Spectral reconstruction in lattice QCD for inclusive and exclusive scattering amplitudes

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for the Baryon Scattering Collaboration (BaSC)

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The Baryon Scattering Collaboration (BaSC)

- Goal: computation of baryon scattering amplitudes from lattice QCD

- Members:
  - DESY: JB, J. Green
  - Mainz: H. Wittig, et al.
  - CMU: C. Morningstar, S. Skinner
  - UNC: A. Nicholson, J. Moscoso
  - LBL: A. Walker-Loud
  - BNL: A. Hanlon
  - MIT: F. Romero-Lopez
  - Intel: B. Hoerz
  - Darmstadt: D. Mohler, B. Cid-Mora

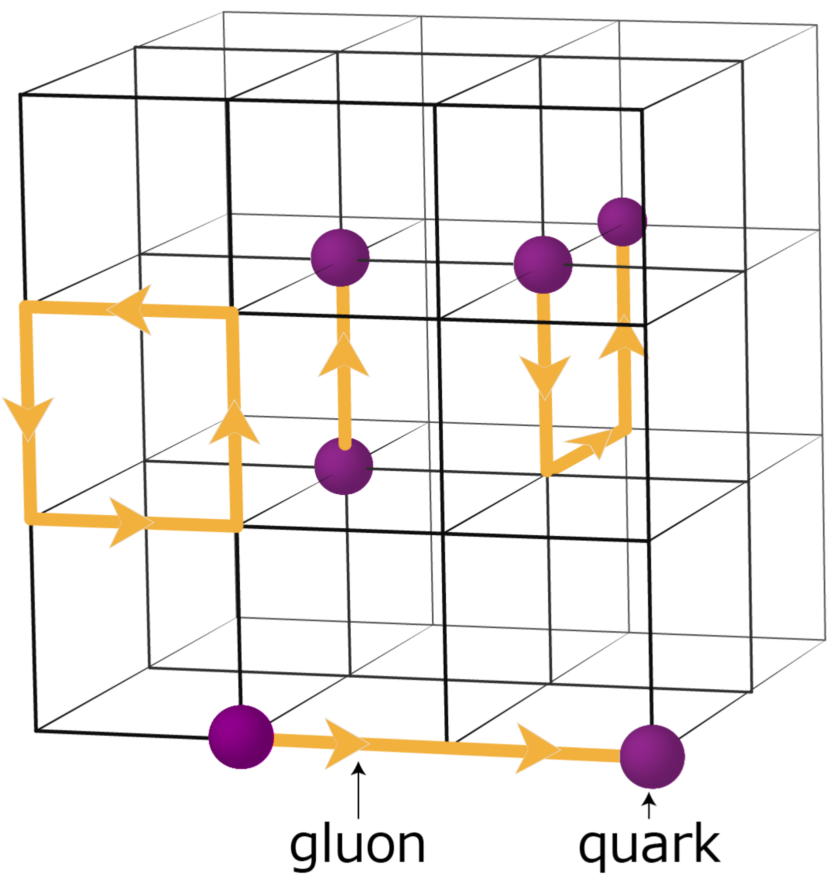
- First project: several meson-baryon and baryon-baryon systems on CLS D200

$$N_\mathrm{f} = 2+1, \quad a = 0.065\mathrm{fm}, \quad m_\pi = 200\mathrm{MeV}, \quad 64^3 \times 128$$

- Talk yesterday by C. Morningstar: nucleon-pion and Lambda(1405)

Lattice QCD: (without electroweak interactions)

- First-principles computer simulation of quarks and gluons
- Euclidean time: $t \to it$
- Data outputs: correlation functions



$$C(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle$$
$$= \sum_n |\langle n|\hat{\mathcal{O}}|\Omega\rangle|^2 \mathrm{e}^{-E_n t}$$
$$= \int \mathrm{d}\omega\, \rho(\omega)\, \mathrm{e}^{-\omega t}$$

- In finite-volume, spectral density is a sum of Dirac-deltas

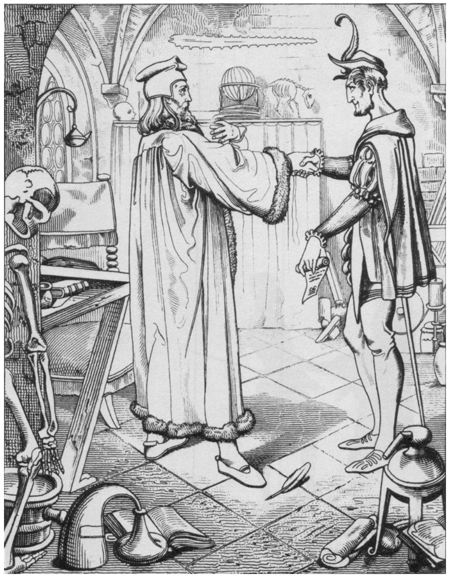
Observables from lattice QCD: Euclidean correlation functions

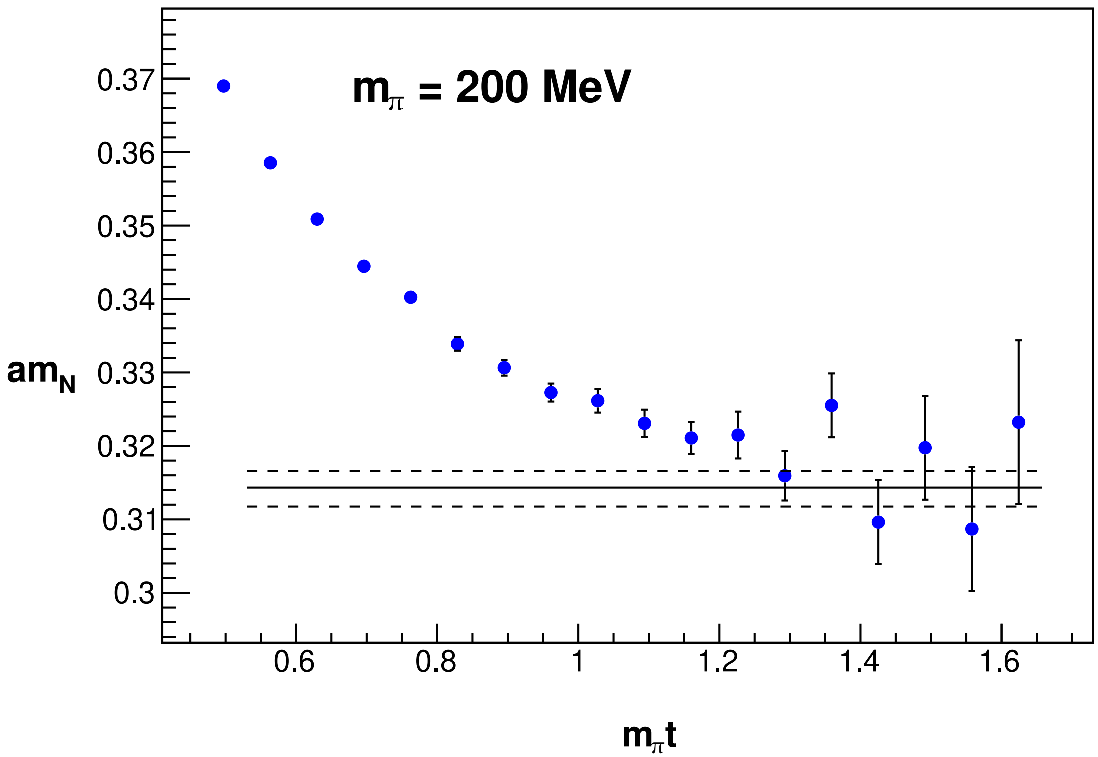
- Large time separation: ground state saturation

$$\lim_{t\to\infty} C_{\rm N}(t) = A\mathrm{e}^{-m_{\rm N}t}\left\{1+O(\mathrm{e}^{-m_\pi t})\right\}$$

- Signal-to-noise problem => ‘Teufelspakt’

$$\lim_{t\to\infty} \frac{C_{\rm N}(t)}{\sigma_{\rm stat}(t)} \propto \mathrm{e}^{-(m_{\rm N}-\frac{3}{2}m_\pi)t}$$





$$m_{\rm eff}(t+0.5a) = \log\left[\frac{C(t)}{C(t+a)}\right]$$

Dotted lines: result of two-state fit model

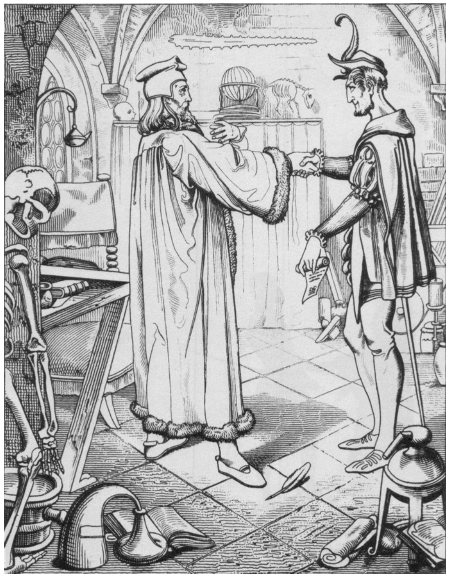
Observables from lattice QCD: Euclidean correlation functions

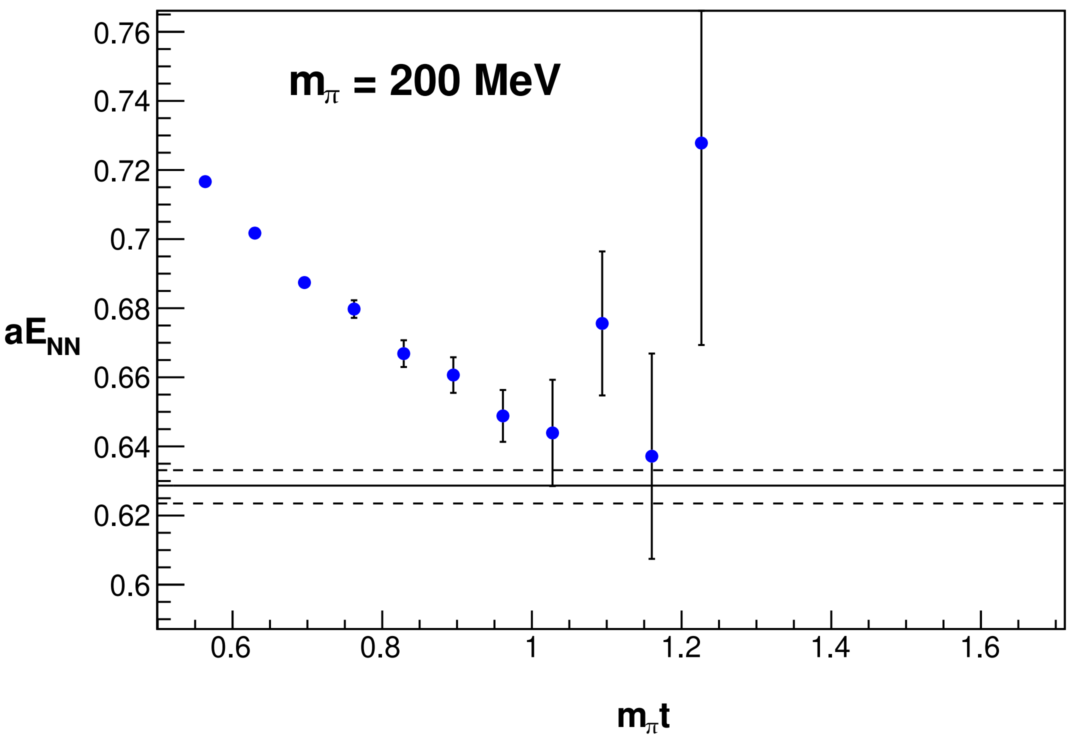
- Large time separation: ground state saturation

$$\lim_{t\to\infty} C_{\rm NN}^{I=0}(t) = A\mathrm{e}^{-E_{\rm NN}^{I=0}t}\left\{1+O(\mathrm{e}^{-m_\pi t})\right\}$$

- Signal-to-noise problem   => 'Teufelspakt'

$$\lim_{t\to\infty} \frac{C_{\rm NN}^{I=0}(t)}{\sigma_{\rm stat}(t)} \propto \mathrm{e}^{-2(m_{\rm N}-\frac{3}{2}m_\pi)t}$$





$$m_{\rm eff}(t+0.5a) = \log\left[\frac{C(t)}{C(t+a)}\right]$$

Dotted lines: 2 x nucleon mass

An alternative to few-state fits at large time:

Goals:

- Use all data, including (precise) early times
- No modeling of excited states

Solution: spectral reconstruction

- From input data

$$C(t) = \int \mathrm{d}\omega\, \mathrm{e}^{-\omega t}\, \rho(\omega), \quad t \in \{2a, 3a, \ldots, 25a\}$$

- Infer

$$D(\alpha,\tau) = \int \mathrm{d}\omega\, \omega^{\alpha}\, \mathrm{e}^{-\omega\tau}\, \rho(\omega), \quad \alpha, \tau\, \mathrm{arbitrary}$$

$$= \int \mathrm{d}\omega\, \sigma(\omega)\, \rho(\omega)$$

Smeared spectral densities à la Backus-Gilbert:

Hansen, Lupo, Tantalo `19; Pijpers+Thompson `92, Backus+Gilbert `68

Seek an estimator of the form:

$$\hat{D}(\alpha,\tau) = \sum_t g_t\, C(t) = \int \mathrm{d}\omega \left(\sum_t g_t \mathrm{e}^{-\omega t}\right) \rho(\omega)$$
$$= \int \mathrm{d}\omega\, \hat{\sigma}(\omega)\, \rho(\omega)$$

Two competing considerations:

- Accuracy: $$A[g] = \int \mathrm{d}\omega\, \{\sigma(\omega) - \hat{\sigma}(\omega)\}^2$$
- Precision: $$B[g] = \mathrm{Var}[\hat{D}] = \sum_{t,t'} g_t\, g_{t'}\, \mathrm{Cov}[C(t), C(t')]$$
- Best estimate by minimizing combination:

$$G_\lambda[g] = (1-\lambda)A[g]/A[0] + \lambda B[g]$$

Test: predicting the effective mass

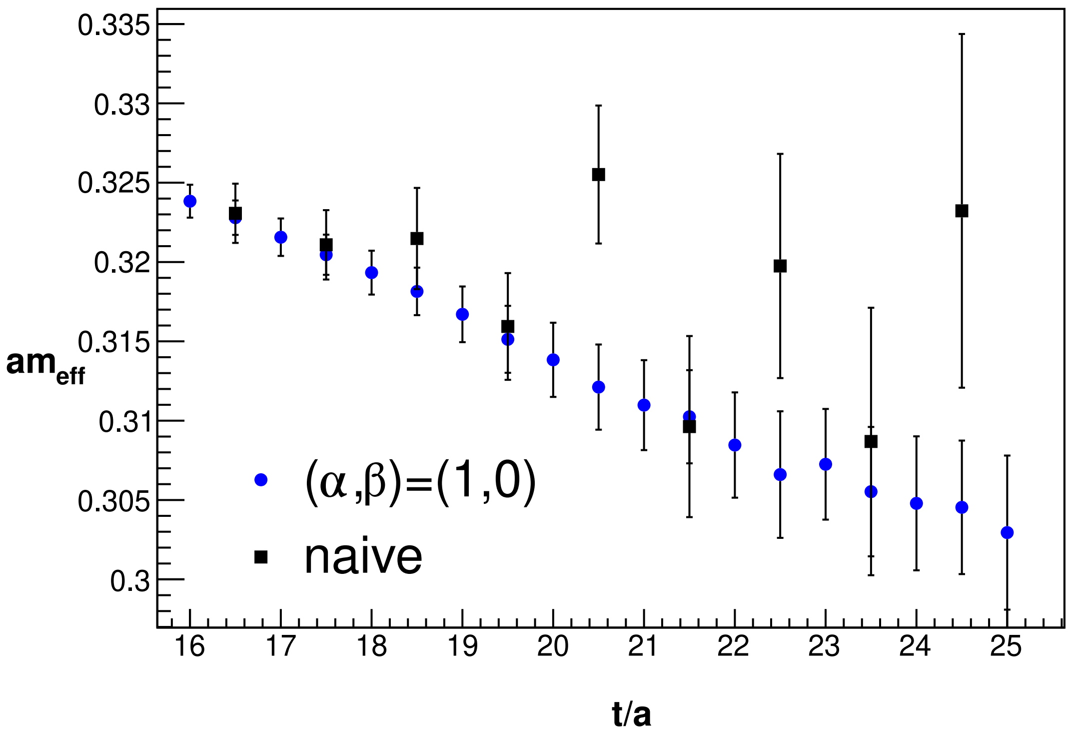
New definition of effective mass:

$$\tilde{m}_{\mathrm{eff}}(\alpha, \beta|\tau) = \left(\frac{D(\alpha, \tau)}{D(\beta, \tau)} \right)^{1/(\alpha-\beta)}$$

Standard definition is special case:

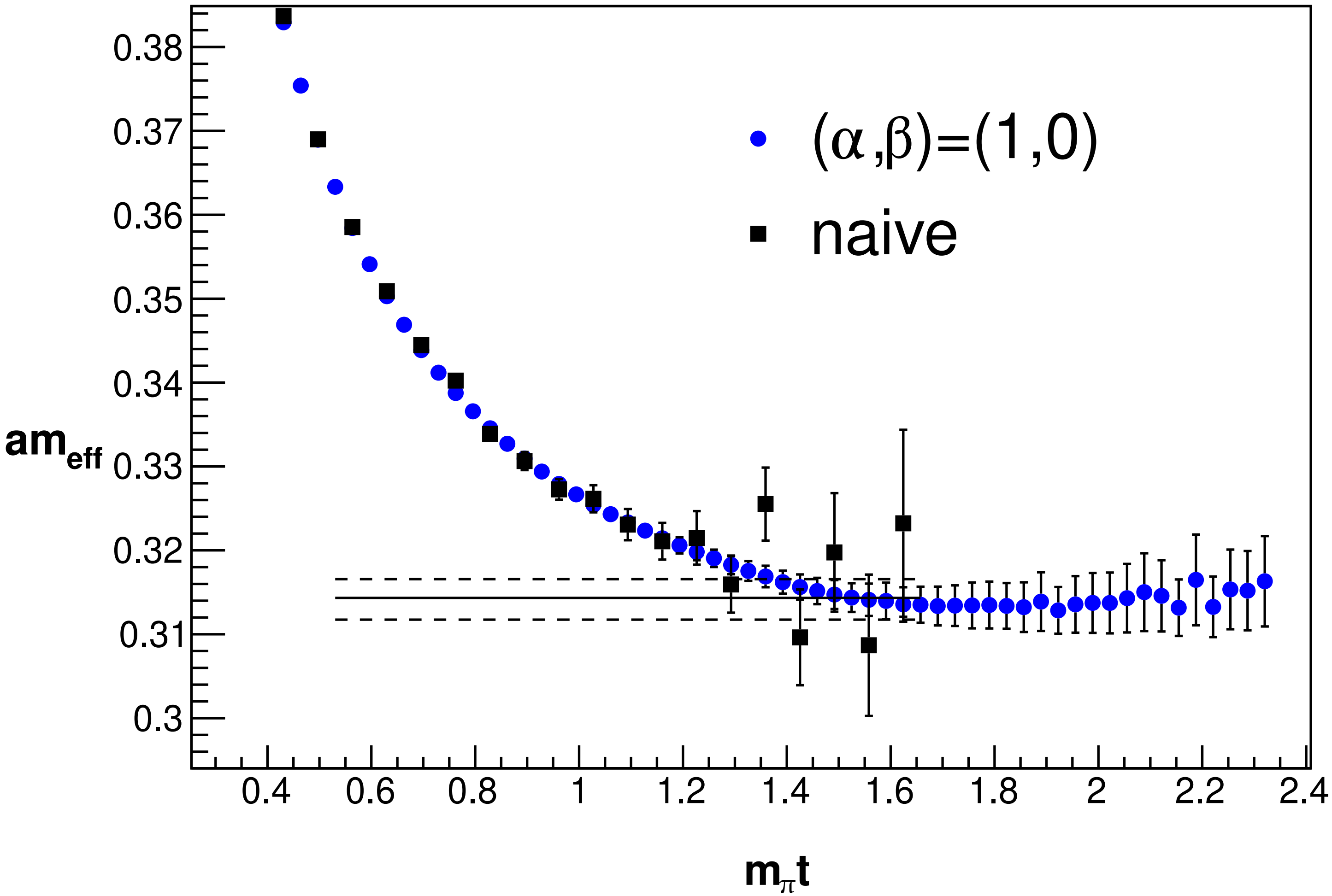
$$\tilde{m}_{\mathrm{eff}}(1, 0|t) = m_{\mathrm{eff}}(t) + \mathrm{O}(a^2)$$

- use timeslices [2a,15a]
- predict [16a, 25a]



Test: comparison with two-state fit

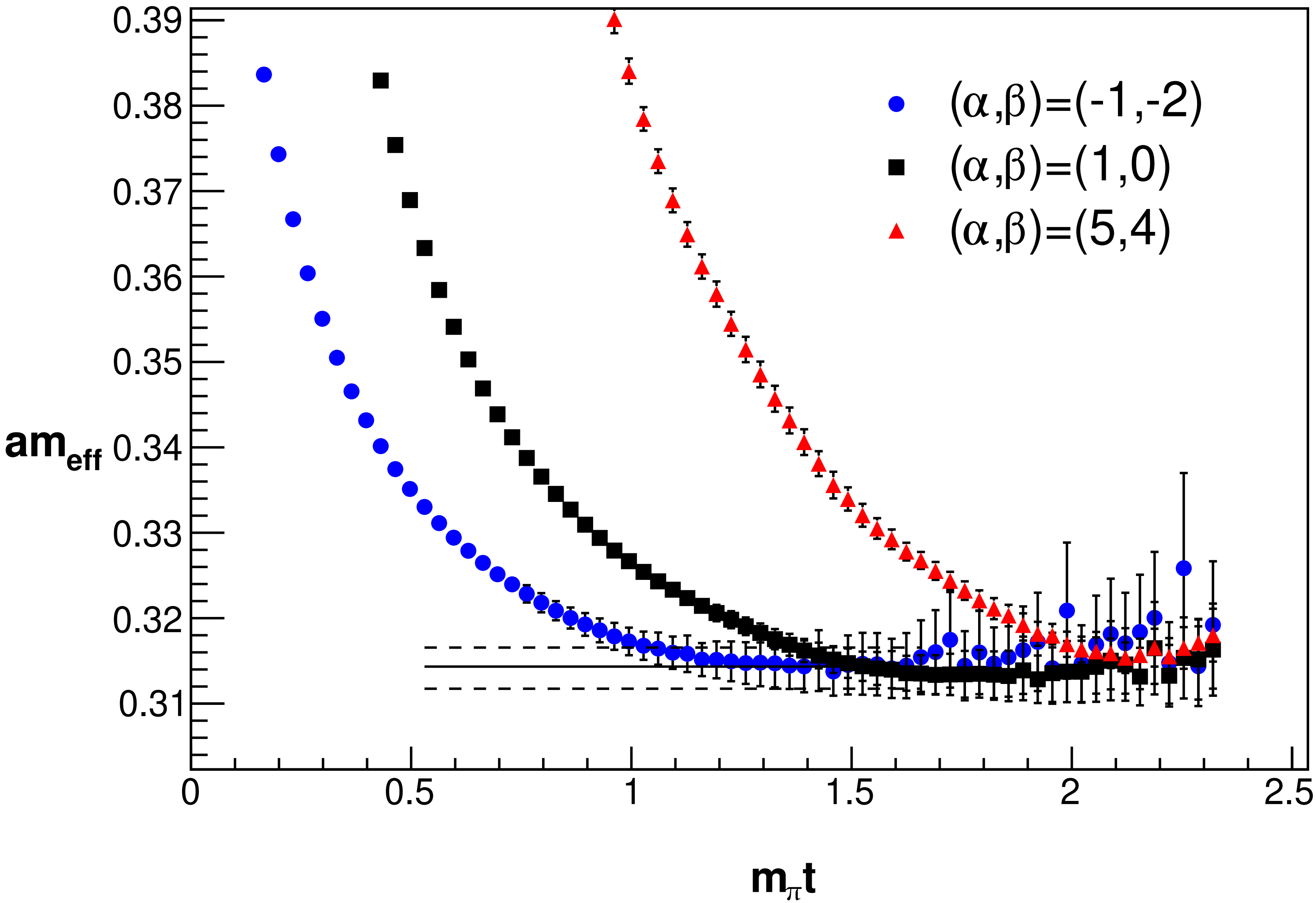
- Using all times [2a, 25a], predict times [26a, 35a]



- Long plateau: systematic error changes by factor of 2

Adjusting excited state contamination:

- Different (α, β) have different excited state systematics



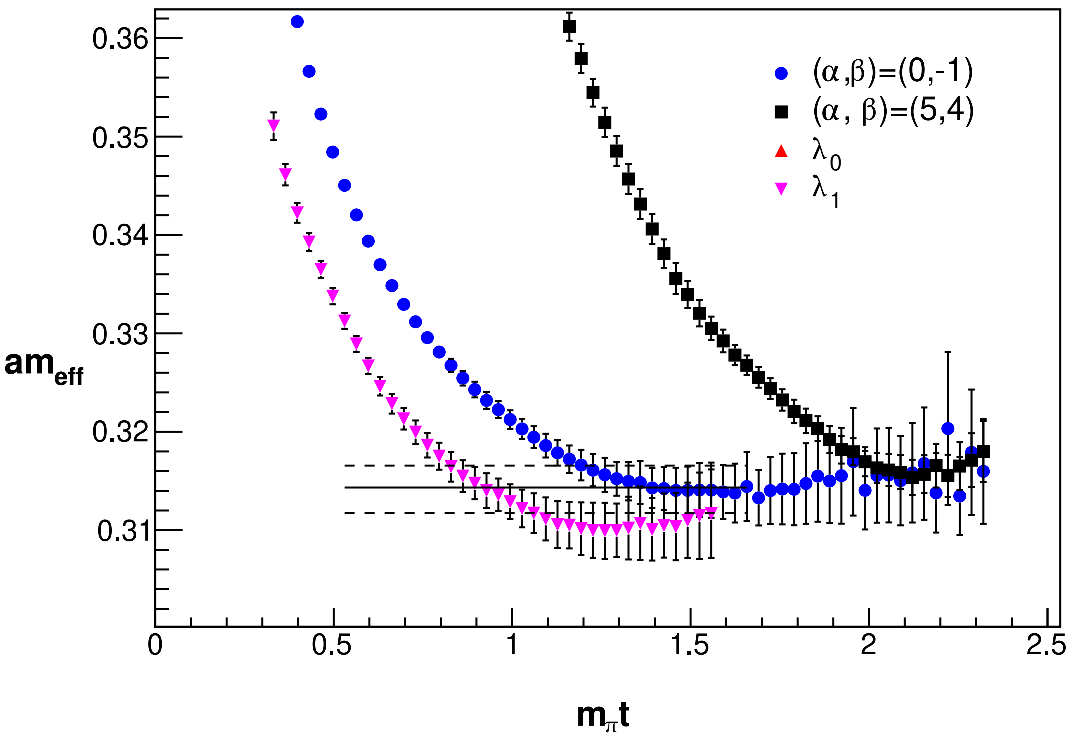
- Further confirmation of plateau value

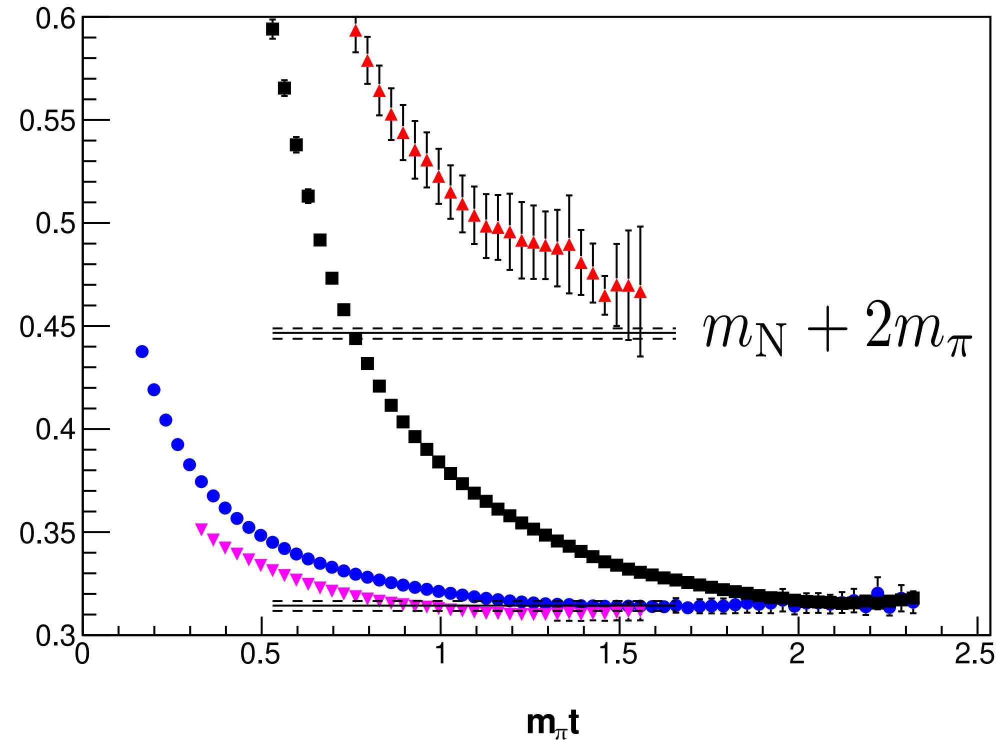
A novel Generalized Eigenvalue Problem (GEVP):

- Two correlation matrices:

$$A_{ij}(t) = D(\alpha_i + \alpha_j, t), \qquad B_{ij}(t) = D(\alpha_i + \alpha_j, t)$$

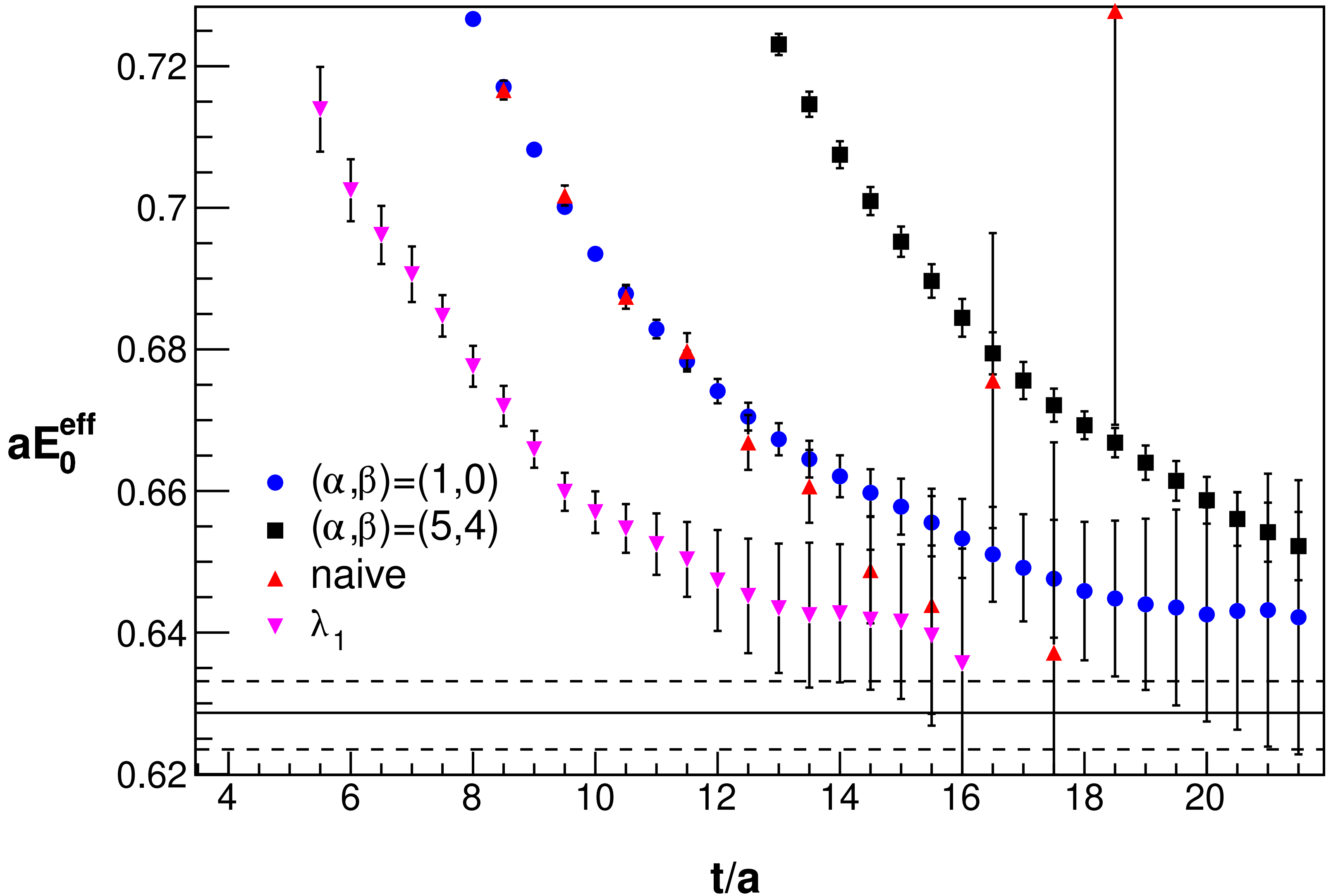
- Equal-time GEVP: $B(t)v_n(t) = \lambda_n(t)A(t)v_n(t)$





Application to nucleon-nucleon:

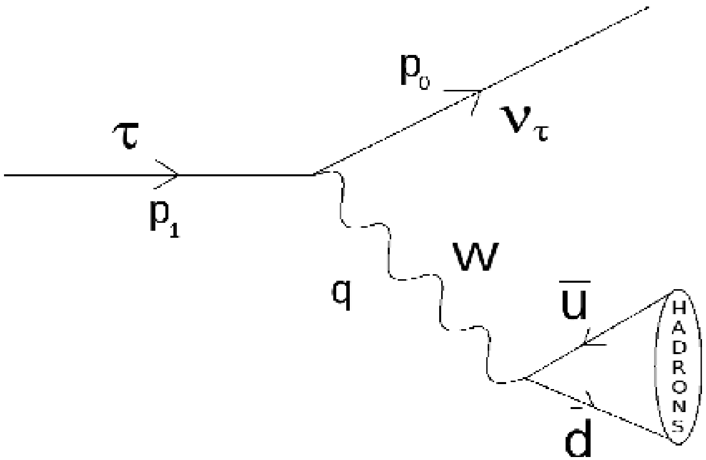
- Same setup as single-nucleon, with 2x2 GEVP

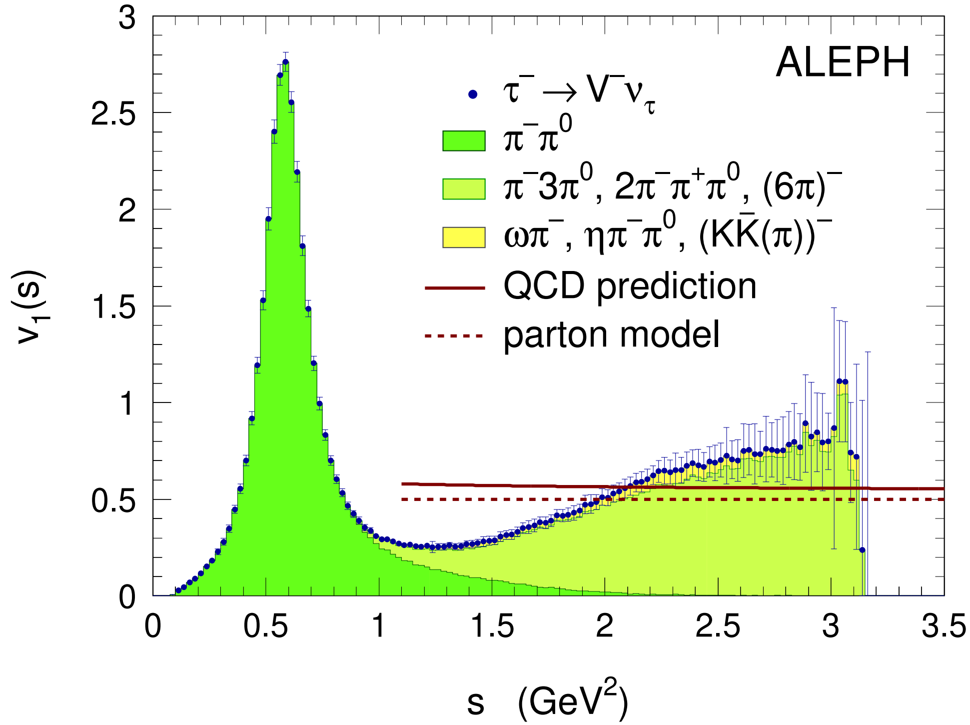


- Mild indication for attraction, but not precise enough. :-(

Inclusive processes in lattice QCD

Hadronic Tau decays:





Decay rate from spectral density:

$$C(t) = \int d^3\boldsymbol{x}\, \langle \Omega | \hat{V}_z^{cc}(\boldsymbol{x})\, \mathrm{e}^{-\hat{H}t}\, \hat{V}_z^{cc}(0)^\dagger | \Omega \rangle$$

$$\propto \int_0^\infty d\omega\, \omega^2 v_1(\omega^2)\, \mathrm{e}^{-\omega t}$$

Smeared spectral densities

- Approximation of Dirac-delta:

$$\lim_{\epsilon \to 0^+} \delta_\epsilon(x) = \delta(x)$$

- Implement $i\epsilon$ -prescription

$$\delta_\epsilon(x) = \frac{i}{x + i\epsilon} = \frac{\epsilon}{x^2 + \epsilon^2} + i\frac{x}{x^2 + \epsilon^2}$$

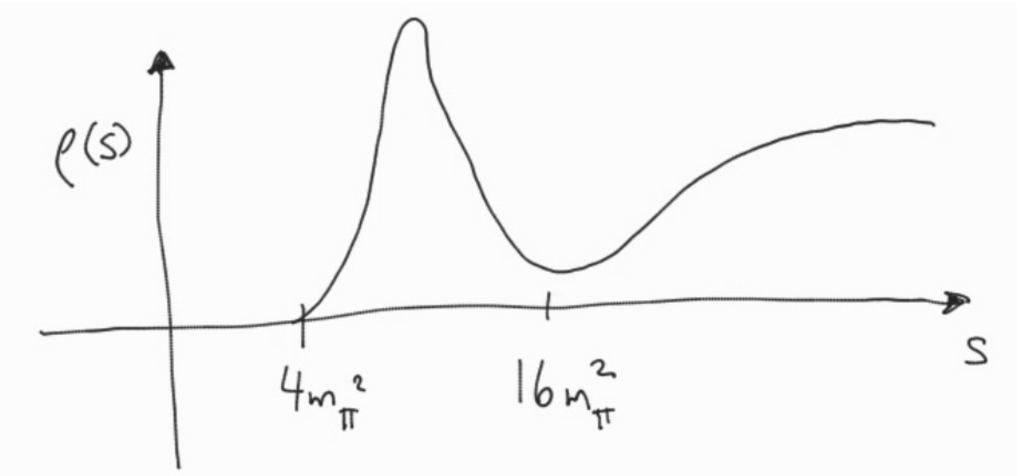
- Cut off high energies (Heaviside approx.):

$$\lim_{\epsilon \to 0^+} \delta_\epsilon(x) = \theta(x)$$

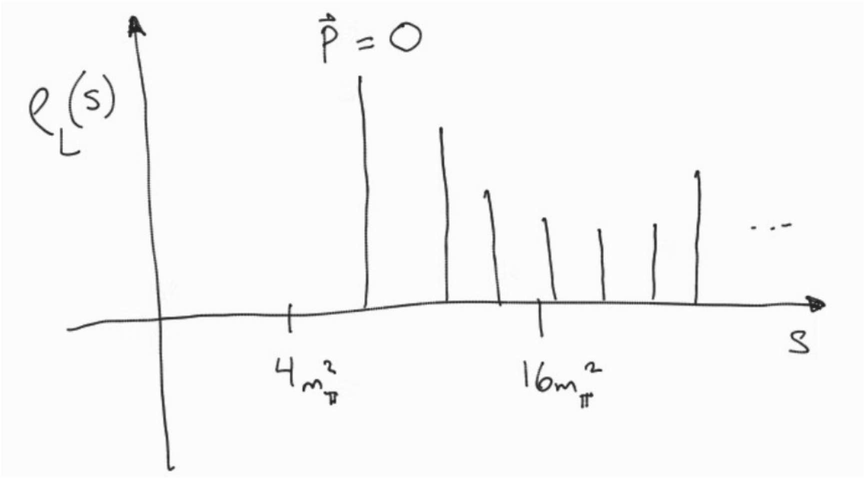
- Dispersion relations, sum rules, ….

Finite vs. infinite volume

Infinite volume: continuous



Finite volume: sum of Dirac-delta peaks.



Not 'close' to infinite volume at finite L!

Masterfield lattice QCD

- Large volumes needed to saturate ordered double limit:

$$v_1(s) = \lim_{\epsilon\to 0^+} \lim_{L\to\infty} v^{\rm g}_{1,\epsilon}(s), \quad v^{\rm g}_{1,\epsilon}(s) = \int {\rm d}\omega \, \frac{{\rm e}^{-\frac{(\omega-\sqrt{s})^2}{2\epsilon^2}}}{\sqrt{2\pi}\epsilon} v_1(\omega^2)$$

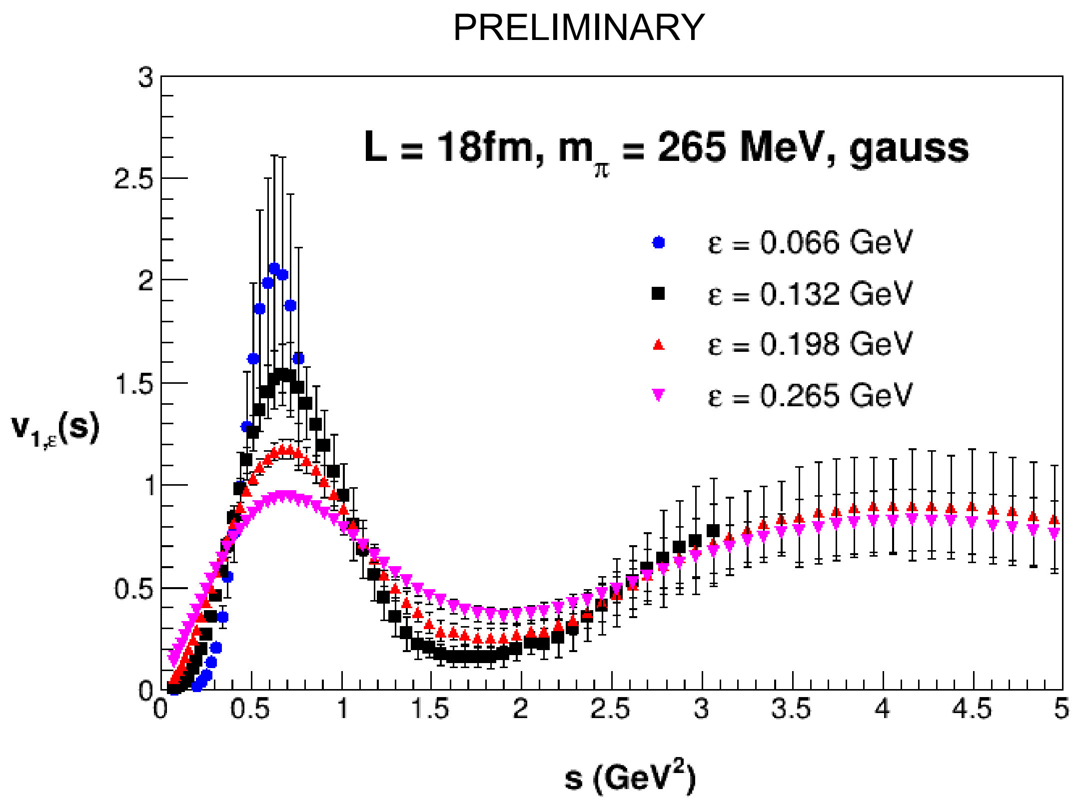
- Relevant idea: masterfield simulation paradigm   M. Lüscher, `17
  - ➔ Only a few gauge configurations
  - ➔ Accrue statistics from separate space-time regions:
    - ➔ O(1000) gauge configs = 6^4 space time regions of size $m_\pi L \approx 3$
- Preliminary application: isovector (axial)vector correlators at

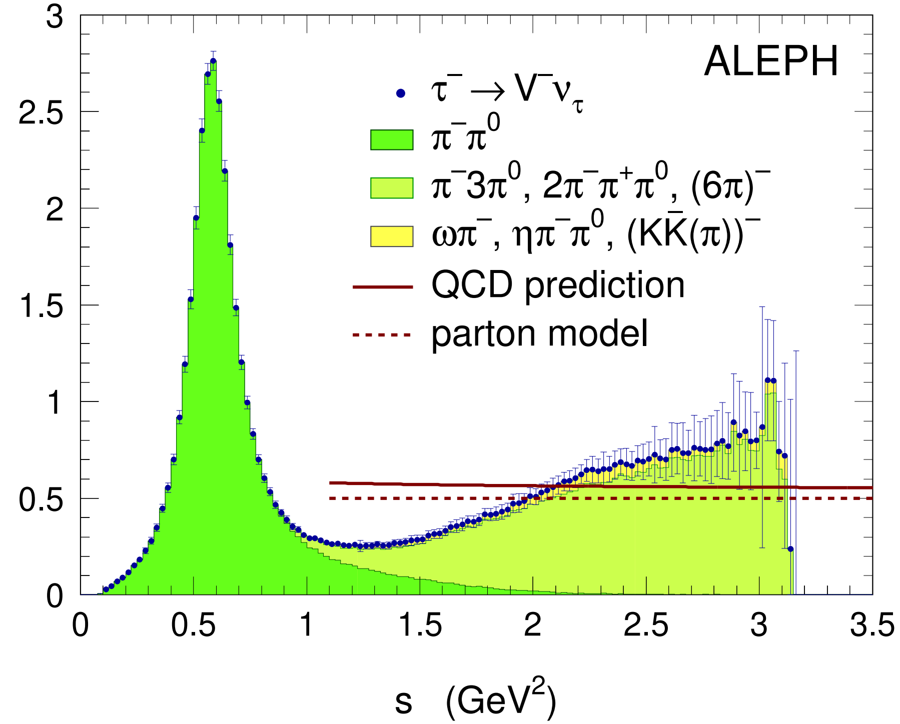
$$N_{\rm f} = 2+1, \quad L = 18{\rm fm},$$

$$a = 0.09{\rm fm}, \quad m_\pi = 265{\rm MeV}$$

M. Bruno, JB, A. Francis, P. Fritzsch, J. Green, M. T. Hansen, M. Lüscher, A. Patella, A. Rago, *in prep.*

Preliminary results





- Comparison to hadronic tau-decay (right) ALEPH collaboration `05
- No extrapolation to zero-width yet
- Mild indication of four-particle effects.

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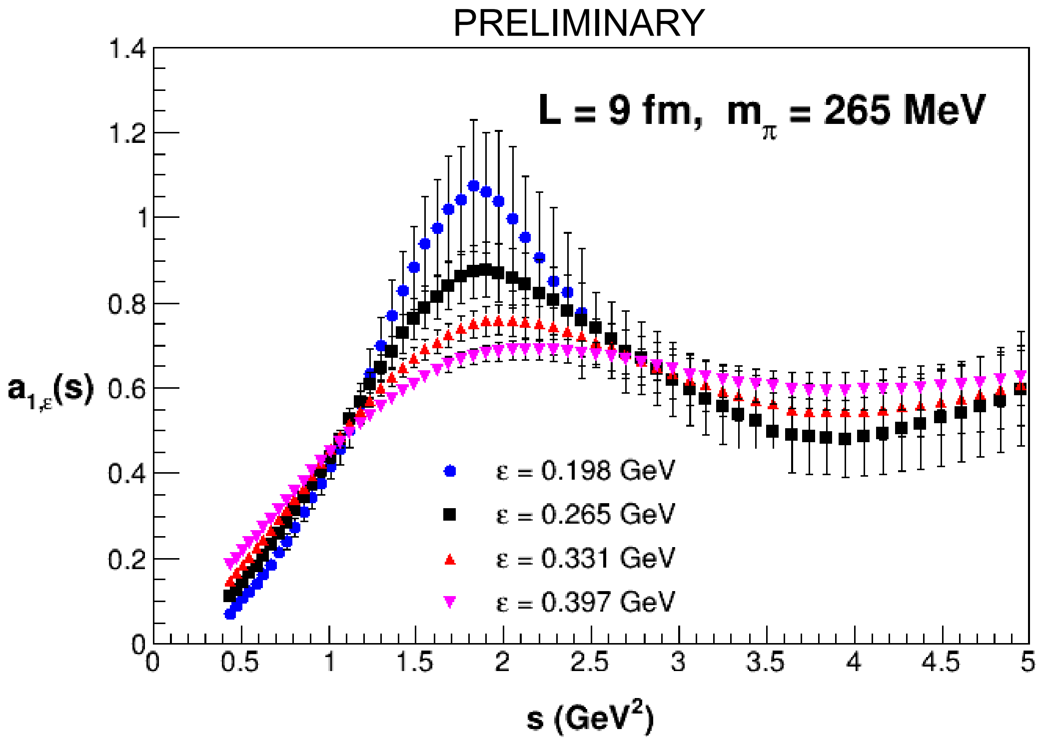
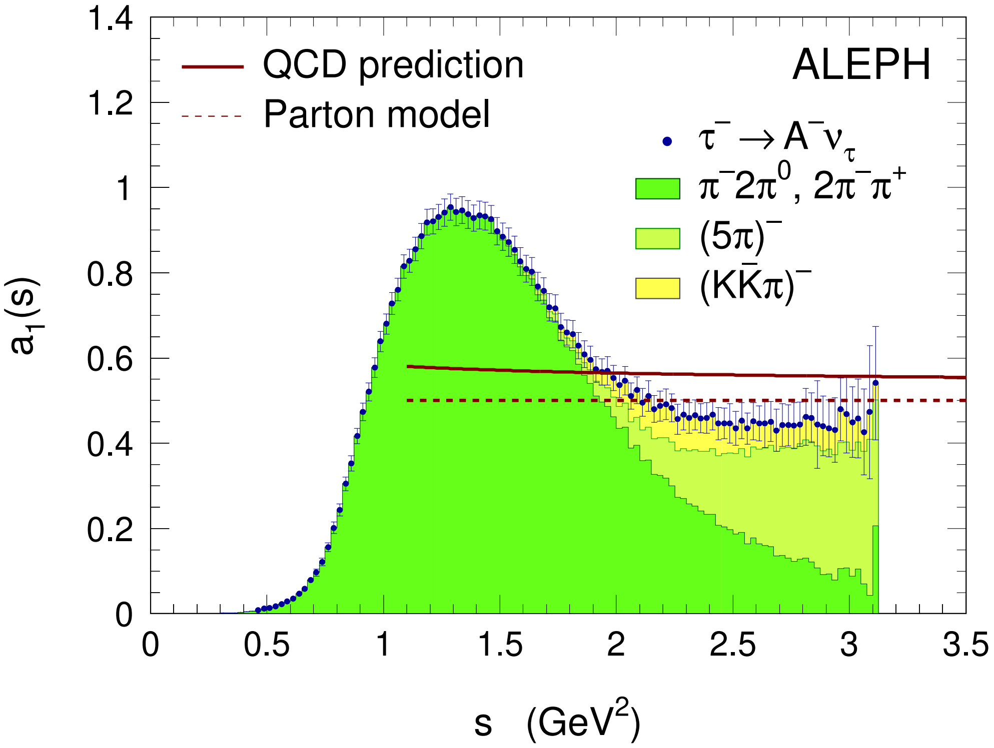
Preliminary results





ALEPH collaboration `05

- Comparison to hadronic tau-decay (right)
- No extrapolation to zero-width yet
- Bump from a1(1260), indication of five pions

M. Bruno, JB, A. Francis, P. Fritzsch, J. Green, M. T. Hansen, M. Lüscher, A. Patella, A. Rago, *in prep.*

Conclusions

- Spectral reconstruction is a promising alternative to standard lattice QCD analysis

- The correlator can be efficiently reconstructed at arbitrary times, with prefactors.
  - ➔ Three-point functions
  - ➔ Thermal effects

- Varying excited state contamination leads to a novel GEVP for spectroscopy.

- Improves nucleon mass, not enough precision for NN

- First lattice QCD computation of inclusive amplitudes are possible!
  - ➔ Large volumes needed for good energy resolution
  - ➔ Masterfield simulation paradigm effective

- Other applications of spectral reconstruction:
  - ➔ SVZ sum rules — T. Ishikawa and S. Hashimoto, Phys.Rev.D 104 (2021) 7, 074521
  - ➔ Inclusive semileptonic B-decays — P. Gambino, et al JHEP 07 (2022) 083
  - ➔ Exclusive scattering amplitudes — JB and M.T. Hansen Phys.Rev.D 100 (2019) 3, 034521