The compositeness of a bound state constrained by a and r_0 and the role of the interaction range

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Outline

1. **Motivation**
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3. **Results**
4. **Summary**

Song, Dai, Oset, "How much is the compositeness of a bound state constrained by a and r_0? The role of the interaction range", EPJA58 (2022) 133

1. Motivation

• To improve the Weinberg's formalism

starting from the pioneer work of Weinberg [Weinberg, "Evidence that the deuteron is not an elementary particle", PR137 (1965) B672] obtained in the limit of very small binding and zero range interaction in r-space.

scattering length effective range

$$a = R\left[\frac{2X_W}{1+X_W} + O(\frac{R_{\rm typ}}{R})\right] \quad r_0 = R\left[-\frac{1-X_W}{X_W} + O(\frac{R_{,\rm typ}}{R})\right]$$

$$f = \frac{1}{k\cot\delta - ik} \approx \frac{1}{-\frac{1}{a} + \frac{1}{2}r_0k^2 - ik} \quad \text{(scattering matrix)}$$

Experimental data for deuteron ($I = 0$, $J = 1$)

$$a = 5.419(7)\ \text{fm}\,,\ r_0 = 1.766(8)\ \text{fm}\,,\ B = 2.224575(9)\ \text{MeV} \text{(small binding)}$$

$\Longrightarrow X_W \simeq 1.68$ **unacceptable**

the actual compositeness should be $X \leq 1$

$$\begin{cases} 1 \text{ scattering length} \\ 2 \text{ the effective range} \\ 3 \text{ the binding energy} \end{cases} \Longrightarrow \text{compositeness} + \text{range of the interaction}$$

determine simultaneously

• Application to three different cases

(small and large binding)

$$\begin{cases} \text{Deuteron} \\ D^*_{s_0}(2317) \\ D^*_{s_1}(2460) \end{cases} \Longrightarrow \quad Z, r_0$$

r_0 the effective range

Z nonmolecular compositeness

$X = 1 - Z$ **molecular compositeness**

2. Formalism

- We start from a potential written in momentum space as

$$\langle \boldsymbol{p}'|V|\boldsymbol{p}\rangle = V(\boldsymbol{p}',\boldsymbol{p}) = V\,\theta(q_{\max}-p')\theta(q_{\max}-p)$$

$q_{\max}$ is the range of the potential in momentum space. Its inverse would provide the range of the interaction in coordinate space (r-space).

- Next we solve the Bethe-Salpeter equation with this potential to obtain the T-matrix (four momentum) $\Longrightarrow$ The q^0 integration is readily done using Cauchy's residues

$$T(\boldsymbol{p}',\boldsymbol{p}) = V(\boldsymbol{p}',\boldsymbol{p}) + \int \frac{d^3\boldsymbol{q}}{(2\pi)^3} V(\boldsymbol{p}',\boldsymbol{q}) T(\boldsymbol{q},\boldsymbol{p}) \frac{w_1(\boldsymbol{q})+w_2(\boldsymbol{q})}{2w_1(\boldsymbol{q})w_2(\boldsymbol{q})} \frac{1}{s-(w_1(\boldsymbol{q})+w_2(\boldsymbol{q}))^2+i\epsilon}$$

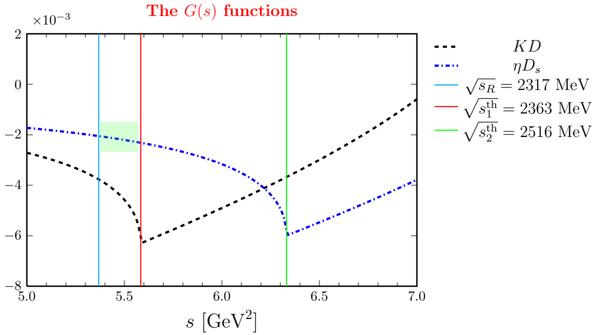
with $w_i(\boldsymbol{q}) = \sqrt{\boldsymbol{q}^2+m_i^2}$. By expanding in a power series

$$T(\boldsymbol{p}',\boldsymbol{p}) = \theta(q_{\max}-p')\theta(q_{\max}-p)\,T \quad \Longrightarrow T = V + VGT$$

$$\Longrightarrow T = [1-VG]^{-1}V \quad \text{(algebraic equation)} \qquad \Uparrow \; G(s)$$

• Energy dependence potential (linear function) $\Longrightarrow$ **meson-meson system**
$\Longrightarrow$ taking an example of $D^*_{s0}(2317)$ **($KD, \eta D_s$)**

$$V = V_{\rm eff} = V_0 + \beta\,(s - s_0)$$



Aceti, Dai, Geng, Oset & Zhang, "Meson-baryon components in the states of the baryon decuplet", EPJA50 (2014) 57 $\Longrightarrow$ how to construct an effective potential $V_{\rm eff}$ and evaluate the compositeness of finding molecular state (one channel)

- The scattering matrix with potential $V_{\rm eff}$ for one channel

$$T(s) = \frac{1}{[V_0 + \beta(s - s_0)]^{-1} - G(s)}$$

it has a pole at s_0 $\Longrightarrow$ hope to investigate

$$V_0^{-1} - G(s_0) = 0, \qquad V_0 = \frac{1}{G(s_0)}$$

- Establish the connection of amplitude $\Longrightarrow$ Quantum Mechanics

$$\frac{1}{C\left\{[\frac{1}{G(s_0)} + \beta(s - s_0)]^{-1} - G(s)\right\}} \approx \frac{1}{-\frac{1}{\mathbf{a}} + \frac{1}{2} r_0 k^2 - ik}$$

$C \Longrightarrow$ normalization constant

$$\mathrm{Im}\, G = -\frac{1}{8\pi}\frac{k}{\sqrt{s}} \quad \Longrightarrow \quad \text{establish the connection}$$

$$8\pi\sqrt{s}\left\{\Big[\frac{1}{G(s_0)}+\beta(s-s_0)\Big]^{-1}-\mathrm{Re}G(s)\right\}\approx\frac{1}{a}-\frac{1}{2}\,r_0\,k^2$$

- at threshold

$$8\pi\sqrt{s_{\mathrm{th}}}\left\{\Big[\frac{1}{G(s_0)}+\beta(s_{\mathrm{th}}-s_0)\Big]^{-1}-\mathrm{Re}G(s_{\mathrm{th}})\right\}=\frac{1}{a}\quad\Longrightarrow\beta$$

- derivative to k^2 at threshold

$$-\frac{1}{2}r_0=\frac{8\pi}{2\sqrt{s_{\mathrm{th}}}}\left[\Big[\frac{1}{G(s_0)}+\beta(s_{\mathrm{th}}-s_0)\Big]^{-1}-\mathrm{Re}G(s)_{\mathrm{th}}\right]\frac{s}{w_1(k)w_2(k)}\Big|_{s_{\mathrm{th}}}$$
$$+8\pi\sqrt{s_{\mathrm{th}}}\left[-\beta\Big[\frac{1}{G(s_0)}+\beta(s_{\mathrm{th}}-s_0)\Big]^{-2}-\frac{\partial\mathrm{Re}[G(s)]}{\partial s}\Big|_{s_{\mathrm{th}}^{+}}\right]\frac{s}{w_1(k)w_2(k)}\Big|_{s_{\mathrm{th}}}$$

- nonmolecular compositeness

$$Z=-g^2\,G(s_0)^2\,\beta$$

$$g^2=\lim_{s\to s_0}(s-s_0)T(s)=\frac{1}{-G(s_0)^2\beta-\frac{\partial G}{\partial s}|_{s_0}}\qquad(\text{L}'\text{Hospital}'\text{s rule})$$

for heavy particles (deuteron)

$$G(E) = \int_{|\boldsymbol{q}|<q_{\max}} \frac{d^3\boldsymbol{q}}{(2\pi)^3} \frac{m_1 m_2}{E_1(\boldsymbol{q})E_2(\boldsymbol{q})} \frac{1}{\sqrt{s} - E_1(\boldsymbol{q}) - E_2(\boldsymbol{q}) + i\epsilon}$$

The potential is now

$$V = V_0 + \beta(E - E_0)$$

Similarly

$$\frac{2\pi E_{\text{th}}}{m_1 m_2}\left\{\left[\frac{1}{G(E_0)} + \beta(E - E_0)\right]^{-1} - \text{Re}G(E_{\text{th}})\right\} = \frac{1}{a}$$

$$-\frac{1}{2}r_0 = \frac{2\pi}{m_1 m_2}\left[[\frac{1}{G(E_0)} + \beta(E_{\rm th} - E_0)]^{-1} - {\rm Re}G(E)_{\rm th} \right] \frac{E}{2E_1(k)E_2(k)}|_{E_{\rm th}}$$

$$+\frac{2\pi E_{\rm th}}{m_1 m_2}\left[-\beta[\frac{1}{G(E_0)} + \beta(E_{\rm th} - E_0)]^{-2} - \frac{\partial {\rm Re}[G(E)]}{\partial E} \right] \frac{E}{2E_1(k)E_2(k)}|_{E_{\rm th}}$$

$$Z = -g^2\, G(E_0)^2\, \beta$$

with

$$g^2 = \lim_{E\to E_0}(E - E_0)T(E) = \frac{1}{-G(E_0)^2\beta - \frac{\partial G}{\partial E}|_{E_0}} \quad \text{(L'Hospital's rule)}$$

$$\beta = \frac{1}{E_{\rm th} - E_0}\left\{ [\frac{1}{a}\frac{1}{2\pi}\frac{m_1 m_2}{m_1 + m_2} + {\rm Re}\, G(E_{\rm th})]^{-1} - \frac{1}{G(E_0)} \right\}$$

3. Results

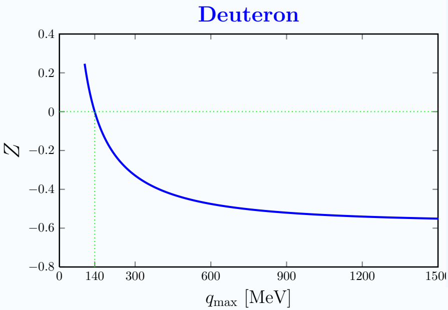
$$\begin{cases} \text{Deuteron} \\ D^*_{s_0}(2317) \\ D^*_{s_1}(2460) \end{cases} \Longrightarrow \quad Z, r_0$$

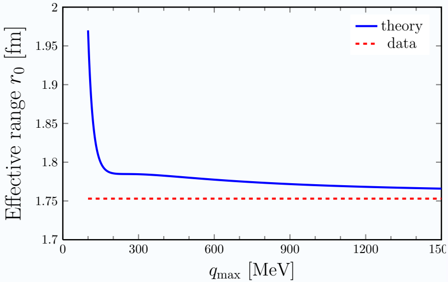
r_0 the effective range

Z nonmolecular compositeness

$X = 1 - Z$ molecular compositeness

Deuteron





a) Starting from $q_{\max} = 100$ MeV, $Z < 0.25$ indicating a **strong molecular** *pn* **component**

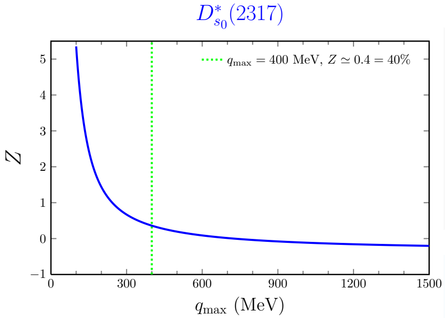
b) $q_{\max} \approx 140$ **MeV** $Z = 0$ indicating that the deuteron is a molecular state

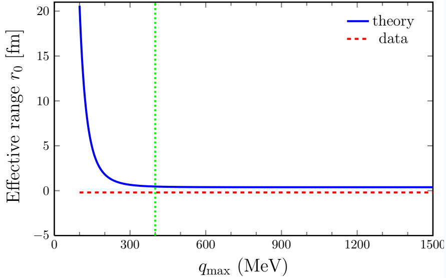
c) beyond $q_{\max} = 140$ MeV, $Z < 0$ negative, discard this situation

1) $q_{\max} \geq 140$ MeV, r_0^{theory} is close to r_0^{exp}, below this value noticeable disagreement

2) $q_{\max}$ is small $\Rightarrow$ indicating that the range of the *NN* interaction in *r*-space is rather large

3) the range of the interaction $\Rightarrow$ a picture of the deuteron far closer to the actual molecular nature than Weinberg's equations





$D^*_{s0}(2317)$

The data from QCD lattice analysis of the finite volume levels by Torres, Oset, Prelovsek, Ramos, JHEP 05 (2015)153

$a(KD) = +1.3 \pm 0.5 \pm 0.1$ fm

$r_0(KD) = -0.1 \pm 0.3 \pm 0.1$ fm

the nominal mass 2317 MeV

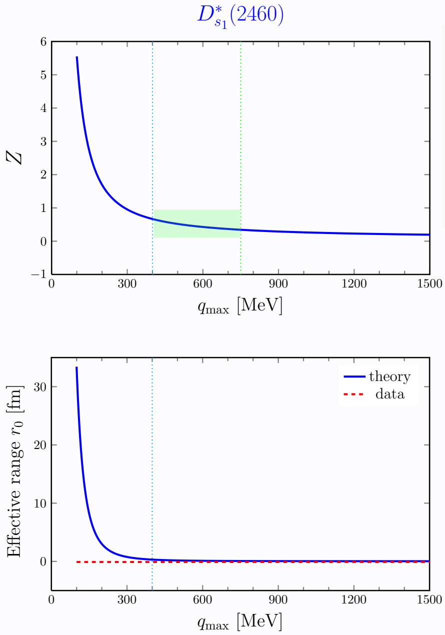
a) $q_{max} > 400$ MeV, **agreement** between r_0^{theory} and r_0^{exp}

b) $\Longrightarrow$ $Z < 0.4$ indicating a DK molecular component with probability larger than 60%

c) in **agreement** with the findings in [JHEP 05 (2015)153] $P(DK) = (72 \pm 13 \pm 5)$

d) $q_{max} \geq 725$ MeV, $Z \leq 0$ $\Longrightarrow$ the the range of the interaction (light vector exchange)

$D^*_{s1}(2460)$



The data from QCD lattice analysis of the finite volume levels by Torres, Oset, Prelovsek, Ramos, JHEP 05 (2015)153

$a(KD^*) = +1.1 \pm 0.5 \pm 0.2$ fm

$r_0(KD^*) = -0.2 \pm 0.3 \pm 0.1$ fm

the nominal mass 2460 MeV

a) $q_{max} > 400$ MeV, **agreement** between r_0^{theory} and r_0^{exp}

b) Z never becomes zero, independent of q_{max}, reaching a value of 0.2 for large q_{max}

c) $0.3 < Z < 0.6$. indicating a KD^* molecular component with probability $\geq 40\%$

d) in **agreement** JHEP05(2015)153 $P(KD^*) = (57 \pm 21 \pm 6)$ $\Longrightarrow$ ηD_s^* channel is mostly responsible for the remaining probability

4. Summary

We **propose an approach** to evaluate simultaneously the effective range and nonmolecular compositeness

$\Longrightarrow$ **improving the Weinberg's formalism** [which was obtained in the limit of very small binding and zero range interaction in r-space]

•The range of the interaction is very important to consider

the combined information of a, r_0 and the binding $\Longrightarrow$ could provide a fair information on the $D^*_{s0}(2317)$ and $D^*_{s1}(2460)$ which are bound by about 40 – 45 MeV

NN interaction (deuteron) has a longer range in r-space than the KD and KD^* in the cases of the $D^*_{s0}(2317)$ and $D^*_{s1}(2460)$ states

•The molecular compositeness

determine simultaneously the value of the compositeness within a certain range, as well as get qualitative information on the range of the interaction for three cases (with small or larger binding)

Thank you