CAN THE TWO-POLE STRUCTURE OF THE $D_0^*(2300)$ BE UNDERSTOOD FROM RECENT LATTICE DATA?

June 8, 2023 | Anuvind Asokan | IAS Forschungszentrum Jülich

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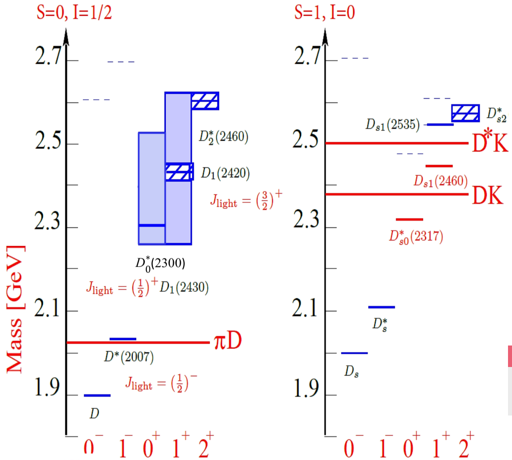
Introduction



- Quark model mesons
  $\Longrightarrow$ quark-anti-quark bound states
- Positive parity c-s scalar($D_{s0}^*(2317)$) and axial vector($D_{s1}(2460)$) much lighter than the quark model prediction
- SU(3) partners: observed as broad bumps in the $D\pi$ and $D^*\pi$ invariant mass in B decays $\rightarrow$ $D_0^*(2300)$ and $D_1(2430)$

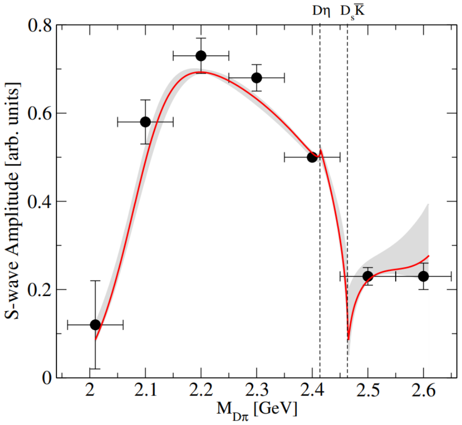
Masses almost equal to the strange counter parts

Quark Modell: M. Di Pierro and E. Eichten, PRD 64 (2001) 114004 however, various UChPT studies find two D_0^* mesons and two D_1 mesons in the energy region of $D_0^*(2300)$ and $D_1(2430)$ respectively

- Two pole picture solves the SU(3) mass peculiarity
- The two D_0^* poles are located at $2105^{+6}_{-8} - i102^{+10}_{-11}$ and $2451^{+35}_{-25} - i134^{+7}_{-8}$
- Lower one SU(3) partner of $D_{s0}^*(2317)$

M.Du et al.,PhysRevD.98.094018,L.Liu et al.,PhysRevD.87.014508...

- Amplitudes consistent with the LHCb data of the three body B meson decays
- But lattice QCD study of $D\pi - D\eta - D_s\bar{K}$ by HadSpec reported only one pole below the $D\pi$ threshold at $m_\pi = 391\ \mathrm{MeV}$



M.Du et al., arXiv:1712.07957v2 [hep-ph]

Is the two pole picture consistent with the lattice data?

Amplitudes from the lattice study

K-matrix parametrisation of the kind

$$K_{ij} = \frac{\left(g_i^{(0)} + g_i^{(1)} s\right)\left(g_j^{(0)} + g_j^{(1)} s\right)}{m^2 - s} + \gamma_{ij}^{(0)} + \gamma_{ij}^{(1)} s,$$

With T-matrix

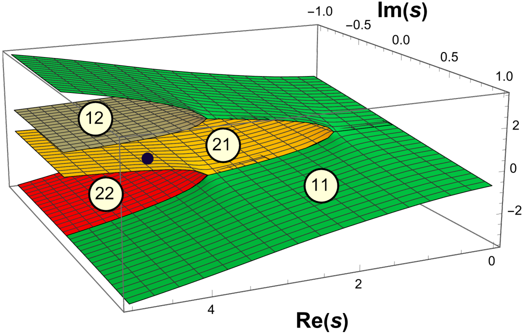
$$T_K(s)_{ij} = \frac{1}{K^{-1}(s)_{ij} + \left(I_{\mathrm{CM}}^{(i)}(s) - I_{\mathrm{CM}}^{(i)}(m^2)\right)\delta_{ij}},$$

where Chew-Mandelstam function given by

$$I_{\mathrm{CM}}^{(i)}(s) = \frac{\rho_i(s)}{\pi}\log\left[\frac{\xi_i(s) + \rho_i(s)}{\xi_i(s) - \rho_i(s)}\right] - \frac{\xi_i(s)}{\pi}\frac{m_2^{(i)} - m_1^{(i)}}{m_1^{(i)} + m_2^{(i)}}\log\frac{m_2^{(i)}}{m_1^{(i)}},$$

$\rightarrow$ 9 different amplitudes with different parameters

Riemann Sheet



- Branch cuts at every channel opening
- Sign of $Im(p_{cm})$ used to label the sheets
- Poles correspond to bound states or resonances
- Poles given by zeros of

$$\det\left(K^{-1}(s) + (I_{\mathrm{CM}}(s) - I_{\mathrm{CM}}(m^2))\right) = 0$$

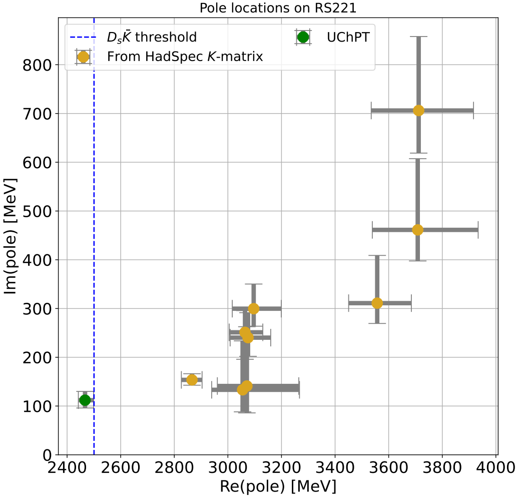
- Crossing from the physical sheet done by adding the discontinuity across the branch cut
- Discontinuity related to imginary part of T-matrix by Schwartz reflection principle

$$\begin{aligned}\mathrm{Disc}\left[T_K(s)\right] &= T_K(s+i\epsilon) - T_K(s - i\epsilon)\\ &= 2i\ \mathrm{Im}[T_K(s+i\epsilon)]\end{aligned}$$

Hidden sheet poles $\Longrightarrow$
On sheets not directly connected at physical sheets

Pole Search

- At $m_\pi = 391\,\mathrm{MeV}$, lowest pole bound state
  - found in all nine parametrizations employed by HadSpec
- Second pole RS211, RS221, and RS222 for almost all parametrizations but
  - they scatter very much
  - also in part located outside fitted region
- Clear correlation between real and imaginary part of the poles
- Extracted pole from the UChPT analysis in line
- Poles located on hidden sheets



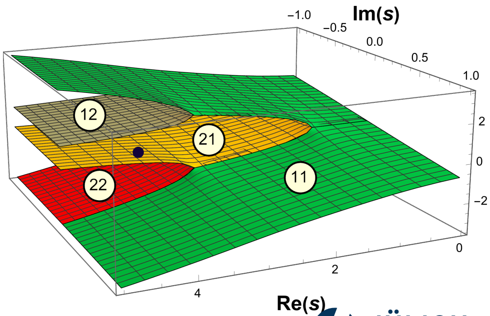
Residues

- Poles characterized by its location and residue
- residue quantifies the couplings of resonances to the various channels given by

$$R_{ij} = \lim_{s \to s_p} (s - s_p) T_{ij}(s).$$

- With the effective coupling is given by

$$g_i^r = R_{ij} / \sqrt{R_{jj}}$$



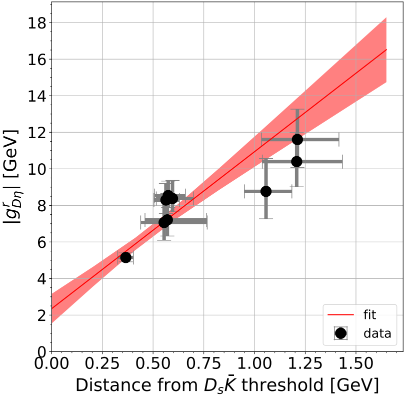
Residue and Threshold Distance

- Effects of hidden pole on physical axis visible at threshold only
- We quantified the distance as

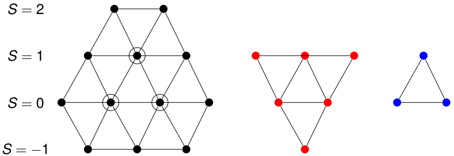
$$Dist = M - M_{thr}$$

to avoid double counting the effect of the couplings

- Effects of poles at threshold encoded in the y-intercept well constrained
- Next step: A parametrization to better constraint the location of the higher pole



SU(3) Symmetry



- $[\bar{3}] \otimes [8] = [1\bar{5}] \oplus [6] \oplus [\bar{3}]$
- SU(3) flavor to isospin basis relation:

$$\begin{pmatrix} |[\bar{3}]\rangle \\ |[6]\rangle \\ |[\overline{15}]\rangle \end{pmatrix} = U \begin{pmatrix} |D\pi\rangle \\ |D\eta\rangle \\ |D_s\bar{K}\rangle \end{pmatrix}$$

- where

$$U = \begin{pmatrix} -3/4 & -1/4 & -\sqrt{3/8} \\ \sqrt{3/8} & -\sqrt{3/8} & -1/2 \\ 1/4 & 3/4 & -\sqrt{3/8} \end{pmatrix}$$

SU(3) Symmetry

Form for K-matrix:

$$K = \left(\frac{g_{\bar{3}}^2}{m_{\bar{3}}^2 - s} + c_{\bar{3}} \right) C_{\bar{3}} + \left(\frac{g_6^2}{m_6^2 - s} + c_6 \right) C_6 + c_{\overline{15}} C_{\overline{15}}$$

the two bare poles are assumed to be in the two SU(3) multiplets with S-wave attractions from LO chiral dynamics

No. of parameters: 5 - 7

- [$\overline{15}$] was found to be repulsive
- T matrix as before
- Subtraction point for Chew-Mandelstam chosen to be $m_{\bar{3}}$

Miguel, et al.,physletb.2017.02.036

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Fitting to Lattice energy Levels

Finite volume T-matrix related to continuum T by

$$\tilde{T}(s) = \frac{1}{T^{-1}(s) - \Delta G(s)},$$

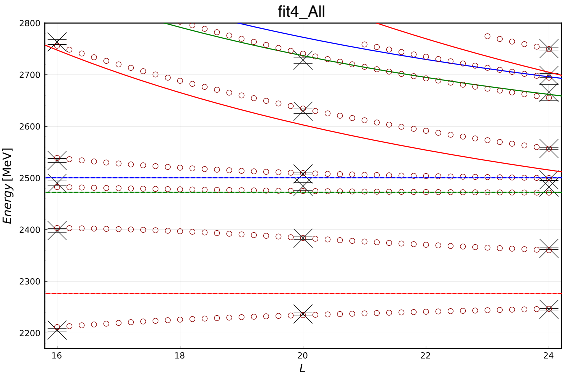
with

$$\Delta G_{ii} = \tilde{G}_{ii}(s) - G_{ii}(s).$$

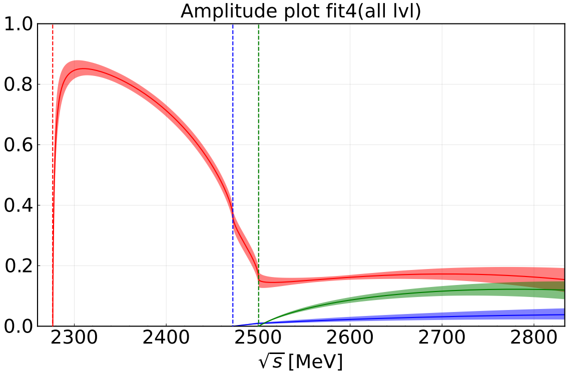
$\tilde{G}_{ii}(s)$ and $G_{ii}(s)$ two meson loop functions in the finite volume and continuum respectively.
The lattice energy levels correspond the zeros of the determinant of $\tilde{T}^{-1}$

Fit Results to all Levels

- Fits done to all energy levels in lattice rest frame
- In our best fit fixed $g_6 = 0$ to omit the explicit pole term of [6] (and thus m_6 is absent).
- χ^2 comparable to the HadSpec amplitudes for the lattice rest frame



$|Amplitude|^2$ Plot



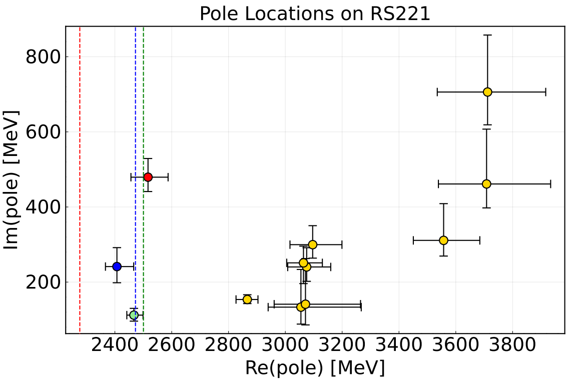
- Shown
  - $D\pi - D\pi$
  - $D\eta - D\eta$
  - $D_s\bar{K} - D_s\bar{K}$
- Lower pole found as bound state same as HadSpec Amplitudes

$$2274.8^{+0.6}_{-0.6}\ \mathrm{MeV}$$

- Higher pole found at

$$2516^{+71}_{-60} - 479^{+38}_{-50} i\ \mathrm{MeV}$$

Pole Location comparison



- HadSpec poles shown in yellow
- SU(3) constrained pole
  - When fitting to all energy levels in rest frame
  - When fitting to the lowest four levels in each volume
- UChPT pole shown in green

Summary

- Various UChPT studies find the experimental structures to be interplay of two D_0^* poles
- Seemingly in contradiction the Lattice study reports one pole
- But we find additional poles on the unphysical sheets in the lattice amplitudes
- They scatter wildly with, their effects on amplitude comparable, because they are located on hidden sheets

> distance from threshold
> balanced by residue

- To extract the location of higher pole from the lattice data we propose use of a SU(3) flavor constrained amplitude

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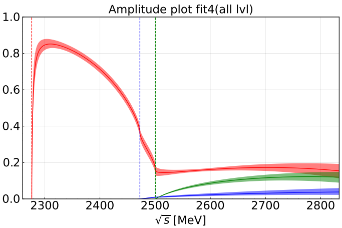
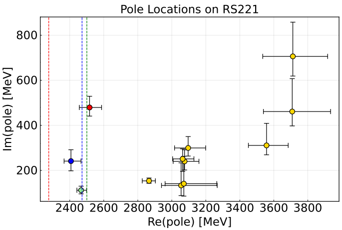
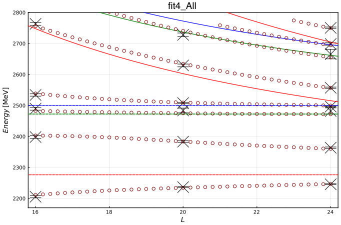
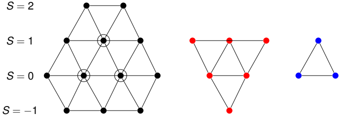
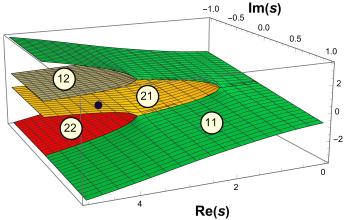


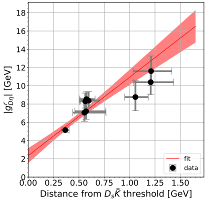




Thank you very much for your attention.







Back Up

relative strengths

$$C_{\bar{3}} = \begin{pmatrix} -3/4 \\ -1/4 \\ -\sqrt{3/8} \end{pmatrix} \begin{pmatrix} -3/4 & -1/4 & -\sqrt{3/8} \end{pmatrix}$$

$$= \frac{3}{8} \begin{pmatrix} 3/2 & 1/2 & \sqrt{3/2} \\ 1/2 & 1/6 & \sqrt{1/6} \\ \sqrt{3/2} & \sqrt{1/6} & 1 \end{pmatrix} ,$$

$$C_{6} = \begin{pmatrix} \sqrt{3/8} \\ -\sqrt{3/8} \\ -1/2 \end{pmatrix} \begin{pmatrix} \sqrt{3/8} & -\sqrt{3/8} & -1/2 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 3/4 & -3/4 & -\sqrt{3/8} \\ -3/4 & 3/4 & \sqrt{3/8} \\ -\sqrt{3/8} & \sqrt{3/8} & 1/2 \end{pmatrix} ,$$

$$C_{15} = \begin{pmatrix} 1/4 \\ 3/4 \\ -\sqrt{3/8} \end{pmatrix} \begin{pmatrix} 1/4 & 3/4 & -\sqrt{3/8} \end{pmatrix}$$

$$= \frac{3}{8} \begin{pmatrix} 1/6 & 1/2 & -\sqrt{1/6} \\ 1/2 & 3/2 & -\sqrt{3/2} \\ -\sqrt{1/6} & -\sqrt{3/2} & 1 \end{pmatrix} .$$

Sheet labels

Table: The notation of the Riemann sheets with the sign of the imaginary part of the c.m. momentum of each channel.

Riemann sheet	Sign of imaginary part of channel momentum		
RS111	$\mathrm{Im}(p_1) > 0$	$\mathrm{Im}(p_2) > 0$	$\mathrm{Im}(p_3) > 0$
RS211	$\mathrm{Im}(p_1) < 0$	$\mathrm{Im}(p_2) > 0$	$\mathrm{Im}(p_3) > 0$
RS221	$\mathrm{Im}(p_1) < 0$	$\mathrm{Im}(p_2) < 0$	$\mathrm{Im}(p_3) > 0$
RS222	$\mathrm{Im}(p_1) < 0$	$\mathrm{Im}(p_2) < 0$	$\mathrm{Im}(p_3) < 0$
RS121	$\mathrm{Im}(p_1) > 0$	$\mathrm{Im}(p_2) < 0$	$\mathrm{Im}(p_3) > 0$
RS112	$\mathrm{Im}(p_1) > 0$	$\mathrm{Im}(p_2) > 0$	$\mathrm{Im}(p_3) < 0$
RS212	$\mathrm{Im}(p_1) < 0$	$\mathrm{Im}(p_2) > 0$	$\mathrm{Im}(p_3) < 0$
RS122	$\mathrm{Im}(p_1) > 0$	$\mathrm{Im}(p_2) < 0$	$\mathrm{Im}(p_3) < 0$

HadSpec Parametrizations

Parametrization	m	$g_i^{(0)}$			$g_i^{(1)}$			$\gamma_{ij}^{(0)}$						$\gamma_{ij}^{(1)}$						χ^2/dof
		1	2	3	1	2	3	11	12	13	22	23	33	11	12	13	22	23	33	
Amplitude 1	✓	✓	✓	✓	-	-	-	✓	✓	✓	✓	-	✓	-	-	-	-	-	-	1.76
Amplitude 2	✓	✓	✓	✓	-	-	-	✓	✓	-	✓	-	✓	-	-	-	-	-	-	1.71
Amplitude 3	✓	✓	✓	✓	-	-	-	✓	-	-	✓	-	✓	-	-	-	-	-	-	1.76
Amplitude 4	✓	✓	✓	✓	-	-	-	-	-	-	✓	-	-	✓	-	-	-	-	-	1.78
Amplitude 5	✓	✓	✓	✓	-	-	-	-	-	-	-	-	-	✓	-	-	✓	-	✓	1.89
Amplitude 6	✓	✓	✓	✓	✓	-	-	✓	-	-	✓	-	✓	-	-	-	-	-	-	1.63
Amplitude 7	✓	✓	✓	✓	✓	-	-	✓	-	-	✓	-	-	-	-	-	-	-	-	1.68
Amplitude 8	✓	✓	✓	✓	✓	-	✓	✓	-	-	✓	-	✓	-	-	-	-	-	-	1.68
Amplitude 9	✓	✓	✓	✓	✓	✓	-	✓	-	-	✓	-	✓	-	-	-	-	-	-	1.66

Poles from Fits

Table: The pole locations from the different fits.

Fits	RS111	RS211	RS221	RS222
Fit 1_4L	$2275.1^{+0.6}_{-0.6} - 0i$	$2515^{+146}_{-18} - 23^{+16}_{-111} i$	$2476^{+136}_{-109} - 253^{+225}_{-120} i$	$2544^{+143}_{-47} - 18^{+18}_{-66} i$
Fit 2_4L	$2274.5^{+0.8}_{-0.7} - 0i$	$2498^{+9}_{-10} - 20^{+7}_{-6} i$	$2503^{+12}_{-13} - 42^{+19}_{-22} i$	$2518^{+19}_{-21} - 63^{+31}_{-44} i$
Fit 3_3L	$2275.1^{+0.6}_{-0.6} - 0i$	$2512^{+22}_{-67} - 50^{+37}_{-20} i$	$2479^{+41}_{-50} - 128^{+103}_{-38} i$	$2571^{+250}_{-135} - 314^{+265}_{-84} i$
Fit 4_4L	$2275.3^{+0.6}_{-0.6} - 0i$	$2518^{+28}_{-17} - 92^{+18}_{-28} i$	$2407^{+59}_{-40} - 241^{+43}_{-50} i$	$2673^{+94}_{-44} - 61^{+19}_{-47} i$
Fit 4_All	$2274.8^{+0.6}_{-0.6} - 0i$	$2681^{+46}_{-33} - -263^{+43}_{-51} i$	$2516^{+71}_{-60} - 479^{+38}_{-50} i$	$3123^{+144}_{-99} - 359^{+86}_{-162} i$

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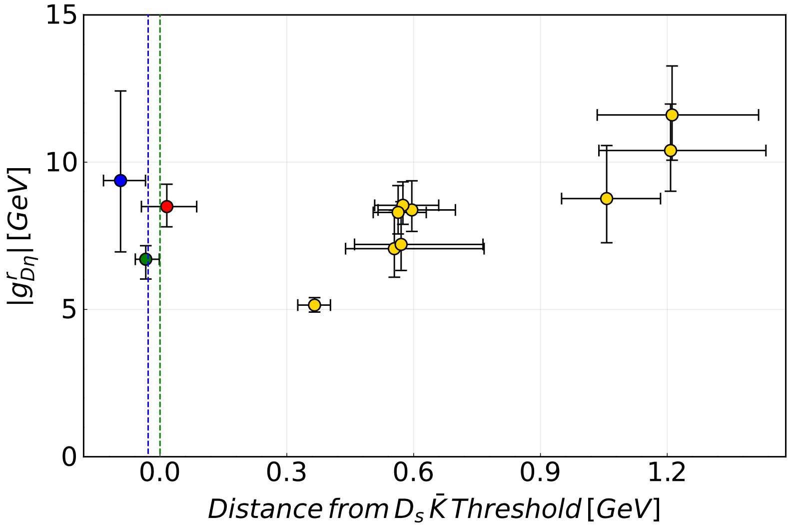
Fit parameters

Table: The best fit values arrived in Fit4_4L and Fit4_all, along with their χ^2/dof. The symbol '-' is used for parameters set to zero (or absent) in the particular fit.

	g_3 [GeV]	$m_{\bar{3}}$ [MeV]	g_6 [GeV]	m_6 [MeV]	$c_{\bar{3}}$	c_6	$c_{\overline{15}}$	χ^2	χ^2/dof
Fit 4_4L	3.16 ± 0.38	2275.3 ± 0.6	-	-	5 ± 2	1.0 ± 0.2	-0.4 ± 0.2	8.2	1.2
Fit 4_All	2.4 ± 0.2	2274.8 ± 0.6	-	-	1.1 ± 0.4	0.54 ± 0.06	-0.26 ± 0.09	29.6	2.1

Residue vs Threshold fit4_4L and fit4_All levels



Sheet Transition

$$T^{-1}_{K,X}(s) = T^{-1}_{K}(s) + \mathrm{Disc}_X[T^{-1}_{K}(s)], \tag{1}$$

$$\mathrm{Disc}_{211}\, T^{-1}_{K} = 2i \begin{bmatrix} -\rho_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{2}$$

$$\mathrm{Disc}_{221}\, T^{-1}_{K} = 2i \begin{bmatrix} -\rho_1 & 0 & 0 \\ 0 & -\rho_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{3}$$