D-wave B_c meson production at LHC

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The talk is based on *Phys. Rev. D 103, 114001 (2021)* [Berezhnoy et al.(2021)Berezhnoy, Belov, and Likhoded].

B_c properties

1. All excitations below the threshold decay into the ground state 1^1S_0.
2. The absence of strong annihilation channels $\Longrightarrow$ the very narrow ground state (practically as B-meson).
3. Spectroscopy can be investigated within the same framework as for $c\bar{c}$ and $b\bar{b}$ quarkoniums.
4. The small total yield comparing to the $c\bar{c}$ and $b\bar{b}$ quarkonia case.
5. The small relative yield of P-wave excitations comparing to the $c\bar{c}$ and $b\bar{b}$ quarkonia case.

B_c family has a spectroscopy similar to $c\bar{c}$ or $b\bar{b}$ quarkonium spectroscopy and decays like B meson

- The main difference in decays (comparing to B meson): the both quarks in B_c are heavy.
- The main difference in spectroscopy (comparing to $c\bar{c}$ and $b\bar{b}$ quarkonia): charge parity can not be determined.

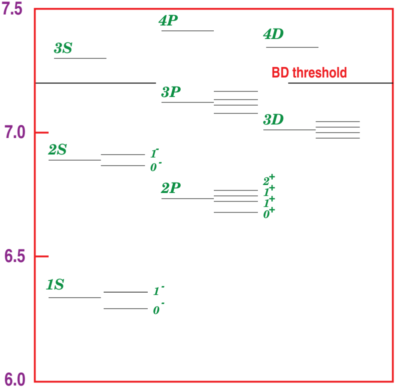
$$h_Q \ \chi_{1Q} \xrightarrow{\text{mixing}} 1^+ \ 1^{+'}$$

$$\begin{aligned}
|2P, 1^{'+}\rangle &= 0.294|S=1\rangle + 0.956|S=0\rangle \\
|2P, 1^{+}\rangle &= 0.956|S=1\rangle - 0.294|S=0\rangle \\
|3P, 1^{'+}\rangle &= 0.371|S=1\rangle + 0.929|S=0\rangle \\
|3P, 1^{+}\rangle &= 0.929|S=1\rangle - 0.371|S=0\rangle
\end{aligned}$$

[Kiselev et al.(1995)Kiselev, Likhoded, and Tkabladze, Gershtein et al.(1995)Gershtein, Kiselev, Likhoded, and Tkabladze]

Spectroscopy

All excitations decay into 1^1S_0.



state	Martin	BT
1^1S_0	6.253	6.264
1^1S_1	6.317	6.337
2^1S_0	6.867	6.856
2^1S_1	6.902	6.899
2^1P_0	6.683	6.700
$2P\ 1^+$	6.717	6.730
$2P\ 1'^+$	6.729	6.736
2^3P_2	6.743	6.747
3^1P_0	7.088	7.108
$3P\ 1^+$	7.113	7.135
$3P\ 1'^+$	7.124	7.142
3^3P_2	7.134	7.153
$3D\ 2^-$	7.001	7.009
3^5D_3	7.007	7.005
3^3D_1	7.008	7.012
$3D\ 2'^-$	7.016	7.012

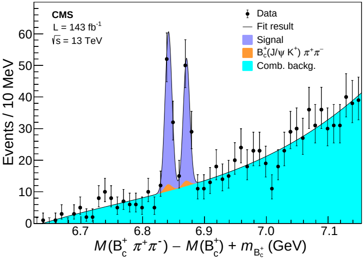
Figure 1: The mass spectrum of $(\bar{b}c)$ with account for the spin-dependent splittings.

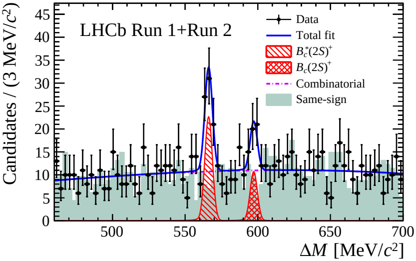
[Gouz et al.(2004)Gouz, Kiselev, Likhoded, Romanovsky, and Yushchenko]

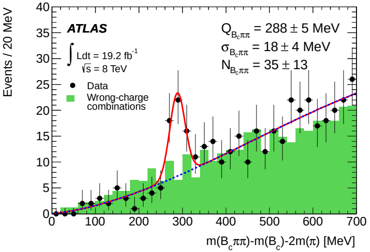
$$M_{B_c} = 6274.9 \pm 0.8 \text{ MeV},$$

$$\tau_{B_c} = 0.507 \pm 0.009 \text{ ps}.$$

$B_c(2S)$ observed in $B_c^{(*)} + \pi\pi$ spectrum







CMS results (reasonable agreement with predictions!)

$$\frac{\sigma(B_c(2S) \to B_c(1S) + \pi^+\pi^-)}{\sigma(B_c)} = (8.2 \pm 1.1)\ \%$$

$$\frac{\sigma(B_c^*(2S))}{\sigma(B_c(2S))} = 1.35 \pm 0.33$$

CMS [Sirunyan et al.(2019)] LHCb [Aaij et al.(2019)] CMS[Sirunyan et al.(2020), CMS(2020)]

$B_c(2S)$: what was measured

Table: The experimental data on $B_c(2S)$

	experiment	ATLAS	CMS	LHCb
	luminosity (energy)	24.1 fb^{-1} (7, 8 TeV)	140 fb^{-1} (13 TeV)	8.7 fb^{-1} (7, 8, 13 TeV)
mass, MeV	2^3S_1 , shifted	6842 ± 6	6842 ± 2	6841 ± 1
	2^1S_0		6871.0 ± 1.6	6872.1 ± 1.6
raw relative yield	2^3S_1		0.0088 ± 0.0014	0.0136 ± 0.0027
	2^1S_0		0.0068 ± 0.0014	0.0063 ± 0.0024
	total	0.18 ± 0.05	0.0156 ± 0.0019	0.0198 ± 0.0036
real relative yield	2^3S_1		(4.69 ± 0.90) %	
	2^1S_0		(3.47 ± 0.71) %	
	total		(8.16 ± 1.1) %	
$N(2^3S_1)/N(2^1S_0)$			1.35 ± 0.33	2.1 ± 0.9

- The registration efficiencies for $\pi^+\pi^-$ are published only by CMS Collaboration, thus the relative yields can not be accurately compared.
- Unexpectedly large yield at ATLAS.

ATLAS [Aad et al.(2014)], CMS [Sirunyan et al.(2019)], LHCb [Aaij et al.(2019)]
See also later CMS publications: [Sirunyan et al.(2020), CMS(2020)]

D-wave states of B_c meson

Predictions for masses of D-wave states of B_c meson (MeV):

State	EQ	GKLT	ZVR	FUI	EFG	GI	MBV	SJSCP	LLLGZ
1^3D_1	7012	7008	7010	7024	7072	7028	6973	6998	7020
$1D_2'$	...	7016	...	...	7079	7036	7003	...	7032
$1D_2$	...	7001	...	...	7077	7041	6974	...	7024
1^1D_2	7009	...	7020	7023	...	...	...	6994	...
1^3D_2	7012	...	7030	7025	...	...	...	6997	...
1^3D_3	7005	7007	7040	7022	7081	7045	7004	6990	7030

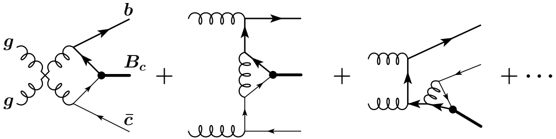
$\sim 20\%$ D could radiate $\pi\pi$ [Eichten and Quigg(1994)] (see also [Asghar et al.(2019)Asghar, Akram, Masud, and Sultan]), and therefore one can expect peaks in the same mass distribution as for $B_c(2S)$

$B_c\pi\pi$-peaks from D states

- one narrow peak at ~ 7000 MeV from $(1D_2, 1D_2')$ states;
- one broad peak at ~ 6930 MeV form shifted and broadened $(1^3D_1, 1D_2, 1D_2', 1^3D_3)$ states.

EQ [Eichten and Quigg(1994)], GKLT [Gershtein et al.(1995)Gershtein, Kiselev, Likhoded, and Tkabladze],
ZVR [Zeng et al.(1995)Zeng, Van Orden, and Roberts], FUI [Fulcher(1999)],
EFG [Ebert et al.(2003)Ebert, Faustov, and Galkin], GI [Godfrey(2004)],
MBV [Monteiro et al.(2017)Monteiro, Bhat, and Vijaya Kumar],
SJSCP [Soni et al.(2018)Soni, Joshi, Shah, Chauhan, and Pandya],
LLLGZ [Li et al.(2019)Li, Liu, Lu, Lü, Gui, and Zhong].

Production of D-wave B_c in hadronic interactions



LO: 36 diagrams
of the order $\mathcal{O}(\alpha_S^4\, v^4)$

- Spin selection for $S = 1$ state:

$$\mathrm{N}(1,1) = |\uparrow\uparrow>$$

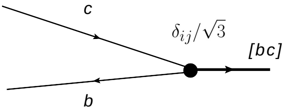
$$\mathrm{N}(1,0) = \frac{1}{\sqrt{2}}\,(|\uparrow\downarrow> + |\downarrow\uparrow>)$$

$$\mathrm{N}(1,-1) = |\downarrow\downarrow>$$

- Spin selection for $S = 0$ state:

$$\mathrm{N}(0,0) = \frac{1}{\sqrt{2}}\,(|\uparrow\downarrow> - |\downarrow\uparrow>)$$

- Colour factor



- Convolution with the wave function of meson. The approximation used:

$$A \sim \int d^3q\,\Psi^*(\boldsymbol{q})\left\{M(p_i,\boldsymbol{q})\big|_{\boldsymbol{q}=0} + q^\alpha\frac{\partial}{\partial q^\alpha}M(p_i,\boldsymbol{q})\big|_{\boldsymbol{q}=0} + \frac{1}{2}q^\alpha q^\beta\frac{\partial^2}{\partial q^\alpha\,\partial q^\beta}M(p_i,\boldsymbol{q})\big|_{\boldsymbol{q}=0} + \cdots\right\},$$

where $\boldsymbol{q}$ — three-momentum of a quark in B_c meson, $\Psi^*(\boldsymbol{q})$ — the wave function of B_c мезона, and M — matrix element for production of four heavy quarks with momenta p_i in gluonic fusion.

Calculations

D-wave spin-singlet $(J = 2,\ j_z = l_z)$:

$$A^{j_z} \sim R''_D(0)\, \epsilon^{\alpha\beta}(j_z)\, \frac{\partial^2 M(\boldsymbol{q})}{\partial q^\alpha \partial q^\beta}\bigg|_{\boldsymbol{q}=0}$$

D-wave spin-triplet $(J = 1, 2, 3;\ j_z = s_z + l_z)$:

$$A^{J j_z} \sim R''_D(0)\, \Pi^{J,\ \alpha\beta\rho}(j_z)\, \frac{\partial^2 M_\rho(\boldsymbol{q})}{\partial q^\alpha \partial q^\beta}\bigg|_{\boldsymbol{q}=0}$$

matrix elements have been summed over J and j_z: $\boxed{\mathbb{P} = \sum_{j_z\, j'_z} A^{j_z} A^{*\, j'_z}}$

and $\boxed{\mathbb{V} = \sum_{J\, J'} \sum_{j_z\, j'_z} \mathcal{A}^{J j_z} \mathcal{A}^{*\, J' j'_z}}$.

Private code

- Calculation of matrix elements with the spin selection
- Numerical differentiating
- Integration over the phase space employing kinematics generator RAMBO
- Convolution with the CT14 PDFs by CTEQ group at the scales $E_T/2 < \mu < 2E_T$:

$$\sigma_{pp} = \int \hat{\sigma}_{gg}(\hat{s}_{gg})\, f_{g1}(x_1)\, f_{g2}(x_2)\, dx_1 dx_2$$

- Cut of the kinematic variables (p_T, η) with respect to the LHCb/CMS conditions and performance of the results in the form of p_T-distributions

Color octet (singlet) NRQCD contributions

The final B_c meson is a superposition of Fock states:

$$|B_c(1^1D_2)\rangle = O(1)|\bar{b}c(^1D_2,\mathbf{1})\rangle + O(v)|\bar{b}c(^1P_1,\mathbf{8})g\rangle + O(v^2)|\bar{b}c(^1S_0,\mathbf{8}\text{ or }\mathbf{1})gg\rangle + \cdots \quad (1)$$

$$|B_c(1^3D_j)\rangle = O(1)|\bar{b}c(^3D_j,\mathbf{1})\rangle + O(v)|\bar{b}c(^3P_{j'},\mathbf{8})g\rangle + O(v^2)|\bar{b}c(^3S_1,\mathbf{8}\text{ or }\mathbf{1})gg\rangle + \cdots \quad (2)$$

- All NRQCD states in (1),(2) contribute to cross section as $O(v^4)$
- The $O(1)$ terms in (1),(2) are well defined:

$$\langle \mathcal{O}_{\mathbf{1}}^{B_c(1^1D_2)}(^1D_2)\rangle \approx \frac{75N_c}{4\pi}|R''_D(0)|^2$$

$$\langle \mathcal{O}_{\mathbf{1}}^{B_c(1^3D_j)}(^3D_j)\rangle \approx \frac{15(2j+1)N_c}{4\pi}|R''_D(0)|^2$$

- The $O(v)$ and $O(v^2)$ terms in (1),(2) are unknown but can be estimated naively under

$$v^2_{\text{eff}} = \frac{\langle E\rangle}{2\mu}$$

Effective value $\boxed{v^2_{\text{eff}} \approx 0.15}$ from

[Gershtein et al.(1995)Gershtein, Kiselev, Likhoded, and Tkabladze]

- Naive velocity scaling:

$$\text{P-wave octet}\begin{cases} \dfrac{\delta_{\bar{b}c}}{\sqrt{3}} \longrightarrow \sqrt{2}\, t^a_{\bar{b}c} \\ |R'_P(0)|^2 \longrightarrow K_{P\mathbf{8}}\cdot|R'_P(0)|^2 \\ K_{P\mathbf{8}} = O(v^2_{\text{eff}}) \end{cases}$$

$$\text{S-wave octet}\begin{cases} \dfrac{\delta_{\bar{b}c}}{\sqrt{3}} \longrightarrow \sqrt{2}\, t^a_{\bar{b}c} \\ |R_S(0)|^2 \longrightarrow K_{S\mathbf{8}}\cdot|R_S(0)|^2 \\ K_{S\mathbf{8}} = O(v^4_{\text{eff}}) \end{cases}$$

$$\text{S-wave singlet}\begin{cases} |R_S(0)|^2 \longrightarrow K_{S\mathbf{1}}\cdot|R_S(0)|^2 \\ K_{S\mathbf{1}} = O(v^4_{\text{eff}}) \end{cases}$$

$gg \to B_c(D) + X$ v.s. $gg \to B_c + X$

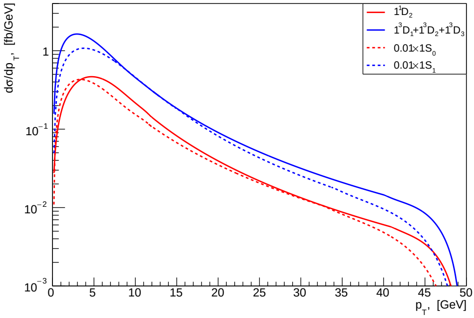
Wave functions by [Eichten and Quigg(2019)]:

$$|R''_D(0)|^2 = 0.0986 \text{ GeV}^7$$

$$|R_S(0)|^2 = 1.994 \text{ GeV}^3$$

Cross sections obtained with $\alpha_s = 0.1$.

energy, $\sqrt{s_{gg}}$, GeV	σ_{gg}, pb	
	$1S$	$1D$
20	1.97	0.009
30	2.90	0.023
50	2.64	0.028
70	1.98	0.024
100	1.44	0.018
150	0.90	0.012

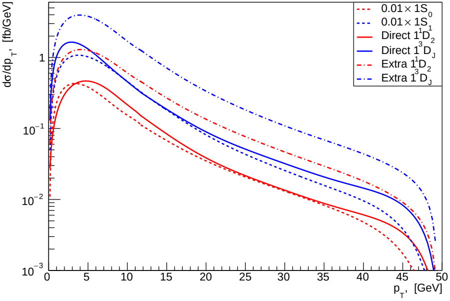


$\sigma(gg \to B_c + X)$ at $\sqrt{s_{gg}} = 100$ GeV.
Red lines: $S = 0$, blue lines: $S = 1$, solid lines: D-waves, dashed lines: S-waves scaled by 0.01

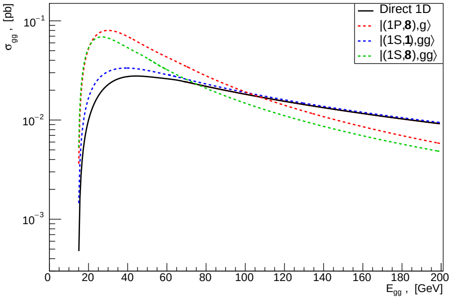
Primary predictions for $B_c(D)$

- $1 \div 2$ % of the direct yield of 1S
- $\sigma(1^3D_1 + 1^3D_2 + 1^3D_3)/\sigma(1^1D_2) \sim (3+5+7)/5 = 3$
- p_T distributions are quite similar to ones for S wave states
- $\sigma(1D)/\sigma(1S)$ ratio is sensible to $|R''_D(0)|^2/|R_S(0)|^2$

Color octet (singlet) contributions to $gg \to B_c(D) + X$



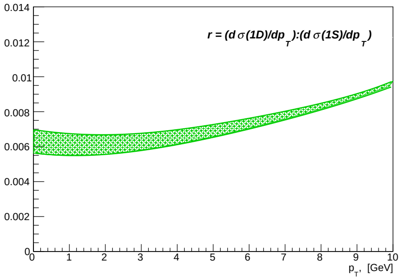
$\sigma\left(gg \to B_c + X\right)$ dependence on transverse momentum at $\sqrt{s}_{gg} = 100$ GeV. Solid lines: direct D-wave production; dashed lines: S-wave states, scaled by 0.01; dashed-dotted lines: extra D-wave states. Red lines: states with $S = 0$, blue lines: states with $S = 1$.



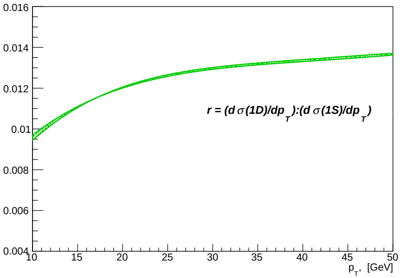
$\sigma\left(gg \to B_c + X\right)$ dependence on gluon-gluon energy. Black line: direct D-wave production; red line: $|(1P, \mathbf{8})g\rangle$ contribution; blue and green lines: $|(1S, \mathbf{1})gg\rangle$ and $|(1S, \mathbf{8})gg\rangle$ contributions correspondingly.

- p_T distribution shapes for $|\bar{b}c(P, \mathbf{8})g\rangle$, $|\bar{b}c(S, \mathbf{8})gg\rangle$, $|\bar{b}c(S, \mathbf{1})gg\rangle$ states nearly reproduce ones for direct D-wave production
- Color octet contributions decrease faster with energy than color singlet ones
- Seems, that the shape of energy dependence is mostly determined by color state of $\bar{b}c$-pair and practically does not depend on its orbital momentum.

$pp \to B_c(D) + X$: yields



r dependence on p_T at different scales for forward kinematics: $2 < \eta < 4.5,\ p_T < 10$ GeV.



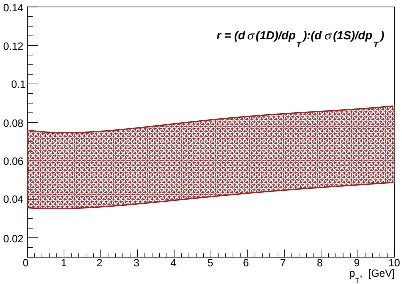
r dependence on p_T at different scales for central kinematics: $|\eta| < 2.5,\ 10$ GeV $< p_T < 50$ GeV.

Relative yields for D-wave B_c mesons for forward and central kinematic regions at LHC.

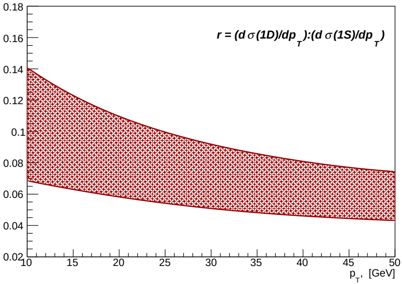
kinematic region	$\sigma\left(1^3S_1\right)/\sigma\left(1^3S_0\right)$	$\sigma\left(1^3D_j\right)/\sigma\left(1^1D_2\right)$	$\sigma\left(1D\right)/\sigma\left(1S\right)$, %
$2 < \eta < 4.5,\ p_T < 10$ GeV	2.4	3.0	0.6 $\div$ 0.7
$\|\eta\| < 2.5,\ 10$ GeV $< p_T < 50$ GeV	2.4	2.3	1.0 $\div$ 1.1

More optimistic values in the approaches considering relativistic corrections.
[Ebert et al.(2011)Ebert, Faustov, and Galkin] — $\sigma\left(1D\right)/\sigma\left(1S\right)$ is approximately 1.6 times higher,
[Martynenko(2019)] — $\sigma\left(1D\right)/\sigma\left(1S\right)$ is approximately 1.5 times higher.

Color octet (singlet) contributions to $B_c(D)$ yield



r dependence on p_T at different scales for forward kinematics: $2 < \eta < 4.5$, $p_T < 10$ GeV. The contributions of $|\bar{b}c(P,\mathbf{8})g\rangle$, $|\bar{b}c(S,\mathbf{8})gg\rangle$ and $|\bar{b}c(S,\mathbf{1})gg\rangle$ states are included.



r dependence on p_T at different scales for central kinematics: $|\eta| < 2.5$, 10 GeV $< p_T < 50$ GeV. The contributions of $|\bar{b}c(P,\mathbf{8})g\rangle$, $|\bar{b}c(S,\mathbf{8})gg\rangle$ and $|\bar{b}c(S,\mathbf{1})gg\rangle$ states are included.

$$K_{P\mathbf{8}} = \mathcal{O}(v_{\textsf{eff}}^2);\quad 0.1 < K_{S\mathbf{8},\mathbf{1}} < 0.2$$

$$K_{S\mathbf{8},\mathbf{1}} = \mathcal{O}(v_{\textsf{eff}}^4);\quad 0.015 < K_{S\mathbf{8},\mathbf{1}} < 0.03$$

- The hadronic relative yield $\sigma(1D)/\sigma(1S)$ is increased by an order of magnitude (three contributions, each of which is $\sim \sigma(1D)_{\textsf{direct}}$)
- The normalization for $|\bar{b}c(P,\mathbf{8})g\rangle$, $|\bar{b}c(S,\mathbf{8})gg\rangle$ and $|\bar{b}c(S,\mathbf{1})gg\rangle$ states is unknown. The naive normalization is only for estimations !

Conclusions

- The $B_c(2S)$ excitations have been observed at LHC in the $B_c\pi^+\pi^-$ spectrum.
- At very large statistics it would be possible to distinguish two peaks in the $B_c\pi^+\pi^-$ mass spectrum: one peak near 7000 MeV formed by $(1D_2,\ 1D_2')$ states and another one near 6930 MeV formed by $(1^3D_1,\ 1D_2,\ 1D_2',\ 1^3D_3)$ states decaying to $B_c^*\pi^+\pi^-$ with further radiative decay $B_c^* \xrightarrow{\gamma} B_c$.
- Also the D-wave B_c excitations could be found in cascade radiative decays $B_c(1D) \xrightarrow{\gamma} B_c(1P) \xrightarrow{\gamma} B_c(1S)$.
- Considering the main color singlet contribution we estimate $B_c(1D)$ states yield in hadronic production as $0.6 \div 1.8\%$ with respect to the direct production of $B_c(1S)$ (approximately $0.4 \div 1.1\%$ with respect to all produced B_c).
- Our estimations for D-wave states of B_c in hadronic production do not contradict the estimations [Cheung and Yuan(1996)] for e^+e^--annihilation.
- The significant experimental excess of the relative yield over the value $0.4 \div 1.1\%$ will indicate an essential contribution of the color octet states to the production.
- We propose to search for D-excitations in $B_c\pi^+\pi^-$ spectrum at LHC at large statics. However we have to conclude that it is quite a challenging experimental task.

Thank for your attention!

We appreciate our colleagues V.O. Galkin and A.P. Martynenko for help and useful discussions on the B_c meson wave functions.

Backup slides

What was expected for $B_c^{(*)}(2S) \to B_c^{(*)} + \pi\pi$

$$B_c(2S) \xrightarrow[\sim 50\%]{\pi^+\pi^-} B_c$$

$$B_c^*(2S) \xrightarrow[\sim 40\%]{\pi^+\pi^-} B_c^*$$

$$\sigma^{2S}/\sigma^{\text{total}} \sim 25\ \%$$

$\sim 10\ \%$ of B_c come from $B_c(2S) \to B_c(1S) + \pi^+\pi^-$

Under assumption that
$|R(B_c^*(2S))(0)| \approx |R(B_c(2S))(0)|$

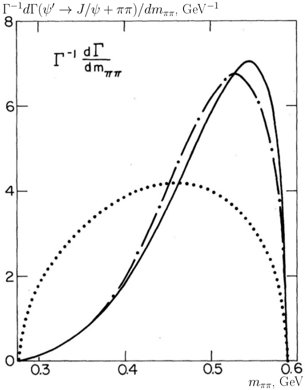
$$\sigma(B_c^*(2S))/\sigma(B_c(2S)) \sim 2.6$$

Relativistic corrections

$|R(B_c^*(2S))(0)|/|R(B_c(2S))(0)| = 0.87$
[Martynenko(2019)]
$|R(B_c^*(2S))(0)|/|R(B_c(2S))(0)| = 0.567$

[Galkin(2019),
Ebert et al.(2011)Ebert, Faustov, and Galkin]

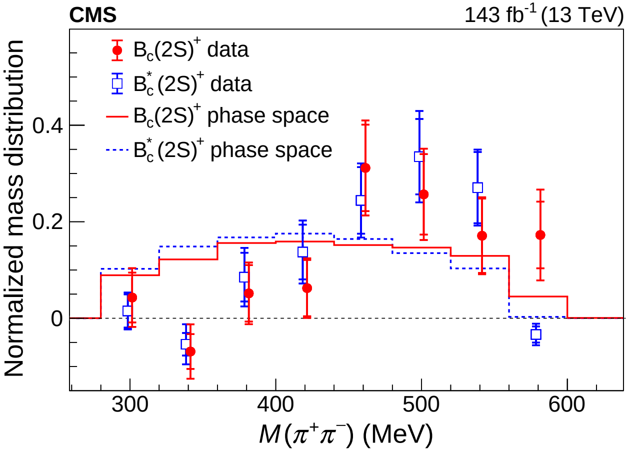
$$\sigma(B_c^*(2S))/\sigma(B_c(2S)) \sim 1 \div 2$$



$$\frac{1}{\Gamma}\frac{d\Gamma}{dm_{2\pi}} \sim \frac{|\mathbf{k}_{\pi\pi}|}{M^2}(2x^2 - 1)\sqrt{x^2 - 1}$$

where $x = m_{\pi\pi}/2m_\pi$ and $\mathbf{k}_{\pi\pi}$ is the momentum of $\pi\pi$-pair in the initial quarkonium rest frame.
[Brown and Cahn(1975), Novikov and Shifman(1981), Voloshin(1975), Voloshin and Zakharov(1980)]

CMS results for $B_c(2S)$: distribution over $M(\pi^+\pi^-)$



CMS [Sirunyan et al.(2019)]

Wave functions for B_c meson

Table: B_c meson wave functions within the quasipotential models.

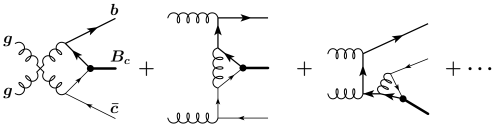
B_c-state	$\|R(0)\|^2$, $\|R''(0)\|^2$ [a]	$\|R(0)\|^2$, $\|R''(0)\|^2$ [b]
1^1S_0	2.68 GeV^3	0.97 GeV^3
1^3S_1	1.09 GeV^3	0.66 GeV^3
1^1D_2	0.078 GeV^7	0.055 GeV^3
1^3D_1	0.314 GeV^7	
1^3D_2	0.098 GeV^7	
1^3D_3	0.061 GeV^7	

[a] [Ebert et al.(2011)Ebert, Faustov, and Galkin, Galkin(2019)],
[b] [Berezhnoy et al.(2019)Berezhnoy, Martynenko, Martynenko, and Sukhorukova, Martynenko(2019)].

$$|R''_D(0)|^2{}_{\text{eff}} = (3|R''_{1^3D_1}(0)|^2 + 5|R''_{1^3D_2}(0)|^2 + 7|R''_{1^3D_3}(0)|^2 + 5|R''_{1^1D_2}(0)|^2)/20,$$

$$|R_S(0)|^2{}_{\text{eff}} = (|R''_{1^1S_0}(0)|^2 + 3|R_{1^3S_1}(0)|^2)/4.$$

D-wave B_c calculation in hadronic interaction



LO: 36 diagrams of the order of $\mathcal{O}(\alpha_S^4\, v^4)$

$$A \sim \int d^3q\, \Psi^*(\boldsymbol{q}) \left\{ T(p_i, \boldsymbol{q})\big|_{\boldsymbol{q}=0} + q^\alpha \frac{\partial}{\partial q^\alpha} T(p_i, \boldsymbol{q})\big|_{\boldsymbol{q}=0} + \frac{1}{2} q^\alpha q^\beta \frac{\partial^2}{\partial q^\alpha \partial q^\beta} T(p_i, \boldsymbol{q})\big|_{\boldsymbol{q}=0} + \cdots \right\}$$

$\boldsymbol{q}$ is quark three momentum in B_c-meson, $\Psi^*(\boldsymbol{q})$ is B_c-meson wave function, and T is the amplitude of four heavy quark gluonic production with momenta p_i. For D wave state an amplitude is proportional to $R''(0)$ and second derivatives of T over $\boldsymbol{q}$.

Spin singlet $(J = 2,\ j_z = l_z)$:

$$A^{j_z} = \frac{1}{2}\sqrt{\frac{15}{8\pi}} R_D''(0) \epsilon^{\alpha\beta}(j_z)\ \frac{\partial^2 M(\boldsymbol{q})}{\partial q^\alpha \partial q^\beta}\bigg|_{\boldsymbol{q}=0}.$$

Spin triplet $(J = 1, 2, 3;\ j_z = s_z + l_z)$:

$$A^{Jj_z} = \frac{1}{2}\sqrt{\frac{15}{8\pi}} R_D''(0) \Pi^{J,\ \alpha\beta\rho}(j_z)\ \frac{\partial^2 M_\rho(\boldsymbol{q})}{\partial q^\alpha \partial q^\beta}\bigg|_{\boldsymbol{q}=0}.$$

$$\Pi^{J,\ \alpha\beta\rho}(j_z) = \sum_{l_z, s_z} \epsilon^{\alpha\beta}(l_z) \epsilon^\rho(s_z) \cdot C^{Jj_z}_{s_z l_z},$$

where $C^{Jj_z}_{s_z l_z}$ are Clebsch-Gordan coefficients.

Amplitude derivatives

$$\mathcal{P}(0,0) = \frac{1}{\sqrt{2}}\{v_+(p_{\bar{b}}+k)\bar{u}_+(p_c-k) - v_-(p_{\bar{b}}+k)\bar{u}_-(p_c-k)\},$$

$$\mathcal{P}(1,S_z) = \begin{cases} \mathcal{P}(1,1) = v_-(p_{\bar{b}}+k)\bar{u}_+(p_c-k) \\ \mathcal{P}(1,0) = \frac{1}{\sqrt{2}}\{v_+(p_{\bar{b}}+k)\bar{u}_+(p_c-k) + v_-(p_{\bar{b}}+k)\bar{u}_-(p_c-k)\} \\ \mathcal{P}(1,-1) = v_+(p_{\bar{b}}+k)\bar{u}_-(p_c-k) \end{cases}$$

$$v_{\lambda_1}(p_{\bar{b}}+k) = (1-\frac{\widehat{k}}{2m_b})v_{\lambda_1}(p_{\bar{b}}),$$

$$\bar{u}_{\lambda_2}(p_c-k) = (1-\frac{\widehat{k}}{2m_c})\bar{u}_{\lambda_2}(p_c),$$

where $p_{\bar{b}} = \frac{m_b}{m_b+m_c}P_{B_c}$, $p_{\bar{c}} = \frac{m_c}{m_b+m_c}P_{B_c}$ and $k(\vec{q})$ ia a Lorentz boost of $(0,\vec{q})$ to the system where B_c momentum is P_{B_c}.

Amplitudes and there derivatives have been calculated numerically as follows:

$$\frac{\partial^2 M}{\partial q_x^2} = \frac{M(p_i,\vec{q}_x) + M(p_i,-\vec{q}_x) - 2M(p_i,0)}{\Delta_x^2}$$

$$\frac{\partial^2 M}{\partial q_x \partial q_y} = \frac{M(p_i,\vec{q}_x+\vec{q}_y) + M(p_i,0) - M(p_i,\vec{q}_x) - M(p_i,\vec{q}_y)}{\Delta_x \Delta_y}$$

Amplitude squared

Matrix elements are summed over J and j_z: $\mathbb{P} = \sum\limits_{j_z\, j'_z} A^{j_z} A^{*\, j'_z}$ and $\mathbb{V} = \sum\limits_{J\, J'} \sum\limits_{j_z\, j'_z} \mathcal{A}^{J j_z} \mathcal{A}^{*\, J' j'_z}$.

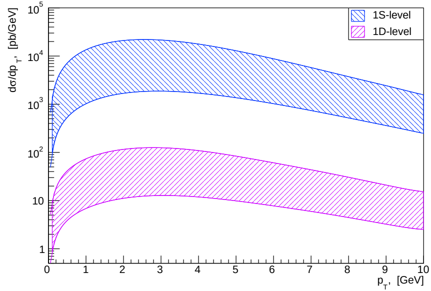
- The amplitude squared for the spin-singlet state with $S = 0$ (1^1D_2):

$$\mathbb{P} = \left(\frac{5}{16\pi}\right) |R''_D(0)|^2 \times$$
$$\left[\left(\left|\frac{\partial^2 M}{\partial k_x^2}\right|^2 + \left|\frac{\partial^2 M}{\partial k_y^2}\right|^2 + \left|\frac{\partial^2 M}{\partial k_z^2}\right|^2\right) + 3\left(\left|\frac{\partial^2 M}{\partial k_x \partial k_y}\right|^2 + \left|\frac{\partial^2 M}{\partial k_x \partial k_z}\right|^2 + \left|\frac{\partial^2 M}{\partial k_y \partial k_z}\right|^2\right)\right.$$
$$\left. - \operatorname{Re}\left(\frac{\partial^2 M}{\partial k_x^2}\frac{\partial^2 M^*}{\partial k_y^2} + \frac{\partial^2 M}{\partial k_x^2}\frac{\partial^2 M^*}{\partial k_z^2} + \frac{\partial^2 M}{\partial k_y^2}\frac{\partial^2 M^*}{\partial k_z^2}\right)\right].$$

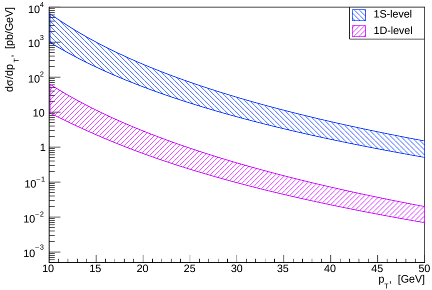
- The sum of amplitudes squared for the spin-triplet states with $S = 1$ (1^3D_1, 1^3D_2, 1^3D_3):

$$\mathbb{V} = |\mathcal{A}(J=1)|^2 + |\mathcal{A}(J=2)|^2 + |\mathcal{A}(J=3)|^2 = \left(\frac{5}{16\pi}\right) |R''_D(0)|^2 \times$$
$$\sum_{s_z}^{-1,0,1}\left[\left(\left|\frac{\partial^2 \mathcal{M}_{s_z}}{\partial k_x^2}\right|^2 + \left|\frac{\partial^2 \mathcal{M}_{s_z}}{\partial k_y^2}\right|^2 + \left|\frac{\partial^2 \mathcal{M}_{s_z}}{\partial k_z^2}\right|^2\right) + 3\left(\left|\frac{\partial^2 \mathcal{M}_{s_z}}{\partial k_x \partial k_y}\right|^2 + \left|\frac{\partial^2 \mathcal{M}_{s_z}}{\partial k_x \partial k_z}\right|^2 + \left|\frac{\partial^2 \mathcal{M}_{s_z}}{\partial k_y \partial k_z}\right|^2\right)\right.$$
$$\left. - \operatorname{Re}\left(\frac{\partial^2 \mathcal{M}_{s_z}}{\partial k_x^2}\frac{\partial^2 \mathcal{M}^*_{s_z}}{\partial k_y^2} + \frac{\partial^2 \mathcal{M}_{s_z}}{\partial k_x^2}\frac{\partial^2 \mathcal{M}^*_{s_z}}{\partial k_z^2} + \frac{\partial^2 \mathcal{M}_{s_z}}{\partial k_y^2}\frac{\partial^2 \mathcal{M}^*_{s_z}}{\partial k_z^2}\right)\right].$$

$pp \to B_c(D) + X$: cross sections



$\sigma\,(pp \to B_c + X)$ dependence on p_T at diff. scales for frontal kinematics: $2 < \eta < 4.5,\ p_T < 10$ GeV.



$\sigma\,(pp \to B_c + X)$ dependence on p_T at diff. scales for central kinematics: $|\eta| < 2.5,\ 10$ GeV $< p_T < 50$ GeV

- Convolution with CT14 PDFs by CTEQ group
  [Dulat et al.(2016)Dulat, Hou, Gao, Guzzi, Huston, Nadolsky, Pumplin, Schmidt, Stump, and Yuan]

$$\sigma_{pp} = \int \sigma_{gg}(\hat{s}_{gg}, \mu) f_{g1}(x_1, \mu) f_{g2}(x_2, \mu) dx_1 dx_2, \qquad E_T/2 < \mu < 2E_T$$

- RAMBO algorithm for Monte-Carlo integraton [Kleiss et al.(1986)Kleiss, Stirling, and Ellis]

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