Renormalization in various schemes of nucleon-nucleon chiral EFT

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Outline

- Explicit renormalization in an EFT: motivation
- Perturbative regime
- Nonperturbative regime: finite and infinite cut off schemes
- $1/r^2$ potential, as a toy-model example
- Summary

Motivation

Explicit renormalization of an EFT in the presence of nonperturbative effects

Heavy quarkonia EFT calculations

Pion-less nuclear EFT

Chiral nuclear EFT,...

Nontrivial issues arise when going beyond leading order

Renormalization of EFT: Motivation. Example of NN

"Perturbative" calculation of the S-matrix, spectrum, etc.

Expansion parameter: (soft scale)/(hard scale) $Q = \dfrac{q}{\Lambda_b} \quad q \in \{|\vec{p}|\,, M_\pi\}\,, \qquad \Lambda_b \sim M_\rho$

$$\mathcal{L}_{\rm eff} = \mathcal{L}_\pi^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)} + \ldots$$

Contains bare parameters (LEC's) $C_i = C_i^r + \delta C_i$

Counter terms δC_i absorb divergent and power-counting breaking contributions

Renormalization: power counting and expansion in terms of renormalized quantities C_i^r

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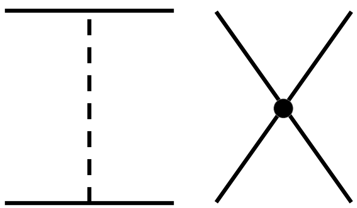
Explicit renormalization of nuclear chiral EFT is complicated by non-perturbative effects.

Power counting for NN chiral EFT

Weinberg, S., **NPB363**, 3 (1991)

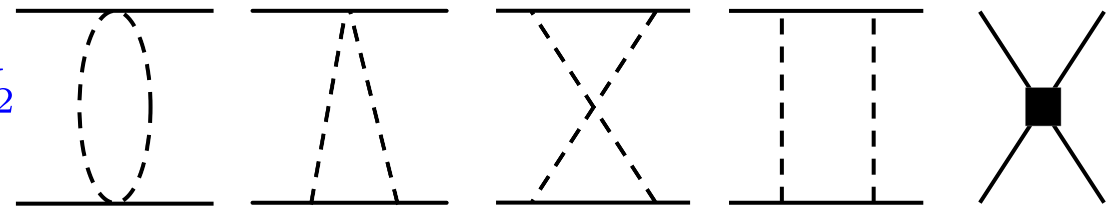
LO

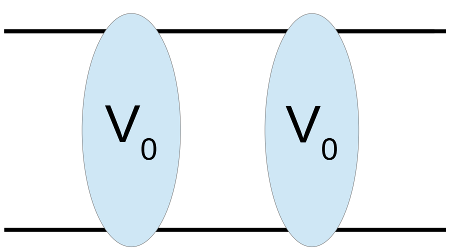
$V_{\mathrm{LO}} = V_0$



NLO

$V_{\mathrm{NLO}} = V_2$





2N-reducible diagrams are enhanced: V_0 must be iterated

$$\text{LO: } T_0 = V_0 + V_0 G V_0 + V_0 G V_0 G V_0 + \ldots$$

$$\text{NLO: } T_2 = V_2 + V_2 G V_0 + V_0 G V_2 + V_2 G V_0 G V_0 + \ldots$$

V_2 is treated perturbatively to have the expansion of the amplitude under control

Regularization

The unregularized amplitude is divergent:

→ Regulator: cutoff Λ

$$\text{LO: } T_0 = V_0 + V_0 G V_0 + V_0 G V_0 G V_0 + \cdots = \sum_{n=0}^{\infty} T_0^{[n]}\,, \qquad T_0^{[n]} \sim \Lambda^n$$

$$\text{NLO: } T_2 = \sum_{m,n=0}^{\infty} \left(V_0 G\right)^m V_2 \left(G V_0\right)^n = \sum_{m,n=0}^{\infty} T_2^{[m,n]}\,, \qquad T_2^{[m,n]} \sim \Lambda^{m+n+2}$$

Positive powers of Λ violate the power counting even if we keep Λ finite

One needs an infinite number of counter terms to absorb them

Power counting restoration. Perturbative analysis. Finite cutoff (of the order of the hard scale).

Perturbative: the series in V_0 is convergent, but the number of iterations is not restricted

$$\text{LO: } T_0^{[n]} = V_0 (G V_0)^n \sim \mathcal{O}(Q^0)$$

Expectation: $\Lambda \approx \Lambda_b : \int \frac{p^{n-1} dp}{(\Lambda_V)^n} \sim \left(\frac{\Lambda}{\Lambda_V}\right)^n \sim \left(\frac{\Lambda_b}{\Lambda_b}\right)^n \sim \mathcal{O}(Q^0)$

G. P. Lepage, nucl-th/9706029
J. Gegelia, **JPG25**, 1681 (1999)

$$\text{NLO: } T_2^{[m,n]} = (V_0 G)^m \, V_2 \, (G V_0)^n \sim \mathcal{O}(Q^0) \neq \mathcal{O}(Q^2)$$

Power-counting violating contributions from large loop momenta: $p \sim \Lambda \, , p' \sim \Lambda \text{ in } V_2(p', p)$

Expectation:power-counting breaking contributions
can be absorbed by lower order contact (counter) terms

Renormalization in the spirit of
Bogoliubov-Parasiuk-Hepp-Zimmermann (BPHZ) procedure
to all orders in V_0:
subtractions in all nested subdiagrams.

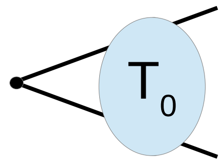
AG, E.Epelbaum, **PRC 105**, 024001 (2022)

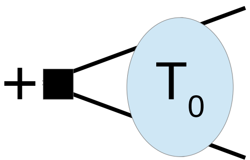
Renormalization at NLO in the nonperturbative case (simplified)

AG, E.Epelbaum, **PRC107**, 044002 (2023)

Distorted wave Born approximation:

$$\mathbb{R}\left(T_2\right)(p) = T_{2;\text{long}}(p) + C_0 \psi_p(r \approx 0)^2 + C_2 \psi_p(r \approx 0) \psi_p'(r \approx 0) + \dots$$

$\psi_p(r \approx 1/\Lambda \approx 0)$ =1+ 

$\psi_p'(r \approx 0)$ =p²+ 

C_i are fixed, e.g., by renormalization conditions at momenta p_i

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$$\det(p_1, p_2) = \begin{vmatrix} \psi_{p_1}(0) & \psi_{p_1}'(0) \\ \psi_{p_2}(0) & \psi_{p_2}'(0) \end{vmatrix} = 0 \quad \Longrightarrow \quad C_0,\, C_2 = \infty$$

Destroys renormalizability if $\det(p_1, p_3 \neq p_2) \neq 0$ (zero is not factorizable)

Typical situation for singular attractive interactions. Wave functions oscillate close to r=0

⇓

Renormalizability constraints on (the short-range part of) the LO potential.
The simplest formulation: LECs must be of natural size (If $\Lambda \sim \Lambda_b$).

Infinite cutoff ($\Lambda >> \Lambda_b$) scheme, "RG invariant"

$\Lambda \to \infty$:

Cutoff independence for each EFT order individually!

A. Nogga, R. Timmermans,
U. van Kolck, **PRC72**, 054006 (2005)
M.P. Valderrama, **PRC84**, 064002 (2011)
B. Long, C. Yang, **PRC84**, 057001 (2011)

All positive powers of Λ cancel:

$$T \approx 1 + \Lambda + \Lambda^2 + \cdots = \frac{1}{1-\Lambda}$$

Motivation: singular potentials in quantum mechanics

$\sim 1/r^n \,, \quad n \geq 2$

W. Frank, D. J. Land and R. M. Spector,
Rev. Mod. Phys. 43, 36 (1971)

Seems to work for the LO amplitude

NN scattering for in the infinite cutoff scheme at NLO. $^3P_0$ partial wave.

Scheme of Long and Yang:

B. Long, C. J. Yang, **PRC84**, 057001 (2011)

$$V^{(2)}(p', p) = V_{2\pi}(p', p) + C_0^{(2)}(\Lambda) p' p + C_2^{(2)}(\Lambda) p' p (p^2 + p'^2)$$

Perturbative NLO: $T^{(2)} = \left[\mathbb{1} + T^{(0)} G\right] V^{(2)} \left[\mathbb{1} + G T^{(0)}\right]$

(avoiding pathologies due to repulsive singular interaction)

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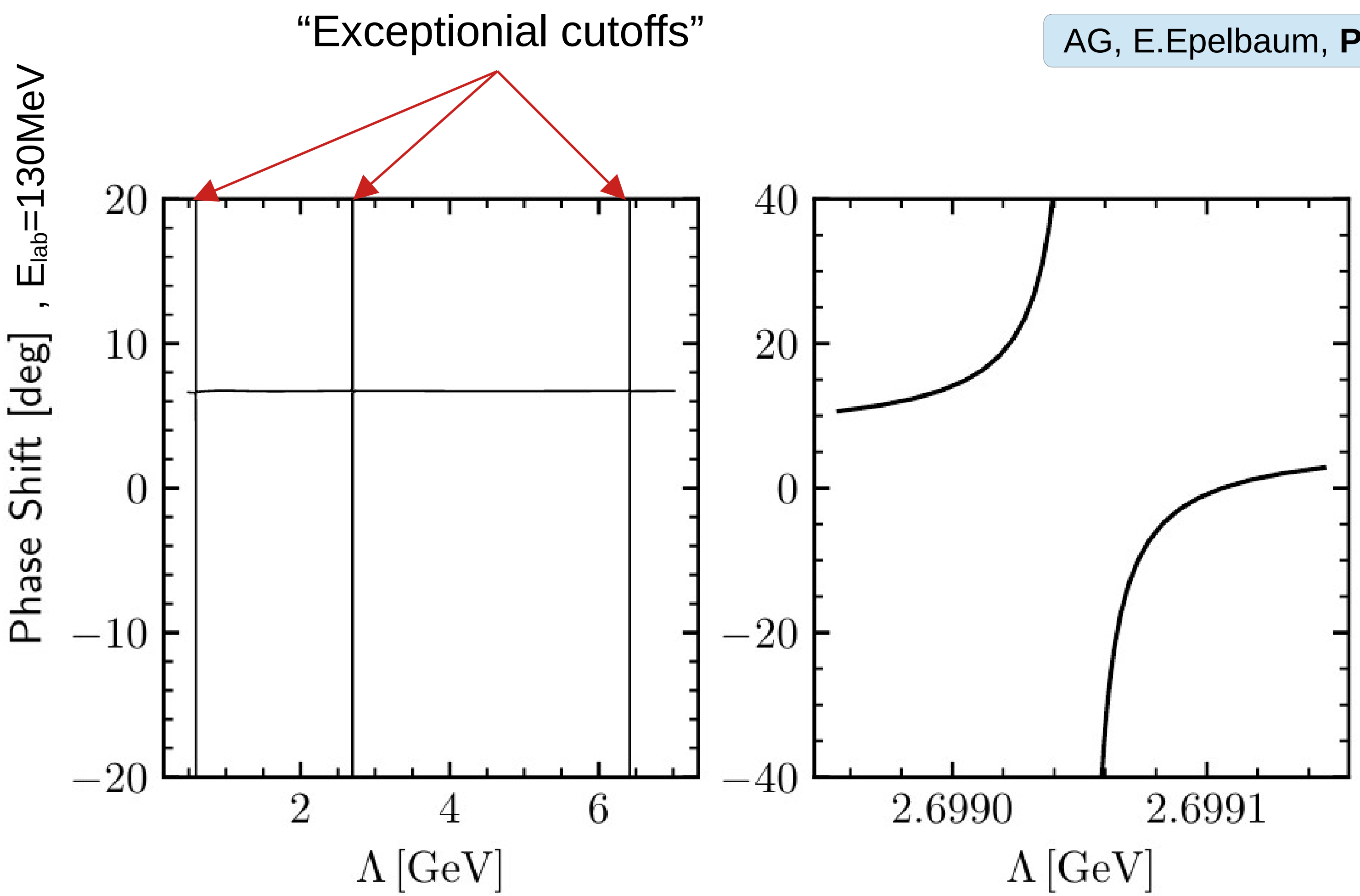
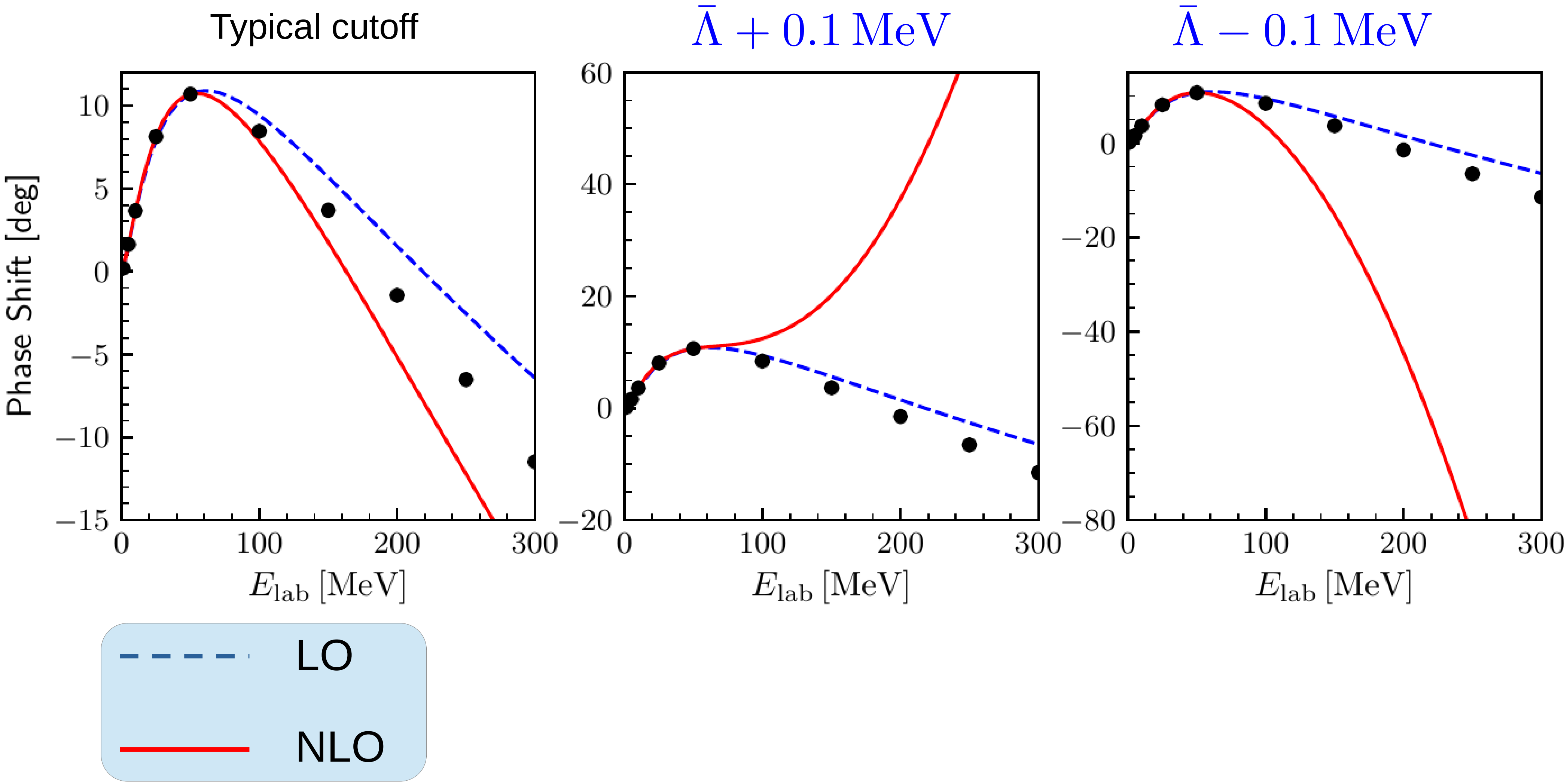
AG, E.Epelbaum, **PRC107**, 034001 (2023)



$^3P_0$ phase shifts. Energy dependence.

“Exceptional” cutoff $\bar{\Lambda} \approx 12\,\mathrm{GeV}$



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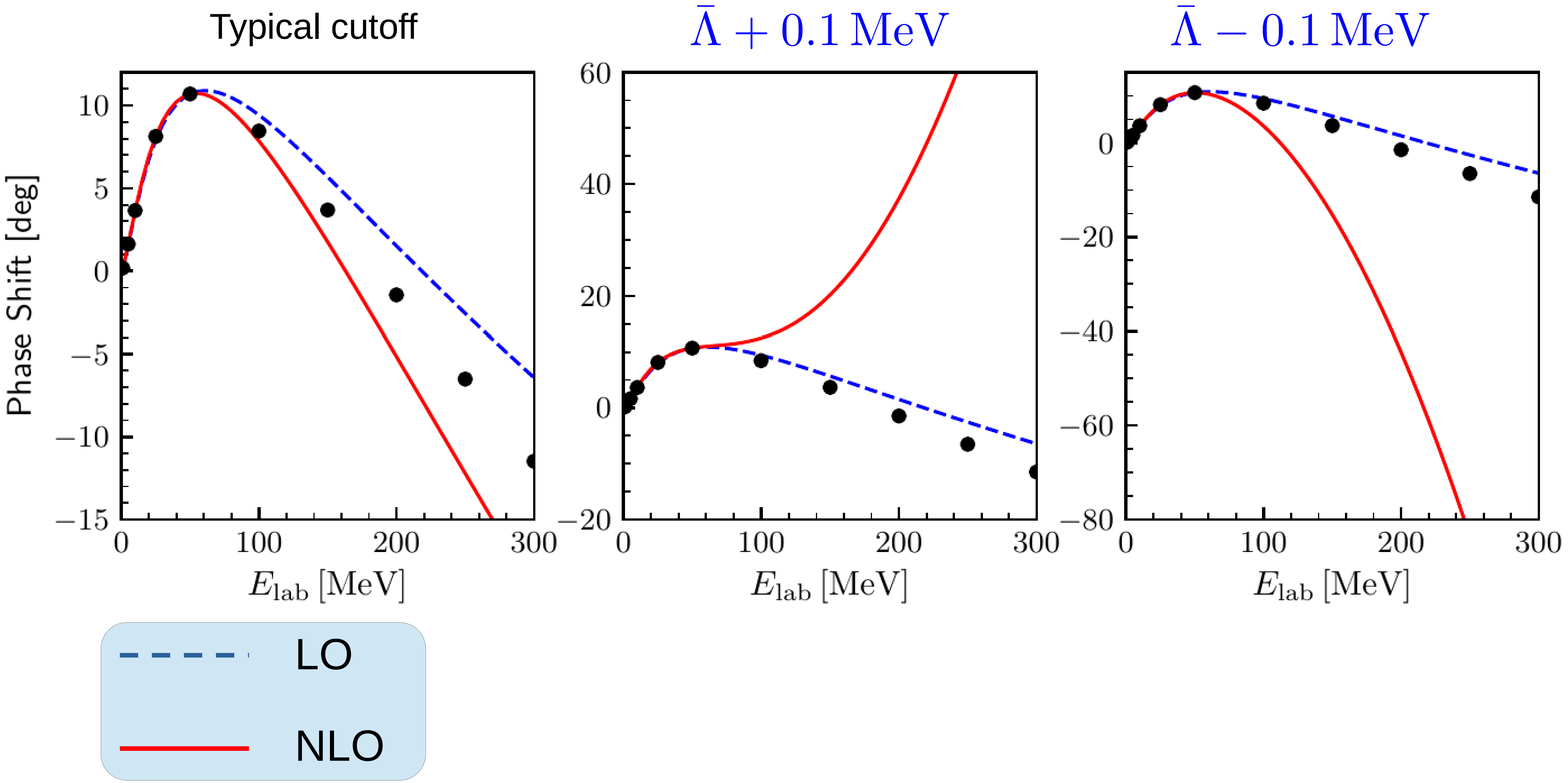

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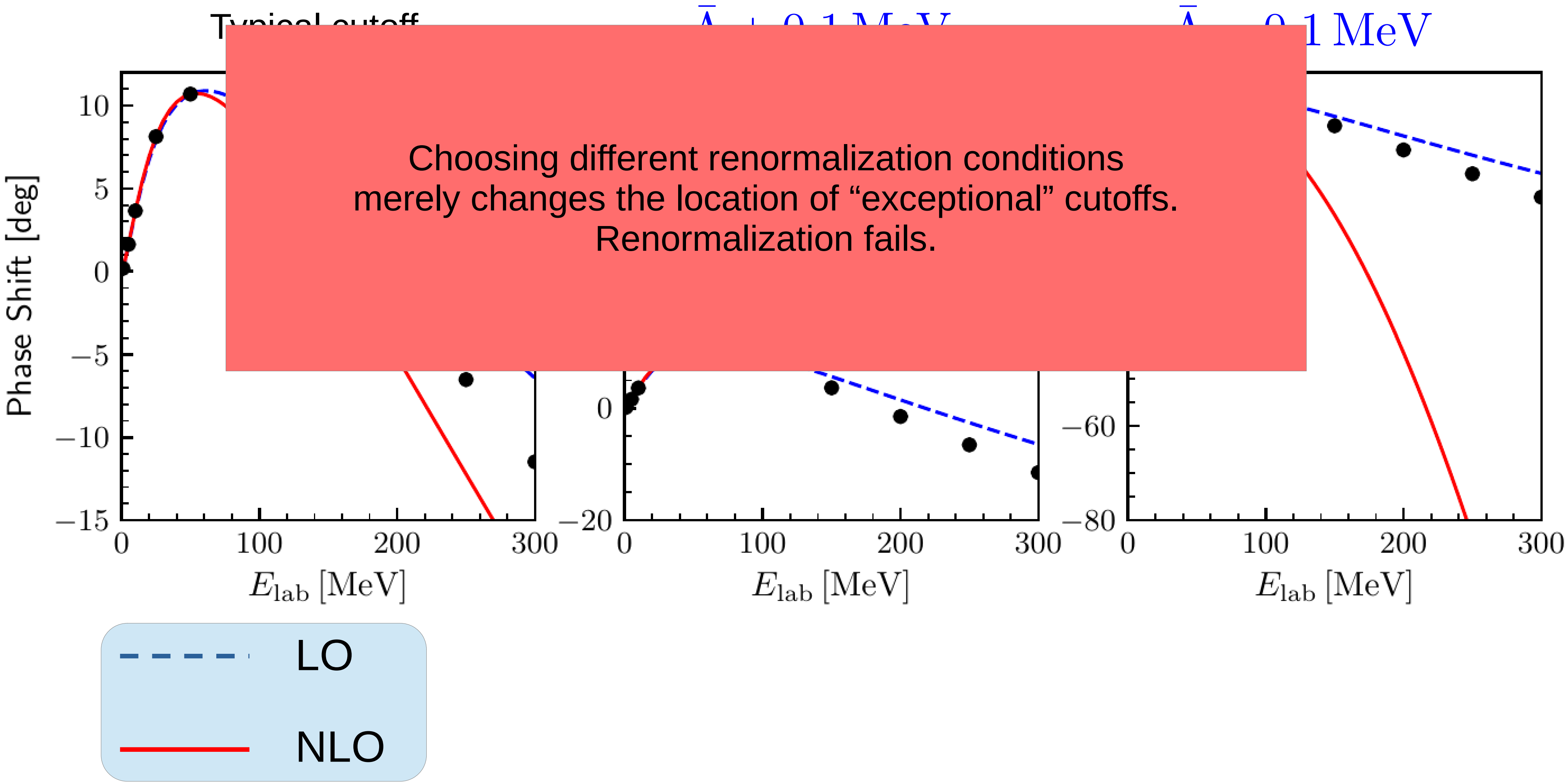

Choosing different renormalization conditions
merely changes the location of "exceptional" cutoffs.
Renormalization fails.

$1/r^2$ potential

Frequently used as the toy model example

Analytically solvable in many cases (scaleless)

Similar short-range behaviour in pionless EFT, 3N system

For pure $1/r^2$ LO potential, $1/r^4$ NLO potential and the sharp cutoff regularization,
There appear no "exceptional" cutoffs (factorization)

B. Long, U. van Kolck,
Annals Phys. 323, 1304 (2008)
AG, E.Epelbaum, **PRC107**, 034001 (2023)

Unique situation.
Any slight modification of the potential (short- or long-range part) or
of a regulator leads to appearance of "exceptional" cutoffs

AG, E.Epelbaum, **PRC107**, 034001 (2023)

Summary

- ✔ Renormalization of an (e.g. NN chiral) EFT in the nonperturbative regime *beyond leading order* imposes constraints on the renormalization scheme, i.e., on a choice of the LO interaction

- ✔ As a consequence, in the infinite cutoff schemes, renormalization beyond LO does not work: “exceptional” cutoffs

- ✔ These findings are also relevant for other systems.
  However, simple toy models such as $1/r^2$ potential might miss some key issues.