Mechanism of production of exotic $X(3872)$ in proton-proton and e^+e^- collisions and its structure

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Contents

Introduction

- $X(3872)$ was discovered by the Belle collaboration.
- We do not know what is the structure of this state.
- We know at present it is $J^\pi = 1^+$, $C = +1$ state, similarly as the well known $\chi_c(1^+)$ quarkonium.
- Different scenarios were proposed:
  (a) $c\bar{c}$ state
  (b) DD^* molecular state
  (c) tetraquark $c\bar{c}q\bar{q}$
  (d) hybrid state
- Can the production of $X(3872)$ in proton-proton and/or e^+e^- collisions provide new information ?

Introduction

- k_t-factorization gives good description of inclusive D-meson distributions (Maciula-Szczurek).
  Higher orders are needed in collinear approach.
- k_t-factorization gives good description of $D^0 - \bar{D}^0$ correlation observables (Maciula-Szczurek).
- k_t-factorization gives good description of $\eta_c(1S)$ production (Babiarz-Schäfer-Szczurek).
- Here we shall consider production of $X(3872)$ taking into account three different approaches for its structure, within the k_t-factorization approach.
- We shall use modern unintegrated gluon distributions.
- A. Cisek, W. Schäfer and A. Szczurek, arXiv:2203.07827, Eur.Phys.J.**C882**, 2022) 1062. "Structure and production mechanism of the enigmatic X(3872) in high-energy hadronic reactions".
- I. Babiarz, R. Pasechnik, W. Schäfer and A. Szczurek, "Probing the structure of $\chi_c(3872)$ with photon transition form factors", Phys. Rev. **D107** (2023) L071503.
- I. Babiarz, R. Pasechnik, W. Schäfer and A. Szczurek. "Light-front approach to axial-vector quarkonium $\gamma^*\gamma^*$, form factors", JHEP ((2022) 054018.

Formalism, structure

- $c\bar{c}$ state, R'(0) extracted from the Eichten-Quigg study (Schrödinger equation)
- DD^* molecule

$$|X(3782)\rangle = \frac{1}{\sqrt{2}}\left(|D\bar{D}^*\rangle + |\bar{D}D^*\rangle\right) \ . \qquad (1)$$

- hybrid approach (Coito, Rupp, van Beveren 2013)

$$|X(3782)\rangle = \alpha|c\bar{c}\rangle + \frac{\beta}{\sqrt{2}}\left(|D\bar{D}^*\rangle + |\bar{D}D^*\rangle\right) \ . \qquad (2)$$

Quarkonium spectra

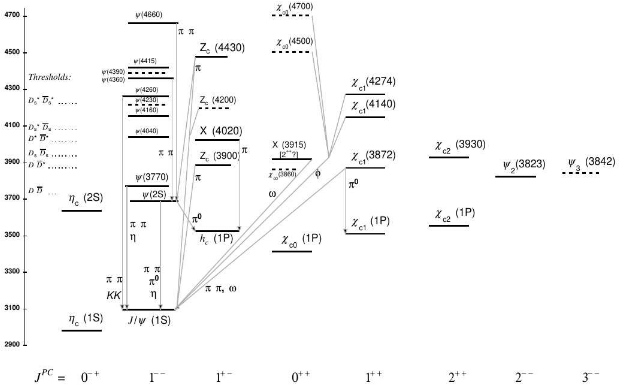


Figure: Mass spectrum of charmonia.

Formalism, production mechanism

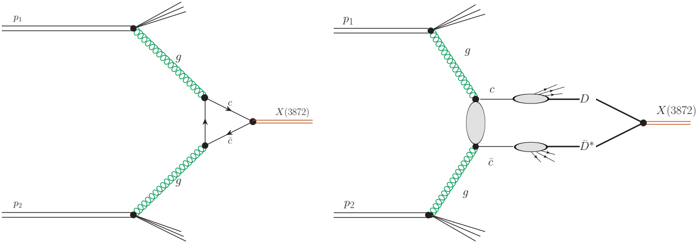


Figure: Generic diagrams for the inclusive process of $X(3872)$ production in proton-proton scattering via two gluons fusion.

adequate for the k_T-factorization calculations

Formalism, production of $c\bar{c}$ state

The inclusive cross section for $X(3872)$-production via the $2 \to 1$ gluon-gluon fusion mode is obtained from

$$\frac{d\sigma}{dyd^2\boldsymbol{p}} = \int \frac{d^2\boldsymbol{q}_1}{\pi \boldsymbol{q}_1^2} \mathcal{F}(x_1, \boldsymbol{q}_1^2, \mu_F^2) \int \frac{d^2\boldsymbol{q}_2}{\pi \boldsymbol{q}_2^2} \mathcal{F}(x_2, \boldsymbol{q}_2^2, \mu_F^2) \delta^{(2)}(\boldsymbol{q}_1 + \boldsymbol{q}_2 - \boldsymbol{p}) \times \frac{\pi}{(x_1 x_2 s)^2} \overline{|\mathcal{M}_{g^*g^* \to X(3872)}|^2} \quad (3)$$

Formalism, production of $c\bar{c}$ state

Here the matrix element squared for the fusion of two off-shell gluons into the $^3P_1$ color singlet $c\bar{c}$ charmonium is (see e.g. Kniehl-Saleev-Vasin for a derivation):

$$\overline{|\mathcal{M}_{g^* g^* \to X(3872)}|^2} = (4\pi\alpha_S)^2 \frac{4|R'(0)|^2}{\pi M_X^3} \frac{\boldsymbol{q}_1^2 \boldsymbol{q}_2^2}{(M_X^2 + \boldsymbol{q}_1^2 + \boldsymbol{q}_2^2)^4}$$

$$\times \quad \Big((\boldsymbol{q}_1^2 + \boldsymbol{q}_2^2)^2 \sin^2\phi + M_X^2 (\boldsymbol{q}_1^2 + \boldsymbol{q}_2^2 - 2|\boldsymbol{q}_1||\boldsymbol{q}_2| \cos\phi) \Big) , \quad (4)$$

where ϕ is the azimuthal angle between $\boldsymbol{q}_1, \boldsymbol{q}_2$.

Formalism, production of $c\bar{c}$ state

The momentum fractions of gluons are fixed as

$$x_{1,2} = m_T \exp(\pm y)/\sqrt{s}, \tag{5}$$

where $m_T^2 = \boldsymbol{p}^2 + M_X^2$.
The derivative of the radial quarkonium wave function at the origin is taken for the first radial p-wave excitation from Eichten-Quigg 2019, $|R'(0)|^2 = 0.1767\,\mathrm{GeV}^5$.
The unintegrated gluon parton distribution functions (gluon uPDFs) are normalized such, that the collinear glue is obtained from

$$xg(x, \mu_F^2) = \int^{\mu_F^2} \frac{d^2\boldsymbol{k}}{\pi \boldsymbol{k}^2} \mathcal{F}(x, \boldsymbol{k}^2, \mu_F^2)\,. \tag{6}$$

The hard scale is taken to be always $\mu_F = m_T$, the transverse mass of the $X(3872)$.

Formalism, production of the molecule

The parton-level differential cross section for the $c\bar{c}$ production, formally at leading-order, reads:

$$\frac{d\sigma(pp \to Q\bar{Q}\,X)}{dy_1 dy_2 d^2\boldsymbol{p}_1 d^2\boldsymbol{p}_2} = \int \frac{d^2\boldsymbol{k}_1}{\pi \boldsymbol{k}_1^2} \mathcal{F}(x_1, \boldsymbol{k}_1^2, \mu_F^2) \int \frac{d^2\boldsymbol{k}_2}{\pi \boldsymbol{k}_2^2} \mathcal{F}(x_2, \boldsymbol{k}_2^2, \mu_F^2)$$
$$\times \delta^{(2)}\left(\boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{p}_1 - \boldsymbol{p}_2\right) \frac{1}{16\pi^2 (x_1 x_2 s)^2} \overline{|\mathcal{M}^{\mathrm{off-shell}}_{g^* g^* \to c\bar{c}}|^2}\,. \qquad (7)$$

where $\mathcal{M}^{\mathrm{off-shell}}_{g^* g^* \to Q\bar{Q}}$ is the off-shell matrix element for the hard subprocess (Catani et al.), we use its implementation from (Maciula-Szczurek)

Formalism, production of the molecule

Here, one keeps exact kinematics from the very beginning and additional hard dynamics coming from transverse momenta of incident partons. Explicit treatment of the transverse momenta makes the approach very efficient in studies of correlation observables. The two-dimensional Dirac delta function assures momentum conservation. The gluon uPDFs are evaluated at longitudinal momentum fractions:

$$x_1 = \frac{m_{T1}}{\sqrt{s}} \exp(+y_1) + \frac{m_{T2}}{\sqrt{s}} \exp(+y_2) \,, \quad (8)$$

$$x_2 = \frac{m_{T1}}{\sqrt{s}} \exp(-y_1) + \frac{m_{T2}}{\sqrt{s}} \exp(-y_2), \quad (9)$$

where $m_{Ti} = \sqrt{p_{Ti}^2 + m_c^2}$ is the quark/antiquark transverse mass.

Formalism, production of the molecule

In the present analysis we employ the heavy c-quark approximation and assume that three-momenta in the pp-cm frame are equal:

$$\vec{p}_D = \vec{p}_c \; . \tag{10}$$

This approximation could be relaxed in future.

We calculate $k_{rel} = \frac{1}{2}\sqrt{M_{c\bar{c}}^2 - 4m_c^2}$.

In the following for illustration we shall therefore assume $k_{max} = 0.2\,\mathrm{GeV}$.

k_{max}^{DD} is then smaller.

The calculation for the SPS molecular scenario is done using the VEGAS algorithm for Monte-Carlo integration.

Production of molecules, probabilities

We now should take into account the fragmentation into D, D^*-mesons. The fragmentation fractions fulfill the sum rule:

$$\sum_i f(c \to H_i) = 1\,. \tag{11}$$

In this formula H_i are the final (after strong decays) hadrons. Therefore the spin-1 D^* mesons should not be included here as it would lead to double counting. The final charmed particles are only those which have only weak decays: $D^+, D^0, D_s^+, \Lambda_c$, etc. The D^0 (or $\bar{D}^0$) are produced directly or come from the decays of spin-1 mesons (see PDG):

$$\mathrm{Br}(D^{*0} \to D^0) \;=\; 1\,, \qquad \mathrm{Br}(D^{*+} \to D^0) = 0.68\,. \tag{12}$$

$$f(c \to D^0) = 0.54 - 0.63\,. \tag{13}$$

The total uncertainties is less than $\sim 10\%$.
The total probability can be decomposed as the sum:

$$f(c \to D^0) = f(c \to D^0)|_{\mathrm{direct}} + f(c \to D^0)|_{\mathrm{feeddown}}\,. \tag{14}$$

Production of molecules, probabilities

The direct component can be approximated as:

$$f(c \to D^0)|_{\mathrm{direct}} \approx f(c \to D^{\pm})|_{\mathrm{direct}} \,, \tag{15}$$

assuming isospin symmetry.
Let us calculate therefore the feeddown probability:

$$\begin{aligned} f(c \to D^0)|_{\mathrm{feeddown}} &= f(c \to D^{*0})\mathrm{Br}(D^{*0} \to D^0) + f(c \to D^{*+})\mathrm{Br}(D^{*+} \to D^0)\,, \\ f(c \to D^+)|_{\mathrm{feeddown}} &= f(c \to D^{*+})\mathrm{Br}(D^{*+} \to D^+)\,. \end{aligned} \tag{16}$$

Then the direct contributions can be calculated from

$$f(c \to D^0)|_{\mathrm{direct}} = f(c \to D^0) - f(c \to D^0)|_{\mathrm{feeddown}} \,, \tag{17}$$

$$f(c \to D^+)|_{\mathrm{direct}} = f(c \to D^+) - f(c \to D^+)|_{\mathrm{feeddown}} \,. \tag{18}$$

Production of molecules, probabilities

$$f(c \to D^0) = f(\bar{c} \to \bar{D}^0) = 0.547 \; , \quad (19)$$
$$f(c \to D^+) = f(\bar{c} \to D^-) = 0.227 \; , \quad (20)$$
$$f(c \to D^{*0}) = f(\bar{c} \to \bar{D}^{*0}) = 0.237 \; , \quad (21)$$
$$f(c \to D^{*+}) = f(\bar{c} \to \bar{D}^{*-}) = 0.237 \; . \quad (22)$$

Here we have assumed that $f(c \to D^{*0}) = f(c \to D^{*+})$.
We then obtain an isospin symmetric result:

$$f(c \to D^0)|_{\mathrm{direct}} = 0.15 \, , \qquad f(c \to D^+)|_{\mathrm{direct}} = 0.15 \, . \quad (23)$$

We summarize that the direct contribution is much smaller than the total one including feeddown.
In the following we shall show results obtained from the total fragmentation fraction as well as using only the direct production fraction for D^0. The arguments are related to lifetime of D^* mesons. This will be discussed below.
The cross section for $c\bar{c}$ production are then multiplied by

$$\frac{1}{2}[f(c \to D^0) f(\bar{c} \to \bar{D}^{*0}) + f(c \to D^{*0}) f(\bar{c} \to \bar{D}^0)] = \begin{cases} 0.036 \text{ direct} \\ 0.13 \text{ including feeddown.} \end{cases} \quad (24)$$

Lifetime of D^* mesons and formation of the molecule

The lifetime of D^* was estimated by theoretical calculation (not measured!) to be $c\tau \approx 2000$ fm (Henriksson et al.).

After such a long time, in general, D^0 from the decay of D^* may be far away from the associated D^{*0} – so formation of the molecule may be difficult.

However, the condition on relative momentum of D and D^* mesons selects mesons flying almost parallel to each other.

In such a system, in a somewhat naive calculation (non-interacting mesons), the probability that together with a D^0 there exists a D^{*0} that has not decayed yet to produce a $X(3872)$ at a time t can be estimated as:

$$P = (1 - \exp(-t/\tau)) \exp(-t/\tau) < 0.25. \tag{25}$$

This suggests that in reality one should rather include only directly produced D^0 (or $\bar{D}^0$). This strongly reduces the cross section and causes that the purely molecular scenario is disfavoured – the corresponding $d\sigma/dp_T$ (see results) is below the experimental data.

Production of molecule via double parton scattering

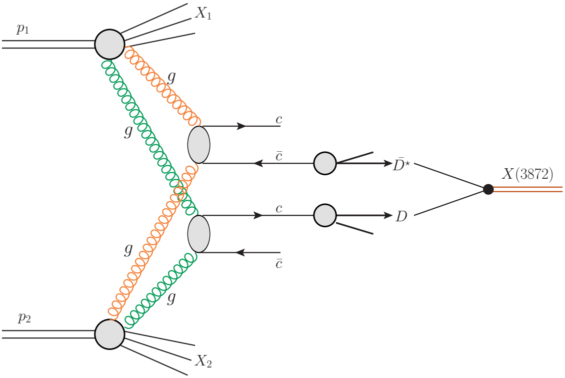


Figure: A generic diagram for the inclusive process of $X(3872)$ production in proton-proton scattering via the double parton scattering mode.

Production via double parton scattering

The corresponding cross section is calculated in the so-called factorized ansatz as:

$$\Delta\sigma = \frac{1}{2\sigma_{\rm eff}} \int \frac{d\sigma_{c\bar{c}}}{dy_1 d^2\boldsymbol{p}_1} \frac{d\sigma_{c\bar{c}}}{dy_2 d^2\boldsymbol{p}_2} \; dy_1 d^2\boldsymbol{p}_1 dy_2 d^2\boldsymbol{p}_2 \Big|_{k_{rel}<k_{max}} . \tag{26}$$

Above the differential distributions of the first and second parton scattering $\frac{d\sigma}{dy_i d^2\boldsymbol{p}_i}$ are calculated in the k_T-factorization approach as explained above. In the following we take $\sigma_{\rm eff} =$ 15 mb (Maciula,Szczurek).

The differential distributions (in p_T of the $X(3872)$ or $y_{\rm diff} = y_1 - y_2$, etc.) are obtained by binning in the appropriate variable.

Production via double parton scattering

We include all possible fusion combinations leading to $X(3872)$:

$$c_1 \longrightarrow D^0, \bar{c}_2 \longrightarrow \bar{D}^{*0} \; , \tag{27}$$

$$c_1 \longrightarrow D^{*0}, \bar{c}_2 \longrightarrow \bar{D}^0 \; , \tag{28}$$

$$\bar{c}_1 \longrightarrow \bar{D}^0, c_2 \longrightarrow D^{*0} \; , \tag{29}$$

$$\bar{c}_1 \longrightarrow \bar{D}^{*0}, c_2 \longrightarrow D^0 \; . \tag{30}$$

This leads to the multiplication factor two times bigger than for the SPS contribution.

Unintegrated gluon distributions

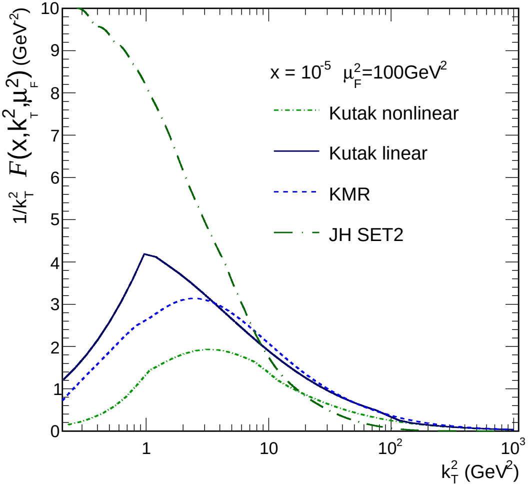
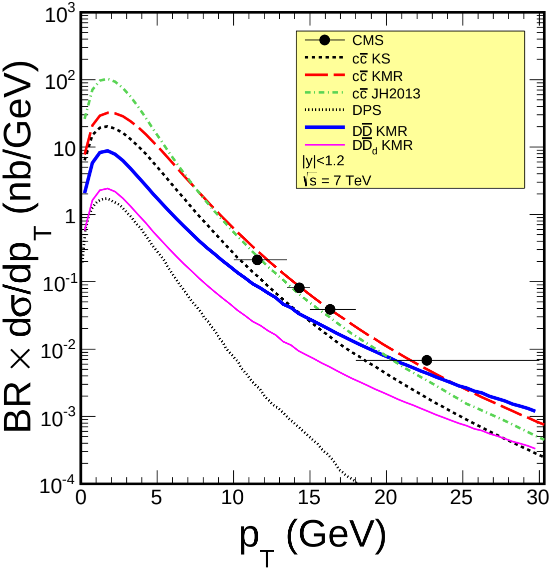
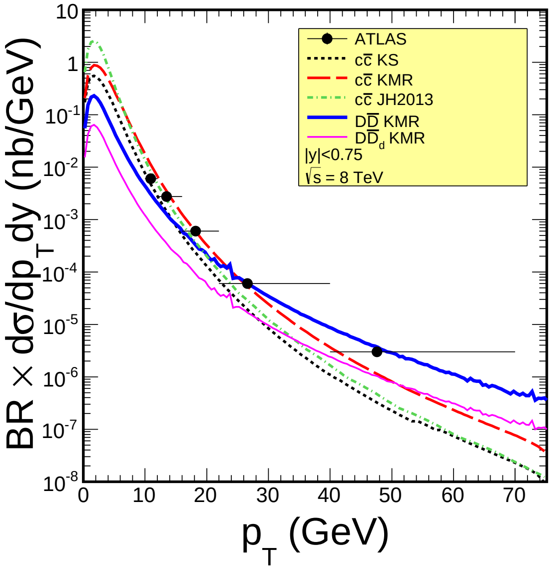


Figure: Some unintegrated gluon distributions as a function of k_t^2 for a given $x = 10^{-5}$ and factorization scale $\mu^2 = 100$ GeV^2.

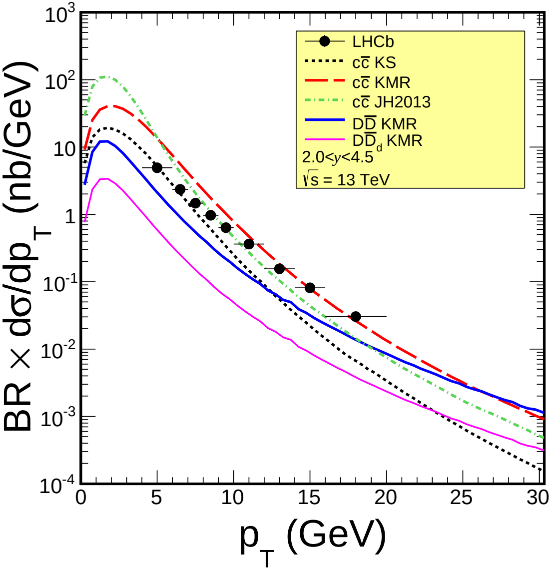
Results, X(3872) production as $c\bar{c}$ state



Results, X(3872) production as $c\bar{c}$ state



Results, X(3872) production as $c\bar{c}$ state



Results, molecular picture

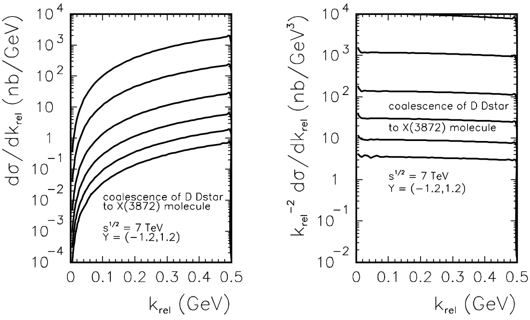


Figure: Distribution in k_{rel} for different windows of $p_{t,c\bar{c}} = p_{t,X}$ (left panel) for the CMS kinematics. In the right panel we show the cross sections divided by k_{rel}^2. In these calculations the KMR UGDF with the MMHT NLO collinear gluon distribution was used.

$p_{t,X}$ (GeV) = (0,5), (5,10), (10,15), (15,20), (20,25), (25,30)

Results, molecular picture

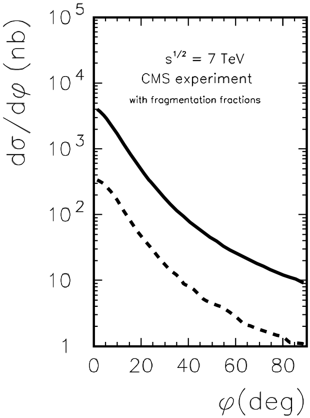


Figure: Azimuthal correlations between $c\bar{c}$ that fullfill the condition $k_{rel} < 0.2$ GeV. Here the CMS cuts were imposed. We show contribution of SPS (solid line) and DPS (dashed line). All D^0 included.

Results, hybrid model

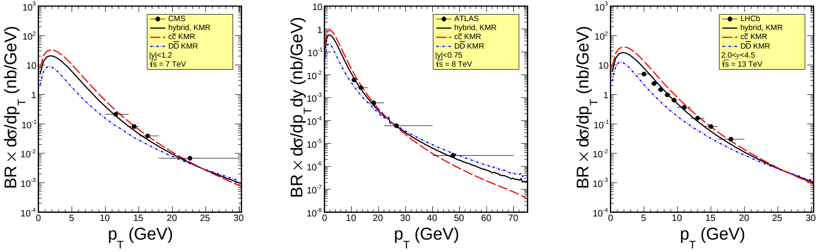


Figure: Transverse momentum distribution of $X(3872)$ for the CMS, ATLAS and LHCb experiments. Shown are results for the KMR UGDF. We show results for different combinations of α and β: (1,0), (0,1), (0.5,0.5).

No effect of D^* lifetime.

Lifetime arguments suggest, however, that rather small probabilities of the molecular component are preferred.

$e^+e^- \rightarrow e^+e^- X(3872)$

- The $X(3872)$ state was discovered by the Belle collaboration in 2003.
- A future Belle-2 could provide a detailed cross sections which could be used to verify the underlying picture.
- There are already some data from Belle-1 (2020) (bound on the reduced width).
- The hadronic reactions suggests large $c\bar{c}$ components of $X(3872)$. There the underlying mechanism is $gg \rightarrow X(3872)$.
- In e^+e^- reaction the underlying mechanism is: $\gamma^*\gamma^* \rightarrow X(3872)$.
  In general, both photons can be virtual. In a first stage one can study $\gamma^*\gamma \rightarrow X(3872)$ (single electron tag events).

$\gamma^*\gamma^* \to 1^{++}$ amplitudes, a reminder

The invariant amplitude can be written as (our third paper):

$$\frac{1}{4\pi\alpha_{em}}\mathcal{M}_{\mu\nu\rho} = i\left(q_1 - q_2 + \frac{Q_1^2 - Q_2^2}{(q_1+q_2)^2}\right)\tilde{G}_{\mu\nu}\frac{M}{2X}F_{TT}(Q_1^2, Q_2^2)$$
$$+ \; i\epsilon_\mu^L \tilde{G}_{\nu\rho}\frac{1}{\sqrt{X}}F_{LT}(Q_1^2, Q_2^2) + i\epsilon_\nu^L \tilde{G}_{\mu\rho}\frac{1}{\sqrt{X}}F_{TL}(Q_1^2, Q_2^2)$$

where

$$\tilde{G}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta \; .$$

The form factors fulfill symmetry relations:

$$F_{TT}(Q_1^2, Q_2^2) = -F_{TT}(Q_2^2, Q_1^2) \; , F_{LT}(Q_1^2, Q_2^2) = -F_{TL}(Q_2^2, Q_1^2) \; .$$

At $Q_1^2 = 0$ or $Q_2^2 = 0$ for $Q\bar{Q}$ state even extra relation:

$$F_{TT}(Q^2, 0) = -\frac{Q}{M}F_{LT}(Q^2, 0) \; .$$

Transition form factors, one virtual photon

Let us consider asymmetric situations $(Q_1^2, 0)$ or $(0, Q_2^2)$.

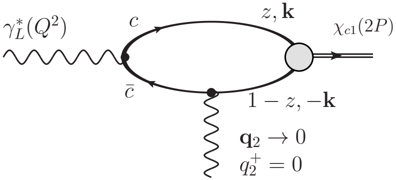


Figure: An illustration of the meson production mechanism in photon-photon fusion in the light-cone dipole picture, with relevant ingredients and kinematics.

Transition form factors, one virtual photon

Due to the Landau-Yang theorem, at least one off-shell photon is required for χ_{c1} production in the photon-photon fusion channel.

We utilize the Light-Front (LF) approach to transition form factors in the Drell-Yan frame (Brodsky 1997).

Here, the longitudinal photon with spacelike virtuality $Q_1^2 \equiv -q_1^2$ carries four-momentum $q_1 = (q_{1+}, q_{1-}, \mathbf{0}_\perp)$, with $q_{1-} = -Q_1^2/(2q_{1+})$ and polarization vector $\varepsilon_L = 1/Q_1\,(q_{1+}, -q_{1-}, \mathbf{0}_\perp)$, while the plus-momentum of the second photon vanishes $q_2^+ = 0$, such that $q_2 = (0, q_{2-}, \boldsymbol{q}_2)$.

In the real photon limit, $Q_2^2 \equiv -q_2^2 = \boldsymbol{q}_2^2 \to 0$, i.e. when its transverse momentum $\boldsymbol{q}_2 \to 0$, the transition amplitude vanishes linearly with $\boldsymbol{q}_2$ enabling us to extract the relevant form factor.

Transition form factor, one real photon

Indeed, in the considered frame, the LF plus component of the EM current is free from parton number changing and instantaneous fermion exchange contributions (Brodsky, 1998).

$$\langle \chi_{c1}(\lambda_A)|J_+(0)|\gamma^*_L(Q^2)\rangle = 2q_{1+}\sqrt{N_c}\int \frac{dz d^2\boldsymbol{k}}{z(1-z)16\pi^3} \times \sum_{\lambda,\bar\lambda} \Psi^{(\lambda_A)*}_{\lambda\bar\lambda}(z,\boldsymbol{k})\,(\boldsymbol{q}_2\cdot\nabla_{\boldsymbol{k}})\Psi^{\gamma_L}_{\lambda\bar\lambda}(z,\boldsymbol{k},Q^2)\,. \quad (32)$$

Transition form factor, one real photon

Above, the summation over the (anti)quark color indices has been performed, $N_c = 3$ is the number of colors in QCD, and we introduced the LF helicity $\lambda_A = \pm 1, 0$ of the axial meson χ_{c1}, as well as $c\bar{c} \to \chi_{c1}$ and $\gamma_L^* \to c\bar{c}$ LFWFs, $\Psi_{\lambda\bar{\lambda}}^{(\lambda_A)*}$ and $\Psi_{\lambda\bar{\lambda}}^{\gamma_L}$, respectively. The integration is over the internal LF momenta of quark (c) and antiquark ($\bar{c}$), namely, the LF momentum fraction $z = k_{c+}/q_{1+}$ of the c-quark and its transverse momentum $\boldsymbol{k}$ as illustrated in the figure. The (anti)quark coupling to the external field conserves the LF helicities $\lambda, \bar{\lambda}$ of quark and antiquark, whose $\pm 1/2$ values are denoted by $\uparrow$ and $\downarrow$, respectively.

Transition form factor, one real photon

Furthermore, it is instructive to utilize the general covariant parametrization of the $\gamma^*\gamma^* \to \chi_{c1}$ amplitude (Babiarz et al.), which is similar to the one found in Poppe and is based on $\gamma^*\gamma^*$ c.m. frame helicity amplitudes. We notice that only one term contributes to the transition amplitude of interest in the limit $Q_2^2 \to 0$,

$$\varepsilon_L^{\mu} n^{-\nu} \mathcal{M}_{\mu\nu\rho} E^{*\rho} \quad \to \quad 4\pi\alpha_{\rm em}\, \tilde{G}_{\nu\rho} n_-^{\nu} E^{*\rho} \frac{F_{\rm LT}(Q^2, 0)}{q_1 \cdot q_2}\,, \quad (33)$$

where $\alpha_{\rm em} = e^2/4\pi$ is the fine structure constant, $E = E(\lambda_A)$ is the polarization vector of the axial meson, $n_- = (0, 1, \mathbf{0}_\perp)$, and $\tilde{G}_{\nu\rho} \equiv \varepsilon_{\nu\rho\alpha\beta} q_1^{\alpha} q_2^{\beta}$.

Transition form factor, one real photon

We choose the LF spin projection $\lambda_A = +1$, and obtain:

$$\langle \chi_{c1}(+1) | J_+(0) | \gamma_L^*(Q^2) \rangle = 2q_{1+} \frac{q_{2x} - iq_{2y}}{\sqrt{2}} \times \frac{\sqrt{4\pi\alpha_{\rm em}} F_{\rm LT}(Q^2, 0)}{Q^2 + M_\chi^2}, \quad (34)$$

in terms of the considered χ_{c1} meson mass
$M_\chi = (3871.65 \pm 0.06)$ MeV
and the photon-meson transition form factor $F_{\rm LT}(Q^2, 0)$.

Transition form factor, one real photon

Combining this expression with Eq. (32) and using the well-known expression for the perturbative LF wave function of the longitudinal photon's $c\bar{c}$ component (Kovchegov-Levin book, 2012),

$$\Psi^{\gamma_L}_{\lambda\bar{\lambda}}(z, \boldsymbol{k}, Q^2) = ee_c \sqrt{z(1-z)}\, \frac{2z(1-z)Q}{\boldsymbol{k}^2 + \epsilon^2}\, \delta_{\lambda,-\bar{\lambda}}\,, \tag{35}$$

with $\epsilon^2 = m_c^2 + z(1-z)Q^2$, charm (anti)quark mass m_c, and the electric charge of the charm quark is $e_c = 2/3$, one arrives at the LFWF representation of the transition form factor:

$$\frac{f_{\rm LT}(Q^2)}{Q^2 + M_\chi^2} = -2\sqrt{2N_c}\, e_c \int \frac{dz d^2\boldsymbol{k}}{16\pi^3}\, \frac{k_x + ik_y}{[\boldsymbol{k}^2 + \epsilon^2]^2} \times \sqrt{z(1-z)} \Big\{ \Psi^{(+1)*}_{\uparrow\downarrow}(z, \boldsymbol{k}) + \Psi^{(+1)*}_{\downarrow\uparrow}(z, \boldsymbol{k}) \Big\}\,. \tag{36}$$

The dimensionless transition from factor $f_{\rm LT}(Q^2) \equiv F_{\rm LT}(Q^2, 0)/Q$ takes a finite value in the limit $Q^2 \to 0$.

The representation of Eq. (36) can also be derived from more general expressions for the transition form factors for two spacelike virtual photons found earlier in Babiarz et al. 2022.

Results for transition form factor

In our analysis, we adopt two different approaches.

- The first approach is based on $c\bar{c}$ radial wave functions in the $c\bar{c}$-pair rest frame obtained by solving the Schrödinger equation for a variety of phenomenologically viable potential models. Then, one appropriately transforms both the resulting radial wave functions (Terentev prescription), and their spin-orbital components (Melosh rotation) in order to describe the boosted meson states in the Drell-Yan frame.
- In the second approach, we have used the LFWFs from numerical results of Li, Maris, Vary. 2017.

Results for transition form factor

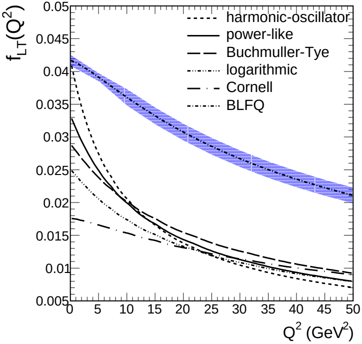


Figure: The dimensionless $\gamma_L^* \gamma \to \chi_{c1}(2P)$ transition form factor $f_{\rm LT}(Q^2)$ found in Eq. (36).

The dimensionless transition from factor $f_{\rm LT}(Q^2) \equiv F_{\rm LT}(Q^2, 0)/Q$ takes a finite value in the limit $Q^2 \to 0$.

Reduced width

Due to Landau-Yang theorem

$$\Gamma_{\gamma\gamma} = 0 \, . \tag{37}$$

Therefore the so-called reduced $\gamma\gamma$ decay width of χ_{c1} given in the limit of the vanishing projectile photon virtuality was introduced

$$\begin{aligned} \tilde{\Gamma}_{\gamma\gamma} &= \lim_{Q^2 \to 0} \frac{M_\chi^2}{Q^2} \Gamma^{\mathrm{LT}}_{\gamma^*\gamma^*}(Q^2, 0, M_\chi^2) \\ &= \frac{\pi \alpha_{\mathrm{em}}^2 M_\chi}{3} \, f_{\mathrm{LT}}^2(0) \, . \end{aligned} \tag{38}$$

Results for reduced width

Table: The reduced width of the $\chi_{c1}(2P)$ state for several models of the charmonium wave functions with specific c-quark mass.

$c\bar{c}$ potential	m_c (GeV)	$f_{\rm LT}(0)$	$\tilde{\Gamma}_{\gamma\gamma}$ (keV)
harmonic oscillator	1.4	0.041	0.36
power-law	1.334	0.033	0.24
Buchmüller-Tye	1.48	0.029	0.18
logarithmic	1.5	0.025	0.14
Cornell	1.84	0.018	0.07
BLFQ	1.6	0.044	0.42

First evidence for the production of $\chi_{c1}(3872)$ in single-tag e^+e^- collisions (Belle 2020) and $X(3872) \to J/\psi\pi^+\pi^-$.
From three measured events, they provided a range for its reduced width,

$$0.02\,\mathrm{keV} < \tilde{\Gamma}_{\gamma\gamma} < 0.5\,\mathrm{keV}. \tag{39}$$

This result has recently been updated by Achasov et al. using a corrected value for the branching ratio $\mathrm{Br}(\chi_{c1}(3872) \to \pi^+\pi^- J/\psi)$ and reads

$$0.024\,\mathrm{keV} < \tilde{\Gamma}_{\gamma\gamma}(\chi_{c1}(3872)) < 0.615\,\mathrm{keV} \tag{40}$$

Results for the reduced width

Using nonrelativistic quark model relations, Achasov et al. 2022 provided the following estimate

$$\tilde{\Gamma}_{\gamma\gamma}(\chi_{c1}(3872)) \approx 0.35\,\mathrm{keV} \div 0.93\,\mathrm{keV}\,. \tag{41}$$

Even with the large dependence on the $c\bar{c}$ potential, all our results, including the BLFQ approach, lie well within the experimentally allowed range.
Therefore, $\gamma^*\gamma$ data do not exclude the $c\bar{c}$ option, although there is some room for a contribution from an additional meson-meson component.
No estimates for the reduced width in the molecular scenario are available.

Conclusions

- The structure of famous $X(3872)$ is not known.
- Can the production of $X(3872)$ in proton-proton scattering be a new source of information ?
- We have calculated production of $X(3872)$ as the $c\bar{c}$ in the k_t-factorization approach within nonrelativistic approach for the $g^*g^* \to X(3872)$ vertex with modern unintegrated distributions.
  A reasonable results have been obtained.
- We have done similar calculation for the DD^* fusion. $c\bar{c}$ is calculated in the k_t-factorization approach. D and $\bar{D}^*$ mesons calculated in infinitly heavy quark approximation.
  A reasonable results have been obtained.
- Having in mind the finite lifetime of D^* mesons we have shown results for both directly produced D^0 and for all D^0, including the feeddown contribution.

Conclusions

- In addition, a hybrid model (mixture of $c\bar{c}$ and molecular component).
  A reasonable results have been obtained.
- All three (naive) approaches describe the LHC data for $pp \to X(3872)$.
- The lifetime argument discussed here for the first time suggests that in reality one should rather include only directly produced D^0 (or $\bar{D}^0$). This strongly reduces the cross section and causes that the purely molecular scenario is disfavoured.
- Therefore in the hybrid scenario the probability of the molecular component should not be too big.

Conclusions for $e^+e^- \to e^+e^- X(3872)$

- We have made predictions for F_{LT} form factors in the $c\bar{c}$ scenario.
  It cannot be verified at present (but Belle-2)
- The obtained so-called $\gamma\gamma$ reduced width is consistent with a poor Belle-1 data In contrast to Li, Li, Vary.
- A possibility to study the transition form factor and reduced width with the EIC.