Neutrinoless Double-Beta Decay from Lattice QCD

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Motivation

- Neutrino mass can arise from either Dirac or Majorana mass term

$$\mathcal{L} \supset -m\bar{\nu}_L\nu_R - \frac{M}{2}\bar{\nu}_R\nu_R^C$$

- ν_R neutral under all charges – Majorana mass term not forbidden
- Majorana mass $\Rightarrow$ ν is own antiparticle

Seesaw Mechanism

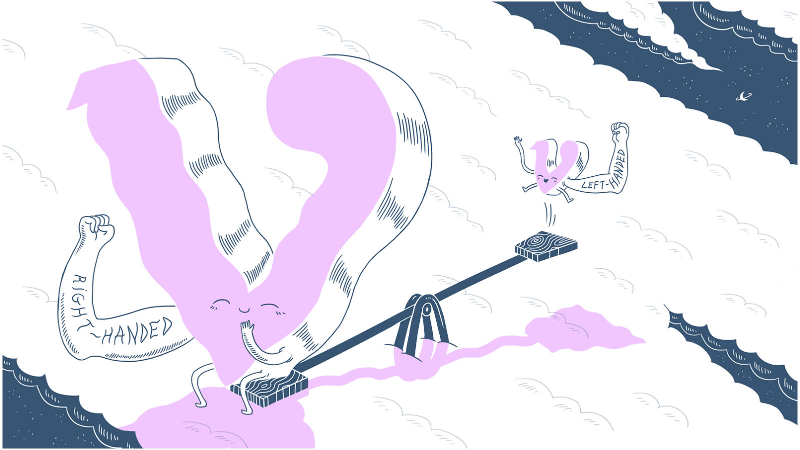


Image credit: Symmetry Magazine, Sandbox Studio, Ana Kova

Leptogenesis

- Universe has more matter than antimatter
- Requires baryon number violation
- Sphalerons: Non-perturbative, high-T SM processes that violate B, L but preserve $B - L$
- Can convert lepton asymmetry into baryon asymmetry
- Majorana neutrinos $\Rightarrow$ lepton number violation

Double-Beta Decay

$$B(Z,A) = \varepsilon_V A - \varepsilon_S A^{2/3} - \varepsilon_C \frac{Z^2}{A^{1/3}} - \varepsilon_{\text{sym}} \frac{(N-Z)^2}{A} + \eta(Z,N) \frac{\Delta}{A^{1/2}}$$

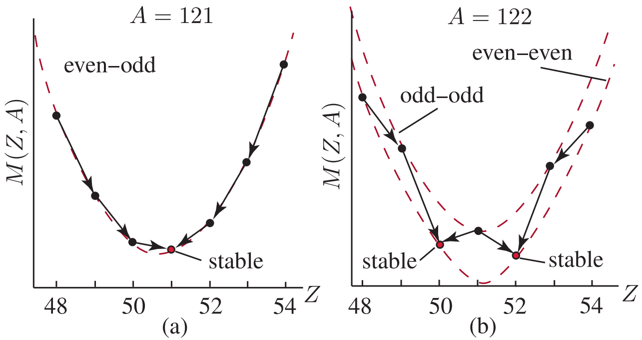
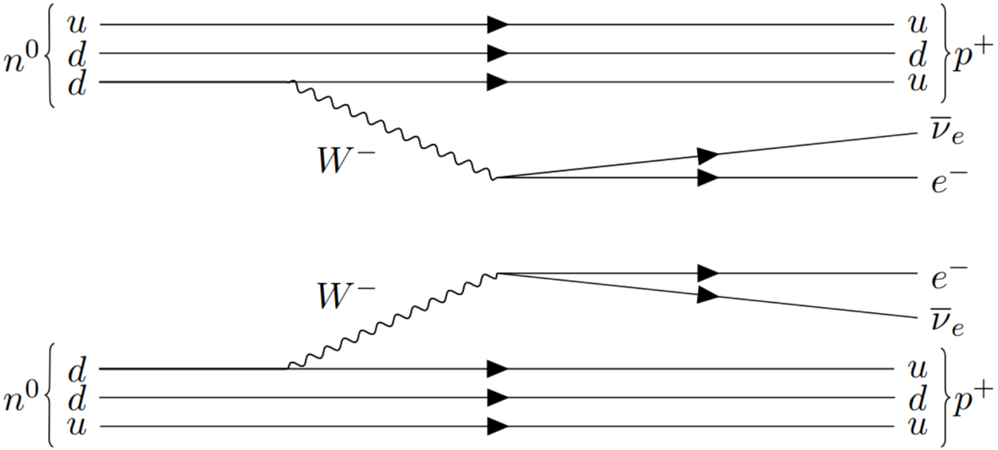
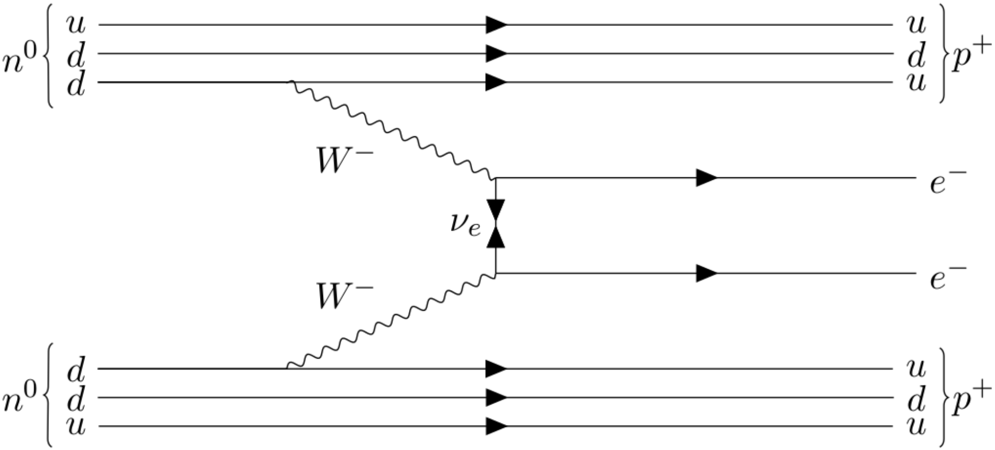


Figure credit: Jaffe and Taylor (2018), after J. Lilley (2001)

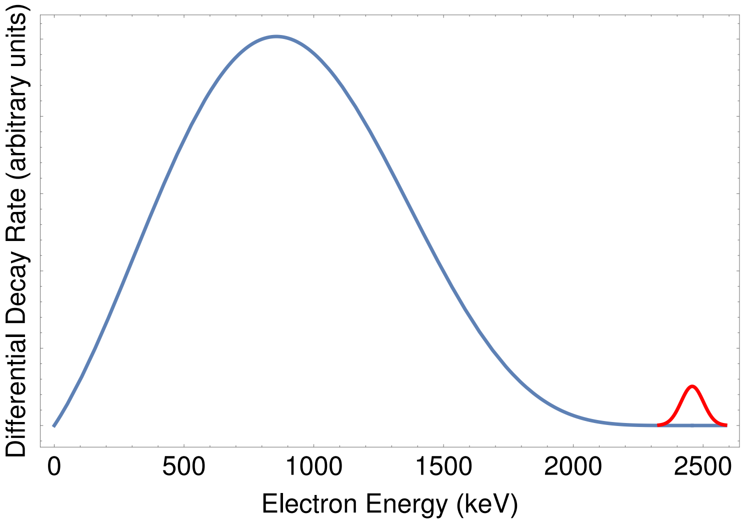
$2\nu\beta\beta$ Diagram



$0\nu\beta\beta$ Diagram



Experimental Signature



Extraction of $m_{\beta\beta}$

$$\left(T_{1/2}^{0\nu}\right)^{-1} \propto |m_{\beta\beta}|^2 G^{0\nu} |M^{0\nu}|^2$$

- $T_{1/2}^{0\nu}$ measured experimentally
- $m_{\beta\beta}$ is effective double-beta neutrino mass
  - $m_{\beta\beta} = |\sum_k U_{ek}^2 m_k|$
- $G^{0\nu}$ is known kinematical factor
- $M^{0\nu}$ is nuclear matrix element
  - Typically estimated with nuclear models (shell models, estimated potentials, etc.)

Nuclear Models

- **Shell Model (SM)**: Nucleons arranged in shells, outer shell(s) studied most closely
- **Quasiparticle random phase approximation (QRPA)**: Hartree-Fock approximation plus collective excitations
- **Energy density functional (EDF)**: Mean field approach (like QRPA) but with additional support for large corrections away from mean field behavior
- **Interacting boson model (IBM)**: Groups nucleons into bosonic pairs to lower effective degrees of freedom
- Subvariants of each model (e.g. density functional used in EDF)

Nuclear Matrix Element Estimates

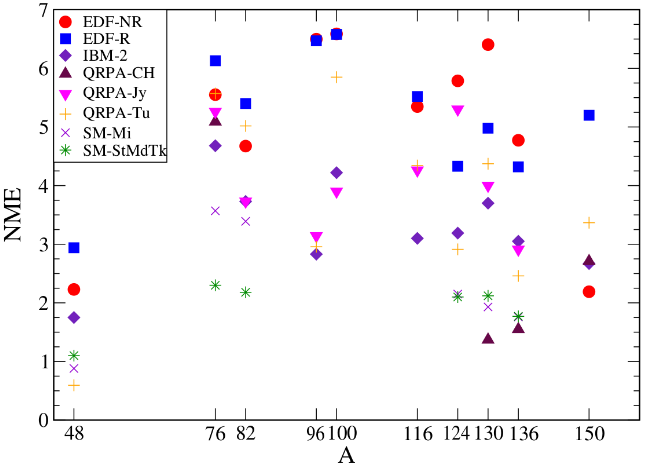


Figure credit: Dolinski, Poon, Rodejohann (1902.04097)

KamLAND-Zen Results

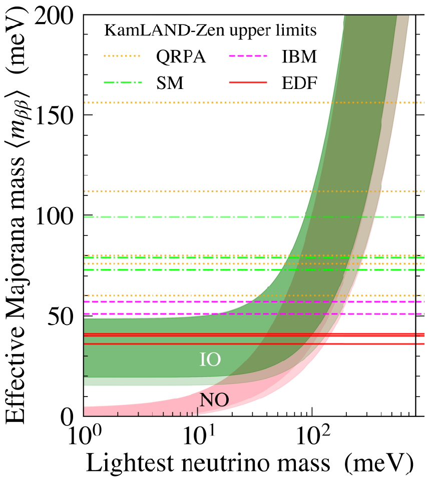


Figure credit: KamLAND-Zen (2203.02139)

nEXO Planned Sensitivity

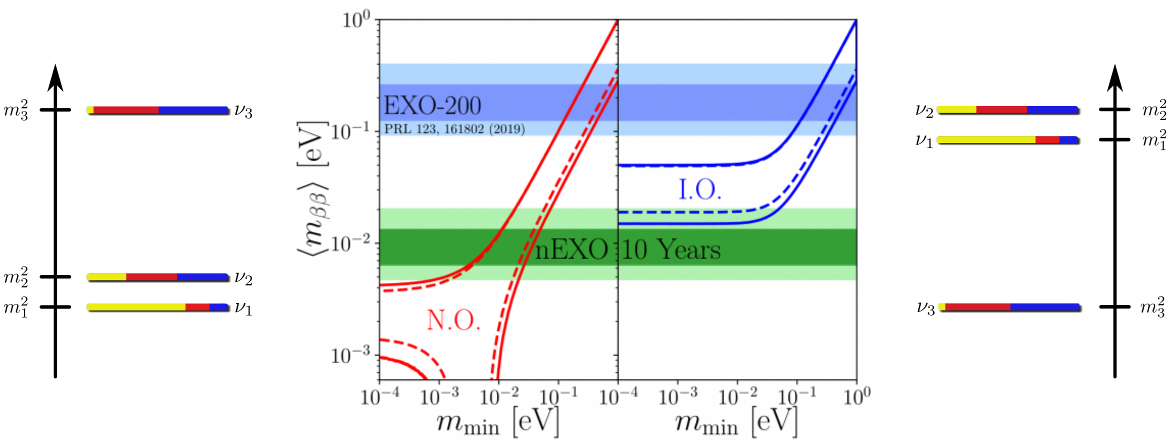


Figure credits: nEXO (2106.16243); Kismalac, Wikimedia Commons

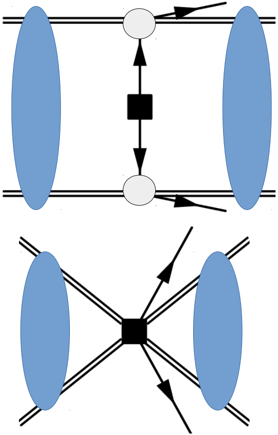
Nuclear Effective Field Theory

- Effective field theory (EFT): Approximate low-energy description of problem
- Relevant degrees of freedom: Nucleons and (maybe) pions
  - Pions can be included (χEFT) or excluded (EFT($\not\pi$))
  - EFT($\not\pi$) requires fewer inputs but only works below $|\mathbf{p}| \leq m_\pi \approx 135$ MeV
- Quark-gluon interactions integrated out to give hadronic couplings
- Successful phenomonologically – can compute binding energies up to ^{132}Sn to within 10–20% (Binder et al., 1512.03802)

Nuclear Effective Field Theory

- Nuclear spectrum requires *NN* and *NNN* contact interactions
  - Could integrate out quark/gluon interactions but difficult
  - Fit to data: *NN* scattering and ^{2}H, ^{3}H binding energies (Bansal et al., 1712.10246)
- For χEFT, also need interactions of $N\pi$, $\pi\pi$, $NN\pi$, etc.
- For weak decays, also need axial and vector nucleon charges

Nuclear EFT for $0\nu\beta\beta$



- Neutrino energy can be hard or soft
- Low-energy contribution factorizes into two SM weak currents
  - Can be computed from existing experimental data
- High-energy intermediate ν outside of EFT validity
- Need contact term to absorb high-energy behavior (Cirigliano et al., 1710.01729, 1802.10097)

Figure credit: 1802.10097

Nuclear EFT for $0\nu\beta\beta$

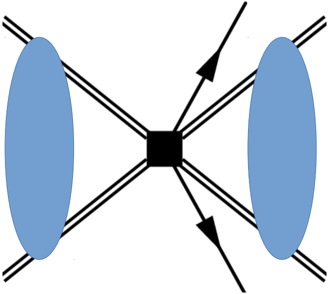


Figure credit: 1802.10097

- EFT contact term unique to $0\nu\beta\beta$
  - No experimental data!
  - Cannot be computed from $2\nu\beta\beta$
- Can be computed using generalization of Cottingham formula
  - Cottingham formula $=$ EM contribution to *np* mass splitting
  - Dominated ($\sim$70%) by elastic intermediate states (π/N pole)
- Dominant contribution can be estimated theoretically (Cirigliano et al., 2102.03371)
  - Likely correct to within 30% but requires model assumptions

Lattice QCD

- First-principles, model-independent solution to hadronic physics
- Only input = Lagrangian of QCD $(\{m_q\}, \alpha_s)$
- Systematically controllable errors
- Can match to EFT, compute low energy constants
- Caveat: Computationally expensive, especially for large systems

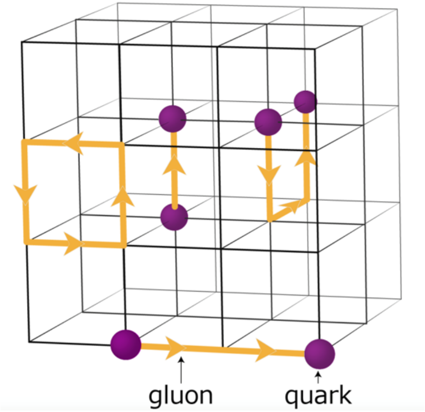


Figure credit: JICFuS, Tsukuba

$0\nu\beta\beta$ for $\pi^- \to \pi^+$

- Simplest to study in pion transition

$$\pi^- \to \pi^+ ee$$

  - Pions give cleaner signal in lattice QCD
  - Fewer quark contractions $\Rightarrow$ cheaper computationally
  - Can control systematic errors (FV, discretization, unphysical m_π)
- Goal: Extract χPT low-energy constant $g_\nu^{\pi\pi}$ (for contact term)

$0\nu\beta\beta$ for $\pi^- \to \pi^+$

$$\mathcal{S}_{\pi\pi} = 1 + \frac{m_\pi^2}{8\pi^2 f_\pi^2}\left(3\log\left(\frac{\mu^2}{m_\pi^2}\right) + 6 + \frac{5}{6}\boldsymbol{g_\nu^{\pi\pi}(\mu)}\right)$$

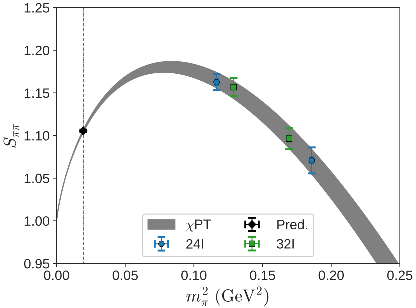


Figure credit: 2004.07404

- Matrix element completely determined up to $g_\nu^{\pi\pi}$
- $g_\nu^{\pi\pi}(\mu = m_\rho)$ measured by two groups
  - $-10.9(8)$ (Tuo, Feng, Jin, 1909.13525)
  - $-10.8(5)$ (Detmold, Murphy, 2004.07404)

Short-Distance $0\nu\beta\beta$

- Standard $0\nu\beta\beta$ paradigm: Two weak currents with light Majorana neutrino

$$(\bar{d}P_L\gamma_\mu u)(x)S_\nu(x-y)(\bar{d}P_L\gamma^\mu u)(y)$$

  - Intermediate neutrino propagates across nuclear scales
  - All operators fully determined by SM
- Some BSM theories predict additional high-energy interactions
- Integrating these out gives contact interaction (Cirigliano et al., 2003.08493)

$$\left(\bar{d}\Gamma_i u\right)\left(\bar{d}\Gamma_j u\right)$$

- Relative sizes of operators (for different i, j) model dependent
- NB: Contact interaction at scale of quarks/gluons
  - Distinct from short-distance effective operator in nuclear EFT

Basis of Short-Distance $0\nu\beta\beta$ Operators

$$\mathcal{O}_1 = (\bar{u}_L\gamma^\mu d_L)\,[\bar{u}_R\gamma_\mu d_R]$$
$$\mathcal{O}_2 = (\bar{u}_R d_L)\,[\bar{u}_R d_L]$$
$$\mathcal{O}_3 = (\bar{u}_L\gamma^\mu d_L)\,[\bar{u}_L\gamma_\mu d_L]$$
$$\mathcal{O}_{1'} = (\bar{u}_L\gamma^\mu d_L]\,[\bar{u}_R\gamma_\mu d_R)$$
$$\mathcal{O}_{2'} = (\bar{u}_R d_L]\,[\bar{u}_R d_L)$$

$$\mathcal{V}_1^\mu = (\bar{u}_L\gamma^\mu d_L)\,[\bar{u}_L d_R]$$
$$\mathcal{V}_2^\mu = \frac{1}{2}(\bar{u}_L\gamma^\mu d_L]\,[\bar{u}_L d_R) - \frac{1}{6}(\bar{u}_L\gamma^\mu d_L)\,[\bar{u}_L d_R]$$
$$\mathcal{V}_3^\mu = (\bar{u}_L\gamma^\mu d_L)\,[\bar{u}_R d_L]$$
$$\mathcal{V}_4^\mu = \frac{1}{2}(\bar{u}_L\gamma^\mu d_L]\,[\bar{u}_R d_L) - \frac{1}{6}(\bar{u}_L\gamma^\mu d_L)\,[\bar{u}_R d_L]$$

- Add $(L \leftrightarrow R)$ to operators where needed
  - Projection to positive parity

Short-Distance $0\nu\beta\beta$ in χEFT

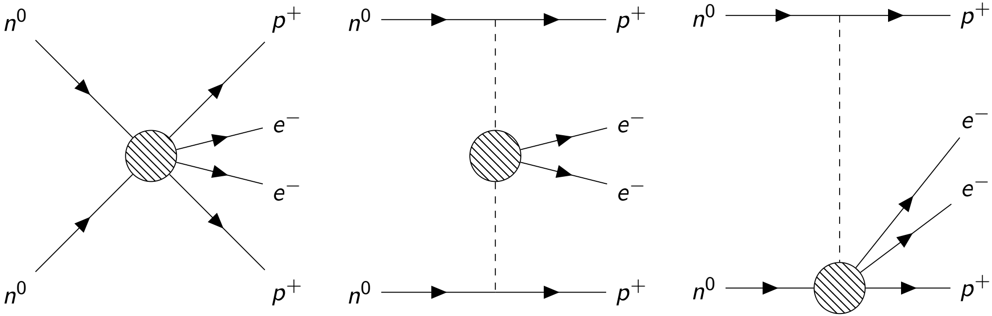
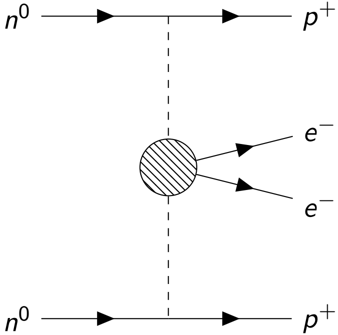


Figure credit: Detmold et al., 2208.05322

Short-Distance $0\nu\beta\beta$ in χEFT

- In Weinberg power counting, dominant effect of short-distance term is through $\pi\pi ee$ interaction
- Can extract coefficient from $\pi^- \to \pi^+ ee$
- Only scalar operators $(\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_{1'}, \mathcal{O}_{2'})$ contribute
    - Vector operators suppressed by m_e/F_π
- NB: Inconsistencies with Weinberg power counting



Short Distance $\pi^- \to \pi^+ ee$ Amplitude

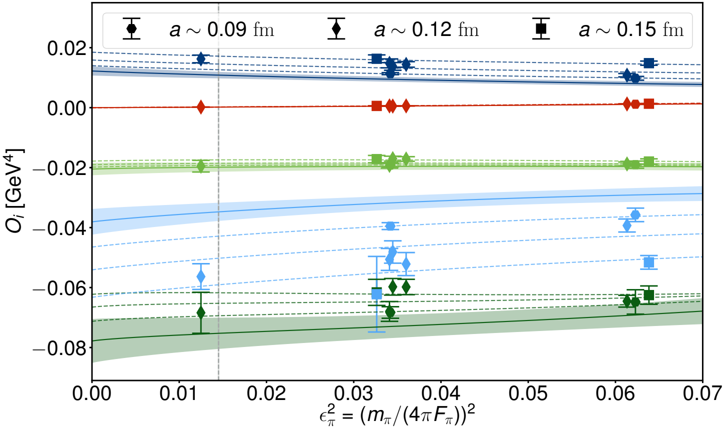


Figure credit: Nicholson et al., 1805.02634

Short Distance $\pi^- \to \pi^+ ee$ Amplitude

Matrix elements for $\langle \pi^+ | \mathcal{O}_i^{\overline{\text{MS}}} | \pi^- \rangle (\mu = 3 \text{ GeV})$, units of 10^{-2} GeV^4:

	CalLat	NPLQCD
$\mathcal{O}_1$	$-1.9(1)$	$-1.3(2)$
$\mathcal{O}_{1'}$	$-7.8(5)$	$-5.4(5)$
$\mathcal{O}_2$	$-3.8(3)$	$-2.5(2)$
$\mathcal{O}_{2'}$	$1.2(1)$	$0.76(8)$
$\mathcal{O}_3$	$0.019(1)$	$0.0087(8)$

- CalLat: Nicholson et al., 1805.02634
- NPLQCD: Detmold, Jay, Murphy, Oare, Shanahan, 2208.05322
- $\mathcal{O}_3$ suppressed by m_π^2/Λ_χ^2 in pionic matrix elements
- Unresolved $\sim 3\sigma$ tension between different calculations

Matrix Elements for $nn \to pp$

- Important to repeat previous calculations for $nn \to pp$ transition
  - EFT coefficient g_{NN} cannot be computed without nucleons
  - Weinberg power counting ($\pi\pi$ interactions dominate short-distance interactions) not fully consistent
  - $\langle pp|\mathcal{O}_3|nn\rangle$ no longer suppressed by m_π^2
  - Vector operators V_i^μ no longer suppressed by m_e
- More difficult computationally
  - Contraction costs $\propto N_q!$ (naïvely)
  - Signal-to-noise problem in lattice QCD

Neutrinoful Double-Beta Decay ($2\nu\beta\beta$)

- Rarest observed Standard Model process
- Experimental data used as inputs or tests of nuclear models of $0\nu\beta\beta$ (Engel, Menéndez, 1610.06548)
- Computed for $nn \to pp$ transition from lattice QCD (Tiburzi et al., 1702.02929)
  - Single lattice spacing and volume
- No intermediate ν prop – weak currents decouple
  - Background field method – quark propagators computed in presence of uniform weak field
- Unphysical quark masses ($m_\pi = 800$ MeV)
  - Dineutron bound at this mass (1508.00886, 1610.04545)
  - Or maybe not! (2108.10835, 2112.04569)

Neutrinoful Double-Beta Decay ($2\nu\beta\beta$)

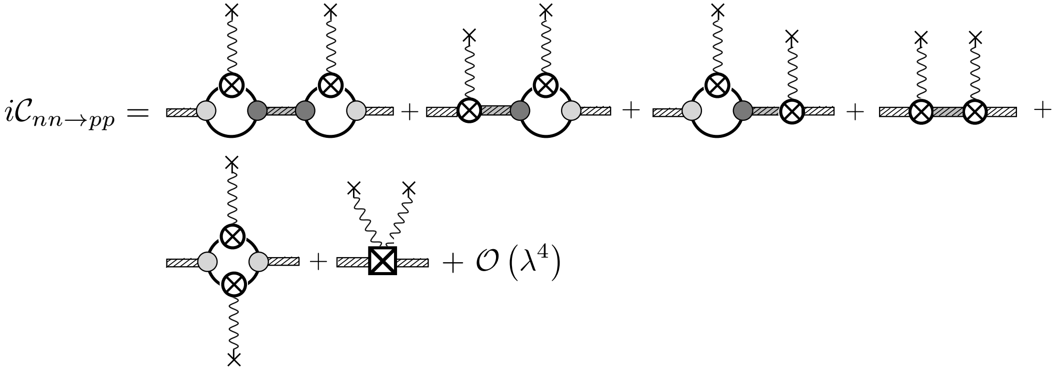


Figure credit: 1702.02929

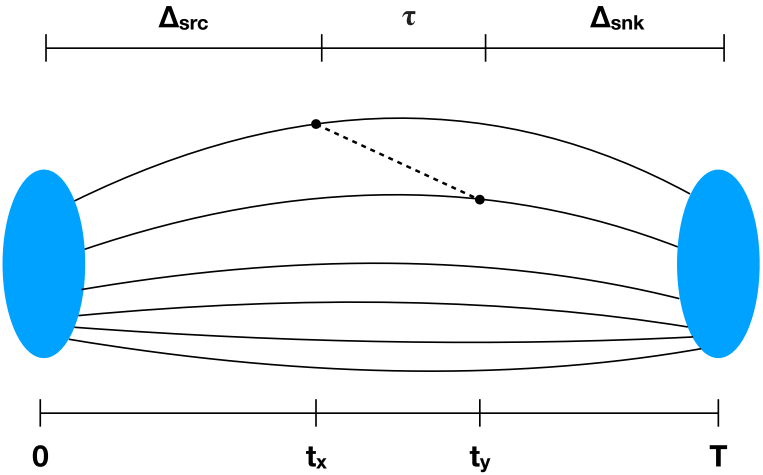
Neutrinoful Double-Beta Decay ($2\nu\beta\beta$)

- Can write full decay amplitude as single-current pieces and two-current LEC $\mathbb{H}_{2,S}$

$$M_{nn\to pp} = -\frac{|M_{pp\to d}|^2}{\Delta} + \frac{Mg_A^2}{4\gamma_s^2} - \mathbb{H}_{2,S}$$

- Computed as $\mathbb{H}_{2,S} = 4.7(2.2)$ fm in 1702.02929
- $\mathbb{H}_{2,S}$ is about 5% correction to full amplitude

Neutrinoless Double-Beta Decay ($0\nu\beta\beta$)



Ongoing work by Detmold, Fu, AVG, Jay, Murphy, Oare, Shanahan

Neutrinoless Double-Beta Decay ($0\nu\beta\beta$)

- Compute matrix element from ratio of 2-point and 4-point correlation functions

$$C_2(t_z) = \int d^3\mathbf{z}\, \langle \mathcal{O}_{nn}(z)\mathcal{O}^\dagger_{nn}(0)\rangle$$

$$C_4(t_z, t_x, t_y) = \int d^3\mathbf{z}\, d^3\mathbf{x}\, d^3\mathbf{y}\, \Gamma^{\text{lept.}}_{\alpha\beta} \langle \mathcal{O}_{pp}(z) J_\alpha(x) J_\beta(y) \mathcal{O}^\dagger_{nn}(0)\rangle S_\nu(x,y)$$

- Signal resolvable for source-sink separations up to about $16a \approx 2.3$ fm
  - Necessary to have such large separations to study large $t_y - t_x$ behavior while suppressing excited states
  - Requires much higher statistics than $2\nu\beta\beta$ calculation (3M propagators versus 49k in $2\nu\beta\beta$)

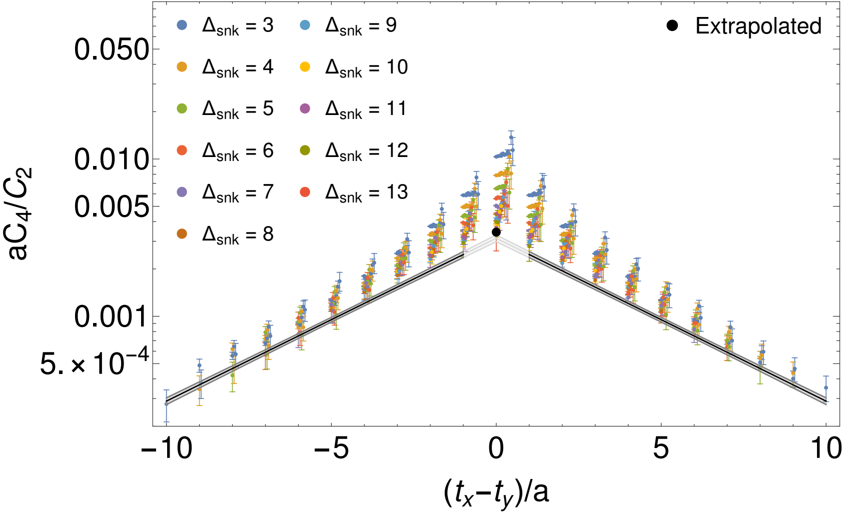
Neutrino Propagator

- Long-distance amplitude contains significant contribution from low-E_ν tail
  - Contribution from separation $\tau = t_y - t_x$ falls off as τ^{-2}
  - Corresponds to large temporal separation between operators
  - Difficult to control (signal-to-noise problem)
- Solution: Use zero-mode subtracted propagator (Davoudi and Kadam, 2012.02083)

$$S_\nu(\tau, \mathbf{z}) = \frac{m_{\beta\beta}}{2L^3} \sum_{\mathbf{q} \in \frac{2\pi}{L}\mathbb{Z}^3 \setminus \{\mathbf{0}\}} \frac{e^{i\mathbf{q}\cdot\mathbf{z}}}{|\mathbf{q}|} e^{-|\mathbf{q}||\tau|}$$

  - Contribution falls off exponentially in τ
  - Correct for zero-mode removal when matching to EFT

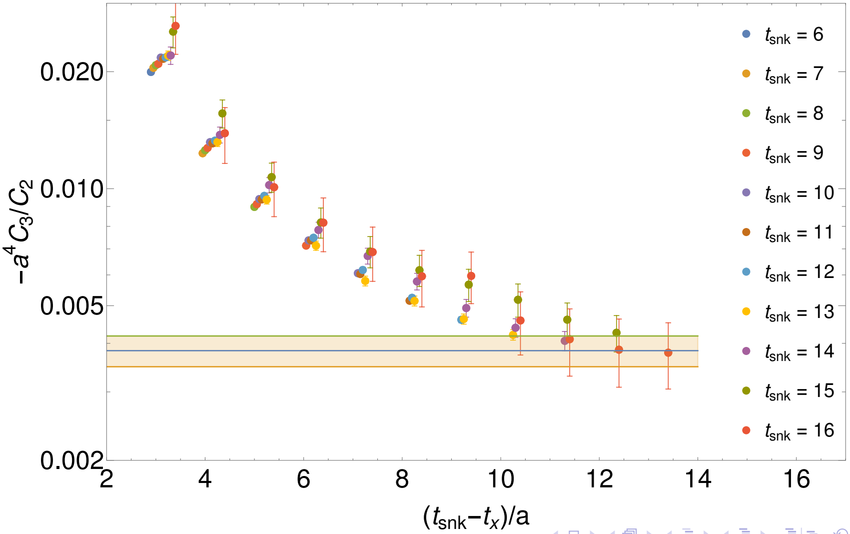
Preliminary Correlator Data



Preliminary Results

- Can estimate plateau value for C_4/C_2 out to $\tau = 9a \approx 1.3$ fm
- Constrain plateau value to exponential in τ
  - Corresponds to single intermediate state $(d + \nu)$
  - Higher-energy intermediate states contribute at small time
    - Fit $\tau = 0$ separately
- Preliminary result: $|\mathcal{M}^{0\nu}| = 0.26(1)$ (stat.) GeV^2
  - Systematics (excited states, renormalization) still under investigation
  - $\approx 10\times$ larger than pion result (≈ 0.019 GeV^2)

Short-Distance Amplitude



Preliminary Results for Scalar Operators

Approximate matrix elements for $\langle f|\mathcal{O}_i|i\rangle$, units of 10^{-2} GeV^4:

	$nn \to pp$	$\pi^- \to \pi^+$ (NPLQCD)
$\mathcal{O}_1$	0.1	$-1.3(2)$
$\mathcal{O}_{1'}$	-1.1	$-5.4(5)$
$\mathcal{O}_2$	-1.0	$-2.5(2)$
$\mathcal{O}_{2'}$	-0.5	0.76(8)
$\mathcal{O}_3$	-2.1	0.0087(8)

- No chiral/continuum extrapolation for $nn \to pp$
- $\mathcal{O}_1, \mathcal{O}_{1'}$ smaller for $nn \to pp$, $\mathcal{O}_3$ larger
- Others are same order of magnitude

Preliminary Results for Vector Operators

Approximate matrix elements for $\langle f|\mathcal{V}_i|i\rangle$, units of 10^{-2} GeV^4:

	$nn \to pp$
$\mathcal{V}_1$	-1
$\mathcal{V}_2$	-0.4
$\mathcal{V}_3$	-0.2
$\mathcal{V}_4$	-0.4

- Essentially absent (suppressed by m_e/Λ_{QCD}) for $\pi^- \to \pi^+$
- Numbers for $nn \to pp$ not renormalized (computation in progress)
- No chiral/continuum extrapolation (as before)
- $\mathcal{V}_i$ slightly smaller than $\mathcal{O}_i$ but not negligible

Future Plans

- Cannot simulate large nuclei on lattice
  - Costs of simulation scale exponentially in number of quarks
  - Very challenging to get beyond ^{4}He on lattice
- Must compare to effective field theory approaches
- Can use EFT coefficients input to current nuclear models
- EFT coefficients ⇒ *ab initio* nuclear calculations
- Improve interpretability of experimental searches

Lattice QCD Details

- Dirac operator $\not{D} + m$ implemented as matrix coupling adjacent vertices of lattice
- Quark propagators computed by inverting Dirac matrix (expensive!)
- Hadronic states created and annihilated by interpolating operators, e.g.

$$\mathcal{O}_n = \varepsilon^{abc} P_+ d_a \left(d_b^T P_+ C \gamma_5 u_c \right)$$

- Correlation functions built from interpolating operators and current insertions

$$C_2(t_z) = \int d^3\mathbf{z} \, \langle \mathcal{O}_{nn}(z) \mathcal{O}^\dagger_{nn}(0) \rangle$$

$$C_4(t_z, t_x, t_y) = \int d^3\mathbf{z} \, d^3\mathbf{x} \, d^3\mathbf{y} \, \Gamma^{\text{lept.}}_{\alpha\beta} \langle \mathcal{O}_{pp}(z) J_\alpha(x) J_\beta(y) \mathcal{O}^\dagger_{nn}(0) \rangle S_\nu(x, y)$$

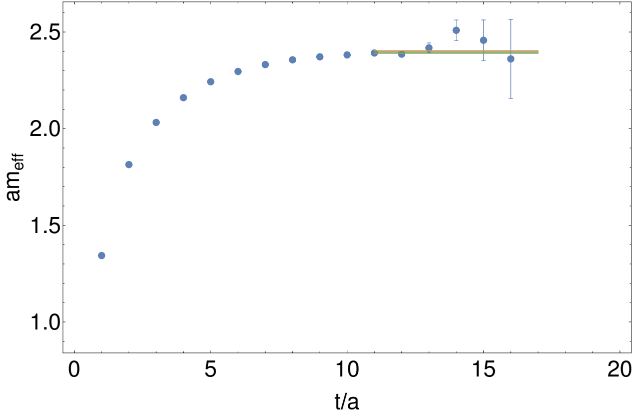
Interpretation of Correlation Functions

- 2-point correlation functions related to energies

$$
\begin{aligned}
\langle \mathcal{O}_H(t)\mathcal{O}_H^\dagger(0)\rangle &= \sum_n \frac{1}{2E_n}\langle 0|\mathcal{O}_H(t)|n\rangle\langle n|\mathcal{O}_H^\dagger(0)|0\rangle \\
&= \sum_n \frac{1}{2E_n}\langle 0|\mathcal{O}_H(0)|n\rangle e^{-E_n t}\langle n|\mathcal{O}_H^\dagger(0)|0\rangle \\
&= \sum_n \frac{1}{2E_n} e^{-E_n t}|\langle 0|\mathcal{O}_H|n\rangle|^2 \\
&\to \frac{1}{2E_0} e^{-E_0 t}|\langle 0|\mathcal{O}_H|H\rangle|^2
\end{aligned}
$$

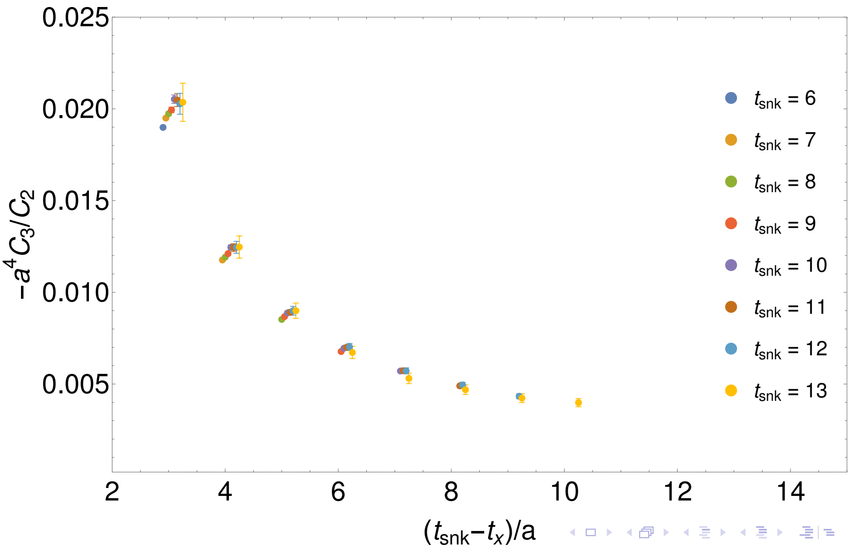
- In Euclidean time, correlation functions exponentially decay instead of oscillating
- At small time, signal has not just ground state H but tower of unwanted excited states with energies $> E_0$

Effective Mass



$$m_{\text{eff}}(t) \equiv \log \frac{C(t)}{C(t+1)} \to E_0$$

Short-Distance Amplitude



Prior Extraction of $M^{0\nu}$