Femtoscopy for $D_0^*(2300)$ and $D_{s0}^*(2317)$ states

HADRON 2023

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HADRON 2023
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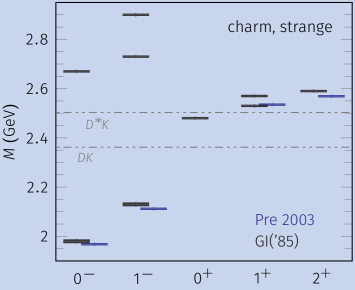
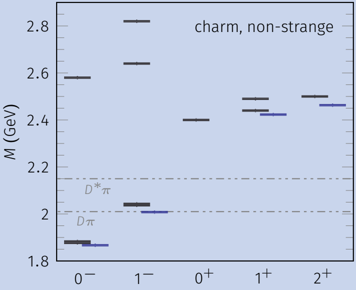
Quark model in the open-charm sector

- Quark model $c\bar{n}$ is still our baseline:

  «In this paper we present the results of a study of light and heavy mesons in soft QCD. We have found that all mesons–from the pion to the upsilon–can be described in a unified framework.» [Godfrey, Isgur, PR,D32,189('85)]





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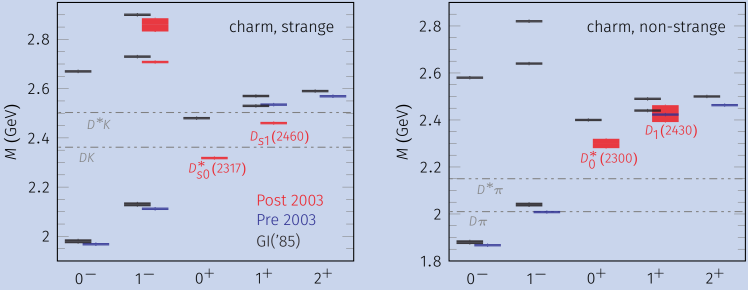


- The discovery of $D^*_{s0}(2317)$ in 2003 (and $D_{s1}(2460)$ later on) is "equivalent" to the discovery of $X(3872)$ in charmonium-like system.

[BABAR, PRL,90,242001('03)]
[CLEO, PR,D68,032002('03)]

Theoretical interpretations

$c\bar{q}$ states

Colangelo, De Fazio, Phys. Lett. B **570**, 180 (2003)
Dai *et al.* Phys. Rev. D **68**, 114011 (2003)
Narison, Phys. Lett. B **605**, 319 (2005)
Bardeen *et al.*, Phys. Rev. D **68**, 054024 (2003)
Lee *et al.*, Eur. Phys. J. C **49**, 737 (2007)
Wang, Wan, Phys. Rev. D **73**, 094020 (2006)

Pure tetraquarks

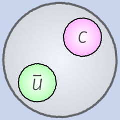
Cheng, Hou, Phys. Lett. B **566**, 193 (2003)
Terasaki, Phys. Rev. D **68**, 011501 (2003)
Chen, Li, Phys. Rev. Lett. **93**, 232001 (2004)
Maiani *et al*,, Phys. Rev. D **71**, 014028 (2005)
Bracco *et al.*, Phys. Lett. B **624**, 217 (2005)
Wang, Wan, Nucl. Phys. A **778**, 22 (2006)

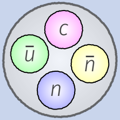
$c\bar{q}$+ tetraquarks or meson–meson

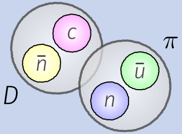
Browder *et al.*, Phys. Lett. B **578**, 365 (2004)
van Beveren, Rupp, Phys. Rev. Lett. **91**, 012003 (2003)

Heavy-light meson–meson molecules

Barnes *et al.*, Phys. Rev. D **68**, 054006 (2003)
Szczepaniak, Phys. Lett. B **567**, 23 (2003)
Kolomeitsev, Lutz, Phys. Lett. B **582**, 39 (2004)
Hofmann, Lutz, Nucl. Phys. A **733**, 142 (2004)
Guo *et al.*, Phys. Lett. B **641**, 278 (2006)
Gamermann *et al.*, Phys. Rev. D **76**, 074016 (2007)
Faessler *et al.*, Phys. Rev. D **76**, 014005 (2007)
Flynn, Nieves, Phys. Rev. D **75**, 074024 (2007)







Meanwhile, in the lattice...

- Masses larger than the physical ones if using $c\bar{s}$ interpolators only.

Bali, Phys. Rev. D **68**, 071501 (2003)
UKQCD Collab., Phys. Lett. B **569**, 41 (2003)

- Masses consistent with $D_0^*(2300)$ and $D_{s0}^*(2317)$ obtained when "meson-meson" interpolators are employed.

Mohler, Prelovsek, Woloshyn, Phys. Rev. D **87**, 034501 (2013)
Mohler *et al.*, Phys. Rev. Lett. **111**, 222001 (2013)

- Close to the physical point: RQCD Collab., Phys. Rev. D **96**, 074501 (2017)

- More complete studies from the HadSpec collaboration:
  - $D\pi$, $D\eta$ and $D_s\bar{K}$ coupled-channel scattering. A bound state with large coupling to $D\pi$ is identified with $D_0^*(2300)$.

    HadSpec Collab., JHEP **1610**, 011 (2016)
  - $D_{s0}^*(2317)$: A bound state is found in the DK channel, with:
    - $\Delta E = 25(3)$ MeV ($m_\pi = 391$ MeV)
    - $\Delta E = 57(3)$ MeV ($m_\pi = 239$ MeV)
    - Compare with experimental, $\Delta E \simeq 45$ MeV (the dependence on m_π does not need to be monotonic!

[HadSpec Collab., JHEP 02 (2021) 100; JHEP 07 (2021) 123]

Lightest 0^+ open-charm situation and puzzles

- $D^*_{s0}(2317)$ $(S, I) = (1, 0)$ $\quad M_{D^*_{s0}(2317)} = 2317.8 \pm 0.5$ MeV (PDG)
- $D^*_0(2300)$ $(S, I) = (0, 1/2)$ $\quad$ Not so well stablished:

	Collab.	M (MeV)	$\Gamma/2$ (MeV)	Ref.
Neutral	Belle	2308 ± 36	138 ± 33	Phys. Rev. D **69**, 112002 (2004)
Neutral	BaBar	2297 ± 22	137 ± 25	Phys. Rev. D **79**, 112004 (2009)
Neutral	FOCUS [?]	2407 ± 41	120 ± 40	Phys. Lett. B **586**, 11 (2004)
Charged	LHCb	2360 ± 34	128 ± 29	Phys. Rev. D **92**, 032012 (2015) ($B^0 \to \bar{D}^0 K^+ \pi^-$)
Charged	LHCb	2349 ± 7	109 ± 9	Phys. Rev. D **92**, 012012 (2015) ($B^0 \to \bar{D}^0 \pi^+ \pi^-$)
Charged	LHCb	2354 ± 13	128 ± 13	Phys. Rev. D **92**, 012012 (2015) ($B^0 \to \bar{D}^0 \pi^+ \pi^-$)
Charged	FOCUS [?]	2403 ± 38	142 ± 21	Phys. Lett. B **586**, 11 (2004)
	PDG	2343 ± 10	115 ± 8	Neutral and charged

Three puzzles

1. **Mass problem:** Why are $D^*_{s0}(2317)$ and $D_{s1}(2460)$ masses much lower than the CQM expectations?
2. **Splittings:** Why $M_{D_{s1}(2460)} - M_{D^*_{s0}(2317)} \simeq M_{D^*} - M_D$ (within a few MeV)?
3. **Hierarchy:** Why $M_{D^*_0(2300)} > M_{D^*_{s0}(2317)}$, *i.e.*, why $c\bar{u}$, $c\bar{d}$ heavier than $c\bar{s}$?

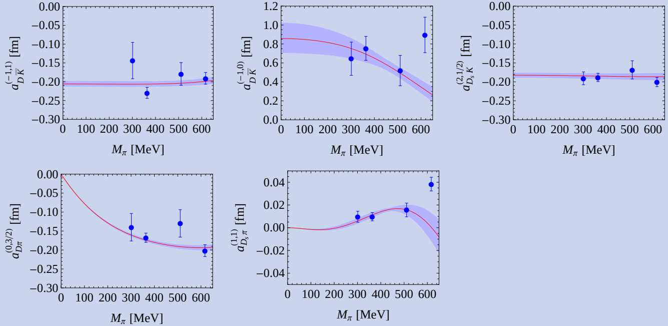
$D\pi$, $D\eta$, $D_s\overline{K}$ scattering amplitudes

- Coupled channel T-matrix: $D\pi$, $D\eta$, $D_s\overline{K}$ scattering $[J^P = 0^+, (S,I) = (0, \frac{1}{2})]$.
- Unitarity: $T^{-1}(s) = V^{-1}(s) - \mathcal{G}(s)$
- Chiral symmetry used to compute the $\mathcal{O}(p^2)$ potential:

$$f^2 V_{ij}(s,t,u) = C^{ij}_{\rm LO}\,\frac{s-u}{4} + \sum_{a=0}^{5} h_a C^{ij}_a(s,t,u)$$

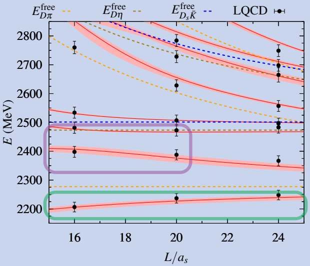
Guo *et al.*, Phys. Lett. B **666**, 251 (2008)
Liu *et al.*, Phys. Rev. D **87**, 014508 (2013)

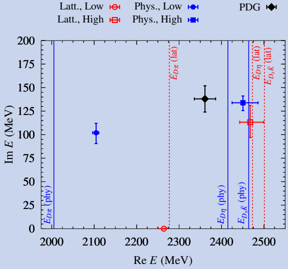
- h_a (LECs), subtraction constants: free parameters previously fixed.
- Fitted to reproduce scattering lengths obtained in a LQCD simulation:



$D\pi$, $D\eta$, $D_s\bar{K}$ and $D^*(2300)$: Comparison with LQCD

MA, P. Fernández-Soler, F.-K. Guo, J. Nieves, Phys. Lett. B **767**, 465 (2017)





- $E_n(L)$ are provided for $D\pi$, $D\eta$, $D_s\bar{K}$ in a recent LQCD simulation. [G. Moir *et al.*, JHEP 1610, 011 (2016)]
- Red Bands: Our amplitude in a finite volume. [MA *et al.*, Phys. Lett. B **767**, 465 (2017)]
- No fit is performed (LECs previously determined)
- Level below threshold, associated with a bound state.
- Second level has large shifts w. r. t. thresholds, non-interacting energy levels.
- For lattice masses, we find a bound state and a resonance.
- For physical masses, both evolve into resonances.

	M (MeV)	$\Gamma/2$ (MeV)
Low pole	2105^{+6}_{-8}	102^{+10}_{-12}
High pole	2451^{+36}_{-26}	134^{+7}_{-8}

- We also study DK, $D_s\eta$, $(S,I)=(1,0)$, $D^*_{s0}(2317)$ bound state: $M = 2315^{+18}_{-28}$ MeV.

The $D^*_0(2300)$ structure actually consists of two different states (with complicated interferences with the thresolds)

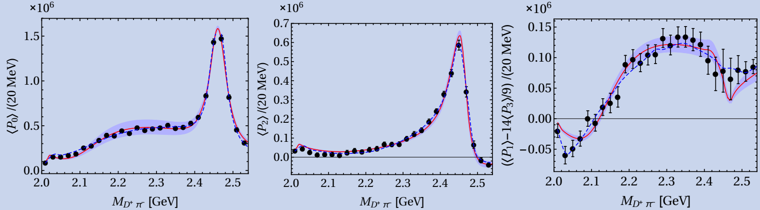
Previously reported in:

Kolomeitsev, Lutz, Phys. Lett. B **582**, 39 (2004)
Guo *et al.*, Phys. Lett. B **641**, 278 (2006)
Guo *et al.*, Eur. Phys. J. A **40**, 171 (2009)

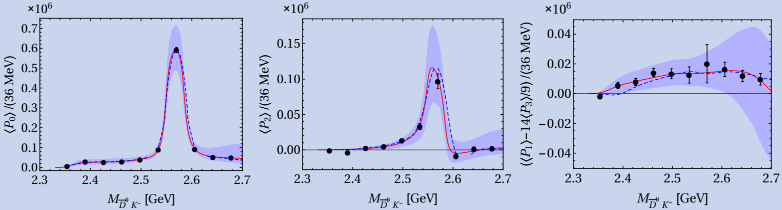
$D\pi$, $D\eta$, $D_s\bar{K}$ and $D^*(2300)$: Comparison with LHCb data

M.-L. Du, MA, P. Fernández-Soler, F.-K. Guo, C. Hanhart, U.-G. Meißner, J. Nieves, D.-L. Yao, PR,D98,094018('18)

- $B^- \to D^+\pi^-\pi^-$ [LHCb Collab., PR,D94,072001('16)]

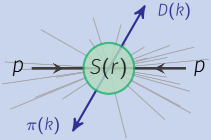


- $B_s^0 \to \bar{D}^0 K^-\pi^+$ [LHCb Collab., PR,D90,072003('14)]



- Rapid movement in $\langle P_{13}\rangle$ [no $D_2(2460)$] at 2.4-2.5 GeV [$D\eta$ and $D_s\bar{K}$].
- Recall: these are the amplitudes with two states in the $D_0^*(2300)$ region, and no fit of the T-matrix parameters is done (production parameters are fitted).

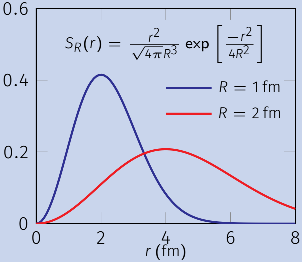
(Very basic introduction to) femtoscopy



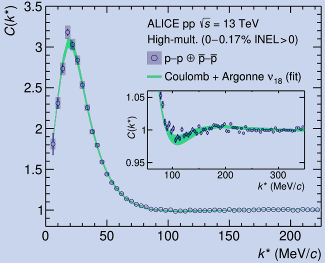
$$C_{\rm exp}(k) = \xi(k)\,\frac{N_{\rm same}(k)}{N_{\rm mixed}(k)}$$

$$C_{\rm th}(k) = \sum_{\ell=0}(2\ell+1)\int dr\, S_R(r)\,|\psi_\ell(r,k)|^2$$

$$\psi_\ell^{[{\rm asy,nr}]}(k,r) = j_\ell(kr) + f_\ell(k)\,\frac{e^{i(kr-\pi\ell/2)}}{r}$$



[ALICE PL,B805,135419('20)]



- Known the source $S(r)$, explore interactions (encoded in the wave function)
- A new method to explore hadron interactions
- Lot of attraction. In this conference:
  - M. Janik [Mon. 11:00]
  - V. Mantovani [Mon. 14:30]
  - D. Mihaylov [Mon. 17:40]
  - This talk [Now]
  - L. Graczykowski [Wed. 14:00]
  - W. Rzesa [Wed. 14:24]
  - L. Serksnyte [Wed. 15:12]
  - E. Oset [Thu. 14:30]
  - M. Lesch [Thu. 15:12]
  - R. Lea [Thu. 15:42]

[Fabbietti, Mantovani, Vázquez-Doce, ARNPS,71,377('21)]

Check those talks for more references!

Open charm femtoscopy

[MA, Nieves, Ruiz–Arriola, 2304.03107]

Channels (S-wave only):

- D_0^* ($S=0, I=1/2$): $D\pi$, $D_s\overline{K}$, $D\eta$.
- D_{s0}^* ($S=1, I=0,1$): $D_s^+\pi^0$, D^0K^+, D^+K^0, $D_s^+\eta$.

$$C_i(s) = 1 + \int_0^\infty dr\, S_R(r) \left[\sum_j \left| \psi_i^j(s,r) \right|^2 - j_0(p_i r)^2 \right]$$

$$\psi_i^j(s,r) = j_0(p_i r)\delta_{ij} + \widetilde{G}_j(s,r) T_{ji}(s)$$

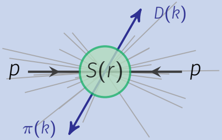
$$T^{-1}(s) = V^{-1}(s) - G(s) \quad \text{(previous slides)}$$

$$\widetilde{G}_j(s,r) = \frac{1}{\pi} \int_{s_{\text{th}}}^\infty ds' \frac{p_j(s')}{8\pi\sqrt{s'}} \frac{j_0(p_j(s')r)}{s - s' + i\epsilon} \theta(\Lambda - p_i(s'))$$

- $C(k)$ in terms of on-shell T-matrix:
  - Feijoo, Vidaña, MA, Nieves, Oset, 2303.06079
- Coupled channels based on:
  - Lednicky, Lyuboshitz($\times 2$), PAN,61,2950('98)
  - Haidenbauer, NPA,981,1('19)
- Cut-off Λ:
  - Only used in $\widetilde{G}(s,r)$, not in $G(s)$
  - Very soft effect, because $\widetilde{G}$ would be convergent (extra $1/p$ from j_0)
  - Take $\Lambda \in [0.6, 0.9]$ GeV

Goal of our paper [MA, Nieves, Ruiz–Arriola, 2304.03107]

To use our previously fixed T-matrices in the open-charm sector to predict correlation functions to be measured



Accesing isospin-definite $C(k)$ from physical channels

[MA, Nieves, Ruiz–Arriola, 2304.03107]

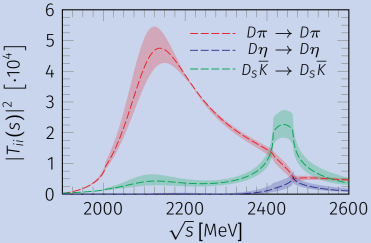
- We are considering $I_z = +\frac{1}{2}$ channels: $D^+\pi^0/D^0\pi^+$, $D^+\eta$, $D_s^+\overline{K}^0$.
- We avoid Coulomb interaction, which shows up in $I_z = -\frac{1}{2}$, through $D^+\pi^-$ (not in $D^0\pi^0$) and $D_s^+K^-$.
- Q: Does it make sense to use isospin channels, when physical ones are measured?
  - Physical channels will have combination of $I = \frac{1}{2}$ and $I = \frac{3}{2}$.
  - Potentially, you can also have Coulomb interaction.

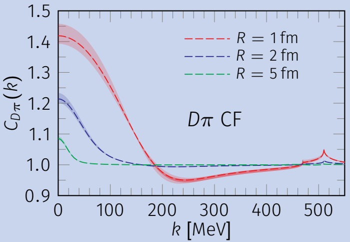
  A: Let us see...
- Inserting isospin decompositions in the amplitudes, we get interesting relations:
  - $I = \dfrac{1}{2}$: $C_{D\pi}^{(1/2)} = 2C_{D^0\pi^+} - C_{D^+\pi^0} = \dfrac{3C_{D^0\pi^+} - C_{D^0\pi^-}}{2}$
  - $I = \dfrac{3}{2}$: $C_{D\pi}^{(3/2)} = 2C_{D^+\pi^0} - C_{D^0\pi^+} = C_{D^0\pi^-}$

Open charm femtoscopy: results $S = 0$, $I = 1/2$ $[D\pi, D\eta, D_s\bar{K}]$

[MA, Nieves, Ruiz–Arriola, 2304.03107]

- Lower pole, peak at 2135 MeV would correspond to $k_{D\pi} = 215$ MeV. However, we find a minimum at $C_{D\pi}(k)$.

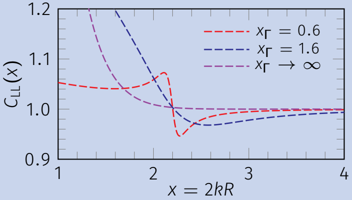




- Take the LL approximation to $C(k)$, and $f(k)$ in terms of $\delta(k)$:

$$C_{\rm LL}(k) = 1 + \frac{2\sin^2\delta(k)}{x^2}\left(e^{-x^2} + \frac{2xF_1(x)}{\sqrt{\pi}}\cot\delta(k)\right)$$

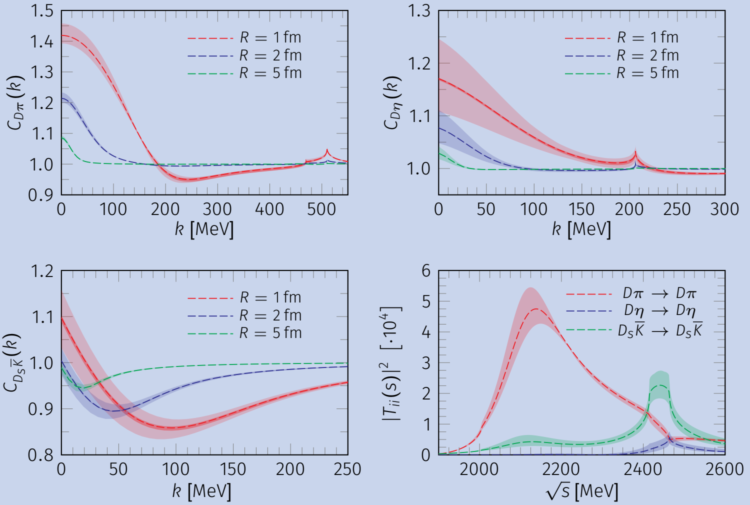
  - For a simple BW: $C_{\rm LL}(k_R) = 1$, $\quad C'_{\rm LL}(k_R) < 0$



Conclusion: the minimum at $k_{D\pi} = 215$ MeV is a clear signature of the lowest pole.

Open charm femtoscopy: results $S = 0$, $I = 1/2$ $[D\pi, D\eta, D_s\overline{K}]$

[MA, Nieves, Ruiz–Arriola, 2304.03107]



- Two different minima at $\sqrt{s} \simeq 2135$ MeV ($D\pi$ CF) and 2475 MeV ($D_s\overline{K}$ CF), produced by the two different D_0^* states, can be observed
- Their observation would constitute a strong additional support of the two-state pattern.

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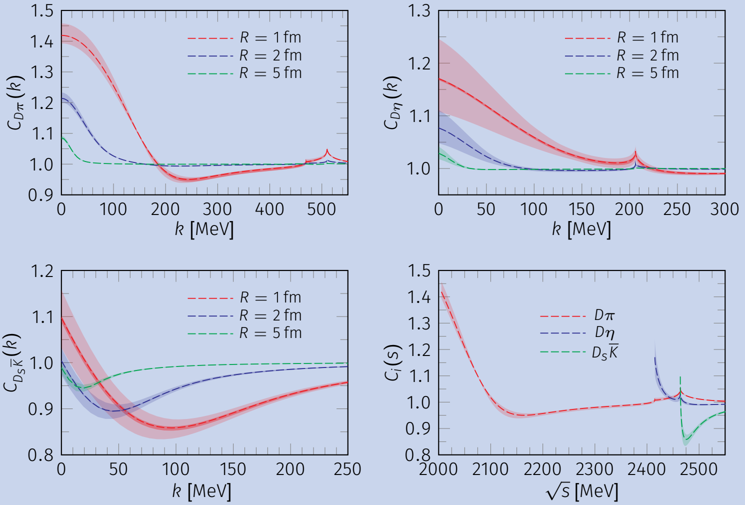

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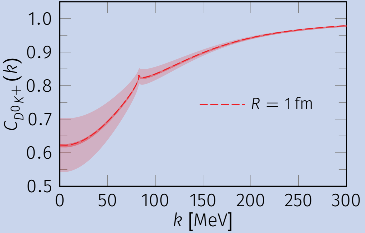
Open charm femtoscopy: results $S=1$, $I=0$ and 1 $[D_s^+\pi^0,\ D^0K^+,\ D^+K^0,\ D_s^+\eta]$

[MA, Nieves, Ruiz–Arriola, 2304.03107]

- $D_{s0}^*(2317)$ is a (mostly) DK $I=0$ bound state, with $B=45\,\text{MeV}$, $p_B=190i\,\text{MeV}$
- Why use physical channels?
  - $E^{\text{th}}_{D^+K^0} - E^{\text{th}}_{D^0K^+} \simeq 9\,\text{MeV}$ $[k_{D^0K^+} \simeq 83\,\text{MeV}]$
  - Also, $C_{D^0K^+} = C_{D^+K^0} = \dfrac{C_0^{DK} + C_1^{DK}}{2}$
- Clear depletion at threshold, related to the presence of $D_{s0}^*(2317)$
- Similar trends in recent works, only small differences in the values:
  - [Liu, Lu, Geng, 2302.01046]
  - [Ikeno, Toledo, Oset, 2305.16431]



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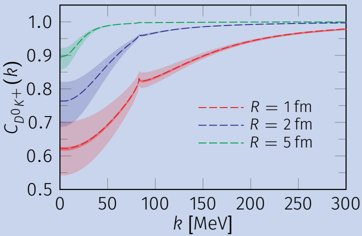
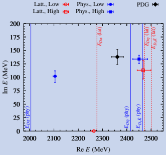
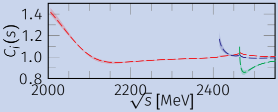


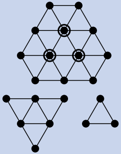
Summary and conclusions

- We have employed unitarized NLO chiral amplitudes to study the open-charm sector.
- The LECs are fixed through LQCD calculations of m_π-dependent scattering lengths.
- The $D_0^*(2300)$ structure is actually produced by two different states (poles), together with complicated interferences with thresholds.
- This two-state structure receives strong support. Without any fit, the amplitudes are compatible with:
  - available LQCD simulations,
  - and experimental data
- This picture, with the lowest D_0^* pole and $D^*(2317)$ being flavour partners nicely solves simultaneously all the puzzles.
- We have used these amplitudes to predict femtoscopy correlation functions in the open-charm sector.
- In particular, we have highlighted the imprints that the two-state structure leaves on $C_{D\pi}$, $C_{D\eta}$, and $C_{D_s\overline{K}}$.
- The measurements of these CFs can shed further light on the (very interesting!) open-charm sector.

Scheme of unitary EFTs relations to LQCD and experiments







Femtoscopic CFs

Spectroscopy (two D_0^*, one D_{s0}^*)

$SU(3)$ limit origin of poles

Predictions for other channels

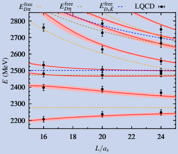
HL Amplitudes:

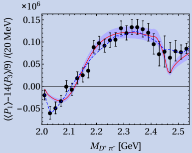
- Chiral & HQSS
- Unitarity

Comparison with LQCD levels

Fixing free parameters

Comparison with LHCb (& BaBar) exp. data





Femtoscopy for $D_0^*(2300)$ and $D_{s0}^*(2317)$ states

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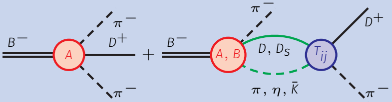
HADRON 2023
20th International Conference on
Hadron Spectroscopy and Structure
Genova Jun. 5-9, 2023

Comparison with experimental data: $B^- \to D^+\pi^-\pi^-$

Du, MA, Fernández-Soler, Guo, Hanhart, Meißner, Nieves, Yao, PR,D98,094018('18)

- $\mathcal{A}(s,z) = \mathcal{A}_0(s) + \sqrt{3}\mathcal{A}_1(s)P_1(z) + \sqrt{5}\mathcal{A}_2(s)P_2(z) + \ldots$
- P-,D-wave as in LHCb paper: D^*, $D^*(2680)$ in P-wave, $D_2^*(2460)$ in D-wave
- S-wave parameterization:

$\mathcal{A}_0(s) =$



$$\mathcal{A}_0(s) = A\Bigg\{E_\pi\left[2 + G_1(s)\left(\frac{5}{3}T_{11}^{1/2}(s) + \frac{1}{3}T^{3/2}(s)\right)\right] + \frac{1}{3}E_\eta G_2(s)T_{21}^{1/2}(s) + \sqrt{\frac{2}{3}}E_{\bar{K}}G_3(s)T_{31}^{1/2}(s)\Bigg\} + BE_\eta G_2(s)T_{21}^{1/2}(s),$$

- Angular moments: $\langle P_\ell\rangle(s) = \int dz\, |\mathcal{A}(s,z)|^2\, P_\ell(z)$

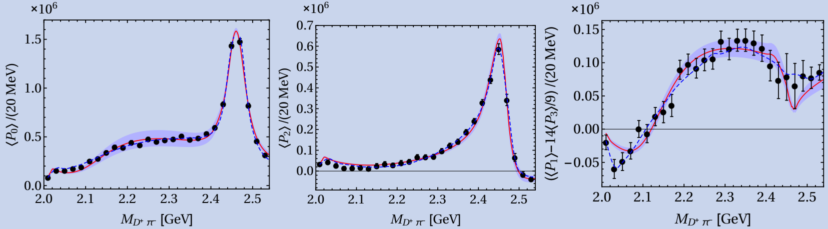
$$\langle P_0\rangle \propto |\mathcal{A}_0|^2 + |\mathcal{A}_1|^2 + |\mathcal{A}_2|^2\ , \qquad \langle P_2\rangle \propto \frac{2}{5}|\mathcal{A}_1|^2 + \frac{2}{7}|\mathcal{A}_2|^2 + \frac{2}{\sqrt{5}}|\mathcal{A}_0|\,|\mathcal{A}_2|\cos(\delta_0 - \delta_2)\ ,$$

$$\langle P_{13}\rangle \equiv \langle P_1\rangle - \frac{14}{9}\langle P_3\rangle \propto \frac{2}{\sqrt{3}}|\mathcal{A}_0|\,|\mathcal{A}_1|\cos(\delta_0 - \delta_1)\ .$$

Comparison with experimental data: $B^- \to D^+\pi^-\pi^-$

Du, MA, Fernández-Soler, Guo, Hanhart, Meißner, Nieves, Yao, PR,D98,094018('18)

Data: LHCb Collab., PR,D94,072001('16)

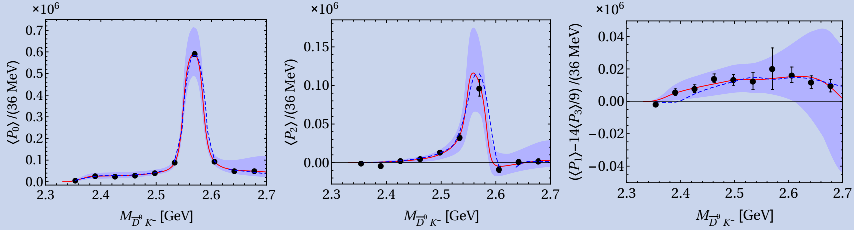


- Parameters: $B/A = -3.6 \pm 0.1$, $a_A = 1.0 \pm 0.1$, $\chi^2/\text{d.o.f.} = 1.7$
- — This work. - - - LHCb. Bands: fit uncertainty
- Good agreement with data & with LHCb fit
- Rapid movement in $\langle P_{13} \rangle$ [no $D_2(2460)$] between 2.4 and 2.5 GeV. Related to $D\eta$ and $D_s\bar{K}$ openings.
- Recall: these are the amplitudes with two states in the $D_0^*(2300)$ region, and no fit of the T-matrix parameters is done.

Comparison with experimental data: $B_s^0 \to \bar{D}^0 K^- \pi^+$

Du, MA, Fernández-Soler, Guo, Hanhart, Meißner, Nieves, Yao, PR,D98,094018('18)

Data: LHCb Collab., PR,D90,072003('14)



- Exactly the same formalism applied to $B_s^0 \to \bar{D}^0 K^- \pi^+$.

$$\mathcal{A}_0(s) = E_K \left[C + \frac{C+A}{2} G_1(s) T_{11}^0(s) + \frac{C-A}{2} G_1(s) T_{11}^1(s) \right] - \frac{1}{\sqrt{3}} \left(\frac{3}{2} B - C \right) E_\eta G_2(s) T_{21}^0(s)$$

- Take same value of B/A, a_A, as before: $C/A = 4.8^{+3.4}_{-1.7}$, $\chi^2/\text{d.o.f.} = 1.6$
- — This work. - - - LHCb. Bands: fit uncertainty
- Good agreement with data & with LHCb fit
- See also Du *et al.*, PRL,126,192001('21)

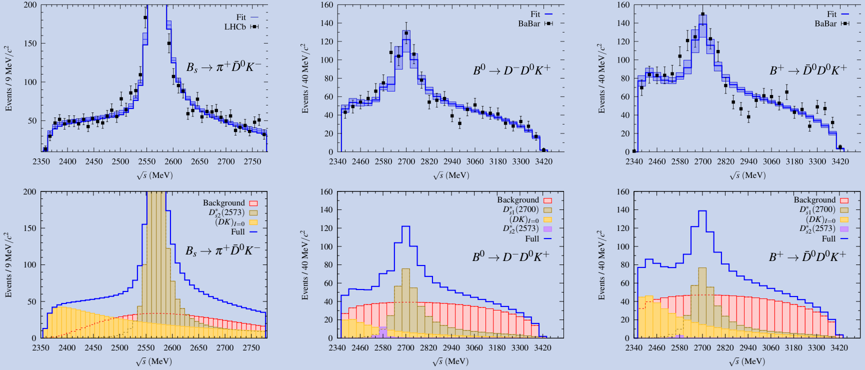
Old analysis of LHCb data

MA, Nieves, Oset, Jido, EPJ,C76,300('16)

Data: LHCb Collab., PR,D90,072003('14)

BaBar Collab., PR,D91,052002('15)

- The LHCb (and BaBar) data had already been analyzed with a similar amplitude



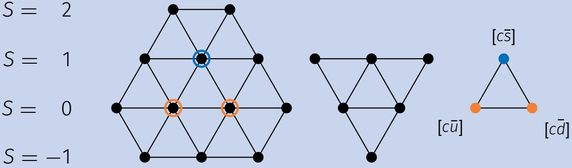
$SU(3)$ light–flavor limit

[M. A. *et al.*, Phys. Lett. B **767**, 465 (2017)]

- $SU(3)$ flavor limit: $m_i \to m = 0.49$ GeV, $M_i \to M = 1.95$ GeV.
- Irrep decomposition: $\bar{3} \otimes 8 = \boxed{\overline{15} \oplus 6 \oplus \bar{3}}$. T and V can be diagonalized:

$$V_d(s) = D^\dagger V(s) D = \text{diag}\left(V_{\overline{15}}(s)\,, V_6(s)\,, V_{\bar{3}}(s)\right) = \boxed{A(s)\,\text{diag}\,(1, -1, -3)}\,,$$

- $\overline{15}$ is repulsive. 6 and $\bar{3}$ are attractive. “Curiously”, $\bar{3}$ admits a $c\bar{q}$ interpretation.



In the $SU(3)$ limit:

- low D_0^* and $D_{s0}^*(2317)$ connect with a bound state in $\bar{3}$
- high D_0^* connects with a virtual state in 6
- See also [Gregory *et al.*, 2106.15391]

State	Channels	(S, I)	$\overline{15}$	6	$\bar{3}$
D_0^*	$D\pi$, $D\eta$, $D_s\bar{K}$	$(0, \frac{1}{2})$	✓	✓	✓
$D_{s0}^*(2317)$	DK, $D_s\eta$	$(1, 0)$	✓	✗	✓

- A recent LQCD calculation by the HadSpec Collaboration finds a similar picture.

[HadSpec Collab., JHEP 02 (2021) 100; JHEP 07 (2021) 123]

Predictions for other sectors: charm

(S,I)	Channels	$\overline{15}$	6	$\bar{3}$	0^+ M	0^+ $\Gamma/2$	1^+ M	1^+ $\Gamma/2$
$(0,\frac{1}{2})$	$D^{(*)}\pi$, $D^{(*)}\eta$, $D_s^{(*)}\bar{K}$	✓	✓	✓	(R) 2105^{+6}_{-8} (R) 2451^{+36}_{-26}	102^{+10}_{-12} 134^{+7}_{-8}	(R) 2240^{+5}_{-6}	93^{+9}_{-9}
$(1,0)$	$D^{(*)}K$, $D_s^{(*)}\eta$	✓	✗	✓	(B) 2315^{+18}_{-28}		(B) 2436^{+16}_{-22}	
$(-1,0)$	$D^{(*)}\bar{K}$	✗	✓	✗	(V) 2342^{+13}_{-41}		–	
$(1,1)$	$D_s^{(*)}\pi$, $D^{(*)}K$	✓	✓	✗	–		–	

- HQSS relates 0^+ ($D_{(s)}P$) and 1^+ ($D^*_{(s)}P$) sectors: similar resonance pattern.
- Two pole structure: higher D_1 pole probably affected by ρ channels.
- $D\bar{K}$ [0^+, $(-1,0)$]: this virtual state (from **6**) has a large impact on the scattering length, $a^{D\bar{K}}_{(-1,0)} \simeq 0.8$ fm. (Rest of scattering lengths are $|a| \simeq 0.1$ fm.)

Predictions for other sectors: bottom

(S,I)	Channels	$\overline{15}$	6	$\bar{3}$	0^+ M	0^+ $\Gamma/2$	1^+ M	1^+ $\Gamma/2$
$(0,\frac{1}{2})$	$\bar{B}^{(*)}\pi$, $\bar{B}^{(*)}\eta$, $\bar{B}_s^{(*)}\bar{K}$	✓	✓	✓	(R) 5537^{+9}_{-11} (R) 5840^{+12}_{-13}	116^{+14}_{-15} 25^{+6}_{-5}	(R) 5581^{+9}_{-11}	115^{+13}_{-15}
$(1,0)$	$\bar{B}^{(*)}K$, $\bar{B}_s^{(*)}\eta$	✓	✗	✓	(B) 5724^{+17}_{-24}		(B) 5768^{+17}_{-23}	
$(-1,0)$	$\bar{B}^{(*)}\bar{K}$	✗	✓	✗	(V–B) thr.		(V–B) thr.	
$(1,1)$	$\bar{B}_s^{(*)}\pi$, $\bar{B}^{(*)}K$	✓	✓	✗	–		–	

- Heavy flavour symmetry relates charm (D) and bottom ($\bar{B}$) sectors.
- $(0,\frac{1}{2})$: B_0^*, two-pole pattern also observed.
- $(-1,0)$: $[B^{(*)}\bar{K}]$: very close to threshold. Relevant prediction. Can be either bound or virtual (6) within our errors.
- $(1,1)$: $[\bar{B}_s\pi, \bar{B}K, 0^+]$, $X(5568)$ channel. No state is found: $\overline{15}$ and 6. If it exists, it is not generated with these $B_s\pi$, $B\bar{K}$ interactions.

  M. A. *et al.*, Phys. Lett. B **757**, 515 (2016); Guo *et al.*, Commun. Theor. Phys. **65**, 593 (2016)
- $(1,0)$: Our results for B_{s0}^* and B_{s1} agree with other results from LQCD:

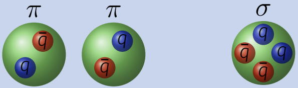
  Lang *et al.*, Phys. Lett. B **750**, 17 (2015); M. A. *et al.* Eur. Phys. J. C**77**, 170 (2017)
- Comparison of 0^+, 1^+ beauty states by Colangelo *et al.*, Phys. Rev. D **86** 054024 (2012): agreement in $(1,0)$ $[b\bar{s}]$, but not in $(0,1/2)$ $[b\bar{q}]$.

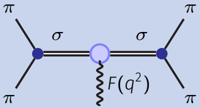
Open questions for the community

- Need of more collaboration between (and simultaneous use of!) different "subcommunities": LQCD, molecular/tetraquarks/QM models...
- Spectroscopy, mixing:
  Specific example of $D_{s0}^*(2317)$, take for granted the presence of a CQM $c\bar{s}$ state. Theoretical possibilities:
  - Genuine $c\bar{s}$, (very) renormalized by DK threshold. Or renormalized by DK interactions themselves?
  - Or, there is a $S=1$, $I=0$ state coming from DK interactions in addition to the $c\bar{s}$ state. If so, where are those two poles? Which is which?

[Cincioglu *et al.*, EPJ,C76,576('16); MA *et al.*, EPJ,C78,722('19)]

- Nature/size:
  - Can we address the question of $4q$, $q\bar{q}$, molecule based on the size of the object?





  - For $\pi\pi$ scattering, σ meson: MA, Oller, PR,D86,034003('12)
    - $\sqrt{\langle r^2\rangle_\sigma^S} \simeq 0.44$ fm vs $\sqrt{\langle r^2\rangle_\pi^S} \simeq 0.81$ fm
  - Perhaps only theoretical? Future lattice QCD calculations?

Briceño *et al.*, PR,D103,114512('21) [and refs. therein]

Connecting $SU(3)$ and physical limits Riemann sheets

Riemann sheets:

$$\mathcal{G}_{ii}(s) \to \mathcal{G}_{ii}(s) + i\,\frac{p_i(s)}{4\pi\sqrt{s}}\,\xi_i$$

$SU(3)$ limit:

$$m_i = m_i^{\rm phy} + x(m - m_i^{\rm phy})\,, \qquad (m = 0.49\ \text{GeV})\,,$$
$$M_i = M_i^{\rm phy} + x(M - M_i^{\rm phy})\,, \qquad (M = 1.95\ \text{GeV})\,.$$

- Physical case ($x = 0$): RS specified by $(\xi_1\xi_2\xi_3)$, $\xi_i = 0$ or 1.
- $SU(3)$ symmetric case ($x = 1$): all channels have the same threshold, so there are only two RS (000) and (111).
- To connect the lower pole with the T_6 virtual state,

$$\xi_3 = x \qquad (1,1,0) \to (1,1,x)$$

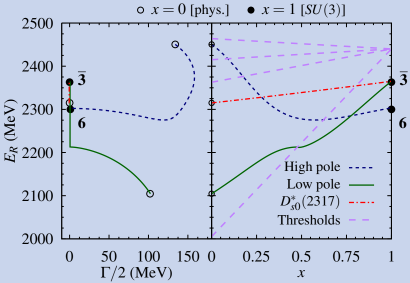
- To connect the lower pole with the $T_{\overline{3}}$ bound state,

$$\xi_1 = 1 - x \qquad (1,0,0) \to (1-x,0,0)$$

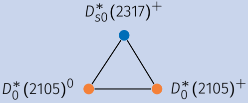
(II)

Connecting physical ($x = 0$) and flavor $SU(3)$ ($x = 1$) limits:

$$m_i = m_i^{\rm phy} + x(m - m_i^{\rm phy})\ , \quad (m = 0.49\ \text{GeV})\ ,$$
$$M_i = M_i^{\rm phy} + x(M - M_i^{\rm phy})\ , \quad (M = 1.95\ \text{GeV})\ .$$



- The high D_0^* connects with a **6** virtual state (unph. RS, below threshold).
- The low D_0^* connects with a $\bar{3}$ bound state (ph. RS, below threshold).
- The $D_{s0}^*(2317)$ also connects with the $\bar{3}$ bound state.



- The low D_0^* and the $D_{s0}^*(2317)$ are $SU(3)$ flavor partners.
- This solves the "puzzle" of $D_{s0}^*(2317)$ being lighter than $D_0^*(2300)$: it is not, the lower D_0^* pole ($M = 2105$ MeV) is lighter.

Form factors in semileptonic $D \to \pi \bar{\ell} \nu_\ell$

D.-L. Yao, P. Fernández-Soler, MA, F.-K. Guo, J. Nieves, Eur. Phys. J. C **78**, 310 (2018)

- General definitions:

$$\frac{\mathrm{d}\Gamma(D \to \pi \bar{\ell} \nu_\ell)}{\mathrm{d}q^2} = \frac{G_F^2}{24\pi^3} |\vec{p}_\pi|^3 |V_{cd}|^2 |f_+(q^2)| \, . \qquad [q^2 = 0 : f_+(0) = f_0(0)]$$

$$\langle \pi(p') | \bar{q} \gamma^\mu Q | D(p) \rangle = f_+(q^2) \left[\Sigma^\mu - \frac{m_D^2 - m_\pi^2}{q^2} q^\mu \right] + f_0(q^2) \frac{m_D^2 - m_\pi^2}{q^2} q^\mu \, ,$$

- "Isospin" form factors, related to $D\pi$, $D\eta$, $D_s\bar{K}$ scattering:

$$\mathcal{F}^{(0,1/2)}(s) \equiv \begin{pmatrix} -\sqrt{\frac{3}{2}} f_0^{D^0 \to \pi^-}(s) \\ -f_0^{D^+ \to \eta}(s) \\ -f_0^{D_s^+ \to K^0}(s) \end{pmatrix} \, , \qquad \mathrm{Im}\mathcal{F}(s) = T^*(s) \Sigma(s) \mathcal{F}(s)$$

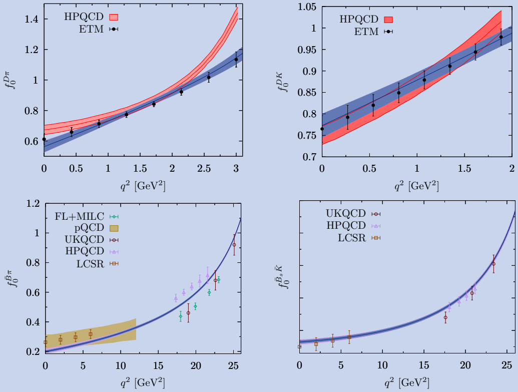
- Write form factors as Omnés matrix times polynomials

$$\mathcal{F}(s) = \Omega(s) \cdot \mathcal{P}(s)$$

- Polynomials fixed so as to reproduce the NLO chiral lagrangian:

$$\mathcal{L}_0 = f_{\mathcal{P}} \left(\mathring{m} \mathcal{P}_\mu^* - \partial_\mu \mathcal{P} \right) u^\dagger J^\mu \, ,$$

$$\mathcal{L}_0 = \beta_1 \mathcal{P}\, u \,(\partial_\mu U^\dagger) J^\mu + \beta_2 (\partial_\mu \partial_\nu \mathcal{P})\, u \,(\partial^\nu U^\dagger)\, J^\mu \, .$$



- Points mostly from LQCD
- Also LCSR for $q^2 \to 0$
- Good agreement in general
- CKM matrix can also be calculated
- Definitive results may differ...

	This work	Exp.
$10^3 \lvert V_{ub} \rvert$	4.3(7)	4.49(24) [Incl.] 3.72(19) [Excl.]
$\lvert V_{cd} \rvert$	0.244(22)	0.220(5)
$\lvert V_{cs} \rvert$	0.945(41)	0.995(16)

Why is $D_0^*(2300)$ interesting?

- Lightest systems to test ChPT with heavy mesons, besides $D^* \to D\pi$.
- $D\pi$ interactions (where it shows up) are relevant, since $D\pi$ appears as a final state in many reactions that are being considered now (*i.e.*, $Z_c(3900)$ and $\bar{D}^* D\pi$)
- $D_0^*(2300)$ is important in weak interactions and CKM parameters:

Flynn, Nieves, Phys. Rev. D **76**, 031302 (2007)

D.-L. Yao, P. Fernández-Soler, MA, F.-K. Guo, J. Nieves, Eur. Phys. J. C **78**, 310 (2018)

  - It determines the shape of the scalar form factor $f_0(q^2)$ in semileptonic $D \to \pi$ decays.
  - Relation to $|V_{cd}|$: $f_+(0) = f_0(0)$ and $\mathrm{d}\Gamma \propto |V_{cd} f_+(q^2)|^2$.
  - Even more interesting: the bottom analogue $|V_{ub}|$.

$D\pi$, $D\eta$, $D_s\overline{K}$ energy levels in a finite volume

- Periodic boundary conditions imposes momentum quantization
- Lüscher formalism:

Commun. Math. Phys. **105**, 153 (1986)

Nucl. Phys. B **354**, 531 (1991)

infinite volume	finite volume
$\vec{q} \in \mathbb{R}^3$	$\vec{q} = \frac{2\pi}{L}\vec{n}, \quad \vec{n} \in \mathbb{Z}^3$
$\int_{\mathbb{R}^3} \frac{\mathrm{d}^3 q}{(2\pi)^3}$	$\frac{1}{L^3} \sum_{\vec{n}\in\mathbb{Z}^3}$

- In practice, changes in the T-matrix: $T(s) \to \widetilde{T}(s,L)$:

Döring *et al.*, Eur. Phys. J. **A** 47, 139 (2011)

$$\mathcal{G}_{ii}(s) \to \widetilde{\mathcal{G}}_{ii}(s,L) = \mathcal{G}_{ii}(s) + \lim_{\Lambda\to\infty} \left(\frac{1}{L^3} \sum_{\vec{n}}^{|\vec{q}\,|<\Lambda} I_i(\vec{q}\,) - \int_0^\Lambda \frac{q^2 \mathrm{d}q}{2\pi^2} \, I_i(\vec{q}\,) \right) ,$$

$$V(s) \to \tilde{V}(s,L) = V(s) ,$$

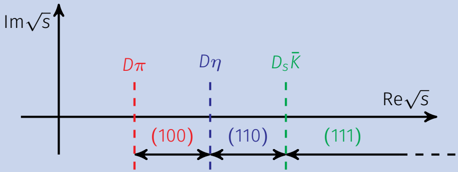
$$T^{-1}(s) \to \widetilde{T}^{-1}(s,L) = V^{-1}(s) - \widetilde{\mathcal{G}}(s,L) ,$$

- Free energy levels: $E^{(i)}_{n,\text{free}}(L) = \omega_{i1}((2\pi n/L)^2) + \omega_{i2}((2\pi n/L)^2)$
- Interacting energy levels $E_n(L)$: $\widetilde{T}^{-1}(E_n^2(L),L) = 0$ (poles of the $\widetilde{T}$-matrix).

T-matrix and analytical continuations

- Normalization: $-ip_{ii}(s)T_{ii}(s) = 4\pi\sqrt{s}\left(\eta_i(s)e^{2i\delta_i(s)} - 1\right)$.
- $\mathcal{G}_{ii}(s) = G(s, m_i, M_i)$, regularized with a subtraction constant $a(\mu)$ ($\mu = 1$ GeV).
- Riemann sheets (RS) denoted as $(\xi_1\xi_2\xi_3)$:

$$\mathcal{G}_{ii}(s) \to \mathcal{G}_{ii}(s) + i\,\frac{p_i(s)}{4\pi\sqrt{s}}\,\xi_i$$



Chiral dynamics and two-state structure(s)

- Other famous two-poles structures rooted in chiral dynamics:

$\Lambda(1405)$ $[\Sigma\pi, N\bar{K}]$

Oller, Meißner, Phys. Lett. B **500**, 263 (2001)
Jido *et al.*, Nucl. Phys. A **725**, 181 (2003)
García-Recio *et al.*, Phys. Lett. B **582**, 49 (2004)
Magas *et al.*, Phys. Rev. Lett. **95**, 052301 (2005)

$K_1(1270)$

Roca *et al.*, Phys. Rev. D **72**, 014002 (2005)
Geng *et al.*, Phys. Rev. D **75**, 014017 (2007)
García-Recio *et al.*, Phys. Rev. D **83**, 016007 (2011)

- Recently, Clymton, Kim, 2305.14812 claim a two-pole structure for $b_1(1235)$.

- Chiral dynamics:
  - Incorporates the $SU(3)$ light-flavor structure,
  - Determines the strength of the interaction,
  - Ensures lightness of Goldstone bosons, which in turn separates generating channels from higher hadronic channels.