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Diabatic Representation for Heavy Mesons

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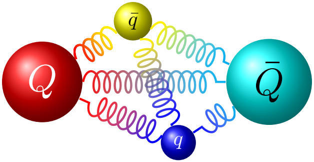
20th International Conference
on Hadron Spectroscopy and Structure
Genova, June 7, 2023

Outline

1. Born-Oppenheimer Approximation
2. String Breaking
3. Diabatic Representation
4. Practical Applications

BORN-OPPENHEIMER APPROXIMATION

BO Approximation for QCD

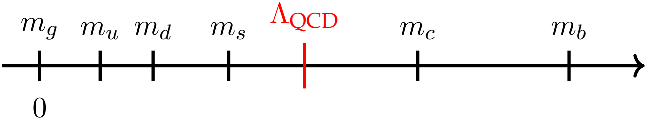


Heavy degrees of freedom

- heavy quarks

Light fields

- gluons
- light quarks



The sharp difference between the energy scales allows to separately solve the heavy quarks and light fields:

1. The energy levels for the light fields with static quarks at distance r, $V_i(r)$, are calculated in lattice QCD.
2. The motion of the heavy quarks is calculated from a Schrödinger equation with $V_i(r)$ as potential.

Watch out for avoided crossings!

If some energy levels show mixing, one has to include also coupling terms between the corresponding channels.

The presence of two static quarks breaks the symmetries of QCD:

- rotations;
- parity;
- charge-conjugation parity;

down to:

- cylindrical symmetry;
- combined CP symmetry.

The quantum numbers are not

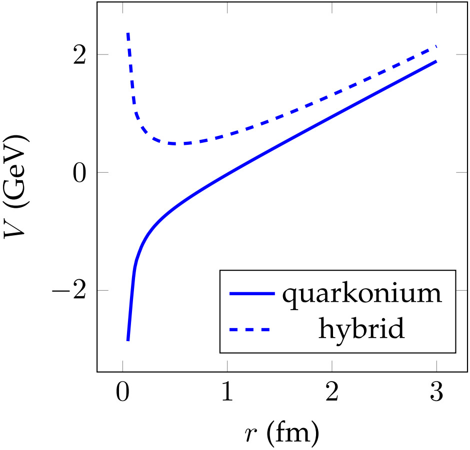
- J total angular momentum;
- P parity;
- C charge-conjugation;

but rather

- λ angular momentum projection on the $Q\bar{Q}$ axis;
- η g or u for positive or negative CP, respectively.

Heavy-Quark Spin Symmetry

The potentials are independent of the spin of the static quarks.



Juge, Kuti, and Morningstar 1999; Bali 2001

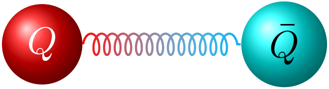
Excited states

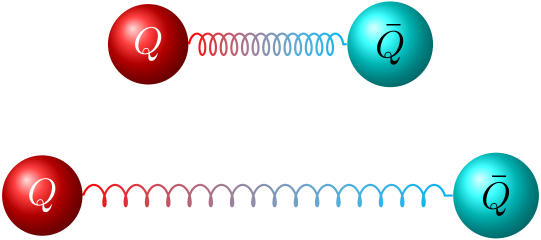
Hybrid potentials

Ground state

Quarkonium potential

STRING BREAKING


Q
$\bar{Q}$


Q
$\bar{Q}$
Q
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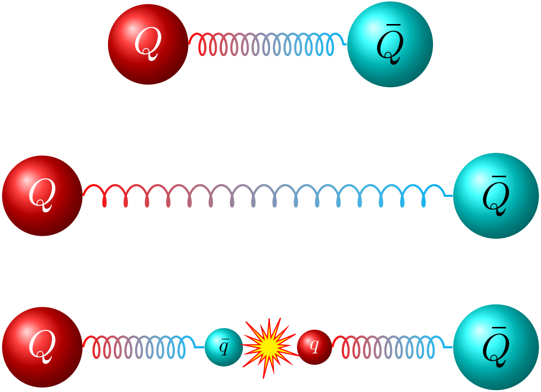
Q
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Q
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Q
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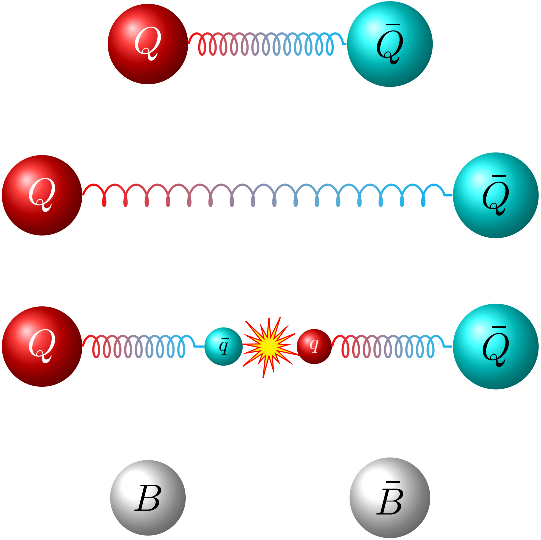
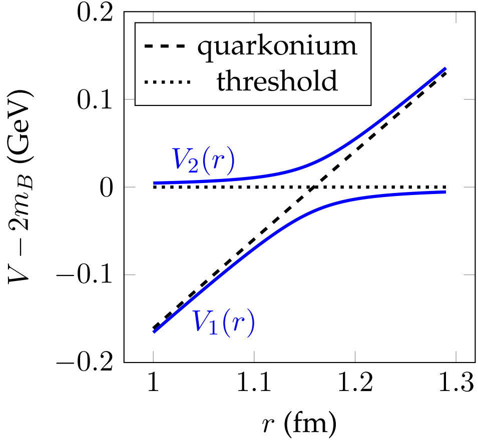
q
$\bar{Q}$



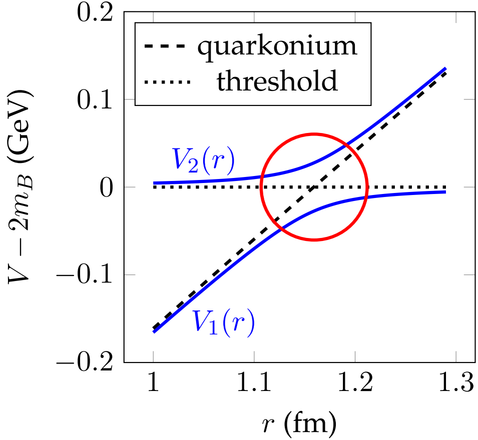
Q
$\bar{Q}$
Q
$\bar{Q}$
Q
$\bar{q}$
q
$\bar{Q}$
B
$\bar{B}$



Bali et al. 2005; Bulava et al. 2019

Open-flavor threshold

Minimum energy for the production of an open-flavor meson pair



Bali et al. 2005; Bulava et al. 2019

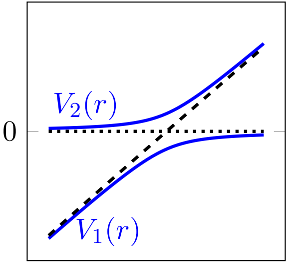
Open-flavor threshold

Minimum energy for the production of an open-flavor meson pair

Avoided crossing

$Q\bar{Q}$ and $B\bar{B}$ mix through string breaking.

Static potentials of mixed $Q\bar{Q}$-$B\bar{B}$ channels can be seen as eigenvalues of an interaction matrix between pure $Q\bar{Q}$ and $B\bar{B}$.



$$\longleftrightarrow \begin{pmatrix} V_{\mathcal{Q}}(r) & g(r) \\ g(r) & 0 \end{pmatrix} = \mathbf{G}(r)$$

$V_{\mathcal{Q}}(r)$ is the quarkonium potential;

$g(r)$ is the string-breaking rate from lattice QCD;

0 is the static-meson pair threshold (by definition).

Why?

The physical mass difference Δ between B and B^* mesons breaks heavy-quark spin symmetry. This effect is particularly important for molecular states.

Correcting the static potentials

1. The string-breaking rate $g(r)$ splits into different transition rates for the $B\bar{B}$, $B\bar{B}^* \pm B^*\bar{B}$, and $B^*\bar{B}^*$ channels with coefficients determined by Dirac algebra.
2. A factor of Δ or 2Δ is added to the threshold energies of the $B\bar{B}^* \pm B^*\bar{B}$ or $B^*\bar{B}^*$ channels.

The interaction matrix depends on the $Q\bar{Q}$ spin state

Q and $\bar{Q}$ have two spin states each, so there are four different interaction matrices $\mathbf{G}^{\eta,\lambda}(r)$ in total:

3 $\eta = g$ ($CP = +$, $Q\bar{Q}$ spin $s = 1$) and projection $\lambda = -1, 0, +1$;

1 $\eta = u$ ($CP = -$, $Q\bar{Q}$ spin $s = 0$) and projection $\lambda = 0$.

Interaction matrix with $\eta = u$ and $\lambda = 0$

$$\mathbf{G}^{u,0}(r) = \begin{pmatrix} V_{\mathcal{Q}}(r) & -\frac{1}{\sqrt{2}}g(r) & -\frac{1}{\sqrt{2}}g(r) \\ -\frac{1}{\sqrt{2}}g(r) & \Delta & 0 \\ -\frac{1}{\sqrt{2}}g(r) & 0 & 2\Delta \end{pmatrix}$$

DIABATIC REPRESENTATION

Why not use the interaction matrices as potentials?

The interaction matrices $\mathbf{G}^{\eta,\lambda}(r)$ are not the multichannel BO potentials. The precise relation between them depends on the particular representation of BO.

Coupled channels in BO can be treated in two representations:

Adiabatic

- mixed $Q\bar{Q}$ and di-meson channels
- diagonal potential matrix

Diabatic

- pure $Q\bar{Q}$ and di-meson channels
- diagonal kinetic energy matrix

$$\sum_{i',\sigma'} \left(-\delta_{i,i'}\delta_{\sigma,\sigma'} \frac{\nabla^2}{m_Q} + V_{i,i'}^{\eta,\sigma,\sigma'}(\vec{r}) \right) \Psi_{i'}^{\eta,\sigma'}(\vec{r}) = E\Psi_i^{\eta,\sigma}(\vec{r})$$

Diabatic channels with total spin s and projection σ

$$\Psi_i^{\eta,\sigma}(\vec{r}) \rightarrow \Psi_{Q\bar{Q}(s)}^{\eta,\sigma}(\vec{r}), \Psi_{B\bar{B}(s)}^{\eta,\sigma}(\vec{r})$$

Diabatic potential matrix

$$V_{i,i'}^{\eta,\sigma,\sigma'}(\vec{r}) = \sum_{\lambda} D_{\sigma,\lambda}^{s_i}(\varphi,\theta,\psi) D_{\sigma',\lambda}^{s_{i'}}(\varphi,\theta,\psi)^* G_{i,i'}^{\eta,\lambda}(r)$$

The total static angular momentum is

$$\vec{S} = \vec{S}_{Q\bar{Q}} + \vec{J}_{\text{light}}$$

with:

$\vec{S}_{Q\bar{Q}}$ the total spin of the heavy quarks;

$\vec{J}_{\text{light}}$ the total angular momentum of the light fields.

Static sources separated by $\vec{r}$ break rotational symmetry

S^2 is not conserved. Only the projection $\vec{S} \cdot \hat{r}$ is.

For quarkonium

$$\vec{J}_{\text{light}} = 0, \qquad \text{so} \qquad \vec{S} = \vec{S}_{Q\bar{Q}}.$$

When the orbital angular momentum $\vec{L}$ is introduced, the total angular momentum is

$$\vec{J} = \vec{L} + \vec{S}$$

The values of L^2 and S^2 may be different for each channel, but it can be checked that J^2 and $\vec{J} \cdot \hat{z}$ are the same.

Reintroducing the motion restores rotational symmetry

With the introduction of orbital angular momentum, one can use angular momentum algebra to derive exact total angular momentum conservation from the diabatic potential matrix.

The spectrum consists of states with definite J^{PC}.

PRACTICAL APPLICATIONS

In practice, for any given J^{PC} configuration, the spectrum is calculated from a radial potential matrix with elements

$$V^{J^{PC}}_{i,i',l,l'}(r) = V^{\eta,J}_{i,i',l,l'}(r) + \delta_{i,i'}\delta_{l,l'}\frac{l(l+1)}{m_Q r^2}.$$

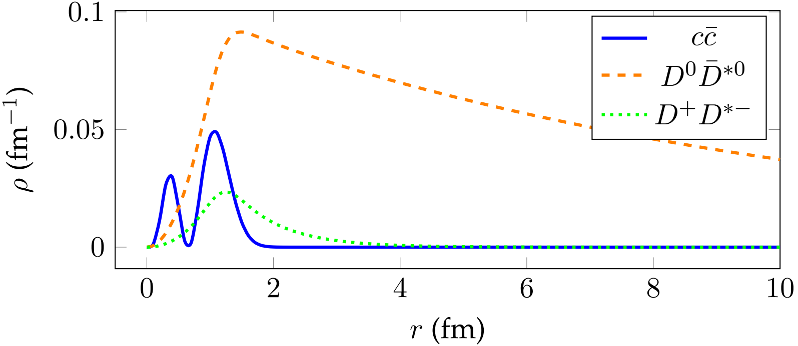
Potential matrix for $J^{PC} = 0^{-+}$

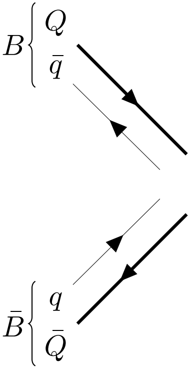
$$\mathbf{V}^{0^{-+}}(r) = \begin{pmatrix} V_Q(r) & \frac{1}{\sqrt{2}}g(r) & \frac{1}{\sqrt{2}}g(r) \\ \frac{1}{\sqrt{2}}g(r) & \Delta + \frac{2}{m_Q r^2} & 0 \\ \frac{1}{\sqrt{2}}g(r) & 0 & 2\Delta + \frac{2}{m_Q r^2} \end{pmatrix}$$

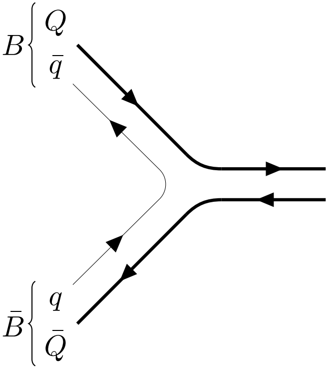
Phenomenological study of $X(3872)$

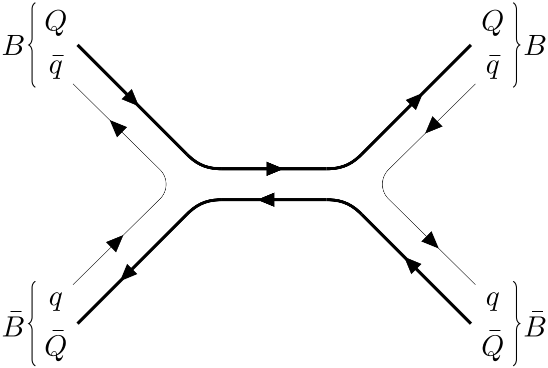
The potential matrix for $J^{PC} = 1^{++}$ can be fine tuned so there is a bound state just below the $D^0\bar{D}^{*0}$ threshold.

Calculated radial probability density $\rho(r)$ for $X(3872)$:




B
Q
$\bar{q}$
$\bar{B}$
q
$\bar{Q}$


B
Q
$\bar{q}$
$\bar{B}$
q
$\bar{Q}$


B
Q
$\bar{q}$
Q
$\bar{q}$
B
$\bar{B}$
q
$\bar{Q}$
q
$\bar{Q}$
$\bar{B}$

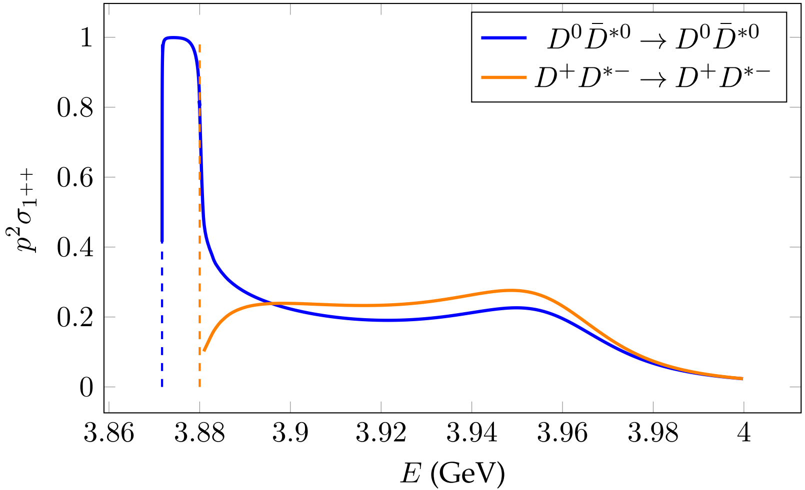
SUMMARY

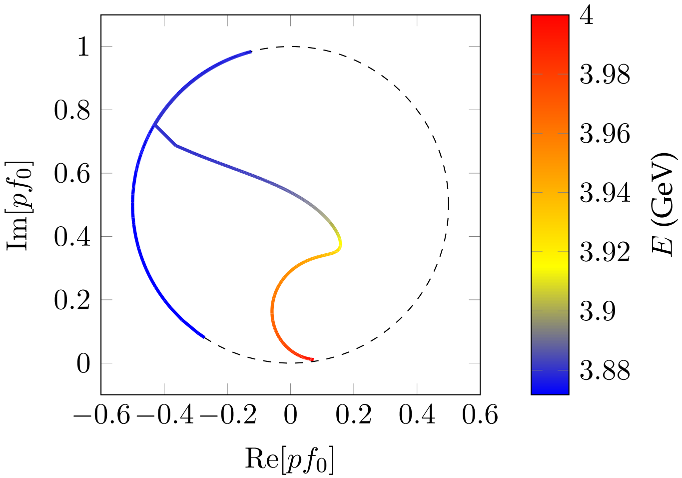
Conclusion

Mesons containing two heavy quarks can be studied *ab initio* using QCD potentials with string breaking.

- The potentials can be directly calculated in lattice QCD with static quark-antiquark and di-meson sources.
- Heavy-quark spin symmetry breaking from meson mass splittings can be consistently taken into account in lattice-QCD potentials.
- Adding orbital angular momentum within the diabatic BO framework ensures exact conservation of total angular momentum.

APPENDIX


$D^0\bar{D}^{*0} \to D^0\bar{D}^{*0}$
$D^+D^{*-} \to D^+D^{*-}$
$p^2\sigma_{1^{++}}$
E (GeV)
0
0.2
0.4
0.6
0.8
1
3.86
3.88
3.9
3.92
3.94
3.96
3.98
4


1
0.8
0.6
0.4
0.2
0
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
Im[pf_0]
Re[pf_0]
4
3.98
3.96
3.94
3.92
3.9
3.88
E (GeV)

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