CAN THE TWO-POLE STRUCTURE OF THE $D_0^*(2300)$ BE UNDERSTOOD FROM RECENT LATTICE DATA?

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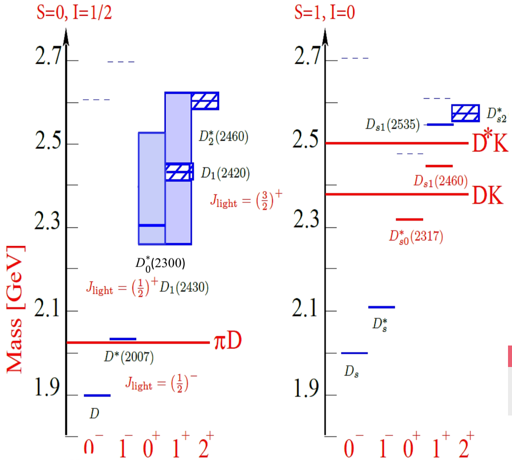
Introduction



- Quark model mesons
  =) quark-anti-quark bound states
- Positive parity c-s scalar($D_{s0}(2317)$) and axial vector($D_{s1}(2460)$) much lighter than the quark model prediction
- SU(3) partners: observed as broad bumps in the D and D invariant mass in B decays !
  $D_0(2300)$ and $D_1(2430)$

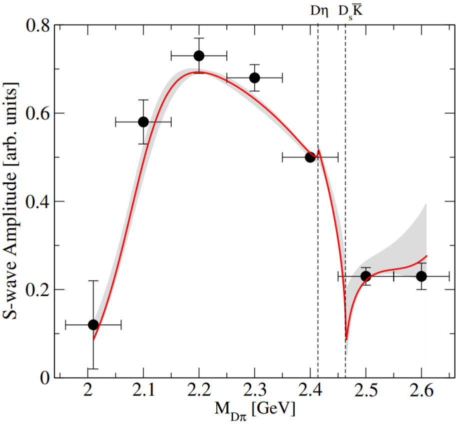
Masses almost equal to the strange counter parts

Quark Modell: M. Di Pierro and E. Eichten, PRD 64 (2001) 114004 however, various UChPT studies find two D_0 mesons and two D_1 mesons in the energy region of $D_0(2300)$ and $D_1(2430)$ respectively

- Two pole picture solves the SU(3) mass peculiarity
- The two D_0 poles are located at 2105^{+6}_{8} $i102^{+10}_{11}$ and 2451^{+35}_{25} $i134^{+7}_{8}$
- Lower one SU(3) partner of $D_{s0}(2317)$

M.Du et al.,PhysRevD.98.094018,L.Liu et al.,PhysRevD.87.014508...

- Amplitudes consistent with the LHCb data of the three body B meson decays
- But lattice QCD study of D D D_sK by HadSpec reported only one pole below the D threshold at m = 391 MeV



M.Du et al., arXiv:1712.07957v2 [hep-ph]

Is the two pole picture consistent with the lattice data?

Amplitudes from the lattice study

K-matrix parametrisation of the kind

$$K_{ij} = \frac{g_i^{(0)} + g_i^{(1)} s \qquad g_j^{(0)} + g_j^{(1)} s}{m^2 \qquad s} + {}_{ij}^{(0)} + {}_{ij}^{(1)} s;$$

With T-matrix

$$T_K(s)_{ij} = \frac{1}{K^{\;1}(s)_{ij} + \quad I_{CM}^{(i)}(s) \qquad I_{CM}^{(i)}(m^2) \qquad {}_{ij}};$$

where Chew-Mandelstam function given by

$$I_{CM}^{(i)}(s) = \frac{{}_i(s)}{} \log \frac{{}_i(s) + \quad {}_i(s)}{{}_i(s) \qquad {}_i(s)} \qquad \frac{{}_i(s)}{} \frac{m_2^{(i)} \qquad m_1^{(i)}}{m_1^{(i)} + m_2^{(i)}} \log \frac{m_2^{(i)}}{m_1^{(i)}};$$

! 9 different amplitudes with different parameters

Riemann Sheet

- Branch cuts at every channel opening
- Sign of $Im(p_{cm})$ used to label the sheets
- Poles correspond to bound states or resonances
- Poles given by zeros of

$$\det \;\; K \quad ^{1}(s) + (I_{CM}(s) \qquad I_{CM}(m^2)) \quad = 0$$

- Crossing from the physical sheet done by adding the discontinuity across the branch cut
- Discontinuity related to imginary part of T-matrix by Schwartz reflection principle

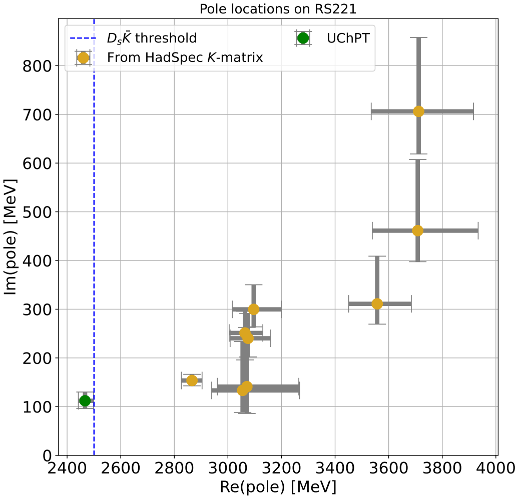
$$\begin{aligned} \mathrm{Disc}\,[T_K(s)] &= T_K(s+i \;\;) \qquad T_K(s \qquad i \;\;) \\ &= 2i \;\; \mathrm{Im}[T_K(s+i \;\;)] \end{aligned}$$

Hidden sheet poles =)

On sheets not directly connected at physical sheets

Pole Search

- At m = 391 MeV, lowest pole bound state
  - found in all nine parametrizations employed by HadSpec
- Second pole RS211, RS221, and RS222 for almost all parametrizations but
  - they scatter very much
  - also in part located outside fitted region
- Clear correlation between real and imaginary part of the poles
- Extracted pole from the UChPT analysis in line
- Poles located on hidden sheets



Residues

- Poles characterized by its location and residue
- residue quantifies the couplings of resonances to the various channels given by

$$R_{ij} = \lim_{s \to s_p} (s - s_p) T_{ij}(s).$$

- With the effective coupling is given by

$$g_i^r = R_{ij} / \sqrt{R_{jj}}$$

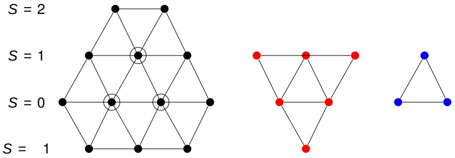
Residue and Threshold Distance

- Effects of hidden pole on physical axis visible at threshold only
- We quantified the distance as

$$Dist = M \quad M_{thr}$$

  to avoid double counting the effect of the couplings
- Effects of poles at threshold encoded in the y-intercept well constrained
- Next step: A parametrization to better constraint the location of the higher pole

SU(3) Symmetry



- $[3] \quad [8] = [15] \quad [6] \quad [3]$
- SU(3) flavor to isospin basis relation:

$$\begin{pmatrix} |[3]\rangle \\ |[6]\rangle \\ |[15]\rangle \end{pmatrix} = U \begin{pmatrix} |D\;\rangle \\ |D\;\rangle \\ |D_s K\rangle \end{pmatrix}$$

- where

$$U = \begin{pmatrix} 3/4 & 1/4 & \sqrt{3/8} \\ \sqrt{3/8} & \sqrt{3/8} & 1/2 \\ 1/4 & 3/4 & \sqrt{3/8} \end{pmatrix}$$

SU(3) Symmetry

Form for K-matrix:

$$K = \left(\frac{g_3^2}{m_3^2 - s} + c_3\right) C_3 + \left(\frac{g_6^2}{m_6^2 - s} + c_6\right) C_6 + c_{\overline{15}}\, C_{\overline{15}}$$

the two bare poles are assumed to be in the two SU(3) multiplets with S-wave attractions from LO chiral dynamics

No. of parameters: 5 - 7

- [15] was found to be repulsive
- T matrix as before
- Subtraction point for Chew-Mandelstam chosen to be m_3

Miguel, et al.,physletb.2017.02.036

Fitting to Lattice energy Levels

Finite volume T-matrix related to continuum T by

$$\tilde{T}(s) = \frac{1}{T^{-1}(s) - G(s)};$$

with

$$G_{ii} = \tilde{G}_{ii}(s) - G_{ii}(s).$$

$\tilde{G}_{ii}(s)$ and $G_{ii}(s)$ two meson loop functions in the finite volume and continuum respectively.
The lattice energy levels correspond the zeros of the determinant of $\tilde{T}^{-1}$

Fit Results to all Levels

- Fits done to all energy levels in lattice rest frame
- In our best fit fixed $g_6 = 0$ to omit the explicit pole term of [6] (and thus m_6 is absent).
- ^{2} comparable to the HadSpec amplitudes for the lattice rest frame

jAmplitudej^2 **Plot**

- Shown
  - $D \quad - D$
  - $D \quad - D$
  - $D_s\bar{K} - D_s\bar{K}$
- Lower pole found as bound state same as HadSpec Amplitudes

$$2274{:}8^{+0{:}6}_{0{:}6}\ \text{MeV}$$

- Higher pole found at

$$2516^{+71}_{60} \quad 479^{+38}_{50} i\ \text{MeV}$$

Pole Location comparison

- HadSpec poles shown in yellow
- SU(3) constrained pole
  - When fitting to all energy levels in rest frame
  - When fitting to the lowest four levels in each volume
- UChPT pole shown in green

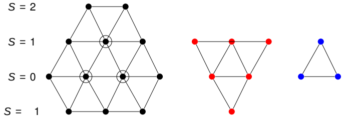
Summary

- Various UChPT studies find the experimental structures to be interplay of two D_0 poles
- Seemingly in contradiction the Lattice study reports one pole
- But we find additional poles on the unphysical sheets in the lattice amplitudes
- They scatter wildly with, their effects on amplitude comparable, because they are located on hidden sheets

distance from threshold
balanced by residue

- To extract the location of higher pole from the lattice data we propose use of a SU(3) flavor constrained amplitude

Thank you very much for your attention.



Back Up

relative strengths

$$C_3 = \begin{pmatrix} 3/4 \\ 1/4 \\ \sqrt{3/8} \end{pmatrix} \begin{pmatrix} 3/4 & 1/4 & \sqrt{3/8} \end{pmatrix} = \frac{3}{8} \begin{pmatrix} 3/2 & 1/2 & \sqrt{3/2} \\ 1/2 & 1/6 & \sqrt{1/6} \\ \sqrt{3/2} & \sqrt{1/6} & 1 \end{pmatrix};$$

$$C_6 = \begin{pmatrix} \sqrt{3/8} \\ \sqrt{3/8} \\ 1/2 \end{pmatrix} \begin{pmatrix} \sqrt{3/8} & \sqrt{3/8} & 1/2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 3/4 & 3/4 & \sqrt{3/8} \\ 3/4 & 3/4 & \sqrt{3/8} \\ \sqrt{3/8} & \sqrt{3/8} & 1/2 \end{pmatrix};$$

$$C_{15} = \begin{pmatrix} 1/4 \\ 3/4 \\ \sqrt{3/8} \end{pmatrix} \begin{pmatrix} 1/4 & 3/4 & \sqrt{3/8} \end{pmatrix} = \frac{3}{8} \begin{pmatrix} 1/6 & 1/2 & \sqrt{1/6} \\ 1/2 & 3/2 & \sqrt{3/2} \\ \sqrt{1/6} & \sqrt{3/2} & 1 \end{pmatrix};$$

Sheet labels

Table: The notation of the Riemann sheets with the sign of the imaginary part of the c.m. momentum of each channel.

Riemann sheet	Sign of imaginary part of channel momentum		
RS111	$\text{Im}(p_1) > 0$	$\text{Im}(p_2) > 0$	$\text{Im}(p_3) > 0$
RS211	$\text{Im}(p_1) < 0$	$\text{Im}(p_2) > 0$	$\text{Im}(p_3) > 0$
RS221	$\text{Im}(p_1) < 0$	$\text{Im}(p_2) < 0$	$\text{Im}(p_3) > 0$
RS222	$\text{Im}(p_1) < 0$	$\text{Im}(p_2) < 0$	$\text{Im}(p_3) < 0$
RS121	$\text{Im}(p_1) > 0$	$\text{Im}(p_2) < 0$	$\text{Im}(p_3) > 0$
RS112	$\text{Im}(p_1) > 0$	$\text{Im}(p_2) > 0$	$\text{Im}(p_3) < 0$
RS212	$\text{Im}(p_1) < 0$	$\text{Im}(p_2) > 0$	$\text{Im}(p_3) < 0$
RS122	$\text{Im}(p_1) > 0$	$\text{Im}(p_2) < 0$	$\text{Im}(p_3) < 0$

HadSpec Parametrizations

Parametrization	m	$g_i^{(0)}$			$g_i^{(1)}$			$\gamma_{ij}^{(0)}$						$\gamma_{ij}^{(1)}$						χ^2/dof
		1	2	3	1	2	3	11	12	13	22	23	33	11	12	13	22	23	33	
Amplitude 1	✓	✓	✓	✓	-	-	-	✓	✓	✓	✓	-	✓	-	-	-	-	-	-	1.76
Amplitude 2	✓	✓	✓	✓	-	-	-	✓	✓	-	✓	-	✓	-	-	-	-	-	-	1.71
Amplitude 3	✓	✓	✓	✓	-	-	-	✓	-	-	✓	-	✓	-	-	-	-	-	-	1.76
Amplitude 4	✓	✓	✓	✓	-	-	-	-	-	-	✓	-	-	✓	-	-	-	-	-	1.78
Amplitude 5	✓	✓	✓	✓	-	-	-	-	-	-	-	-	-	✓	-	-	✓	-	✓	1.89
Amplitude 6	✓	✓	✓	✓	✓	-	-	✓	-	-	✓	-	✓	-	-	-	-	-	-	1.63
Amplitude 7	✓	✓	✓	✓	✓	-	-	✓	-	-	✓	-	-	-	-	-	-	-	-	1.68
Amplitude 8	✓	✓	✓	✓	✓	-	✓	✓	-	-	✓	-	✓	-	-	-	-	-	-	1.68
Amplitude 9	✓	✓	✓	✓	✓	✓	-	✓	-	-	✓	-	✓	-	-	-	-	-	-	1.66

Poles from Fits

Table: The pole locations from the different fits.

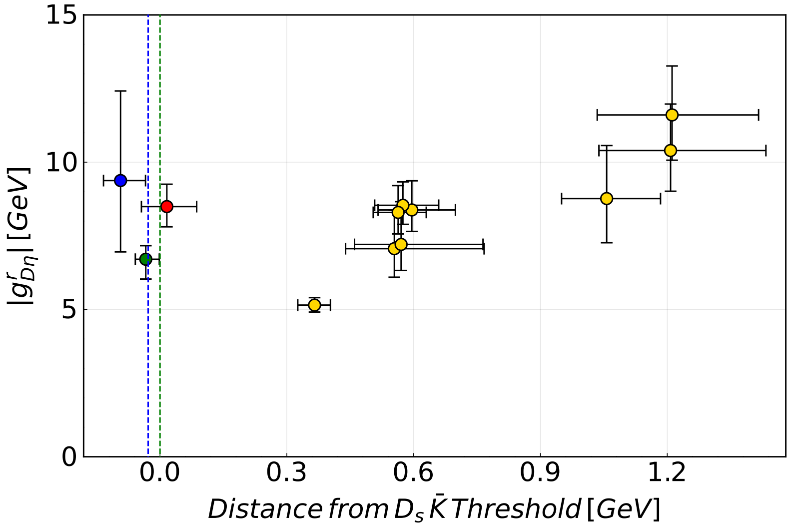
Fits	RS111		RS211		RS221		RS222	
Fit 1_4L	$2275{:}1^{+0{:}6}_{0{:}6}$	$0i$	2515^{+146}_{18}	$23^{+16}_{111}i$	2476^{+136}_{109}	$253^{+225}_{120}i$	2544^{+143}_{47}	$18^{+18}_{66}i$
Fit 2_4L	$2274{:}5^{+0{:}8}_{0{:}7}$	$0i$	2498^{+9}_{10}	$20^{+7}_{6}i$	2503^{+12}_{13}	$42^{+19}_{22}i$	2518^{+19}_{21}	$63^{+31}_{44}i$
Fit 3_3L	$2275{:}1^{+0{:}6}_{0{:}6}$	$0i$	2512^{+22}_{67}	$50^{+37}_{20}i$	2479^{+41}_{50}	$128^{+103}_{38}i$	2571^{+250}_{135}	$314^{+265}_{84}i$
Fit 4_4L	$2275{:}3^{+0{:}6}_{0{:}6}$	$0i$	2518^{+28}_{17}	$92^{+18}_{28}i$	2407^{+59}_{40}	$241^{+43}_{50}i$	2673^{+94}_{44}	$61^{+19}_{47}i$
Fit 4_All	$2274{:}8^{+0{:}6}_{0{:}6}$	$0i$	2681^{+46}_{33}	$263^{+43}_{51}i$	2516^{+71}_{60}	$479^{+38}_{50}i$	3123^{+144}_{99}	$359^{+86}_{162}i$

Fit parameters

Table: The best fit values arrived in Fit4_4L and Fit4_all, along with their ^{2}=dof. The symbol '-' is used for parameters set to zero (or absent) in the particular fit.

	g_3 [GeV]	m_3 [MeV]	g_6 [GeV]	m_6 [MeV]	c_3	c_6	$c_{\overline{15}}$	^{2}	^{2}=dof
Fit 4_4L	3:16 0:38	2275:3 0:6	-	-	5 2	1:0 0:2	0:4 0:2	8.2	1:2
Fit 4_All	2:4 0:2	2274:8 0:6	-	-	1:1 0:4	0:54 0:06	0:26 0:09	29:6	2:1

Residue vs Threshold fit4_4L and fit4_All levels



Sheet Transition

$$T_{K;X}^{\;1}(s) = T_K^{\;1}(s) + \mathrm{Disc}_X[T_K^{\;1}(s)]; \qquad (1)$$

$$\mathrm{Disc}_{211}\, T_K^{\;1} = 2i \begin{bmatrix} {}_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \qquad (2)$$

$$\mathrm{Disc}_{221}\, T_K^{\;1} = 2i \begin{bmatrix} {}_1 & 0 & 0 \\ 0 & {}_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}: \qquad (3)$$