Interpretation of the Ω_c decay into $\pi^+ K^- \Xi$ from the $\Omega(2012)$ molecular perspective

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R. Pavao and E. Oset, Eur. Phys. J. C78, 857 (2018).
N. Ikeno, G. Toledo, and E. Oset, Phys. Rev. D 101, 094016 (2020).
N. Ikeno, W. H. Liang, G. Toledo, and E. Oset, Phys.Rev.D 106, 034022 (2022).

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Discovery of Ω(2012): Excited state of Ω with S=-3

In **2018**, Belle reported a new state Ω(2012) state: PRL121(2018)052003

$$M_{\Omega(2012)} = 2012.4 \pm 0.7 \pm 0.6\ \mathrm{MeV}$$
$$\Gamma_{\Omega(2012)} = 6.4^{+2.5}_{-2.0} \pm 1.6\ \mathrm{MeV}$$

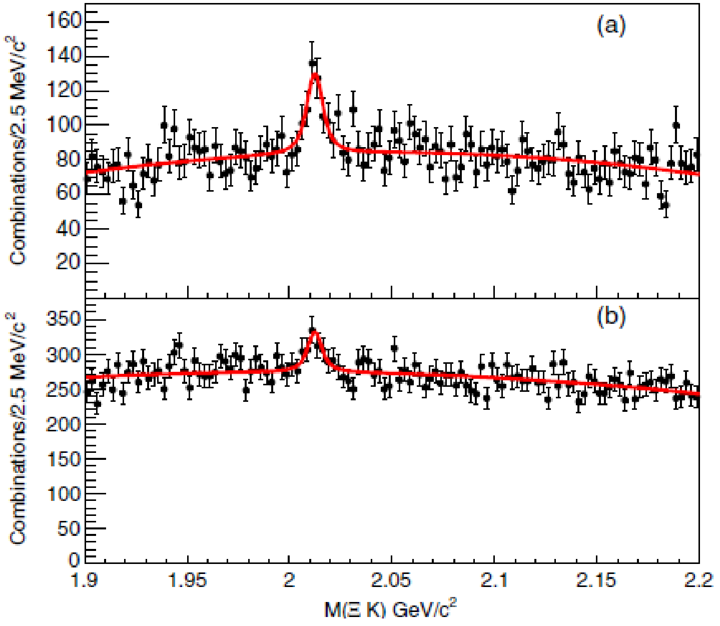


FIG. 2. The (a) $\Xi^0 K^-$ and (b) $\Xi^- K^0_S$ invariant mass distributions in data taken at the $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ resonance energies.

This prompted many theoretical studies of the Ω(2012) nature

- Quark model pictures
- Molecular pictures based on the meson-baryon interaction

- Only $\bar{K}\Xi^*(1530)$ state:
  - Y. H. Lin and B. S. Zou, Phys. Rev. D 98, 056013 (2018).
- Coupled channels $\bar{K}\Xi^*(1530)$, ηΩ, $\bar{K}\Xi$
  - M. P. Valderrama, Phys. Rev. D 98, 054009 (2018).
  - Y. Huang, M. Z. Liu, J. X. Lu, J. J. Xie, and L. S. Geng, Phys. Rev. D 98, 076012 (2018).
  - R. Pavao and E. Oset, Eur. Phys. J. C 78, 857 (2018).
  - M. V. Polyakov, H. D. Son, B. D. Sun, and A. Tandogan, Phys. Lett. B 792, 315 (2019). .....

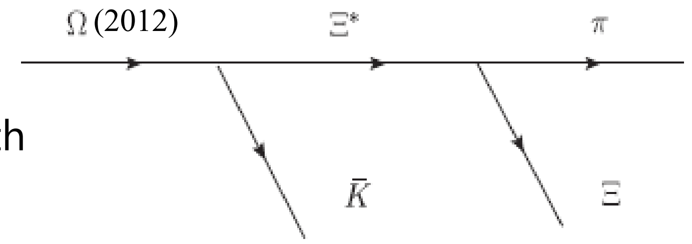
Ω(2012) is dynamically generated as a molecular state from the $K\Xi^*$ and ηΩ coupled channels interaction

Belle experimental data

- In **2019**, Belle reported a ratio of the Ω(2012) decay: PRD100(2019)032006

$$\mathcal{R}^{\Xi\pi K}_{\Xi K} = \frac{\mathcal{B}(\Omega(2012) \to \Xi(1530)(\to \Xi\pi)K)}{\mathcal{B}(\Omega(2012) \to \Xi K)} < 11.9\%$$

$\Omega(2012) \to \pi\bar{K}\Xi$

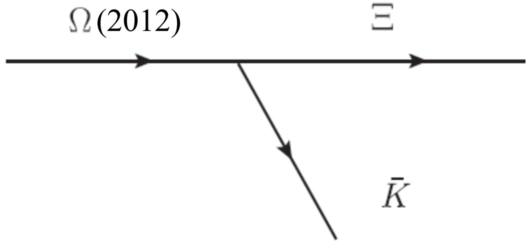


Three-body ΞπK decay width is significantly smaller than that of two-body ΞK decay width

=> Challenging result for the molecular picture nature

- In **2022**, a reanalysis of data (different cut): arXiv:2207.03090

$$\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}} = 0.97 \pm 0.24 \pm 0.07$$



=> Possibility of strong support for the molecular picture

- In **2021**, Ω(2012) has been observed by the Ω_c decay: PRD104(2021)052005

$$\frac{\mathcal{B}(\Omega_c^0 \to \pi^+ \Omega(2012)^-) \times \mathcal{B}(\Omega(2012)^- \to K^-\Xi^0)}{\mathcal{B}(\Omega_c^0 \to \pi^+ K^- \Xi^0)} = (9.6 \pm 3.2 \pm 1.8)\%,$$

Our study:
- ✓ Study a mechanism for $\Omega_c \to \pi^+\Omega(2012) \to \pi^+(K^-\Xi^0)$ reaction from Ω(2012) molecular perspective
- ✓ Discuss the consistency of the molecular picture with the Belle results

Formalism: Coupled channels approach

R. Pavao and E. Oset, EPJC78(2018)
N. Ikeno, G. Toledo, E. Oset, PRD(2020)

3 channels: $\boxed{\bar{K}\Xi^*,\ \eta\Omega \text{ (}s\text{-wave)},\ \bar{K}\Xi \text{ (}d\text{-wave)}}$

- Bethe-Salpeter equation:

$$T = [1 - VG]^{-1} V$$

- Transition potential: Ω^* $J^P=3/2^-$

$$\begin{array}{cc} & \begin{array}{ccc} \bar{K}\Xi^* & \eta\Omega & \bar{K}\Xi \end{array} \\ V = & \begin{pmatrix} 0 & 3F & \alpha q_{\rm on}^2 \\ 3F & 0 & \beta q_{\rm on}^2 \\ \alpha q_{\rm on}^2 & \beta q_{\rm on}^2 & 0 \end{pmatrix} \begin{array}{c} \bar{K}\Xi^* \\ \eta\Omega \\ \bar{K}\Xi \end{array} \end{array}$$

$$F = -\frac{1}{4f^2}(k^0 + k'^0) \qquad q_{\rm on} = \frac{\lambda^{1/2}\left(s, m_{\bar{K}}^2, m_{\Xi}^2\right)}{2\sqrt{s}}$$

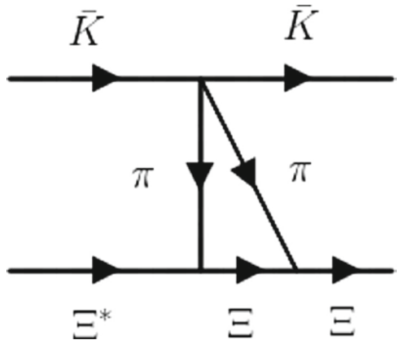
k^0, k'^0 the energies of initial and final states

- Diagonal potential is null
- Non-diagonal potential is nonzero.

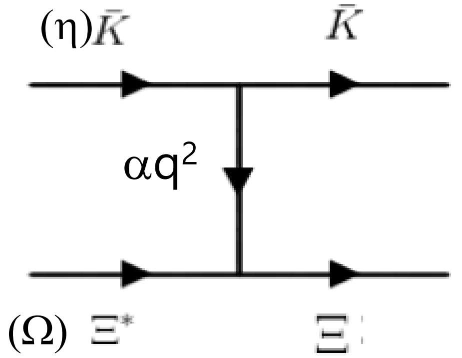
- s-wave potentials between $\bar{K}\Xi^*$ and $\eta\Omega$: taken from chiral Lagrangian of

S. Sarkar, E. Oset, M.J. Vicente Vacas, NPA750(2005) 294
E.E. Kolomeitsev, M.F.M. Lutz, PLB 585 (2004) 243–252

- d-wave potential between $\bar{K}\Xi$ and $\bar{K}\Xi^*$ or $\eta\Omega$: described in terms of α, β: free parameters



A possible d-wave diagram for the $\bar{K}\Xi^*$ -> $\bar{K}\Xi$ transition



We do not make a model
Estimates done by M. P. Valderrama, PRD98,054009 (2018).

$G_{K^- \Xi^*}$ function accounting for $\Xi^* \to \pi\Xi$ decay

- Meson-Baryon loop function G:

$$G(\sqrt{s}) = \begin{pmatrix} G_{\bar{K}\Xi^*}(\sqrt{s}) & 0 & 0 \\ 0 & G_{\eta\Omega}(\sqrt{s}) & 0 \\ 0 & 0 & G_{\bar{K}\Xi}(\sqrt{s}) \end{pmatrix}$$

q_{max}: cut off parameter

For s-wave channel

$$G_i(\sqrt{s}) = \int_{|\boldsymbol{q}|<q_{\max}} \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_i(\boldsymbol{q})} \frac{M_i}{E_i(\boldsymbol{q})} \frac{1}{\sqrt{s} - \omega_i(\boldsymbol{q}) - E_i(\boldsymbol{q}) + i\epsilon}$$

for $i = \bar{K}\Xi^*, \eta\Omega$

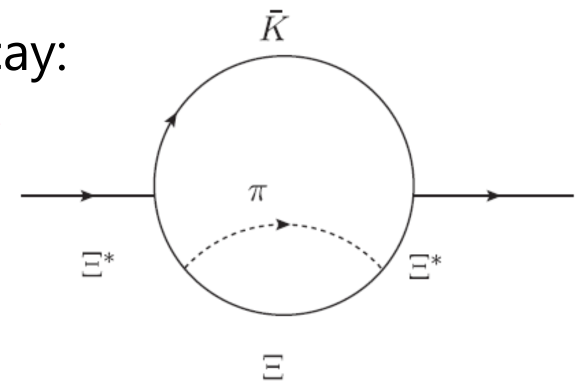
For d-wave channel

$$G_{\bar{K}\Xi}(\sqrt{s}) = \int_{|\boldsymbol{q}|<q'_{\max}} \frac{d^3q}{(2\pi)^3} \frac{(q/q_{\rm on})^4}{2\omega_{\bar{K}}(\boldsymbol{q})} \frac{M_\Xi}{E_\Xi(\boldsymbol{q})} \frac{1}{\sqrt{s} - \omega_{\bar{K}}(\boldsymbol{q}) - E_\Xi(\boldsymbol{q}) + i\epsilon}$$

- We take into account the Ξ^* mass distribution due to its width for $\Xi^* \to \pi\Xi$ decay:

  $G_{K^-\Xi^*}$ is convolved with the Ξ^* mass distribution: $\Omega(2012) \to \pi K\Xi$ decay

$$\tilde{G}_{\bar{K}\Xi^*}(\sqrt{s}) = \frac{1}{N} \int_{M_{\Xi^*}-\Delta M_{\Xi^*}}^{M_{\Xi^*}+\Delta M_{\Xi^*}} d\tilde{M} \left(-\frac{1}{\pi}\right) \mathrm{Im}\left(\frac{1}{\tilde{M} - M_{\Xi^*} + i\frac{\Gamma_{\Xi^*}}{2}}\right) G_{\bar{K}\Xi^*}(\sqrt{s}, m_{\bar{K}}, \tilde{M})$$



=> Comparison of the $\Omega(2012)$ with/without convolution gives us the estimate of $\Omega(2012)$ decay width into $\bar{K}\Xi$ and $\pi\bar{K}\Xi$ decay channels :

$$\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}} = \frac{\Gamma_{\Omega^*\to\pi\bar{K}\Xi}}{\Gamma_{\Omega^*\to\bar{K}\Xi}} = \frac{\Gamma_{\Omega^*,\mathrm{con}} - \Gamma_{\Omega^*,\mathrm{non}}}{\Gamma_{\Omega^*,\mathrm{non}}}$$

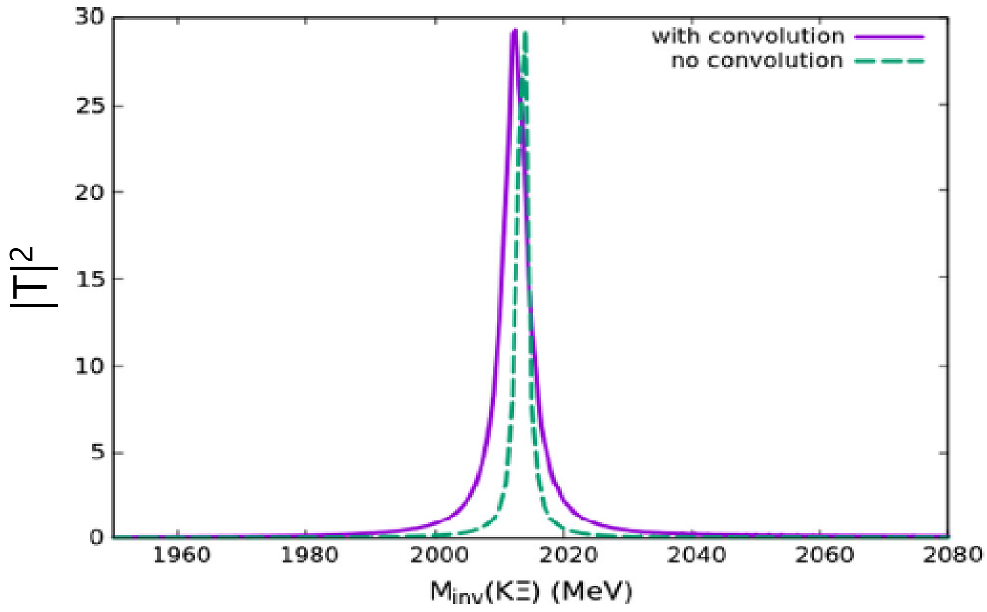
Γ_{con} : $G_{K^-\Xi^*}$ with convolution (accounts for $K\Xi$ and $\pi K\Xi$ decays)
Γ_{non} : $G_{K^-\Xi^*}$ without convolution (only for $K\Xi$ decay)

Calculated Ω(2012) mass, width, and decay ratio

We make a fit to the experimental data by changing the α, β, q_{max} parameters.

$$M_{\Omega(2012)} = 2012.4 \pm 0.7 \pm 0.6 \text{ MeV} \qquad \Gamma_{\Omega(2012)} = 6.4^{+2.5}_{-2.0} \pm 1.6 \text{ MeV}$$

- Result by R. Pavao and E. Oset, EPJC78(2018)



α (MeV^{-3})	β (MeV^{-3})	$q_{\max} = q'_{\max}$ (MeV)
4.0×10^{-8}	1.5×10^{-8}	735

- Result with convolution

$$m_{\Omega^*} = 2012.37 \text{ MeV}, \quad \Gamma_{\Omega^*} = 6.24 \text{ MeV}.$$

- Result without convolution

$$m_{\Omega^*}^{(\text{no conv.})} = 2013.5 \text{ MeV}, \quad \Gamma_{\Omega^*}^{(\text{no conv.})} = 3.2 \text{ MeV}.$$

$$\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}} = \frac{\Gamma_{\Omega^*\to\pi\bar{K}\Xi}}{\Gamma_{\Omega^*\to\bar{K}\Xi}} = \frac{\Gamma_{\Omega^*,\text{con}} - \Gamma_{\Omega^*,\text{non}}}{\Gamma_{\Omega^*,\text{non}}} = 0.95$$

=> Good agreement with the latest Belle result $\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}} = 0.97 \pm 0.24 \pm 0.07$

Limitation of calculated ratio R

We also find an acceptable solution in terms of natural values for the q_{max}, α, β parameters which reproduce fairly well the experimental data in 2019 $\frac{\Gamma_\Omega(\pi\bar{K}\Xi)}{\Gamma_{\Omega,\bar{K}\Xi}} < 11.9\,\%$

- Results of Set1-3 by N. Ikeno, G. Toledo, and E. Oset, PRD101, 094016 (2020)

  Similar results in J. Lu, C. Zeng, E. Wang, J. Xie, and L. Geng, Eur. Phys. J. C, 361 (2020)

	Pavao,&Oset	Set 1	Set 2	Set 3
$q_{\rm max}(\bar{K}\Xi^*)$ [MeV]	735	735	775	735
$q_{\rm max}(\eta\Omega)$ [MeV]	735	735	710	750
α $[10^{-8}\ \mathrm{MeV}^{-3}]$	4.0	-8.7	-8.7	-11.0
β $[10^{-8}\ \mathrm{MeV}^{-3}]$	1.5	18.3	18.3	20.0
M_R [MeV]	2012.4	2012.7	2012.7	2012.6
Γ_R [MeV]	6.24	7.3	7.7	8.2
$\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}}$	0.95	0.109	0.104	0.109

The molecular picture was pushed to the limit and showed that the ratio R could be made as small as 10%, but not smaller than 10%

Couplings g_i of different channels

- The couplings g_i of the $\Omega(2012)$ to the different channels are obtained from the residue of the T-matrix of the pole in the second Riemann sheet. Close to the pole

$$T_{ij} = \frac{g_i g_j}{z - z_R} \,(z, \text{complex energy};\ z_R, \text{complex pole position}) \qquad g_i^2 = \lim_{z \to z_R} (z - z_R) T_{ii};\ \ g_j = g_i \frac{T_{ij}}{T_{ii}}|_{z=z_R}.$$

- We also show the wave function at the origin for the s-wave states, wf($g_i G_i$), and the probability of each channel $-g_i^2 \frac{\partial G_i}{\partial \sqrt{s}}$

$$\sum_i (-) g_i^2 \frac{\partial G}{\partial \sqrt{s}} = 1,$$

	$\bar{K}\Xi^*$ (2027)	$\eta\Omega$ (2220)	$\bar{K}\Xi$ (1812)
g_i	$1.86 - i0.02$	$3.52 - i0.46$	$-0.42 + i0.12$
g_i (Pavao, Oset)	$2.01 + i0.02$	$2.84 - i0.01$	$-0.29 + i0.04$
$\mathrm{wf}_i(g_i G_i)$	$-34.05 - i1.10$	$-30.66 + i3.67$	...
$-g_i^2 \frac{\partial G_i}{\partial \sqrt{s}}$	$0.57 + i0.10$	$0.25 - i0.06$	...

- D. Gamermann, J. Nieves, E. Oset, and E. R. Arriola, PRD 81, 014029 (2010).
- F. Aceti, L.R. Dai, L.S. Geng, E. Oset and Y. Zhang, EPJA50, 57 (2014)
- S. Weinberg, Phys. Rev. 130, 776 (1963).
- T. Hyodo, D. Jido and A. Hosaka, PRC85,015201 (2012).
- T. Hyodo, Int. J. Mod. Phys. A28, 1330045 (2013).
- T. Sekihara, PRC104, 035202 (2021), etc.

The strength of the *wf* and the probability dominates for the $\bar{K}\Xi^*$ state.

Note, however, that the $\eta\Omega$ channel is required to bind Ω^* state since the diagonal potential of the $\bar{K}\Xi^*$ channel is zero and hence cannot produce any bound state by itself.

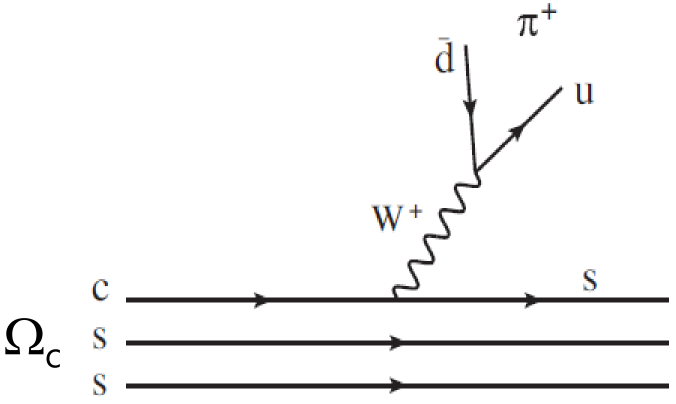
The $\Omega_c \to \pi^+\Omega(2012) \to \pi^+(K^-\Xi^0)$ reaction

- Belle result: $$\frac{\mathcal{B}(\Omega_c^0 \to \pi^+ \Omega(2012)^-) \times \mathcal{B}(\Omega(2012)^- \to K^- \Xi^0)}{\mathcal{B}(\Omega_c^0 \to \pi^+ K^- \Xi^0)} = (9.6 \pm 3.2 \pm 1.8)\%,$$

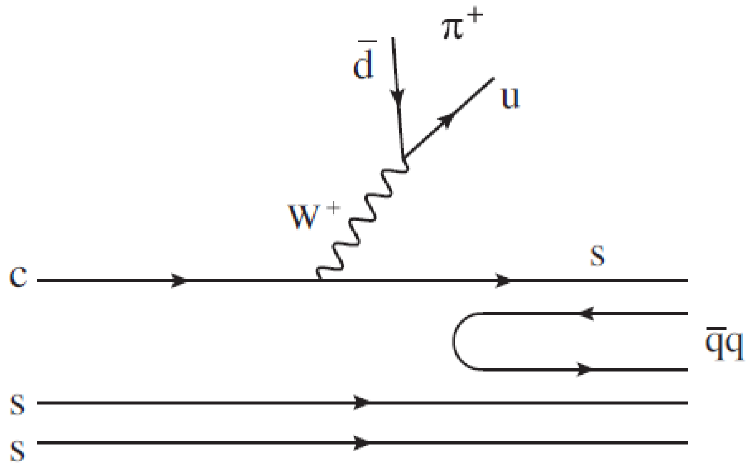
- We study a mechanism for $\Omega_c \to \pi^+\Omega(2012)$ production through an external emission weak decay, where the $\Omega(2012)$ is dynamically generated from the interaction of $\bar{K}\Xi^*$ and $\eta\Omega$, with $\bar{K}\Xi$ as the main decay channel.

N. Ikeno, W. H. Liang, G. Toledo, and E. Oset, PRD 106, 034022 (2022).

Microscopic picture for $\Omega_c \to \pi^+ sss$



Hadronization of an ss pair



$$sss \to \sum_i s\bar{q}_i q_i ss = \sum_i P_{3i} q_i ss$$

where $$P \equiv \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \sqrt{\frac{2}{3}}\eta' \end{pmatrix}$$

$$sss \to K^- \underline{uss} + \bar{K}^0 \underline{dss} - \frac{\eta}{\sqrt{3}} \underline{sss}$$

We obtain $\bar{K}\Xi^*$, $\bar{K}\Xi$, $\eta\Omega$

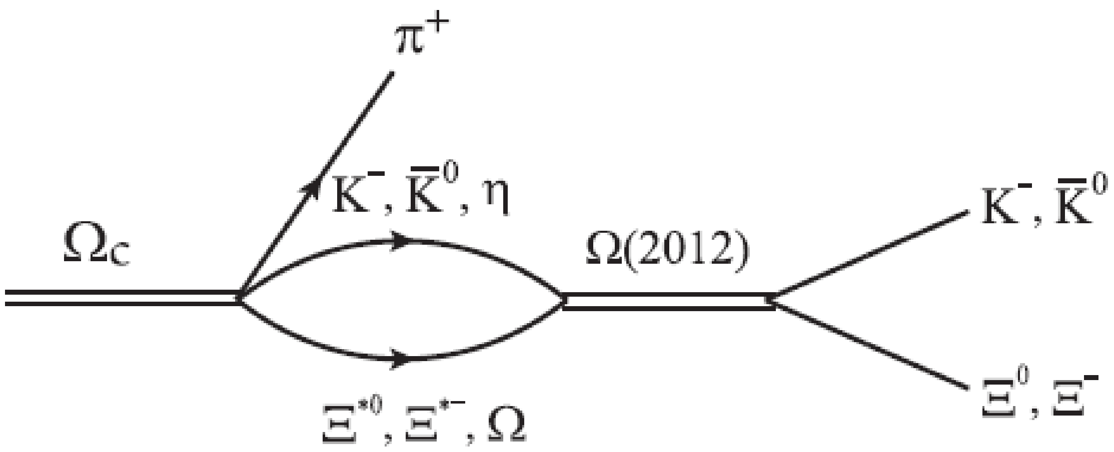
- Weak interaction vertices: $V_P = C(q^0 + \vec{\sigma} \cdot \vec{q})$

$\mathcal{L}_{W,\pi} \sim W^\mu \partial_\mu \phi$. $\mathcal{L}_{\bar{q}Wq} \sim (\bar{q}_{\text{fin}} W_\mu \gamma^\mu (1 - \gamma_5) q_{\text{in}}$

C: unknown constant

K. Miyahara, et al., PRC95, 035212 (2017) :
V. R. Debastiani, et al., PRD97, 094035 (2018)

The $\Omega_c \to \pi^+\Omega(2012) \to \pi^+(K^-\Xi^0)$ reaction



$$\frac{\mathcal{B}(\Omega_c^0 \to \pi^+ \Omega(2012)^-) \times \mathcal{B}(\Omega(2012)^- \to K^-\Xi^0)}{\mathcal{B}(\Omega_c^0 \to \pi^+ K^- \Xi^0)} = (9.6 \pm 3.2 \pm 1.8)\%,$$

$\pi^+ K\Xi^*$ and $\pi^+\eta\Omega$ are produced
=> $K\Xi^*$ and $\eta\Omega$ interact and produce $\Omega(2012)$
=> Later $\Omega(2012)$ decays into $K\Xi$

$\bar{K}\Xi$ mass distribution for Ω_c decay:

$$\frac{d\Gamma_{\rm signal}}{dM_{\rm inv}(K^-\Xi^0)} = \frac{1}{(2\pi)^3}\frac{M_\Xi}{M_{\Omega_c}} p_\pi \tilde{p}_{K^-} \sum |\bar{t}|^2,$$

$$p_{\pi^+} = \frac{\lambda^{1/2}(M_{\Omega_c}^2, m_\pi^2, M_{\rm inv}^2(K^-\Xi^0))}{2M_{\Omega_c}}$$

$$\tilde{p}_{K^-} = \frac{\lambda^{1/2}(M_{\rm inv}^2(K^-\Xi^0), m_{K^-}^2, M_{\Xi^0}^2)}{2M_{\rm inv}(K^-\Xi^0)}$$

Amplitude

$$t = (-\sqrt{2}W(\bar{K}^-\Xi^{*0})G_{\bar{K}\Xi^*}(M_{\rm inv})g_{R,\bar{K}\Xi^*} + W(\eta\Omega)G_{\eta\Omega}(M_{\rm inv})g_{R,\eta\Omega})\frac{1}{M_{\rm inv} - M_R + i\frac{\Gamma_R}{2}}\left(-\frac{1}{\sqrt{2}}\right)g_{R,\bar{K}\Xi}$$

W: Weight for the matrix elements of $\Omega_c \uparrow\uparrow\uparrow$ going to π^+ and the different final states

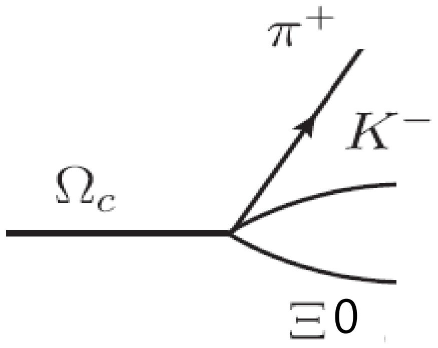
R stands for $\Omega(2012)$ resonance

$g_{R,\bar{K}\Xi^*}$, $g_{R,\eta\Omega}$, $g_{R,\bar{K}\Xi}$: Couplings to $\Omega(2012)$

We use the same values obtained from the $\Omega(2012)$ study

Background for $\Omega_c \to \pi^+ K^- \Xi^0$ without going through the $\Omega(2012)$

(1) Direct reaction:

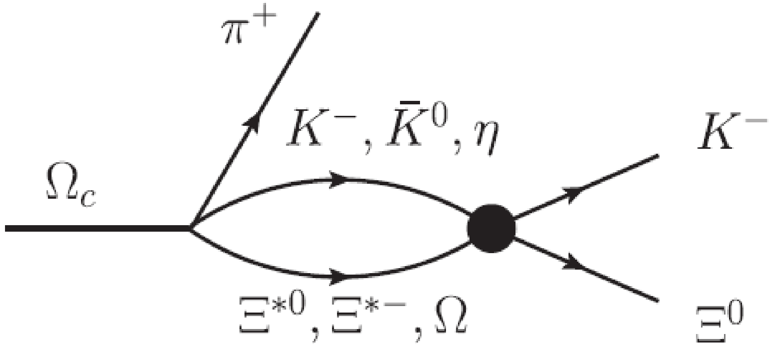


$$\frac{d\Gamma^{(1)}_{\text{bac}}}{dM_{\text{inv}}(K^-\Xi^0)} = \frac{1}{(2\pi)^3}\frac{M_\Xi}{M_{\Omega_c}} p_\pi \tilde{p}_{K^-} \sum |\bar{t}^{(1)}_{\text{bac}}|^2$$

$$\sum |\bar{t}^{(1)}_{\text{bac}}|^2 = C^2 \frac{4}{27}\vec{q}^2.$$

$$q = p_{\pi^+} = \frac{\lambda^{1/2}(M^2_{\Omega_c}, m^2_\pi, M^2_{\text{inv}}(K^-\Xi^0))}{2M_{\Omega_c}}$$

(2) Reaction through intermediate states:



$$\frac{d\Gamma^{(2)}_{\text{bac}}}{dM_{\text{inv}}(K^-\Xi^0)} = \frac{1}{(2\pi)^3}\frac{M_\Xi}{M_{\Omega_c}} p_{\pi^+} \tilde{p}_{K^-} \sum |\bar{t}^{(2)}_{\text{bac}}|^2$$

$$t^{(2)}_{\text{bac}} = W(K^-\Xi^{*0}) \cdot G_{\bar{K}\Xi^*}(M_{\text{inv}}) \cdot \alpha \vec{p}^{\,2}_{\bar{K}} - \frac{1}{\sqrt{2}} W(\eta\Omega) \cdot G_{\eta\Omega}(M_{\text{inv}}) \cdot \beta \vec{p}^{\,2}_{\bar{K}}$$

$$V = \begin{array}{cc} & \begin{array}{ccc} \bar{K}\Xi^* & \eta\Omega & \bar{K}\Xi \end{array} \\ & \begin{pmatrix} 0 & 3F & \alpha q^2_{\text{on}} \\ 3F & 0 & \beta q^2_{\text{on}} \\ \alpha q^2_{\text{on}} & \beta q^2_{\text{on}} & 0 \end{pmatrix} \begin{array}{c} \bar{K}\Xi^* \\ \eta\Omega \\ \bar{K}\Xi \end{array} \end{array}$$

$$V_{\bar{K}\Xi^* \to \bar{K}\Xi} = \alpha \vec{q}^{\,2}_{\bar{K}},$$

$$V_{\eta\Omega \to \bar{K}\Xi} = \beta \vec{q}^{\,2}_{\bar{K}}$$

α, β : Potential parameters

We use the same values obtained from the $\Omega(2012)$ study

Calculated total $\pi^+K^-\Xi^0$ production

N. Ikeno, W. H. Liang, G. Toledo, and E. Oset, PRD 106, 034022 (2022).

- We define ratios:

$$R_1 \equiv \frac{\Gamma^{(1)}_{\rm bac}}{\Gamma_{\rm signal}} = \frac{\Gamma^{(1)}_{\rm bac}/\Gamma_{\Omega_c}}{\Gamma_{\rm signal}/\Gamma_{\Omega_c}} = \frac{\mathcal{B}^{(1)}_{\rm bac}}{\mathcal{B}_{\rm signal}} = 0.16 \sim 0.47 \quad \Rightarrow \quad \mathcal{B}^{(1)}_{\rm bac} = R_1 \mathcal{B}_{\rm signal},$$

$$R_2 \equiv \frac{\Gamma^{(2)}_{\rm bac}}{\Gamma_{\rm signal}} = \frac{\Gamma^{(2)}_{\rm bac}/\Gamma_{\Omega_c}}{\Gamma_{\rm signal}/\Gamma_{\Omega_c}} = \frac{\mathcal{B}^{(2)}_{\rm bac}}{\mathcal{B}_{\rm signal}} = 0.10 \sim 0.20 \quad \Rightarrow \quad \mathcal{B}^{(2)}_{\rm bac} = R_2 \mathcal{B}_{\rm signal}$$

	R_1	R_2	$R_1 + R_2$
Pavao,&Oset	0.45	0.17	0.62
Set 1	0.23	0.16	0.39
Set 2	0.21	0.12	0.33
Set 3	0.21	0.20	0.41

N. Ikeno, G. Toledo, E. Oset, PRD(2020)
R. Pavao and E. Oset, EPJC78(2018)

- Total $\pi^+K^-\Xi^0$ production stemming from the molecular picture

$$\mathcal{B}[\Omega_c \to \pi^+ K^- \Xi^0]_{\rm th} = \mathcal{B}_{\rm signal} + \mathcal{B}^{(1)}_{\rm bac} + \mathcal{B}^{(2)}_{\rm bac} = \mathcal{B}_{\rm signal}(1 + R_1 + R_2)$$

<- Unknown constant C is implicitly included in $B_{\rm signal}$

$$\frac{\mathcal{B}[\Omega_c \to \pi^+ K^- \Xi^0]_{\rm th}}{\mathcal{B}[\Omega_c \to \pi^+ K^- \Xi^0]_{\rm exp}} = \boxed{\frac{\mathcal{B}_{\rm signal}}{\mathcal{B}[\Omega_c \to \pi^+ K^- \Xi^0]_{\rm exp}}}(1 + R_1 + R_2)$$

Ratio measured by Belle

$$\frac{\mathcal{B}(\Omega_c^0 \to \pi^+ \Omega(2012)^-) \times \mathcal{B}(\Omega(2012)^- \to K^- \Xi^0)}{\mathcal{B}(\Omega_c^0 \to \pi^+ K^- \Xi^0)} = (9.6 \pm 3.2 \pm 1.8)\%,$$

$$= (9.6 \pm 3.2 \pm 1.8)(1 + R_1 + R_2)\% \quad = 12.2\text{–}15.7\%$$

with 38% uncertainty

=> Based on the molecular picture, we obtain only about 12–20% of the total production with the three modes evaluated to produce $\pi^+K^-\Xi^0$, one resonant and two nonresonant

Two other sources we did not consider

- There are two other sources we did not consider: ~85% of the total production

$\boxed{\Omega_c \to \Xi^0 \bar{K}^{*0} \to \Xi^0 K^- \pi^+}$ (PDG data)

(Γ_7, Γ_8 of PDG data)

$$\frac{\Gamma_8}{\Gamma_7} = \frac{\mathcal{B}[\Xi^0 \bar{K}^{*0} \to \Xi^0 K^- \pi^+]}{\mathcal{B}[\Xi^0 K^- \pi^+]} = \frac{0.68 \pm 0.16}{1.20 \pm 0.18} = \underline{0.57 \pm 0.16},$$

=> This means that about 60% of the $\Omega_c \to \pi^+ K^- \Xi^0$ decay comes from the $\Xi^0 K^{*-} \to \pi^+ K^- \Xi^0$ decay, which is not a part of our calculation

$\boxed{\Omega_c \to \pi^+ \Omega^* \to \pi^+ K^- \Xi^0}$ (Quark model)

Ω^* are any kind of excited $\Omega(sss)$ states

We can rely upon a theoretical Quark model calculation

K. L. Wang, Q. F. Lü, J. J. Xie, and X. H. Zhong, PRD107, 034015 (2023)

$$\frac{\mathcal{B}[\Omega_c \to \pi^+ \Omega(1^2P_{3/2^-}) \to \pi^+ K^- \Xi^0]}{\mathcal{B}[\Omega_c \to \pi^+ K^- \Xi^0]} \simeq 0.08,$$

$$\frac{\mathcal{B}[\Omega_c \to \pi^+ \Omega(1^2P_{1/2^-}) \to \pi^+ K^- \Xi^0]}{\mathcal{B}[\Omega_c \to \pi^+ K^- \Xi^0]} \simeq 0.11,$$

$$\frac{\mathcal{B}[\Omega_c \to \pi^+ \Omega(1^4D_{1/2^+}) \to \pi^+ K^- \Xi^0]}{\mathcal{B}[\Omega_c \to \pi^+ K^- \Xi^0]} \simeq 0.04$$

$$\frac{\mathcal{B}[\Omega_c \to \pi^+ \Omega(1^4D_{3/2^+}) \to \pi^+ K^- \Xi^0]}{\mathcal{B}[\Omega_c \to \pi^+ K^- \Xi^0]} \simeq 0.02$$

Summing all these contributions, we find a fraction of

$$\frac{\mathcal{B}[\Omega_c \to \pi^+ \Omega^* \to \pi^+ K^- \Xi^0]}{\mathcal{B}[\Omega_c \to \pi^+ K^- \Xi^0]} \simeq \underline{0.25}.$$

We find a total fraction of

$$\boxed{0.82 \pm 0.16 \simeq 66 - 98\%.}$$

=> The consistency of the molecular picture with all data

Summary

- We have studied the molecular picture for the $\Omega(2012)$ state with the coupled channels $\bar{K}\Xi^*$, $\eta\Omega$, $\bar{K}\Xi$

- We also have studied the $\Omega_c \to \pi^+\Omega(2012) \to \pi^+K^-\Xi^0$ reaction

- We found that the obtained results are consistent with all Belle results

  - $\Omega(2012)$ mass, width, and the ratio R of the $\Xi\pi K$ width to the ΞK width

  - All three channels account for about (12–20)% of the total $\Omega_c \to \pi^+K^-\Xi^0$ decay rate

- We should note that the molecular structure of $\Omega(2012)$ is mostly a $\bar{K}\Xi^*$ bound state. However, it requires the interaction with the $\eta\Omega$ channel to bind, while neither the $\bar{K}\Xi^*$ nor the $\eta\Omega$ states would be bound by themselves.

- New information on experimental data is most welcome.

W: Weight for the matrix elements of $\Omega_c \uparrow\uparrow\uparrow$ going to π^+ and the different final states

$$K^-\Xi^{*0}(S_z = 3/2):\ W = \frac{1}{\sqrt{3}} C(q^0 + q_z),$$

$$\bar{K}^0\Xi^{*-}(S_z = 3/2):\ W = \frac{1}{\sqrt{3}} C(q^0 + q_z),$$

$$\eta\Omega(S_z = 3/2):\ W = -\frac{1}{\sqrt{3}} C(q^0 + q_z),$$

$$K^-\Xi^{*0}(S_z = 1/2):\ W = \frac{1}{3} C q_+,$$

$$\bar{K}^0\Xi^{*-}(S_z = 1/2):\ W = \frac{1}{3} C q_+,$$

$$\eta\Omega(S_z = 1/2):\ W = -\frac{1}{3} C q_+,$$

$$K^-\Xi^{0}(S_z = 1/2):\ W = \frac{\sqrt{2}}{3} C q_+$$

$$\bar{K}^0\Xi^{-}(S_z = 1/2):\ W = -\frac{\sqrt{2}}{3} C q_+,$$

$$V_P = C(q^0 + \vec{\sigma}\cdot\vec{q})$$

the matrix element of

$$\vec{\sigma}\cdot\vec{q} = \sigma_+ q_- + \sigma_- q_+ + \sigma_z q_z,$$

$$\sigma_+ = \frac{1}{2}(\sigma_x + i\sigma_y), \qquad \sigma_- = \frac{1}{2}(\sigma_x - i\sigma_y),$$

$$q_+ = q_x + iq_y, \qquad q_- = q_x - iq_y,$$

q^0 is the energy of the π^+ and q its three-momentum

We can take the z direction in the π^+ direction, and when integrating over the π^+ angles, we get the angle averaged values of q_z^2, $q^0 q_z$, and $|q_+|^2$,

$$q_z^2 \to \frac{1}{3}\vec{q}^2, \qquad q^0 q_z \to 0, \qquad |q_+|^2 = q_x^2 + q_y^2 \to \frac{2}{3}\vec{q}^2.$$

internal emission

We can also have $\bar{K}^0\pi^+\Xi^-$, $\bar{K}^0\pi^+\Xi^{*-}$ production via internal emission, although suppressed by a color factor around 1/3.

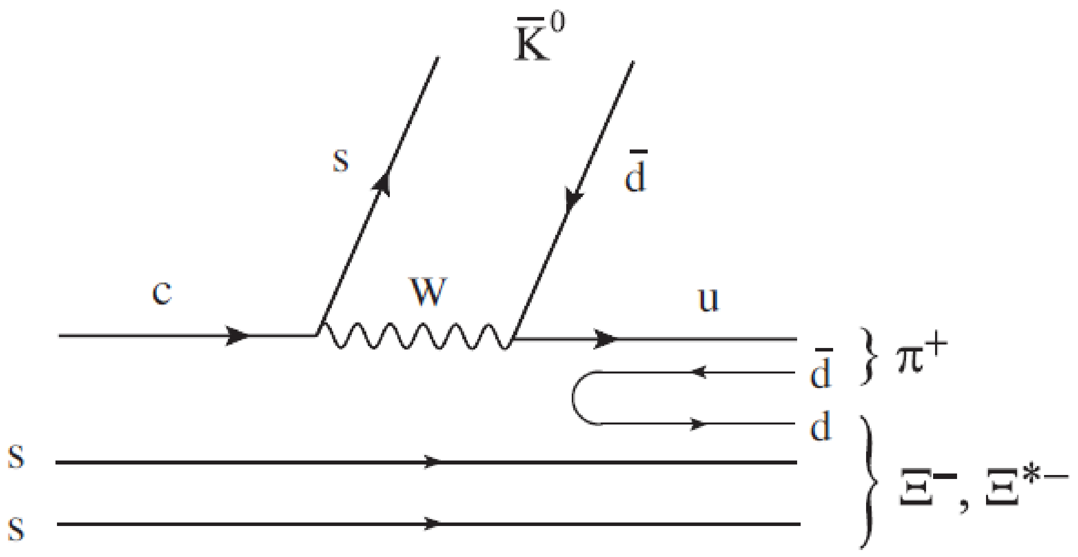


FIG. 4. Mechanism for internal emission for $\bar{K}^0\pi^+\Xi^-$, $\bar{K}^0\pi^+\Xi^{*-}$.

If we look at the $\pi^+K^-\Xi^0$ final state production, we just have an external emission