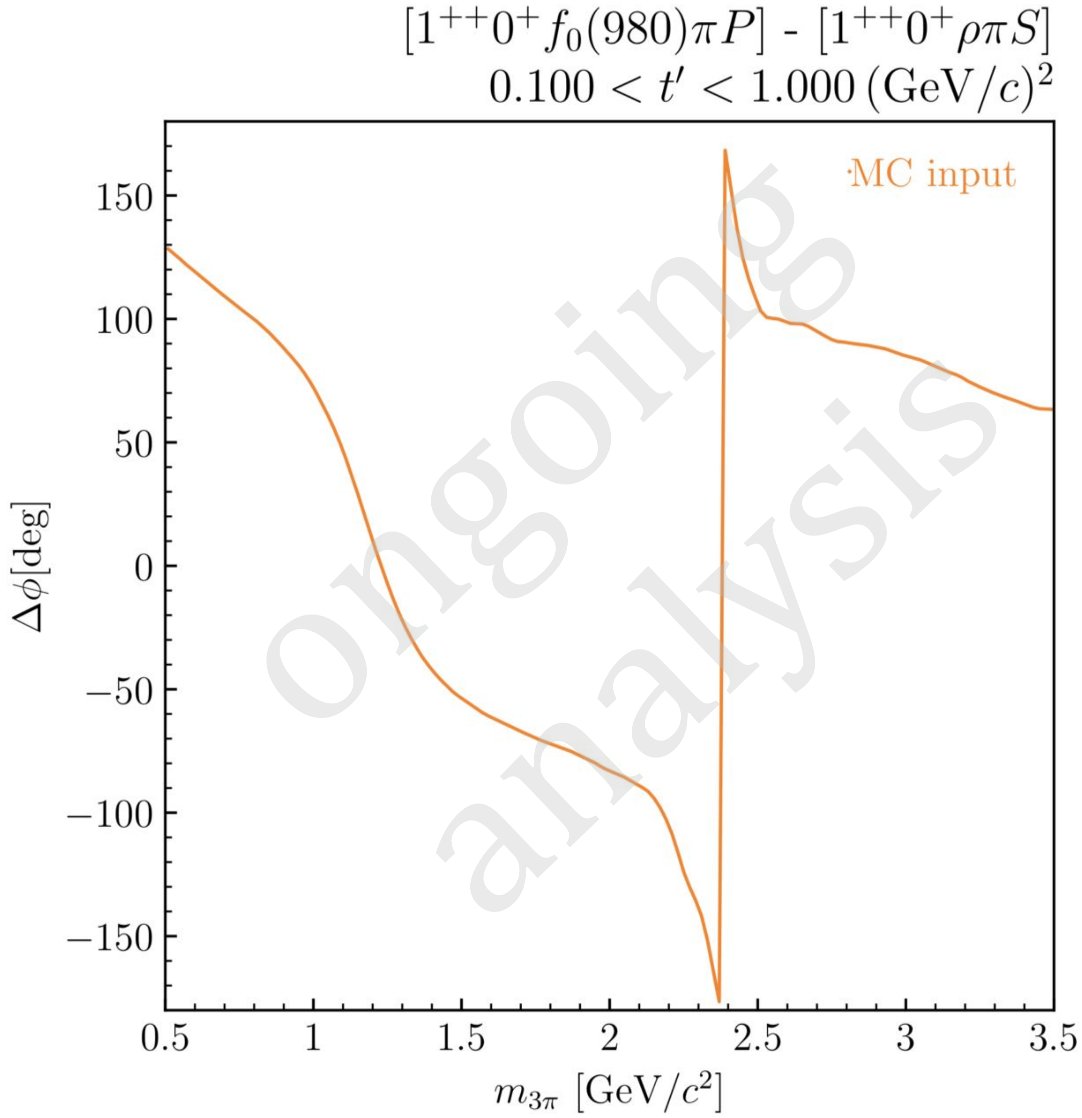

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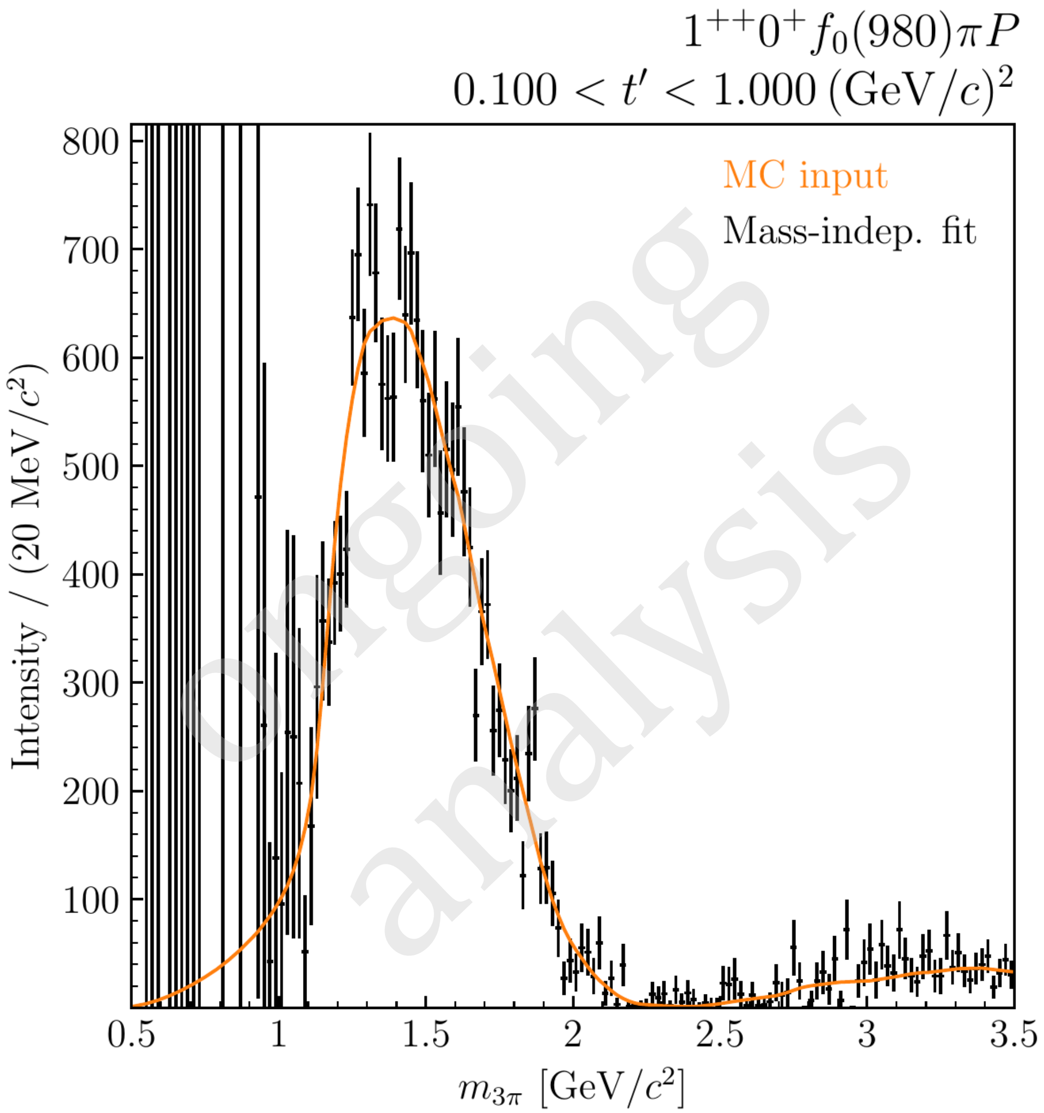
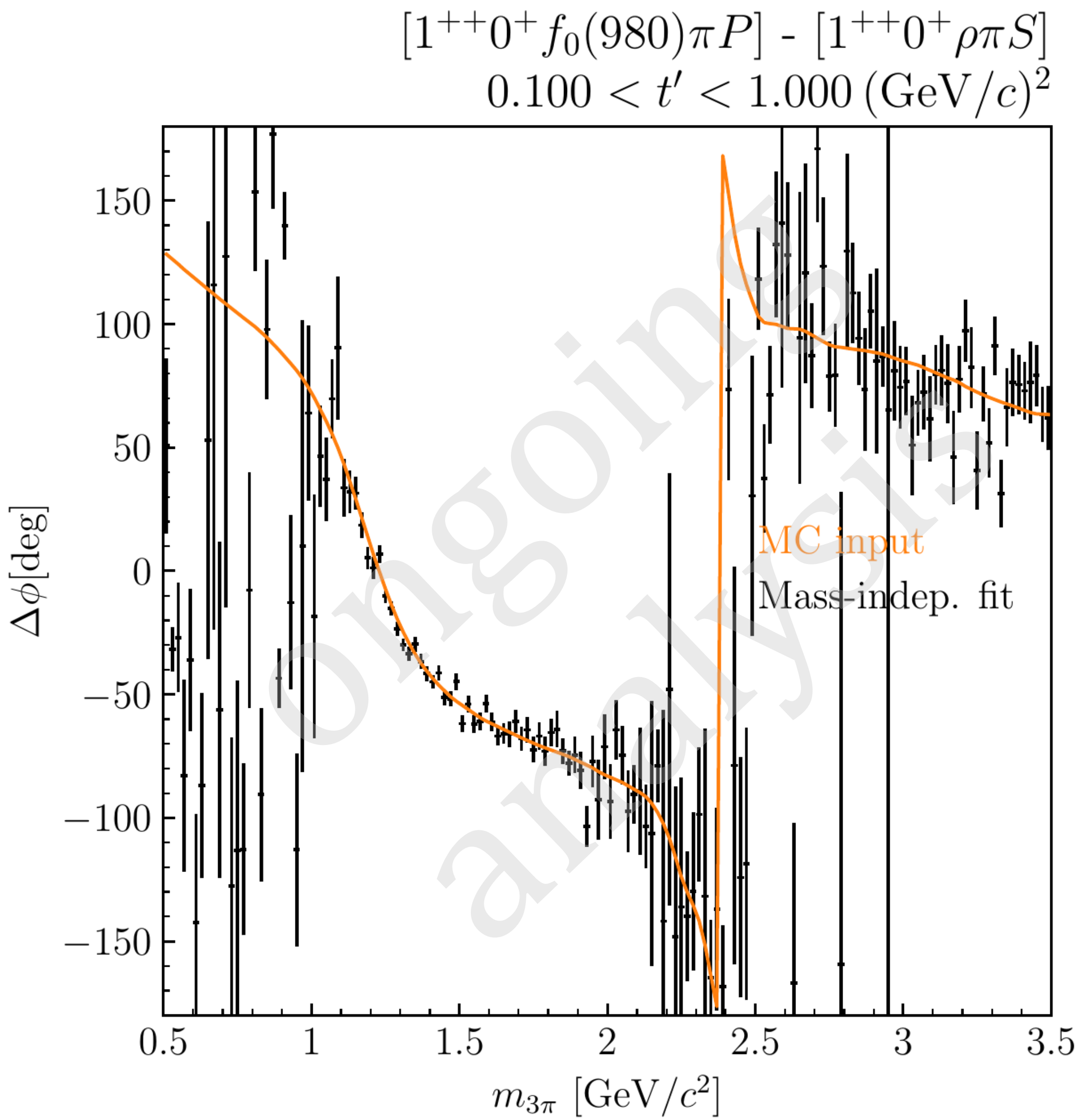
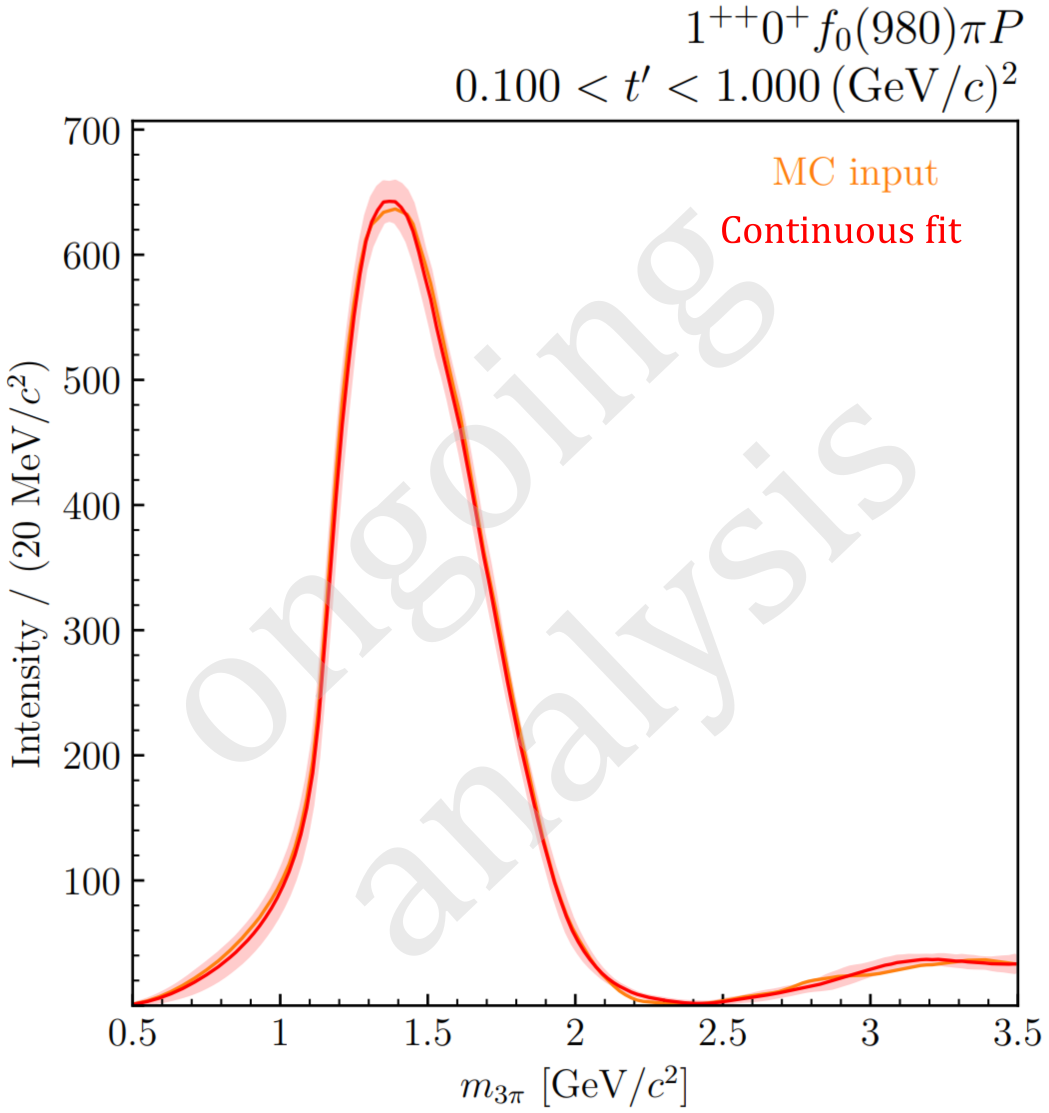
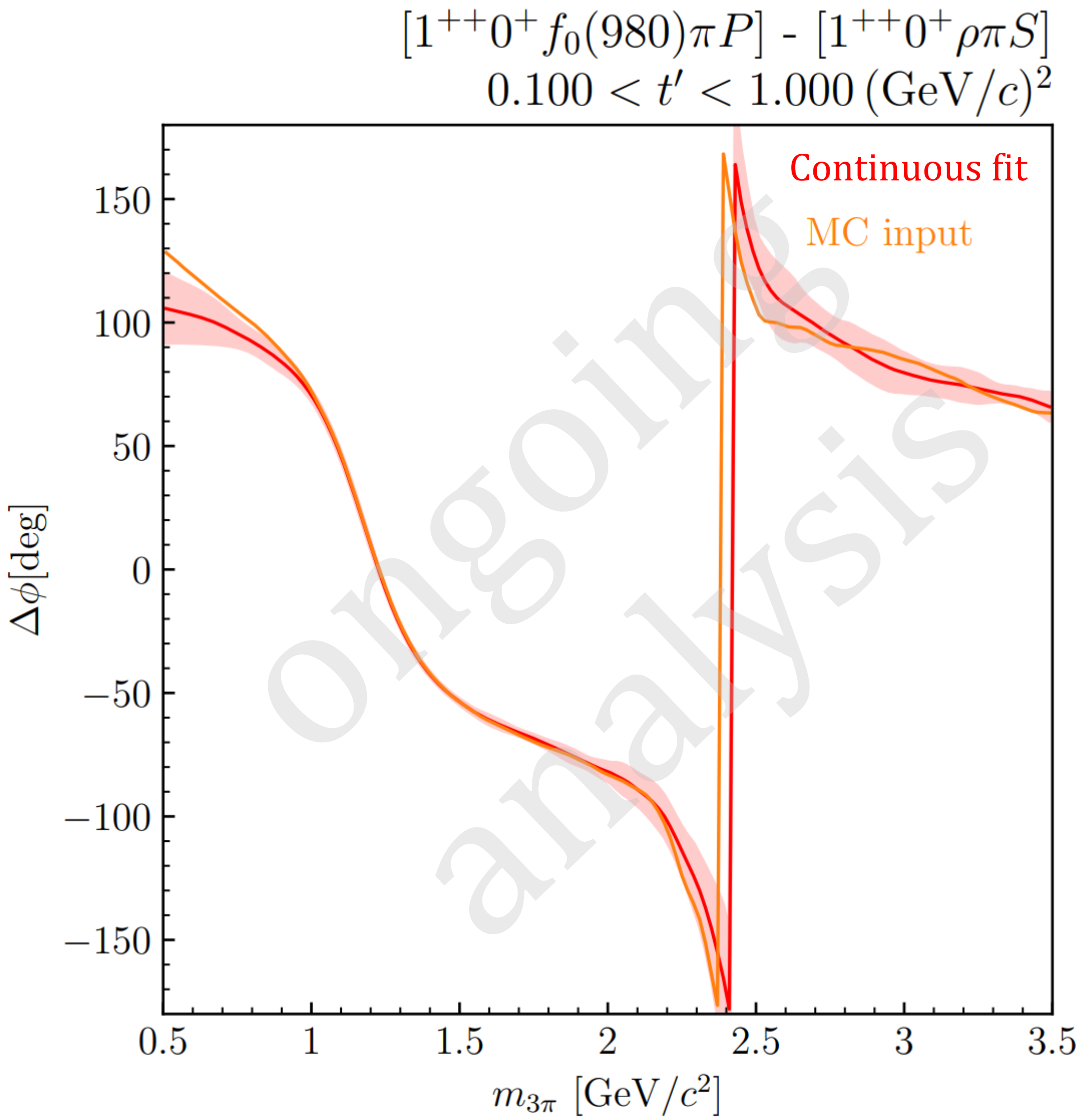


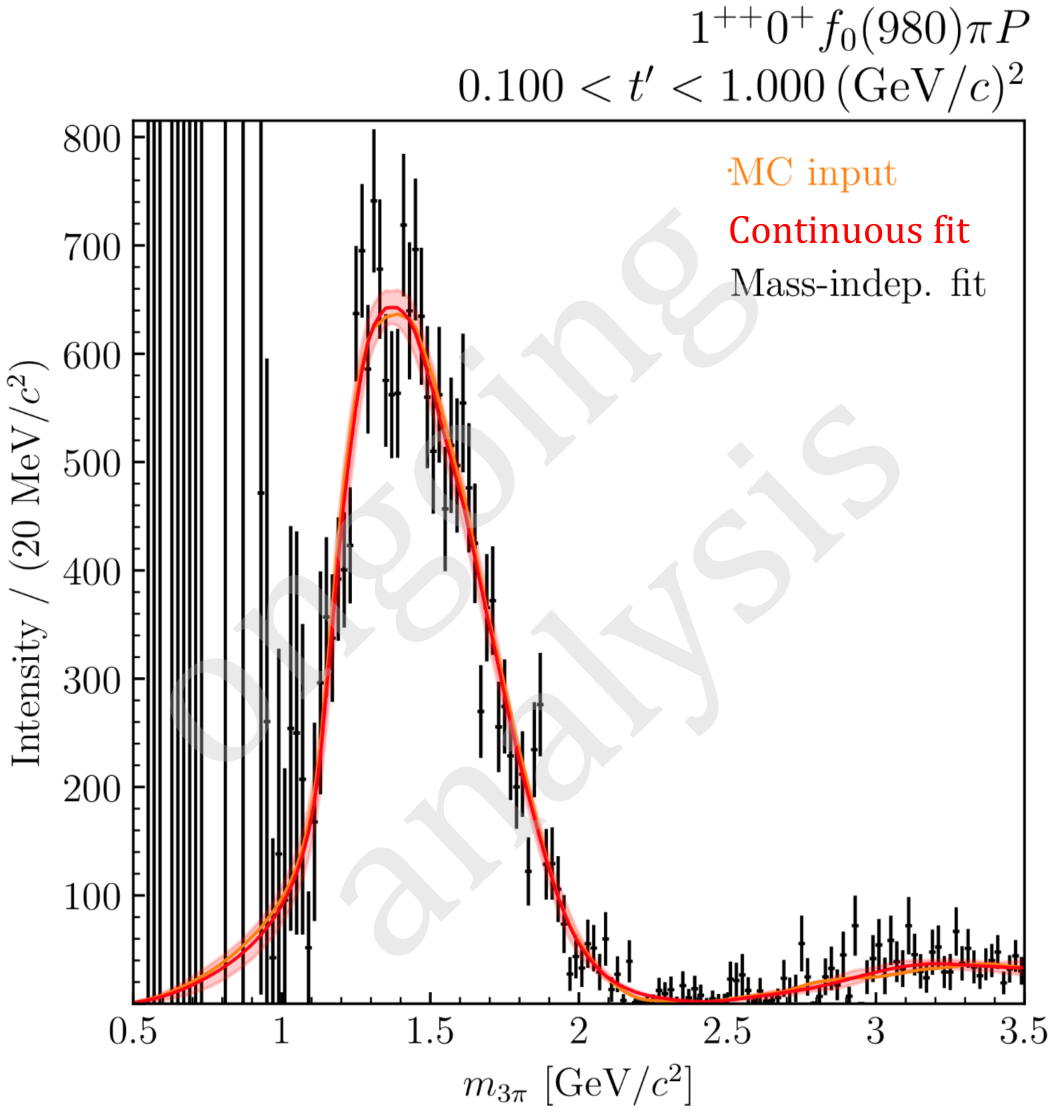


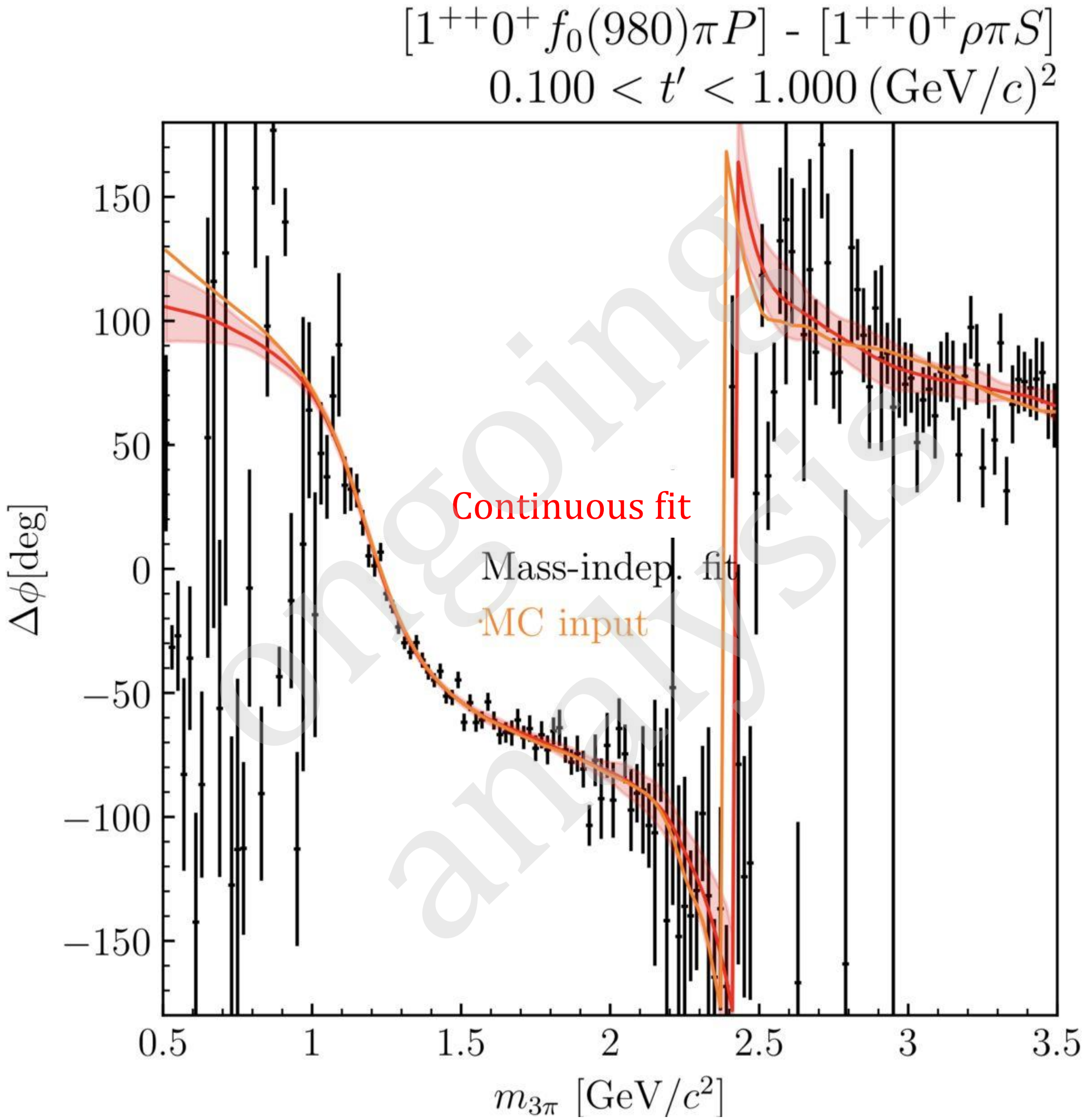
Input-Output Study





Input-Output Study





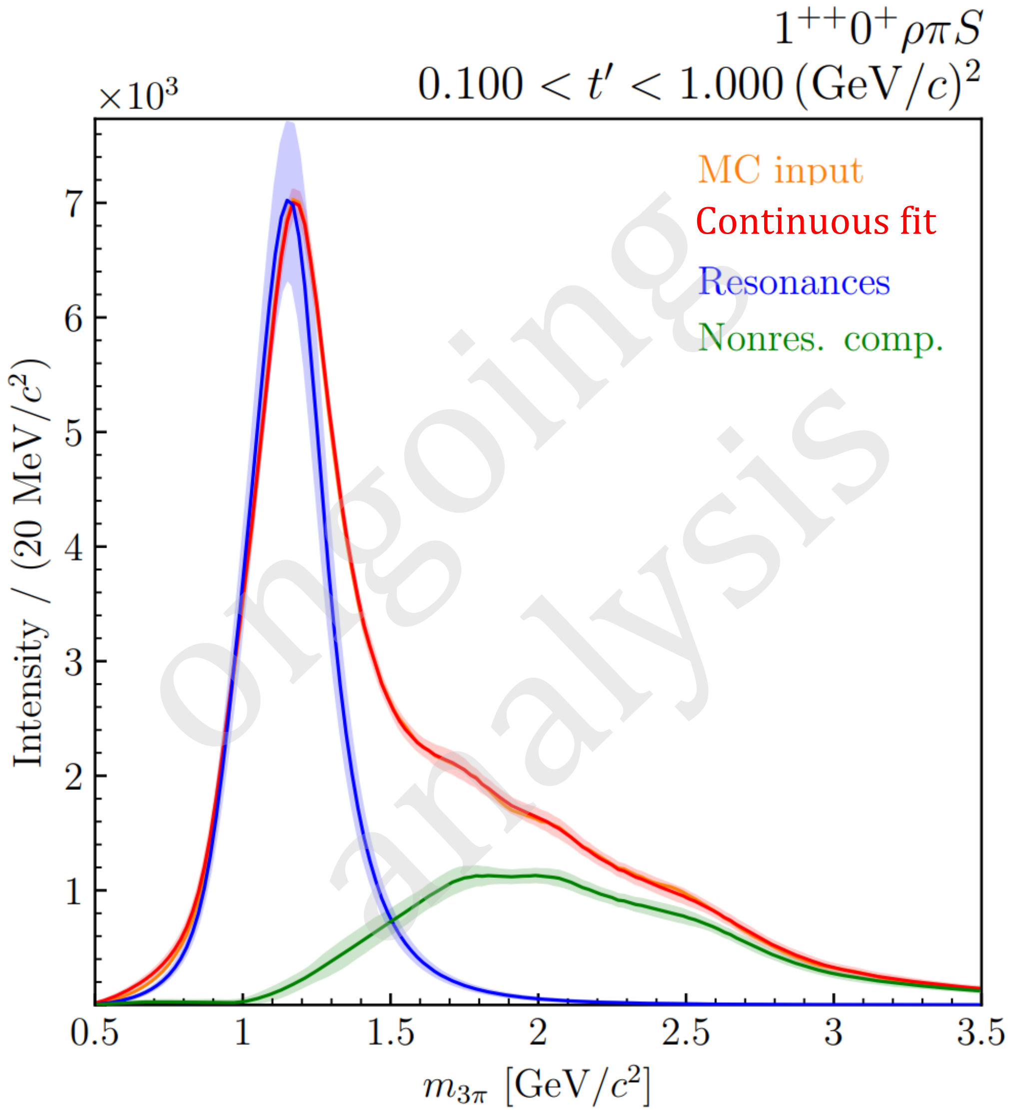
Single-Step Resonance Model Fit

We can go one step further!

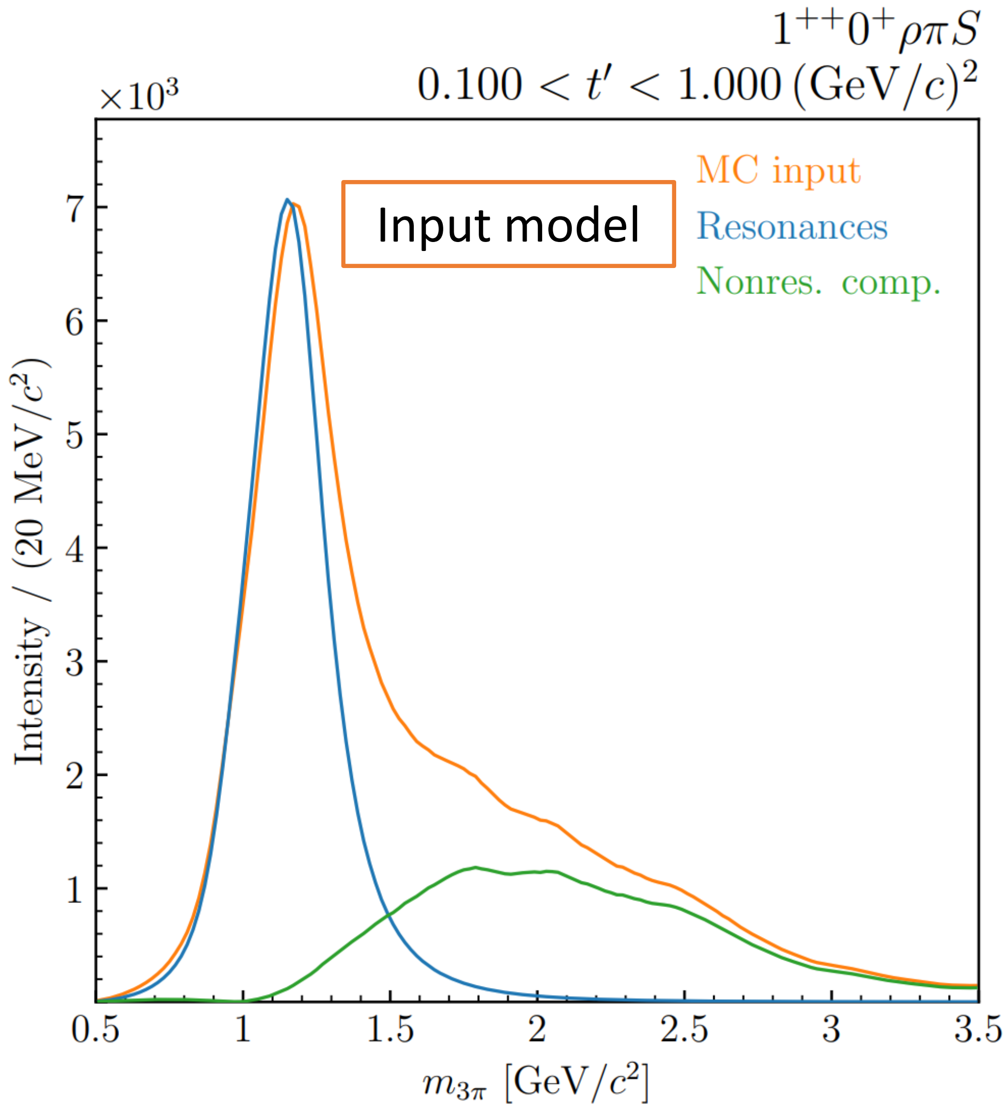
In **selected waves**:

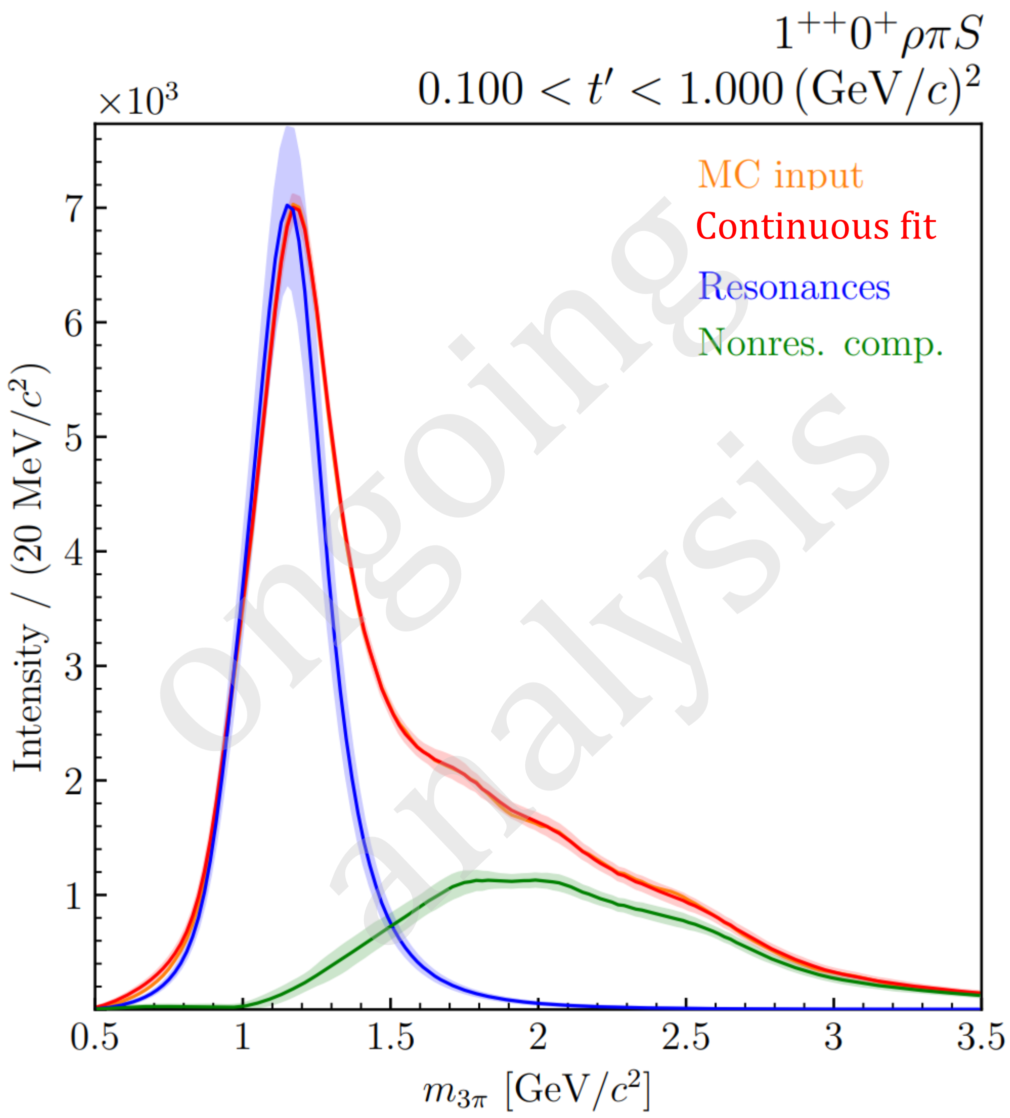
- add **resonant part**
  (e.g. as sum of Breit-Wigner distributions)
- use NIFTy as **flexible non-res. background**
- amplitude described by coherent sum

→ extract resonance parameters **in a single fit**



Single-Step Resonance Model Fit





Conclusion

High-precision data from COMPASS in $\pi^-\pi^-\pi^+$ and $K_S^0 K^-$ allow in-depth study of a_J and π_J states

Ambiguities appear in the partial-wave decomposition of two-body states

- Ambiguous amplitudes are **continuous** and can be calculated
- PWD fit/finite data has an effect on ambiguous solutions
- **Choice of included partial waves** may suppress the ambiguities

NIFTy: new approach to partial-wave analysis

- **Includes continuity, kinematics and regularization**
- Overcomes limitations of conventional approach
- Can include resonance-model fit
- Demonstrated in Monte Carlo pseudodata studies

julien.beckers@tum.de

Outlook

We can combine both presented topics

→ Apply NIFTy method on ambiguity problem in $K_S^0 K^-$

- Use continuity constraints to separate ambiguous solutions over entire mass range
- Improve fit quality

Partial-wave analysis using NIFTy model is being successfully applied on real data

Thank you for your attention!

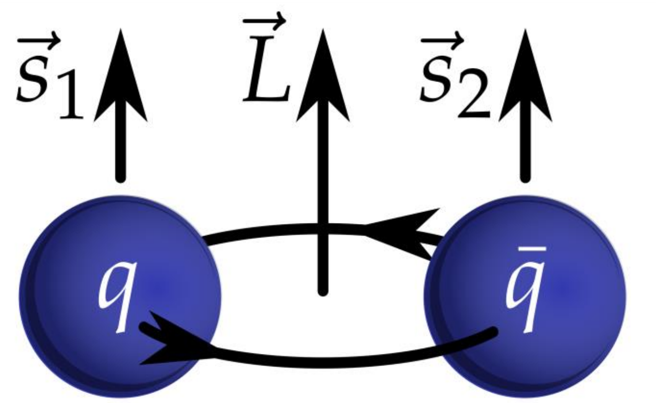
BACKUP

QCD in the Resonance Region

- At low energies (hadron regime): **QCD not solvable perturbatively**

- Theory: rely on models and effective theories, e.g. **quark model** (hadrons as bound states of **valence quarks**)

- Experimentally: **precision measurements** of hadronic states and search for so-called **exotic states** (forbidden in the quark model)



From Prog.Part.Nucl.Phys. 113 (2020) 103755

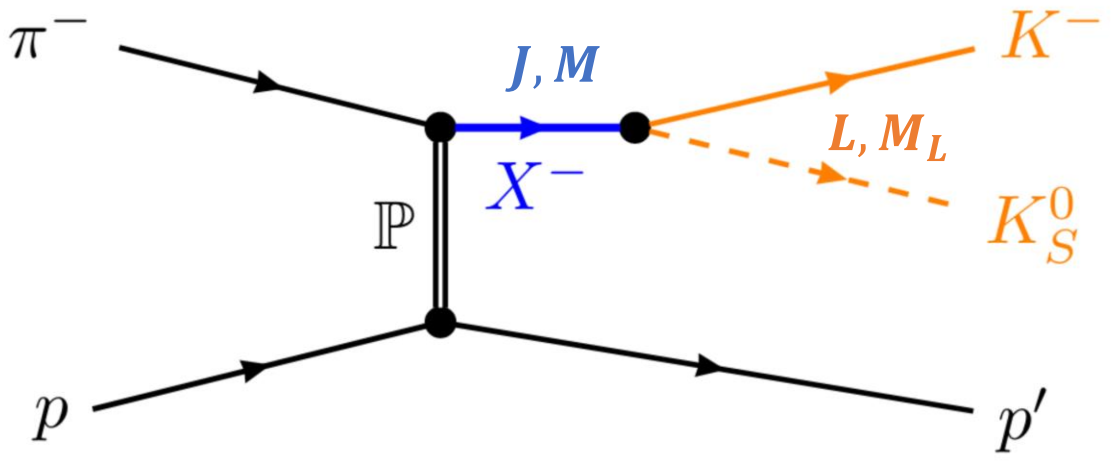
Partial-Wave Decomposition

$$I(m_X, t'; \theta, \phi) = \frac{dN}{dm_X dt' d\theta d\phi} = \left|M_{fi}\right|^2$$

- Separate process amplitude into **partial waves**
  - Spin $\boldsymbol{J}$ and spin-projection $\boldsymbol{M}$

$$\boldsymbol{J} = \boldsymbol{L}, \boldsymbol{M} = \boldsymbol{M_L}$$

$$\boldsymbol{P} = \boldsymbol{C} = (-1)^{\boldsymbol{J}}$$



Partial wave:
specific $(\boldsymbol{J^{PC}M})$

Partial-Wave Decomposition

$$I(m_X, t'; \theta, \phi) = \frac{dN}{dm_X dt' d\theta d\phi} = \left| \sum_J T_J(m_X, t')\, \psi_J(\theta, \phi) \right|^2$$

- Separate process amplitude into **partial waves**
  - Spin $\boldsymbol{J}$ and spin-projection $\boldsymbol{M}$

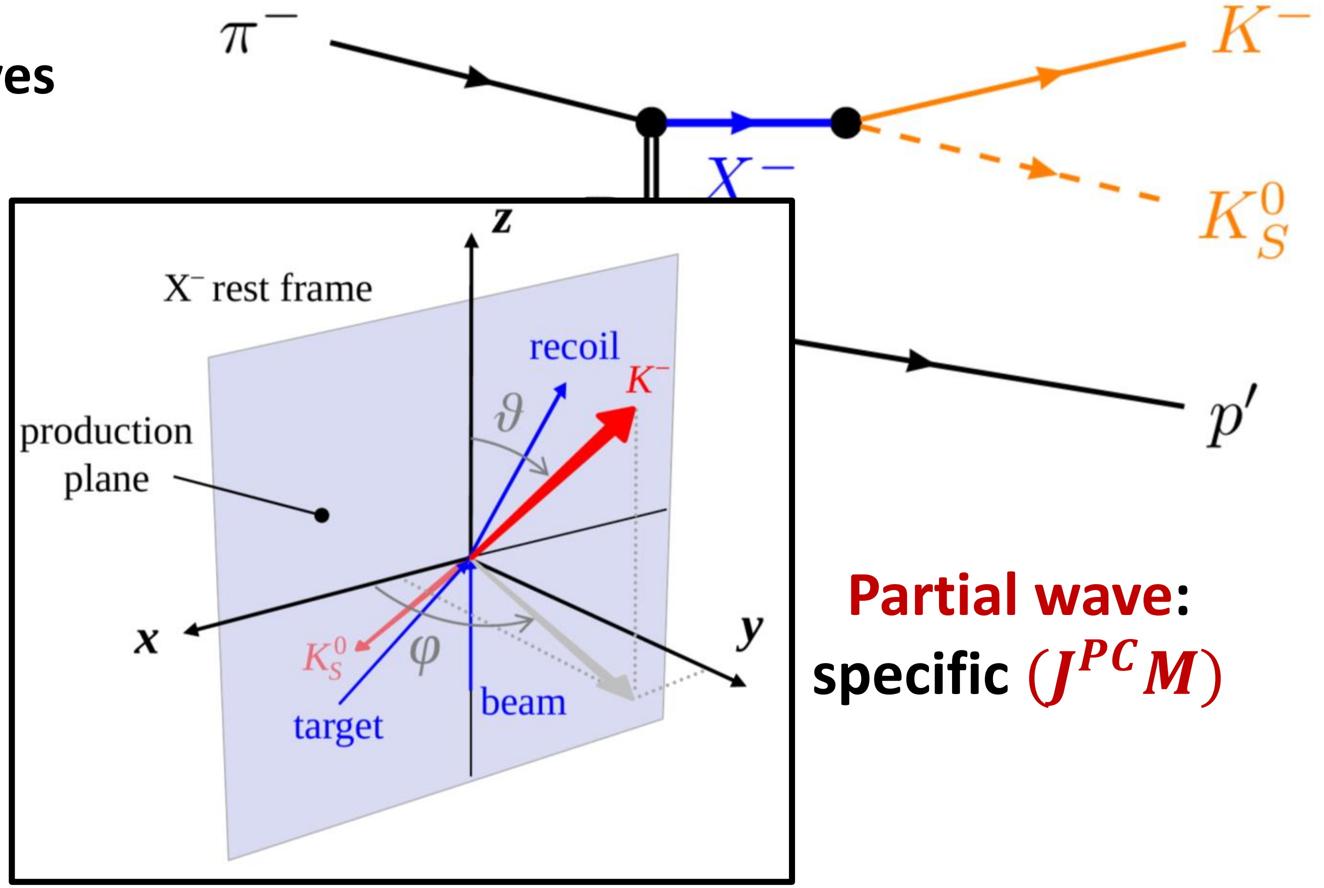
- Production, propagation and decay of X^-

$$\boldsymbol{T_{JM}(m_X, t')}$$

$$\boldsymbol{\psi_J(\theta, \phi) = Y_J^1(\theta, \phi)}$$

$$\boldsymbol{M = 1}$$

$(\text{reflectivity basis}, \varepsilon = -1 \text{ suppressed} \rightarrow \mathrm{M} \neq 0\ )$



Partial wave:
specific $(J^{PC}M)$

Partial-Wave Decomposition

$$I(m_X, t'; \theta, \phi) = \frac{dN}{dm_X\,dt'\,d\theta\,d\phi} = \left| \sum_J T_J(m_X, t')\, \psi_J(\theta, \phi) \right|^2$$

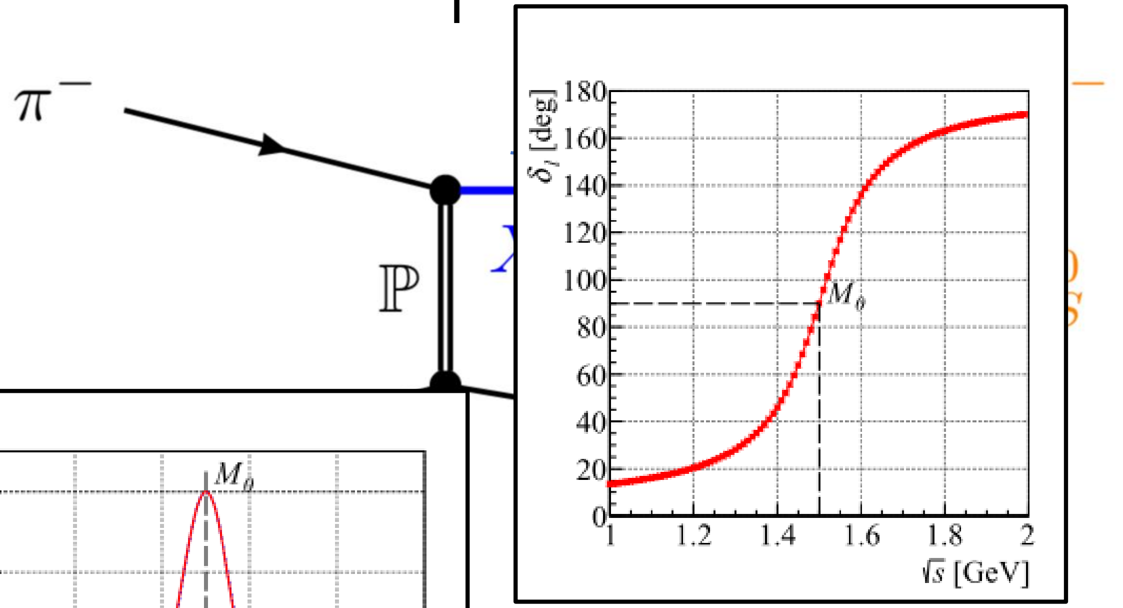
- Separate process amplitude into **partial waves**
  - Spin $\boldsymbol{J}$ and spin-projection $\boldsymbol{M}$
- Production, propagation and decay of X^-

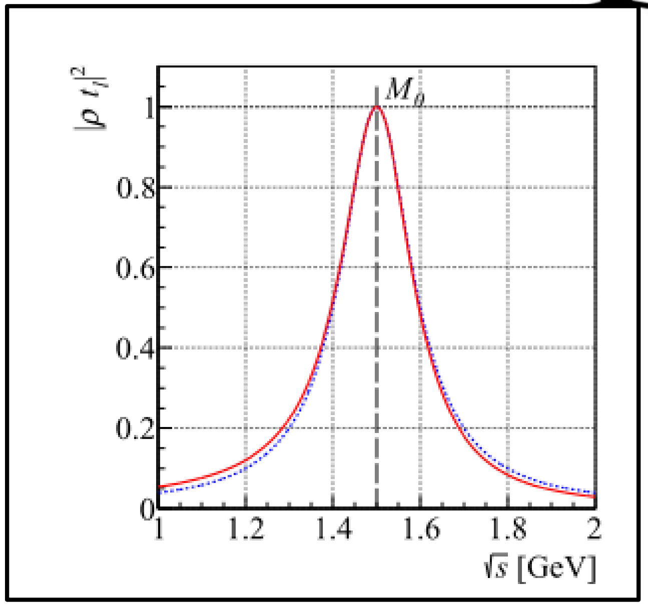
$$\boldsymbol{T_J(m_X, t') = P(m_X, t')\, D(m_X)}$$

$$\boldsymbol{\psi_J(\theta, \phi) = Y_J^M(\theta, \phi)}$$

$$\boldsymbol{M = 1}$$

- Fit $I(m_X, t'; \theta, \phi)$ to data **in $(\mathbf{m_X}, \mathbf{t'})$ bins**:
- Choose finite set of $\{J^{PC}\}$





Partial wave:
specific (J^{PC})

From B. Ketzer et al., Prog.Part.Nucl.Phys. 113 (2020) 103755, Light-Meson Spectroscopy with COMPASS

Ambiguities in Incoherent Sectors

$$\boldsymbol{\varepsilon = \pm 1}: \quad I(m_X, t'; \theta, \phi) = \left| \sum_{JM} T^{+}_{JM}(m_X, t')\, \psi^{+}_{JM}(\theta, \phi) \right|^2 + \left| \sum_{JM} T^{-}_{JM}(m_X, t')\, \psi^{-}_{JM}(\theta, \phi) \right|^2$$

$$a_0^- = \sum_{J=0}^{J^-_{\max}} T^-_{J0}\, Y_J^0(\theta, 0) \qquad \varepsilon = -1, M = 0$$

$$a_1^- = \sum_{J=1}^{J^-_{\max}} T^-_{J1}\, Y_J^1(\theta, 0) \qquad \varepsilon = -1, M = 1$$

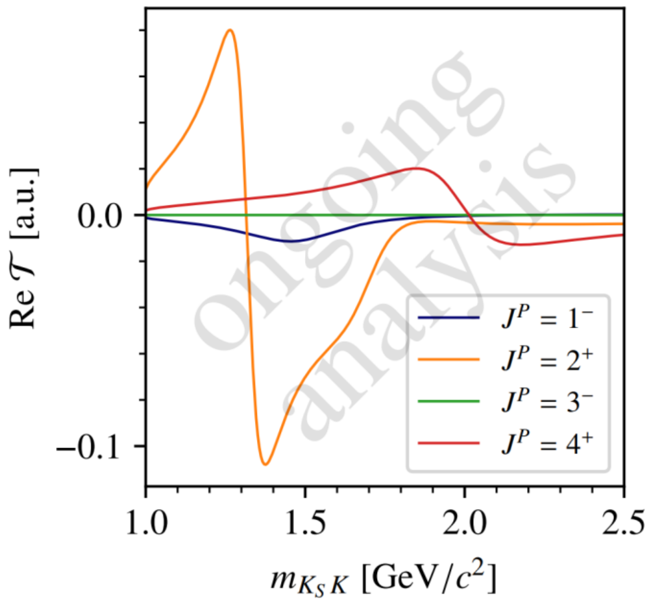
$$a_1^+ = \sum_{J=1}^{J^+_{\max}} T^+_{J1}\, Y_J^1(\theta, 0) \qquad \varepsilon = +1, M = 1$$

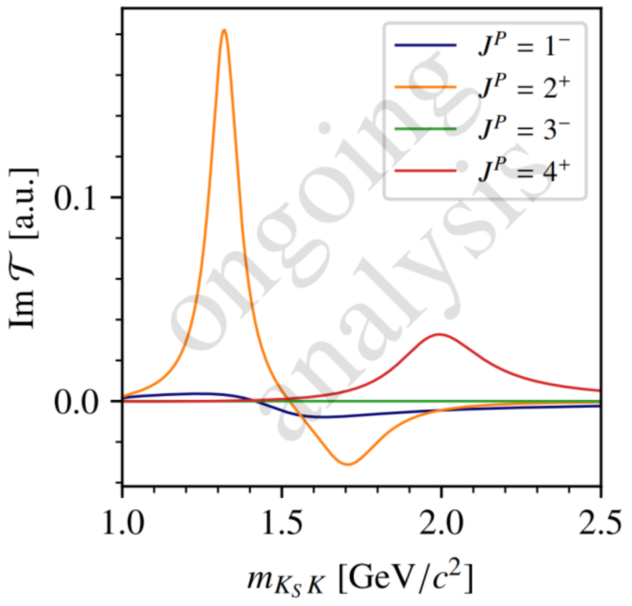
- $a_s^- = a_0^- + a_1^-$, then same procedure as for a single sector
- New amplitudes for $\varepsilon = +1$: $|a_1^+|^2 = |a_1^-|^2 - \text{const.}$ → **positivity requirement**!

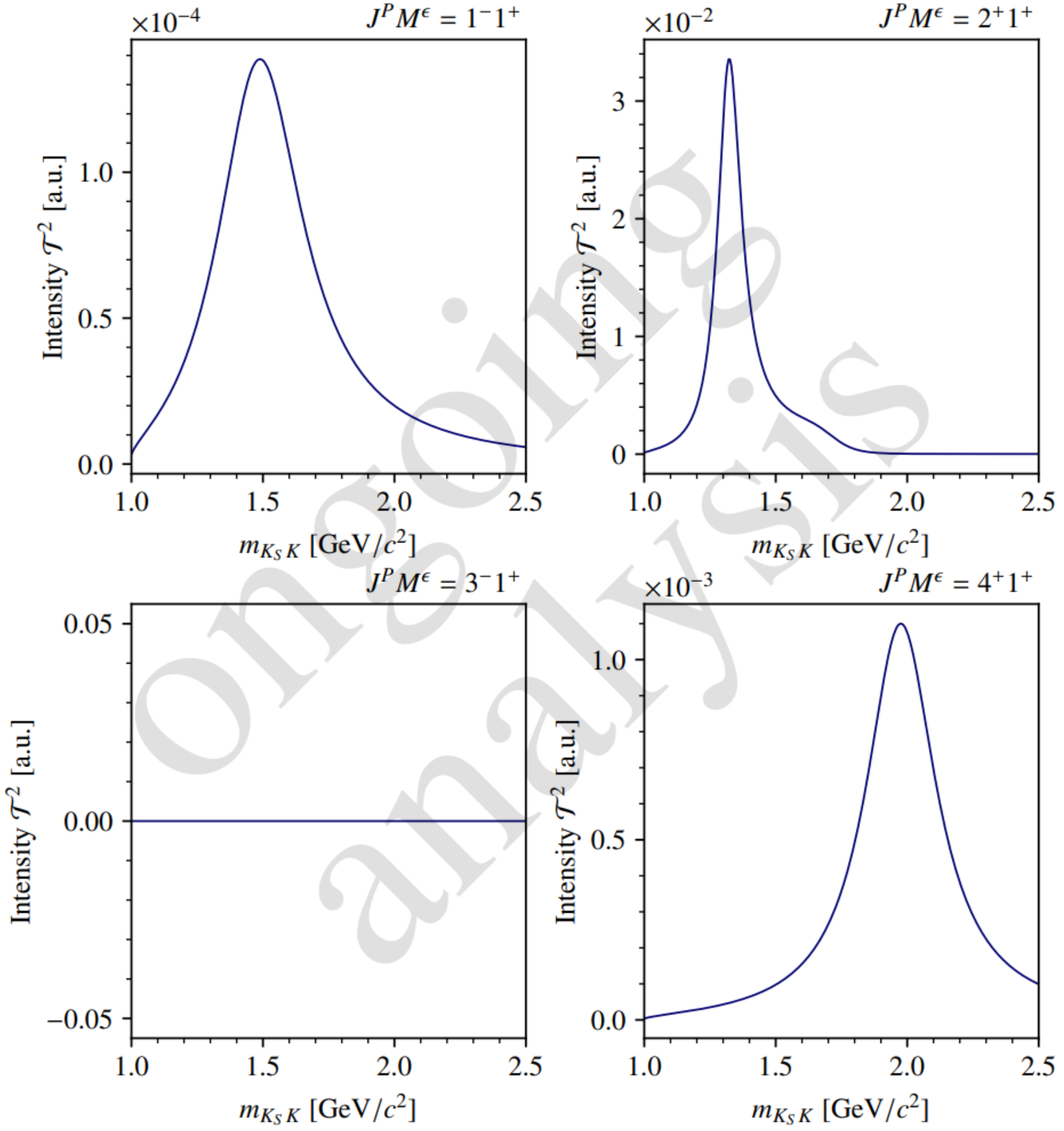
Continuous Amplitude Model

I. Continuous intensity model

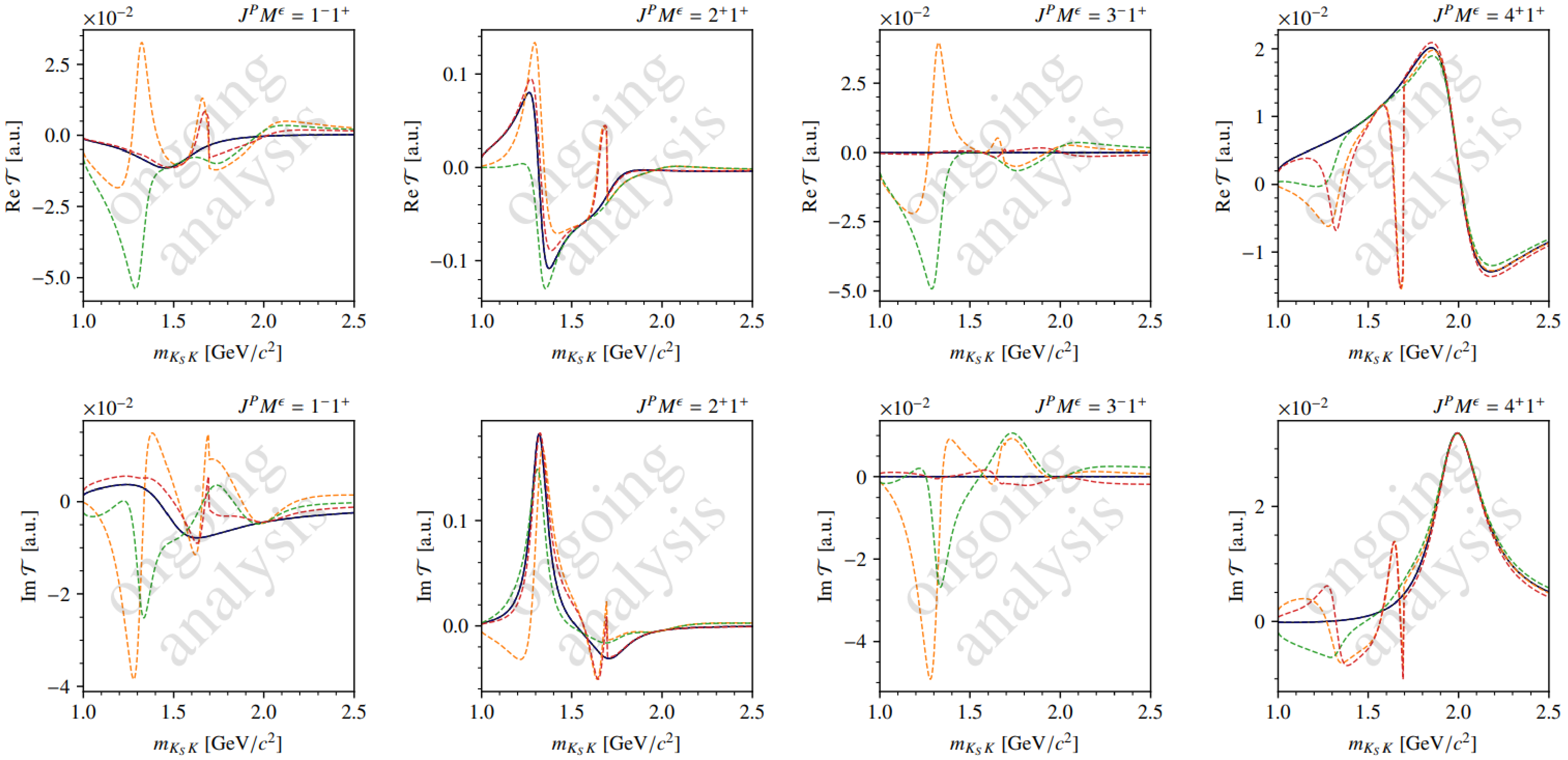
- create a model for the amplitudes in four waves
- m_X-dependence by Breit-Wigner amplitudes

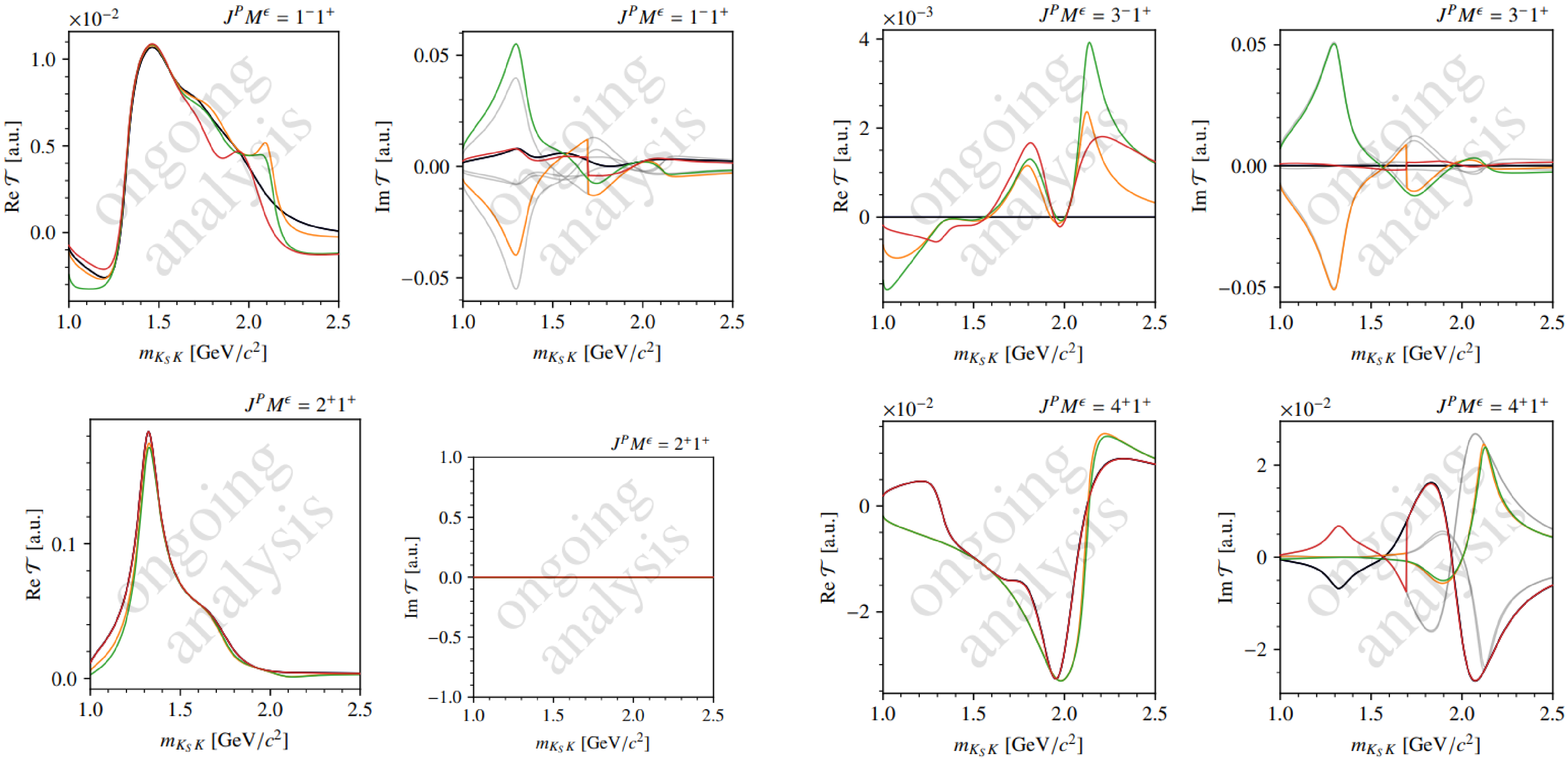




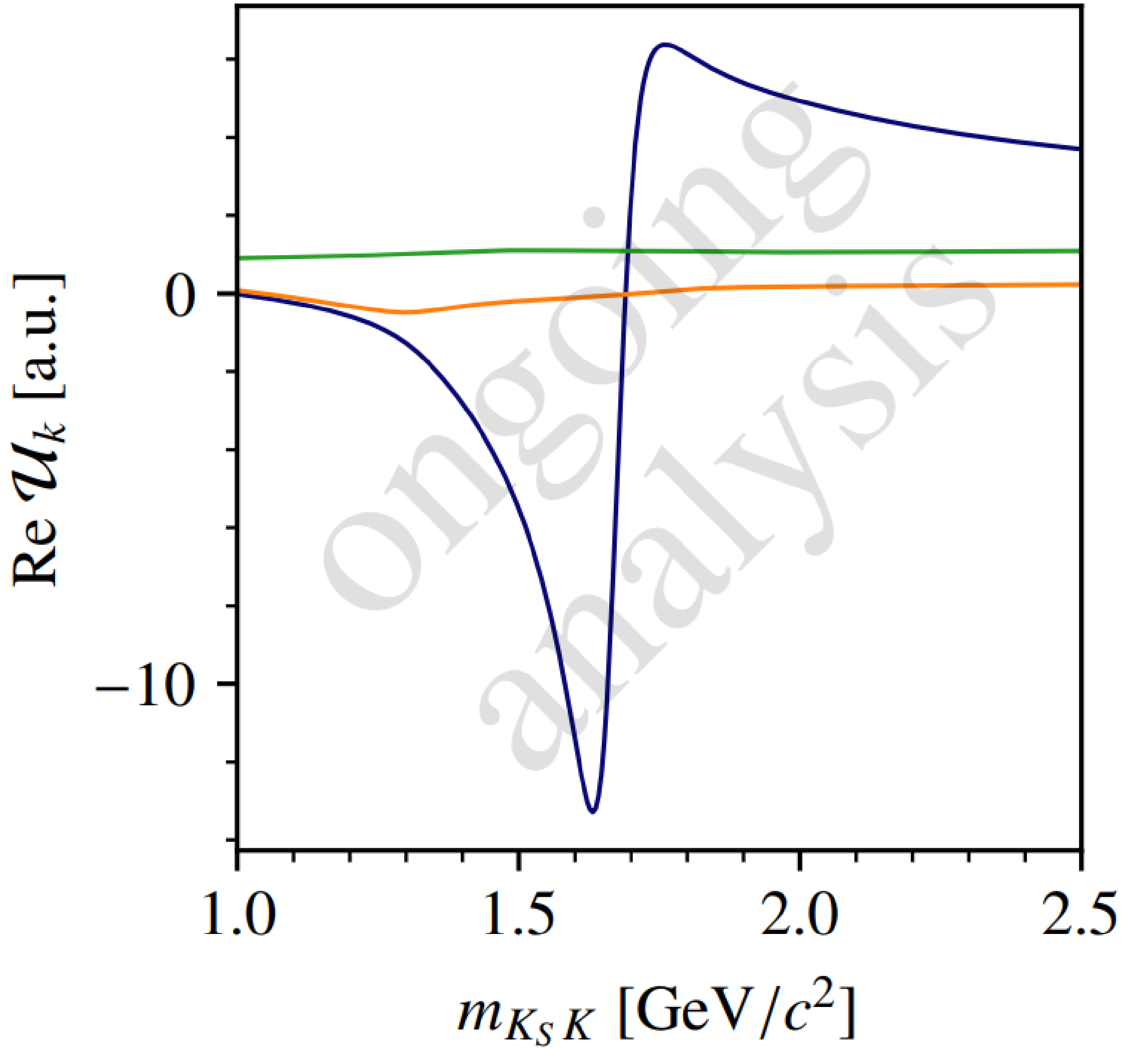


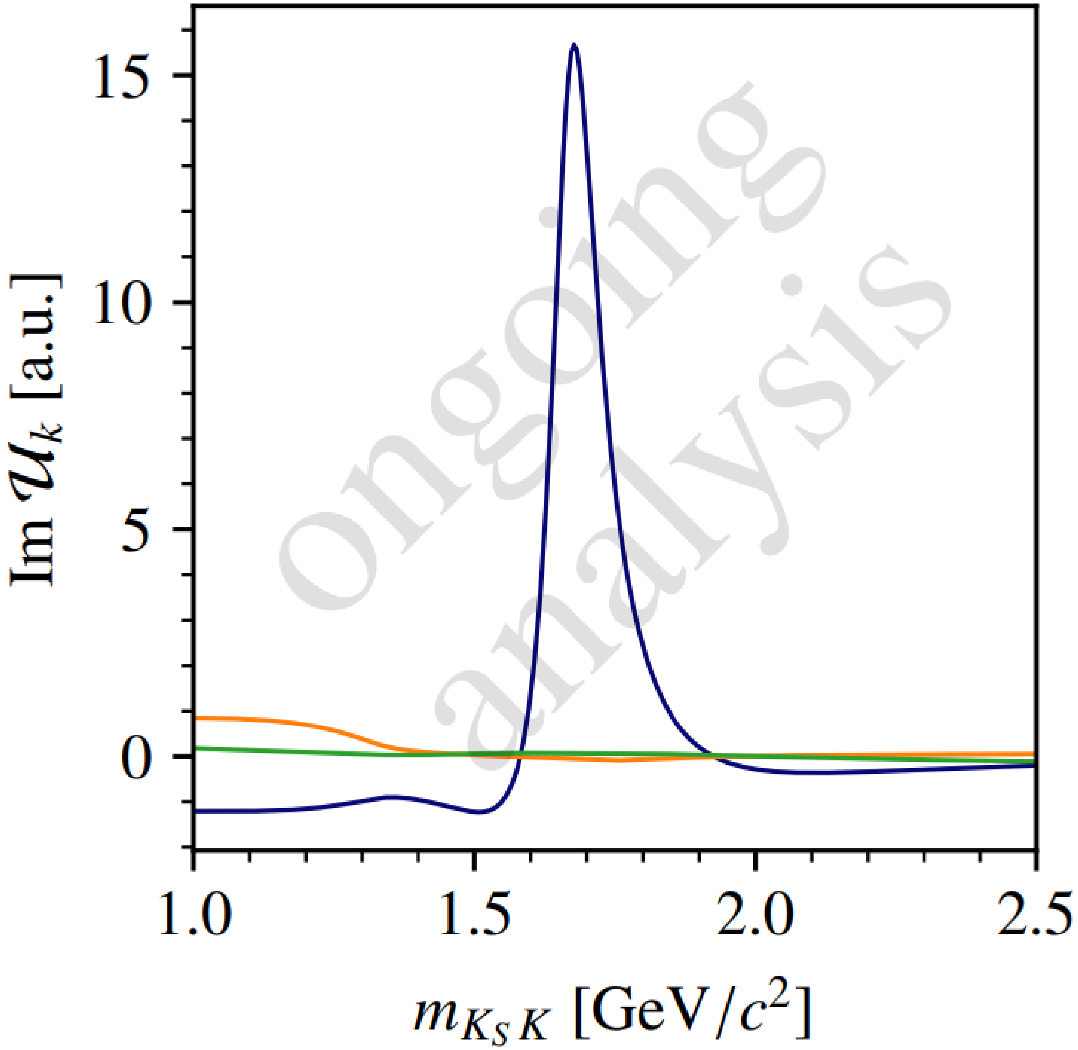
J^P	resonance content	m_0 [GeV/c^2]	Γ_0 [GeV/c^2]	c	ϕ
1^-	$\rho(1450)$	1.465	0.400	0.0564	1.8023
2^+	$a_2(1320)$	1.3181	0.1098	1	0
	$a_2(1700)$	1.698	0.265	0.1480	π
3^-	None	x	x	x	x
4^+	$a_4(1970)$	1.967	0.324	0.1274	6.0072

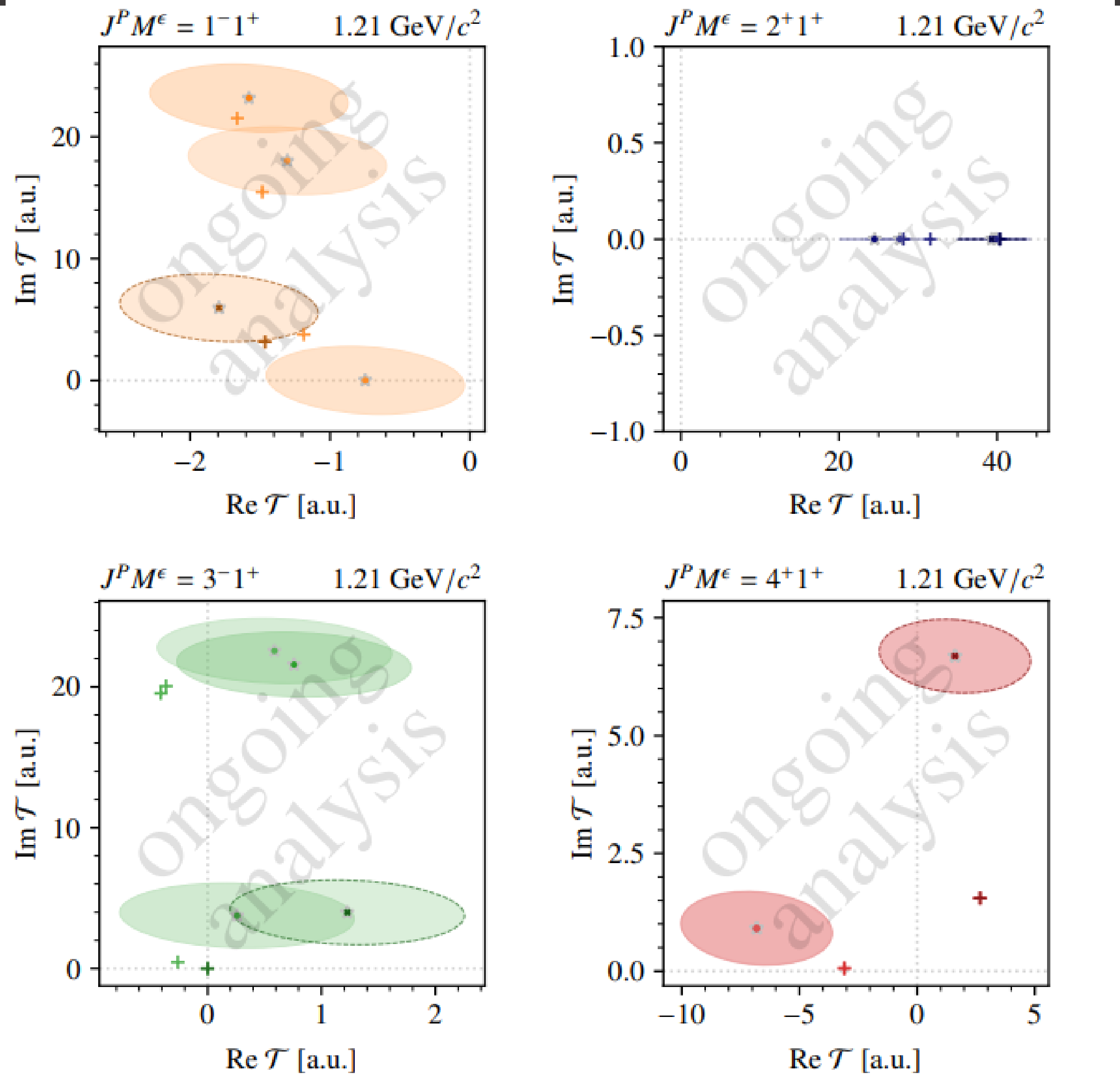

$\times 10^{-2}$ $J^P M^\epsilon = 1^-1^+$
$\mathrm{Re}\,\mathcal{T}$ [a.u.]
$J^P M^\epsilon = 2^+1^+$
$\times 10^{-2}$ $J^P M^\epsilon = 3^-1^+$
$\times 10^{-2}$ $J^P M^\epsilon = 4^+1^+$
$\mathrm{Im}\,\mathcal{T}$ [a.u.]
$m_{K_S K}$ [GeV/c^2]
ongoing analysis

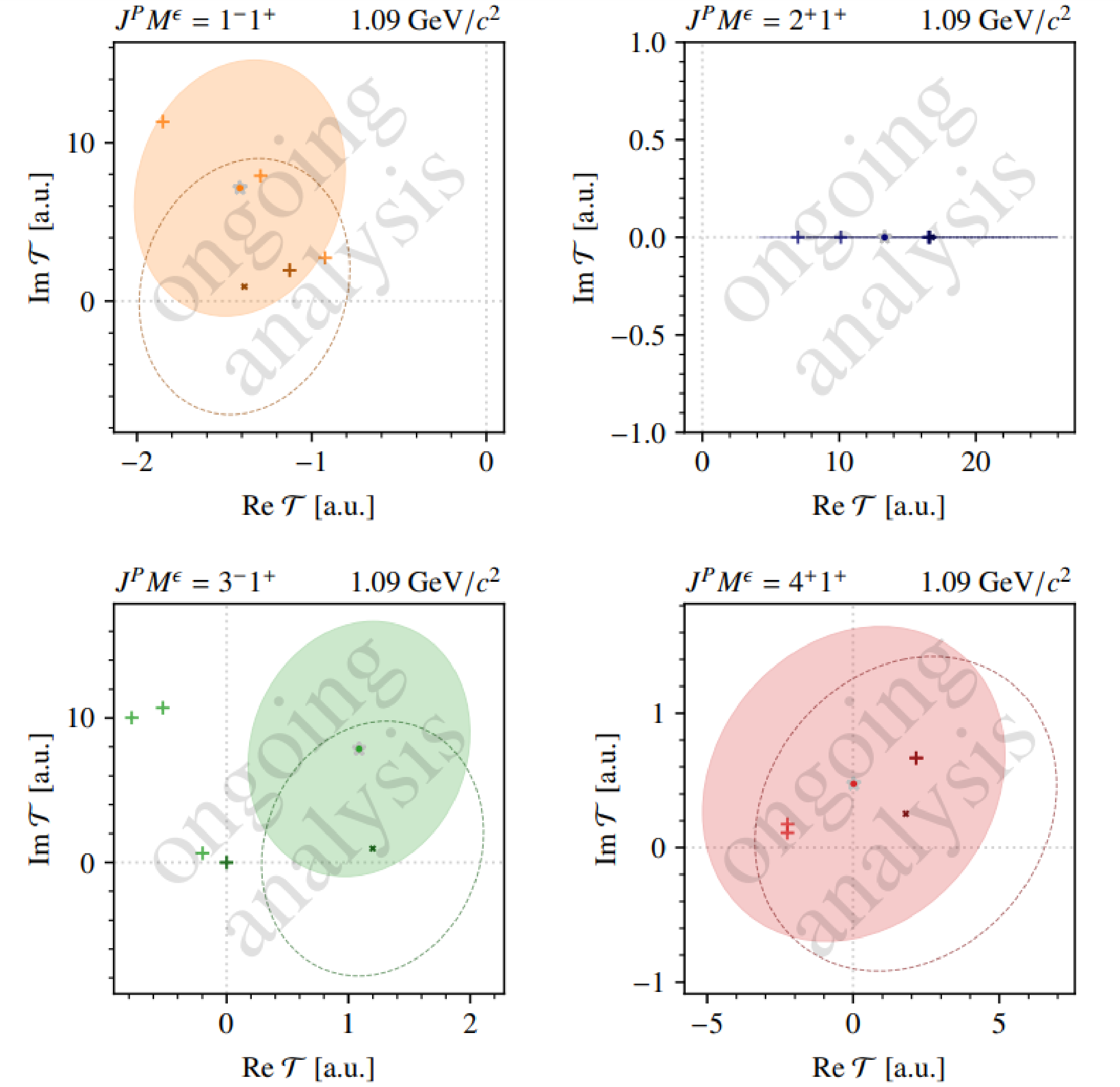


$J^P M^\epsilon = 1^- 1^+$
$\times 10^{-2}$
$\mathrm{Re}\,\mathcal{T}$ [a.u.]
$\mathrm{Im}\,\mathcal{T}$ [a.u.]
$J^P M^\epsilon = 3^- 1^+$
$\times 10^{-3}$
$J^P M^\epsilon = 2^+ 1^+$
$J^P M^\epsilon = 4^+ 1^+$
$m_{K_S K}$ [GeV/c^2]
ongoing analysis


0
−10
1.0
1.5
2.0
2.5
Re $\mathcal{U}_k$ [a.u.]
$m_{K_S K}$ [GeV/c^2]
ongoing analysis


15
10
5
0
1.0
1.5
2.0
2.5
Im $\mathcal{U}_k$ [a.u.]
$m_{K_S K}$ [GeV/c^2]
ongoing analysis


$J^P M^\epsilon = 1^- 1^+$ 1.21 GeV/c^2
$J^P M^\epsilon = 2^+ 1^+$ 1.21 GeV/c^2
$J^P M^\epsilon = 3^- 1^+$ 1.21 GeV/c^2
$J^P M^\epsilon = 4^+ 1^+$ 1.21 GeV/c^2
Im $\mathcal{T}$ [a.u.]
Re $\mathcal{T}$ [a.u.]
ongoing analysis


$J^P M^\epsilon = 1^- 1^+$ 1.09 GeV/c^2
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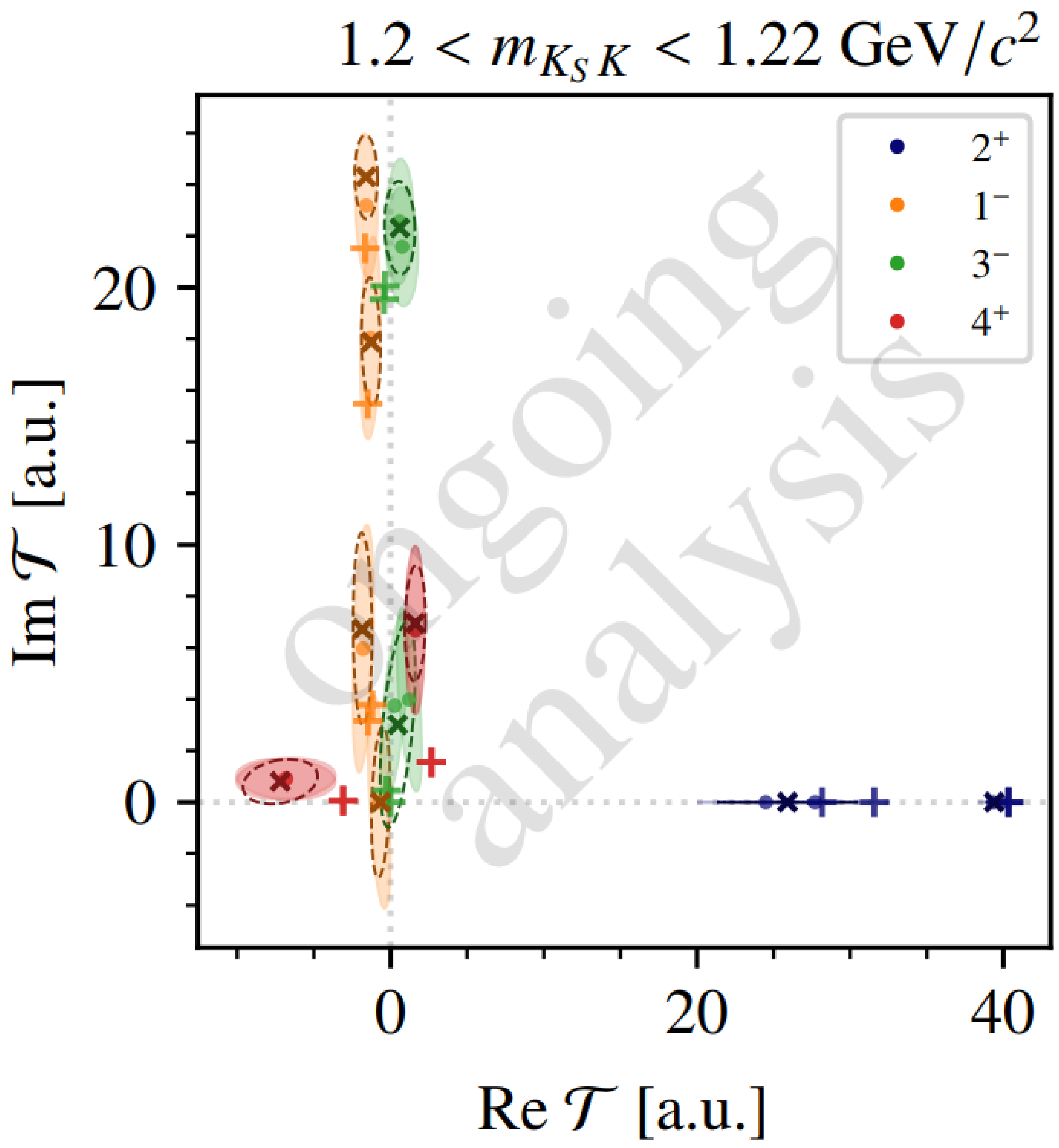
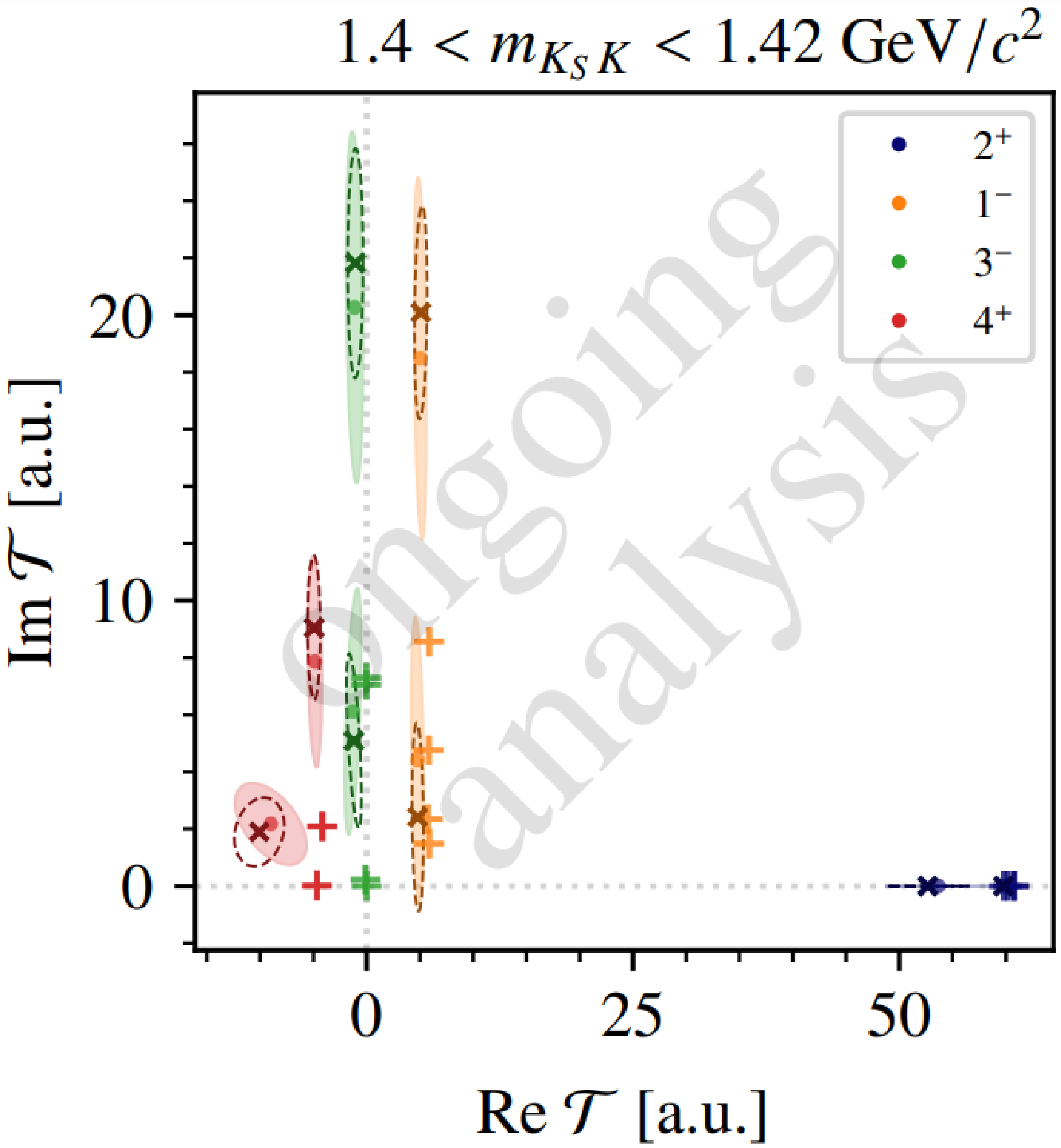
Im $\mathcal{T}$ [a.u.]
Re $\mathcal{T}$ [a.u.]
ongoing analysis



$1.2 < m_{K_S K} < 1.22\ \mathrm{GeV}/c^2$
Im $\mathcal{T}$ [a.u.]
Re $\mathcal{T}$ [a.u.]
2^+
1^-
3^-
4^+
Ongoing analysis



$1.4 < m_{K_S K} < 1.42\ \mathrm{GeV}/c^2$
Im $\mathcal{T}$ [a.u.]
Re $\mathcal{T}$ [a.u.]
2^+
1^-
3^-
4^+
Ongoing analysis

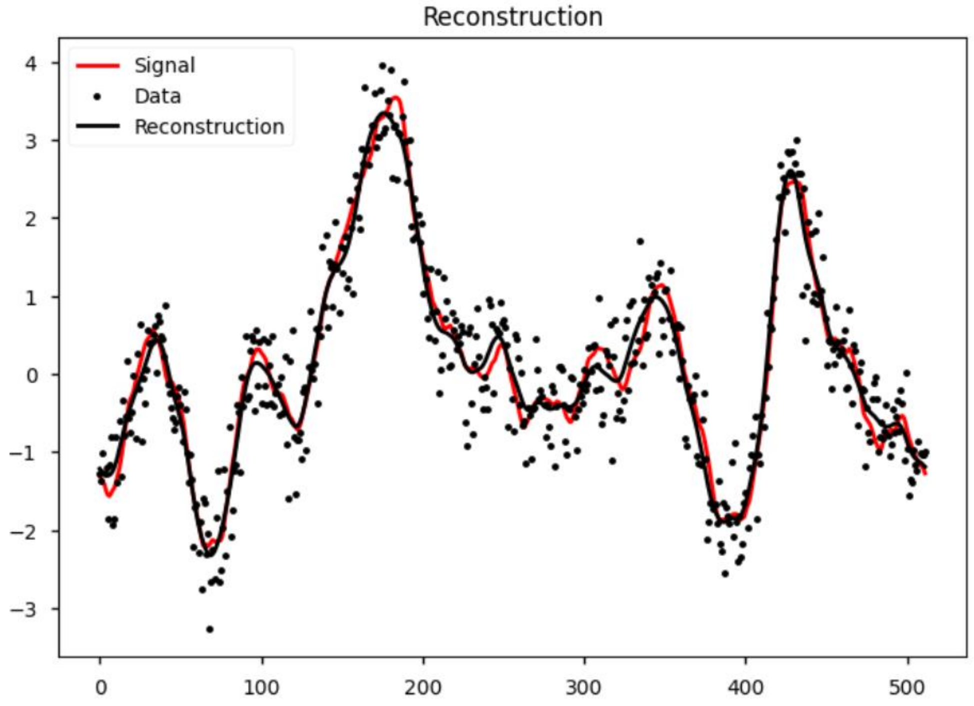
Implementation: NIFTy Framework

Framework by team from the Max-Planck Institute for Astrophysics:

NIFTY: “**N**umerical **I**nformation **F**ield **T**heory”

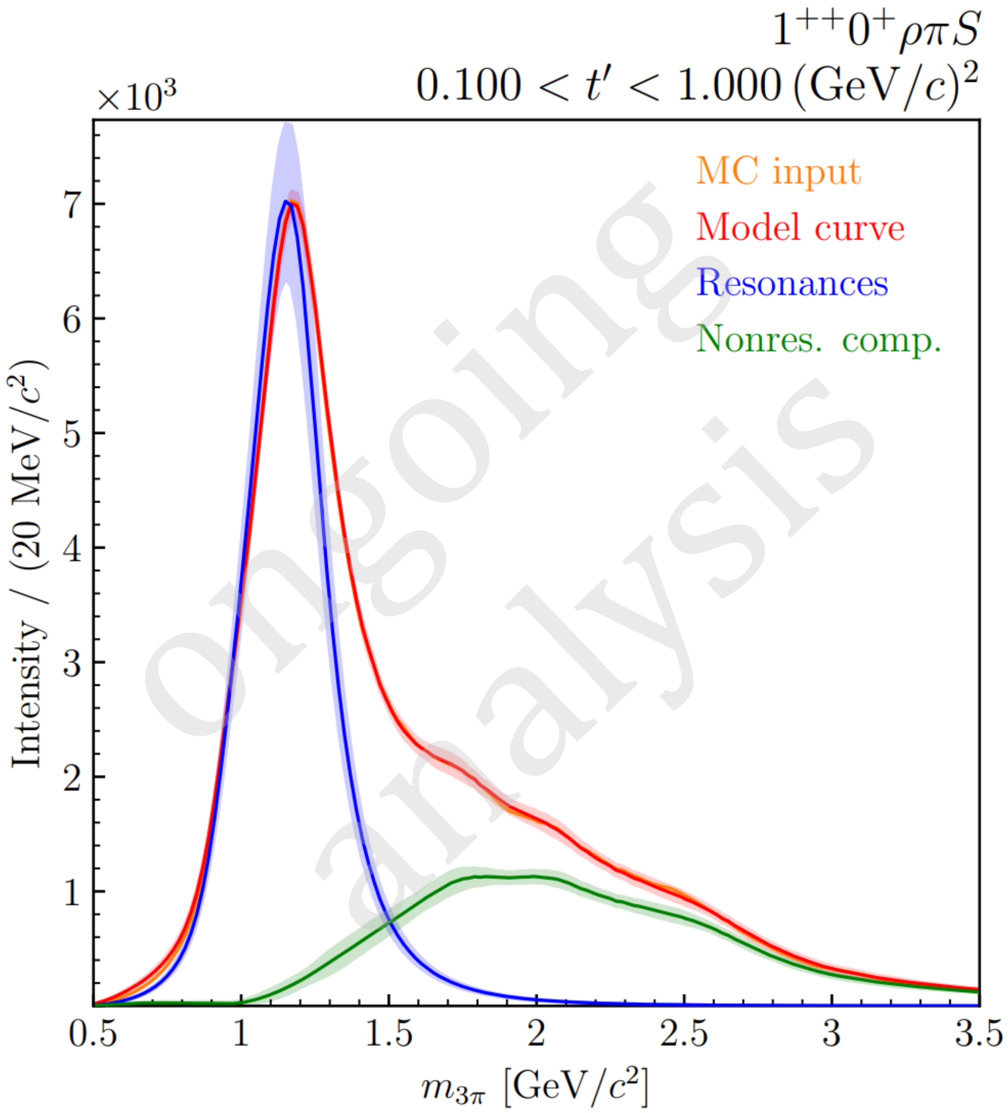
- Provides continuous non-parametric models

https://ift.pages.mpcdf.de/nifty/

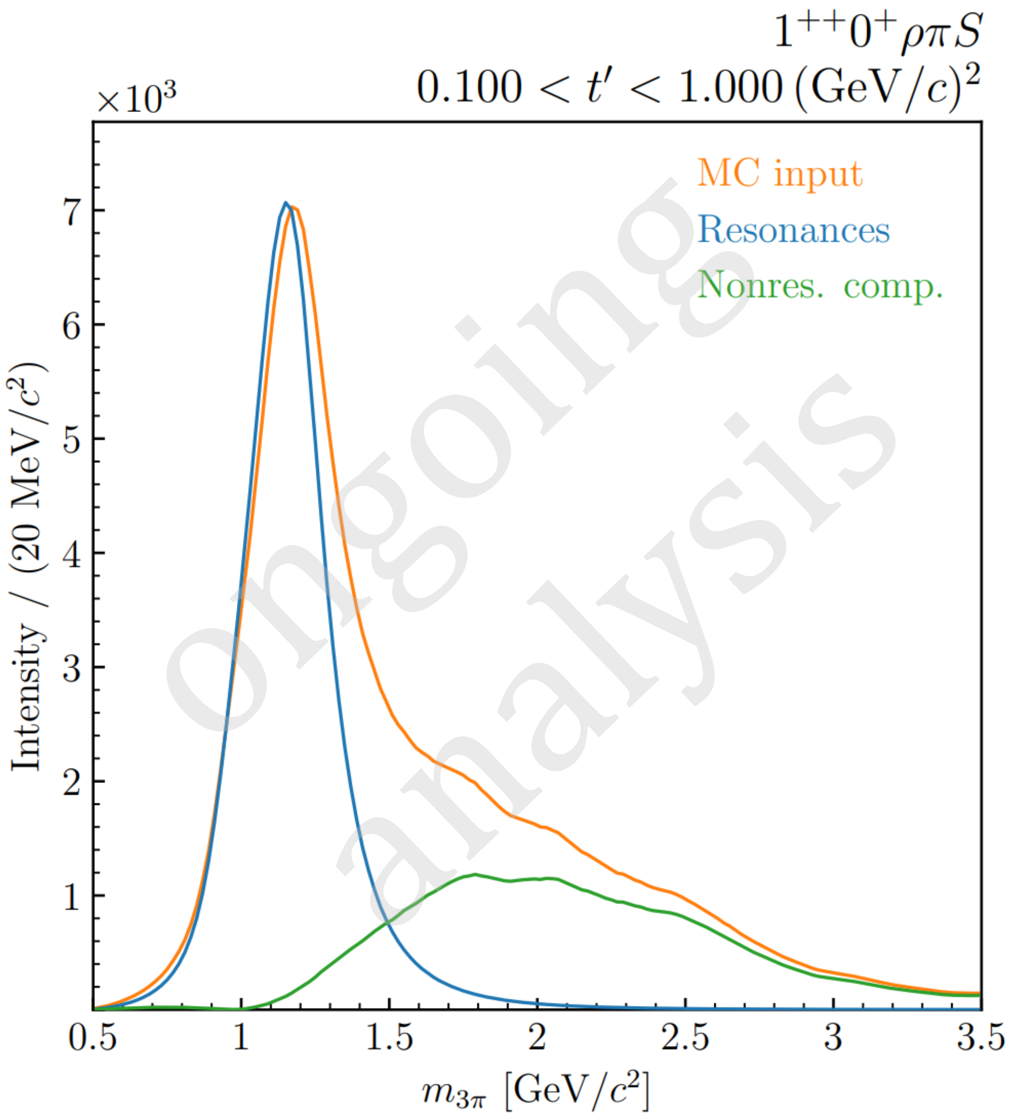


https://ift.pages.mpcdf.de/nifty/user/getting_started_0.html

This work is done in collaboration with Jakob Knollmueller (TUM / ORIGINS Excellence Cluster)

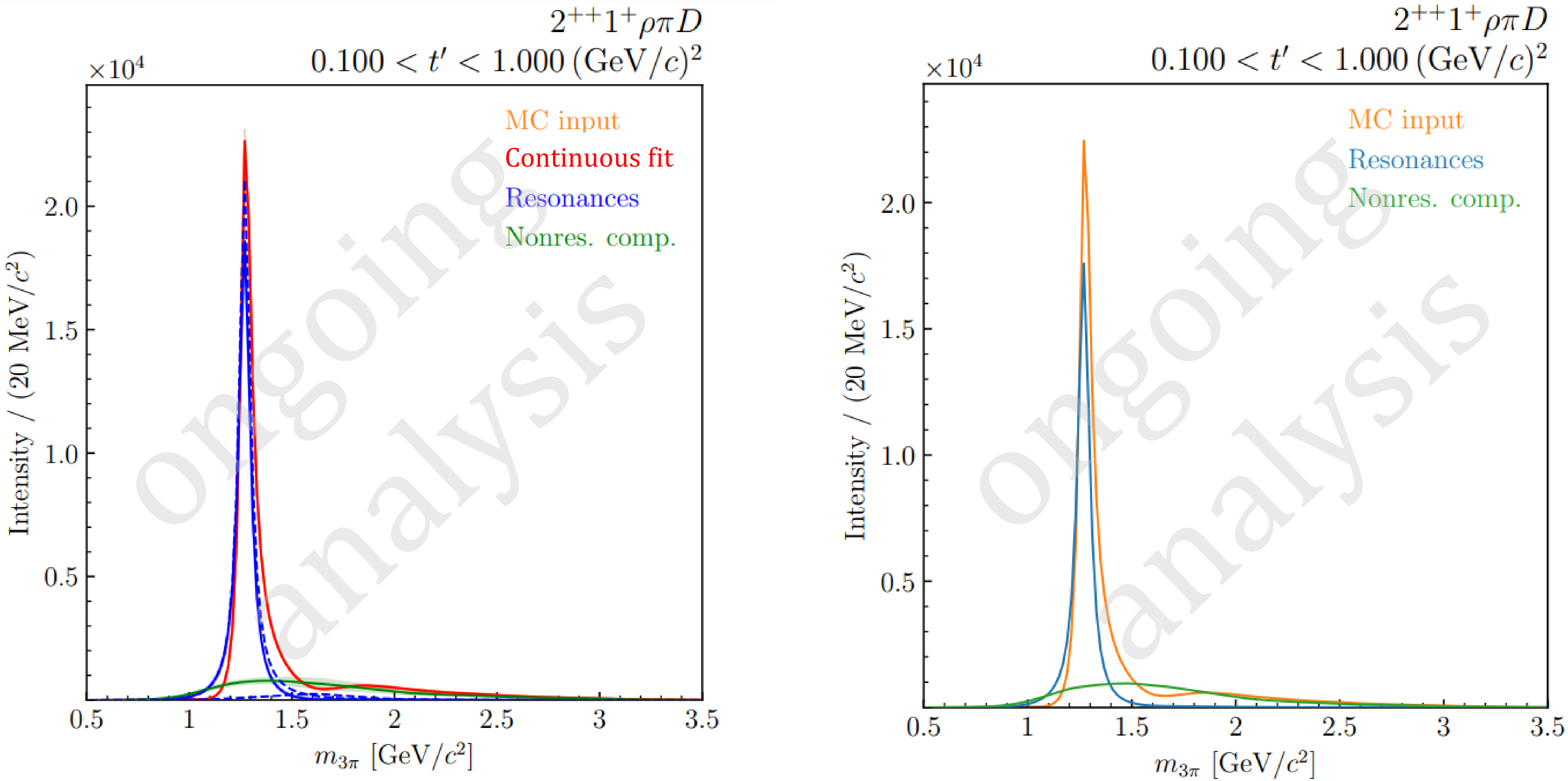


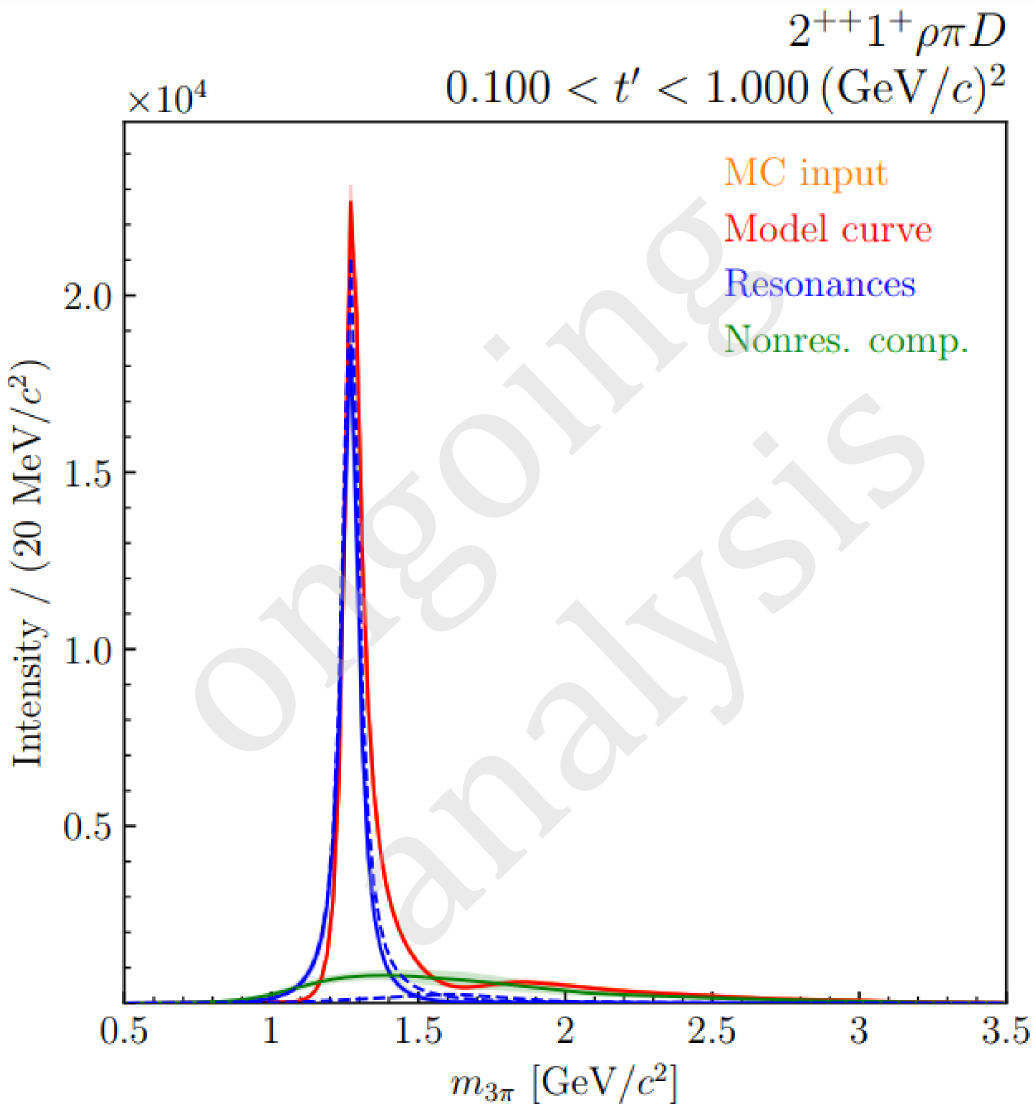
$1^{++}0^{+}\rho\pi S$
$0.100 < t' < 1.000\,(\mathrm{GeV}/c)^2$
$\times 10^3$
MC input
Model curve
Resonances
Nonres. comp.
Intensity / (20 MeV/c^2)
$m_{3\pi}$ [GeV/c^2]
ongoing analysis

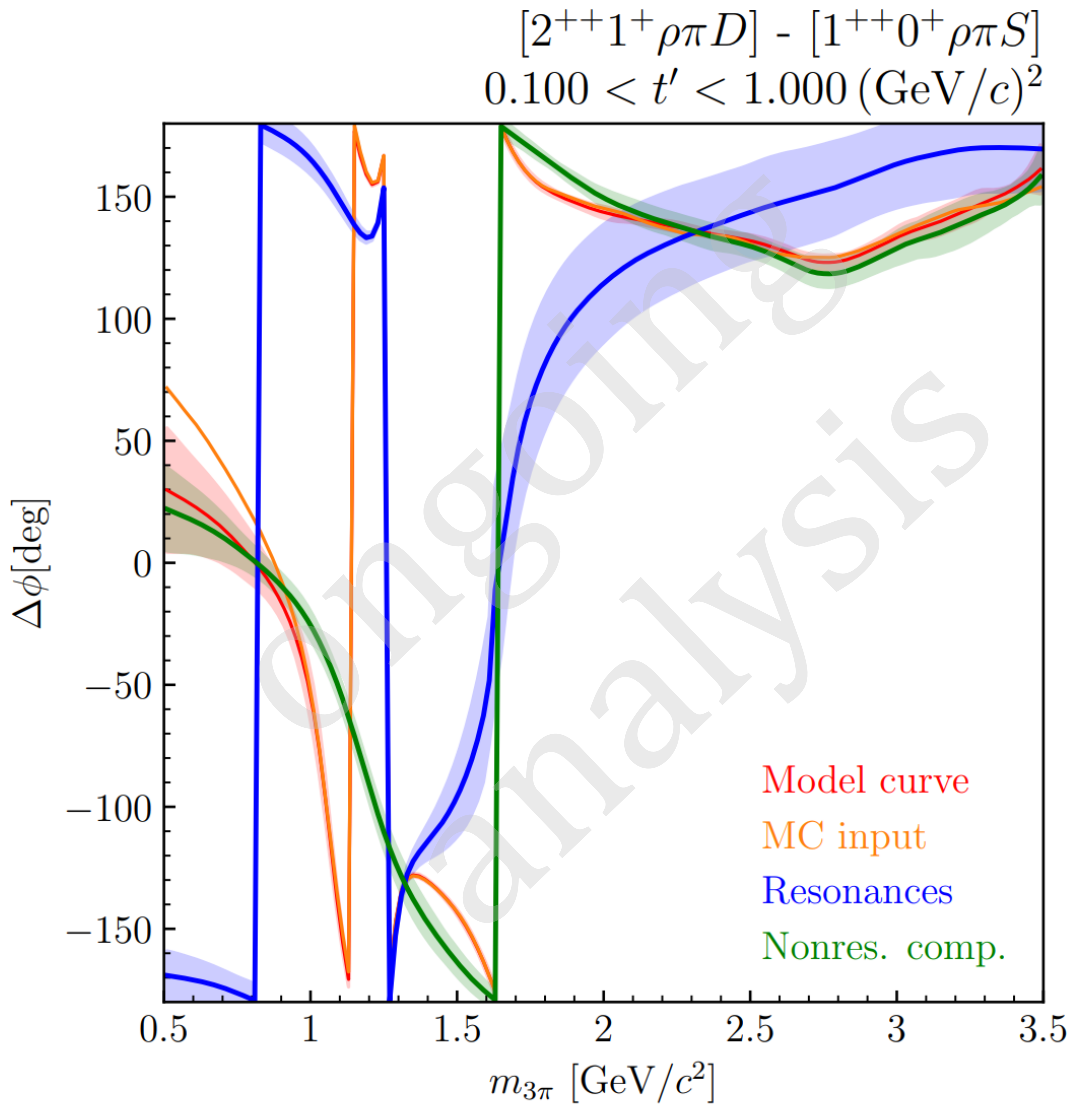


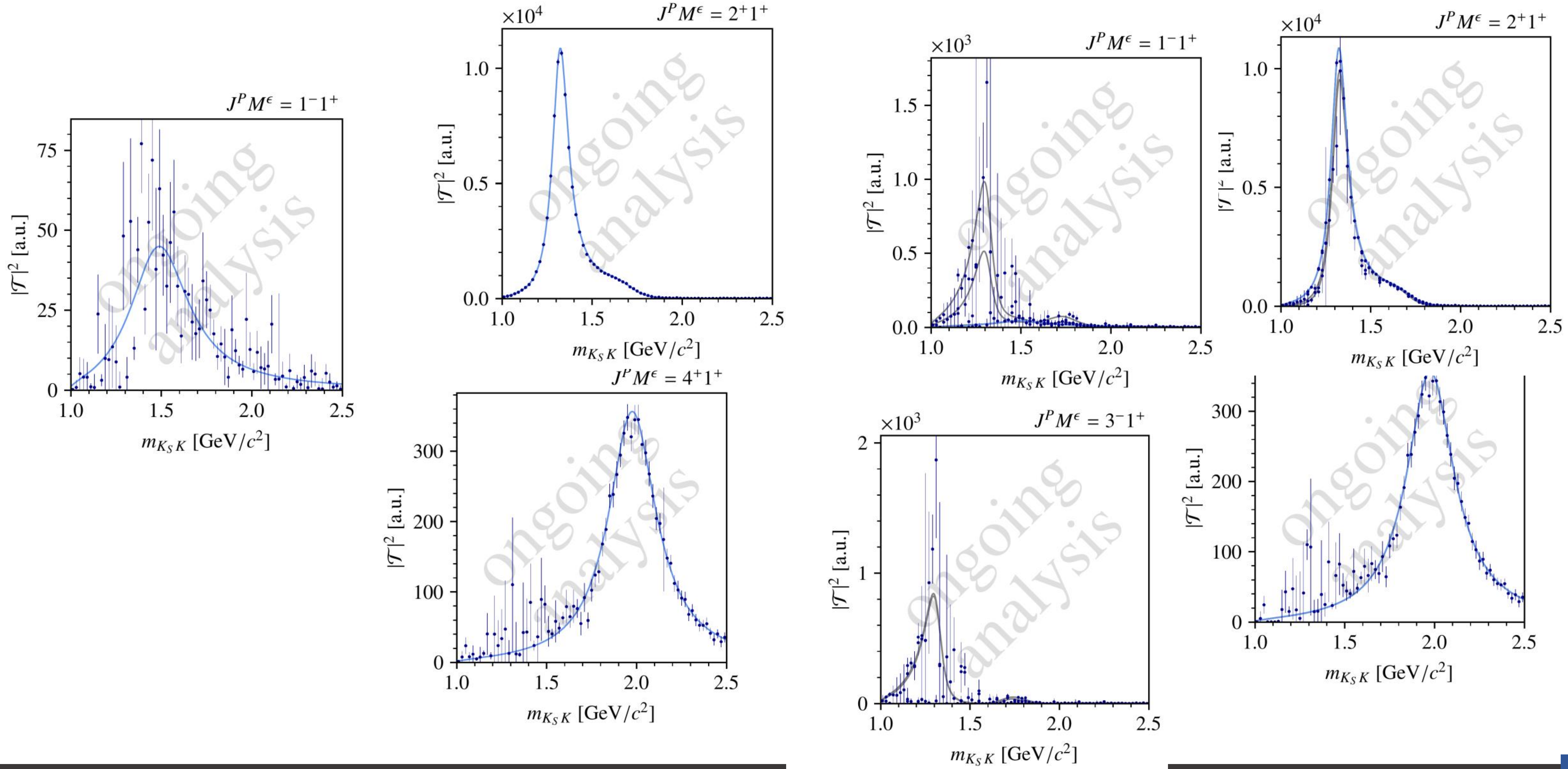
$1^{++}0^{+}\rho\pi S$
$0.100 < t' < 1.000\,(\mathrm{GeV}/c)^2$
$\times 10^3$
MC input
Resonances
Nonres. comp.
Intensity / (20 MeV/c^2)
$m_{3\pi}$ [GeV/c^2]
ongoing analysis

Input-Output Study




$2^{++}1^{+}\rho\pi D$
$0.100 < t' < 1.000\,(\mathrm{GeV}/c)^2$
$\times 10^4$
MC input
Model curve
Resonances
Nonres. comp.
ongoing analysis
Intensity / (20 MeV/c^2)
$m_{3\pi}$ [GeV/c^2]


$[2^{++}1^{+}\rho\pi D]$ - $[1^{++}0^{+}\rho\pi S]$
$0.100 < t' < 1.000\,(\mathrm{GeV}/c)^2$
ongoing analysis
Model curve
MC input
Resonances
Nonres. comp.
$\Delta\phi$[deg]
$m_{3\pi}$ [GeV/c^2]


$J^P M^\epsilon = 1^-1^+$
$J^P M^\epsilon = 2^+1^+$
$J^P M^\epsilon = 4^+1^+$
$J^P M^\epsilon = 1^-1^+$
$J^P M^\epsilon = 2^+1^+$
$J^P M^\epsilon = 3^-1^+$
$\times 10^4$
$\times 10^3$
$|\mathcal{T}|^2$ [a.u.]
$m_{K_S K}$ [GeV/c^2]
ongoing analysis

Outlook: NIFTy on KsK- pseudodata