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Hyperon electromagnetic form factors in VMD model

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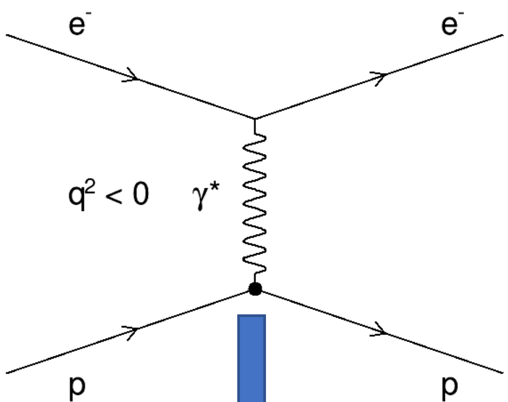
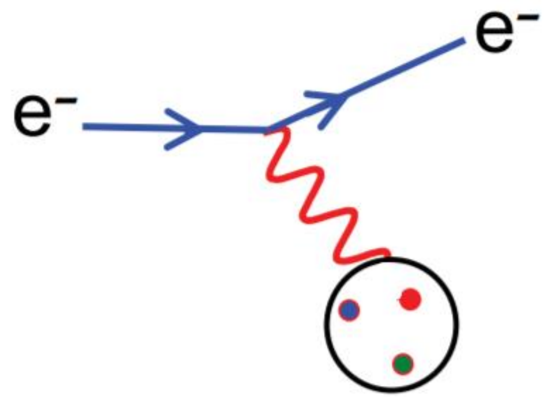
HADRON 2023

Outline

- Introduction： electromagnetic form factors
- The model: Vector Meson Dominance
- Hyperon electromagnetic form factors
- Summary

Electromagnetic form factors (space-like)





$$\langle p_f | \hat{J}^{\mu}(0) | p_i \rangle = \overline{u}(p_f) \left[F_1(q^2)\gamma^{\mu} - F_2(q^2)\frac{\sigma^{\mu\nu}q_{\nu}}{2M} \right] u(p_i)$$

$$\Gamma^{\mu}(q^2) = \gamma^{\mu} F_1^p(q^2) + i\frac{F_2^p(q^2)}{2M_p}\sigma^{\mu\nu}q_{\nu}$$

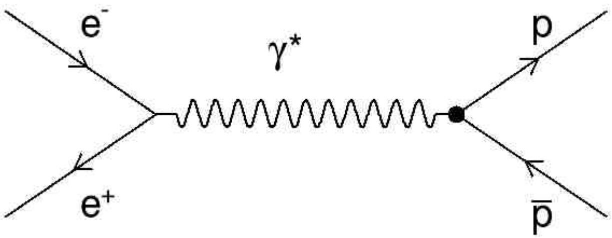
F_1^N : Dirac form factor
F_2^N : Pauli form factor

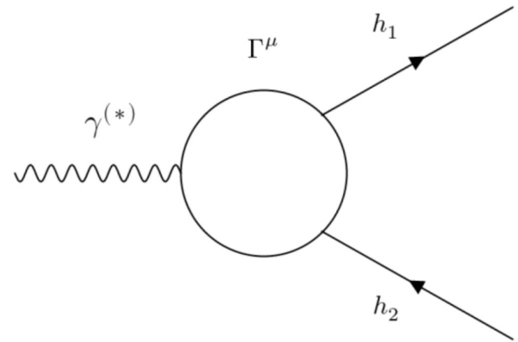
$$G_E^N(Q^2) = F_1^N(Q^2) - \tau F_2^N(Q^2) \ , \ G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2) \ , \ \tau = \frac{Q^2}{4M_N^2}$$

$$F_1^p(0) = 1 \, , F_1^n(0) = 0 \, , \ F_2^p(0) = \kappa_p \, , F_2^n(0) = \kappa_n$$

S. Pacetti, R. Baldini Ferroli and E. Tomasi-Gustafsson, ``Proton electromagnetic form factors: Basic notions, present achievements and future perspectives," **Phys. Rept. 550-551, 1-103 (2015).**

Electromagnetic form factors (time-like)

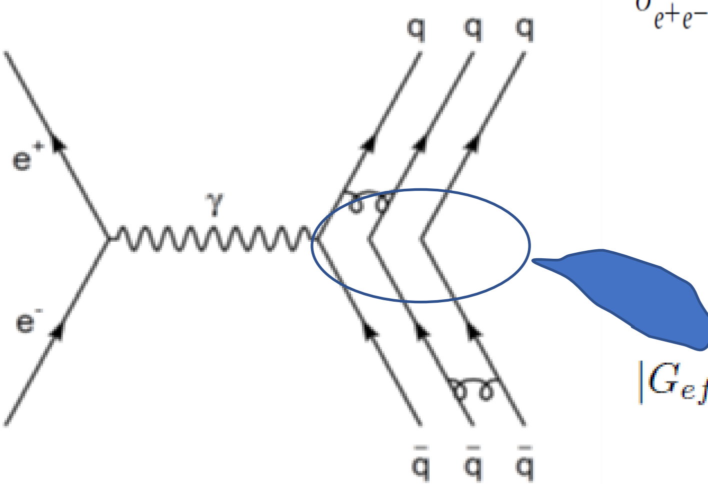




$$\left(\frac{d\sigma}{d\Omega}\right)^{th}_{e^+e^-\to N\bar{N}} = \frac{\alpha^2\beta}{4q^2} C_N(q^2)\left\{|G_M^N(q^2)|^2(1+\cos^2\theta)+|G_E^N(q^2)|^2\frac{1}{\tau}\sin^2\theta\right\}$$

$$\sigma^{th}_{e^+e^-\to N\bar{N}} = \frac{\alpha^2\beta}{4q^2} C_N(q^2)\int d\Omega\left[|G_M^N(q^2)|^2(1+\cos^2\theta)+|G_E^N(q^2)|^2\frac{\sin^2\theta}{\tau}\right]$$

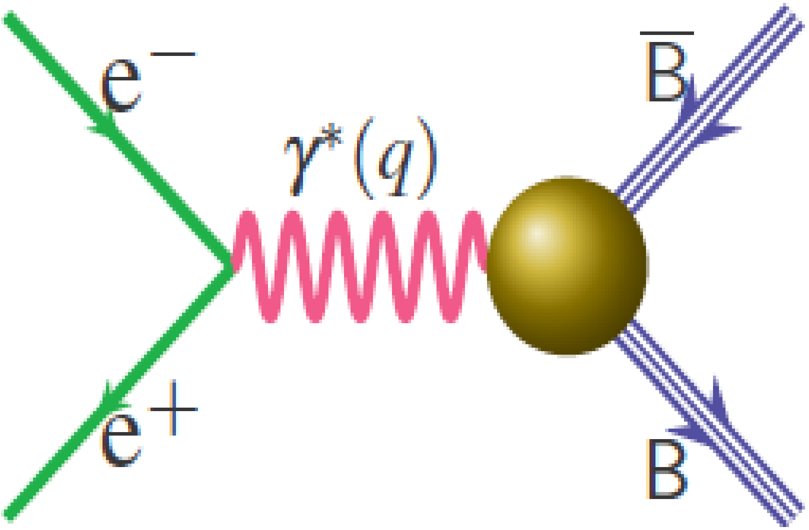
$$= \frac{4\pi\alpha^2\beta}{3q^2} C_N(q^2)\left[|G_M^N(q^2)|^2+\frac{|G_E^N(q^2)|^2}{2\tau}\right].$$



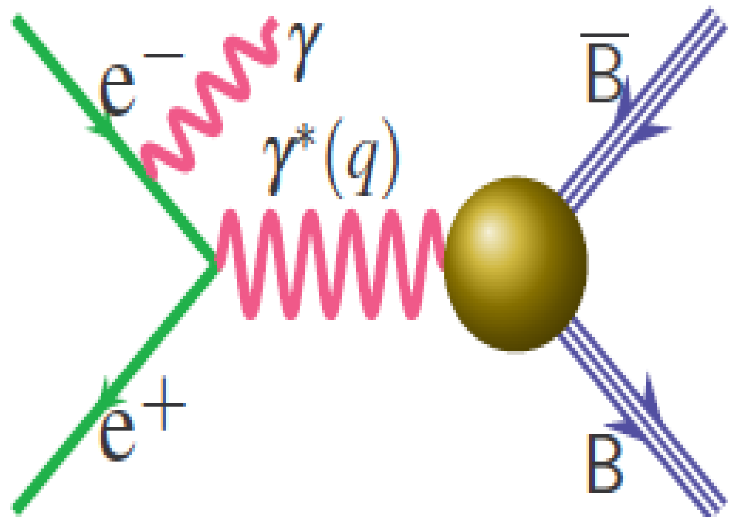
$$|G_{eff}(q^2)| = \sqrt{\frac{\sigma(q^2)}{\sigma_{point}(q^2)}} = \sqrt{\frac{|G_M(s)|^2+\frac{2M^2}{s}|G_E(s)|^2}{1+\frac{2M^2}{s}}}$$

Experimental measurements (time-like)

See more details @
Plenary talk on June 6,
By Xiao-Rong Zhou



Energy Scan



Initial State Radiation

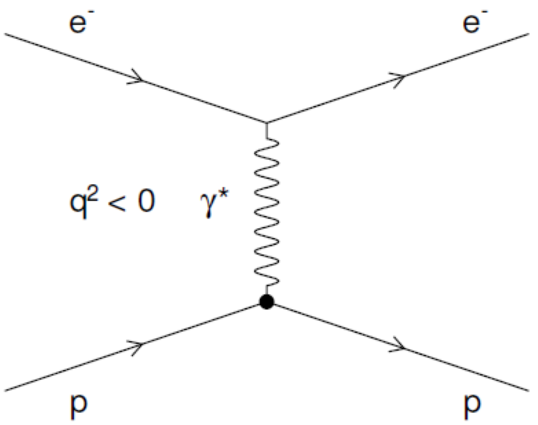
Both techniques can be used at BESIII.

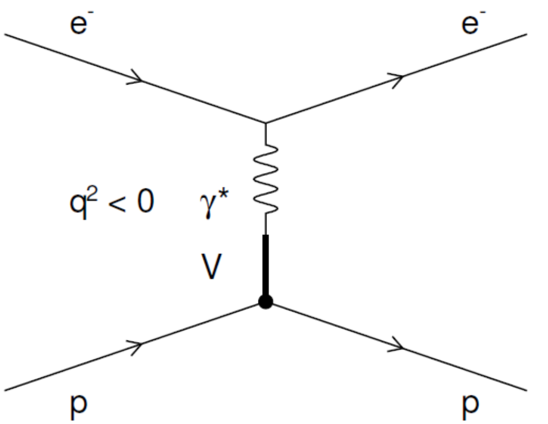
See also the talk on June 7:

Recent results of baryon electromagnetic form factors at BESIII

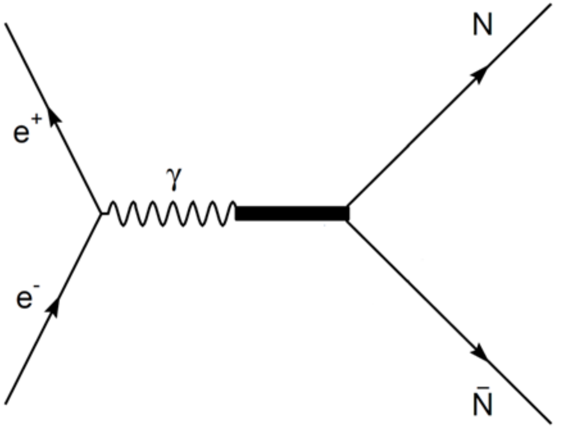
Weiping Wang (On behalf of the BESIII Collaboration)

VMD: vector meson dominance model



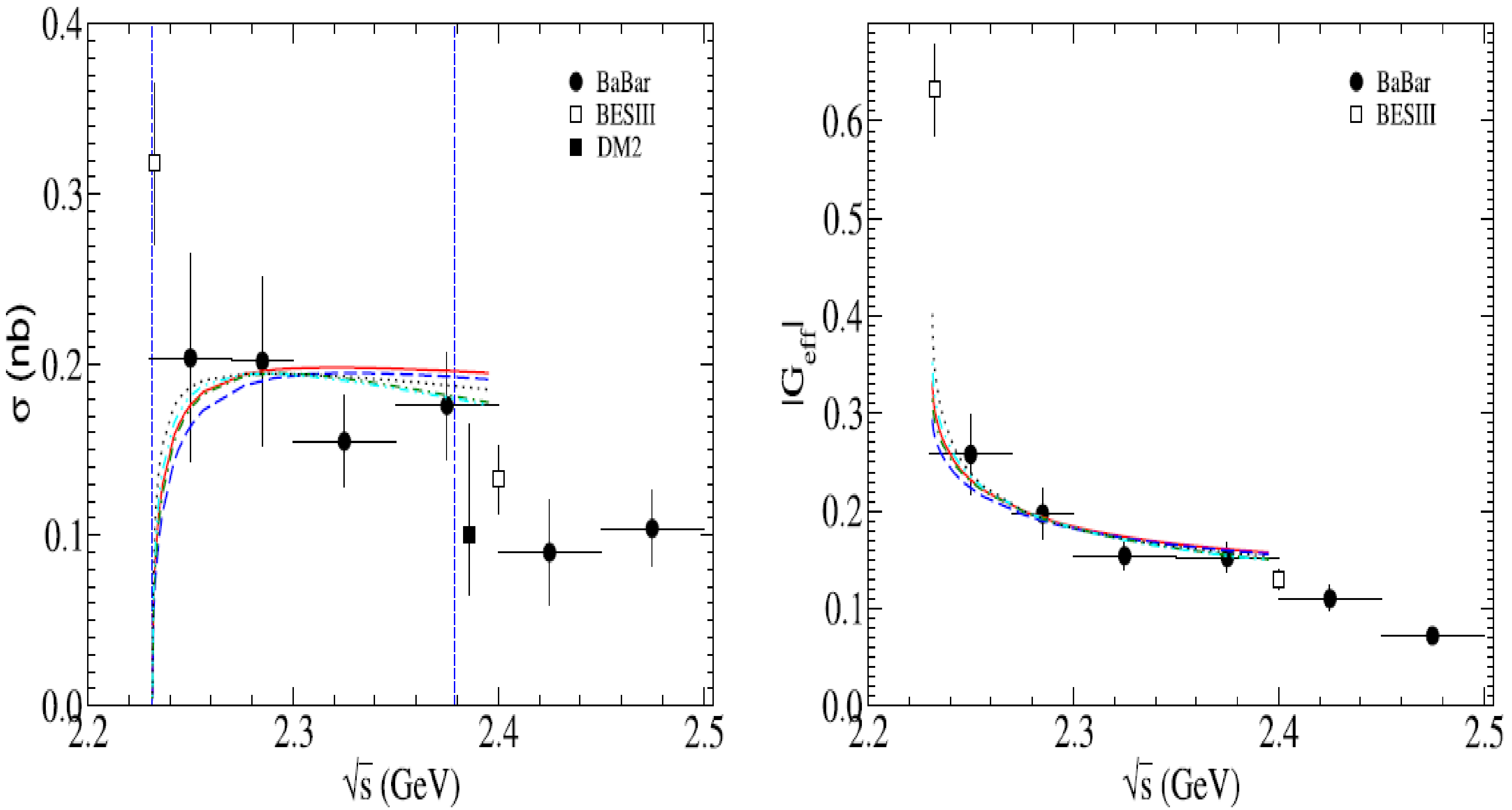


$$\mathcal{L}_{V\gamma} = \sum_V \frac{eM_V^2}{f_V} V_\mu A^\mu$$



$$L_{\mathrm{NNV}} = g\bar{\psi}\gamma_\mu\psi\varphi_V^\mu + \frac{\kappa}{4m}\bar{\psi}\sigma_{\mu\nu}\psi\left(\partial^\mu\varphi_V^\nu - \partial^\nu\varphi_V^\mu\right)$$

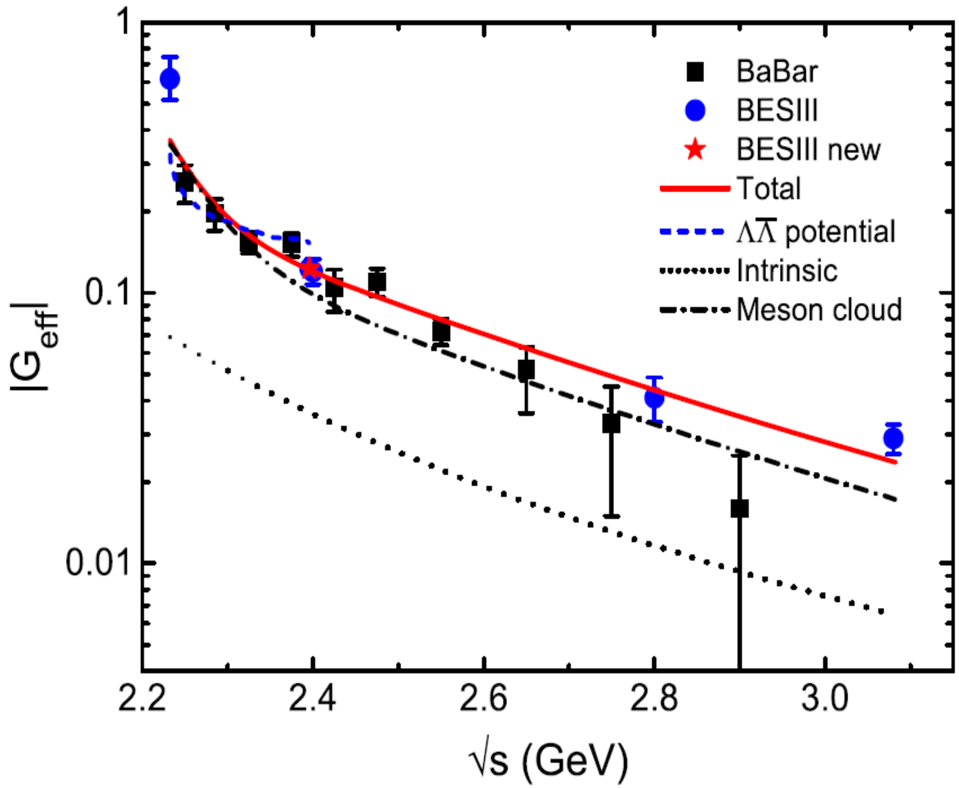
Λ EMFFs in final state interactions



J. Haidenbauer and U. G. Meißner, Phys. Lett. B 761, 456-461(2016).

Λ EMFFs in VMD

Y. Yang, D. Y. Chen and Z. Lu,
Phys. Rev. D 100, 073007 (2019).

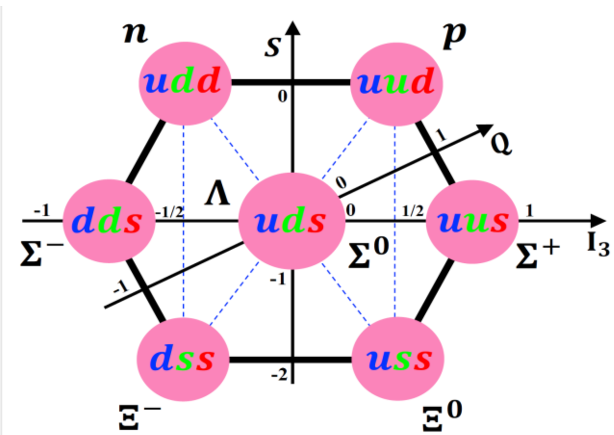


fit. In the present scenario, there are 16 experimental data and 10 free parameters. The value of intrinsic parameter γ is fitted to be 0.336 GeV^{-2} and the other parameters are summarized in Table II. It should be noticed that $g(q^2)$

$$g(q^2) = \frac{1}{(1-\gamma q^2)^2}$$

$$\gamma_N = \frac{1}{0.71\ \mathrm{GeV}^2} = 1.408\ \mathrm{GeV}^{-2}$$

State	Mass	Width	State	Mass	Width
$\omega(782)$ [55]	782	8.1	$\phi(1020)$ [56]	1019	4.2
$\omega(1420)$ [57]	1418	104	$\phi(1680)$ [57]	1674	165
$\omega(1650)$ [57]	1679	121	$\phi(2170)$ [58]	2171	128



Λ EMFFs in VMD (New proposal)

$$
\begin{aligned}
\mathrm{F}_1(\mathrm{Q}^2) &= \mathrm{g}(\mathrm{Q}^2)[-\beta_\omega - \beta_\phi + \beta_\omega \frac{\mathrm{m}_\omega^2}{\mathrm{m}_\omega^2 + \mathrm{Q}^2} + \beta_\phi \frac{\mathrm{m}_\phi^2}{\mathrm{m}_\phi^2 + \mathrm{Q}^2} + \beta_\mathrm{x} \frac{\mathrm{m}_\mathrm{x}^2}{\mathrm{m}_\mathrm{x}^2 + \mathrm{Q}^2}] \\
\mathrm{F}_2(\mathrm{Q}^2) &= \mathrm{g}(\mathrm{Q}^2)[(\mu_\Lambda - \alpha_\phi) \frac{\mathrm{m}_\omega^2}{\mathrm{m}_\omega^2 + \mathrm{Q}^2} + \alpha_\phi \frac{\mathrm{m}_\phi^2}{\mathrm{m}_\phi^2 + \mathrm{Q}^2} + \alpha_\mathrm{x} \frac{\mathrm{m}_\mathrm{x}^2}{\mathrm{m}_\mathrm{x}^2 + \mathrm{Q}^2}] \\
\mathrm{g}(\mathrm{Q}^2) &= 1/(1 + \gamma \mathrm{Q}^2)^2
\end{aligned}
$$

$$\boxed{Q^2 \to -q^2}$$

$$
\begin{aligned}
\frac{m_\omega^2}{m_\omega^2 + Q^2} &\to \frac{m_\omega^2}{m_\omega^2 - q^2 - i m_\omega \Gamma_\omega} \\
\frac{m_\phi^2}{m_\phi^2 + Q^2} &\to \frac{m_\phi^2}{m_\phi^2 - q^2 - i m_\phi \Gamma_\phi} \\
\frac{m_x^2}{m_x^2 + Q^2} &\to \frac{m_x^2}{m_x^2 - q^2 - i m_x \Gamma_x},
\end{aligned}
$$

$$
\begin{aligned}
G_E(q^2) &= F_1(q^2) + \tau F_2(q^2) \\
G_M(q^2) &= F_1(q^2) + F_2(q^2)
\end{aligned}
$$

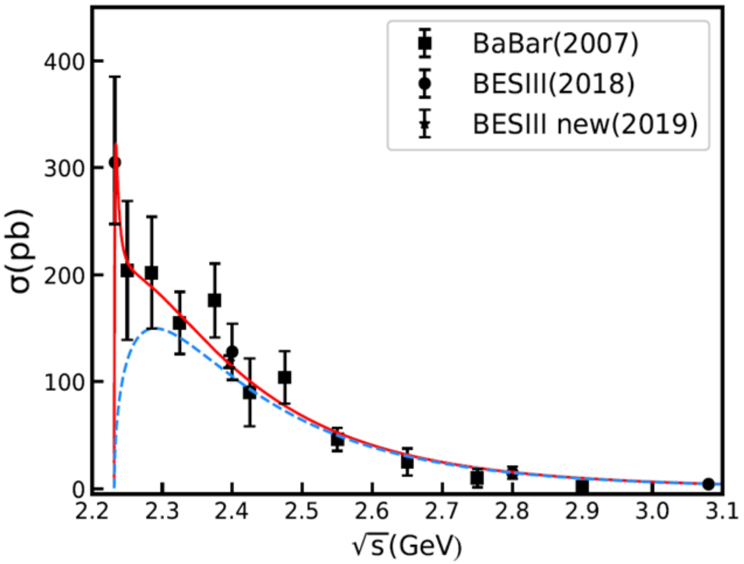
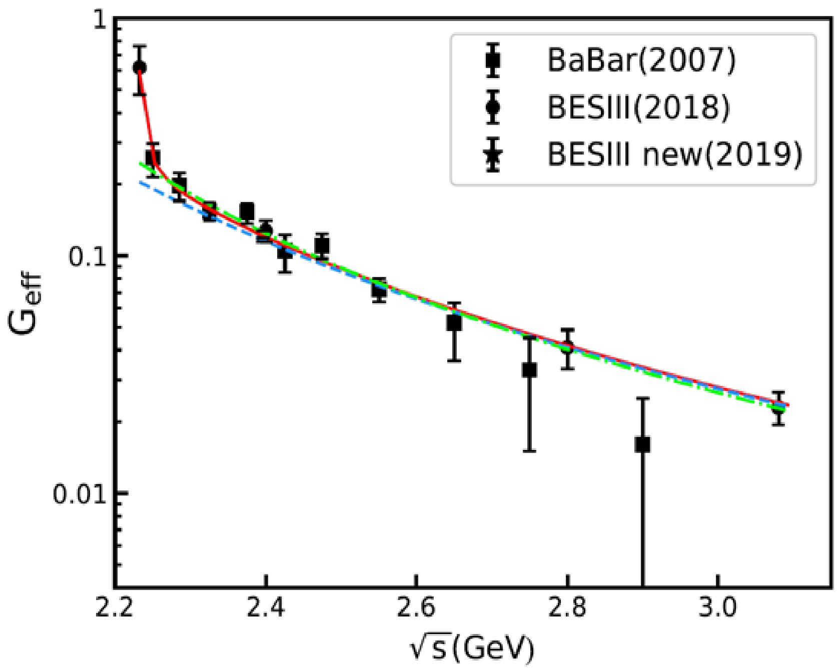


Figure: Cross section of the reaction $e^+e^- \to \bar{\Lambda}\Lambda$.

Z. Y. Li, A. X. Dai and J. J. Xie,
Chin. Phys. Lett. 39, 011201 (2022).



The red solid curve represents the total contributions from ω, ϕ and $X(2231)$, while the blue dashed curve stands for the results without the contribution from the new $X(2231)$ state. The green-dash-dotted curve stands for the fitted results with the effective form factor as in .

$$\boxed{G_{\text{eff}} = C_0 g(q^2) = \frac{C_0}{(1-\gamma q^2)^2}}$$

Table: Values of model parameters determined in this work.

Parameter	Value	Parameter	Value
γ (GeV^{-2})	0.43	β_ω	-1.13
β_ϕ	1.35	α_ϕ	-0.40
β_x	0.0015	m_x (MeV)	2230.9
Γ_x (MeV)	4.7		

New state X(2231) ?

Z. Y. Li, A. X. Dai and J. J. Xie, Chin. Phys. Lett. 39, 011201 (2022).

Flatte function

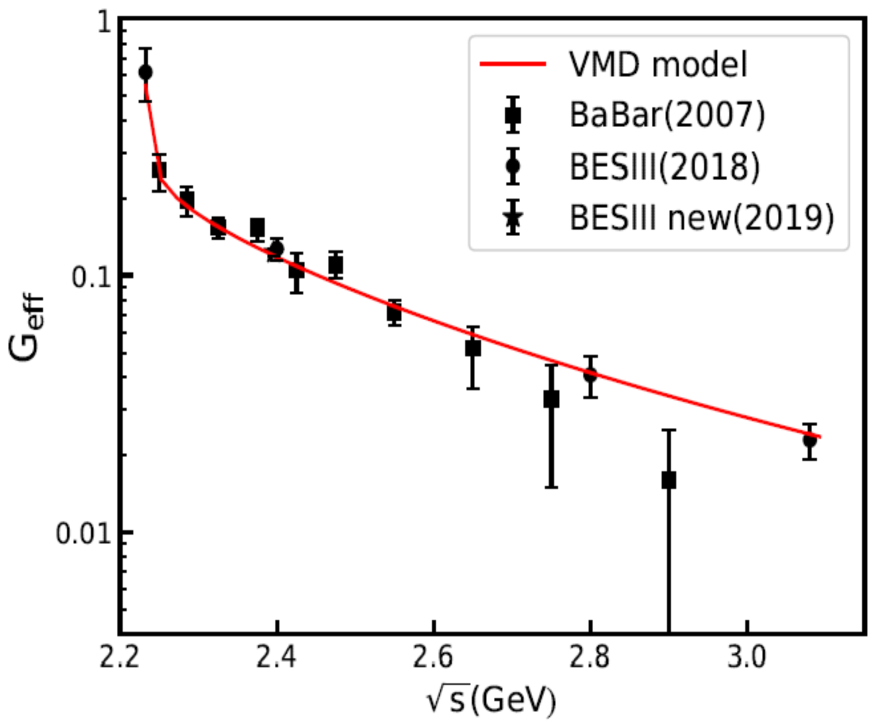


Figure: Fitting result of $|G_{eff}|$ with Flatte.

S.M. Flatte, Phys. Lett. B 63, 224-227 (1976).

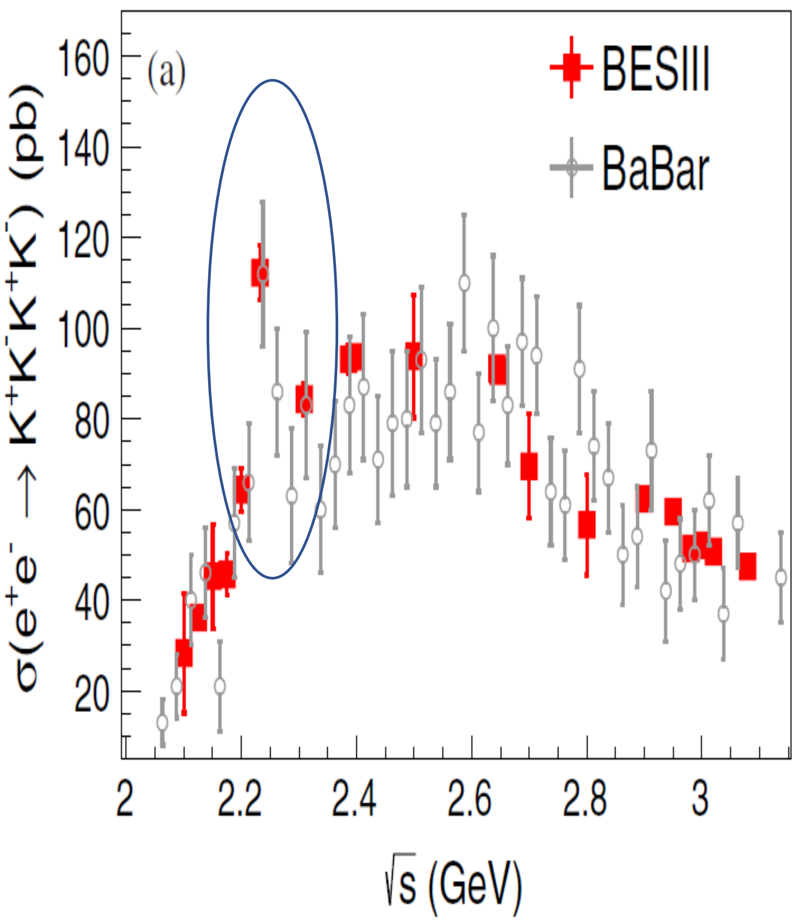
On the other hand, if one takes a Flatté form for the total decay width of $\omega(1420)$, $\omega(1650)$, $\phi(1680)$, and $\phi(2170)$, the experimental data can also be well reproduced with a strong coupling of these resonances to the $\Lambda\bar{\Lambda}$ channel.

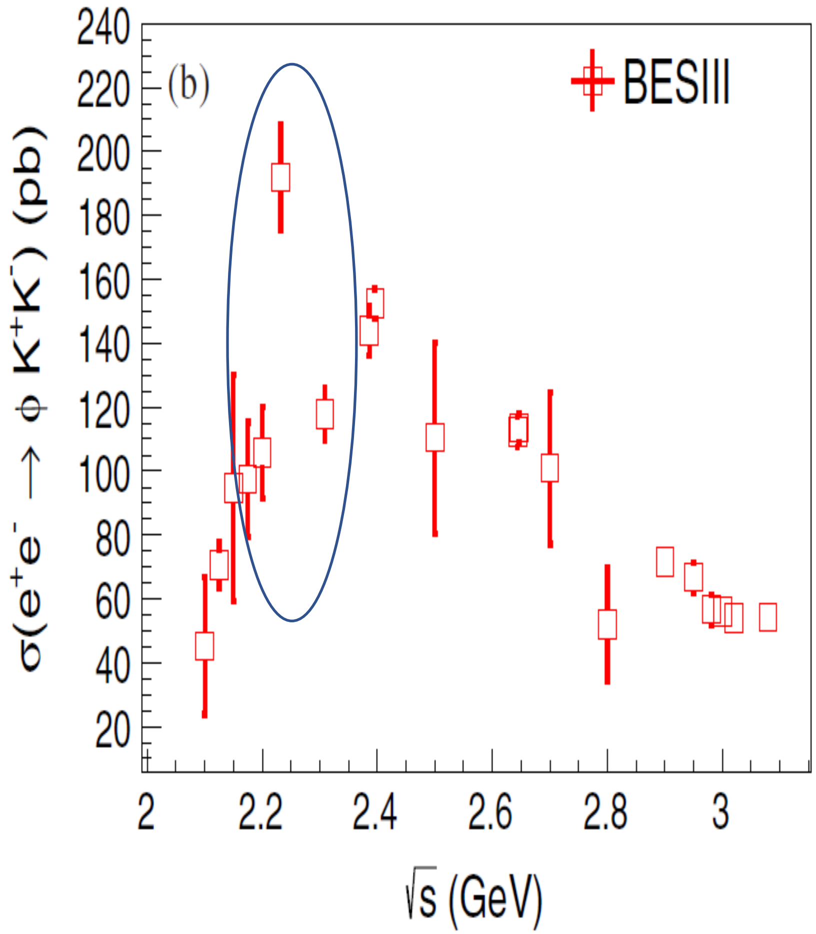
$$\Gamma_x = \Gamma_0 + \Gamma_{\Lambda\bar{\Lambda}}(s) \qquad \Gamma_{\Lambda\bar{\Lambda}} = \frac{g^2}{4\pi}\sqrt{\frac{s}{4} - M_\Lambda^2}$$

Parameter	Value	Parameter	Value
γ (GeV^{-2})	0.57 ± 0.21	$\beta_{\omega\phi}$	-0.3 ± 0.31
β_x	-0.03 ± 0.09	m_x (MeV)	2237.7 ± 50.2
Γ_0 (MeV)	$8.8^{+75.9}_{-8.8}$	$g_{\Lambda\bar{\Lambda}}$	3.0 ± 1.9

Z. Y. Li, A. X. Dai and J. J. Xie, Chin. Phys. Lett. 39, 011201 (2022).

Where is X(2231)?

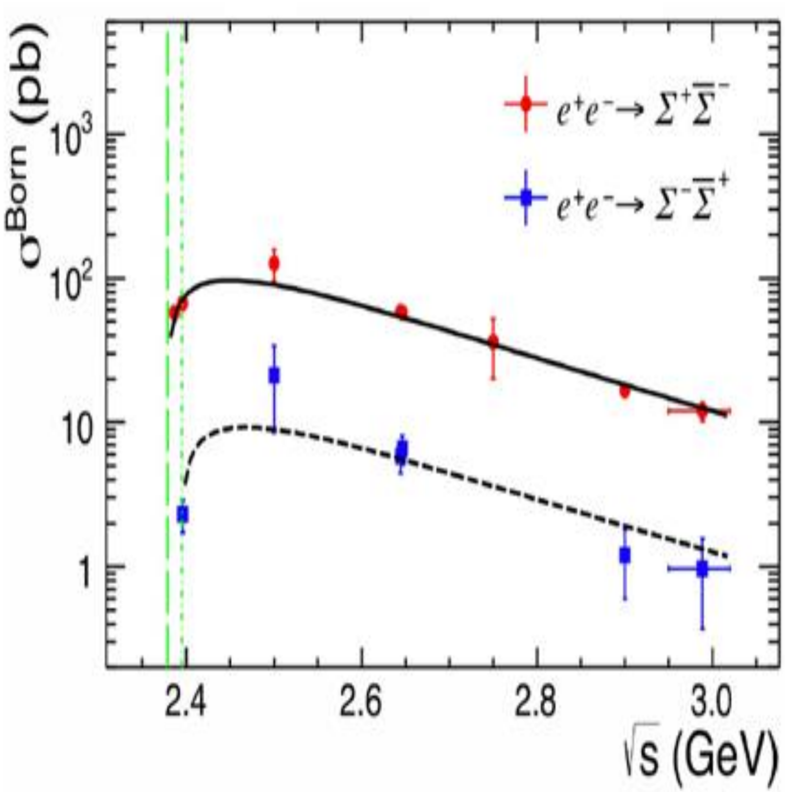


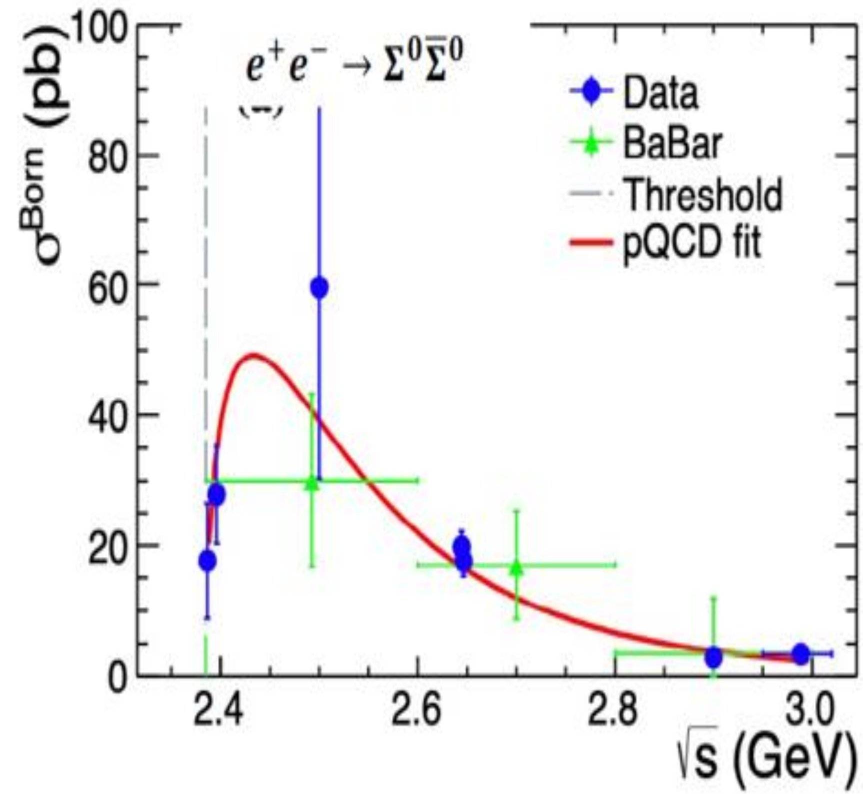


M. Ablikim, et al., Phys. Rev. D 100, 032009(2019).

Σ EMFFs

$$\sigma(s) = \frac{4\pi\alpha^2\beta}{3s}C(s)\left[1+\frac{2M^2}{s}\right]|G_{eff}(s)|^2 = \sigma_{point}(s)|G_{eff}(s)|^2.$$





BESIII, **Phys. Lett. B 814, 136110 (2021)； Phys. Lett. B 831, 137187 (2022).**

The ratio $\Sigma^+\bar{\Sigma}^-$: $\Sigma^0\bar{\Sigma}^0$: $\Sigma^-\bar{\Sigma}^+$ is about 9.7 ± 1.3 : 3.3 ± 0.7 : 1.

Σ^+, Σ^-, and Σ^0 EMFFs （VMD）

$$|\Sigma^+\bar{\Sigma}^-\rangle = \frac{1}{\sqrt{2}}|1,0\rangle + \frac{1}{\sqrt{3}}|0,0\rangle + \frac{1}{\sqrt{6}}|2,0\rangle$$

$$|\Sigma^-\bar{\Sigma}^+\rangle = -\frac{1}{\sqrt{2}}|1,0\rangle + \frac{1}{\sqrt{3}}|0,0\rangle + \frac{1}{\sqrt{6}}|2,0\rangle$$

$$|\Sigma^0\bar{\Sigma}^0\rangle = -\frac{1}{\sqrt{3}}|0,0\rangle + \sqrt{\frac{2}{3}}|2,0\rangle$$

Isospin decomposition

$$F_1^{\Sigma^+} = g(q^2)(f_1^{\Sigma^+} + \frac{\beta_\rho}{\sqrt{2}}B_\rho - \frac{\beta_{\omega\phi}}{\sqrt{3}}B_{\omega\phi}),$$

$$F_2^{\Sigma^+} = g(q^2)(f_2^{\Sigma^+}B_\rho - \frac{\alpha_{\omega\phi}}{\sqrt{3}}B_{\omega\phi}),$$

$$F_1^{\Sigma^-} = g(q^2)(f_1^{\Sigma^-} - \frac{\beta_\rho}{\sqrt{2}}B_\rho - \frac{\beta_{\omega\phi}}{\sqrt{3}}B_{\omega\phi}),$$

$$F_2^{\Sigma^-} = g(q^2)(f_2^{\Sigma^-}B_\rho - \frac{\alpha_{\omega\phi}}{\sqrt{3}}B_{\omega\phi}),$$

$$F_1^{\Sigma^0} = g(q^2)(\frac{\beta_{\omega\phi}}{\sqrt{3}} - \frac{\beta_{\omega\phi}}{\sqrt{3}}B_{\omega\phi}),$$

$$F_2^{\Sigma^0} = g(q^2)\mu_{\Sigma^0}B_{\omega\phi},$$

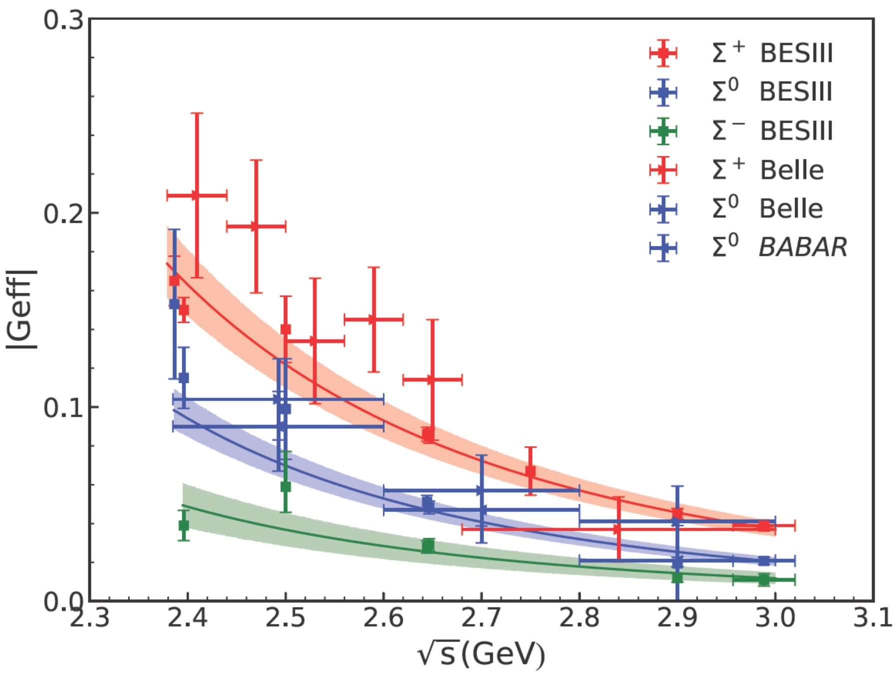
$$B_\rho = \frac{m_\rho^2}{m_\rho^2 - q^2 - im_\rho\Gamma_\rho},$$

$$B_{\omega\phi} = \frac{m_{\omega\phi}^2}{m_{\omega\phi}^2 - q^2 - im_{\omega\phi}\Gamma_{\omega\phi}},$$

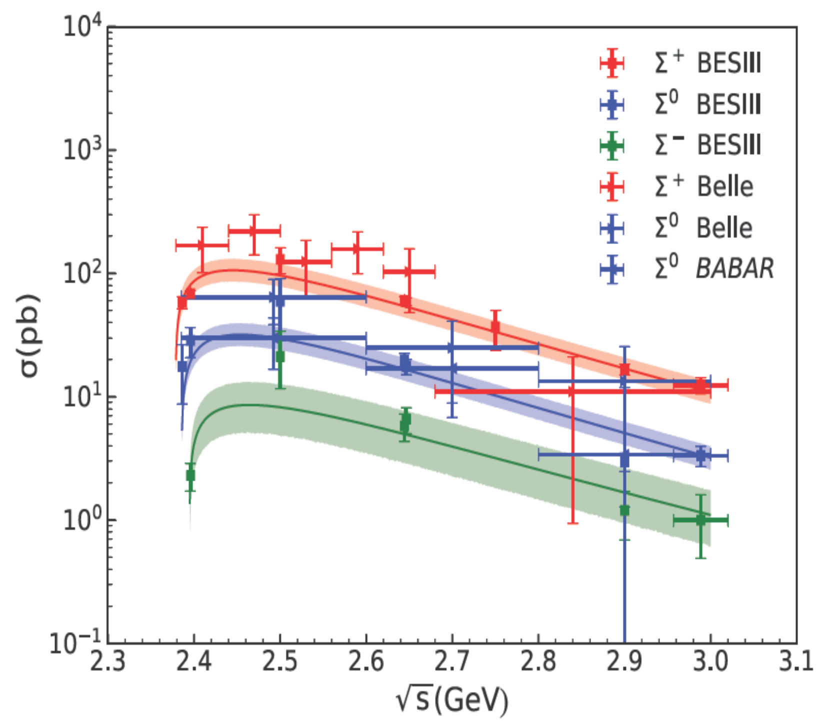
$$f_1^{\Sigma^+} = 1 - \frac{\beta_\rho}{\sqrt{2}} + \frac{\beta_{\omega\phi}}{\sqrt{3}}, \quad f_2^{\Sigma^+} = 2.112 + \frac{\alpha_{\omega\phi}}{\sqrt{3}},$$

$$f_1^{\Sigma^-} = -1 + \frac{\beta_\rho}{\sqrt{2}} + \frac{\beta_{\omega\phi}}{\sqrt{3}}, \quad f_2^{\Sigma^-} = -0.479 + \frac{\alpha_{\omega\phi}}{\sqrt{3}}$$

Σ^+, Σ^-, and Σ^0 EMFFs （VMD）



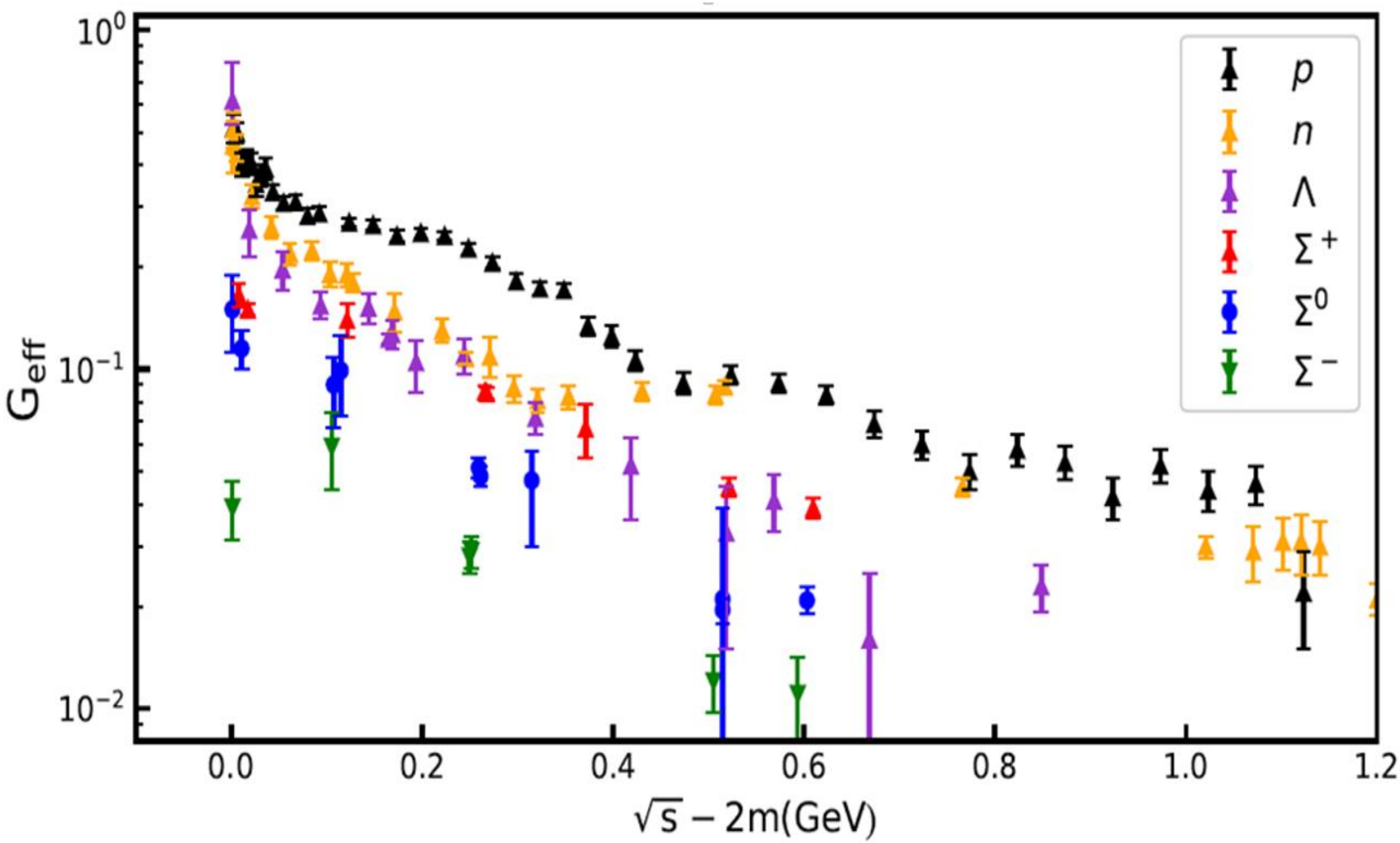
Parameter	Value	Parameter	Value
γ (GeV^{-2})	0.527 ± 0.024	$\alpha_{\omega\phi}$	-3.18 ± 0.77
$\beta_{\omega\phi}$	-0.08 ± 0.06	β_ρ	1.63 ± 0.07



With one γ, we can describe all the current experimental data on Σ^+, Σ^-, and Σ^0 EMFFs .

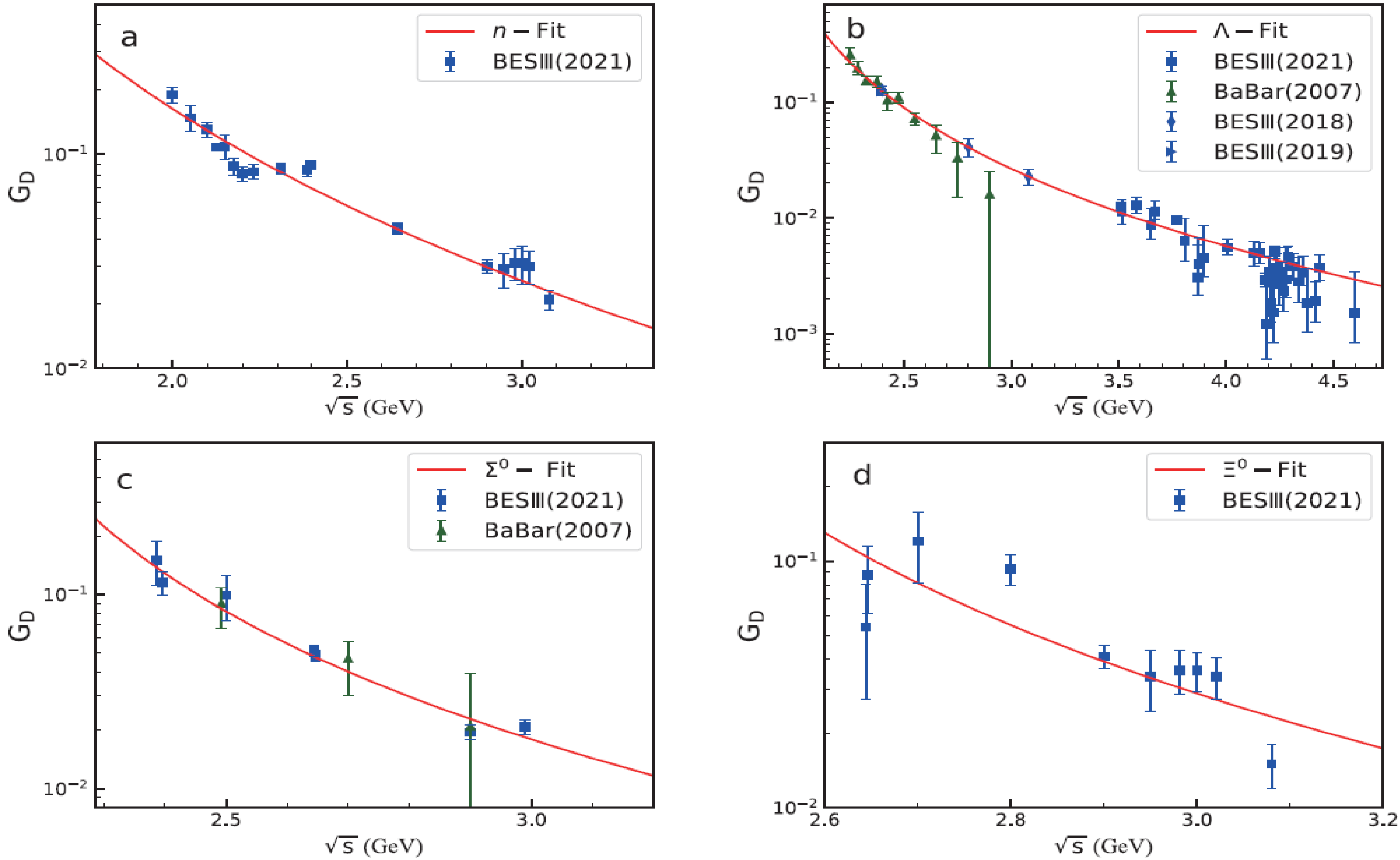
Bing Yan, Cheng Chen, and J. J. Xie, **Phys. Rev. D107, 076008 (2023).**

Dipole behavior of baryon effective form factors



$$G_D(q^2) = \frac{c_0}{(1 - \gamma q^2)^2}$$

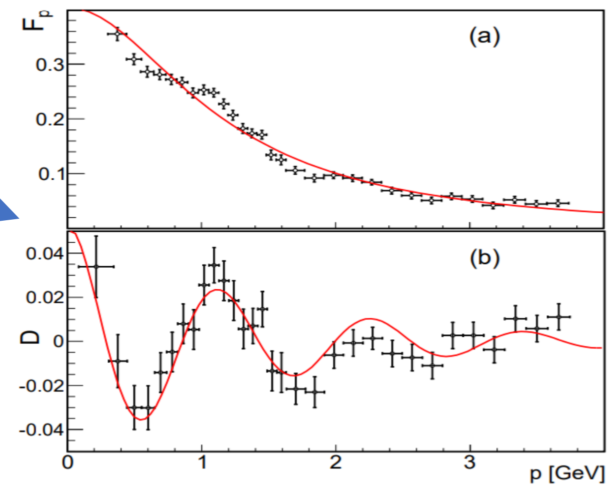
Parameter	n	Λ	Σ^0	Ξ^0
γ	1.41 (fixed)	0.34 ± 0.08	0.26 ± 0.01	0.21 ± 0.02
c_0	3.48 ± 0.06	0.11 ± 0.01	0.033 ± 0.007	0.023 ± 0.008
χ^2/dof	4.3	2.4	1.1	3.0



Oscillation of baryon effective form factors

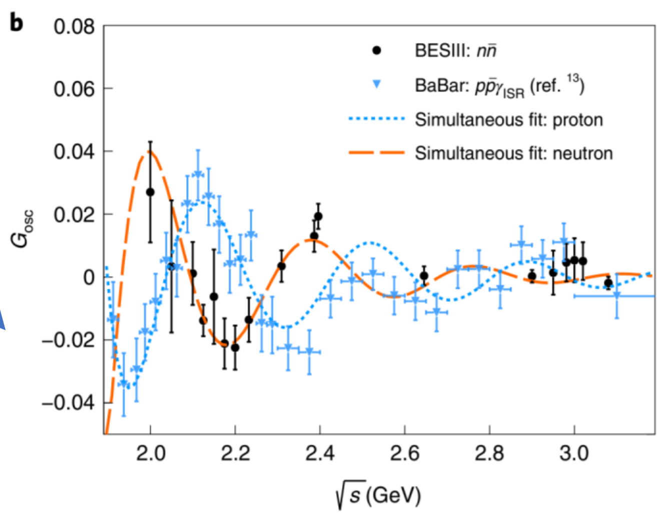
2015, Andrea Bianconi et al., Phys. Rev. Lett., 2015, 114(23): 232301.

$$F_{3p}(s) = \frac{F_0}{\left(1+\frac{s}{m_a^2}\right)\left(1-\frac{s}{m_0^2}\right)^2},$$
$$F_{osc}(p(s)) = Ae^{-Bp}\cos(Cp+D).$$



2021, BESIII Collaboration,Nature Phys., 2021, 17(11): 1200-1204.

$$F_{osc}^{n,p} = A^{n,p}\exp\left(-B^{n,p}p\right)\cos\left(Cp+D^{n,p}\right)$$



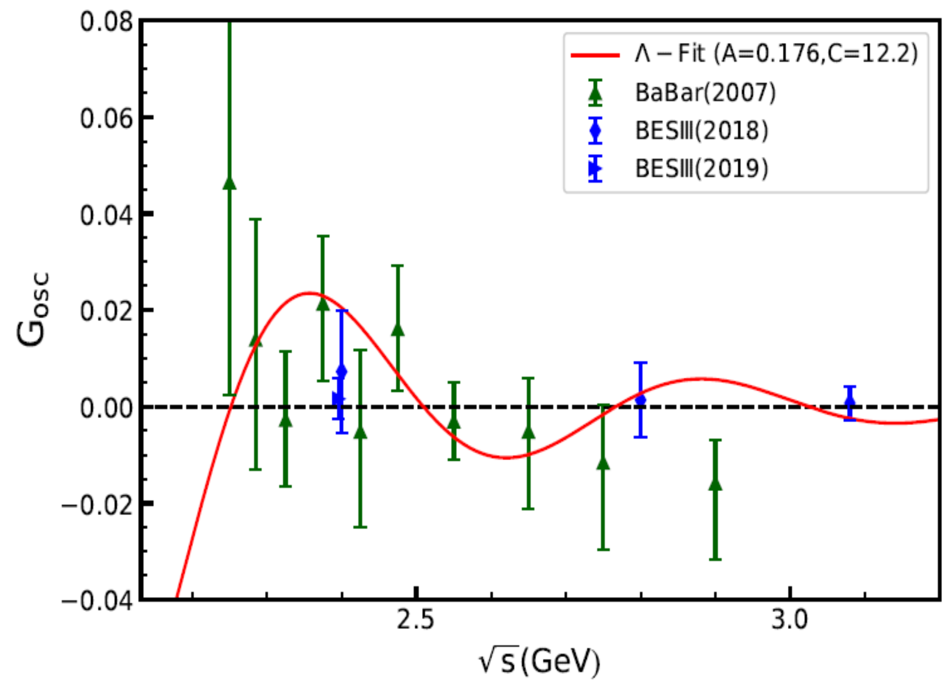
New parametrization

$$G_{osc} = A \cdot \frac{c_0}{(1 - \gamma \cdot s)^2} \cdot \cos\left(C \cdot \sqrt{s} + D\right) \qquad G_D(q^2) = \frac{c_0}{(1 - \gamma q^2)^2}$$

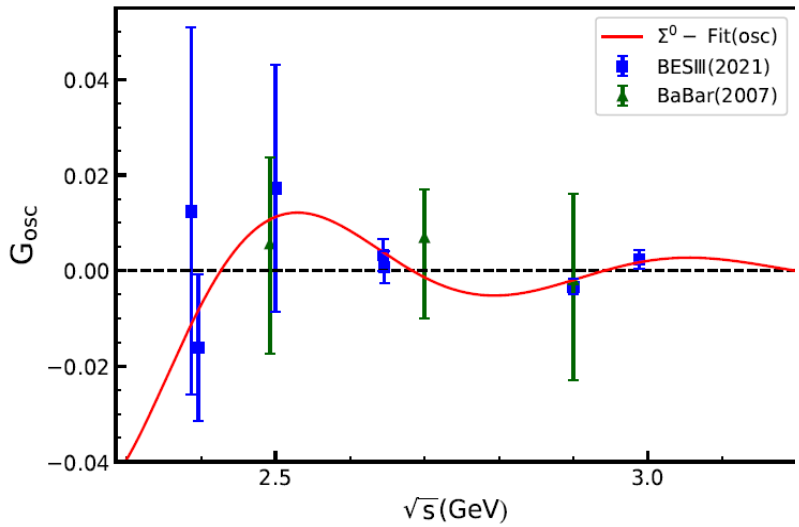
$$\begin{aligned} G_{\text{eff}}(s) &= G_{\text{D}}(s) + G_{\text{osc}}(s) \\ &= \frac{c_0}{(1-\gamma s)^2}\left(1 + A\cos(C\sqrt{s} + D)\right) \end{aligned}$$

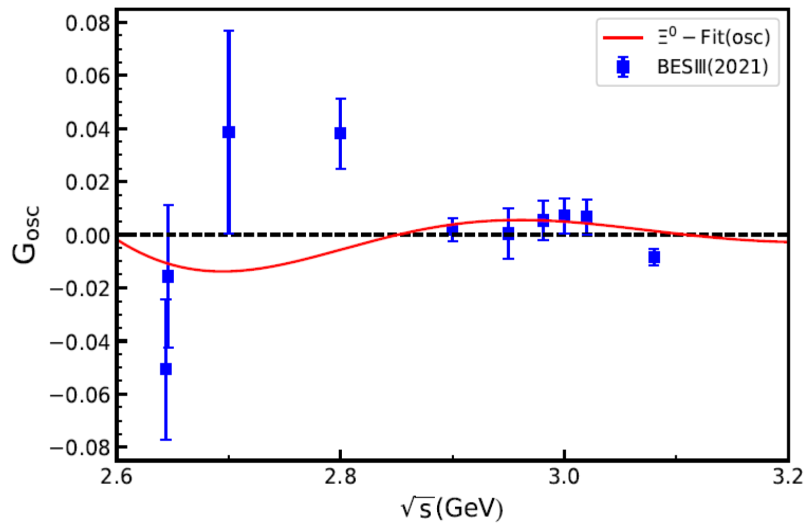
$$\boxed{data = G_{eff} = G_D + G_{osc}}$$

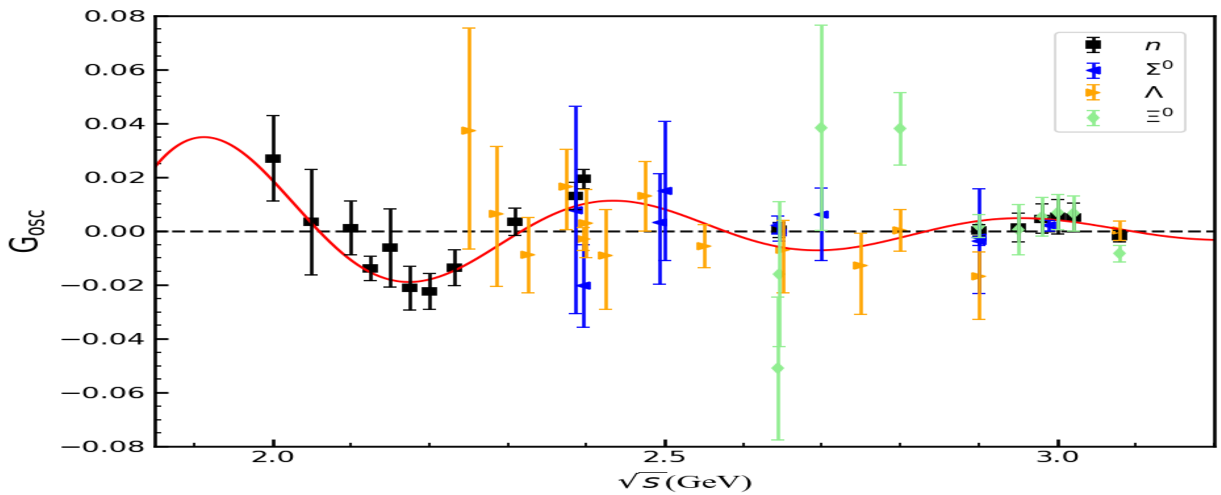
$$\rightarrow \boxed{G_{osc} = data - G_D}$$



Numerical results







A.X. Dai, Z.Y. Li, L. Chang and J.J. Xie, Chin. Phys. C 46, 073104 (2022).

Summary

1、 Threshold enhancement

a）, Final state interaction　　　b), Flatte （strong coupling）

Vector mesons

2、 Oscillation of baryon effective form factors

a), Phenomenology　　　b), Mechanism unknown

See the talk on June 6:

Light flavor vector mesons between 2 and 3 GeV at BESIII

Dong Liu *on behalf of BESIII Collaboration*

$$g(q^2) = \frac{1}{(1-\gamma q^2)^2}$$

Thank you very much for your attention!

e-Print: 2206.01494 [nucl-th]

New insights into the oscillation of the nucleon electromagnetic form factors

Qin-He Yang[1,2], Ling-Yun Dai[1,2,*] Di Guo[1,2], Johann Haidenbauer[3], Xian-Wei Kang[4,5], and Ulf-G. Meißner[6,3,7]

PHYSICAL REVIEW D **105**, L071503 (2022)

Letter

Timelike nucleon electromagnetic form factors: All about interference of isospin amplitudes

Xu Cao,[1,2,*] Jian-Ping Dai,[3,†] and Horst Lenske[4,‡]

PHYSICAL REVIEW D **107**, L091502 (2023)

Letter

Toy model to understand the oscillatory behavior in timelike nucleon form factors

Ri-Qing Qian,[1,2,3,4,*] Zhan-Wei Liu,[1,2,3,4,†] Xu Cao,[2,3,5,6,‡] and Xiang Liu[1,2,3,4,§]

PHYSICAL REVIEW LETTERS **128**, 052002 (2022)

New Insights into the Nucleon's Electromagnetic Structure

Yong-Hui Lin,[1] Hans-Werner Hammer,[2,3] and Ulf-G. Meißner[1,4,5]