SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL
FROM QUARKS AND GLUONS TO HADRONS AND NUCLEI

Cluster of Excellence PRISMA+
Precision Physics, Fundamental Interactions and Structure of Matter

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Deutsche Forschungsgemeinschaft DFG

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Neutrons and nuclei as a precision laboratory for Vud and CKM unitarity

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HADRON 2023, Genova, June 5-9, 2023

Understanding β Decays: A Cornerstone of the Standard Model

Existence of neutrinos to explain the continuous β spectrum (Pauli, 1930)

Contact theory of β decay (Fermi, 1933)

Parity violation in β decay (Lee, Yang 1956 & Wu 1957)

V - A theory (Sudarshan & Marshak and Gell-Mann & Feynman, 1957)

Radiative corrections to 4-Fermi theory: important step to the Standard Model

RC to muon decay UV finite for V-A —> $G_F = G_\mu = 1.1663788(7) \times 10^{-5} GeV^{-2}$

But RC to neutron decay - log UV divergent!

UV behavior of β decay rate at 1-loop (Sirlin, 1967) $\frac{\alpha}{2\pi} P^0 d^3p \; 3[1+2\bar{Q}] \ln(\Lambda/M)$

$\bar{Q}$: average charge of fields involved: $1 + 2\bar{Q}_{\mu,\nu_\mu} = 0$ but $1 + 2\bar{Q}_{n,p} = 2$

Standard Model with massive W,Z-bosons (Glashow-Salam-Weinberg, 1967)

Precision, Universality and CKM unitarity

In SM the same coupling of W-boson to leptons and hadrons, $G_V = G_\mu$

Before RC were included: $G_V \sim 0.98 G_\mu$

Large $\log(M_Z/M_p)$ in RC for neutron —> $G_V \sim 0.95 G_\mu$

Kaon and hyperon decays? ($\Delta S = 1$) — even lower rates!

Cabibbo: strength shared between 2 generations

$$|G_V^{\Delta S=0}| = \cos\theta_C G_\mu$$

$$|G_V^{\Delta S=1}| = \sin\theta_C G_\mu$$

Cabibbo unitarity: $\cos^2\theta_C + \sin^2\theta_C = 1$

Kobayashi & Maskawa: 3 flavors + CP violation — CKM matrix V

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

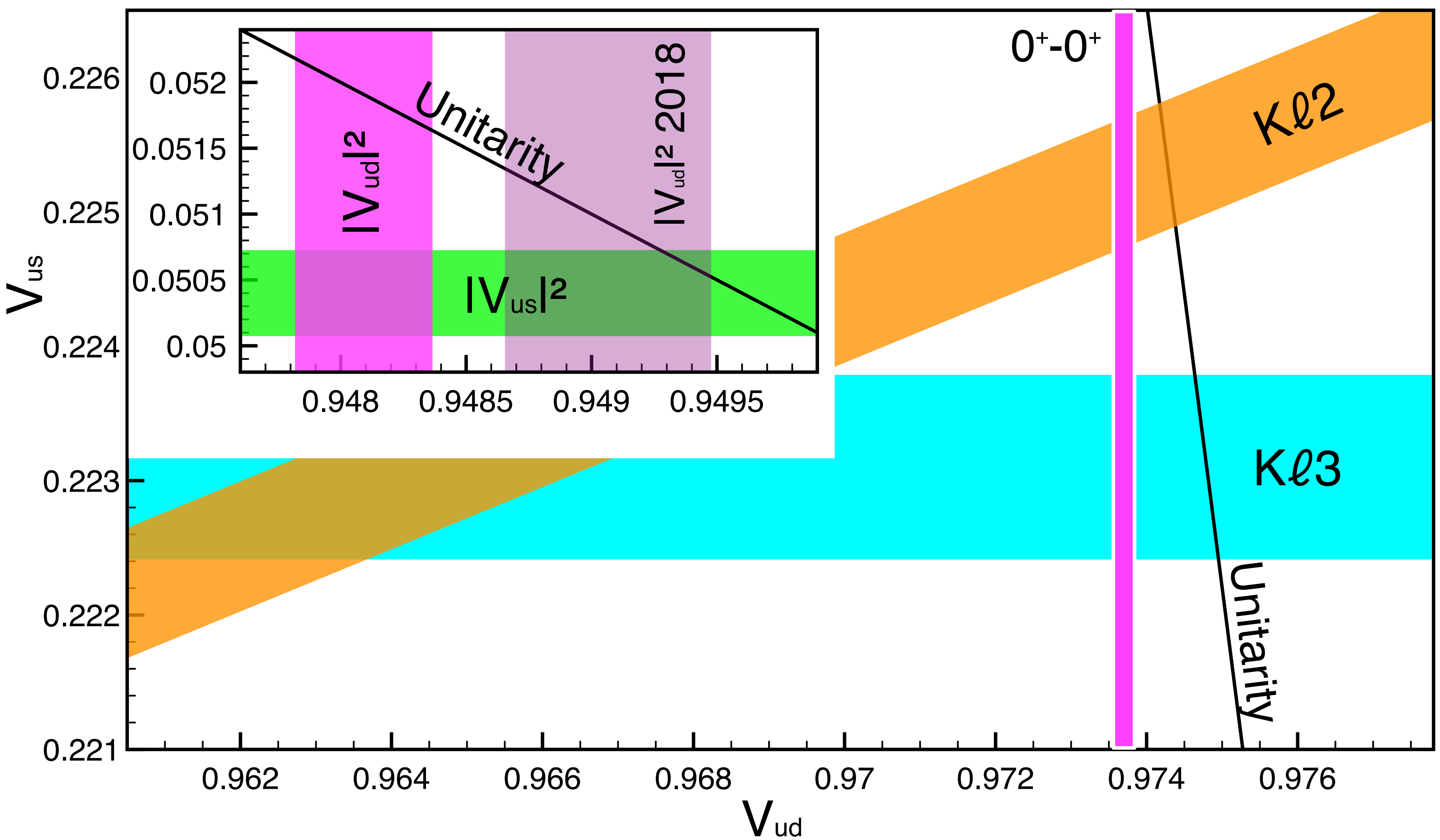
CKM unitarity - completeness of the SM: $VV^\dagger = \mathbf{1}$

Top row unitarity constraint: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

Detailed understanding of β decays largely shaped the Standard Model

Status of top-row CKM unitarity

$$\underset{\sim 0.95}{|V_{ud}|^2} + \underset{\sim 0.05}{|V_{us}|^2} + \underset{\sim 10^{-5}}{\cancel{|V_{ub}|^2}} = 0.9985(6)_{V_{ud}}(4)_{V_{us}}$$



Inconsistencies between measurements of V_{ud} and V_{us} and SM predictions

Main reason for Cabibbo-angle anomaly: shift in V_{ud} (and small uncertainties?)

Status of V_{ud}

Theory: Major reduction of uncertainties in the past few years

Universal correction Δ_R^V to free and bound neutron decay

Identified 40 years ago as the bottleneck for precision improvement

Novel approach dispersion relations + experimental data + EFT + lattice QCD

Δ_R^V uncertainty: factor 2 reduction

δ_{NS} uncertainty: factor 3 increase!!!

C-Y Seng et al., Phys.Rev.Lett. 121 (2018) 24, 241804;
C-Y Seng, MG, M.J. Ramsey-Musolf, Phys.Rev. D 100 (2019) 1, 013001;
MG, Phys.Rev.Lett. 123 (2019) 4, 042503;
C-Y Seng, X. Feng, MG, L-C Jin, Phys.Rev. D 101 (2020) 11, 111301;
A. Czarnecki, B. Marciano, A. Sirlin, Phys.Rev. D 100 (2019) 7, 073008

RC to semileptonic pion decay

δ Factor 3 reduction

X. Feng, MG, L-C Jin, P-X Ma, C-Y Seng, Phys.Rev.Lett. 124 (2020) 19, 192002

Experiment

$g_A = -1.27641(56)$

Factor 4 reduction

$g_A = -1.2677(28)$

PERKEO-III *B. Märkisch et al, Phys.Rev.Lett. 122 (2019) 24, 242501*

aSPECT *M. Beck et al, Phys. Rev. C101 (2020) 5, 055506*

$\tau_n = 877.75(28)^{+16}_{-12}$

Factor 2-3 reduction

$\tau_n = 887.7(2.3)$

UCNτ *F. M. Gonzalez et al. Phys. Rev. Lett. 127 (2021) 162501*

BL1 (NIST) *Yue et al, PRL 111 (2013) 222501*

Status of V_{ud}

0^+-0^+ nuclear decays: long-standing champion

$$|V_{ud}|^2 = \frac{2984.43s}{\mathscr{F}t(1+\Delta_R^V)}$$

$$|V_{ud}^{0^+-0^+}| = 0.97370\,(1)_{exp,\,nucl}\,(3)_{NS}\,(1)_{RC}[3]_{total}$$

Nuclear uncertainty x 3

Neutron decay: discrepancies in lifetime τ_n and axial charge g_A; competitive!

$$|V_{ud}|^2 = \frac{5024.7\text{ s}}{\tau_n(1+3g_A{}^2)(1+\Delta_R)}$$

Single best measurements only

$$|V_{ud}^{\text{free n}}| = 0.9733\,(2)_{\tau_n}\,(3)_{g_A}\,(1)_{RC}[4]_{total}$$

PDG average

$$|V_{ud}^{\text{free n}}| = 0.9733\,(3)_{\tau_n}\,(8)_{g_A}\,(1)_{RC}[9]_{total}$$

RC not a limiting factor: more precise experiments a-coming

Pion decay $\pi^+ \to \pi^0 e^+ \nu_e$: theoretically cleanest, experimentally tough

$$|V_{ud}|^2 = \frac{0.9799}{(1+\delta)}\frac{\Gamma_{\pi\ell 3}}{0.3988(23)\,\text{s}^{-1}}$$

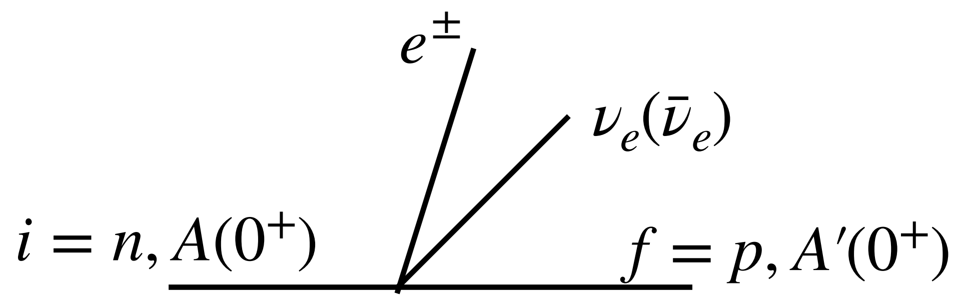
$$|V_{ud}^{\pi\ell 3}| = 0.9739\,(27)_{exp}\,(1)_{RC}$$

Future exp: 1 o.o.m. (PIONEER)

RC to nuclear beta decay: overall setup

RC to nuclear beta decay: overall setup

Tree-level amplitude



$\sim V_{ud}$

Radiative corrections to tree-level amplitude $\sim \alpha/2\pi \approx 10^{-3}$

Precision goal for V_{ud} extraction 1×10^{-4}

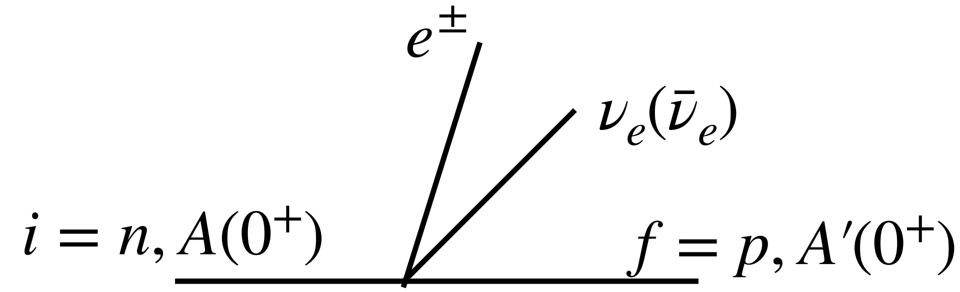
Electron carries away energy E < Q-value of a decay

E-dep RC: $\frac{\alpha}{2\pi}\left(\frac{E}{\Lambda}, \ln\frac{E}{\Lambda}, \dots\right)$

Energy scales Λ

RC to nuclear beta decay: overall setup

Tree-level amplitude



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Energy scales Λ

Λ

Weak boson scale
$M_Z, M_W \sim 90\,\mathrm{GeV}$

Hadronic scale
$\Lambda_{\mathrm{had}} = 300\,\mathrm{MeV}$

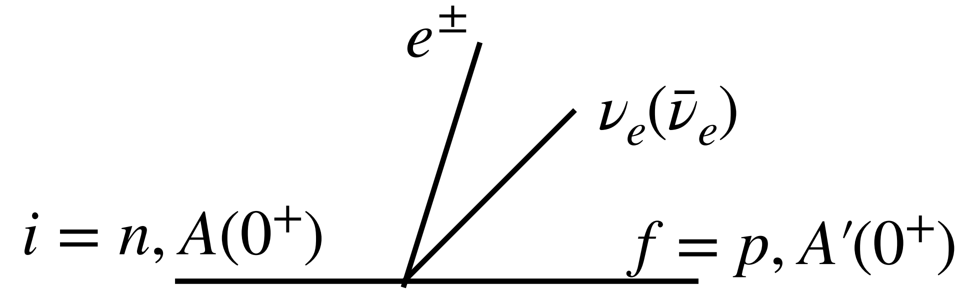
Nuclear scale
$\Lambda_{\mathrm{nuc}} = 10 - 30\,\mathrm{MeV}$

Decay Q-value (endpoint energy)
$Q_{if} = M_i - M_f = 1 - 10\,\mathrm{MeV}$

Electron mass
$m_e \approx 0.5\,\mathrm{MeV}$

RC to nuclear beta decay: overall setup

Tree-level amplitude



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Energy scales Λ

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Weak boson scale
$M_Z, M_W \sim 90\,\mathrm{GeV}$

Hadronic scale
$\Lambda_{\mathrm{had}} = 300\,\mathrm{MeV}$

Same for all nuclei

Nuclear scale
$\Lambda_{\mathrm{nuc}} = 10 - 30\,\mathrm{MeV}$

Nuclear structure dependent
(QCD)

Decay Q-value (endpoint energy)
$Q_{if} = M_i - M_f = 1 - 10\,\mathrm{MeV}$

Nucleus-specific

Electron mass
$m_e \approx 0.5\,\mathrm{MeV}$

Nuclear structure independent
(QED)

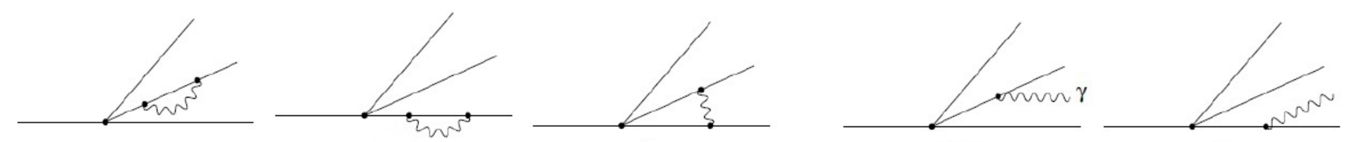
RC to beta decay: overall setup

Generically: only IR and UV extremes feature large logarithms!
Works by Sirlin (1930-2022) and collaborators: all large logs under control

IR: Fermi function + Sirlin function

Fermi function: resummation of $(Z\alpha)^n$ —> Dirac - Coulomb problem

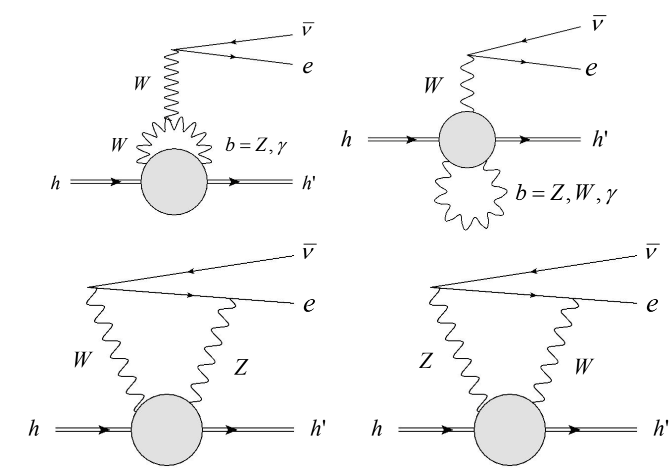
Sirlin function (outer correction):
All IR-div. pieces beyond Coulomb distortion



UV: large EW logs + pQCD corrections

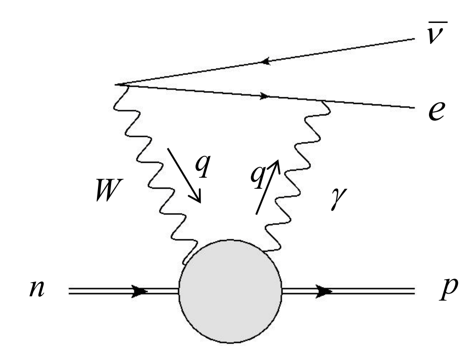
Inner RC:
energy- and model-independent

W,Z - loops
UV structure of SM



γW-box: sensitive to all scales

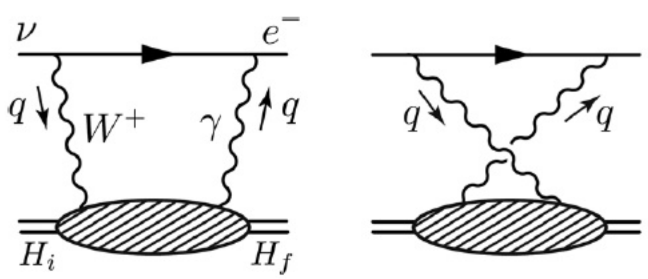
New method for computing EW boxes: dispersion theory
Combine exp. data with pQCD, lattice, EFT, ab-initio nuclear



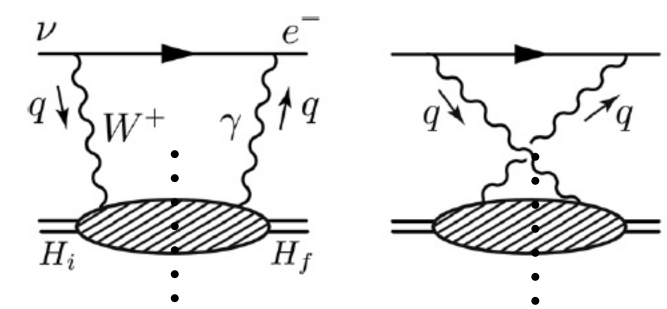
Dispersion Formalism for γW-box

γW-box from dispersion relations

Model-dependent part or RC: γW-box



Generalized Compton tensor
time-ordered product — complicated!



Commutator (Im part) - only on-shell
hadronic states — related to data

$$\int dx e^{iqx} \langle H_f(p) \,|\, T\{J^\mu_{em}(x) J^{\nu,\pm}_W(0)\} \,|\, H_i(p)\rangle$$

Generalized (non-diagonal) Compton amplitudes

$$\int dx e^{iqx} \langle H_f(p) \,|\, [J^\mu_{em}(x), J^{\nu,\pm}_W(0)] \,|\, H_i(p)\rangle$$

Interference structure functions

Physics of taming model dependence with dispersion relations:

virtual photon polarizes the nucleon/nucleus;
Long- and intermediate-range part of the box sensitive to hadronic **polarizabilities**
Polarizabilities related to the excitation spectrum via dispersion relation
(Cf. Kramers-Kronig)

Universal RC from dispersion relations

Interference γW structure functions $$\mathrm{Im}T^{\mu\nu}_{\gamma W} = \ldots + \frac{i\varepsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta}{2(pq)}F_3^{\gamma W}(x,Q^2)$$

After some algebra (isospin decomposition, loop integration)

$$\Box^{b,\mathrm{e}}_{\gamma W}(E_e) = \frac{\alpha}{\pi}\int_0^\infty dQ^2 \frac{M_W^2}{M_W^2+Q^2}\int_{\nu_{\mathrm{thr}}}^\infty \frac{d\nu'}{\nu'}\frac{\nu'+2\sqrt{\nu'^2+Q^2}}{(\nu'+\sqrt{\nu'^2+Q^2})^2}\frac{F_{3,-}(\nu',Q^2)}{Mf_+(0)} + \mathcal{O}(E_e^2)$$

$$\Box^{b,\mathrm{o}}_{\gamma W}(E_e) = \frac{2\alpha E_e}{3\pi}\int_0^\infty dQ^2 \int_{\nu_{\mathrm{thr}}}^\infty \frac{d\nu'}{\nu'}\frac{\nu'+3\sqrt{\nu'^2+Q^2}}{(\nu'+\sqrt{\nu'^2+Q^2})^3}\frac{F_{3,+}(\nu',Q^2)}{Mf_+(0)} + \mathcal{O}(E_e^3)$$

Advantage to previous approach (Marciano & Sirlin):

- Explicit 2-fold integral, isospin decomposition and energy dependence

Nachtmann moments play a role in DIS

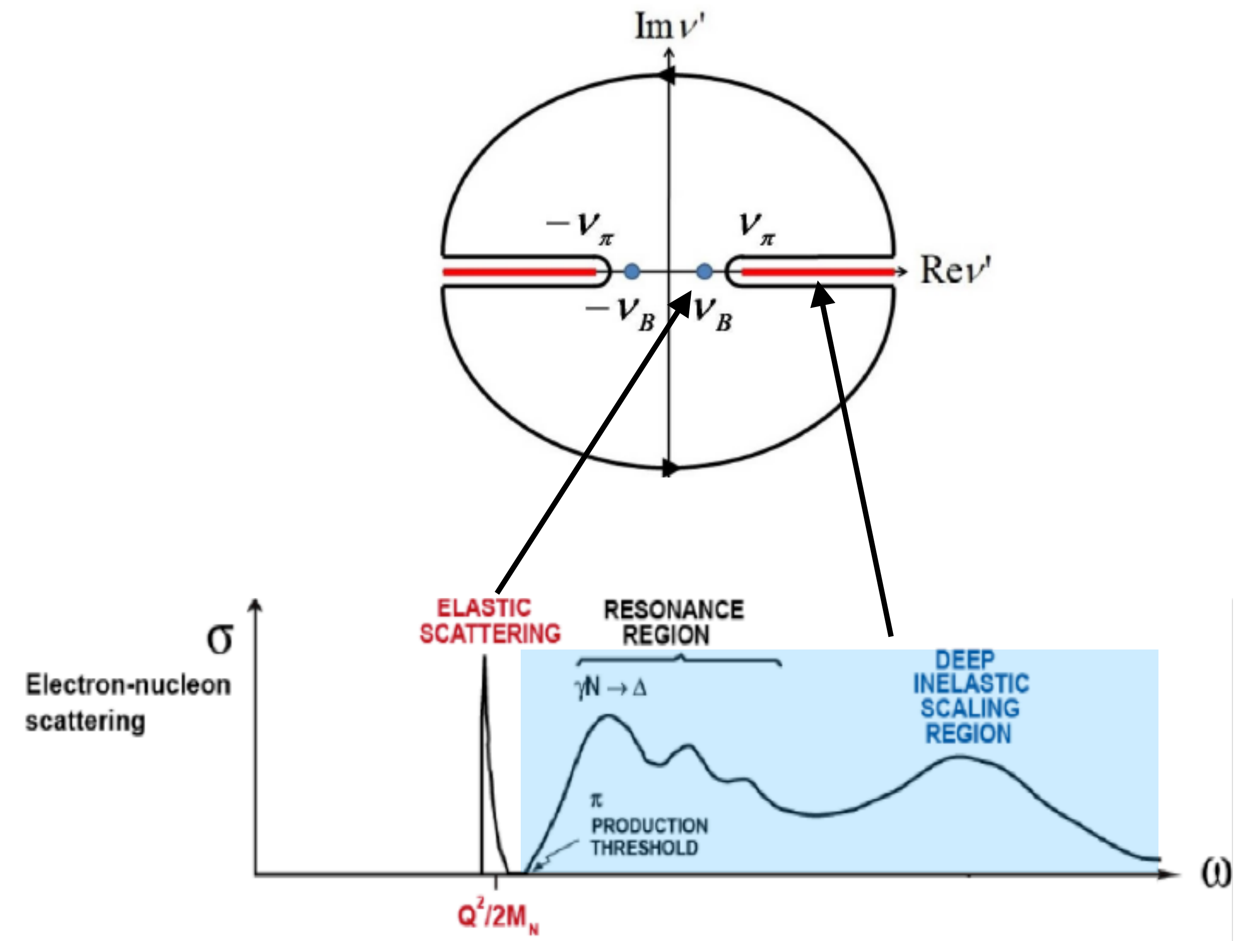
$$M_3(n,Q^2) = \frac{n+1}{n+2}\int_0^1 \frac{dx\xi^n}{x^2}\frac{2x(n+1)-n\xi}{n+1}F_3(x,Q^2), \qquad \xi = \frac{2x}{1+\sqrt{1+4M^2x^2/Q^2}}$$

Hiding the nu-integration in the Nachtmann moments:

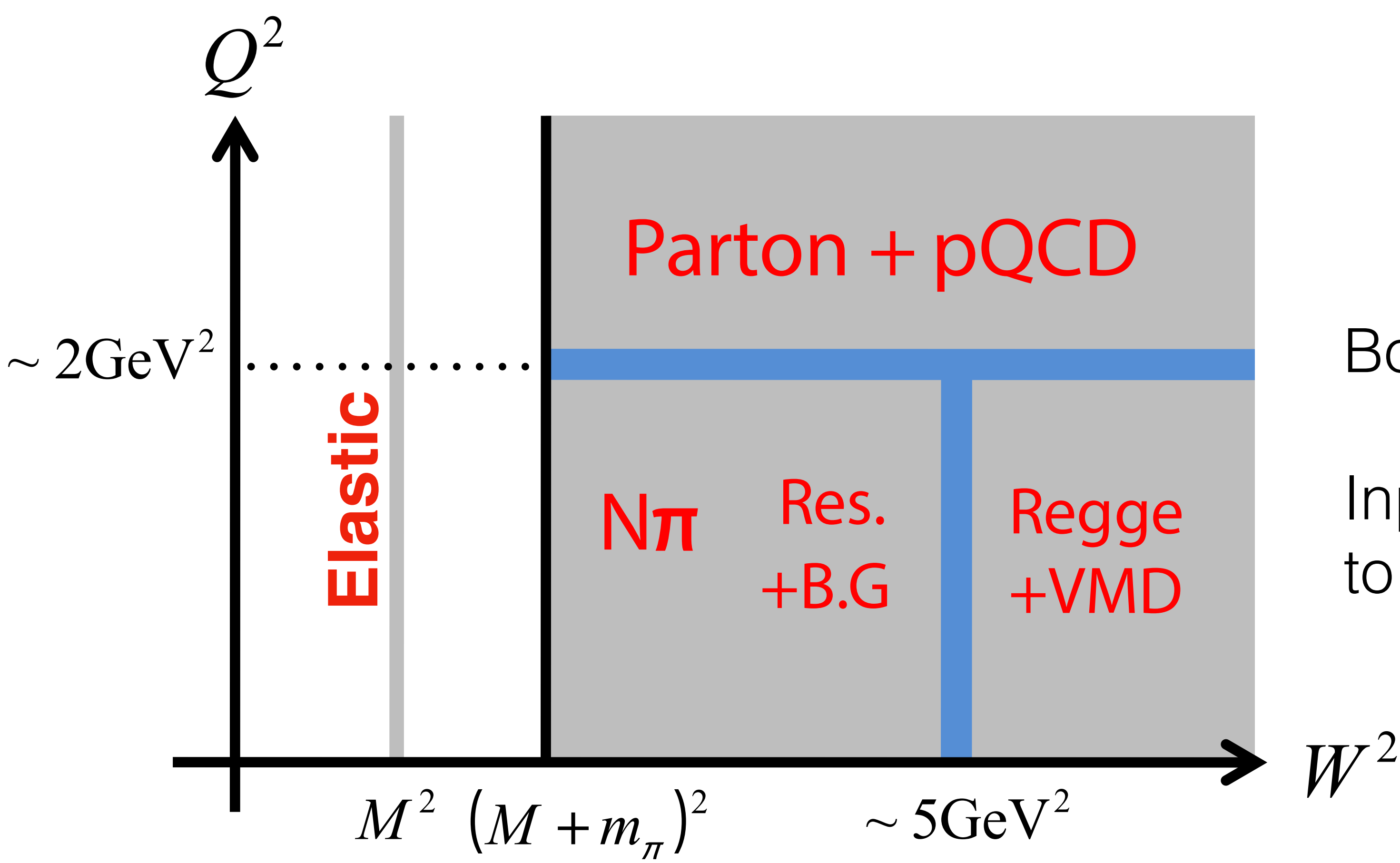
$$\Box^{b}_{\gamma W}(E_e) = \frac{3\alpha}{2\pi}\int_0^\infty \frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}\left[M_{3,-}(1,Q^2) + \frac{8E_eM}{9Q^2}M_{3,+}(2,Q^2)\right] + \mathcal{O}(E_e^2)$$

Input into dispersion integral

Dispersion in energy: $W^2 = M^2 + 2M\nu - Q^2$
scanning hadronic intermediate states



Dispersion in Q²:
scanning dominant physics pictures



Boundaries between regions - approximate

Input in DR related (directly or indirectly) to experimentally accessible data

Input into dispersion integral - $\nu/\bar{\nu}$ data

Mixed CC-NC γW SF (no data) <—> Purely CC WW SF (inclusive neutrino data)

Isospin symmetry: vector-isoscalar current related to vector-isovector current

Only useful if we know the physical mechanism (Born, DIS, Regge, Resonance,…)

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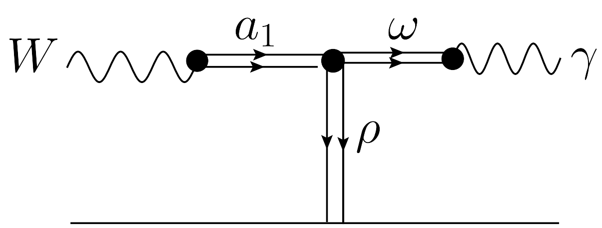
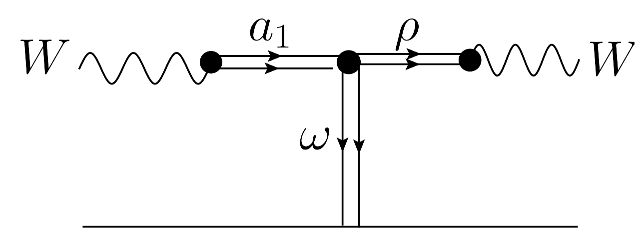
Were able to identify the missing part with Regge (multiparticle continuum)





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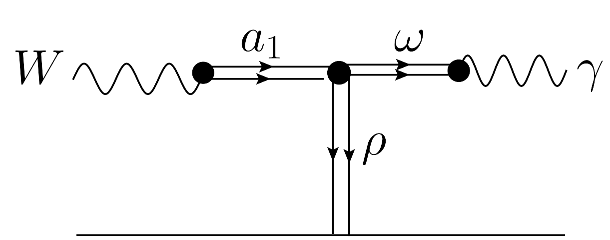
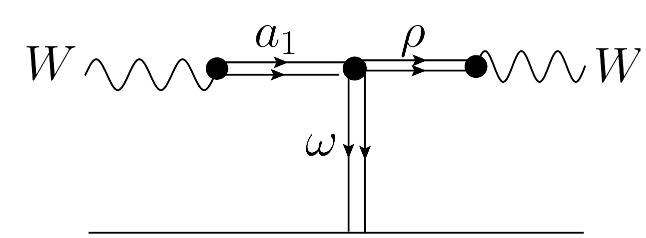
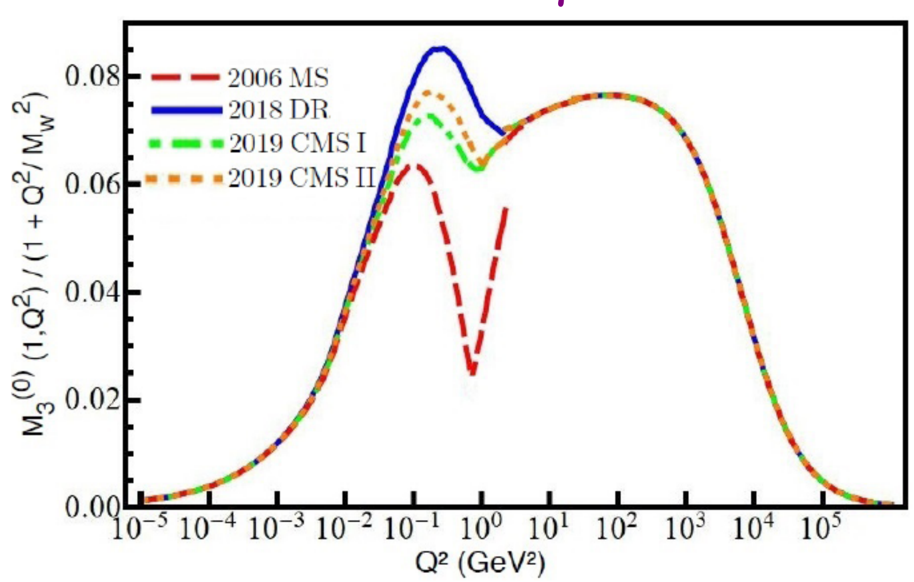
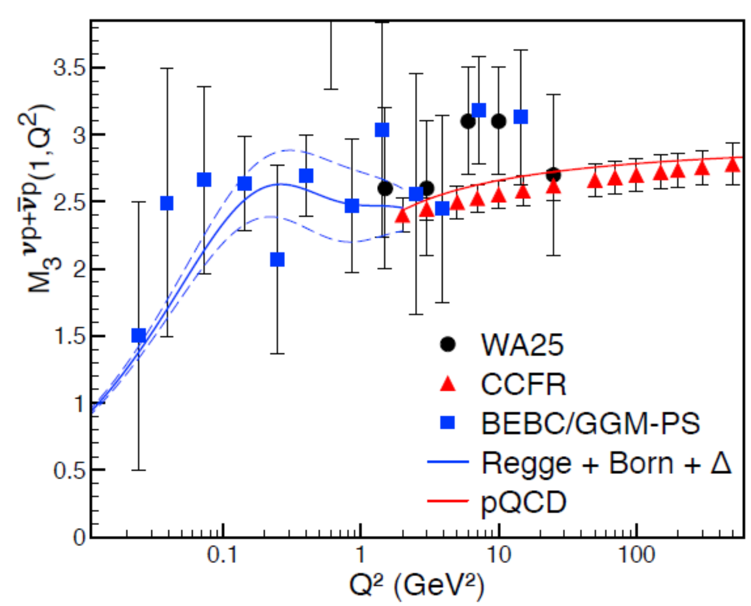




Free neutron γW box



Neutrino scattering data



Marciano, Sirlin 2006: $\Delta_R^V = 0.02361(38)$ —> $|V_{ud}| = 0.97420(10)_{Ft}(18)_{RC}$

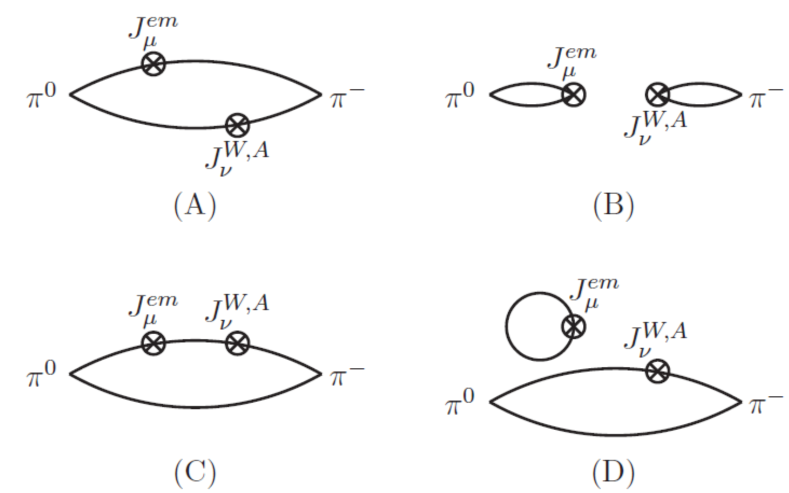
DR (Seng et al. 2018): $\Delta_R^V = 0.02467(22)$ —> $|V_{ud}| = 0.97370(10)_{Ft}(10)_{RC}$

γW-box from DR + Lattice QCD input

Currently available neutrino data at low Q^2 - low quality;
Look for alternative input — compute Nachtmann moment $M_3^{(0)}$ on the lattice

First direct LQCD computation $\pi^- \to \pi^0 e^- \nu_e$

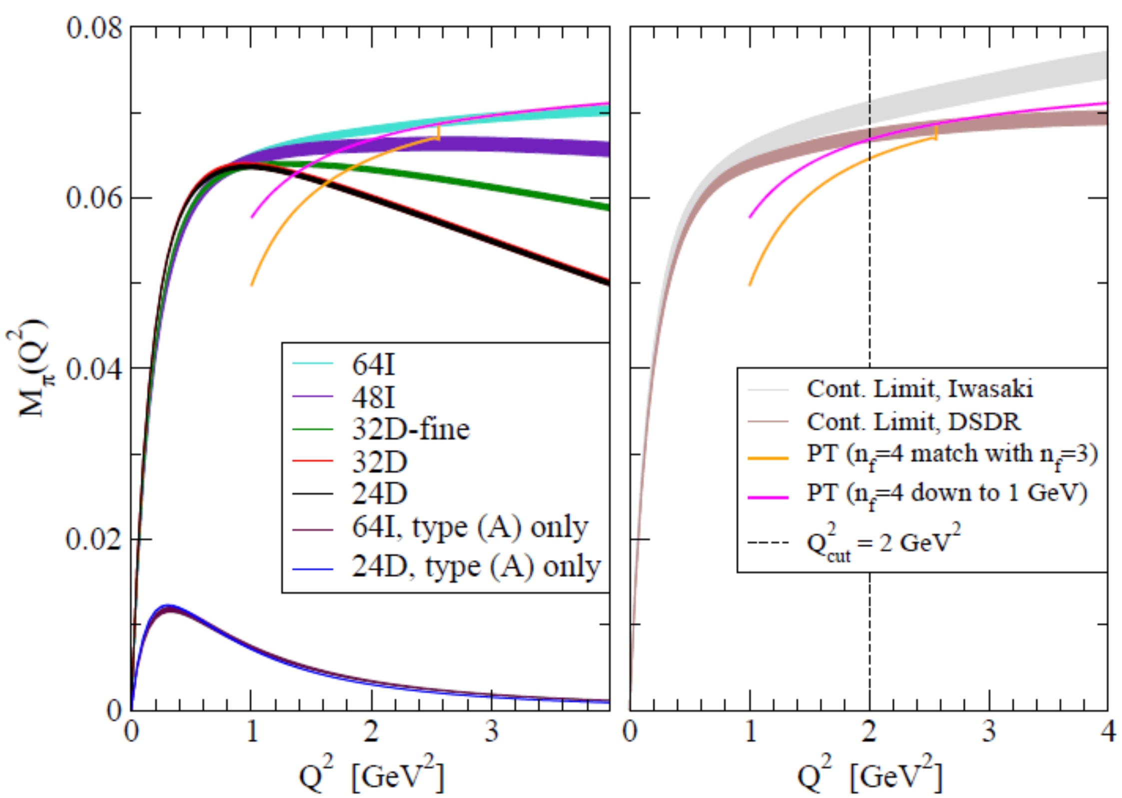
Feng, MG, Jin, Ma, Seng 2003.09798



Quark contraction diagrams

5 LQCD gauge ensembles at physical pion mass
Generated by RBC and UKQCD collaborations
w. 2+1 flavor domain wall fermion

Match onto pQCD at $Q^2 \sim 2\,\mathrm{GeV}^2$



(before cont. extrapolation) (after cont. extrapolation)

$$\square_{\gamma W}^{VA,\pi} = 2.830(11)_{\rm stat}(26)_{\rm sys}$$

Independent calculation by Los Alamos group

Yoo et all, 2305.03198

$$\square_{\gamma W}^{VA,\pi} = 2.810(26)_{\rm stat+sys}$$

First lattice QCD calculation of γW-box

Direct impact for pion decay $\pi^+ \to \pi^0 e^+ \nu_e$

$$|V_{ud}|^2 = \frac{0.9799}{(1+\delta)} \frac{\Gamma_{\pi\ell 3}}{0.3988(23)\ \mathrm{s}^{-1}}$$

Previous calculation of δ — in ChPT

Cirigliano, Knecht, Neufeld and Pichl, EPJC 2003

Significant reduction of the uncertainty!

$$\delta: \quad 0.0334(10)_{\mathrm{LEC}}(3)_{\mathrm{HO}} \to 0.0332(1)_{\gamma W}(3)_{\mathrm{HO}}$$

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Analogous result for $K\ell 3$ decays:

$$\Box_{\gamma W}^{VA,K} = 2.437(44)_{\mathrm{stat+sys}}$$

Ma, Feng, MG, Jin, Seng 2102.12048

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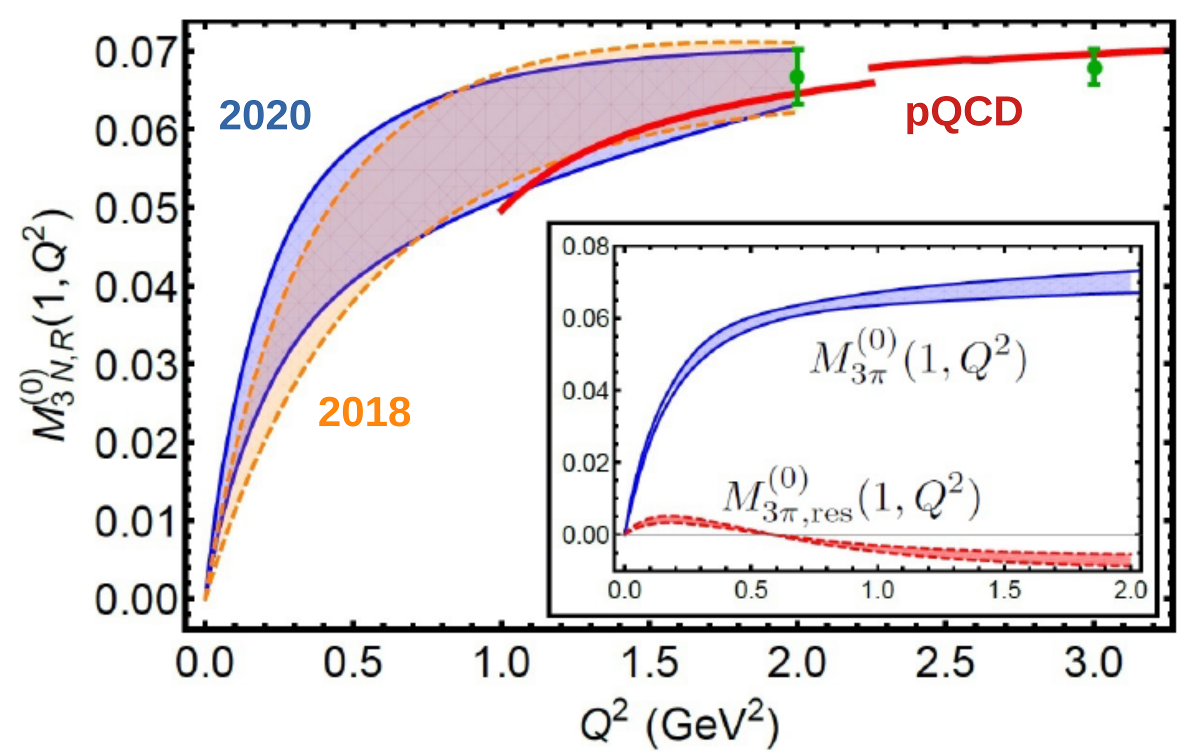
Indirectly constrains the free neutron γW-box
— requires some phenomenology
Based on Regge universality & factorization

Seng, MG, Feng, Jin, 2003.11264



Independent confirmation of the empirical DR result AND uncertainty

$$\Delta_R^V = 0.02467(22)_{\mathrm{DR}} \to 0.02477(24)_{\mathrm{LQCD+DR}}$$

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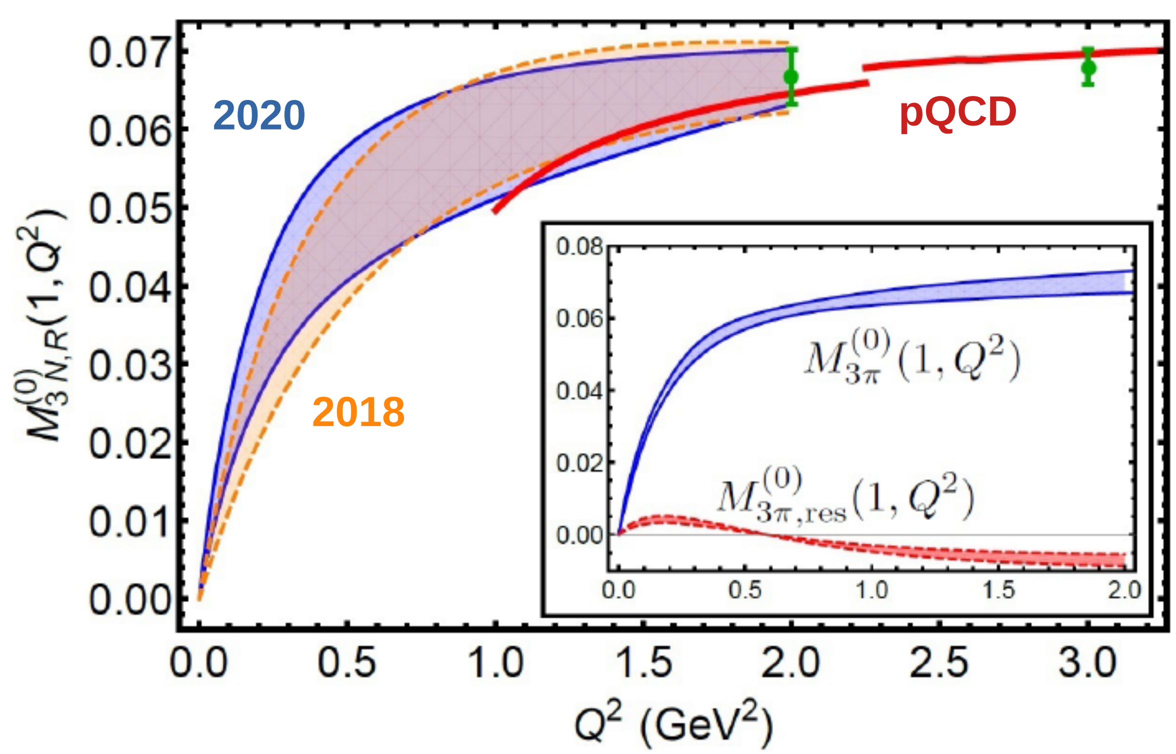
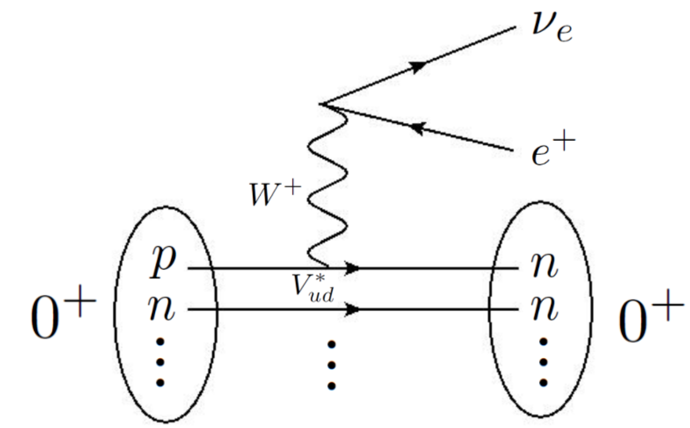


Direct LQCD calculation on the neutron underway by two groups!

Superallowed nuclear decays

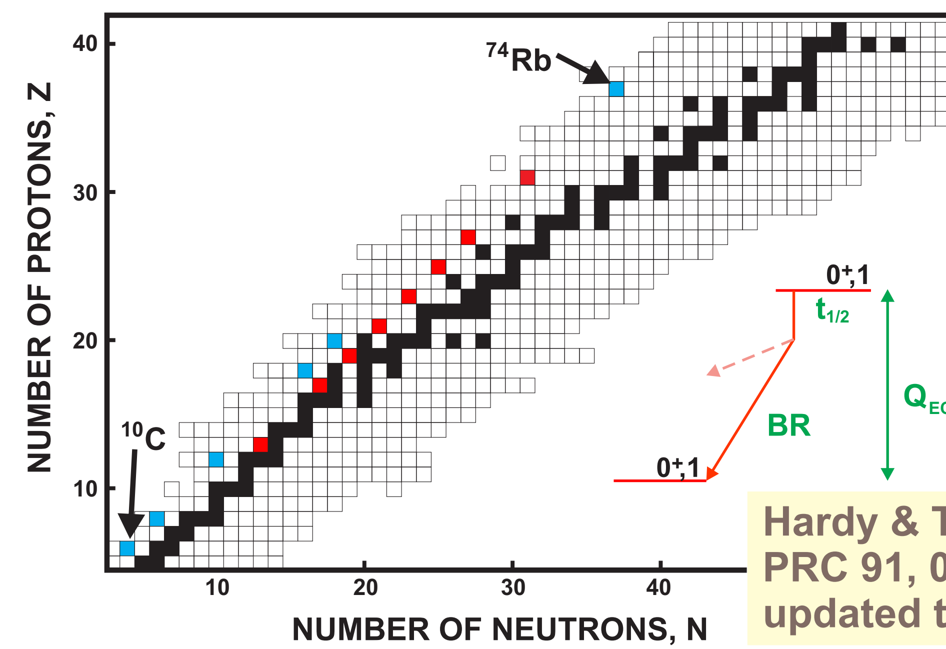
Precise V_{ud} from superallowed decays

Superallowed 0^+-0^+ nuclear decays:
- only conserved vector current
- many decays
- all rates equal modulo phase space

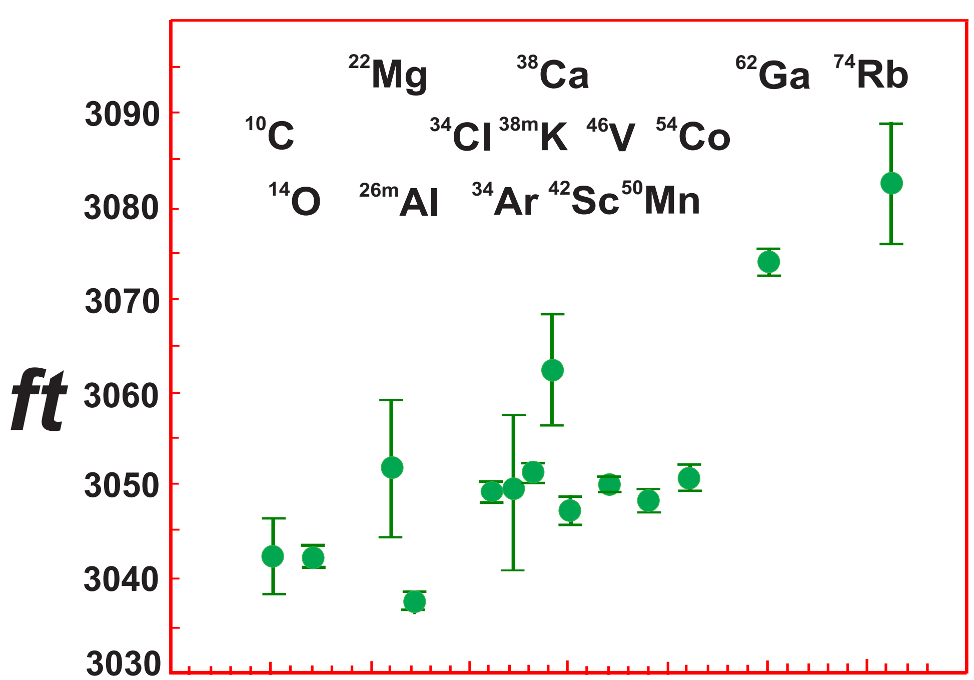


Experiment: **f** - phase space (Q value) and **t** - partial half-life ($t_{1/2}$, branching ratio)

- **8 cases with *ft*-values measured to <0.05% precision; 6 more cases with 0.05-0.3% precision.**
- **~220 individual measurements with compatible precision**



Hardy & Towner
PRC 91, 025501 (2015)
updated to 2017



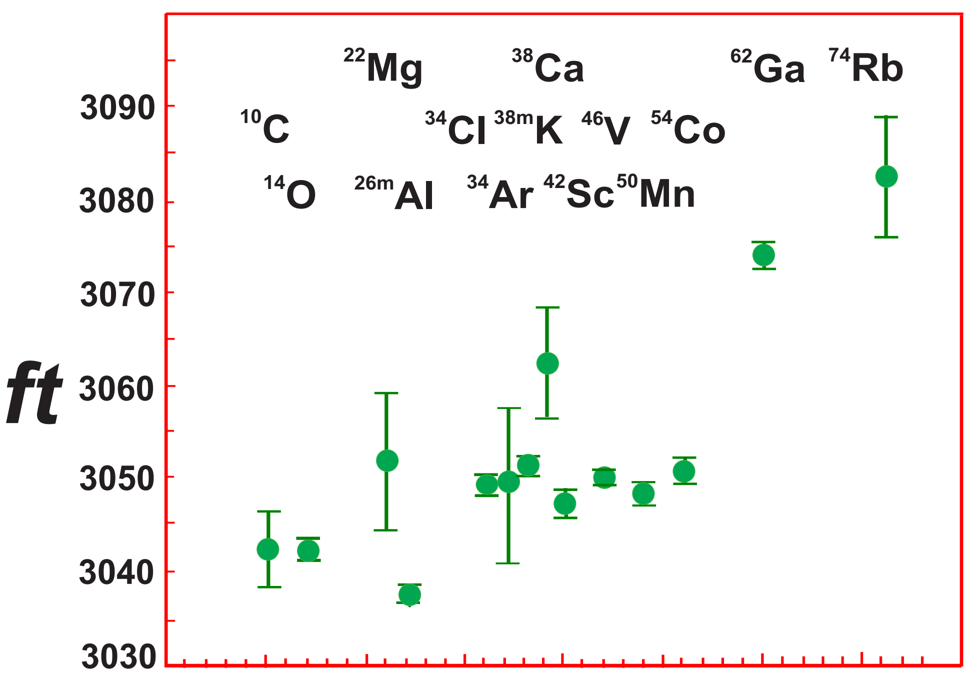
ft values: same within ~2% but not exactly!
Reason: SU(2) slightly broken
a. RC (e.m. interaction does not conserve isospin)
b. Nuclear WF are not SU(2) symmetric
(proton and neutron distribution not the same)

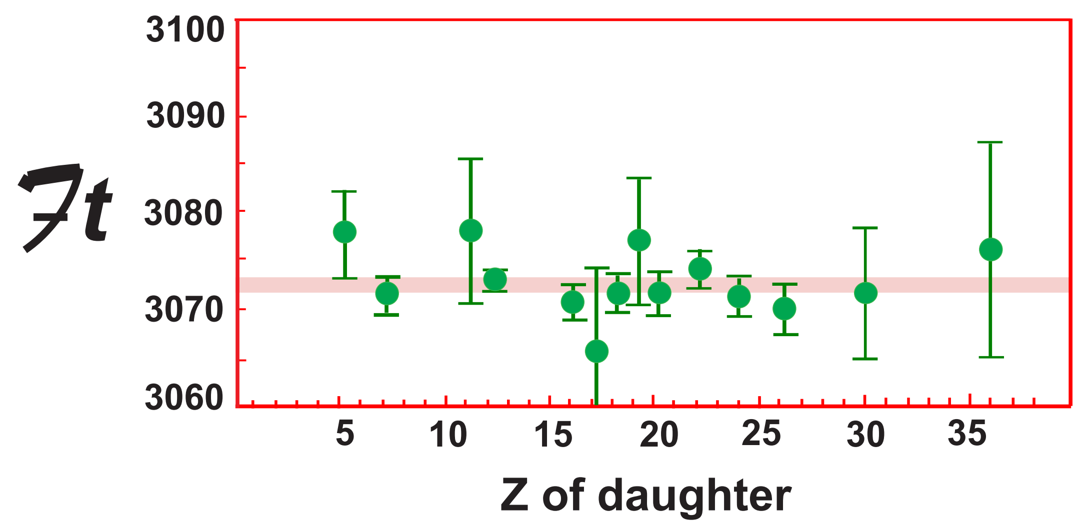
Splitting the γW-box into Universal and Nuclear Parts

To obtain Vud —> absorb all decay-specific corrections into universal **Ft**

$$ft(1 + \mathrm{RC} + \mathrm{ISB}) = \mathscr{F}\mathrm{t}(1 + \Delta_R^V) = \mathrm{ft}(1 + \delta'_R)(1 - \delta_C + \delta_{NS})(1 + \Delta_R^V)$$

Outer: QED Isospin-breaking Nuclear structure Universal inner





Average of 14 decays - 0.02%

$$\overline{\mathcal{F}t} = 3072.1 \pm 0.7$$

Hardy, Towner 1973 - 2020

Splitting the γW-box into Universal and Nuclear Parts

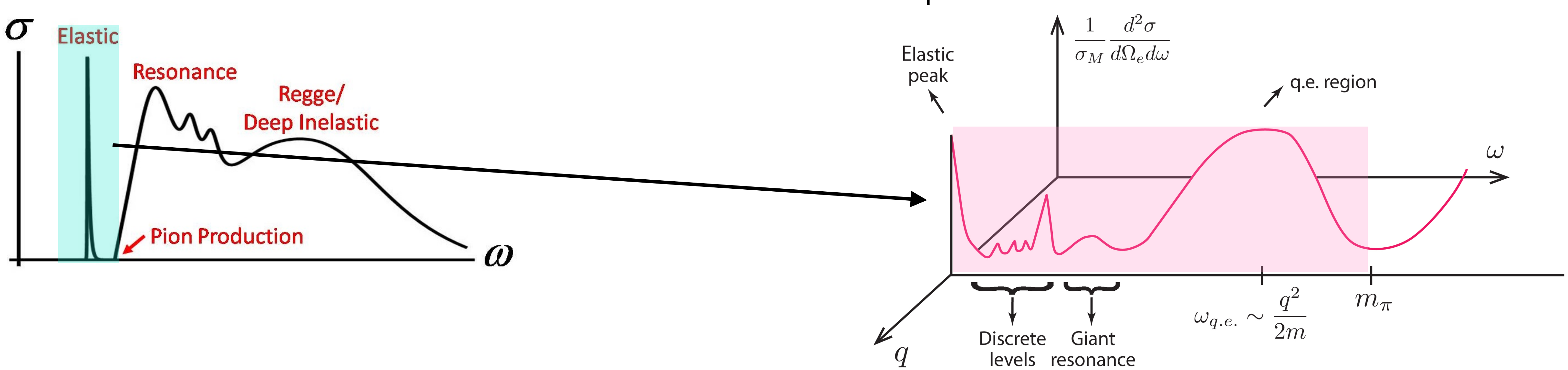
NS correction reflects extraction of the free box $\delta_{\rm NS} = 2[\,\Box_{\gamma W}^{\rm VA,\,nucl} - \Box_{\gamma W}^{\rm VA,\,free\,n}\,]$

RC on a free neutron $\Delta_R^V \propto F_3^{\rm free\,n} \propto \int dx e^{iqx} \sum_X \langle p\,|\,J_{em}^{\mu,(0)}(x)\,|\,X\rangle\langle X\,|\,J_W^{\nu,+}(0)\,|\,n\rangle$

RC on a nucleus $\Delta_R^V + \delta_{NS} \propto F_3^{\rm Nucl.} \propto \int dx e^{iqx} \sum_{X'} \langle A'\,|\,J_{em}^{\mu,(0)}(x)\,|\,X'\rangle\langle X'\,|\,J_W^{\nu,+}(0)\,|\,A\rangle$

Input in the DR for the universal RC

Input in the DR for the RC on a nucleus



δ_{NS} from DR with energy dependence averaged over the spectrum

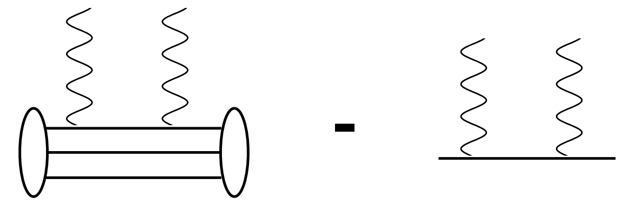
$$\delta_{NS} = \frac{2\alpha}{\pi M}\int_0^{\rm few\,GeV^2} dQ^2 \int_{\nu_{thr}}^{\nu_\pi} \frac{d\nu}{\nu}\left[\frac{\nu+2q}{(\nu+q)^2}\left(F_3^{(0)\,Nucl.} - F_3^{(0),\,B}\right) + \frac{2\langle E\rangle}{3}\frac{\nu+3q}{(\nu+q)^3}F_3^{(-)\,Nucl.}\right]$$

Splitting the γW-box into Universal and Nuclear Parts

Need to know the full nuclear Green's function $T^{\gamma W\ \mathrm{nuc}}_{\mu\nu} \sim \sum_{k,\ell} \langle f | J^{W}_{\mu}(k)\, G_{\mathrm{nuc}}\, J^{\mathrm{EM}}_{\nu}(\ell) | i \rangle$

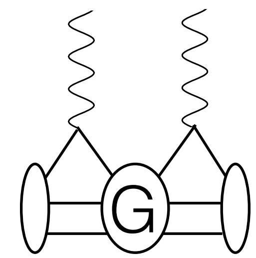
$\delta_{\mathrm{NS}} =$

(A) same active nucleon



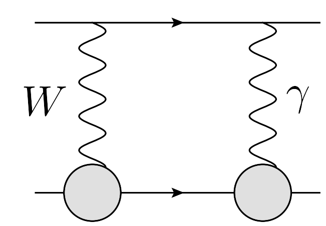
Modified Born

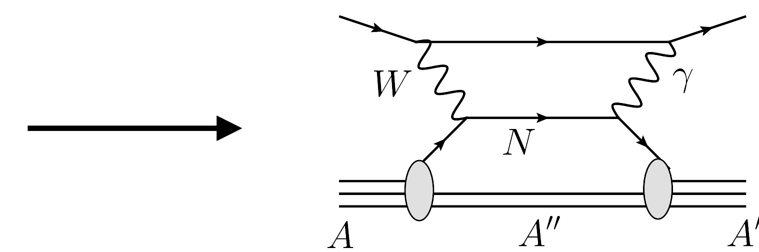
(B) two nucleons correlated by G



Specifically nuclear effect

Elastic nucleon box —> single N QE knockout





Splitting the γW-box into Universal and Nuclear Parts

Need to know the full nuclear Green's function $T^{\gamma W\,\mathrm{nuc}}_{\mu\nu} \sim \sum_{k,\ell} \langle f | J^W_\mu(k)\, G_{\mathrm{nuc}}\, J^{\mathrm{EM}}_\nu(\ell) | i \rangle$

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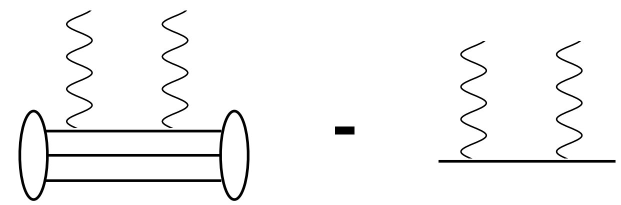

Modified Born

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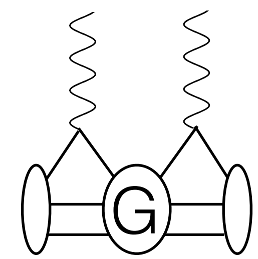

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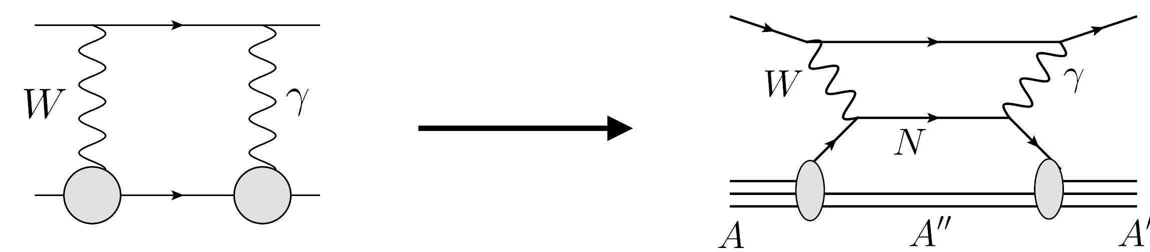


QE contribution from DR: $\delta^{\mathrm{QE}}_{\mathrm{NS}} = \delta^{\mathrm{QE},\,0}_{\mathrm{NS}} + \langle E \rangle \delta^{\mathrm{QE},\,1}_{\mathrm{NS}}$

$$\delta_{NS} = \frac{2\alpha}{\pi NM} \int_0^{\text{few GeV}^2} dQ^2 \int_{\nu_{thr}}^{\nu_\pi} \frac{d\nu}{\nu} \left[\frac{\nu + 2q}{(\nu+q)^2} \left(F_3^{(0)\,QE} - F_3^{(0),\,B} \right) + \frac{2\langle E \rangle}{3} \frac{\nu + 3q}{(\nu+q)^3} F_3^{(-)\,QE} \right]$$

HT value 2018: Old estimate: $\delta\mathcal{F}t = -(1.8 \pm 0.4)s \quad + \quad (0 \pm 0)s$

$\mathcal{F}t = 3072.1(7)s$ New estimate: $\delta\mathcal{F}t = -(3.5 \pm 1.0)s \quad + \quad (1.6 \pm 0.5)s$

C-Y Seng, MG, M J Ramsey-Musolf 1812.03352

MG 1812.04229

Nuclear structure uncertainty tripled! $\mathcal{F}t = (3072 \pm 2)s$

Ab-Initio $\delta_{\rm NS}$

Only a naive warm-up calculation — ab-initio $\delta_{\rm NS}$ necessary!

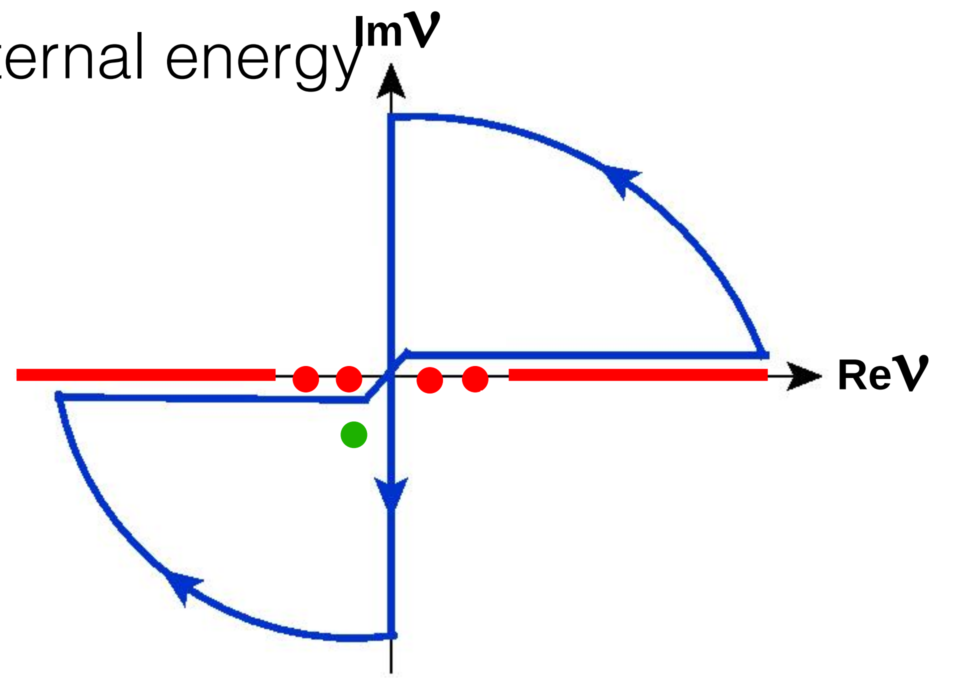
Dispersion theory of $\delta_{\rm NS}$: isospin structure + multipole expansion

Seng, MG 2211.10214

Interesting effects detected:

Mixed isospin structure due to 2B currents (absent for n, $\pi e3$)

Residue contribution if 0^+ state is not g.s.: anomalous threshold
Normal threshold: nuclear excitation spectrum separated from external state by finite energy gap — only virtual;
if there are states below — can go on-shell even without external energy



Residue contribution: contains parts singular at $E_e = 0$
—> should contribute to outer correction δ_R'

Currently, effort on light systems C-10, O-14

Accessible to NCSM, GFMC, CC, …
Important cross checks should become possible soon

With Michael Gennari, Petr Navratil, Mehdi Drissy, Garrett King, Saori Pastore

Status of δ_C

Isospin symmetry breaking in superallowed β-decay

Tree-level Fermi matrix element

$$M_F = \langle f | \tau^+ | i \rangle$$

τ^+ — Isospin operator

$|i\rangle$, $|f\rangle$ — members of T=1 isotriplet

If isospin symmetry were exact, $M_F \to M_0 = \sqrt{2}$

Isospin symmetry is broken in nuclear states
(e.g. Coulomb, nucleon mass difference, …)

In presence of isospin symmetry breaking (ISB):

$$|M_F|^2 = |M_0|^2 (1 - \delta_C)$$

ISB correction is crucial for V_{ud} extraction

HT: shell model with *phenomenological* Woods-Saxon potential locally adjusted to:

- Masses of the isotriplet T=1, 0^+ (IMME)
- Neutron and proton separation energies
- Known proton radii of stable isotopes

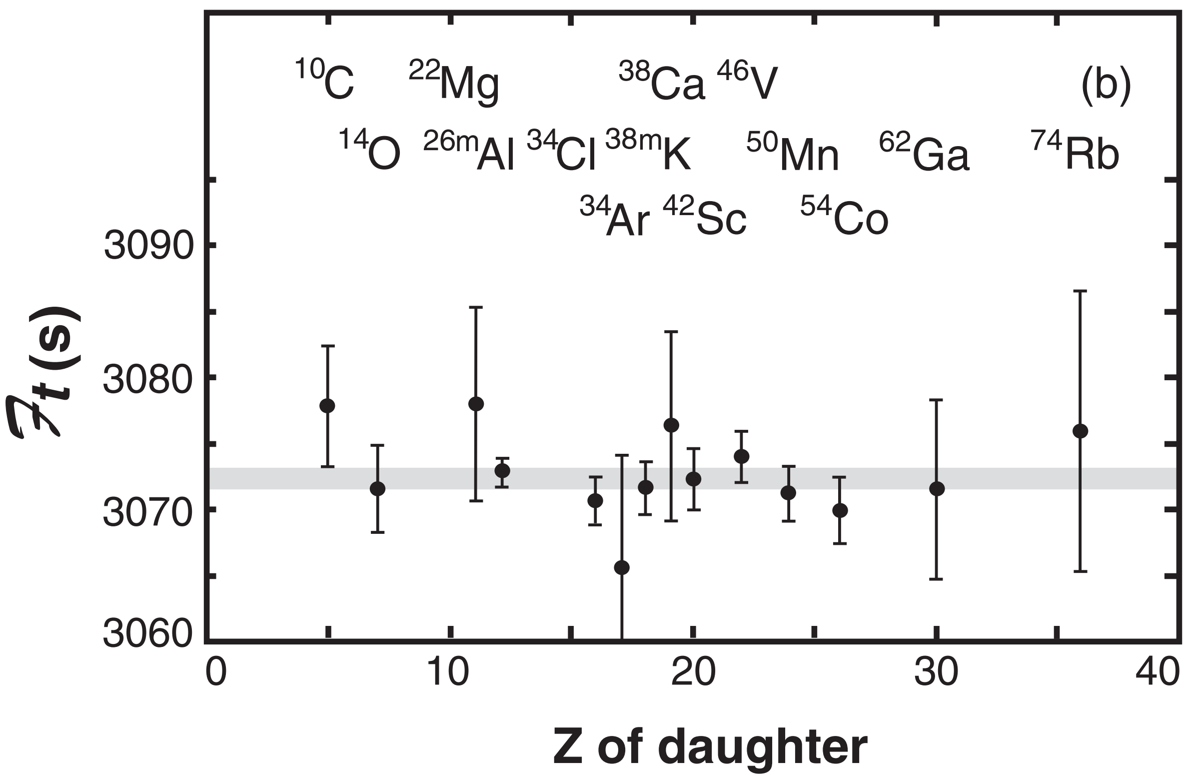
TABLE X. Corrections δ'_R, δ_{NS}, and δ_C that are applied to experimental ft values to obtain $\mathcal{F}t$ values.

Parent nucleus	δ'_R (%)	δ_{NS} (%)	δ_{C1} (%)	δ_{C2} (%)	δ_C (%)
$T_z = -1$					
^{10}C	1.679	−0.345(35)	0.010(10)	0.165(15)	0.175(18)
^{14}O	1.543	−0.245(50)	0.055(20)	0.275(15)	0.330(25)
^{18}Ne	1.506	−0.290(35)	0.155(30)	0.405(25)	0.560(39)
^{22}Mg	1.466	−0.225(20)	0.010(10)	0.370(20)	0.380(22)
^{26}Si	1.439	−0.215(20)	0.030(10)	0.405(25)	0.435(27)
^{30}S	1.423	−0.185(15)	0.155(20)	0.700(20)	0.855(28)
^{34}Ar	1.412	−0.180(15)	0.030(10)	0.665(55)	0.695(56)
^{38}Ca	1.414	−0.175(15)	0.020(10)	0.745(70)	0.765(71)
^{42}Ti	1.427	−0.235(20)	0.105(20)	0.835(75)	0.940(78)
$T_z = 0$					
^{26m}Al	1.478	0.005(20)	0.030(10)	0.280(15)	0.310(18)
^{34}Cl	1.443	−0.085(15)	0.100(10)	0.550(45)	0.650(46)
^{38m}K	1.440	−0.100(15)	0.105(20)	0.565(50)	0.670(54)
^{42}Sc	1.453	0.035(20)	0.020(10)	0.645(55)	0.665(56)
^{46}V	1.445	−0.035(10)	0.075(30)	0.545(55)	0.620(63)
^{50}Mn	1.444	−0.040(10)	0.035(20)	0.610(50)	0.645(54)
^{54}Co	1.443	−0.035(10)	0.050(30)	0.720(60)	0.770(67)
^{62}Ga	1.459	−0.045(20)	0.275(55)	1.20(20)	1.48(21)
^{66}As	1.468	−0.060(20)	0.195(45)	1.35(40)	1.55(40)
^{70}Br	1.486	−0.085(25)	0.445(40)	1.25(25)	1.70(25)
^{74}Rb	1.499	−0.075(30)	0.115(60)	1.50(26)	1.62(27)

J. Hardy, I. Towner, Phys.Rev. C 91 (2014), 025501

$$\delta_C \sim 0.17\% - 1.6\% \,!$$

ISB vs. scalar interactions?

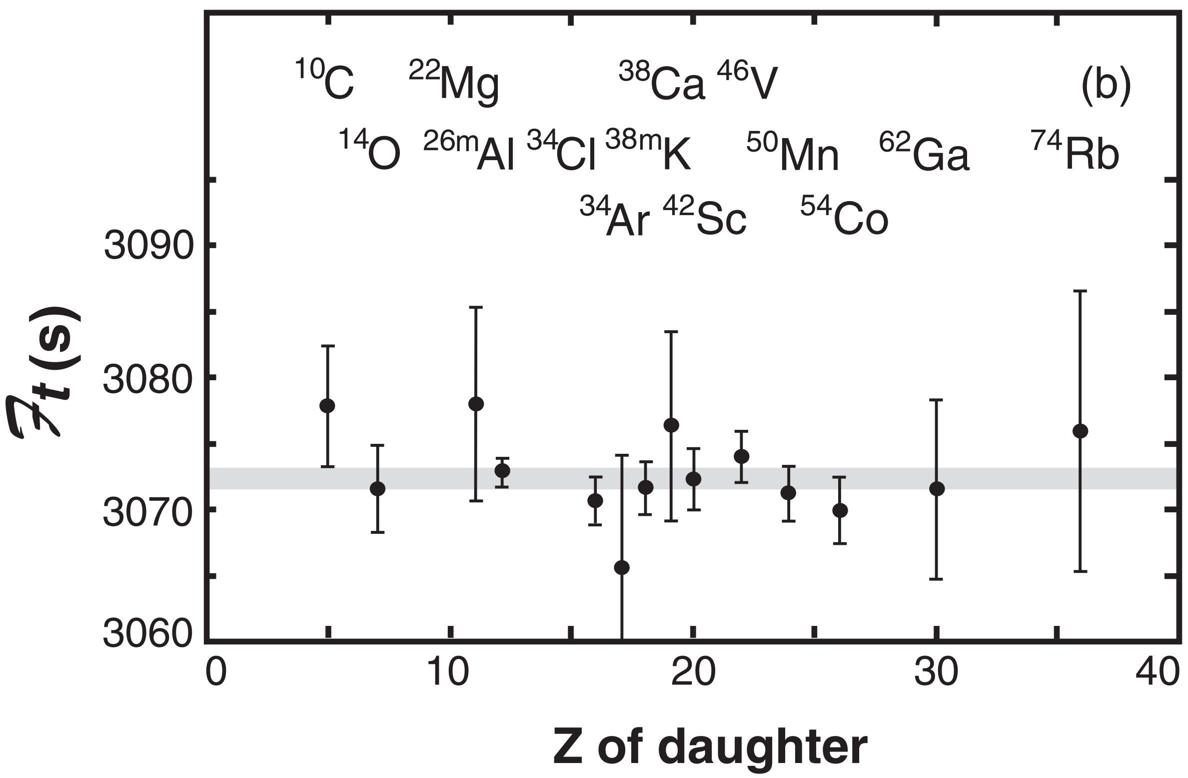


Once all corrections are included:
CVC —> Ft constant

δ_C particularly important for alignment!

Fit to 14 transitions:
Ft constant within 0.02% if using SM-WS

ISB vs. scalar interactions?



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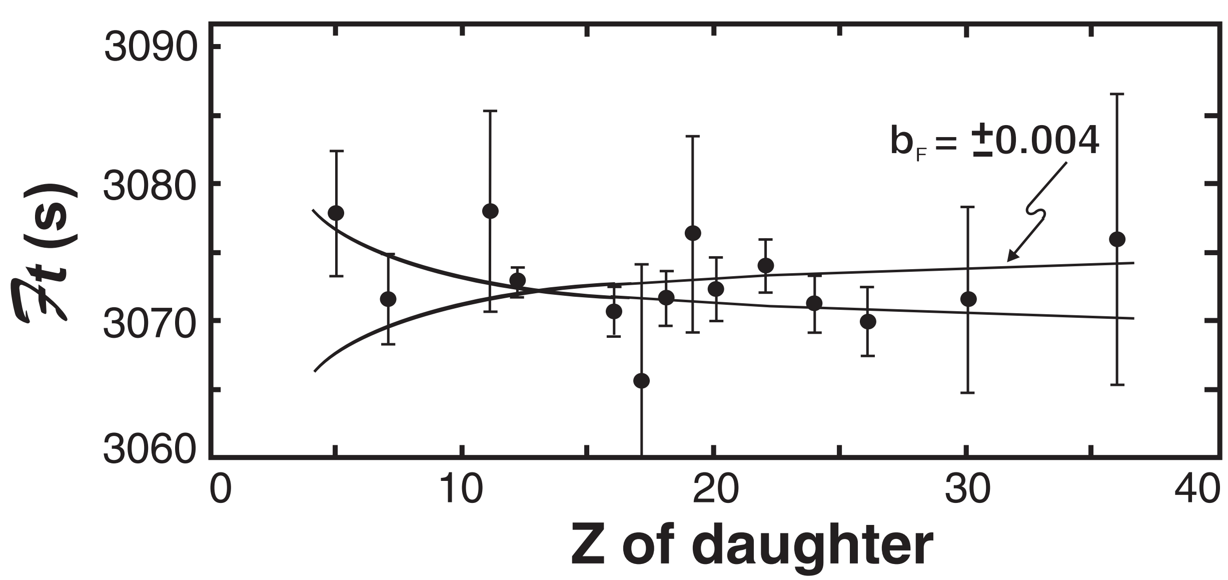


If BSM scalar currents present: "Fierz interference" b_F

$$\mathscr{F}t^{SM} \to \mathscr{F}t^{SM}\left(1 + b_F \frac{m_e}{\langle E_e \rangle}\right)$$

$Q_{EC} \uparrow$ with Z —> effect of $b_F \downarrow$ with Z
Introduces nonlinearity in the Ft plot

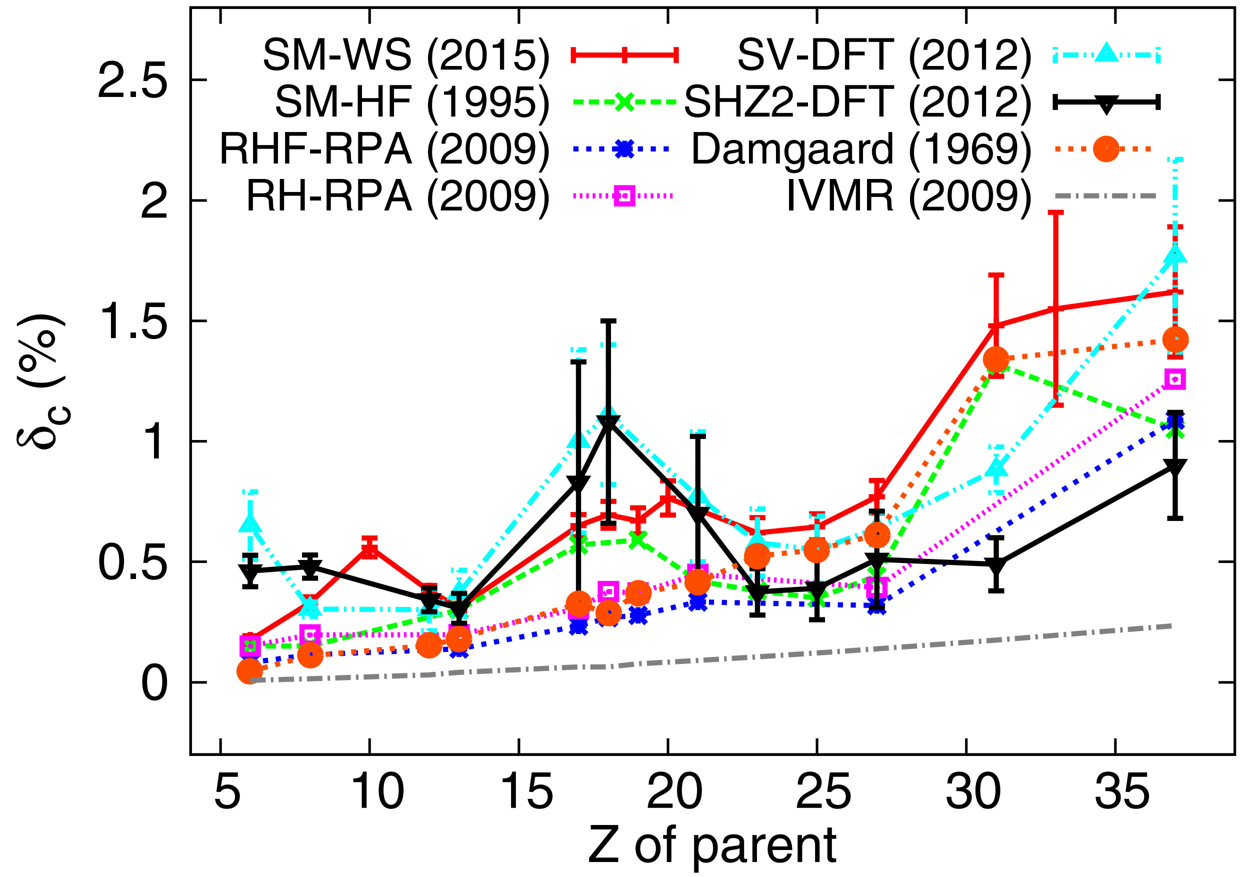
$b_F = -0.0028(26)$ ~ consistent with 0

Nuclear model comparison for δ_C

J. Hardy, I. Towner, Phys.Rev. C 91 (2014), 025501

			RPA				
	SM-WS	SM-HF	PKO1	DD-ME2	PC-F1	IVMR[a]	DFT
$T_z = -1$							
^{10}C	0.175	0.225	0.082	0.150	0.109	0.147	0.650
^{14}O	0.330	0.310	0.114	0.197	0.150		0.303
^{22}Mg	0.380	0.260					0.301
^{34}Ar	0.695	0.540	0.268	0.376	0.379		
^{38}Ca	0.765	0.620	0.313	0.441	0.347		
$T_z = 0$							
^{26m}Al	0.310	0.440	0.139	0.198	0.159		0.370
^{34}Cl	0.650	0.695	0.234	0.307	0.316		
^{38m}K	0.670	0.745	0.278	0.371	0.294	0.434	
^{42}Sc	0.665	0.640	0.333	0.448	0.345		0.770
^{46}V	0.620	0.600					0.580
^{50}Mn	0.645	0.610					0.550
^{54}Co	0.770	0.685	0.319	0.393	0.339		0.638
^{62}Ga	1.475	1.205					0.882
^{74}Rb	1.615	1.405	1.088	1.258	0.668		1.770
χ^2/ν	1.4	6.4	4.9	3.7	6.1		4.3[b]

L. Xayavong, N.A. Smirnova, Phys.Rev. C 97 (2018), 024324



HT: χ^2 as criterion to prefer SM-WS; V_{ud} and limits on BSM strongly depend on nuclear model

Nuclear community embarked on ab-initio δ_C calculations (NCSM, GFMC, CC, IMSRG)
Especially interesting for light nuclei accessible to different techniques!

Phenomenological constraints on δ_C?

Idea: δ_C dominated by Coulomb repulsion between protons (hence C)

Coulomb interaction generates both δ_C and ISB combinations of nuclear radii

Auerbach 0811.4742; 2101.06199; Seng, MG 2208.03037; 2304.03800; 2212.02681

Nuclear Hamiltonian: $H = H_0 + V_{\text{ISB}} \approx H_0 + V_C$

Coulomb potential for uniformly charged sphere $\quad V_C \approx -\frac{Ze^2}{4\pi R_C^3} \sum_{i=1}^{A} \left(\frac{1}{2} r_i^2 - \frac{3}{2} R_C^2 \right) \left(\frac{1}{2} - \hat{T}_z(i) \right)$

ISB due to IV monopole, $V_{\text{ISB}} \approx \frac{Ze^2}{8\pi R^3} \sum_i r_i^2 \hat{T}_z(i) = \frac{Ze^2}{8\pi R^3} \hat{M}_0^{(1)}$

Same operator generates nuclear radii $\quad R_{p/n,\phi} = \sqrt{\frac{1}{X} \langle \phi | \sum_{i=1}^{A} r_i^2 \left(\frac{1}{2} \mp \hat{T}_z(i) \right) | \phi \rangle}$

Phenomenological constraints on δ_C?

$0^+, T=1, T_z=-1$ → $0^+, T=1, T_z=0$ → $0^+, T=1, T_z=1$

E.g. ${}^{42}_{22}\mathrm{Ti} \rightarrow {}^{42}_{21}\mathrm{Sc} \rightarrow {}^{42}_{20}\mathrm{Ca}$

ISB-sensitive combinations of radii: Wigner-Eckart theorem

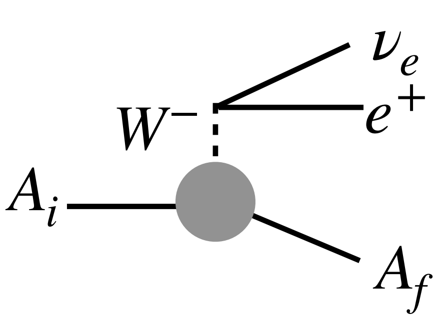
$$\Delta M_A^{(1)} \equiv \langle f | M_{+1}^{(1)} | i \rangle + \langle f | M_0^{(1)} | f \rangle$$

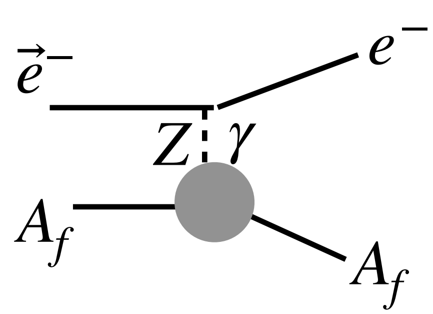
$$\Delta M_B^{(1)} \equiv \frac{1}{2}\left(Z_1 R_{p,1}^2 + Z_{-1} R_{p,-1}^2\right) - Z_0 R_{p,0}^2$$

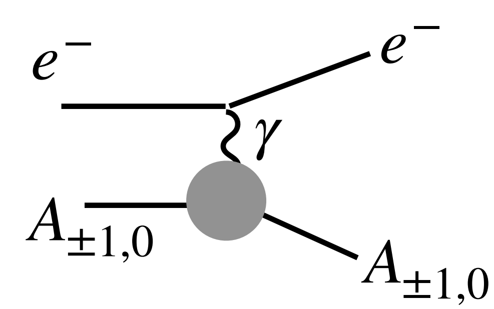
Transition radius From β spectrum

Neutron skin From PVES

Charge radii from atomic spectra and electron scattering







$F_{CW}(Q^2) = 1 - R_{CW}^2 Q^2/6 + \ldots$

$$A^{PV} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\frac{Q_W}{Z}\frac{F_{NW}(Q^2)}{F_{Ch}(Q^2)}$$

$F_{Ch}(Q^2) = 1 - R_{Ch}^2 Q^2/6 + \ldots$

Since N ≠ Z for $T_z = \pm 1$ factors $Z_{\pm1,0}$ remove the symmetry energy to isolate ISB
(Usually PVES —> neutron skins —> symmetry energy —> nuclear EOS —> nuclear astrophysics)

Electroweak radii constrain ISB in superallowed β-decay

For numerical analysis: lowest isovector monopole resonance dominates
One ISB matrix element, one energy splitting

Model for δ_C $\rightarrow$ prediction for $\Delta M^{(1)}_{A,B}$

Transitions	δ_C (%)					$\Delta M_A^{(1)}$ (fm^2)					$\Delta M_B^{(1)}$ (fm^2)				
	WS	DFT	HF	RPA	Micro	WS	DFT	HF	RPA	Micro	WS	DFT	HF	RPA	Micro
^{26m}Al $\rightarrow$ ^{26}Mg	0.310	0.329	0.30	0.139	0.08	-2.2	-2.3	-2.1	-1.0	-0.6	-0.12	-0.12	-0.11	-0.05	-0.03
^{34}Cl $\rightarrow$ ^{34}S	0.613	0.75	0.57	0.234	0.13	-5.0	-6.1	-4.6	-1.9	-1.0	-0.17	-0.21	-0.16	-0.06	-0.04
^{38m}K $\rightarrow$ ^{38}Ar	0.628	1.7	0.59	0.278	0.15	-5.4	-14.6	-5.1	-2.4	-1.3	-0.15	-0.42	-0.15	-0.07	-0.04
^{42}Sc $\rightarrow$ ^{42}Ca	0.690	0.77	0.42	0.333	0.18	-6.2	-6.9	-3.8	-3.0	-1.6	-0.15	-0.17	-0.09	-0.07	-0.04
^{46}V $\rightarrow$ ^{46}Ti	0.620	0.563	0.38	/	0.21	-5.8	-5.3	-3.6	/	-2.0	-0.12	-0.11	-0.08	/	-0.04
^{50}Mn $\rightarrow$ ^{50}Cr	0.660	0.476	0.35	/	0.24	-6.4	-4.6	-3.4	/	-2.4	-0.12	-0.09	-0.06	/	-0.04
^{54}Co $\rightarrow$ ^{54}Fe	0.770	0.586	0.44	0.319	0.28	-7.8	-5.9	-4.4	-3.2	-2.8	-0.13	-0.10	-0.07	-0.05	-0.05

Can discriminate models if independent information on nuclear radii is available
ΔM_A from measured radii —> test models for δ_C

Charge radii across superallowed isotriplets?
Some are known (but difficult — unstable isotopes, some g.s. are not 0^+)
Typically, precision is not enough to make a quantitative statement — need to improve!

Summary: Status of V_{ud} and top-row CKM unitarity

3-sigma CKM unitarity deficit established

Significant shift in V_{ud} due to shift in Δ_R^V

EW boxes: DR + Exp. + Lattice QCD+ ChPT +…

Calculation for Δ_R^V confirmed by several groups

Formalism applied to $K\ell 3$ decays;

Puzzles: $K\ell 2$ - $K\ell 3$, Beam-Bottle n-lifetime

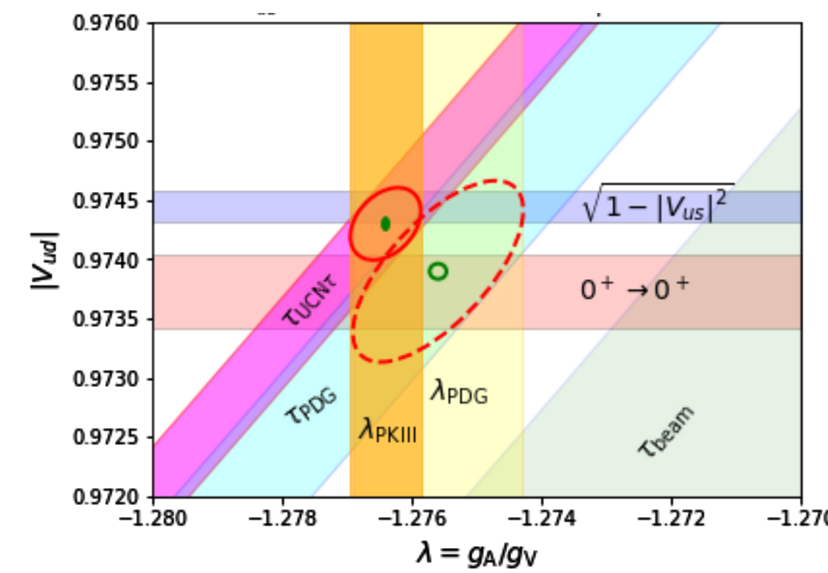
Unified universal RC Δ_R^V and nuclear correction $\delta_{\rm NS}$

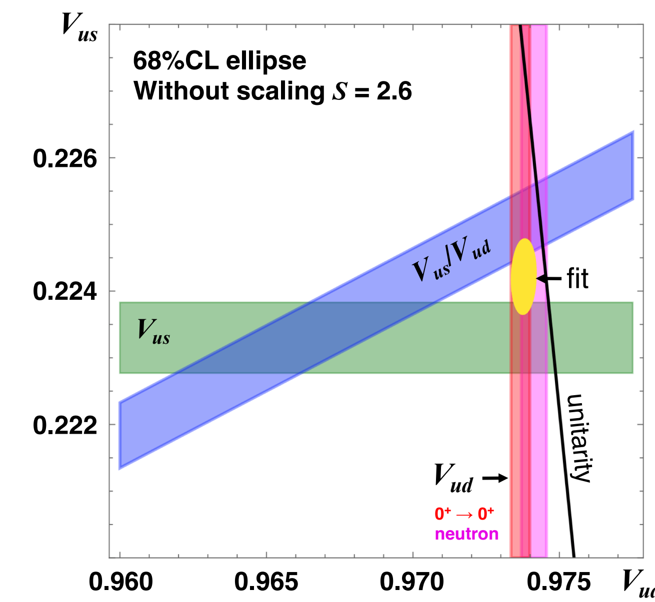
Both SM (V_{ud}) and BSM (b_F) tests depend on δ_C and $\delta_{\rm NS}$

Direct lattice QCD evaluation of the γW-box

Modern ab initio theory of δ_C and $\delta_{\rm NS}$ underway!

BSM: RH currents across light and strange quarks may resolve all puzzles





Status of V_{us}

V_{us} Status and Outlook

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{\mathrm{EW}} \, |V_{us}|^2 \, |f_+^{K^0\pi^-}(0)|^2 \, I_{K\ell}(\lambda_{K\ell}) \left(1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{\mathrm{EM}}\right)$$

with $K \in \{K^+, K^0\}$; $\ell \in \{e, \mu\}$, and:

C_K^2 1/2 for K^+, 1 for K^0

S_{EW} Universal SD EW correction (1.0232)

Inputs from experiment:

$\Gamma(K_{\ell 3(\gamma)})$ Rates with well-determined treatment of radiative decays:

- Branching ratios
- Kaon lifetimes

$I_{K\ell}(\{\lambda\}_{K\ell})$ Integral of form factor over phase space: λs parameterize evolution in t

Inputs from theory:

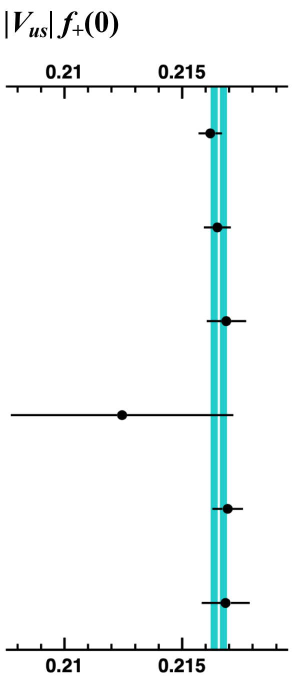
$f_+^{K^0\pi^-}(0)$ Hadronic matrix element (form factor) at zero momentum transfer ($t = 0$)

$\Delta_K^{SU(2)}$ Form-factor correction for $SU(2)$ breaking

$\Delta_{K\ell}^{\mathrm{EM}}$ Form-factor correction for long-distance EM effects

V_{us} from Kl3 decays

$|V_{us}| f_+(0)$ from world data: 2022 update

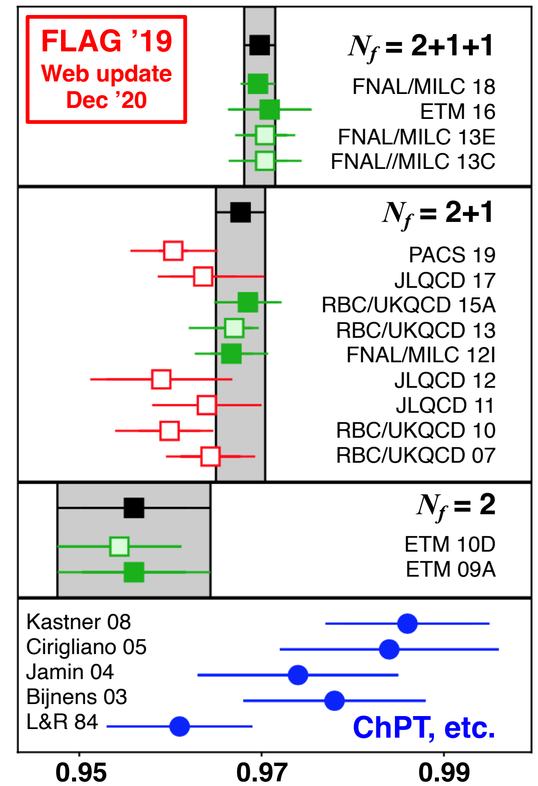


		% err	BR	τ	Δ	Int
$K_L e3$	0.2162(5)	0.23	0.09	0.20	0.02	0.05
$K_L \mu 3$	0.2165(6)	0.26	0.15	0.18	0.02	0.07
$K_S e3$	0.2169(8)	0.39	0.38	0.02	0.02	0.05
$K_S \mu 3$	0.2125(47)	2.2	2.2	0.02	0.02	0.08
$K^\pm e3$	0.2169(6)	0.30	0.27	0.06	0.11	0.05
$K^\pm \mu 3$	0.2168(10)	0.47	0.45	0.06	0.11	0.08

Approx. contrib. to % err from: BR, τ, Δ, Int

Average: $|V_{us}| f_+(0)$ = 0.21656(35) χ^2/ndf = 1.89/5 (86%)

Evaluations of $f_+(0)$



FLAG '21 averages:

N_f = 2+1+1 $f_+(0)$ = 0.9698(17)

Uncorrelated average of:

FNAL/MILC 18: HISQ, 5sp, $m_\pi \to$ 135 MeV, new ensembles added to FNAL/MILC 13E

ETM 16: TwMW, 3sp, $m_\pi \to$ 210 MeV, full q^2 dependence of f_+, f_0

N_f = 2+1 $f_+(0)$ = 0.9677(27)

Uncorrelated average of:

FNAL/MILC 12I: HISQ, $m_\pi \sim$ 300 MeV

RBC/UKQCD 15A: DWF, $m_\pi \to$ 139 MeV

JLQCD 17 not included because only single lattice spacing used

ChPT $f_+(0)$ = 0.970(8)

Ecker 15, Chiral Dynamics 15: Calculation from Bijnens 03, with new LECs from Bijnens, Ecker 14

$K_{\mu 3}$

$f_+(0)$ = 0.9698(17)

N_f = 2+1+1

V_{us} = 0.22330(35)$_{\text{exp}}$(39)$_{\text{lat}}$(8)$_{\text{IB}}$

$\Delta^{(1)}_{\text{CKM}}$ = −0.00176(16)$_{\text{exp+IB}}$(17)$_{\text{lat}}$(51)$_{ud}$ = −3.1σ

V_{us} / V_{ud} from Kl2 decays

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2(\gamma)}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2(\gamma)}} m_{K^\pm}}\right)^{1/2} \frac{1-m_\mu^2/m_{\pi^\pm}^2}{1-m_\mu^2/m_{K^\pm}^2}\left(1-\frac{1}{2}\delta_{\mathrm{EM}}-\frac{1}{2}\delta_{SU(2)}\right)$$

Inputs from experiment:

From $K^\pm$ BR fit:

BR($K^\pm_{\mu 2(\gamma)}$) = 0.6358(11)

$\tau_{K\pm}$ = 12.384(15) ns

From PDG:

BR($\pi^\pm_{\mu 2(\gamma)}$) = 0.9999

$\tau_{\pi\pm}$ = 26.033(5) ns

Inputs from theory:

δ_{EM} Long-distance EM corrections

$\delta_{SU(2)}$ Strong isospin breaking
$f_K/f_\pi \rightarrow f_{K\pm}/f_{\pi\pm}$

f_K/f_π Ratio of decay constants

Cancellation of lattice-scale uncertainties from ratio

NB: Most lattice results already corrected for $SU(2)$-breaking: $f_{K\pm}/f_{\pi\pm}$

V_{us} / V_{ud} from Kl2 decays

Giusti et al.
PRL 120 (2018)

First lattice calculation of EM corrections to P_{l2} decays

- Ensembles from ETM
- N_f = 2+1+1 Twisted-mass Wilson fermions

$\delta_{SU(2)} + \delta_{EM} = -0.0122(16)$

- Uncertainty from quenched QED included (0.0006)

Compare to ChPT result from Cirigliano, Neufeld '11:

$\delta_{SU(2)} + \delta_{EM} = -0.0112(21)$

Di Carlo et al.
PRD 100 (2019)

Update, extended description, and systematics of Giusti et al.

$\delta_{SU(2)} + \delta_{EM} = -0.0126(14)$

$$|V_{us}/V_{ud}| \times f_K/f_\pi = 0.27679(28)_{BR}(20)_{corr}$$

Lattice results for f_K/f_π

N_f = 2+1+1 — f_K/f_π = 1.1978(22) S = 1.1

Average is problematic with correlations assumed by FLAG, dominated by FNAL/MILC17 (symmetrized)

ETM 21 New!	**1.1995(44)(7)**
TM quarks, 3sp, $m_\pi \to$ physical Not yet in FLAG '21 average! Replaces ETM 14E in our average	
Miller 20	1.1964(44)
FNAL/MILC17	1.1980($^{+13}_{-19}$)
HPQCD13A	1.1948(15)(18)

Share ensembles
Partially correlated uncertainties using FLAG prescription

N_f = 2+1 — f_K/f_π = 1.1946(34)*

QCDSF/UKQCD17	1.192(10)(13)
BMW16	1.182(10)(26)
RBC/UKQCD14B	1.1945(45)
BMW10	1.192(7)(6)
HPQCD/UKQCD07	1.198(2)(7)

* MILC10 omitted from average because unpublished

$K_{\mu 2}$

f_K/f_π = 1.1978(22)
N_f = 2+1+1

$$V_{us}/V_{ud} = 0.23108(23)_{exp}(42)_{lat}(16)_{IB}$$
$$V_{us} = 0.22504(28)_{exp}(41)_{lat}(06)_{ud}$$
$$\Delta^{(2)}_{CKM} = -0.00098(13)_{exp}(19)_{lat}(53)_{ud} \quad = -1.8\sigma$$

$$\Delta V_{us}(K_{\mu 3} - K_{\mu 2}) = -0.0174(73) \quad -2.4\sigma$$

Existing data from BNL865, KTeV, ISTRA+, KLOE, NA48, NA48/2
Upcoming data from KLOE-2 and NA62

Cabibbo Angle Anomaly as a BSM Signal

Cabibbo Angle Anomaly as a BSM Signal

Leptonic interactions

$$\mathcal{L}_{CC}^{(\mu)} = -\frac{G_F^{(0)}}{\sqrt{2}}\left(1+\epsilon_L^{(\mu)}\right)\ \bar{e}\gamma^\rho(1-\gamma_5)\nu_e \cdot \bar{\nu}_\mu\gamma_\rho(1-\gamma_5)\mu\ +\ \ldots$$

$$\frac{G_F^{(\mu)}V_{ud}}{\sqrt{2}}\left(1-\epsilon_L^{(\mu)}\right)$$

Semi-leptonic interactions

$$\begin{aligned}
\mathcal{L}_{\rm CC} \ = \ -\frac{G_F^{(0)}V_{ud}}{\sqrt{2}} \times \Big[& \left(\delta^{ab}+\epsilon_L^{ab}\right)\ \bar{e}_a\gamma_\mu(1-\gamma_5)\nu_b\cdot\bar{u}\gamma^\mu(1-\gamma_5)d \\
+ & \ \epsilon_R^{ab}\ \bar{e}_a\gamma_\mu(1-\gamma_5)\nu_b\cdot\bar{u}\gamma^\mu(1+\gamma_5)d \\
+ & \ \epsilon_S^{ab}\ \bar{e}_a(1-\gamma_5)\nu_b\cdot\bar{u}d \\
- & \ \epsilon_P^{ab}\ \bar{e}_a(1-\gamma_5)\nu_b\cdot\bar{u}\gamma_5 d \\
+ & \ \epsilon_T^{ab}\ \bar{e}_a\sigma_{\mu\nu}(1-\gamma_5)\nu_b\cdot\bar{u}\sigma^{\mu\nu}(1-\gamma_5)d\ \Big] + \text{h.c.}
\end{aligned}$$

$\varepsilon_i \sim (v/\Lambda)^2$

For global analysis of beta decays in this framework see:

Falkowski, Gonzalez-Alonso, Naviliat-Cuncic, 2010.13797

Cabibbo Angle Anomaly as a BSM Signal

Connect beta decays to UV physics via EFT: Wilson coeffs. of 4-fermion operators

$$
\begin{aligned}
|\bar{V}_{ud}|^2_{0^+\to 0^+} &= |V_{ud}|^2 \left(1 + 2\left(\epsilon_L^{ee} + \epsilon_R - \epsilon_L^{(\mu)}\right) + c^S_{0^+}(Z)\ \epsilon_S^{ee}\right) \\
|\bar{V}_{ud}|^2_{n\to pe\bar{\nu}} &= |V_{ud}|^2 \left(1\ +\ 2\left(\epsilon_L^{ee} + \epsilon_R - \epsilon_L^{(\mu)}\right) + c_n^S \epsilon_S^{ee} + c_n^T \epsilon_T^{ee}\right) \\
|\bar{V}_{us}|^2_{Ke3} &= |V_{us}|^2 \left(1 + 2\left(\epsilon_L^{ee(s)} + \epsilon_R^{(s)} - \epsilon_L^{(\mu)}\right)\right) \\
|\bar{V}_{ud}|^2_{\pi e3} &= |V_{ud}|^2 \left(1 + 2\left(\epsilon_L^{ee} + \epsilon_R - \epsilon_L^{(\mu)}\right)\right) \\
|\bar{V}_{us}|^2_{K_{\mu 2}} &= |V_{us}|^2 \left(1 + 2\left(\epsilon_L^{\mu\mu(s)} - \epsilon_R^{(s)} - \epsilon_L^{(\mu)}\right) - 2\frac{B_0}{m_\ell}\,\epsilon_P^{\mu\mu(s)}\right) \\
|\bar{V}_{ud}|^2_{\pi_{\mu 2}} &= |V_{ud}|^2 \left(1 + 2\left(\epsilon_L^{\mu\mu} - \epsilon_R - \epsilon_L^{(\mu)}\right) - 2\frac{B_0}{m_\ell}\,\epsilon_P^{\mu\mu}\right)
\end{aligned}
$$

Three distinct Cabibbo unitarity deficits may be defined

$$\Delta^{(1)}_{\mathrm{CKM}} = \left|V_{ud}^{\beta}\right|^2 + \left|V_{us}^{K_{\ell 3}}\right|^2 - 1$$ V_{us} from $K_{\ell 3}$ + V_{ud} from β decays

$$\Delta^{(2)}_{\mathrm{CKM}} = \left|V_{ud}^{\beta}\right|^2 + \left|V_{us}^{K_{\ell 2}/\pi_{\ell 2},\beta}\right|^2 - 1$$ V_{us}/V_{ud} from $K_{\mu 2}$ + V_{ud} from β decays

$$\Delta^{(3)}_{\mathrm{CKM}} = \left|V_{ud}^{K_{\ell 2}/\pi_{\ell 2},K_{\ell 3}}\right|^2 + \left|V_{us}^{K_{\ell 3}}\right|^2 - 1$$ V_{us} from $K_{\ell 3}$ + V_{us}/V_{ud} from $K_{\mu 2}$

Cabibbo Angle Anomaly as a BSM Signal

RH currents in ud- and us-sectors

V_{ud}, V_{us}, V_{ud}/V_{us} overconstrained, can solve all tensions

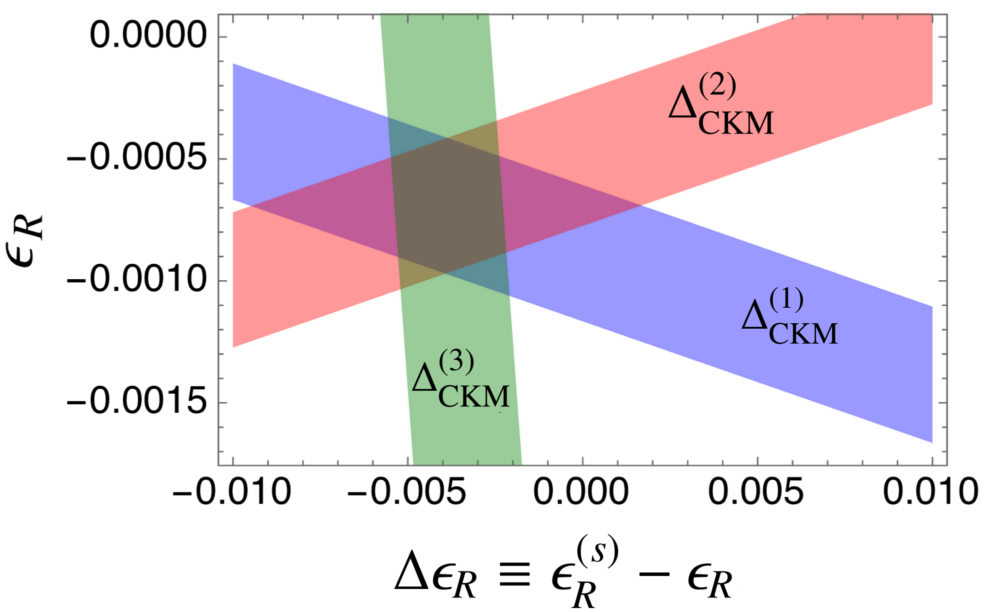
$$
\begin{aligned}
|\bar{V}_{ud}|^2_{0^+\to 0^+} &= |V_{ud}|^2 \left(1 + 2\,\epsilon_R\right) \\
|\bar{V}_{ud}|^2_{n\to pe\bar{\nu}} &= |V_{ud}|^2 \left(1 + 2\,\epsilon_R\right) \\
|\bar{V}_{us}|^2_{Ke3} &= |V_{us}|^2 \left(1 + 2\,\epsilon_R^{(s)}\right) \\
|\bar{V}_{ud}|^2_{\pi e3} &= |V_{ud}|^2 \left(1 + 2\,\epsilon_R\right) \\
|\bar{V}_{us}|^2_{K\mu 2} &= |V_{us}|^2 \left(1 - 2\,\epsilon_R^{(s)}\right) \\
|\bar{V}_{ud}|^2_{\pi\mu 2} &= |V_{ud}|^2 \left(1 - 2\,\epsilon_R\right)
\end{aligned}
$$

Cabibbo Angle Anomaly as a BSM Signal

RH currents in ud- and us-sectors

V_{ud}, V_{us}, V_{ud}/V_{us} overconstrained, can solve all tensions

Cirigliano et al, 2208.11707



$$\Delta^{(1)}_{\rm CKM} = 2\epsilon_R + 2\Delta\epsilon_R V_{us}^2,$$
$$\Delta^{(2)}_{\rm CKM} = 2\epsilon_R - 2\Delta\epsilon_R V_{us}^2,$$
$$\Delta^{(3)}_{\rm CKM} = 2\epsilon_R + 2\Delta\epsilon_R(2 - V_{us}^2)$$

$$\boxed{\begin{aligned}\epsilon_R &= -0.69(27)\times 10^{-3}\\ \Delta\epsilon_R &= -3.9(1.6)\times 10^{-3}\end{aligned}}$$

Λ_R ~ 5-10 TeV 2.5σ effect

$$|\bar V_{ud}|^2_{0^+\to 0^+} = |V_{ud}|^2\left(1+2\,\epsilon_R\right)$$
$$|\bar V_{ud}|^2_{n\to pe\bar\nu} = |V_{ud}|^2\left(1+2\,\epsilon_R\right)$$
$$|\bar V_{us}|^2_{Ke3} = |V_{us}|^2\left(1+2\,\epsilon_R^{(s)}\right)$$
$$|\bar V_{ud}|^2_{\pi e3} = |V_{ud}|^2\left(1+2\,\epsilon_R\right)$$
$$|\bar V_{us}|^2_{K\mu 2} = |V_{us}|^2\left(1-2\,\epsilon_R^{(s)}\right)$$
$$|\bar V_{ud}|^2_{\pi\mu 2} = |V_{ud}|^2\left(1-2\,\epsilon_R\right)$$

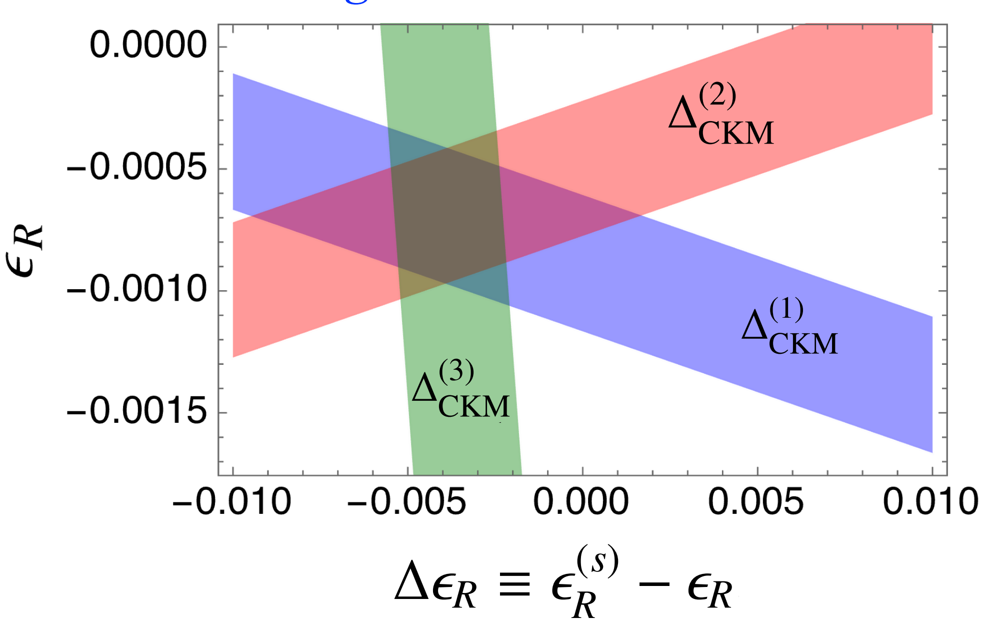
Cabibbo Angle Anomaly as a BSM Signal

RH currents in ud- and us-sectors

$$
\begin{aligned}
|\bar{V}_{ud}|^2_{0^+\to 0^+} &= |V_{ud}|^2\left(1+2\,\epsilon_R\right)\\
|\bar{V}_{ud}|^2_{n\to pe\bar{\nu}} &= |V_{ud}|^2\left(1+2\,\epsilon_R\right)\\
|\bar{V}_{us}|^2_{Ke3} &= |V_{us}|^2\left(1+2\,\epsilon_R^{(s)}\right)\\
|\bar{V}_{ud}|^2_{\pi e3} &= |V_{ud}|^2\left(1+2\,\epsilon_R\right)\\
|\bar{V}_{us}|^2_{K\mu 2} &= |V_{us}|^2\left(1-2\,\epsilon_R^{(s)}\right)\\
|\bar{V}_{ud}|^2_{\pi\mu 2} &= |V_{ud}|^2\left(1-2\,\epsilon_R\right)
\end{aligned}
$$

V_{ud}, V_{us}, V_{ud}/V_{us} overconstrained,
can solve all tensions

Cirigliano et al, 2208.11707



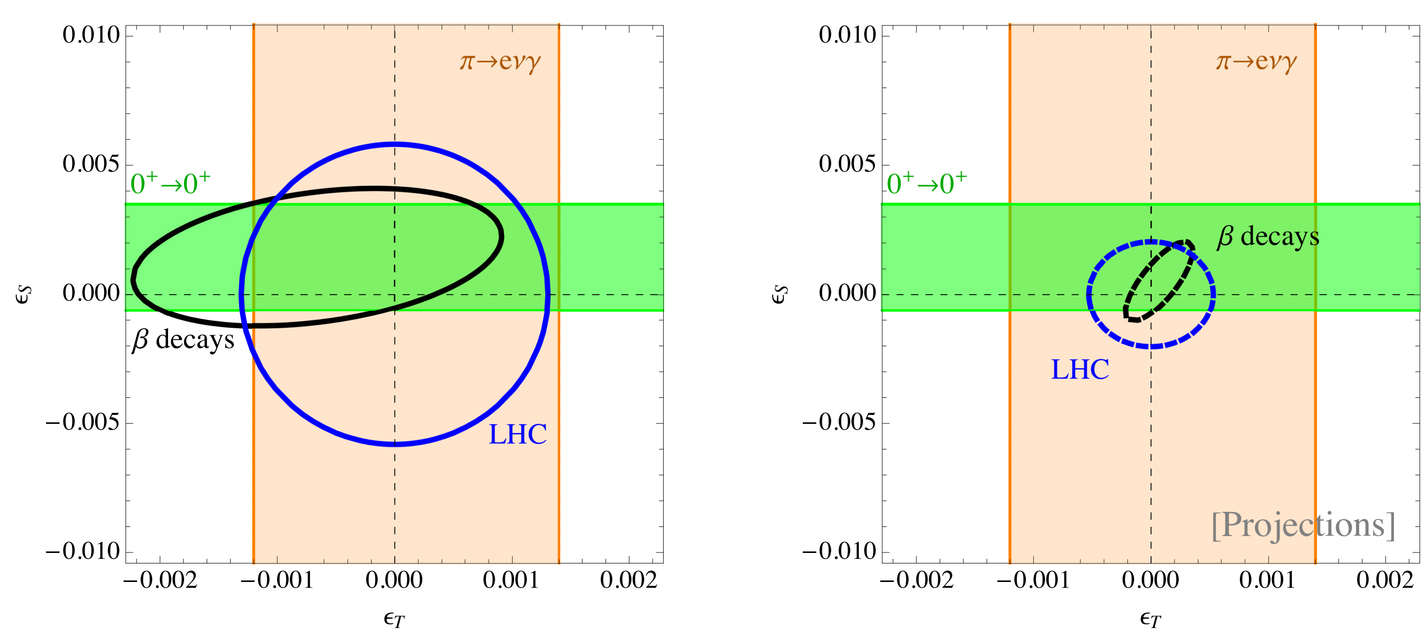
$$
\begin{aligned}
\Delta^{(1)}_{\rm CKM} &= 2\epsilon_R + 2\Delta\epsilon_R V_{us}^2,\\
\Delta^{(2)}_{\rm CKM} &= 2\epsilon_R - 2\Delta\epsilon_R V_{us}^2,\\
\Delta^{(3)}_{\rm CKM} &= 2\epsilon_R + 2\Delta\epsilon_R (2 - V_{us}^2)
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
\epsilon_R &= -0.69(27)\times 10^{-3}\\
\Delta\epsilon_R &= -3.9(1.6)\times 10^{-3}
\end{aligned}
}
$$

Λ_R ~ 5-10 TeV   2.5σ effect

Beta decay vs. LHC on S,T
Complementarity now and in the future!

Gonzalez-Alonso et al 1803.08732



Summary: Status of V_{ud} and top-row CKM unitarity

3-sigma CKM unitarity deficit established

Significant shift in V_{ud} due to shift in Δ_R^V

EW boxes: DR + Exp. + Lattice QCD+ ChPT +…

Calculation for Δ_R^V confirmed by several groups

Formalism applied to $K\ell 3$ decays;

Puzzles: $K\ell 2$ - $K\ell 3$, Beam-Bottle n-lifetime

Unified universal RC Δ_R^V and nuclear correction $\delta_{\rm NS}$

Both SM (V_{ud}) and BSM (b_F) tests depend on δ_C and $\delta_{\rm NS}$

Direct lattice QCD evaluation of the γW-box

Modern ab initio theory of δ_C and $\delta_{\rm NS}$ underway!

BSM: RH currents across light and strange quarks may resolve all puzzles