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Institut de Ciències del Cosmos  
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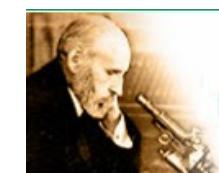
# Rare $B$ decays in the LHC era

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Javier Virto

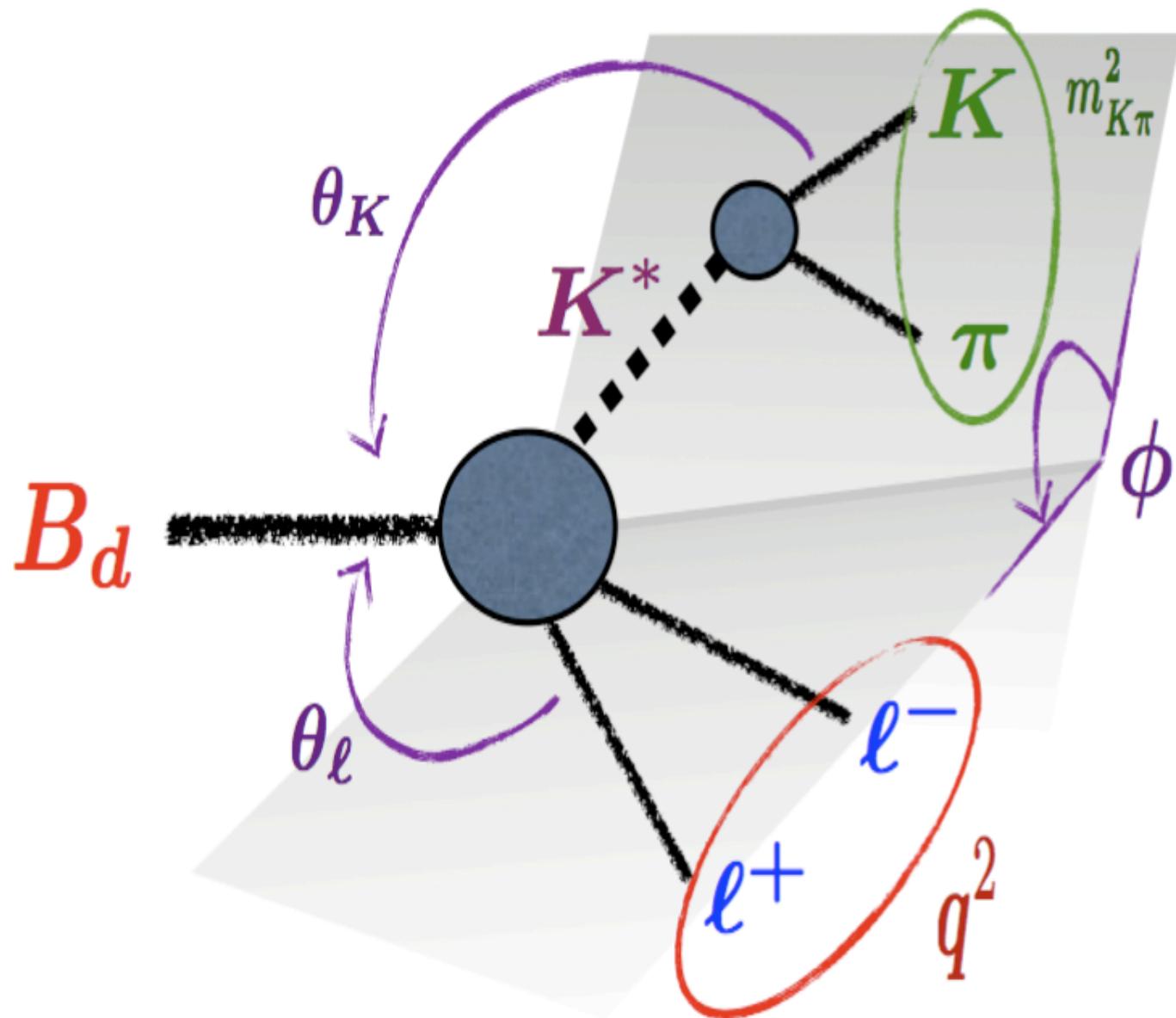
Universitat de Barcelona

HADRON 2023 – June 9th, 2023



Investigación  
Programa  
Ramón y Cajal

Will be discussing these type of decays:



## Observation of $J/\psi p$ Resonances Consistent with Pentaquark States in $\Lambda_b^0 \rightarrow J/\psi K^- p$ Decays #1

LHCb Collaboration • Roel Aaij (CERN) et al. (Jul 13, 2015)

Published in: *Phys.Rev.Lett.* 115 (2015) 072001 • e-Print: [1507.03414](#) [hep-ex]

[pdf](#) [links](#) [DOI](#) [cite](#) [claim](#)

[reference search](#) [1,545 citations](#)

## Test of lepton universality using $B^+ \rightarrow K^+ \ell^+ \ell^-$ decays #2

LHCb Collaboration • Roel Aaij (NIKHEF, Amsterdam) et al. (Jun 25, 2014)

Published in: *Phys.Rev.Lett.* 113 (2014) 151601 • e-Print: [1406.6482](#) [hep-ex]

[pdf](#) [DOI](#) [cite](#) [claim](#)

[reference search](#) [1,297 citations](#)

## Test of lepton universality with $B^0 \rightarrow K^{*0} \ell^+ \ell^-$ decays #3

LHCb Collaboration • R. Aaij (CERN) et al. (May 16, 2017)

Published in: *JHEP* 08 (2017) 055 • e-Print: [1705.05802](#) [hep-ex]

[pdf](#) [links](#) [DOI](#) [cite](#) [datasets](#) [claim](#)

[reference search](#) [1,220 citations](#)

## Measurement of the ratio of branching fractions $\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \tau^- \bar{\nu}_\tau)/\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \mu^- \bar{\nu}_\mu)$ #4

LHCb Collaboration • Roel Aaij (CERN) et al. (Jun 29, 2015)

Published in: *Phys.Rev.Lett.* 115 (2015) 11, 111803, *Phys.Rev.Lett.* 115 (2015) 15, 159901 (erratum) • e-Print: [1506.08614](#) [hep-ex]

[pdf](#) [links](#) [DOI](#) [cite](#) [claim](#)

[reference search](#) [1,147 citations](#)

## Angular analysis of the $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decay using $3 \text{ fb}^{-1}$ of integrated luminosity #5

LHCb Collaboration • Roel Aaij (CERN) et al. (Dec 14, 2015)

Published in: *JHEP* 02 (2016) 104 • e-Print: [1512.04442](#) [hep-ex]

[pdf](#) [links](#) [DOI](#) [cite](#) [datasets](#) [claim](#)

[reference search](#) [901 citations](#)

## Measurement of Form-Factor-Independent Observables in the Decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ #6

LHCb Collaboration • R Aaij (NIKHEF, Amsterdam) et al. (Aug 7, 2013)

Published in: *Phys.Rev.Lett.* 111 (2013) 191801 • e-Print: [1308.1707](#) [hep-ex]

[pdf](#) [DOI](#) [cite](#) [claim](#)

[reference search](#) [738 citations](#)

SM is GIM/CKM and loop suppressed:

$$= \frac{g^2}{M_W^2} \sum_i v_{ib} v_{is}^* F(x_i)$$

$$x_i = m_i^2/M_W^2$$

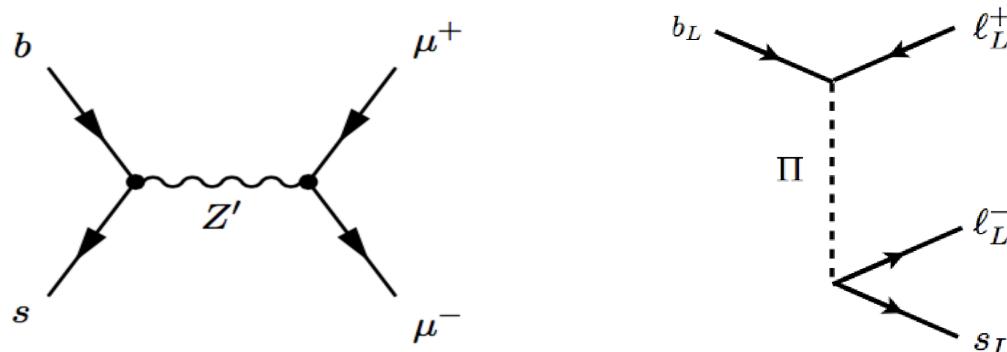
$$F(x_i) = F(0) + x_i \cdot F'(0) + \dots$$

$$\sum_i v_{ib} v_{is}^* = 0$$

$$= \frac{g^2}{M_W^2} v_{tb} v_{ts}^* f(x_t) + \mathcal{O}(x_u - x_c)$$

4%      Loop  $\simeq \frac{1}{(4\pi)^2}$

Competes (potentially) with tree-level BSM contributions...



An order of magnitude estimate gives:

$$\frac{1}{(4\pi)^2} \frac{g^4}{M_W^2} \frac{m_t^2}{M_W^2} V_{tb} V_{ts}^* \sim \frac{1}{\Lambda_{NP}^2} \quad \Rightarrow \quad \Lambda_{NP} \sim 35 \text{ TeV}$$

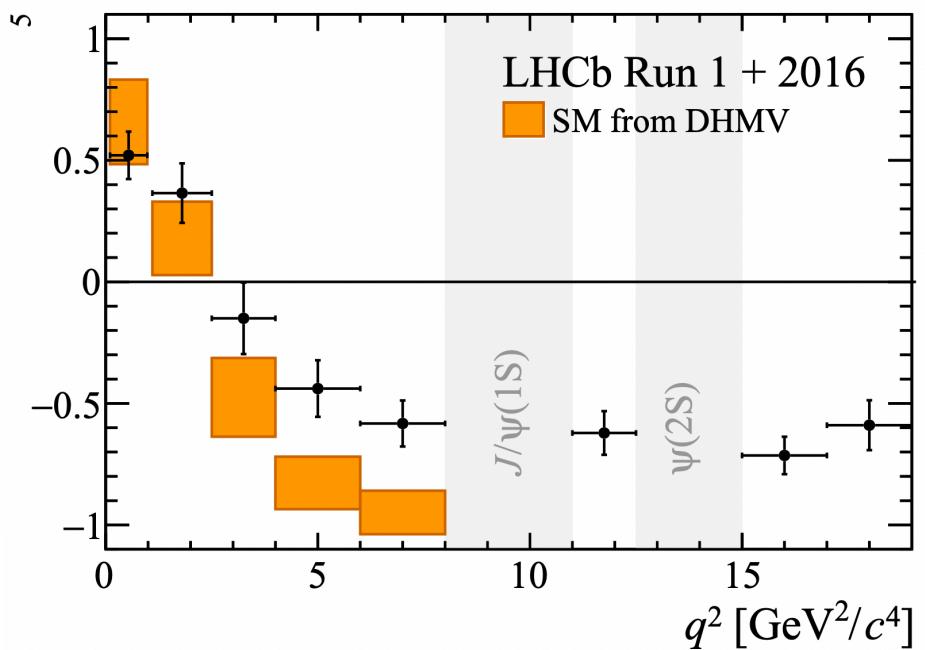
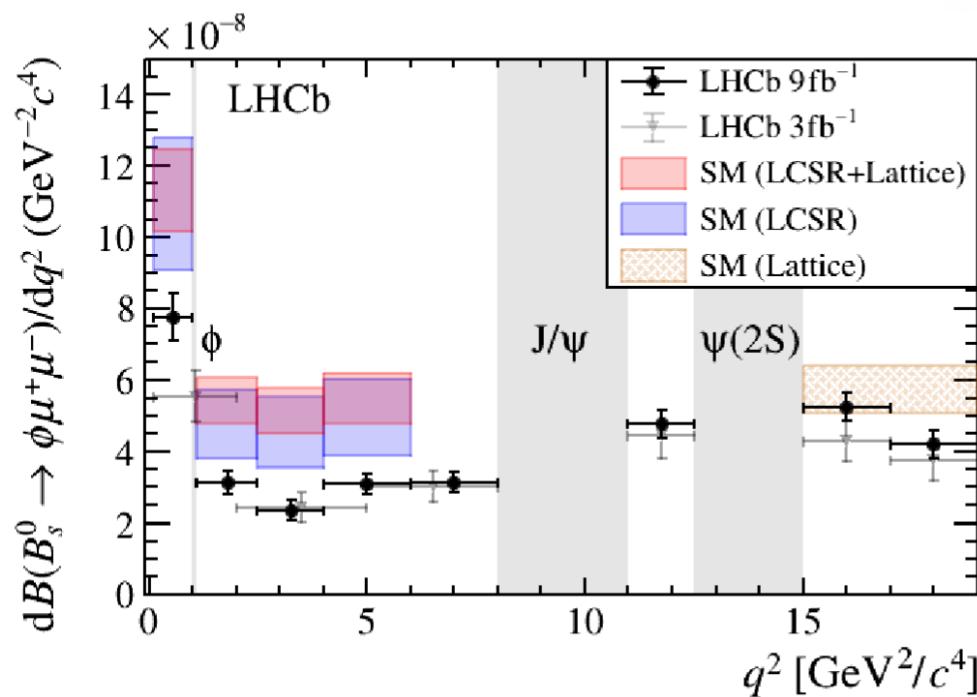
But in practice the important measure is **experimental + theoretical errors**

Experimentally the key is statistics \*\*\*

[\*\*\* It is actually a bit harder than that...]

# Tensions in $b \rightarrow s\mu^+\mu^-$ : BRs and AOs

Angular observables



Branching fractions

# Tensions in $b \rightarrow s\mu^+\mu^-$ : Two roads

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## ► New Physics in $b \rightarrow se^+e^-$

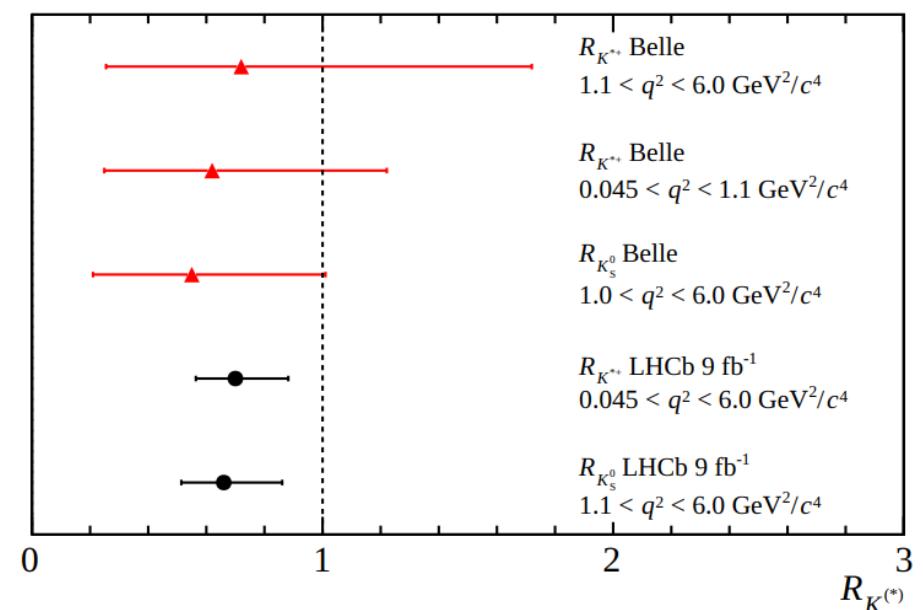
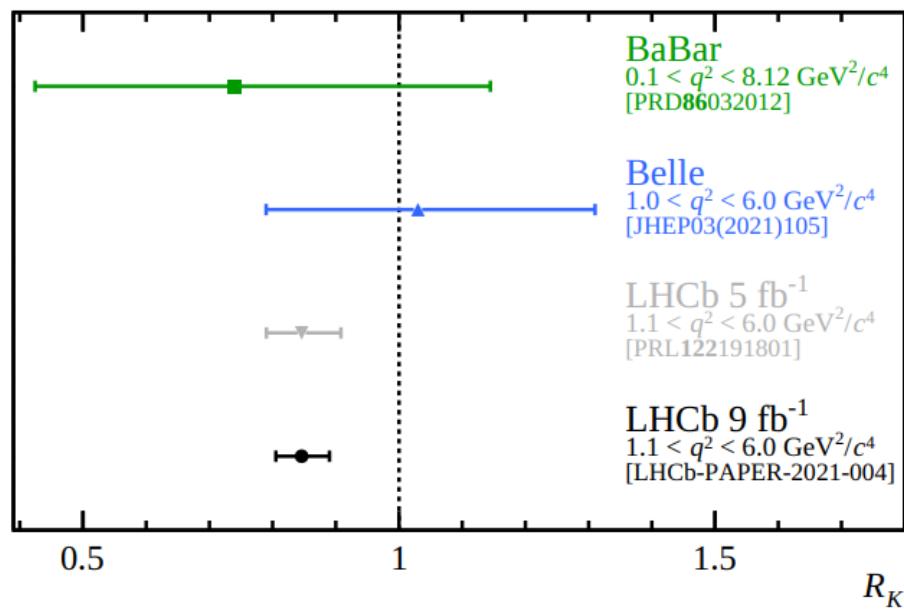
- Should see same type of deviations in both modes
- Theoretically there is no particular advantage

## ► Lepton-Flavour Non-Universality

- Theoretically clean (smoking gun)
- Possible correlations to  $b \rightarrow c\tau\nu$
- Possible connection to LFV and effects elsewhere
- Did not seem so well motivated at the time, but we got over it

# Lepton-Flavour Non-Universality @ 2021

LHCb 2103.11769, LHCb 2110.09501



$$R_H = \frac{\int_{\text{bin}} dq^2 \ BR(B \rightarrow H \mu^+ \mu^-)}{\int_{\text{bin}} dq^2 \ BR(B \rightarrow He^+ e^-)} \quad \xrightarrow{\text{--- SM ---}} \quad \simeq 1$$

Hiller, Kruger

► Anomalies in  $b \rightarrow s\mu^+\mu^-$ : (LHCb, Belle, ATLAS, CMS)

$$P'_5(B \rightarrow K^*\mu^+\mu^-) \sim 3\sigma; \quad \mathcal{B}(B_s \rightarrow \phi\mu^+\mu^-) \sim 2\sigma$$

$$\mathcal{B}(B \rightarrow K^{(*)}\mu^+\mu^-) \sim 1\sigma; \quad \text{many others} \sim 0\sigma$$

Combined (180 Observables)  $\sim 5\sigma$

Significantly alleviated if

Descotes, Matias, Virto 2013

$$\mathcal{L}_{NP} \simeq (35 \text{ TeV})^{-2} [\bar{s}\gamma_\nu P_L b][\bar{\mu}\gamma^\nu \mu]$$

► LFNU: (LHCb, Belle)

$$R_K, R_{K^*} \gtrsim 2\sigma; \quad Q_5 \equiv P'_{5\mu} - P'_{5e} \gtrsim 1\sigma$$

Combined  $\sim 4\sigma$

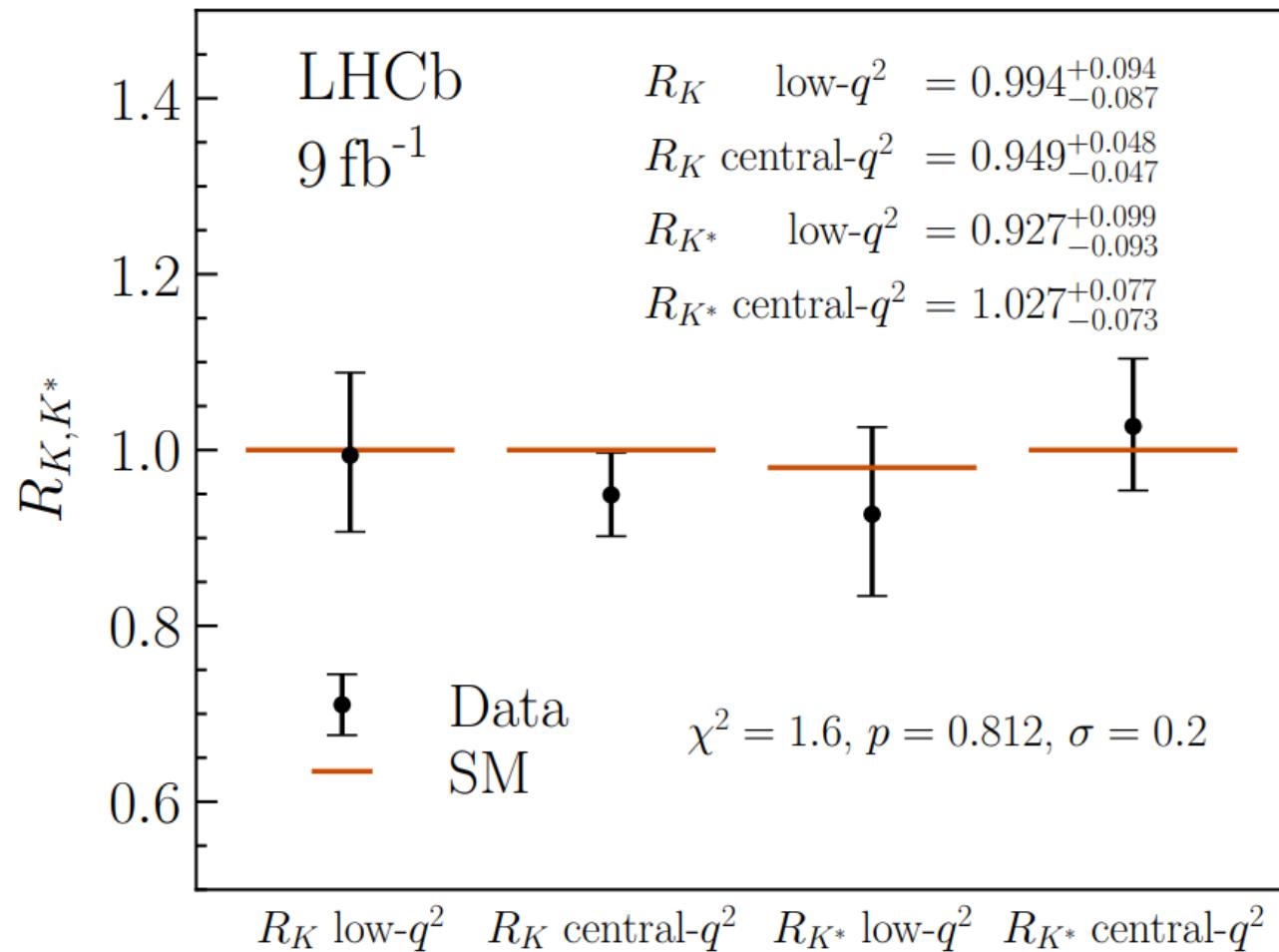
Consistent<sup>(\*)</sup> with  $b \rightarrow s\mu^+\mu^-$

Alonso, Camalich, Grinstein 2014

NP interpretation requires accurate TH predictions of  $B \rightarrow M\ell^+\ell^-$  obs

# Lepton-Flavour Non-Universality @ 2022

LHCb 2212.09153



► Anomalies in  $b \rightarrow s\mu^+\mu^-$ : (LHCb, Belle, ATLAS, CMS)

$$P'_5(B \rightarrow K^*\mu^+\mu^-) \sim 3\sigma ; \quad \mathcal{B}(B_s \rightarrow \phi\mu^+\mu^-) \sim 2\sigma$$

$$\mathcal{B}(B \rightarrow K^{(*)}\mu^+\mu^-) \sim 1\sigma ; \quad \text{many others} \sim 0\sigma$$

Combined (180 Observables)  $\sim 5\sigma$

Significantly alleviated if

Descotes, Matias, Virto 2013

$$\mathcal{L}_{NP} \simeq (35 \text{ TeV})^{-2} \left\{ [\bar{s}\gamma_\nu P_L b][\bar{\mu}\gamma^\nu \mu] + [\bar{s}\gamma_\nu P_L b][\bar{e}\gamma^\nu e] \right\}$$

► LFNU: (LHCb, Belle)

$$R_K, R_{K^*} \gtrsim 2\sigma ; \quad Q_5 \equiv P'_{5\mu} - P'_{5e} \gtrsim 1\sigma$$

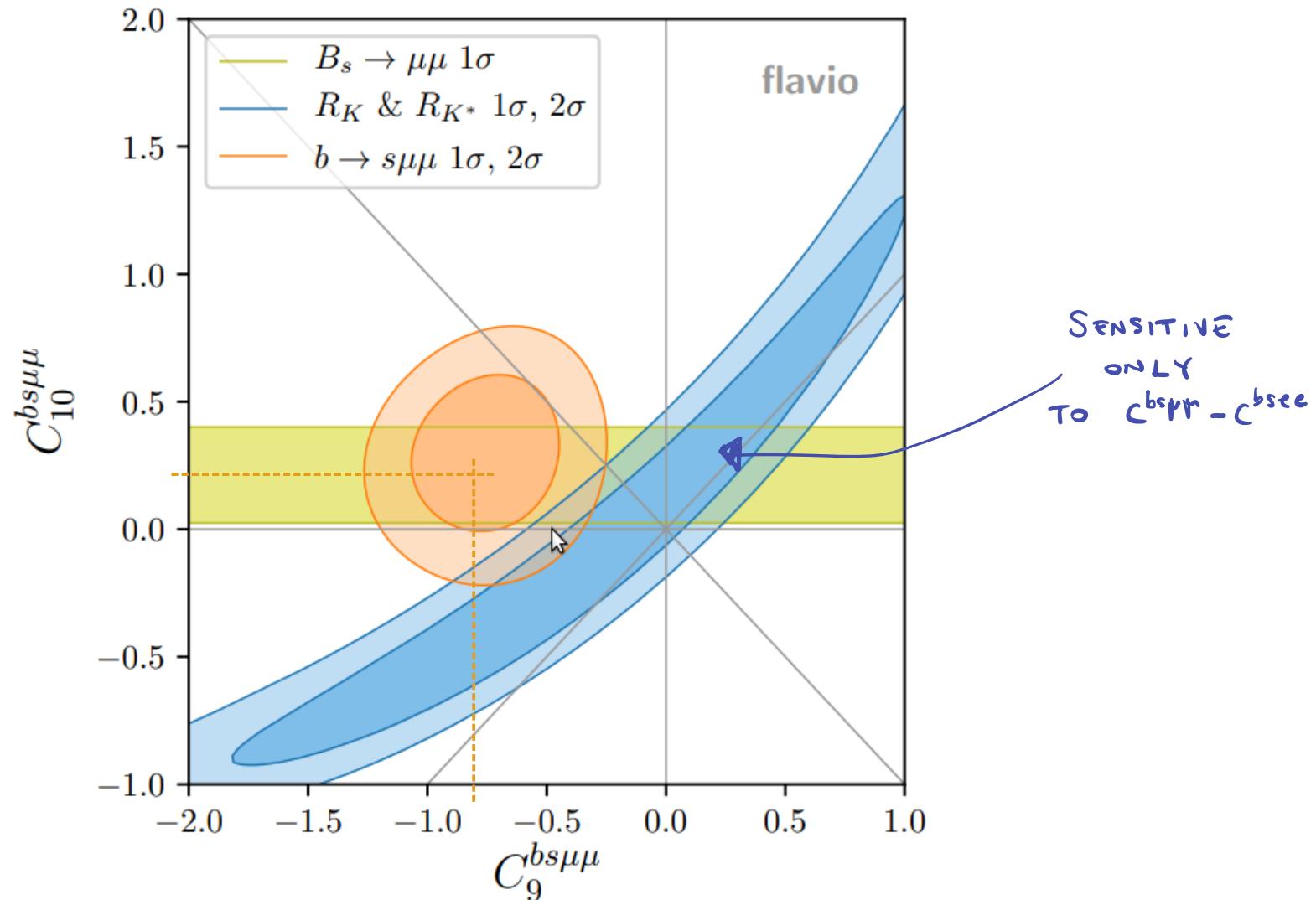
Consistent<sup>(\*)</sup> with  $b \rightarrow s\mu^+\mu^-$

Combined  $\sim 4\sigma$

Alonso, Camalich, Grinstein 2014

NP interpretation requires accurate TH predictions of  $B \rightarrow M\ell^+\ell^-$  obs

Greljo, Salko, Smolkovic, Stangl 2022



## Theory Calculations:

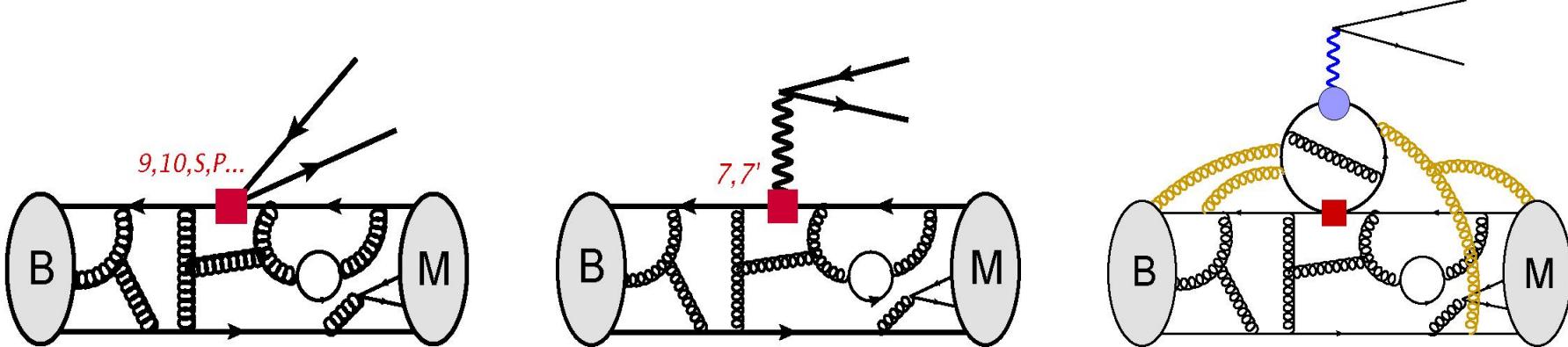
$$\mathcal{L}_{EFT} = \mathcal{L}_{QCD+QED} + \sum_i C_i \mathcal{O}_i$$

$C_i$  = Calculated through a perturbative matching calculation

$$\mathcal{A}(B \rightarrow f) = \sum_i \underbrace{C_i}_{BSM} \underbrace{\langle f | T\{\dots \mathcal{O}_i \dots\} | B \rangle}_{QCD}$$

$\langle \mathcal{O}_i \rangle$  = Non-perturbative and difficult to calculate

# Anatomy of $B \rightarrow M_\lambda \ell^+ \ell^-$ EFT Amplitudes



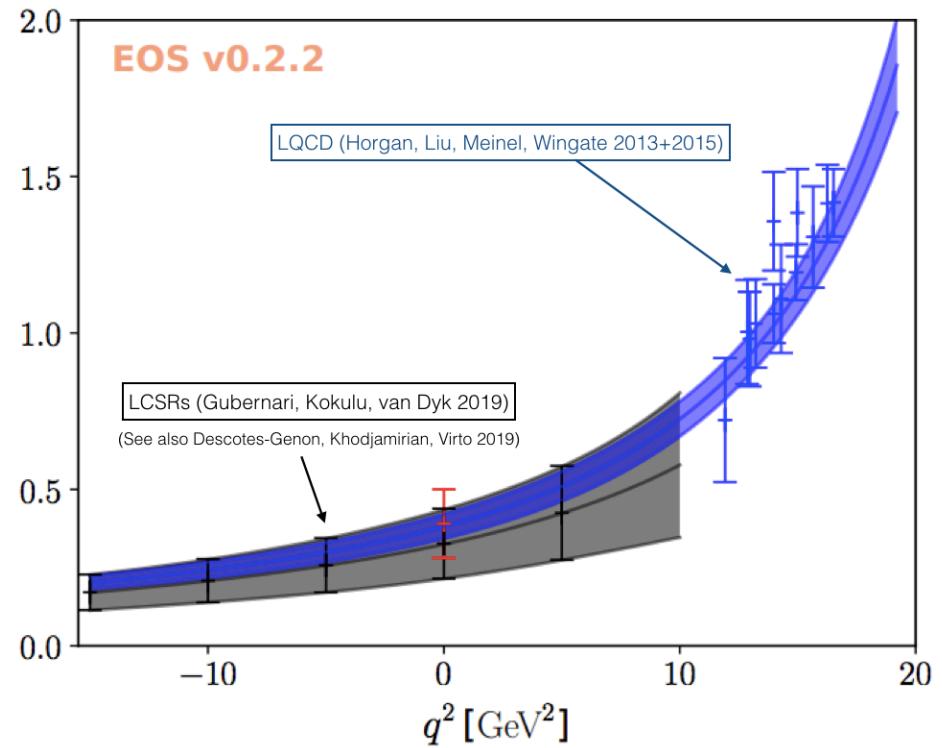
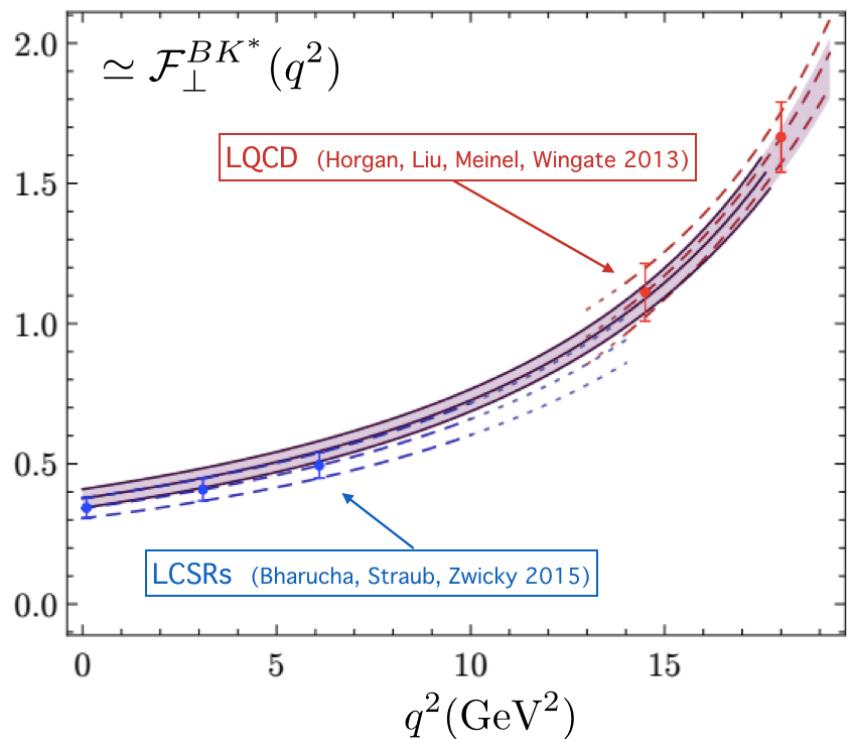
$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[ C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

- Local (Form Factors):  $\mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{M}_\lambda(k) | \bar{s} \Gamma_\lambda^{(T)} b | \bar{B}(k+q) \rangle$
- Non-Local:  $\mathcal{H}_\lambda(q^2) = i \mathcal{P}_\mu^\lambda \int d^4x e^{iq \cdot x} \langle \bar{M}_\lambda(k) | T\{j_{\text{em}}^\mu(x), \mathcal{C}_i \mathcal{O}_i(0)\} | \bar{B}(q+k) \rangle$

In both cases the “modern” strategy is

- ▶ **Calculate/extract** the form factors in optimal/feasible kinematic regions (not necessarily physical or the regions we are interested in)  
→ “data”
- ▶ **Parametrize**  $q^2$  dependence by means of a rigorous **analytic expansion**
- ▶ **Fit** the (truncated) parametrization to the “data”
- ▶ Control the truncation error by means of a **dispersive bound**.

# Local Form Factors

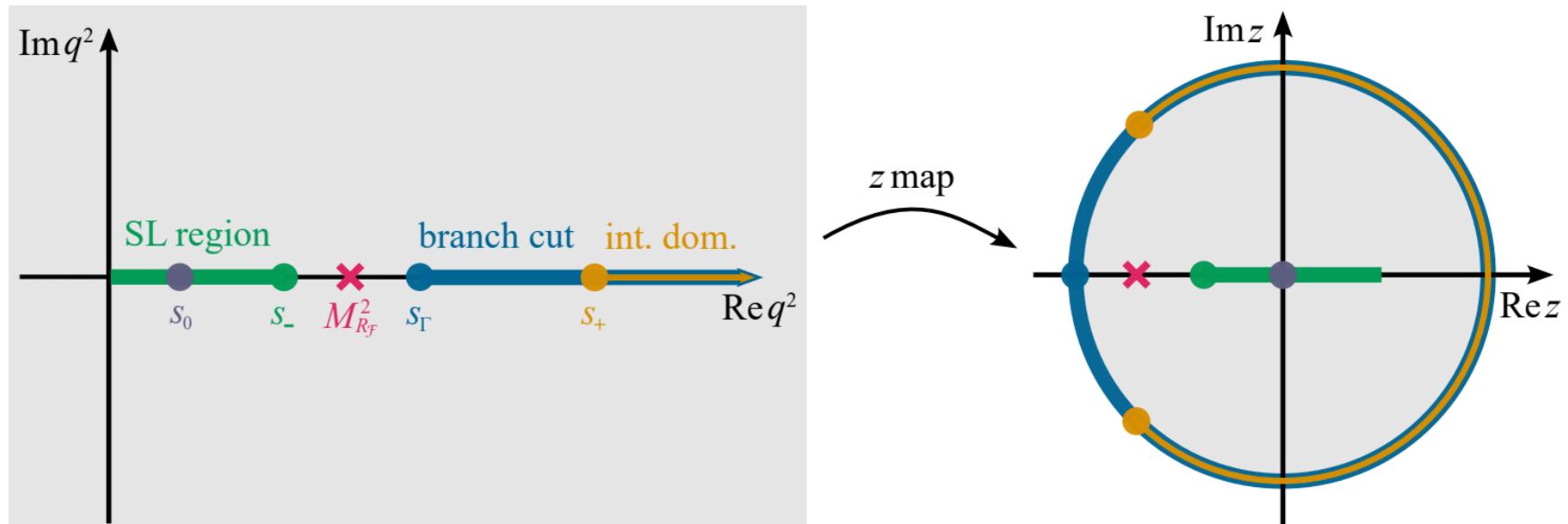


- ▶ Two main approaches: (1) **Lattice QCD** (large  $q^2$   $\star\star\star$ ) (2) **LCSRs** (low  $q^2$ )
- ▶ Two approaches to **LCSRs**, in terms of (1)  $K^*$  LCDAs (2)  $B$  LCDAs
- ▶  $q^2$  dependence parametrized via a (dispersively-bounded) z-expansion

# Form Factors : $q^2$ -dependence from analyticity

Bourrely, Caprini, Lellouch; Boyd, Grinstein, Lebed; Caprini, Lellouch, Neubert; ...

- Conformal mapping: 
$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$



- "z-parametrization":  $\widehat{\mathcal{F}}_\lambda^{(T)}(q^2(z))$  is analytic in  $|z| < 1$  ( $|z_{\text{phys}}| < 0.15$ )

$$\mathcal{F}_\lambda^{(T)}(q^2) = \frac{1}{(q^2 - m_{B_S^*}^2)} \sum_k \alpha_k z(q^2)^k$$

# Form Factors : Dispersive Bounds (BGL + improvement)

Boyd, Grinstein, Lebed 1997; Bharucha, Feldmann, Wick 2014, Gubernari, Reboud, van Dyk, Virto 2023

1. One starts with the two-point function

$$\Pi_{\Gamma}^{\mu\nu}(q) \equiv i \int d^4x e^{iq\cdot x} \langle 0 | T\{J_{\Gamma}^{\mu}(x) J_{\Gamma}^{\dagger,\nu}(0)\} | 0 \rangle = \sum_{\lambda=t,\perp,\parallel,0} \epsilon_{\lambda}^{\mu} \epsilon_{\lambda}^{\nu*} \Pi_{\Gamma}^{(\lambda)}(q^2)$$

2. The **invariant functions** fulfil a once-subtracted dispersion relation:

$$\chi_{\Gamma}^{(\lambda)}(Q^2) = \left[ \frac{\partial}{\partial q^2} \right] \Pi_{\Gamma}^{(\lambda)}(q^2) \Big|_{q^2=Q^2} = \frac{1}{\pi} \int_0^{\infty} ds \frac{\text{Im} \Pi_{\Gamma}^{(\lambda)}(s)}{(s - Q^2)^2} .$$

3. The function  $\chi_{\Gamma}^{(\lambda)}(Q^2)$  can be calculated in an OPE at a suitable subtraction point  $Q^2$

Bharucha, Feldmann, Wick 2014

4. The discontinuity of  $\Pi_{\Gamma}^{(\lambda)}(q^2)$  is the spectral function:

$$\text{Im} \Pi_{\Gamma}^{(\lambda)}(s) \sim \sum_H \langle 0 | J^{\mu} | H \rangle \langle H | J^{\nu\dagger} | 0 \rangle \sim f_{B_S^*}^2 + |F^{BK}|^2 + |F^{BK^*}|^2 + |F^{Bs\phi}|^2 + \dots$$

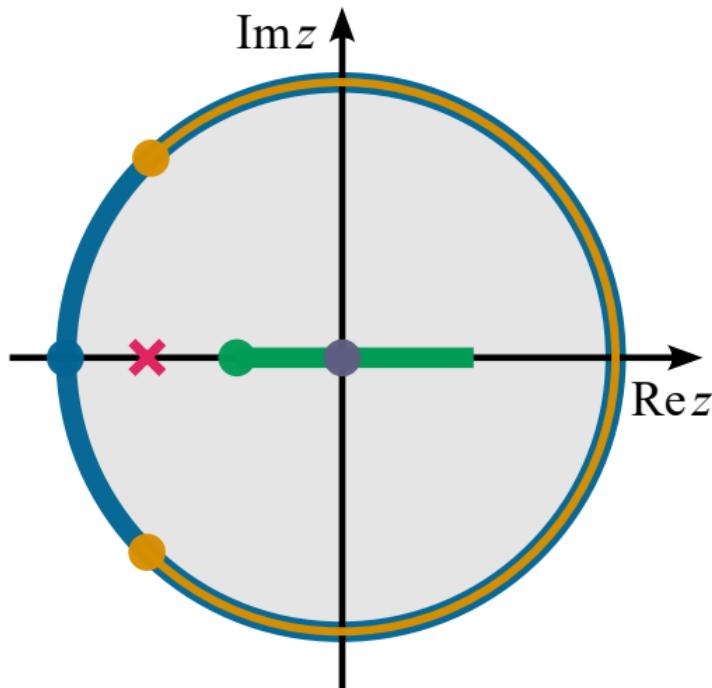
(up to phase-space functions...)

# Form Factors : Dispersive Bounds (BGL + improvement)

Flynn, Jüttner, Tsang 2023; Gubernari, Reboud, van Dyk, Virto 2023

In order to simplify the bound, it is thus convenient to reparametrize:

$$\hat{\mathcal{F}}_{\lambda}^{B \rightarrow M}(q^2) = \mathcal{B}_{\mathcal{F}}(z)\phi_{\mathcal{F}}(z)\mathcal{F}_{\lambda}^{B \rightarrow M}(q^2) = \sum_k \alpha_k^{\mathcal{F}} p_k^{\mathcal{F}}(z)$$



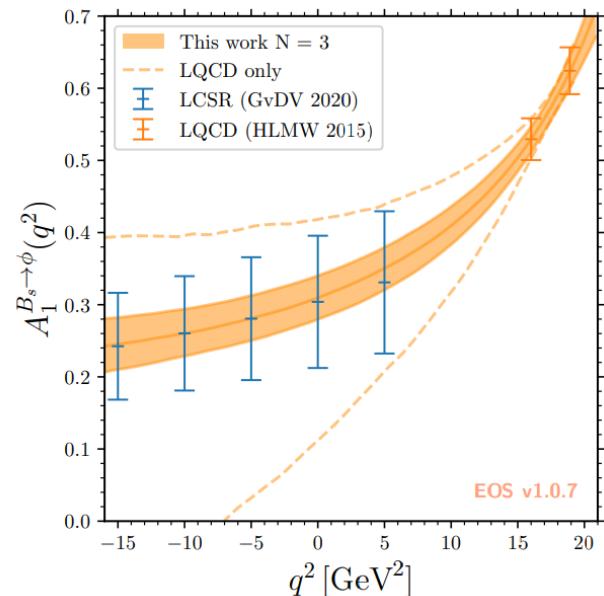
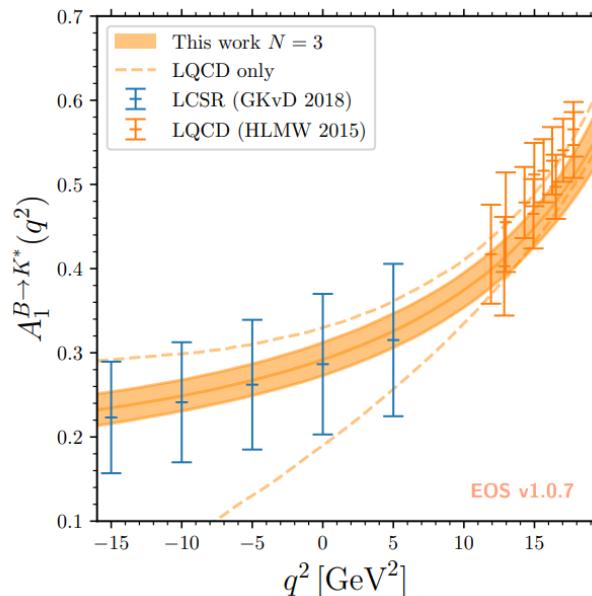
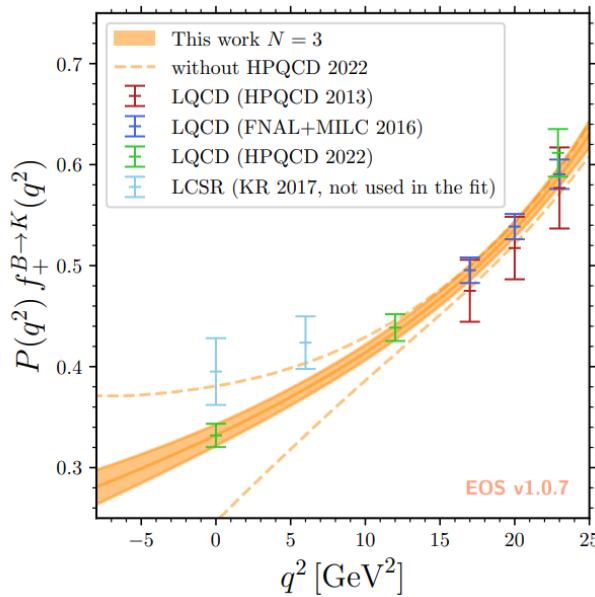
$$\int_{-\alpha_{\mathcal{F}}}^{+\alpha_{\mathcal{F}}} d\theta p_m^{\mathcal{F}}(e^{i\theta})p_n^{\mathcal{F}}(e^{-i\theta}) = \delta_{mn}$$

$$\sum_{B \rightarrow M} \int_{-\alpha_{\mathcal{F}}}^{+\alpha_{\mathcal{F}}} d\theta \left| \hat{\mathcal{F}}_{\lambda}^{B \rightarrow M}(e^{i\theta}) \right|^2 < 1$$

$$\boxed{\sum_{\mathcal{F}, k} |\alpha_k^{\mathcal{F}}|^2 < 1}$$

# Local Form Factors: Results

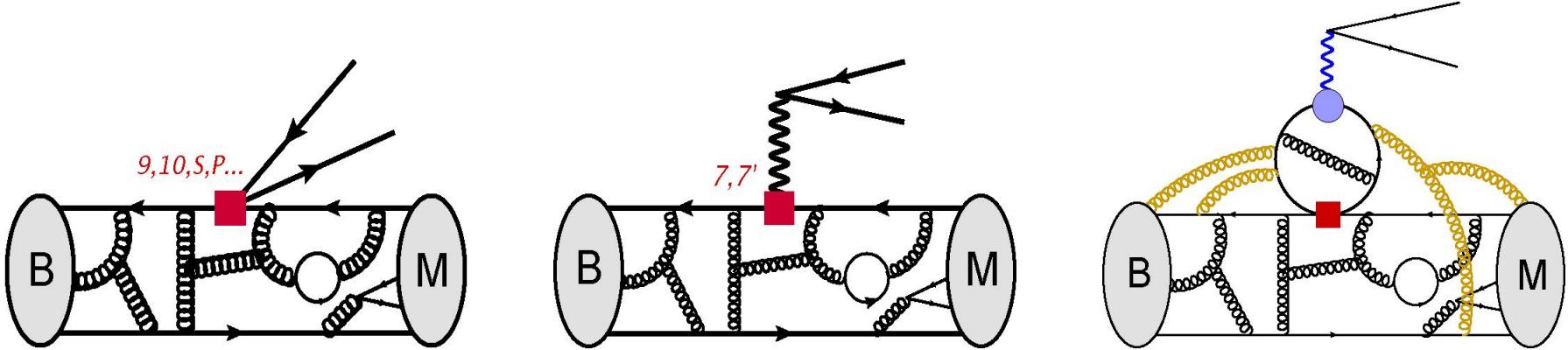
Gubernari, Reboud, van Dyk, Virto 2023



Truncate the series expansion to  $N = 2, 3, 4$

Uncertainties stable for  $N > 2$

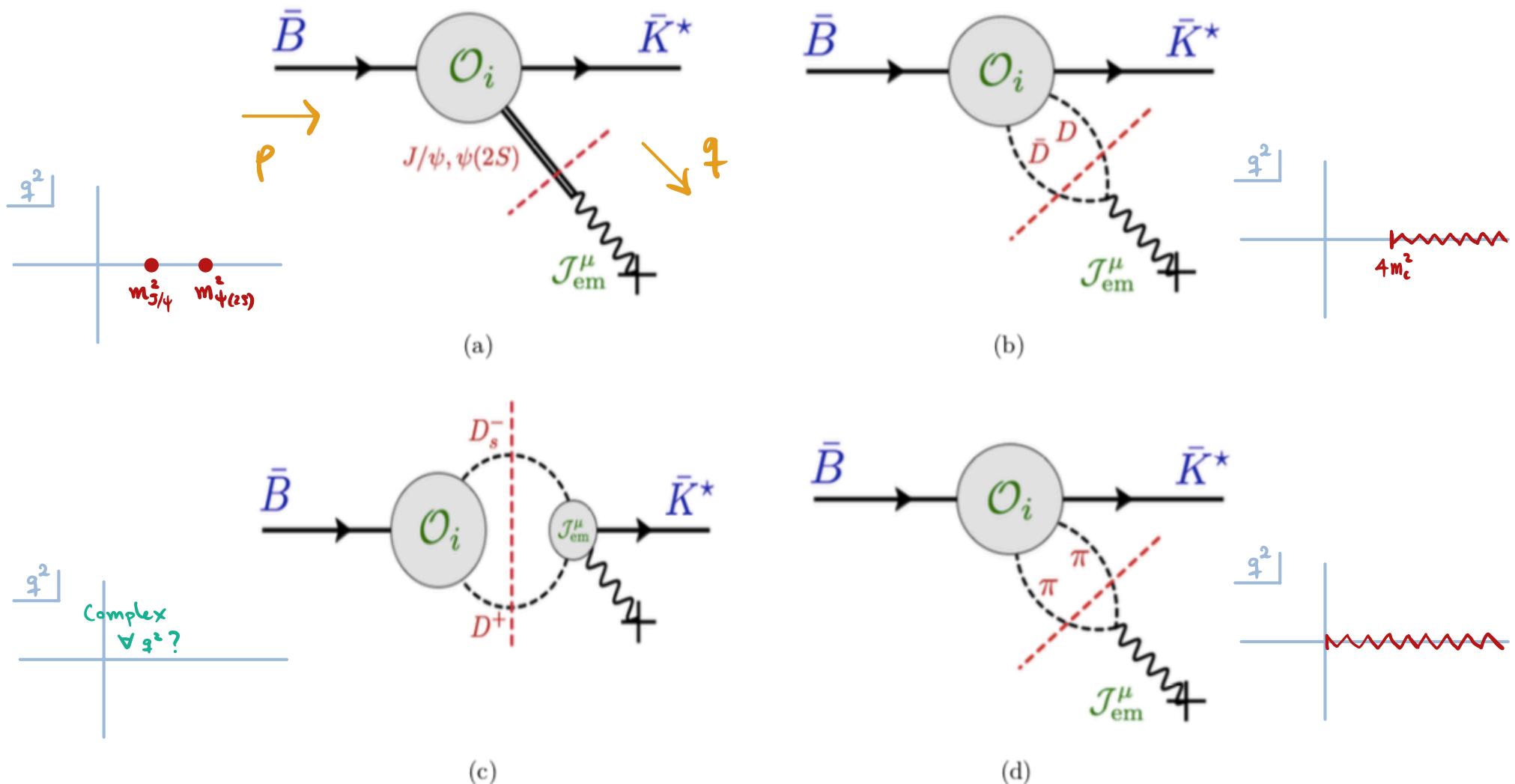
# Non-Local Form Factors



$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[ C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

- Local (Form Factors) :  $\mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{M}_\lambda(k) | \bar{s} \Gamma_\lambda^{(T)} b | \bar{B}(k+q) \rangle$
- Non-Local :  $\mathcal{H}_\lambda(q^2) = i \mathcal{P}_\mu^\lambda \int d^4x e^{iq \cdot x} \langle \bar{M}_\lambda(k) | \mathcal{T} \{ \mathcal{J}_{\text{em}}^\mu(x), \mathcal{C}_i \mathcal{O}_i(0) \} | \bar{B}(q+k) \rangle$

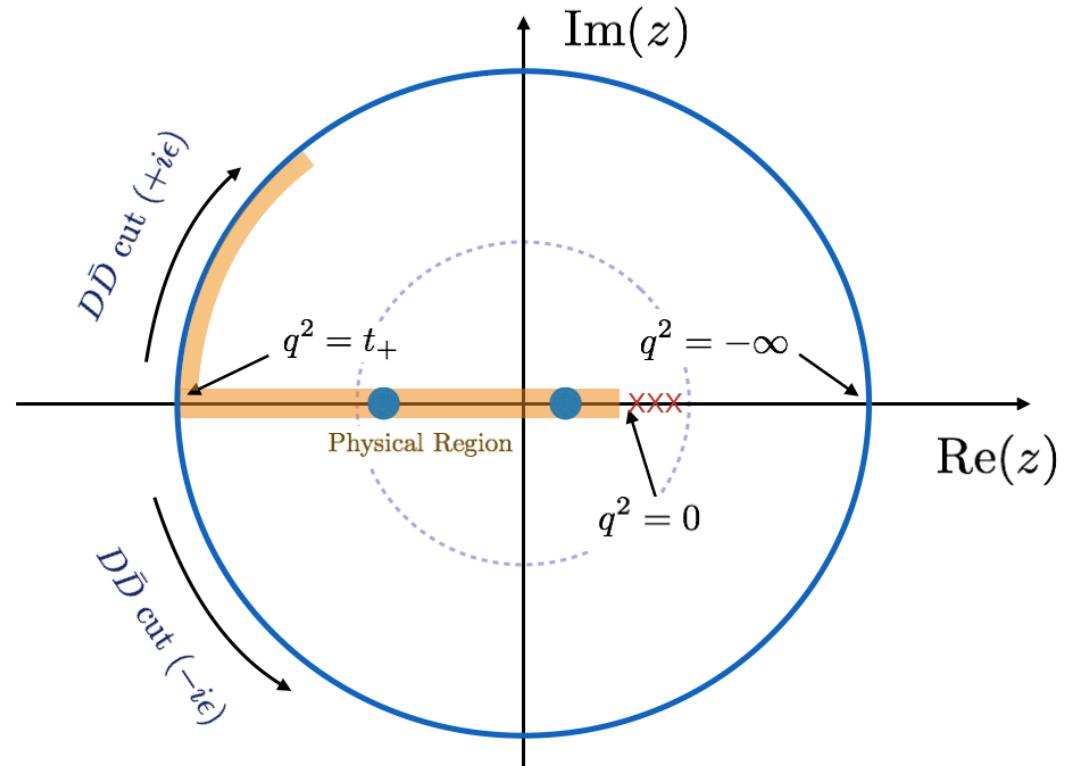
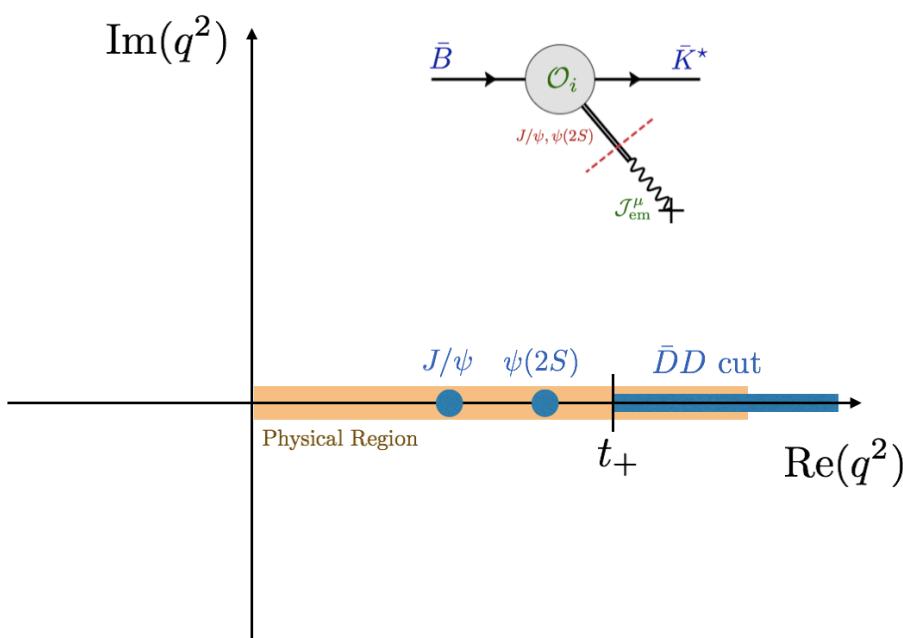
# Non-Local Form Factors: Analytic structure



# Analytic continuation to physical $q^2$

Bobeth, Chrzaszcz, van Dyk, Virto 2017

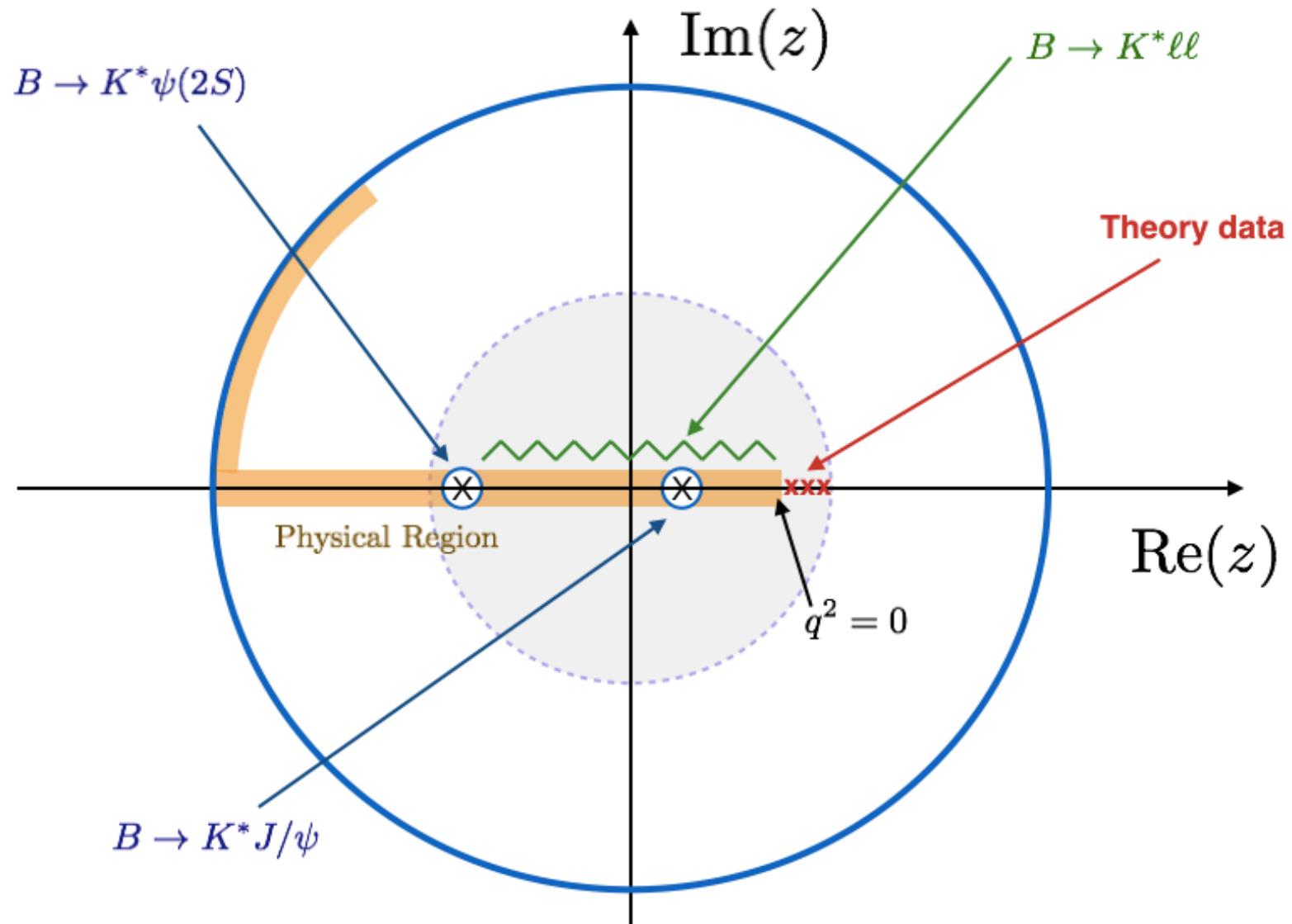
$z$ -parametrisation for  $\mathcal{H}_\lambda(q^2)$



- $\hat{\mathcal{H}}_\lambda(q^2(z)) = (q^2 - M_{J/\psi}^2)(q^2 - M_{\psi(2S)}^2) \mathcal{H}_\lambda(q^2)$  is analytic in  $|z| < 1$
- Taylor expand  $\hat{\mathcal{H}}_\lambda(z)$  around  $z = 0$ : 
$$\hat{\mathcal{H}}_\lambda(z) = \left[ \sum_{k=0}^K \alpha_k^{(\lambda)} z^k \right] \mathcal{H}_\lambda(z)$$
- Expansion needed for  $|z| < 0.52$  ( $-7 \text{ GeV}^2 \leq q^2 \leq 14 \text{ GeV}^2$ )

# Fit to $z$ -parametrisation

Bobeth, Chrzaszcz, van Dyk, Virto 2017



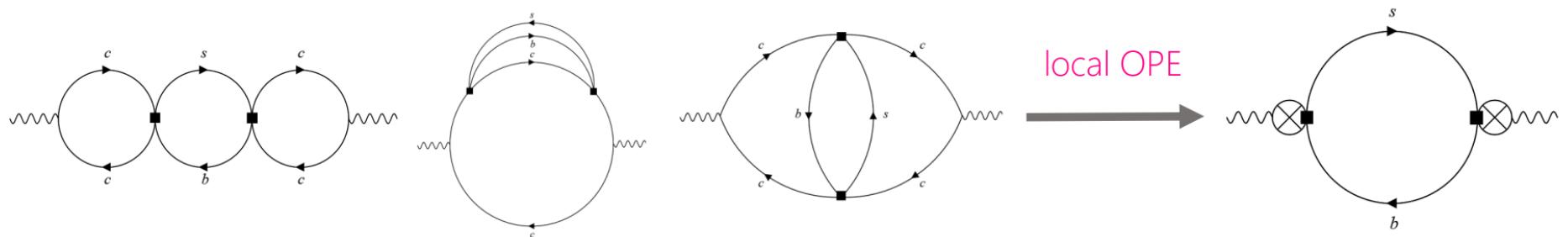
## 1. Use the correlation function

$$\Pi(q) \equiv i \int d^4x e^{iq \cdot x} \langle 0 | T \{ O^\mu(q; x), O^{\mu, \dagger}(q; 0) \} | 0 \rangle$$

where

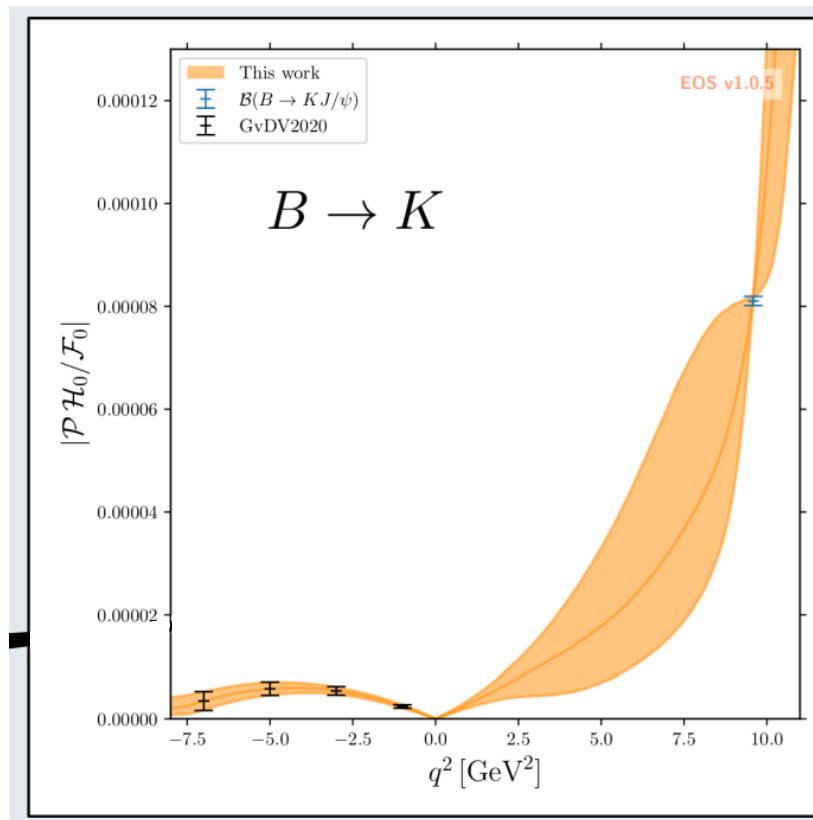
$$O^\mu(q; x) = -i \int d^4y e^{+iq \cdot y} T \{ j_{\text{em}}^\mu(x + y), (C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2)(x) \}$$

## 2. Calculate in OPE region



3. Note that (skematic)  $\text{Im} \Pi(q^2) \sim \sum_\lambda |\mathcal{H}_\lambda|^2 + \text{positive}$

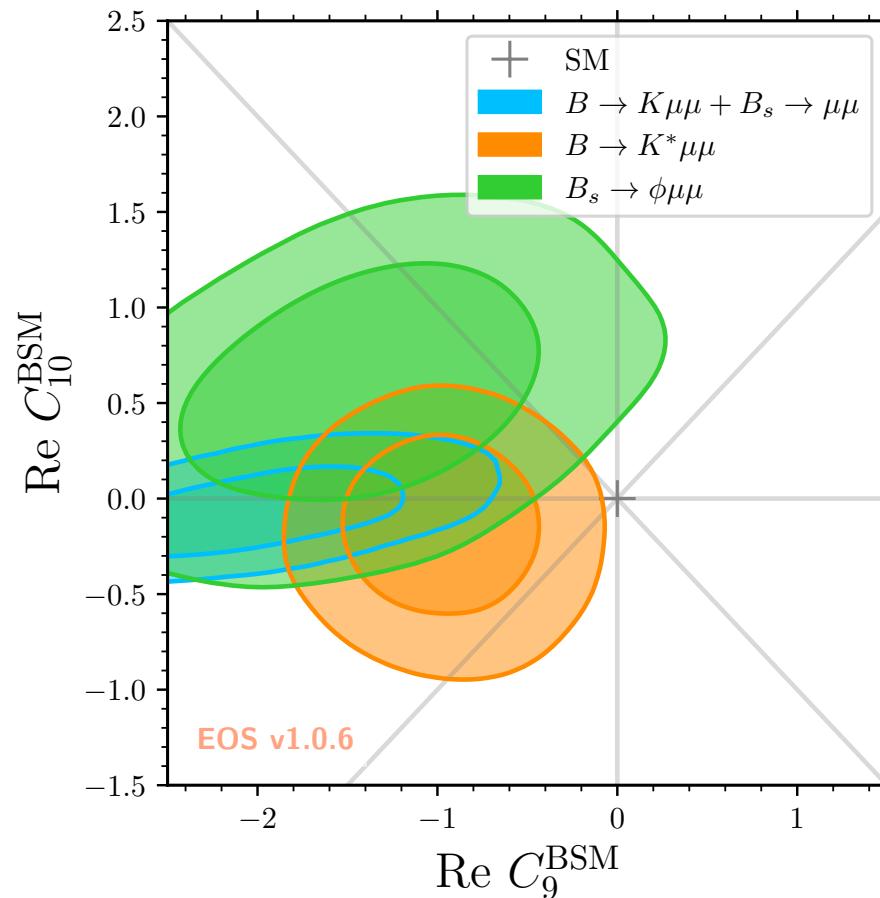
Non-local form factor fitted to LCOPE and  $J/\psi$  data



Use under-constrained fit ( $N = 5$ ) which saturates dispersive bound

All p-values are larger than 11%

Proof-of-concept fit to three modes separately.



Updated blue contour with:

new HPQCD'22 [2207.12468] form factors  
new  $B_s \rightarrow \mu\mu$  average [CMS 2212.10311]

# Summary

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Anomalies in  $b \rightarrow s\mu\mu$  recently extended to  $b \rightarrow see$

The understanding of rare  $B$  decay measurements requires the knowledge of **local and non-local form factors**

$$\mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{M}_\lambda(k) | \bar{s} \Gamma_\lambda^{(T)} b | \bar{B}(k+q) \rangle$$

$$\mathcal{H}^\mu(q^2) = i \int d^4x e^{iq \cdot x} \langle \bar{M}_\lambda(k) | T\{ j_{\text{em}}^\mu(x), [\bar{b}_L \gamma^\nu c_L] [\bar{c}_L \gamma_\nu s](0) \} | \bar{B}(q+k) \rangle$$

Currently based on:

1. Calculation
2. Analytic parametrization
3. Dispersive bound

If you are into hadronic physics, you can try to get involved

END

$B$  mesons mix and decay due to  $\mathcal{L}_{\text{Weak}} + \mathcal{L}_{\text{BSM}}$ ?

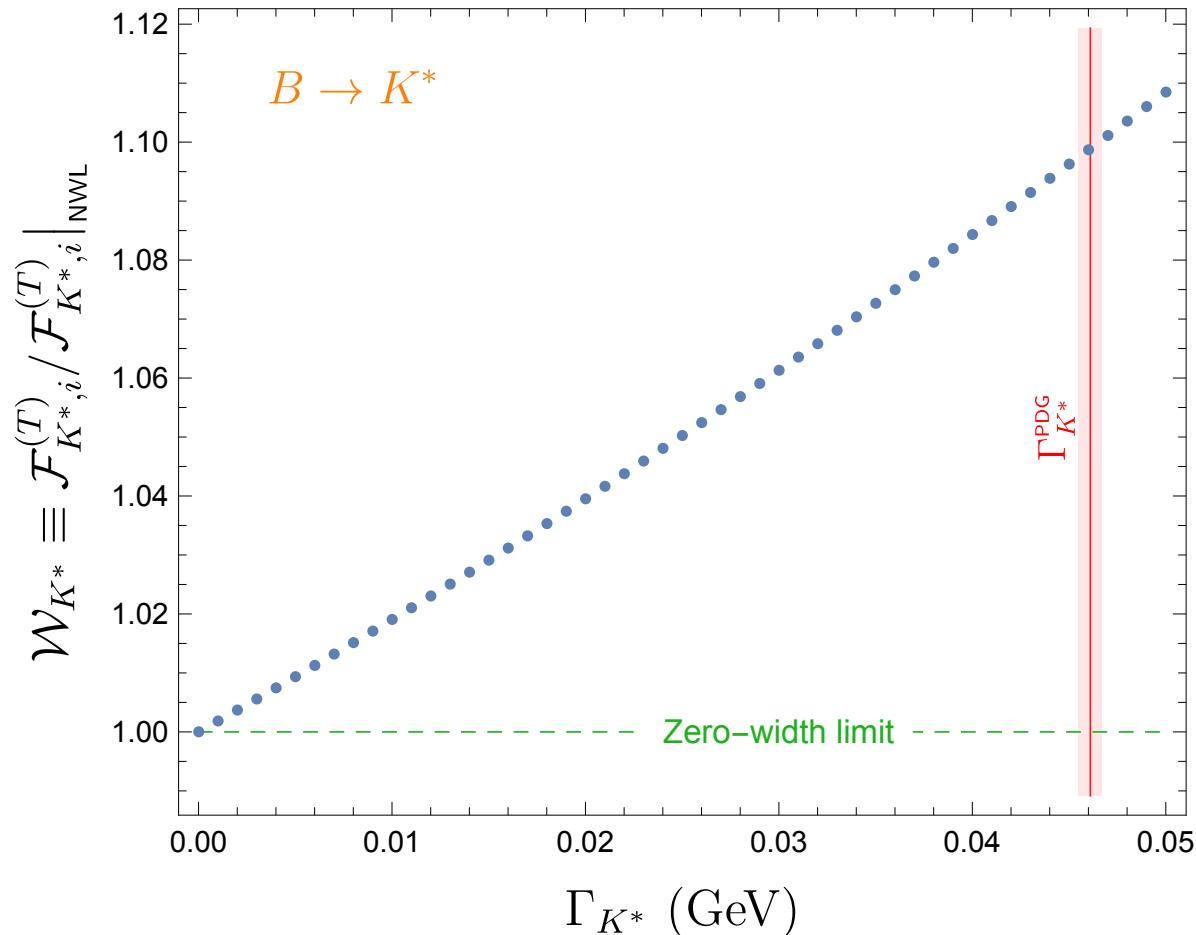
For  $m_B \ll M_W, M_{\text{BSM}}$  we use an EFT :  $\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{QCD+QED}} + \sum_i C_i \mathcal{O}_i$

Class	Flavour structure	Number of Ops.	Other flavours	ADM	Example process
Class I	$\bar{s}b \bar{s}b$	5+3	$\bar{d}b \bar{d}b$	$\hat{\gamma}_I$	$B_q - \bar{B}_q$ mixing
Class II	$\bar{u}b \bar{\ell} \nu_{\ell'}$	$(2 + 3) \times 9$	$\bar{c}b \bar{\ell} \nu_{\ell'}$	$\hat{\gamma}_{II}$	$\bar{B}_d \rightarrow \pi^+ \mu^- \bar{\nu}$
Class III	$\bar{s}b \bar{u}c$	10+10	$\bar{s}b \bar{c}u$ $\bar{d}b \bar{u}c$ $\bar{d}b \bar{c}u$	$\hat{\gamma}_{III}$	$B^- \rightarrow \bar{D}^0 K^-$
Class IV	$\bar{s}b \bar{s}d$	5+5	$\bar{d}b \bar{d}s$ $\bar{b}s \bar{b}d$	$\hat{\gamma}_{IV}$	$B^- \rightarrow \bar{K}^0 K^-$
Class V	$\bar{s}b \bar{q}q$ $\bar{s}b F, \bar{s}b G$ $\bar{s}b \bar{\ell}\ell$	57+57	$\bar{d}b \bar{q}q$ $\bar{d}b F, \bar{d}b G$ $\bar{d}b \bar{\ell}\ell$	$\hat{\gamma}_V$	$\bar{B}_d \rightarrow D^+ D_s^-$ $\bar{B}_d \rightarrow X_s \gamma$ $B^- \rightarrow K^- \mu^+ \mu^-$
Class Vb	$\bar{s}b \bar{\ell}\ell', \ell \neq \ell'$	$(5 + 5) \times 6$	$\bar{d}b \bar{\ell}\ell'$	$\hat{\gamma}_{Vb}$	$\bar{B}_s \rightarrow \mu^- \tau^+$
Class V $\nu$	$\bar{s}b \bar{\nu}_{\ell} \nu_{\ell'}$	$(1 + 1) \times 9$	$\bar{d}b \bar{\nu}_{\ell} \nu_{\ell'}$	zero	$B^- \rightarrow K^- \bar{\nu}\nu$

Aebischer, Fael, Greub, Virto 2017

# Form factors for unstable mesons (e.g., $K^*$ ): width effects

Descotes-Genon, Khodjamirian, Virto 2019

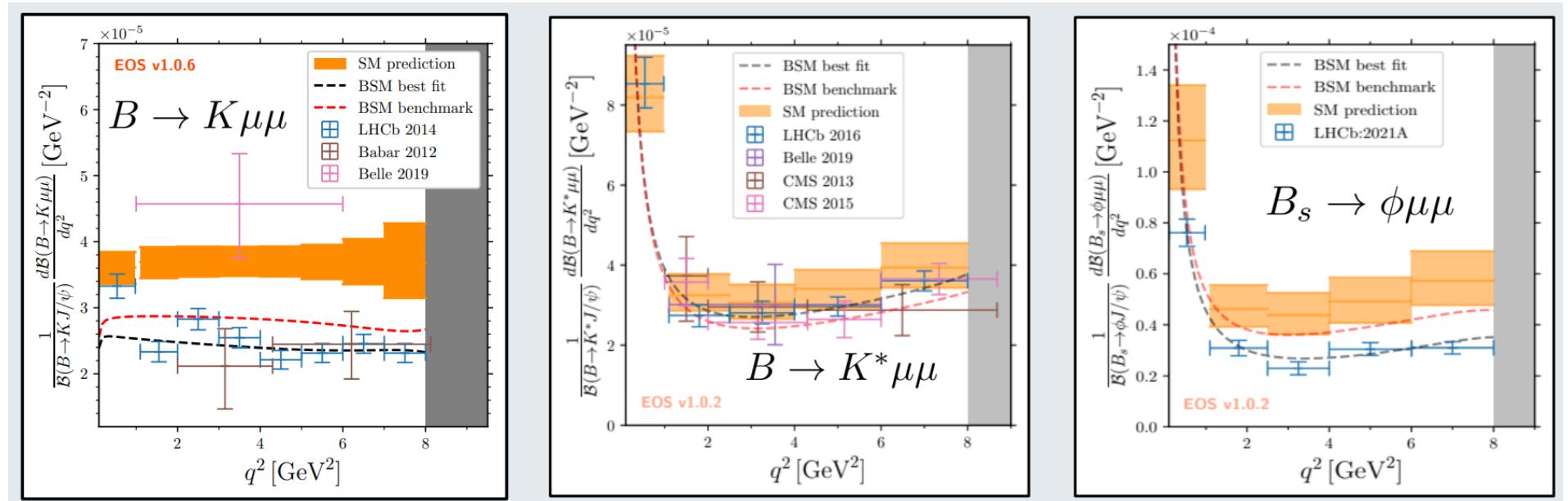


Crucial input:  $\tau \rightarrow K\pi\nu$

$$\begin{aligned}\mathcal{W}_{K^*} &\approx 1 + 1.9 \frac{\Gamma_{K^*}}{m_{K^*}} \\ \mathcal{W}_{K^*} &= 1.09 \pm 0.01\end{aligned}$$

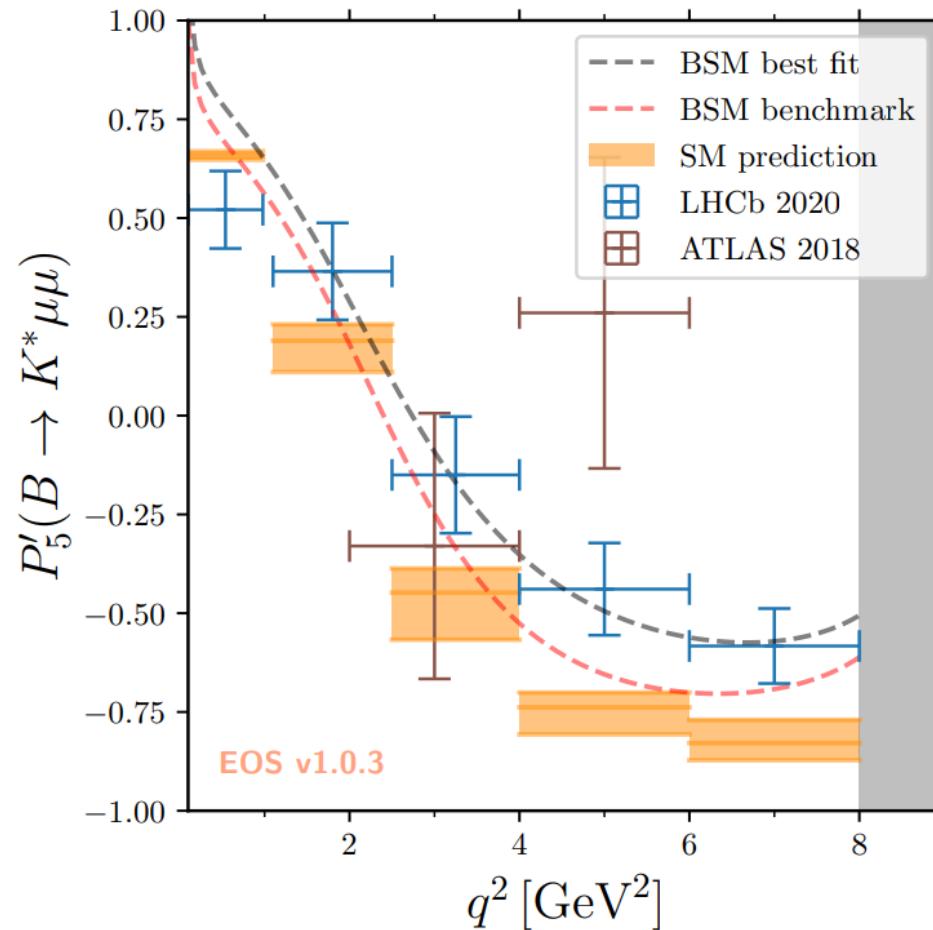
⇒ BRs are corrected by a factor  $|\mathcal{W}_{K^*}|^2 \simeq 1.2$  (increasing anomalies)

## Branching Fractions



Conservatively accounting for the non-local form factors does not improve agreement with data

## Angular observables



Conservatively accounting for the non-local form factors does not improve agreement with data