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# Radiative corrections in semileptonic tau decays

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- The tau lepton is the only known lepton that is heavy enough to decay into hadrons.
- Semileptonic tau decays are a valuable tool for studying QCD hadronization at low energies.

$H^-$	$\begin{array}{c} \text{Precision} \ [\mathcal{B}_H] \\ \text{PDG} \ 2022 \end{array}$	Rad. Corr.	Application
$\pi^{-}$	0.5%	✓ +	LFU, NP
$K^{-}$	1.4%	$\checkmark$ +	$V_{us},  { m LFU},  { m NP}$
$\pi^-\pi^0$	0.4%	$\checkmark$ +	$ \rho,  \rho',  \cdots,  (g-2)_{\mu},  \mathrm{NP} $
$K^-K^0$	2.3%	×	$ ho', \cdots, \operatorname{NP}$
$ar{K}^0\pi^-$	1.7%	$\checkmark$ +	$K^*, V_{us}, C\!\!/ \mathbf{P}, \mathbf{NP}$
$K^{-}\pi^{0}$	3.5%	$\checkmark$ +	$K^*, V_{us}, NP$
$K^-\eta$	5.2%	×	$K^*,  \mathrm{NP}$
$\pi^{-}\pi^{+}\pi^{-}$	0.5%	×	$a_1$
$\pi^- 2\pi^0$	1.1%	×	$a_1$

Decker and Fikemeier '95, Arroyo-Ureña et al '21
 Cirigliano et al '01, Flores-Tlalpa et al '06, Miranda and Roig '20
 Antonelli et al '13, Flores-Baéz and Morones-Ibarra '13

Short-Distance corrections: Sirlin '78; Marciano-Sirlin '93

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		Т	his worl	k

- The EM radiative corrections require the inclusion of virtual and real photons.
- The SI contributions for the Kπ channel were studied in Phys. Rev.
   D 88 (2013) 7, 073009 and JHEP 10 (2013) 070 .
- The virtual-photon corrections are IR divergent. The virtual loops, which treat the K and  $\pi$  as point-like, induce a shift to the form factors:  $\bar{f}_{\pm,0}^{K\pi}(s) \rightarrow \bar{f}_{\pm,0}^{K\pi}(s) + \delta \bar{f}_{\pm,0}^{K\pi}(s, u)$
- In ChPT, three diagrams contribute to this effect.



Phys. Lett. B 513 (2001) 361-370

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- The SI part of the radiative process is introduced by means of the Low theorem.

$$\mathcal{M}^{\gamma} = \underbrace{\frac{\mathcal{M}}{k}}_{k} + \mathcal{M}_{1}k^{0} + k\mathcal{M}_{2} + \dots$$

- The EM radiative corrections require the inclusion of virtual and real photons.
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$$\mathcal{M}_{\tau}^{\gamma} = \mathcal{M}_{\tau}^{(0)'} e\left[\frac{p^{+} \cdot \epsilon(k)}{p^{+} \cdot k} - \frac{p_{\tau} \cdot \epsilon(k)}{p_{\tau} \cdot k}\right]$$

• The leading Low-term is fully determined by the amplitude of the non-radiative decay.



# Amplitude

• The most general structure is given by

JHEP 08 (2002) 002 Phys. Rev. D 95 (2017) 5, 054015 Phys. Rev. D 102 (2020) 114017





• The hadronic matrix elements is

$$H^{\nu}(p_{-},p_{0}) = C_{V}F_{+}(t)Q^{\nu} + C_{S}\frac{\Delta_{-0}}{t}q^{\nu}F_{0}(t), \qquad t=q^{2}$$
 where

$$q^{\nu} = (p_{-} + p_{0})^{\nu}, \ Q^{\nu} = (p_{-} - p_{0})^{\nu} - \frac{\Delta_{-0}}{t}q^{\nu} \text{ and } \Delta_{ij} = m_{i}^{2} - m_{j}^{2}$$

# Amplitude

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 The vector and axial-vector terms can be split into two parts, structure-independent (SI) and structure-dependent (SD), according to the Low and Burnett-Kroll theorems.

$$V^{\mu\nu} = V^{\mu\nu}_{SI} + V^{\mu\nu}_{SD}$$
$$A^{\mu\nu} = A^{\mu\nu}_{SD}$$

• At low-energies, the SM of EW and strong interactions is described by an EFT known as Chiral Perturbation Theory (ChPT).

#### SI contributions

• The **SI** contribution reads

JHEP 08 (2002) 002 Phys. Rev. D 95 (2017) 5, 054015 Phys. Rev. D 102 (2020) 114017

$$V_{SI}^{\mu\nu} = \frac{H^{\nu}(p_{-} + k, p_{0})(2p_{-} + k)^{\mu}}{2k \cdot p_{-} + k^{2}} + \left\{ -C_{V}F_{+}(t') - \frac{\Delta_{-0}}{t'} [C_{S}F_{0}(t') - C_{V}F_{+}(t')] \right\} g^{\mu\nu} \\ -C_{V} \frac{F_{+}(t') - F_{+}(t)}{k \cdot (p_{-} + p_{0})} Q^{\nu} q^{\mu} + \frac{\Delta_{-0}}{tt'} \{2 [C_{S}F_{0}(t') - C_{V}F_{+}(t')] - \frac{C_{S}t'}{k \cdot (p_{-} + p_{0})} [F_{0}(t') - F_{0}(t)] \} q^{\mu} q^{\nu} \\ \hline \mathbf{T}^{-} \rightarrow \mathbf{K}^{-} \mathbf{\pi}^{0} \mathbf{\gamma} \mathbf{\nu}_{\mathbf{T}} \\ p_{-} \rightarrow p_{K} \qquad C_{V} = \frac{1}{\sqrt{2}} \qquad \mathbf{T}^{-} \rightarrow \mathbf{K}^{0} \mathbf{\pi}^{-} \mathbf{\gamma} \mathbf{\nu}_{\mathbf{T}} \\ p_{0} \rightarrow p_{\pi} \qquad C_{V} = 1 \\ \Delta_{-0} \rightarrow \Delta_{K\pi} \qquad C_{S} = \frac{1}{\sqrt{2}} \qquad \mathbf{T}^{-} \rightarrow \mathbf{K}^{-} \mathbf{K}^{0} \mathbf{\gamma} \mathbf{\nu}_{\mathbf{T}} \\ \mathbf{T}^{-} \rightarrow \mathbf{K}^{-} \mathbf{K}^{0} \mathbf{\gamma} \mathbf{\nu}_{\mathbf{T}} \\ C_{V,S} \rightarrow -C_{V,S} \qquad \mathbf{T}^{-} \mathbf{K}^{-} \mathbf{K}^{0} \mathbf{\gamma} \mathbf{\nu}_{\mathbf{T}} \\ C_{V} = -1 \\ C_{S} = -1 \qquad \mathbf{W} \text{erecover the usual definition of } H^{\nu}_{K\pi} \\ \end{array}$$

#### SI contributions

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$$V_{\mathrm{SI}}^{\mu\nu} = \frac{H^{\nu}(p_{-}+k,p_{0})(2p_{-}+k)^{\mu}}{2k \cdot p_{-}+k^{2}} + \left\{ \begin{array}{c} C_{V}F_{+}(t') - \frac{\Delta_{-0}}{t'} \left[ C_{S}F_{0}(t') - C_{V}F_{+}(t') \right] \right\} g^{\mu\nu} \\ -C_{V} \frac{F_{+}(t') - F_{+}(t)}{k \cdot (p_{-}+p_{0})} Q^{\nu}q^{\mu} + \frac{\Delta_{-0}}{tt'} \left\{ 2 \left[ C_{S}F_{0}(t') - C_{V}F_{+}(t') \right] - \frac{C_{S}t'}{k \cdot (p_{-}+p_{0})} \left[ F_{0}(t') - F_{0}(t) \right] \right\} q^{\mu}q^{\nu} \\ \hline \mathbf{T}^{-} \rightarrow \mathbf{K}^{-} \mathbf{\pi}^{0} \boldsymbol{\gamma} \boldsymbol{\nu}_{\mathbf{T}} \\ p_{-} \rightarrow p_{K} \\ p_{0} \rightarrow p_{K} \\ \Delta_{-0} \rightarrow \Delta_{K\pi} \\ \hline \mathbf{\chi}^{-} \mathbf{\chi}^{0} \\ \mathbf{\chi}^{-} \mathbf{\chi}^{0} \\ \hline \mathbf{T}^{-} \rightarrow \mathbf{K}^{-} \mathbf{K}^{0} \boldsymbol{\gamma} \boldsymbol{\nu}_{\mathbf{T}} \\ \hline \mathbf{\chi}^{-} \mathbf{\chi}^{-} \mathbf{\chi}^{-} \mathbf{\chi}^{-} \mathbf{\chi}^{0} \\ \hline \mathbf{\chi}^{-} \mathbf{\chi}^{-} \mathbf{\chi}^{-} \mathbf{\chi}^{0} \\ \hline \mathbf{\chi}^{-} \mathbf{\chi}^{-} \mathbf{\chi}^{-} \mathbf{\chi}^{0} \mathbf{\chi}^{-} \mathbf{\chi}^{-} \\ \hline \mathbf{\chi}^{-} \mathbf{\chi}^{-} \mathbf{\chi}^{-} \mathbf{\chi}^{0} \\ \hline \mathbf{\chi}^{-} \mathbf{\chi}^{-} \mathbf{\chi}^{-} \mathbf{\chi}^{-} \mathbf{\chi}^{0} \\ \hline \mathbf{\chi}^{-} \mathbf{\chi}^{-} \mathbf{\chi}^{-} \mathbf{\chi}^{0} \\ \hline \mathbf{\chi}^{-} \mathbf{\chi}^{-} \mathbf{\chi}^{-} \mathbf{\chi}^{0} \\ \hline \mathbf{\chi}^{-} \mathbf{\chi}^{-} \mathbf{\chi}^{0} \mathbf{\chi}^{0} \\ \hline \mathbf{\chi}^{-} \mathbf{\chi}^{0} \\ \hline \mathbf{\chi}^{-} \mathbf{\chi}^{0} \\ \hline \mathbf{\chi}^{-} \mathbf{\chi}^{0} \\ \hline \mathbf{\chi}^{-} \mathbf{\chi}^{0} \\ \hline \mathbf{\chi}^{0} \\ \hline$$

#### SD contributions

• At  $O(p^4)$  in ChPT with resonances (RChT), the vector form factors  $V^{\mu\nu}$  are saturated by the exchange of vector and axial-vector resonances:



#### SD contributions

• At  $O(p^4)$  in ChPT, the axial form factors,  $A^{\mu\nu}$ , get contributions from the Wess-Zumino-Witten functional:



#### Decay spectrum



• The differential rate for the radiative decays in the tau rest frame is given by

$$d\Gamma = \frac{(2\pi)^4}{4m_\tau} \sum_{\rm spin} \overline{|\mathcal{M}|^2} d\Phi_4$$

• To be sensitive to the dynamics of the radiative transition, a cut on photon energies is necessary.



 $E_{oldsymbol{\gamma}}^{ ext{cut}}=300\, ext{MeV}$ 



#### Branching ratio





The **SD contributions** are important even below <u>100 MeV</u>.

	$E_{\gamma}^{\mathrm{cut}}$	Low	SI	RχT	_
$\mathcal{B}_{K^-\pi^0}$	$100 { m MeV} \\ 300 { m MeV} \\ 500 { m MeV} \end{cases}$	$3.4 \times 10^{-6}$ $6.2 \times 10^{-7}$ $7.4 \times 10^{-8}$	$3.0 \times 10^{-6}$ $3.4 \times 10^{-7}$ $3.5 \times 10^{-8}$	$\begin{array}{c} 3.8(3)\times 10^{-6}\\ 9.4(3.1)\times 10^{-7}\\ 3.3(1.8)\times 10^{-7} \end{array}$	~ 10 % ~ 34 % ~ 78 %
$\mathcal{B}_{ar{K}^0\pi^-}$	$100 { m MeV} \\ 300 { m MeV} \\ 500 { m MeV} \end{cases}$	$2.6 \times 10^{-5}$ $6.2 \times 10^{-6}$ $1.0 \times 10^{-6}$	$1.4 \times 10^{-5}$ $1.1 \times 10^{-6}$ $7.1 \times 10^{-8}$	$\begin{array}{c} 1.6(0) \times 10^{-5} \\ 1.9(2) \times 10^{-6} \\ 2.4(4) \times 10^{-7} \end{array}$	~ 63 % ~ 226 % ~ 317 %
$\mathcal{B}_{K^-K^0}$	$100 { m MeV} \\ 300 { m MeV} \\ 500 { m MeV} \end{cases}$	$\begin{array}{c} 5.3 \times 10^{-7} \\ 4.8 \times 10^{-8} \\ 3.7 \times 10^{-10} \end{array}$	$3.7 \times 10^{-7}$ $1.9 \times 10^{-8}$ $3.0 \times 10^{-10}$	$9.4(2.6) \times 10^{-7} 3.1(1.4) \times 10^{-7} 2.9(1.8) \times 10^{-8}$	~ 44 % ~ 85 % ~ 99 %

• The Low's approximation is not sufficient to describe these decays for photon energies above ~100 MeV.

	$E_{\gamma}^{\mathrm{cut}}$	Low	SI	RχT	_
$\mathcal{B}_{K^-\pi^0}$	$100 { m MeV}$ $300 { m MeV}$ $500 { m MeV}$	$3.4 \times 10^{-6}$ $6.2 \times 10^{-7}$ $7.4 \times 10^{-8}$	$\begin{array}{c} 3.0 \times 10^{-6} \\ 3.4 \times 10^{-7} \\ 3.5 \times 10^{-8} \end{array}$	$\begin{array}{c} 3.8(3)\times 10^{-6}\\ 9.4(3.1)\times 10^{-7}\\ 3.3(1.8)\times 10^{-7} \end{array}$	~ 21 % ~ 64 % ~ 89 %
$\mathcal{B}_{ar{K}^0\pi^-}$	$100 { m MeV} \\ 300 { m MeV} \\ 500 { m MeV} \end{cases}$	$2.6 \times 10^{-5}$ $6.2 \times 10^{-6}$ $1.0 \times 10^{-6}$	$1.4 \times 10^{-5}$ $1.1 \times 10^{-6}$ $7.1 \times 10^{-8}$	$1.6(0) \times 10^{-5}$ $1.9(2) \times 10^{-6}$ $2.4(4) \times 10^{-7}$	~ 12 % ~ 42 % ~ 70 %
$\mathcal{B}_{K^-K^0}$	$100 { m MeV} \\ 300 { m MeV} \\ 500 { m MeV} \end{cases}$	$5.3 \times 10^{-7}$ $4.8 \times 10^{-8}$ $3.7 \times 10^{-10}$	$\begin{array}{c} 3.7\times 10^{-7} \\ 1.9\times 10^{-8} \\ 3.0\times 10^{-10} \end{array}$	$9.4(2.6) \times 10^{-7} 3.1(1.4) \times 10^{-7} 2.9(1.8) \times 10^{-8}$	~ 61 % ~ 94 % ~ 99 %

- The Low's approximation is not sufficient to describe these decays for photon energies above ~100 MeV.
- These decays are an excellent probe for testing SD effects.

#### **Radiative corrections**

• The photon-inclusive double differential rate can be written as

$$\frac{d\Gamma}{dt}\Big|_{PP(\gamma)} = \frac{G_F^2 |V_{uD} F_+(0)|^2 S_{EW} m_\tau^3}{768\pi^3 t^3} \left(1 - \frac{t}{m_\tau^2}\right)^2 \lambda^{1/2} (t, m_-^2, m_0^2) \left[C_V^2 |\tilde{F}_+(t)|^2 \left(1 + \frac{2t}{m_\tau^2}\right) \lambda (t, m_-^2, m_0^2) + 3C_S^2 \Delta_{-0}^2 |\tilde{F}_0(t)|^2 \right] G_{EM}(t)$$
+  $3C_S^2 \Delta_{-0}^2 |\tilde{F}_0(t)|^2 \left] G_{EM}(t)$ 
• For simplicity, we split the contributions to the decay width as
$$\frac{d\Gamma}{dt}\Big|_{PP(\gamma)} = \frac{d\Gamma}{dt}\Big|_{PP} + \frac{d\Gamma}{dt}\Big|_{III} + \frac{d\Gamma}{dt}\Big|_{IV/III} + \frac{d\Gamma}{dt}\Big|_{rest}, \quad G_{EM}(t) = G_{EM}^{(0)}(t) + \delta G_{EM}(t)$$

$$\sum_{\substack{q = 0 \\ q = 0}}^{2s} \int_{0}^{2s} \int_{0$$

#### **Radiative corrections**

• The differential decay width can be written as

JHEP 08 (2002) 002

JHEP 10 (2013) 070



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#### **Radiative corrections**

• Integrating upon t, we get

$$\Gamma_{PP(\gamma)} = \frac{G_F^2 S_{EW} m_{\tau}^5}{96\pi^3} \left| V_{uD} F_+(0) \right|^2 I_h^{\tau} \left( 1 + \delta_{EM}^{hh} \right)^2$$

where

$$I_{h}^{\tau} = \frac{1}{8m_{\tau}^{2}} \int_{t_{thr}}^{m_{\tau}^{2}} \frac{dt}{t^{3}} \left(1 - \frac{t}{m_{\tau}^{2}}\right)^{2} \lambda^{1/2}(t, m_{-}^{2}, m_{0}^{2}) \left[C_{V}^{2} |\tilde{F}_{+}(t)|^{2} \left(1 + \frac{2t}{m_{\tau}^{2}}\right) \lambda(t, m_{-}^{2}, m_{0}^{2}) + 3C_{S}^{2} \Delta_{-0}^{2} |\tilde{F}_{0}(t)|^{2}\right].$$

#### Electromagnetic corrections to hadronic tau decays in %

$\delta_{ m EM}$	Ref. [33]	$G_{EM}^{(0)}(t)$ $\delta G_{EM}$		$\delta G_{EM}(t)$			
		Model 1	Model 2	SI	SI + 2F	SI + 3F	$\delta^{K^{-}\pi^{0}} - (0.000^{+0.008})0)$
$K^-\pi^0$	-0.20(20)	-0.019	-0.137	-0.001	+0.006	+0.010	$\delta_{\rm EM} = -(0.009_{-0.118})/0$ $s\bar{K}^0\pi^-$ (0.166±0.010)
$\bar{K}^0\pi^-$	-0.15(20)	-0.086	-0.208	-0.098	-0.085	-0.080	$\delta_{\rm EM}^{\rm II, \pi} = -(0.166^{+0.016}_{-0.122})\%$
$K^{-}K^{0}$	—	-0.046	-0.223	-0.012	+0.003	+0.016	$\delta_{\rm EM}^{K^-K^0} = -(0.030^{+0.026}_{-0.179})\%$
$\pi \pi^{\circ}$	_	-0.196	-0.363	-0.010	-0.002	+0.010	$\delta_{\rm EM}^{\pi^-\pi^0} = -(0.186^{+0.024}_{-0.169})\%$

JHEP 10 (2013) 070

 $\delta^{K^{-}\eta} = -\left(0.026^{+0.024}_{-0.162}\right)\% \quad \delta^{K^{-}\eta'} = -\left(0.304^{+0.380}_{-0.030}\right)\%$ 

#### Impact of radiative corrections on NP

• The low-energy Lagrangian that describes the semileptonic strangeness-conserving ( $\Delta S=0$ ) or -changing ( $\Delta S=1$ ) charge current transitions reads, (D = d, s)

$$\mathcal{L}_{eff} = -\frac{G_F V_{uD}}{\sqrt{2}} \Big[ (1 + \epsilon_L^{\tau}) \bar{\tau} \gamma_\mu (1 - \gamma_5) \nu_\tau \cdot \bar{u} \gamma^\mu (1 - \gamma_5) D + \epsilon_R^{\tau} \bar{\tau} \gamma_\mu (1 - \gamma_5) \nu_\tau \cdot \bar{u} \gamma^\mu (1 + \gamma_5) D \\ + \bar{\tau} (1 - \gamma_5) \nu_\tau \cdot \bar{u} (\epsilon_S^{\tau} - \epsilon_P^{\tau} \gamma_5) D + \epsilon_T^{\tau} \bar{\tau} \sigma_{\mu\nu} (1 - \gamma_5) \nu_\tau \cdot \bar{u} \sigma^{\mu\nu} (1 - \gamma_5) D \Big] + \text{h.c.}$$

The one meson decay width is given by

 $G_F \tilde{V}_{uD}^e = G_F \left( 1 + \epsilon_L^e + \epsilon_R^e \right) V_{uD}$ Phys.Rev.D 104 (2021) 9, L091502

$$\Gamma(\tau^{-} \to P^{-}\nu_{\tau}) = \frac{G_{F}^{2} |\tilde{V}_{uD}^{e}|^{2} f_{\pi}^{2} m_{\tau}^{3}}{16\pi} \left(1 - \frac{m_{\pi}^{2}}{m_{\tau}^{2}}\right)^{2} (1 + \delta_{em}^{\tau P} + 2\Delta^{\tau P}), \qquad \qquad \delta_{\tau\pi} = -0.24(56)\%$$

$$\Delta^{\tau P} = \epsilon_{L}^{\tau} - \epsilon_{L}^{e} - \epsilon_{R}^{\tau} - \epsilon_{R}^{e} - \frac{m_{P}^{2}}{m_{\tau}(m_{u} + m_{D})\epsilon_{P}^{\tau}} \implies \Delta^{\tau\pi} = -0.14(72) \cdot 10^{-2},$$

$$\Delta^{\tau K} = -1.02(86) \cdot 10^{-2},$$

• The partial decay width for two-meson decays is

$$\frac{d\Gamma}{dt}\Big|_{PP(\gamma)} = \frac{G_F^2 |\tilde{V}_{uD}^e|^2 m_\tau^3 S_{EW}}{384\pi^3 t} \left(1 - \frac{t}{m_\tau^2}\right)^2 \lambda^{1/2} (t, m_P^2, m_{P'}^2) \\
\times \left[ \left(G_{EM}(t) + 2(\epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e)\right) X_{VA} + \epsilon_S^\tau X_S + \epsilon_T^\tau X_T + (\epsilon_S^\tau)^2 X_{S^2} + (\epsilon_T^\tau)^2 X_{T^2} \right]$$

#### Impact of radiative corrections on NP

• The low-energy Lagrangian that describes the semileptonic strangeness-conserving ( $\Delta S=0$ ) or -changing ( $\Delta S=1$ ) charge current transitions reads, (D = d, s)

$$\mathcal{L}_{eff} = -\frac{G_F V_{uD}}{\sqrt{2}} \Big[ (1 + \epsilon_L^{\tau}) \bar{\tau} \gamma_\mu (1 - \gamma_5) \nu_\tau \cdot \bar{u} \gamma^\mu (1 - \gamma_5) D + \epsilon_R^{\tau} \bar{\tau} \gamma_\mu (1 - \gamma_5) \nu_\tau \cdot \bar{u} \gamma^\mu (1 + \gamma_5) D \\ + \bar{\tau} (1 - \gamma_5) \nu_\tau \cdot \bar{u} (\epsilon_S^{\tau} - \epsilon_P^{\tau} \gamma_5) D + \epsilon_T^{\tau} \bar{\tau} \sigma_{\mu\nu} (1 - \gamma_5) \nu_\tau \cdot \bar{u} \sigma^{\mu\nu} (1 - \gamma_5) D \Big] + \text{h.c.}$$

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$$\Delta^{\tau P} = \epsilon_{L}^{\tau} - \epsilon_{L}^{e} - \epsilon_{R}^{\tau} - \epsilon_{R}^{e} - \frac{m_{P}^{2}}{m_{\tau}(m_{u} + m_{D})\epsilon_{P}^{\tau}} \qquad \qquad \Delta^{\tau\pi} = -0.14(72) \cdot 10^{-2},$$

$$\Delta^{\tau K} = -1.02(86) \cdot 10^{-2},$$

• The partial decay width for two-meson decays is

#### Global fit ( $\Delta S=0$ )

• The chi-squared function to be minimized in our fits is

$$\chi^2 = \sum_k \left( \frac{\bar{N}_k^{\rm th} - \bar{N}_k^{\rm exp}}{\sigma_{\bar{N}_k^{\rm exp}}} \right)^2 + \left( \frac{BR_{\pi\pi}^{\rm th} - BR_{\pi\pi}^{\rm exp}}{\sigma_{BR_{\pi\pi}^{\rm exp}}} \right)^2 + \left( \frac{BR_{KK}^{\rm th} - BR_{KK}^{\rm exp}}{\sigma_{BR_{KK}^{\rm exp}}} \right)^2 + \left( \frac{BR_{\pi\pi}^{\rm th} - BR_{\pi\pi}^{\rm exp}}{\sigma_{BR_{\pi\pi}^{\rm exp}}} \right)^2 \,,$$

The constraints for the non-standard interactions

$$\begin{pmatrix} \epsilon_{L}^{\tau} - \epsilon_{L}^{e} + \epsilon_{R}^{\tau} - \epsilon_{R}^{e} \\ \epsilon_{R}^{\tau} + \frac{m_{\pi}^{2}}{2m_{\tau}(m_{u} + m_{d})} \epsilon_{P}^{\tau} \\ \epsilon_{S}^{\tau} \\ \epsilon_{T}^{\tau} \end{pmatrix} = \begin{pmatrix} 0.0 \pm 0.6 \pm 0.6 \pm 0.4 \pm 0.1 \pm 1.7 \pm 0.0 \\ 0.1 \pm 0.5 \pm 3.4 \pm 0.0 \pm 0.9 \pm 0.0 \\ 1.0.3 \pm 0.5 \pm 3.4 \pm 0.0 \pm 0.9 \pm 0.0 \\ 10.3 \pm 0.5 \pm 25.0 - 0.1 \pm 0.9 \pm 0.0 \\ 0.4 \pm 0.2 \pm 4.4 \pm 0.1 \pm 1.1 \pm 0.0 \\ 0.4 \pm 0.2 \pm 4.4 \pm 0.1 \pm 1.1 \pm 0.0 \\ 0.4 \pm 0.2 \pm 4.4 \pm 0.1 \pm 1.1 \pm 0.0 \\ 0.4 \pm 0.2 \pm 4.4 \pm 0.1 \pm 1.1 \pm 0.0 \\ 0.4 \pm 0.2 \pm 4.4 \pm 0.1 \pm 1.1 \pm 0.0 \\ 0.4 \pm 0.2 \pm 4.4 \pm 0.1 \pm 1.1 \pm 0.0 \\ 0.4 \pm 0.2 \pm 4.4 \pm 0.1 \pm 1.1 \pm 0.0 \\ 0.4 \pm 0.2 \pm 4.4 \\ 0.1 \pm 1.1 \pm 0.0 \\ 0.4 \pm 0.2 \pm 4.4 \\ 0.1 \pm 1.1 \pm 0.0 \\ 0.4 \pm 0.2 \pm 4.4 \\ 0.1 \pm 1.1 \pm 0.0 \\ 0.4 \pm 0.2 \pm 4.4 \\ 0.1 \pm 1.1 \pm 0.0 \\ 0.4 \pm 0.2 \pm 4.4 \\ 0.4 \pm 0.4 \\$$

Phys.Lett.B 804 (2020) 135371

# Global fit ( $\Delta S=1$ )

• The chi-squared function to be minimized in our fits is

$$\chi^2 = \sum_k \left( \frac{\bar{N}_k^{\rm th} - \bar{N}_k^{\rm exp}}{\sigma_{\bar{N}_k^{\rm exp}}} \right)^2 + \left( \frac{BR_{K\pi}^{\rm th} - BR_{K\pi}^{\rm exp}}{\sigma_{BR_{K\pi}^{\rm exp}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm exp}}{\sigma_{BR_{K\eta}^{\rm exp}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm exp}}{\sigma_{BR_{K\eta}^{\rm exp}}} \right)^2 + \left( \frac{BR_{K\pi}^{\rm th} - BR_{K\eta}^{\rm exp}}{\sigma_{BR_{K\eta}^{\rm exp}}} \right)^2 + \left( \frac{BR_{K\pi}^{\rm th} - BR_{K\eta}^{\rm exp}}{\sigma_{BR_{K\eta}^{\rm exp}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm exp}}{\sigma_{BR_{K\eta}^{\rm exp}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm exp}}{\sigma_{BR_{K\eta}^{\rm exp}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm exp}}{\sigma_{BR_{K\eta}^{\rm exp}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm exp}}{\sigma_{BR_{K\eta}^{\rm exp}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm exp}}{\sigma_{BR_{K\eta}^{\rm exp}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm exp}}{\sigma_{BR_{K\eta}^{\rm exp}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm exp}}{\sigma_{BR_{K\eta}^{\rm exp}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm exp}}{\sigma_{BR_{K\eta}^{\rm exp}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm exp}}{\sigma_{BR_{K\eta}^{\rm exp}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm exp}}{\sigma_{BR_{K\eta}^{\rm exp}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm exp}}{\sigma_{BR_{K\eta}^{\rm exp}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm exp}}{\sigma_{BR_{K\eta}^{\rm exp}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm exp}}{\sigma_{BR_{K\eta}^{\rm exp}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm exp}}{\sigma_{BR_{K\eta}^{\rm exp}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm exp}}{\sigma_{BR_{K\eta}^{\rm th}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm th}}{\sigma_{BR_{K\eta}^{\rm th}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm th}}{\sigma_{BR_{K\eta}^{\rm th}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm th}}{\sigma_{BR_{K\eta}^{\rm th}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm th}}{\sigma_{BR_{K\eta}^{\rm th}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm th}}{\sigma_{BR_{K\eta}^{\rm th}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm th}}{\sigma_{BR_{K\eta}^{\rm th}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm th}}{\sigma_{BR_{K\eta}^{\rm th}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm th}}{\sigma_{BR_{K\eta}^{\rm th}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm th}}{\sigma_{BR_{K\eta}^{\rm th}}} \right)^2 + \left( \frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm th}}{\sigma_{BR_{K$$

The constraints for the non-standard interactions

Phys.Lett.B 804 (2020) 135371

# Global fit

- Assuming MFV, we can perform a global fit that includes both, the strangeness-conserving and -changing sectors.
- The constraints for the non-standard interactions

$$\begin{pmatrix} \epsilon_L^{\tau} - \epsilon_L^e + \epsilon_R^{\tau} - \epsilon_R^e \\ \epsilon_R^{\tau} \\ \epsilon_P^{\tau} \\ \epsilon_S^{\tau} \\ \epsilon_T^{\tau} \end{pmatrix} = \begin{pmatrix} 2.7 \pm 0.5 + 2.3 + 0.4 \pm 0.0 \pm 0.3 + 0.0 \pm 0.0 \pm 0.3 + 0.0 \\ 7.1 \pm 4.7 + 1.2 \pm 0.9 \pm 1.8 \pm 0.2 + 12.3 \pm 0.0 \\ 7.1 \pm 4.7 + 1.2 \pm 0.9 \pm 1.8 \pm 0.2 + 12.3 \pm 0.0 \\ -7.7 \pm 6.1 \pm 0.0 + 1.3 + 2.4 \pm 0.0 + 4.1 \\ 5.3 + 0.6 + 2.0 + 0.1 \pm 2.3 \pm 0.0 \pm 0.1 + 0.1 \\ 5.3 + 0.6 + 2.0 + 0.1 \pm 0.0 \pm 0.0 \pm 0.1 + 0.1 \\ -0.2 \pm 0.2 + 3.6 + 0.1 \pm 0.0 \pm 0.0 \pm 0.4 + 0.5 + 0.2 \\ -0.2 \pm 0.2 + 3.6 + 0.1 \pm 0.0 \pm 0.0 \pm 0.4 + 0.5 + 0.2 \\ 1 & 0.235 \\ 1 \end{pmatrix} \times 10^{-2}, \ \rho_{ij} = \begin{pmatrix} 1 & 0.056 & 0.000 & -0.270 & -0.402 \\ 1 & -0.997 & -0.015 & -0.023 \\ 1 & 0.000 & 0.000 \\ 1 & 0.235 \\ 1 & 0.235 \\ 1 \end{pmatrix}$$

$$\text{[at } \mu = 2 \text{ GeV in the } \overline{\text{MS scheme}} \qquad \qquad \chi^2/d.o.f \sim 1.49$$

Stat. fit, VFF, CKM, Rad. Cors., TFF, m<sub>q</sub>, Rad. Cors.

$$\epsilon_{L}^{\tau} - \epsilon_{L}^{e} + \epsilon_{R}^{\tau} - \epsilon_{R}^{e} \\ \epsilon_{L}^{e} - \epsilon_{L}^{e} + \epsilon_{R}^{\tau} - \epsilon_{R}^{e} \\ \epsilon_{T}^{e} \\ \epsilon_{$$

Phys.Lett.B 804 (2020) 135371

#### Conclusions

- At the current level of precision achieved in semileptonic tau decays, radiative corrections are required to test the SM and to extract information about NP.
- We study the electromagnetic corrections to the hadronic tau decays. Although the model-independent contribution for the Kπ modes was already available, a survey of the structure-dependent one was missing in the literature.
- Our results bridge this gap and decrease the uncertainty by a factor ~ 2, enabling more precise tests of NP.

#### References

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- F.V. Flores-Baéz and J.R. Morones-Ibarra. "Model Independent Electromagnetic corrections in hadronic τ decays", Phys. Rev. D 88 (2013) 7, 073009
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# $|V_{us}|$ from $\tau$ decays

• All  $|V_{us}|$  determinations based on measured  $\tau$  branching fractions are lower than both the kaon and the CKM-unitarity determinations.

