Explore N(1520) transition form factors with non-perturbative dispersion theory at low energy

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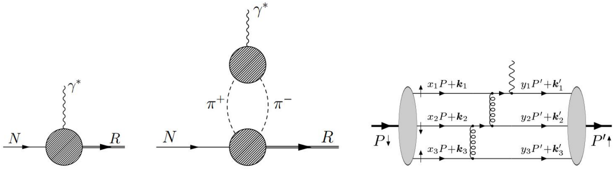
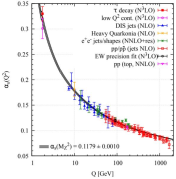
Nucleon electromagnetic structure

We try to understand the structure of the nucleon.



Nucleon transition form factors

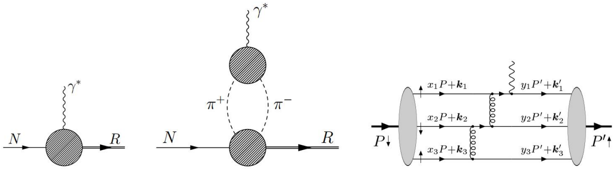


QCD running coupling

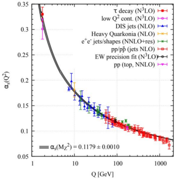
How large is $\langle 0|\, qqq \,|N\rangle$ and $\langle 0|$ Meson Baryon $|N\rangle$, quantitatively?

Nucleon electromagnetic structure

We try to understand the structure of the nucleon.



Nucleon transition form factors

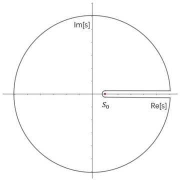


QCD running coupling

How large is $\langle 0|\, qqq \,|N\rangle$ and $\langle 0|$ Meson Baryon $|N\rangle$, quantitatively?

Need model-independent tool $\rightarrow$ Dispersion theory

Dispersion theory in a nutshell



Unitarity cut $[4m_\pi^2, \infty)$

Axiomatic QFT

$\rightarrow$ Form factors are analytic functions in the complex plane.

Unitarity+analyticity

$\rightarrow$ the location of cut, branch point, singularities...

Cauchy integral Formula:

$$F(q^2) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{\mathrm{Im}\, F(s)}{s - q^2 - i\epsilon} ds\,.$$

Previous studies on TFFs by Uppsala group

$\Sigma(J^P = \frac{1}{2}^+) \to \Lambda(J^P = \frac{1}{2}^+)$ (Granados, Leupold, Perotti) [1]
Nucleon isovector form factors (Leupold) [2]
$\Sigma^*(J^P = \frac{3}{2}^+) \to \Lambda(J^P = \frac{1}{2}^+)$ (Junker, Leupold, Perotti, Vitos) [3]
$\Delta(J^P = \frac{3}{2}^+) \to N(J^P = \frac{1}{2}^+)$ (Aung, Leupold, Perotti)(in progress)
Quark mass dependence of nucleon EMFF
(An, Alvarado, Leupold, Alvarez-Ruso) $\to$ Talk on Wednesday.

p	$1/2^+$	****
n	$1/2^+$	****
$N(1440)$	$1/2^+$	****
$N(1520)$	$3/2^-$	****
$N(1535)$	$1/2^-$	****
$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1685)$		*
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	***
$N(1720)$	$3/2^+$	****
$N(1860)$	$5/2^+$	**
$N(1875)$	$3/2^-$	***
$N(1880)$	$1/2^+$	**
$N(1895)$	$1/2^-$	**

Nucleon excited states [4]

Branching ratios of N^* by PDG:

1. $N\pi$ — $55-65\%$
2. $\Delta(1232)\pi$, (S-wave) — $15-23\%$
3. $\Delta(1232)\pi$, (D-wave) — $7-11\%$
4. $N\rho$, $S=\frac{3}{2}$, (S-wave) — $10-16\%$
5. $N\rho$, $S=\frac{1}{2}$, (D-wave) — $0.2-0.4\%$
6. $N\rho$, $S=\frac{3}{2}$, (D-wave) — ≈ 0
7. $N\eta$ — $0.07-0.08\%$

Our Strategy: include low energy degrees of freedom (N, Δ, π, ρ) as model-independent as possible.

$N^*(1520)$ TFFs

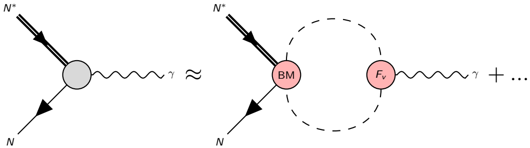
$N^*(1520)$ $I = 1/2$ and $J^P = 3/2^-$.

$$\langle N | j_\mu | N^* \rangle = e\, \bar{u}_N(p_N)\, \Gamma_{\mu\nu}(q)\, u^\nu_{N^*}(p_{N^*})$$

with

$$\Gamma^{\mu\nu}(q) := i \left(\gamma^\mu q^\nu - \not{q} g^{\mu\nu}\right) m_N F_1(q^2) + \sigma^{\mu\alpha} q_\alpha q^\nu F_2(q^2) + \\ + i \left(q^\mu q^\nu - q^2 g^{\mu\nu}\right) F_3(q^2)\,,$$

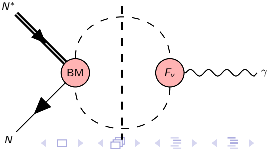
where $q^\mu := p^\mu_{N^*} - p^\mu_N$. In this work, we focus on isovector TFFs:



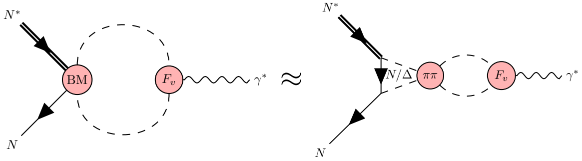
Imaginary part:

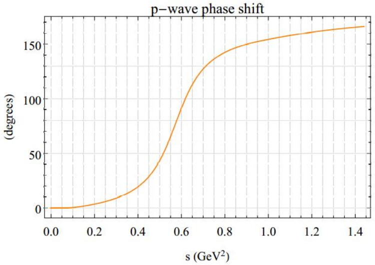
1. Pion vector form factor: F_v
2. Baryon-meson exchanges: BM

Imaginary part $\xrightarrow[\text{relation}]{\text{Dispersion}}$ full amplitude



$N^*(1520)$ TFFs





Pion p-wave phase-shift δ_1.

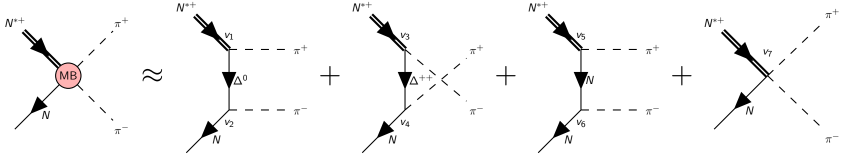
δ_1 contains ρ meson information

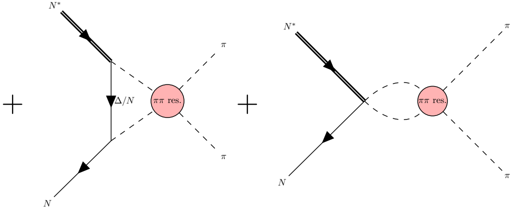
$$f_1(s) = \frac{sin\delta_1(s)}{\sqrt{1-\frac{4m_\pi^2}{s}}}\frac{\sqrt{s}}{2}$$

$$F_v(s) \approx \Omega(s) = exp[\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\delta_1(s')}{s'(s'-s)}]$$

Theory Input

- Fit to hadronic decay data provides v_1, v_3, v_5 and v_7.
- Chiral perturbation theory gives v_2, v_4, v_6.



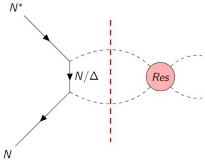


Cuts, Poles and Singularities

Anomalous threshold condition

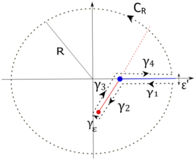
$m^2_{exc} < \frac{1}{2}(m^2_{N^*} + m^2_N - 2m^2_\pi)$

$m_{exc} = m_N$ (see back up slides for rigorous derivation)



Cutkosky cutting rules

N exchange

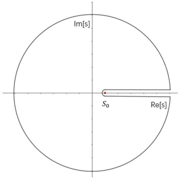


The first Riemann sheet includes an unitarity and an anomalous part.

$T(s) = \frac{1}{2\pi i}\int_{4m_\pi 2}^{\infty} \frac{disc_{UNI}\, T(z)}{z-c} dz + \frac{1}{2\pi i}\int_\gamma \frac{d\gamma}{dt} \frac{disc_{ANOM}\, T((\gamma(t))}{\gamma(t)-s} dt$

Δ exchange

Two singularities on the second Riemann Sheet

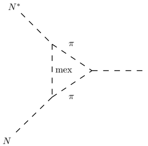


Unitarity cut $[4m^2_\pi, \infty)$

Comparison with 1-loop scalar-triangle

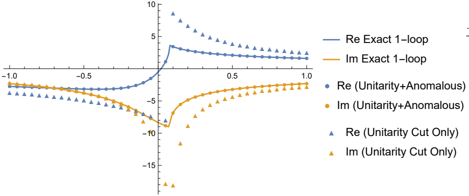
How do we make sure we are right about the analytic structures?

$\rightarrow$ use 1-loop scalar triangle (G. 't Hooft, M. Veltman) as a toy calculation for **double-check**!

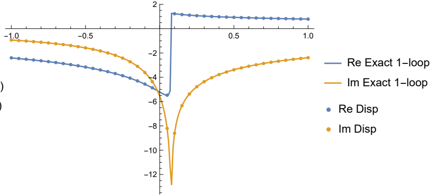


$$= T(s) = \frac{1}{2\pi i}\int_{4m_\pi{}^2}^{\infty} \frac{disc_{UNI}\, T(z)}{z-c}\, dz + \frac{1}{2\pi i}\int_{\gamma} \frac{d\gamma}{dt}\, \frac{disc_{ANOM}\, T((\gamma(t))}{\gamma(t)-s}\, dt$$

Our dispersive relation for the scalar triangle perfectly matches the analytic results:



Nucleon exchange



Δ exchange

Form factor dispersion relation

Subtracted dispersion relations for TFFs:

$$F_i(q^2) = F_i(0) + \frac{q^2}{12\pi} \int_{4m_\pi^2}^{\Lambda^2} \frac{\mathrm{d}s}{\pi} \, \frac{T_i(s)\, p_{\mathrm{c.m.}}^3(s)\, F_\pi^{V\,*}(s)}{s^{3/2}\,(s - q^2 - i\epsilon)} + F_i^{\mathrm{anom}}(q^2) \text{ for } i = 1, 2, 3.$$

$T_i \sim N^* N \to 2\pi$ amplitudes calculated from Muskhelishvili-Omnès formalism:

$$\begin{aligned} T_i(s) \quad = \quad & K_i(s) + \Omega(s)\, P_i + T_i^{\mathrm{anom}}(s) \\ & + \Omega(s)\, s \int_{4m_\pi^2}^{\infty} \frac{\mathrm{d}s'}{\pi} \, \frac{K_i(s')\, \sin\delta(s')}{|\Omega(s')|\,(s' - s - i\epsilon)\, s'} \,. \end{aligned}$$

$P_{i=1,2,3}$ are fit parameters (contact term interactions).
Fix P_i by matching to $\Gamma_{N\rho}$ S, D-wave decay widths (backup slides).

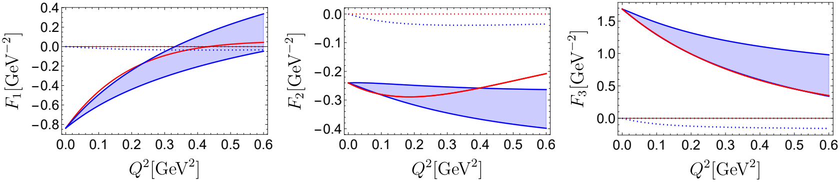
Theory meets experiments: Space-like TFFs (preliminary)

Isovector TFFs $:= \frac{1}{2}(F_i^{\text{proton}} - F_i^{\text{neutron}})\quad i = 1,2,3.$

Only quantitative input: subtraction constants $P_{1,2,3}$

$\Rightarrow$ fix hadronic $N^*N \to \pi\pi$ amplitudes

$\Rightarrow\ N^*(1520) \to N\gamma^*$



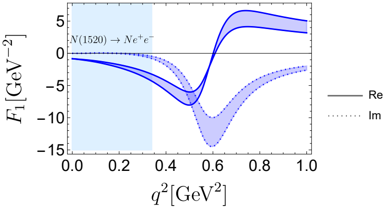
(a) F1 (b) F2 (c) F3

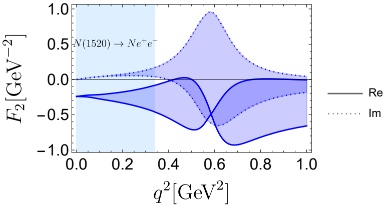
Red: Model dependent parametrization for isovector TFFs (Jlab data for proton, MAID model for neutron)

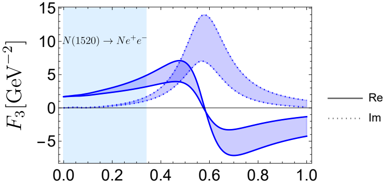
Blue: this work

Full lines: Real part, Dashed lines: Imaginary part.

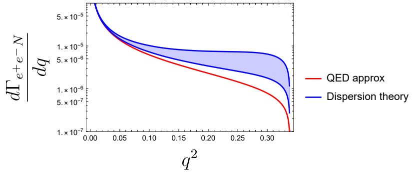
Theory predictions: Time-like TFFs (preliminary)







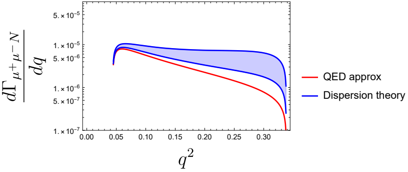
$N^* \to Ne^+e^-$
(Assuming isovector dominance)

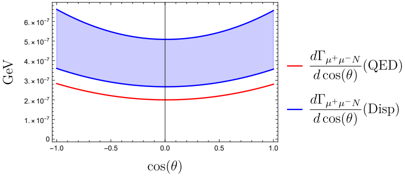


Our prediction:
$\Gamma_{N^* \to Nee} \approx (4.6, 5.2)\,\mathrm{keV}$
$\Gamma_{N^* \to Nee}(\mathrm{QED}) \approx 4.4\,\mathrm{keV}$
Results can be tested by HADES.

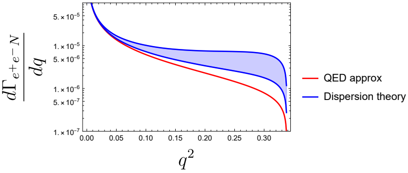
Theory predictions: Time-like TFFs (preliminary)

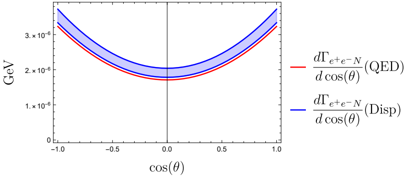
$N^* \to N\mu^+\mu^-$





$N^* \to Ne^+e^-$



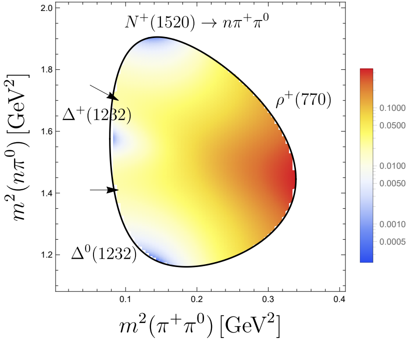


Muonic Dalitz decay prediction:
$\Gamma_{N^* \to N\mu^+\mu^-} \approx (0.6, 1.1)$ keV
$\Gamma_{N^* \to N\mu^+\mu^-}$(QED) ≈ 0.4 keV

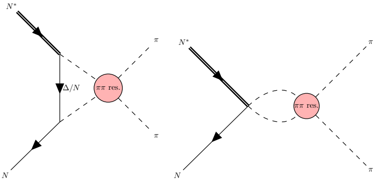
Electronic Dalitz decay prediction:
$\Gamma_{N^* \to Ne^+e^-} \approx (4.6, 5.2)$ keV
$\Gamma_{N^* \to Ne^+e^-}$(QED) ≈ 4.4 keV

Theory predictions: Hadronic Dalitz decay $N^* \to N\pi\pi$

Our dispersive prediction $N^+(1520) \to n\pi^+\pi^0$ as an example



$$\frac{d\Gamma}{dm^2_{N\pi}\, dm^2_{\pi\pi}}$$

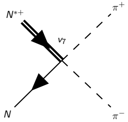


$\to$ can be used to test the quality of isobar model predictions.

$$\begin{aligned} T_i(s) &= K_i(s) + \Omega(s)\, P_i + T_i^{\text{anom}}(s) \\ &\quad + \Omega(s)\, s \int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{K_i(s') \sin\delta(s')}{|\Omega(s')|\,(s' - s - i\epsilon)\, s'} . \end{aligned} \tag{1}$$

Outlook

1. In progress: $N^+ \to p\pi^+\pi^-$, $N^+ \to p\pi^0\pi^0$ including pion s-wave re-scattering dispersively (σ meson).
2. Cross-check the fit parameters (e.g. sign of $N^*N\pi = +0.655$) with isobar model experts (e.g. Jülich-Bonn-Washington model).
3. Determination of $P_{1,2,3}$ (short-distance physics) from QCD-based functional methods (Bethe-Salpeter equation and Dyson-Schwinger Eqs) $\to$ collaboration with Gernot Eichmann (Graz U.) and Christian S. Fischer (Giessen U.).



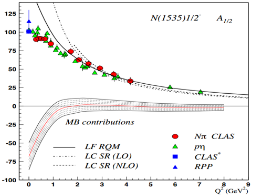
Tool box for non-perturbative QCD

Quark-gluon-based methods:

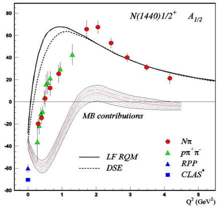
1. **Dyson-Schwinger Equations**
   Infinitely many coupled non-perturbative Eqs. for quark gluon correlators.
2. **QCD light-cone sum rules**
   Operator product expanding correlators around light-cone $x^2 \sim 0 \rightarrow \frac{\Lambda^2_{QCD}}{Q^2} \approx 0$.
3. **Lattice QCD**
   Unable to calculate TFFs due to multi-particle state contamination and many other technical reasons.

Hadron-based methods:

1. **Chiral perturbation theory**
   works very well for Goldstone-bosons by non-linear realisation $SU_L(2) \otimes SU_R(2)/SU_V(2)$
2. **Dispersion theory**

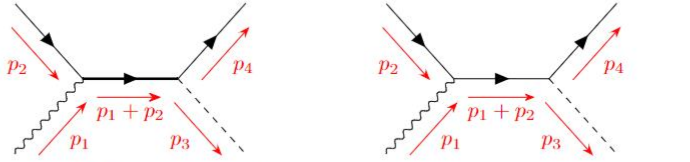


Relativistic quark model and light cone sum rules calculation [5] for TFF of $N \rightarrow N(1535)$.



Dyson-Schwinger Equation prediction [5] TFF of $N \rightarrow N(1440)$

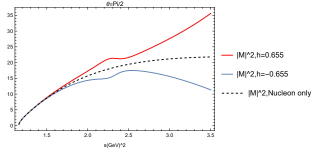
Fixing coupling constants with isobar models



(a) N^* exchange in s-channel (b) N exchange in s-channel

$\gamma N \to \pi N$ s-channel scattering amplitude.

Two amplitudes are constructive (destructive) at $s = (mN^*)^2$ if $h > 0 (h < 0)$.



$\gamma N \to \pi N$ amplitude

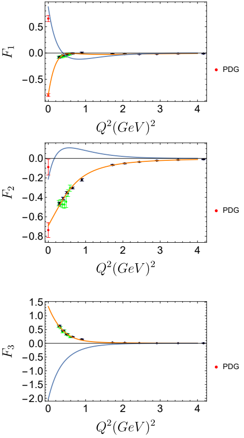
Experimental data on space-like TFFs

We can only calculate isovector TFFs:$=\frac{1}{2}(F_i^{proton} - F_i^{neutron}), i = 1, 2, 3$

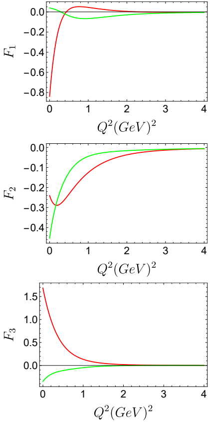
F1

F2

F3



Orange: Proton data (J-lab). Blue: Neutron estimates (MAID)



Isovector TFFs:$=\frac{1}{2}(F_i^{proton} - F_i^{neutron})$
Isoscalar TFFs:$=\frac{1}{2}(F_i^{proton} + F_i^{neutron})$

No error estimate from MAID data
F1 has a complicated structure at low and high energy.

F2 is not iso-vector dominant at low Q^2 and it probably has large uncertainty

F3 has simple behaviour at low and high energy and is dominant at low energy$\rightarrow$
not measurable at the photon point.

Fixing input parameters

$N^*N\pi \rightarrow h = 0.655$
$N^*N\Delta \rightarrow H_1 = 0.28, H_2 = -5.6$
How to fix subtraction constants $P_{1,2,3}$?

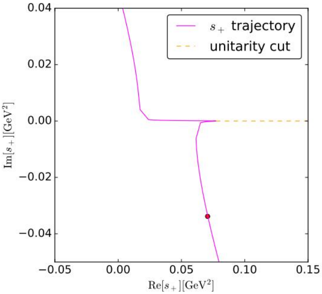
Method A: Fit $P_{1,2,3}$ to slopes of TFF data
Pro: easy to do.
Con: no data exist in $0 < Q^2 \leq 0.25\text{GeV}^2$!!

Method B: Fit to the $N(1520) \rightarrow N\rho$ hadronic decay widths and make use of slopes of TFFs.
Contact terms: P_1, P_2, P_3
$\rightarrow$ 8 possible sign choices.
$dF_1(Q^2)/dQ^2 > 0 \rightarrow$ choose $P_1 > 0$
$N(1520) \rightarrow N\rho$ fit suggests $\rightarrow P_2 \approx 0$
$dF_3(Q^2)/dQ^2 < 0 \rightarrow$ choose $P_3 < 0$
$\rightarrow$ predict $F_{1,2,3}(Q^2)$

Branching ratios of N^* in PDG:

1. $N\pi$ 55 − 65%
2. $\Delta\pi$(S-wave)15 − 23%
3. $\Delta\pi$(D-wave)7 − 11%
4. $N\rho$, $(S = \frac{3}{2}, L = 0)$ 10 − 16%
5. $N\rho$, $(S = \frac{1}{2}, L = 2)$ 0.2 − 0.4%
6. $N\rho$, $(S = \frac{3}{2}, L = 2)$ ≈ 0

Anomalous singularity

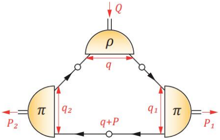


Trajectory of a singularity in the complex plane[6]

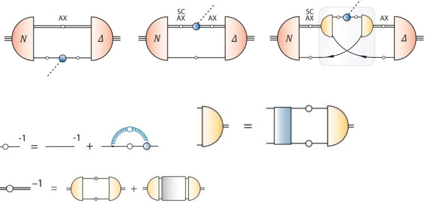
Calculating the couplings in quark di-quark approach in Dyson-Schwinger equations and Bethe-Salpeter equations

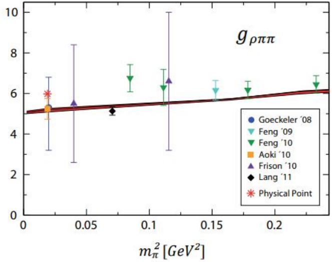
State of art method: Rainbow ladder + quark-di quark approximation (Valentin Mader et al.).

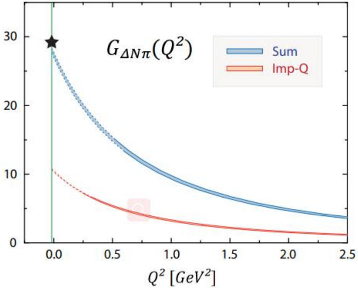
$\rho \to \pi\pi$ transition matrix elements



$\Delta \to N\pi$ transition matrix elements

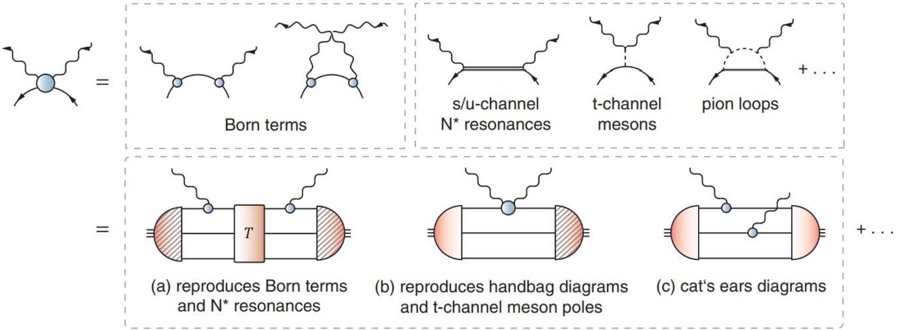






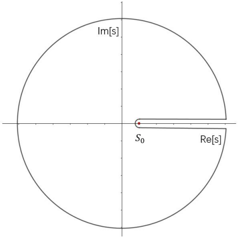
Match the hadrons with quarks and gluons

Compton scattering amplitudes (Gernot Eichmann et al):



Dispersion theory in a nutshell

Example: Pion vector form factor



Unitarity cut $[4m_\pi^2, \infty)$

$$
\begin{aligned}
S &= 1 + iT \\
\text{Unitarity } SS^\dagger &= 1 + i(T - T^\dagger) + |T|^2 = 1 \\
\to 2ImT &= |T|^2 \\
\to ImT_{A\to B} &= \frac{1}{2}\sum_x T_{A\to x} T^\dagger_{x\to B}
\end{aligned} \tag{2}
$$

Simplest example: $A = |\gamma^*\rangle$, $B = \left|\pi^-(p_1)\pi^+(p_2)\right\rangle$.

$$
\Rightarrow T_{\gamma^*\to\pi^-\pi^+} = eA^\mu \underbrace{\langle\pi^-(p_1)\pi^+(p_2)|j^\mu|0\rangle}_{(p_1^\mu - p_2^\mu)F_v(s)} \tag{3}
$$

$$
T_{\gamma^*\to x} = eA^\mu \langle x|j^\mu|0\rangle \tag{4}
$$

$$
ImF_v(s)(p_1^\mu - p_2^\mu) = \frac{1}{2}\sum_x \langle\pi^-(p_1)\pi^+(p_2)|x\rangle^* \langle x|j^\mu|0\rangle \tag{5}
$$

$|x\rangle = 2pions(s \geq 4m_\pi^2), 4pions(s \geq 16m_\pi^2), 2kaons(s \geq 4m_K^2), \ldots$

Dispersion relation

Cauchy integral formula:

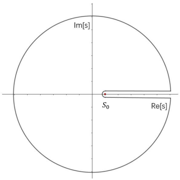
$$F_v(s) = \frac{1}{2\pi i}\int_{s_0=4m_\pi^2}^{\infty} dz \frac{lim_{\epsilon\to 0}[F_v(z+i\epsilon) - F_v(z-i\epsilon)]}{z-s} \tag{6}$$

Schwarz Reflection Principle: $F_v(z-i\epsilon) = F_v(z+i\epsilon)^*$

$$F_v(s) = \frac{1}{\pi}\int_{s_0=4m_\pi^2}^{\infty} dz \frac{Im[F_v(z+i\epsilon)]}{z-s} \text{ Dispersion relation} \tag{7}$$

Consider only the 2 pion contribution

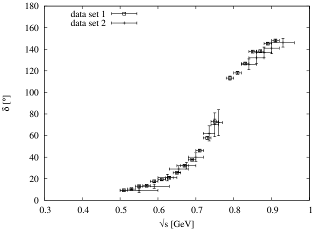
$$2ImF_v(q^2)(p_1^\mu - p_2^\mu) \approx \int d\tau_{2\pi}^{'} \underbrace{\langle\pi^-(p_1)\pi^+(p_2)|\pi^-(p_1^{'})\pi^+(p_2^{'})\rangle^*}_{\text{Pion rescattering amplitude}} \underbrace{\langle\pi^-(p_1^{'})\pi^+(p_2^{'})|j^\mu|0\rangle}_{F_v(q^2)(p_1^{'\mu} - p_2^{'\mu})} \tag{8}$$



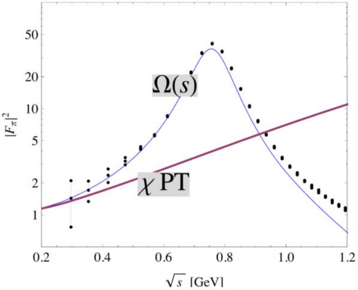
Only pion p-wave re-scattering amp.
$f_1(s) = \frac{sin\delta_1(s)}{\sqrt{1-\frac{4m_\pi^2}{s}}}\frac{\sqrt{s}}{2}$ contributes!
$\Rightarrow f_1(s)$ parametrized by phase-shift δ_1
$\delta_1 \Rightarrow$ well measured by experiments!

Dispersion relation

$$\delta_1 \text{ contains } \rho \text{ meson information} \xrightarrow{\text{Dispersion relation}}$$
$$F_v(s) \approx \Omega(s) = exp[\frac{s}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\delta_1(s')}{s'(s'-s)}]$$



Pion p-wave phase shift [7]



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