The role of meson interactions in the $D_s^+ \to \pi^+\pi^+\pi^-\eta$ decay

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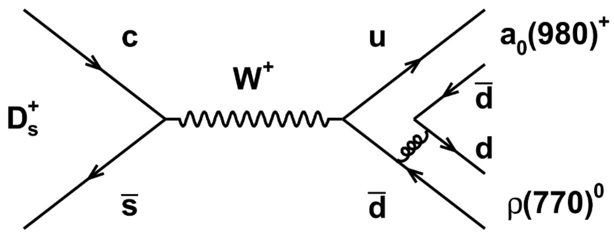
Outline

- Introduction
- Theoretical formalism
- Results and discussion
- Summary

Introduction

- Experiment from *BESIII collaboration Phys.Rev.D 104 071101(2021)*

Study of the Decay $D_s^+ \to \pi^+\pi^+\pi^-\eta$ and Observation of the W-annihilation Decay $D_s^+ \to a_0(980)^+\rho^0$



- Dynamic generation:
  -
  - …

- Motivation and goal: reproduce the six invariant mass distributions

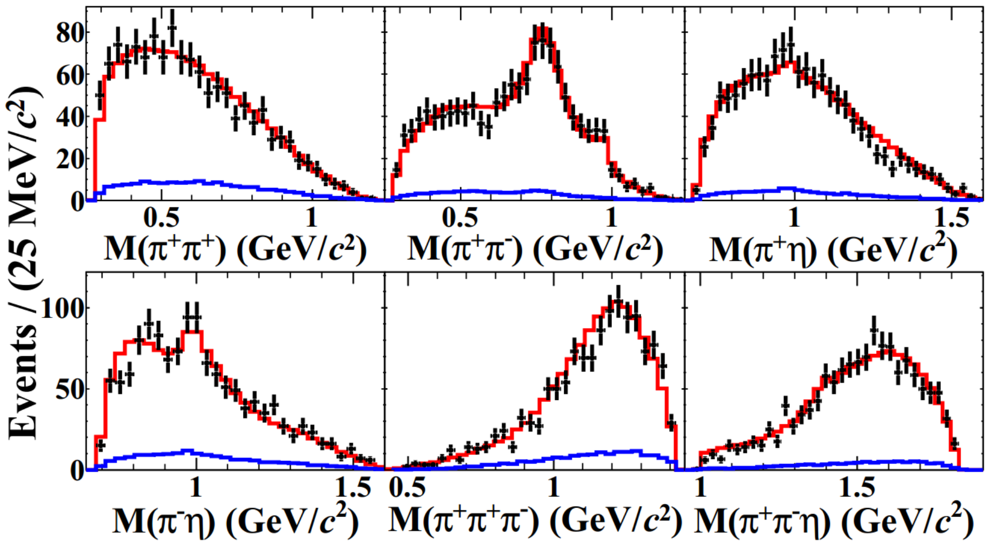


TABLE I. Phases and FFs for various intermediate processes. The first and the second uncertainties are statistical and systematic, respectively.

Amplitude	Phase	FF(%)
$a_1(1260)^+(\rho(770)^0\pi^+)\eta$	0.0(fixed)	$55.4 \pm 3.9 \pm 2.0$
$a_1(1260)^+(f_0(500)\pi^+)\eta$	$5.0 \pm 0.1 \pm 0.1$	$8.1 \pm 1.9 \pm 2.1$
$a_0(980)^+\rho(770)^0$	$2.5 \pm 0.1 \pm 0.1$	$6.7 \pm 2.5 \pm 1.5$
$\eta(1405)(a_0(980)^-\pi^+)\pi^+$	$0.2 \pm 0.2 \pm 0.1$	$0.7 \pm 0.2 \pm 0.1$
$\eta(1405)(a_0(980)^+\pi^-)\pi^+$	$0.2 \pm 0.2 \pm 0.1$	$0.7 \pm 0.2 \pm 0.1$
$f_1(1420)(a_0(980)^-\pi^+)\pi^+$	$4.3 \pm 0.2 \pm 0.4$	$1.9 \pm 0.5 \pm 0.3$
$f_1(1420)(a_0(980)^+\pi^-)\pi^+$	$4.3 \pm 0.2 \pm 0.4$	$1.7 \pm 0.5 \pm 0.3$
$[a_0(980)^-\pi^+]_S\pi^+$	$0.1 \pm 0.2 \pm 0.2$	$5.1 \pm 1.2 \pm 0.9$
$[a_0(980)^+\pi^-]_S\pi^+$	$0.1 \pm 0.2 \pm 0.2$	$3.4 \pm 0.8 \pm 0.6$
$[f_0(980)\eta]_S\pi^+$	$1.4 \pm 0.2 \pm 0.3$	$6.2 \pm 1.7 \pm 0.9$
$[f_0(500)\eta]_S\pi^+$	$2.5 \pm 0.2 \pm 0.3$	$12.7 \pm 2.6 \pm 2.0$

Outline

- Introduction
- **Theoretical formalism**

1. Hadronization of one pair
2. Rescattering from the tree level   component
3. Two hadronizations: external and internal emission
4. Rescattering from the tree level   component
5. Evaluation of the differential cross section: integral

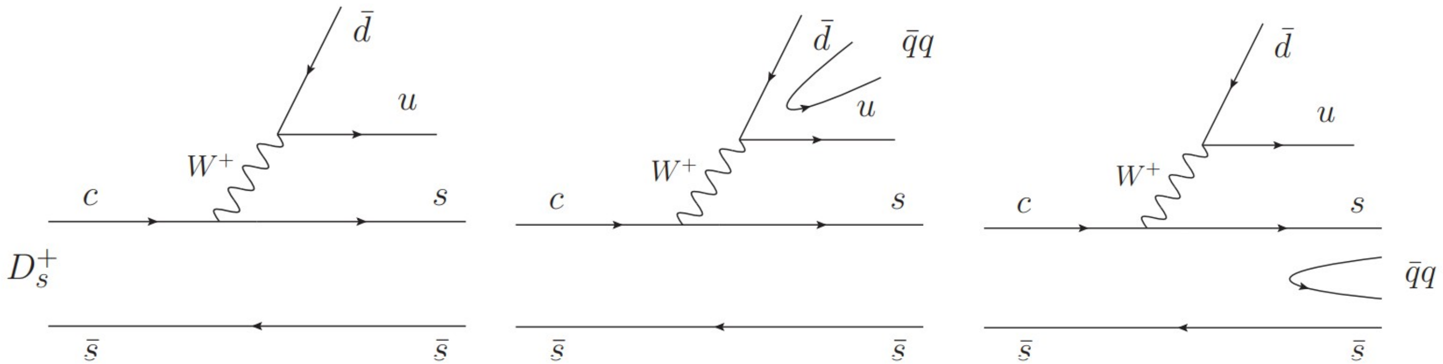
- Results and discussion
- Summary

1. Hadronization of one pair

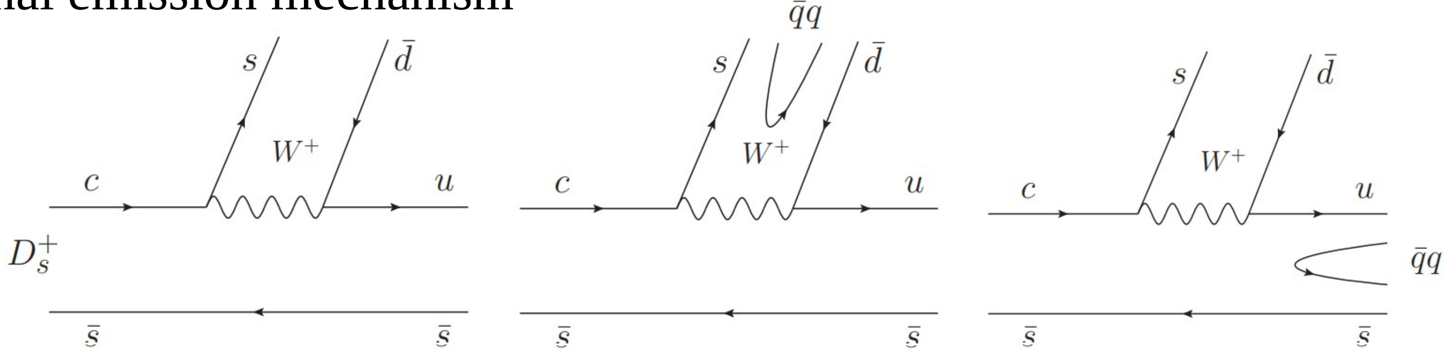
✓ Final states: (**PPV**).

$$P=\begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{3}}+\frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{3}}+\frac{\eta'}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}}+\frac{2\eta'}{\sqrt{3}} \end{pmatrix}, \quad V=\begin{pmatrix} \frac{\rho^0}{\sqrt{2}}+\frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{-\rho^0}{\sqrt{2}}+\frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}$$

➢ External emission mechanism



➢ Internal emission mechanism

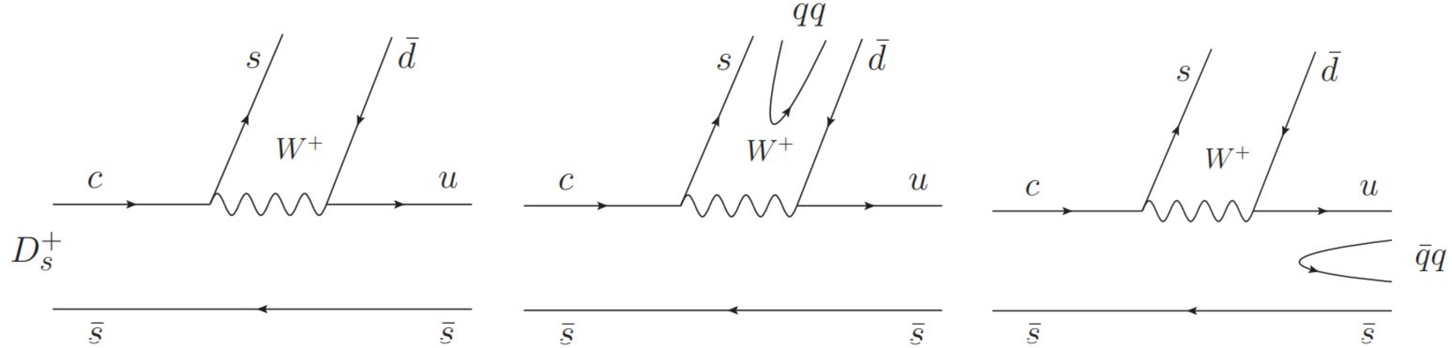


1. Hadronization of one pair

✓ Final states: **(PPV)**.

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \frac{2\eta'}{\sqrt{3}} \end{pmatrix}, \quad V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{-\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}$$

➢ Internal emission mechanism



◆ Hadronize options

1. Hadronize $s\bar{d}$ with PP and $u\bar{s}$ is a vector;
2. Hadronize $u\bar{s}$ to PP and $s\bar{d}$ is a vector;
3. Hadronize $s\bar{d}$ to VP, PV and $u\bar{s}$ is a pseudoscalar;
4. Hadronize $u\bar{s}$ to VP, PV and $s\bar{d}$ is a pseudoscalar.

◆ Example

$$s\bar{d} \to \sum_i s\bar{q}_i q_i \bar{d} = \sum_i P_{3i}P_{i2} = (P^2)_{32}$$
$$= K^-\pi^+ + (-\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}})\bar{K}^0 - \frac{\eta}{\sqrt{3}}\bar{K}^0$$
$$= K^-\pi^+ - \frac{\pi^0}{\sqrt{2}}\bar{K}^0$$

component gives

G-parity considerations

$$G = Ce^{-i\pi I_2}; \qquad e^{-i\pi I_2}|I, I_3\rangle = (-1)^{I-I_3}|I, -I_3\rangle$$

π , ω G= -
η , ρ G= +

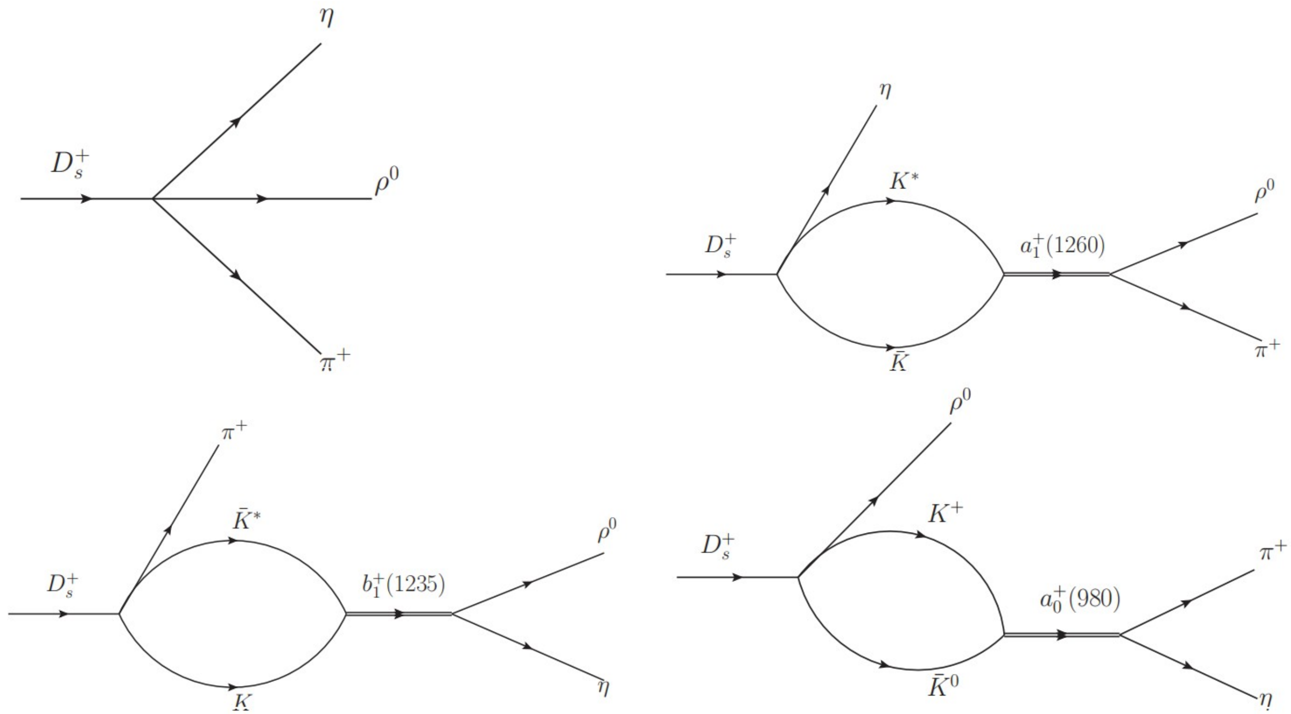
	K^+	K^0	$\bar{K}^0$	K^-	K^{*+}	K^{*0}	$\bar{K}^{*0}$	K^{*-}
$G(K_i)$	$\bar{K}^0$	$-K^-$	$-K^+$	K^0	$-\bar{K}^{*0}$	K^{*-}	K^{*+}	$-K^{*0}$

$$D_s^+ \to \pi^+\pi^+\pi^-\eta$$

The final state has negative parity. Only final states with negative parity must be kept

1. Hadronization of one pair

- Ingredients



- Suitable states and the weights ()

C: Global normalization factor

$$|H1\rangle \equiv C\Big[-\sqrt{\frac{2}{3}}\eta\rho^0\pi^+ + \frac{\eta}{\sqrt{3}}(1+\alpha+\beta)(\bar{K}^{*0}K^+ - K^{*+}\bar{K}^0)$$
$$+\sqrt{2}\beta\rho^0K^+\bar{K}^0 + \alpha\pi^+(K^{*-}K^+ - K^{*0}\bar{K}^0) - \gamma\pi^+(\bar{K}^{*0}K^0 - K^{*+}K^-)\Big]$$

- Couplings

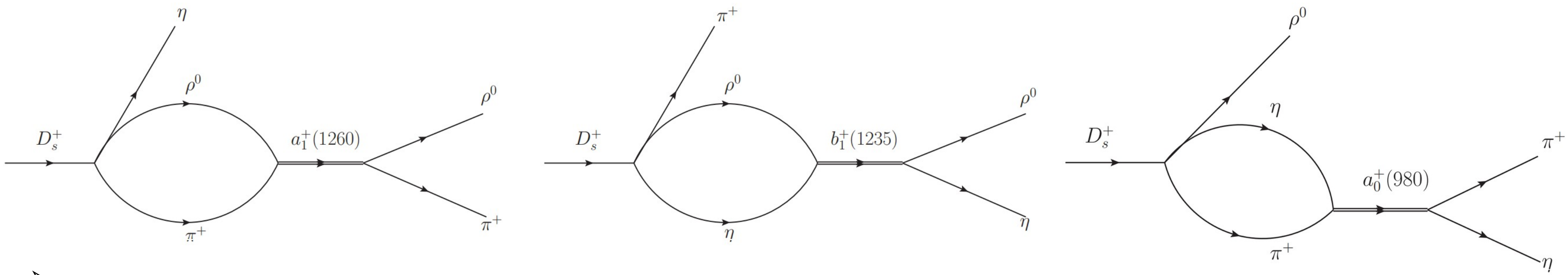
L. Roca et al., Phys. Rev. D **72**, 014002 (2005)

a_1		b_1	
$g_{\bar{K}^* K}$	$g_{\rho\pi}$	$g_{K^*\bar{K}}$	$g_{\rho\eta}$
$1872 - i1486$	$-3.795 + i2330$	$-3041 + i498$	$6172 - i75$

- Amplitude

$$t_{H1}(\pi^+\rho^0\eta) = C\Big[-\sqrt{\frac{2}{3}} + \frac{\eta}{\sqrt{3}}(1+\alpha+\beta)G_{K^*\bar{K}}(M_{\rm inv}(\rho^0\pi^+))\frac{g_{a_1,K^*\bar{K}}\,g_{a_1,\rho\pi}}{M^2_{\rm inv}(\rho^0\pi^+) - M^2_{a_1} + iM_{a_1}\Gamma_{a_1}}$$
$$- \sqrt{2}\beta G_{K\bar{K}}(M_{\rm inv}(\pi^+\eta))\frac{g_{a_0,K\bar{K}}\,g_{a_0,\pi\eta}}{M^2_{\rm inv}(\pi^+\eta) - M^2_{a_0} + iM_{a_0}\Gamma_{a_0}}$$
$$- (\alpha+\gamma)G_{K^*\bar{K}}(M_{\rm inv}(\rho^0\eta))\frac{g_{b_1,K^*\bar{K}}\,g_{b_1,\rho\eta}}{M^2_{\rm inv}(\rho^0\eta) - M^2_{b_1} + iM_{b_1}\Gamma_{b_1}}\Big]$$

- Ingredients



- Couplings

*L. Roca et al.,Phys. Rev. D **72**, 014002 (2005)*

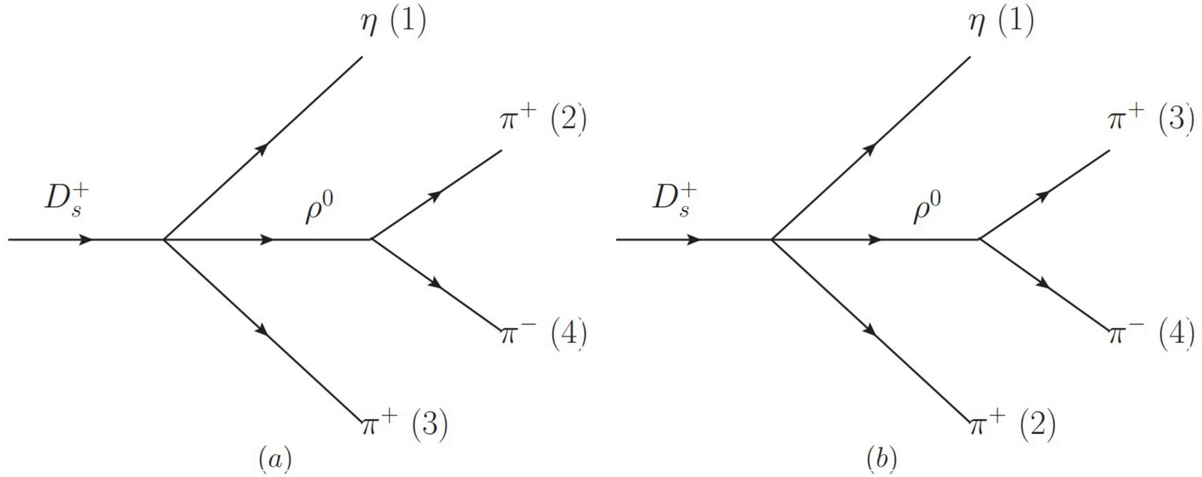
a_1		b_1	
$g_{\bar{K}^* K}$	$g_{\rho\pi}$	$g_{K^* \bar{K}}$	$g_{\rho\eta}$
$1872 - i1486$	$-3.795 + i2330$	$-3041 + i498$	$6172 - i75$

- Amplitude

$$t_{\rm RES}(\rho^0\pi^+\eta) = -C\sqrt{\frac{2}{3}}G_{\rho\pi}(M_{\rm inv}(\rho^0\pi^+))\frac{\frac{1}{\sqrt{2}}g_{a_1,\rho\pi}\frac{1}{\sqrt{2}}g_{a_1,\rho\pi}}{M^2_{\rm inv}(\rho^0\pi^+) - M^2_{a_1} + iM_{a_1}\Gamma_{a_1}}$$
$$- C\sqrt{\frac{2}{3}}G_{\rho\eta}(M_{\rm inv}(\rho^0\eta))\frac{g_{b_1,\rho\eta}g_{b_1,\rho\eta}}{M^2_{\rm inv}(\rho^0\eta) - M^2_{b_1} + iM_{b_1}\Gamma_{b_1}} - C\sqrt{\frac{2}{3}}G_{\pi^+\eta}(M_{\rm inv}(\pi^+\eta))\frac{g_{a_0,\pi\eta}g_{a_0,\pi\eta}}{M^2_{\rm inv}(\pi\eta) - M^2_{a_0} + iM_{a_0}\Gamma_{a_0}}$$

2. Rescattering from the tree level component

- Symmetrized amplitude with decaying to two pions.



- Amplitude

$$t_\rho = -C\left[P_{D_s}\cdot(p_4-p_2)\frac{1}{M_{\rm inv}^2(\rho,a)-M_\rho^2+iM_\rho\Gamma_\rho}t^{(a)}+P_{D_s}\cdot(p_4-p_3)\frac{1}{M_{\rm inv}^2(\rho,b)-M_\rho^2+iM_\rho\Gamma_\rho}t^{(b)}\right]$$

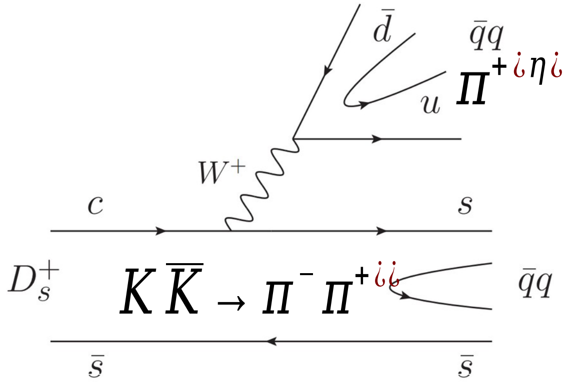
- propagator: the sum over the polarization

$$\sum_{pol} P_D^\mu \epsilon_\mu \epsilon_\nu (p_2-p_4)^\nu = P_D^\mu\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{m_{\rho^2}}\right)¿$$

; q()=

3. Two hadronizations: external emission

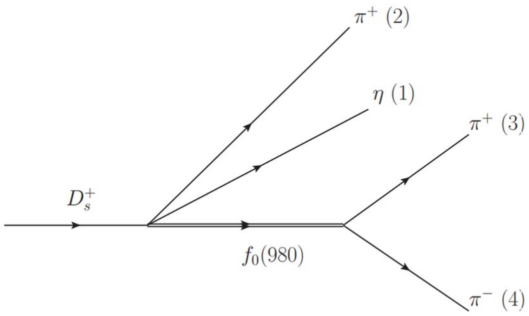
◆ Two hadronizations with external emission: contribution

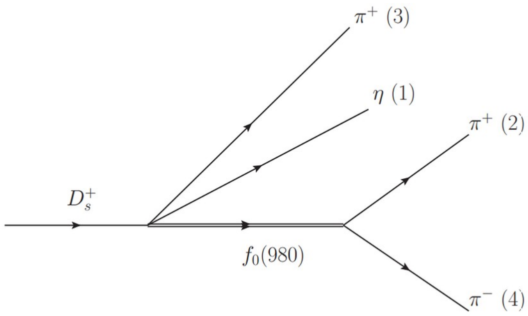


✓ Final states:

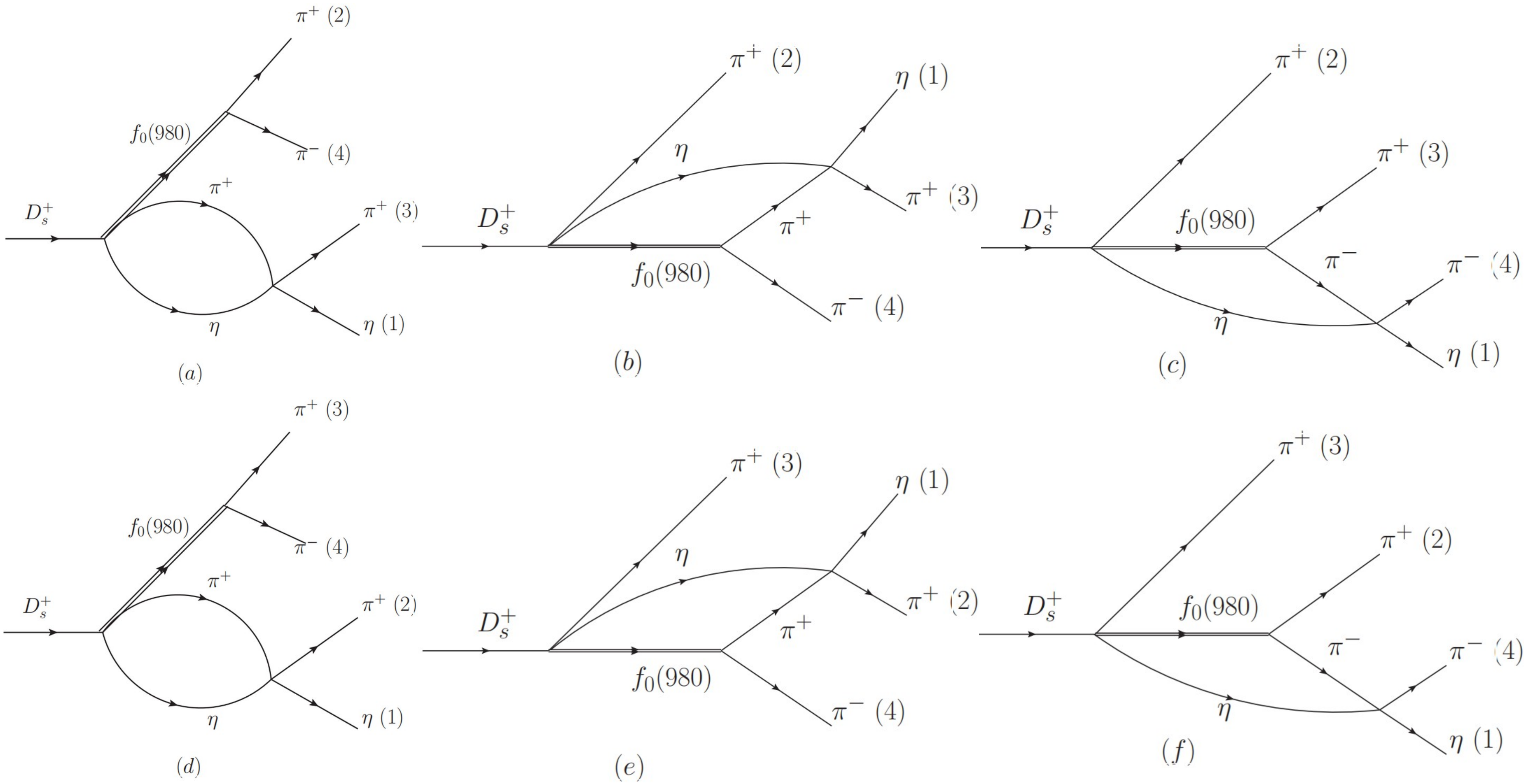
1. Four pseudoscalars(**PPPP)**
2. G-parity

◆ Diagrams stemming from two hadronizations producing the resonance.



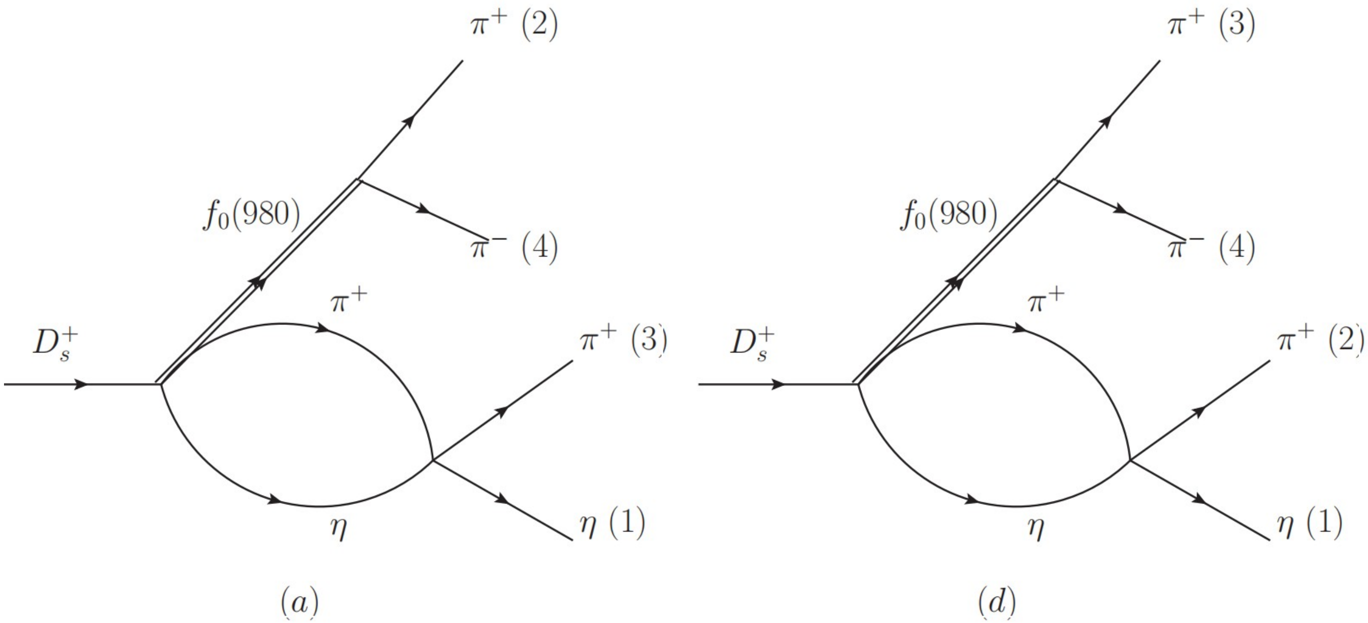


◆ Ingredients:



➢ Amplitude:

- **Ingredient 1:**

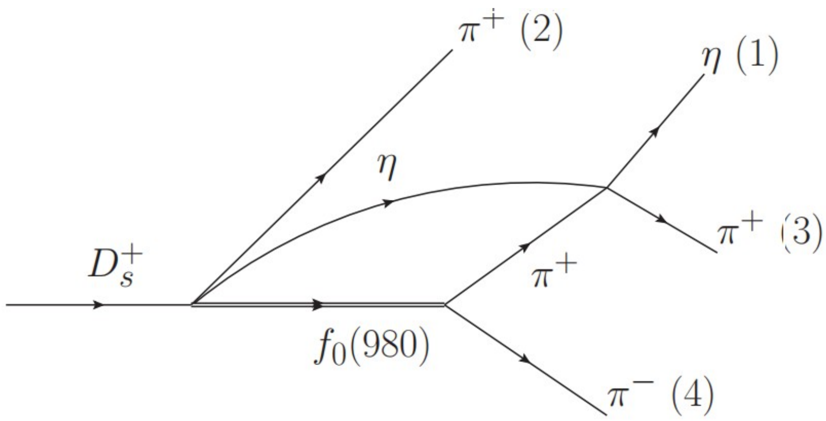


- Amplitude and the weight ()

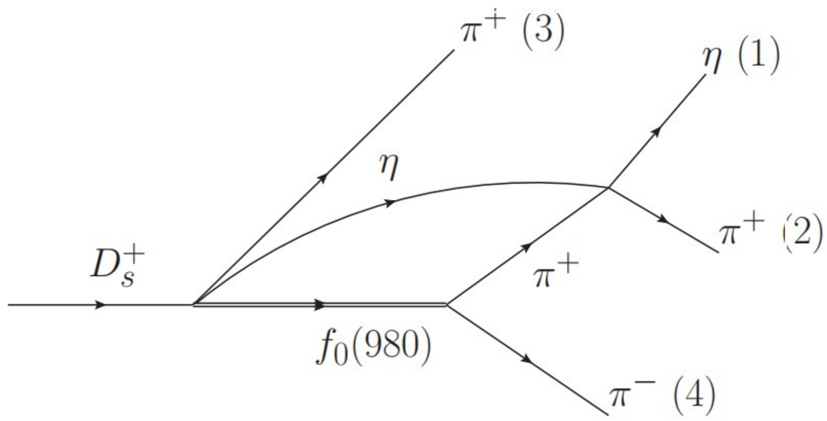
$$t_{2a}(f_0) = C\mu\tilde{D}_{f_0}\left(M_{\rm inv}(\pi^+(2)\pi^-)\right)G_{\pi\eta}\left(M_{\rm inv}(\pi^+(3)\eta)\right)t_{\pi^+\eta,\pi^+\eta}\left(M_{\rm inv}(\pi^+(3)\eta)\right)$$

$$t_{2d}(f_0) = C\mu\tilde{D}_{f_0}\left(M_{\rm inv}(\pi^+(3)\pi^-)\right)G_{\pi\eta}\left(M_{\rm inv}(\pi^+(2)\eta)\right)t_{\pi^+\eta,\pi^+\eta}\left(M_{\rm inv}(\pi^+(2)\eta)\right)$$

- **Ingredient 2:**



(b)



(e)

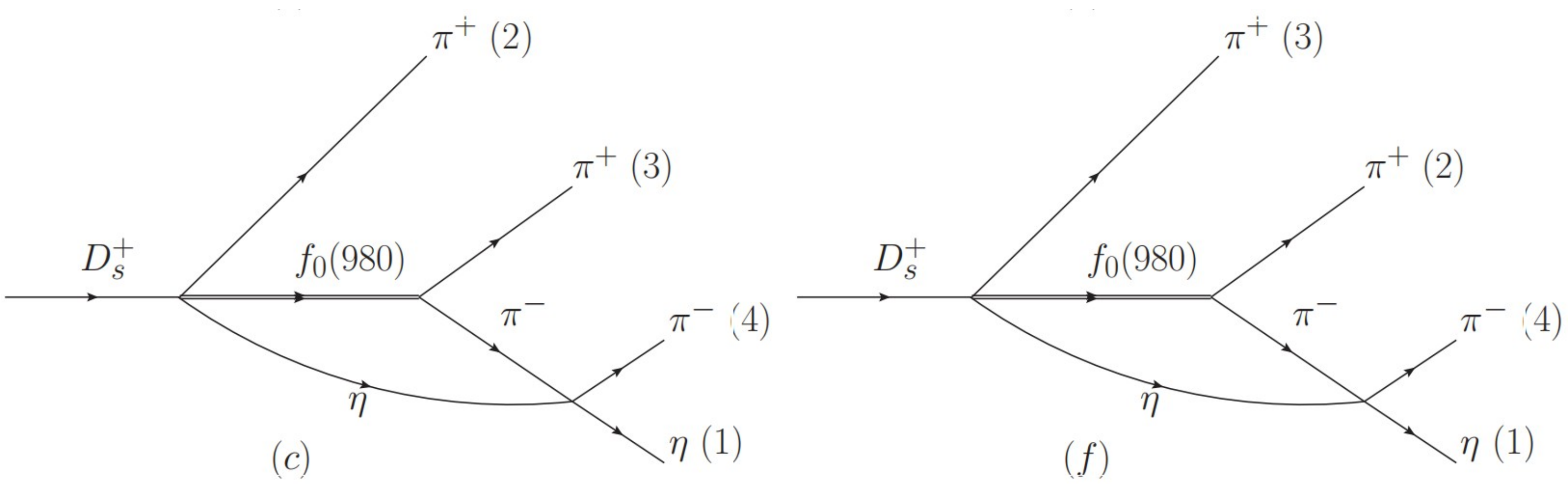
- Amplitude and the weight ()

$$t_{2b}(f_0) = C\mu \tilde{D}_{f_0}\left(M_{\text{inv}}(\pi^-\pi^+_{\text{int}})\right) G_{\pi\eta}\left(M_{\text{inv}}(\pi^+(3)\eta)\right) t_{\pi^+\eta,\pi^+\eta}\left(M_{\text{inv}}(\pi^+(3)\eta)\right)$$

$$t_{2c}(f_0) = C\mu \tilde{D}_{f_0}\left(M_{\text{inv}}(\pi^+(3)\pi^-_{\text{int}})\right) G_{\pi\eta}\left(M_{\text{inv}}(\pi^-\eta)\right) t_{\pi^-\eta,\pi^-\eta}\left(M_{\text{inv}}(\pi^-\eta)\right)$$

4. Rescattering from the tree level component

➢ **Ingredient 3:**



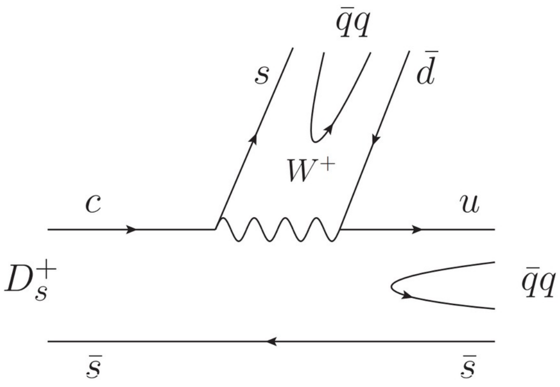
➢ Amplitude and the weight ()

$$t_{2e}(f_0) = C\mu \tilde{D}_{f_0}\left(M_{\text{inv}}(\pi^-\pi^+_{\text{int}})\right) G_{\pi\eta}\left(M_{\text{inv}}(\pi^+(2)\eta)\right) t_{\pi^+\eta,\pi^+\eta}\left(M_{\text{inv}}(\pi^+(2)\eta)\right)$$

$$t_{2f}(f_0) = C\mu \tilde{D}_{f_0}\left(M_{\text{inv}}(\pi^+(2)\pi^-_{\text{int}})\right) G_{\pi\eta}\left(M_{\text{inv}}(\pi^-\eta)\right) t_{\pi^-\eta,\pi^-\eta}\left(M_{\text{inv}}(\pi^-\eta)\right)$$

3. Two hadronizations: internal emission

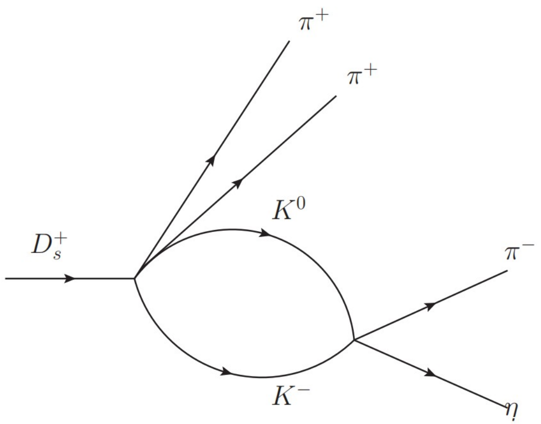
◆ Decay emission



◆ Mechanism contribution of

Final states

1. directly 4 PPPP
2. G-parity



➢ Amplitude and the weight ()

$$t_{DIE} = C \nu G_{K\bar{K}}(M_{inv}(\pi^- \eta)) t_{\pi^- \eta, K^0 K^-}$$

◆ Full amplitude:

$$\boldsymbol{t = t_{H1} + t_{RES} + t_1(f_0) + t_2(f_0) + t_{DIE}}$$

- The width for the decay into

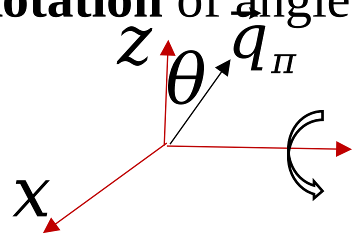
$$\Gamma = \frac{1}{2M_{D_s}} \frac{1}{2} \int \frac{d^3 p_\eta}{(2\pi)^3} \frac{1}{2E_\eta} \int \frac{d^3 p_{\pi^+}}{(2\pi)^3} \frac{1}{2E_{\pi^+}} \int \frac{d^3 p'_{\pi^+}}{(2\pi)^3} \frac{1}{2E'_{\pi^+}} \int \frac{d^3 p_{\pi^-}}{(2\pi)^3} \frac{1}{2E_{\pi^-}} (2\pi)^4 \delta^{(4)}(P - p_\eta - p_{\pi^+} - p'_{\pi^+} - p_{\pi^-}) |t|^2$$

- Define the in the *z* direction:
  - condition allows us to obtain the relation of and variables:

$$\cos\theta \equiv A = \frac{1}{2P_\pi p_\eta} \left[(M_{D_s} - E_\eta - E_{\pi^+} - E'_{\pi^+})^2 - m^2_{\pi^-} - \mathbf{P}^2_\pi - \mathbf{p}^2_\eta \right]$$

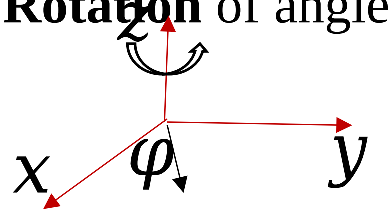
- Define the in the *z* direction:

a. **Rotation** of angle in the *xz* plane, i.e. the along to *y*-axis,



$$R_\theta \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \sin\theta \\ 0 \\ \cos\theta \end{pmatrix} \quad R_\theta \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad R_\theta \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \sin\theta \\ 0 \\ \cos\theta \end{pmatrix}$$

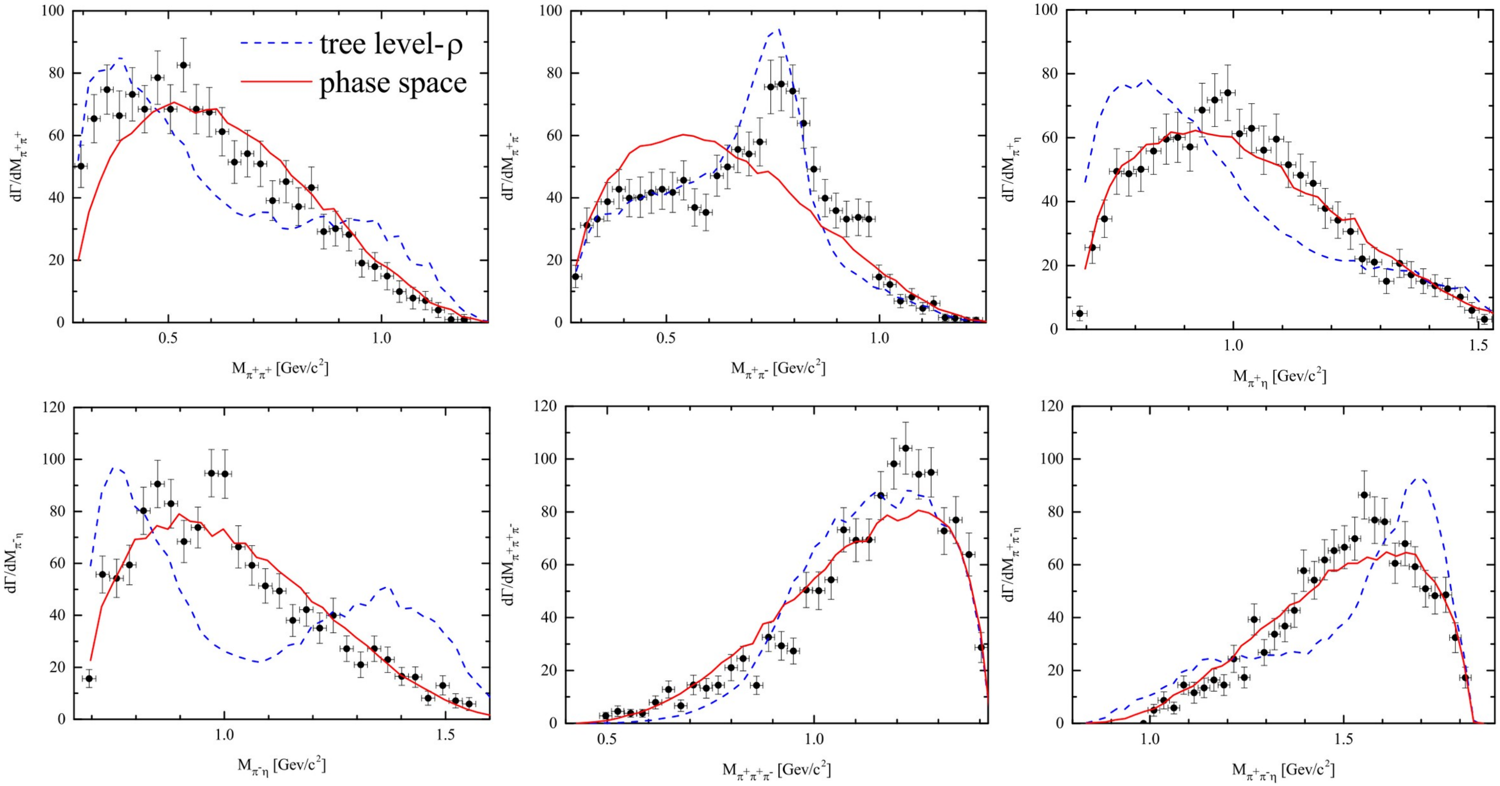
b. **Rotation** of angle in the *xy* plane, i.e. the along to *z*-axis.



$$R_\phi \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\phi \\ \sin\phi \\ 0 \end{pmatrix} \quad R_\phi \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{pmatrix} \quad R_\phi \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Results| Phase space and the tree-level of ρ

- Phase space: missing all the different structures which are visible in the experimental data.

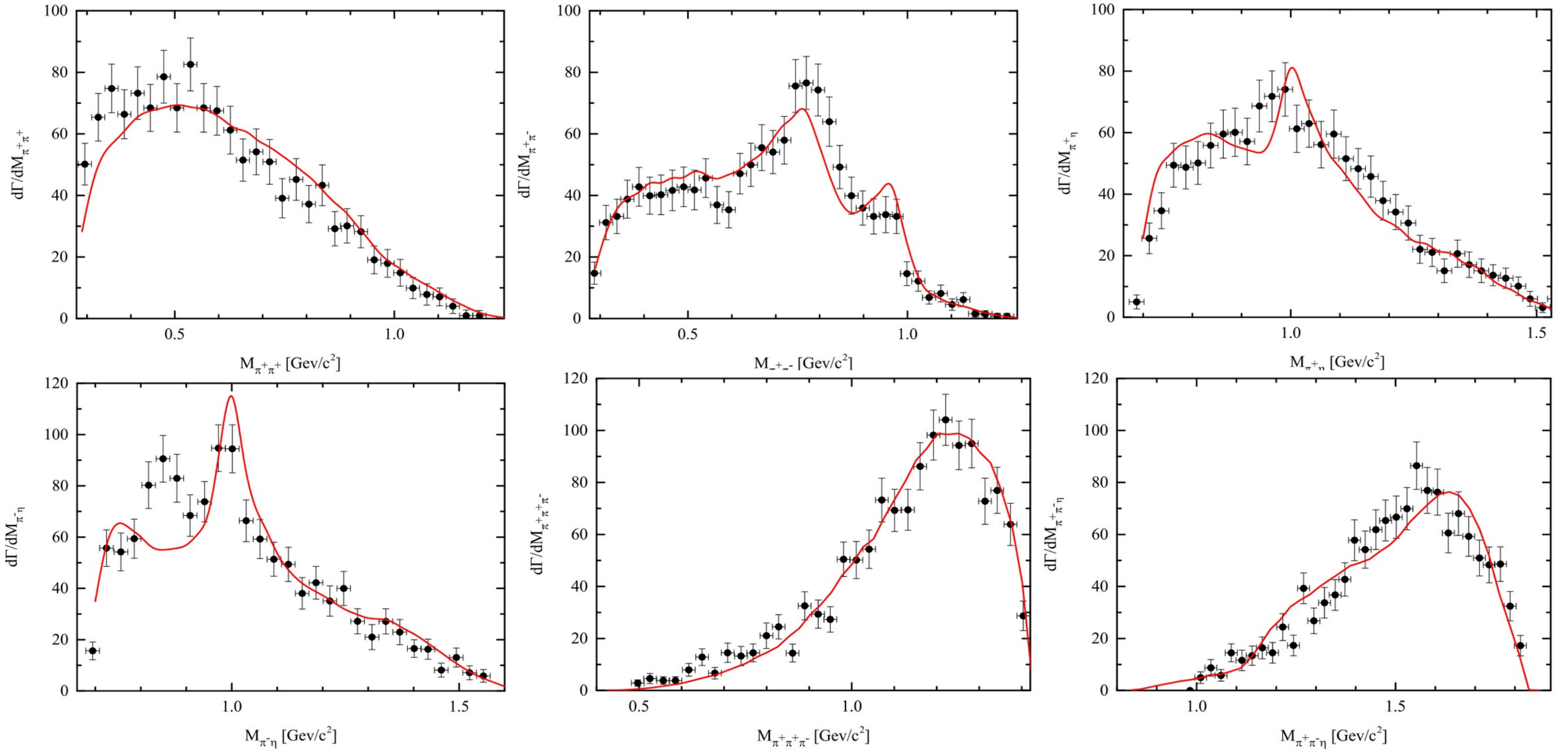


- the contribution of the term alone at the tree level.

1. peak in the distribution
2. A broad bump in at 0.5 GeV. Not ! (term)

Results| the different mass distributions

◆ Formalism comes from the systematic consideration of all possible mechanisms

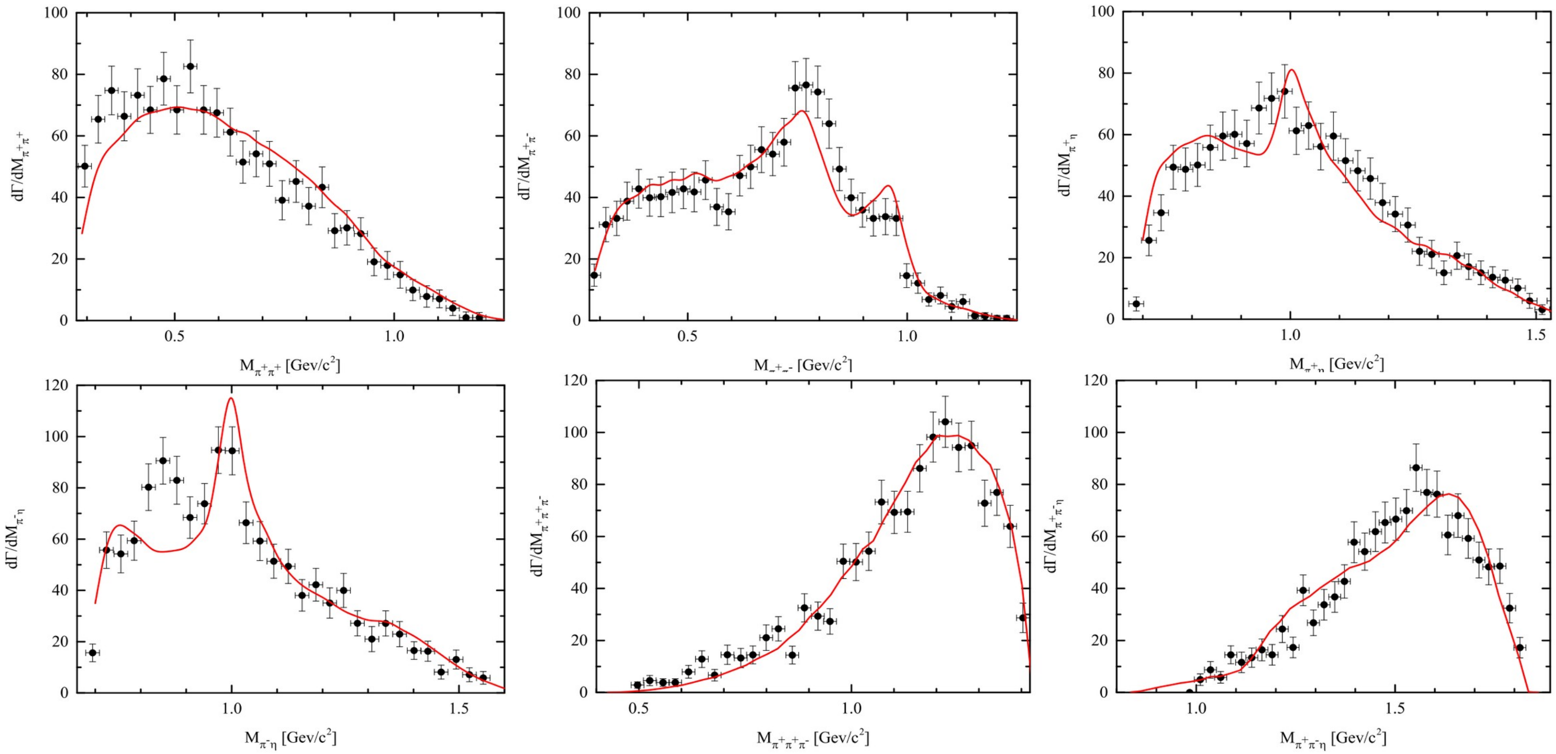


◆ A best fit to all the six mass distributions

1.
2. The improvement over the mass distributions of the tree level term is remarkable.

Results| the different mass distributions

◆ Formalism comes from the systematic consideration of all possible mechanisms



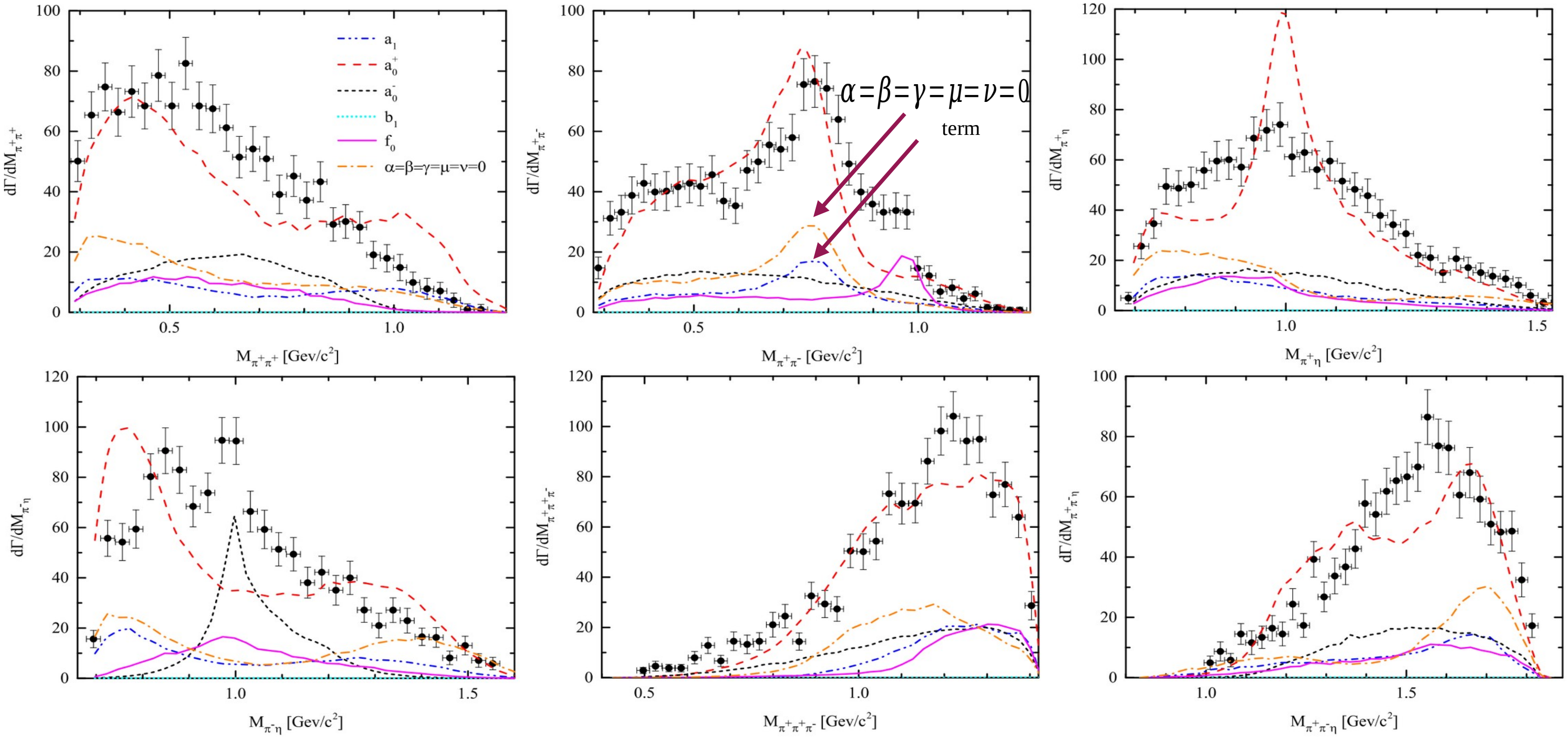
◆ The low and high energy bump of the mass distribution is well reproduced

3. , , peak in the , distribution;
4. distributions are also in good agreement with the experiment.

!! A peak around 0.85 GeV that we cannot reproduce and do not know its dynamical origin

Results| The contributions of , , , , , and external emission

◆ The contribution of the different resonances

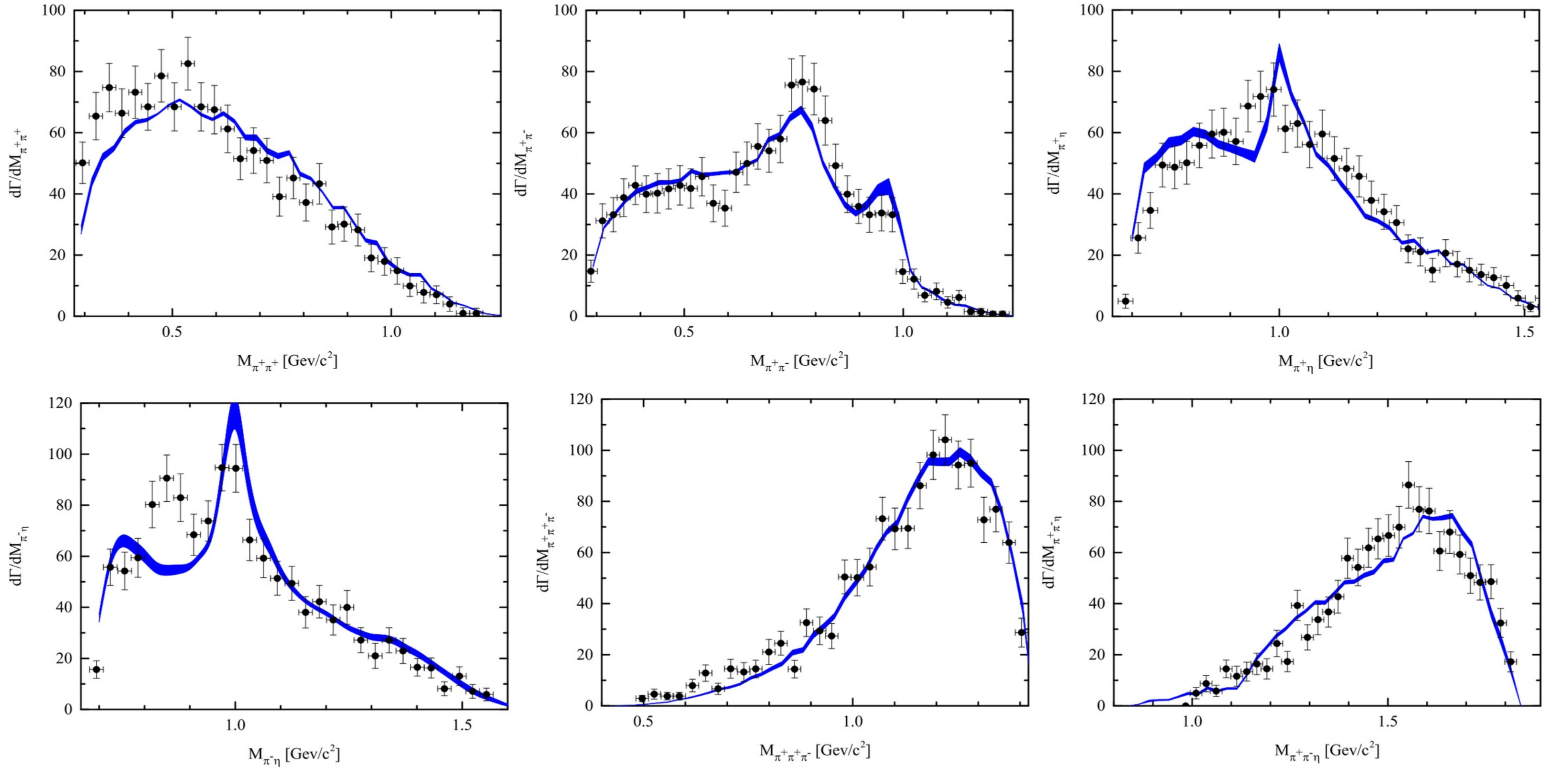


◆ The contributions to the mass distributions: interferences among the different amplitudes

1. The dominant contribution comes from
2. term: peak appears
3. The strength of is practically negligible(mechanisms excitation without experiment!).

Results| Theoretical error bands

◆ Theoretical error bands vs. experimental data



◆ The parameters are different for each fit

1. The bands are so narrow: small uncertainties in the fit results.
2. Take care of the value of the parameters: the strong correlations of the parameters.

Summary

◆ The study of reaction: final state interaction of pairs of mesons

1. The first time of → in quark level with external and internal emission.
2. We have made a theoretical study of the reaction taking into account the **final state** interaction of pairs of mesons. (the filter of **G-parity**)
3. A support for the amplitudes studied by **chiral unitary approach**: we obtain a good reproduction of the 6 invariant mass distributions with **less freedom(5) than** the experiment analysis(19).
4. In these mass distributions: we could clearly see peaks for the , , , , in reasonable agreement with experiment.
5. The fit parameters have large uncertainties but there are strong correlations between them such that the uncertainties in the fit are small.