[261/154]

HADRON 2023

June, 2023

K-matrix analysis of bottomonium in e^+e^-

Eric Swanson

ExoHad

Exotic Hadrons Topical Collaboration

Nils Hüsken

Ryan Mitchell

N. Hüsken, R.E. Mitchell, E.S. Swanson, K-matrix Analysis of e^+e^- Annihilation in the Bottomonium Region, 2204.11915

Motivation

Seek a comprehensive unitary analysis of all channels $e^+e^- \rightarrow$ bottom

New data makes thresholds evident, fits should move beyond Breit-Wigners.

Previous $\Upsilon(5S)$ branching fractions were estimated by measuring ee -> <final state> near $\Upsilon(5S)$ and dividing by ee -> all. This only works if $\Upsilon(5S)$ is produced in isolation (now known to be false!).

Three-body final states can provide additional constraints to the fit.

Formalism

Many channels + thresholds, desire for unitarity => K-matrix formalism

Chew-Mandelstam function

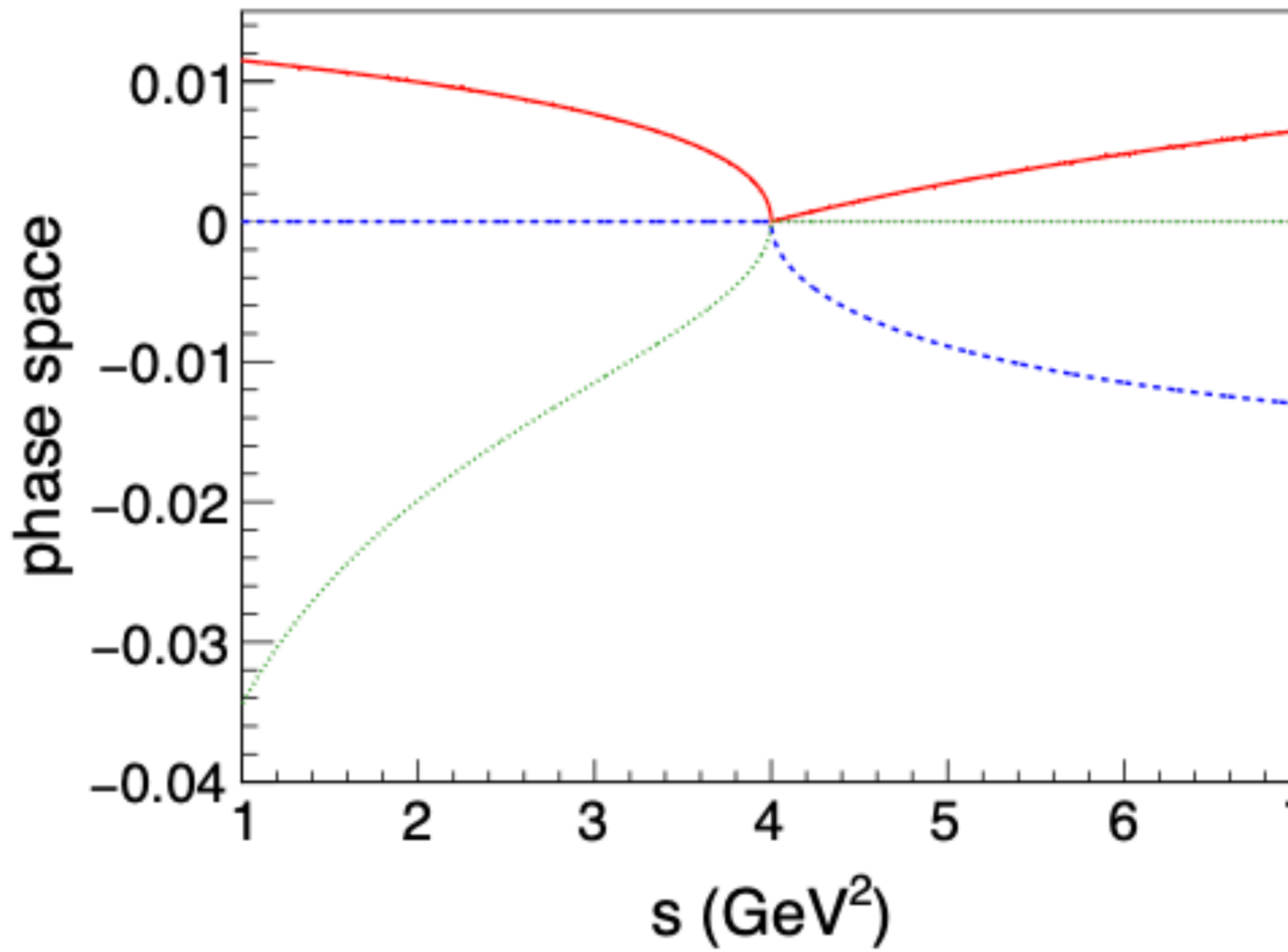
$$K^{-1} = \mathscr{M}^{-1} + \overbrace{i\rho - R}^{-C}$$

$$\mathscr{M} = (1 + KC)^{-1}K = K(1 + CK)^{-1} = K(K + KCK)^{-1}K$$

$$\rho_{\mu,\nu} = \delta_{\mu,\nu} \frac{k_\nu(s)}{8\pi S_\nu \sqrt{s}}.$$

Formalism

Chew-Mandelstam function



red= Re(C)
blue = Im(C)
green=Im(rho)

Formalism

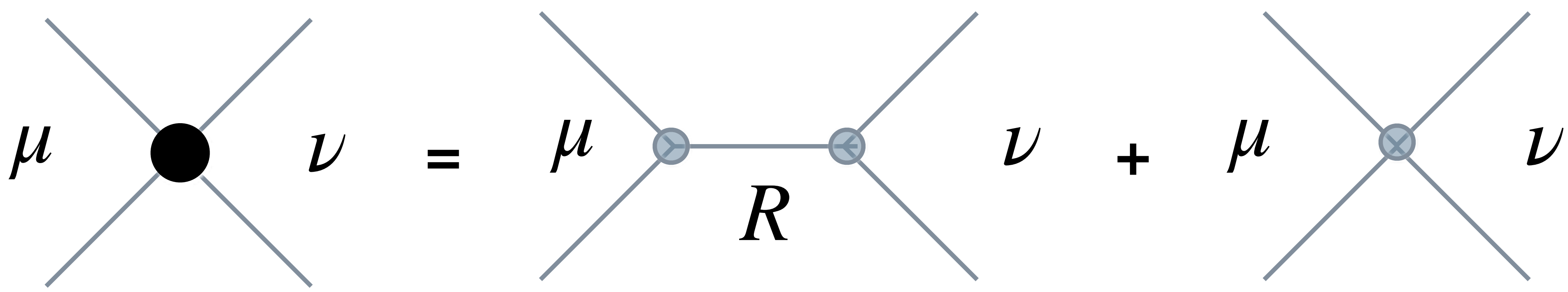
$$\mathcal{M}_{\mu,ee} = \sum_{\nu} (1 + \hat{K}\hat{C})^{-1}_{\mu,\nu} \, P_\nu$$

Aitchison "P-vector"
hat = reduced channel space

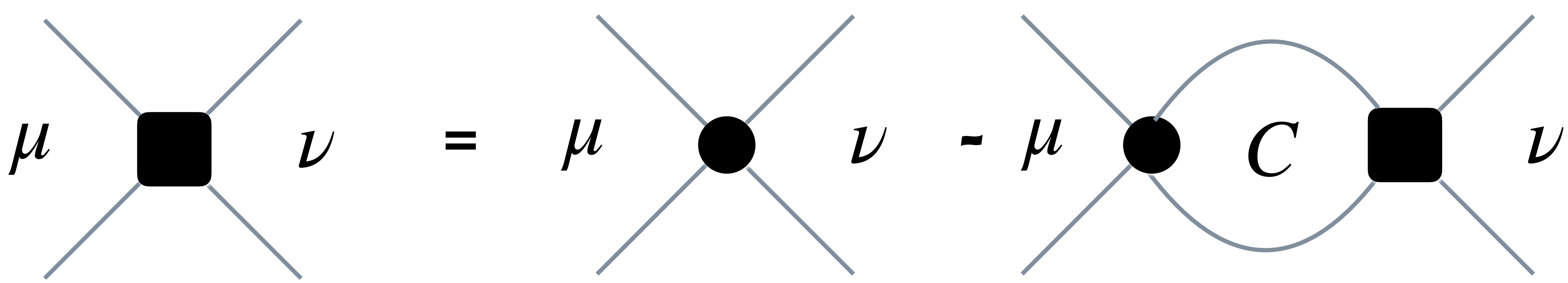
$$K_{\mu,\nu} = \sum_R \frac{g_{R:\mu} g_{R:\nu}}{m_R^2 - s} + f_{\mu,\nu}$$

Formalism

K



$\mathscr{M}$



Formalism

Incorporate three-body channels

i. implement as quasi two-body (as is usually done)
ii. implement as a perturbative three-body channel

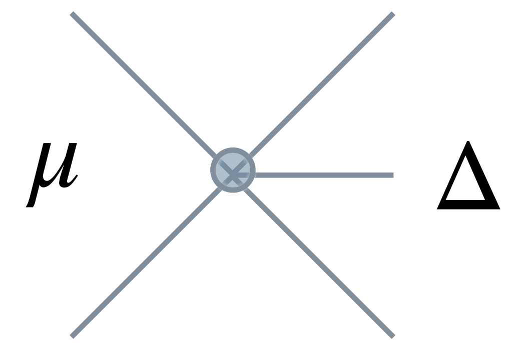
$$\mathcal{M}_{\Delta,ee} = \sum_{\nu} F_{\nu}^{(\Delta)} (1 + \hat{C}\hat{K})^{-1}_{\nu,ee}$$

similar to Aitchison, but for the final state

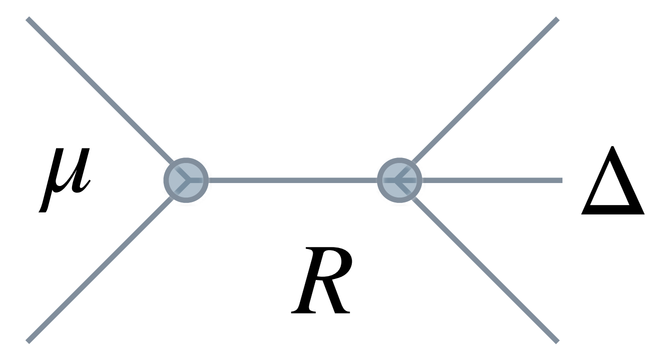
Formalism

Examples of the "F vector"

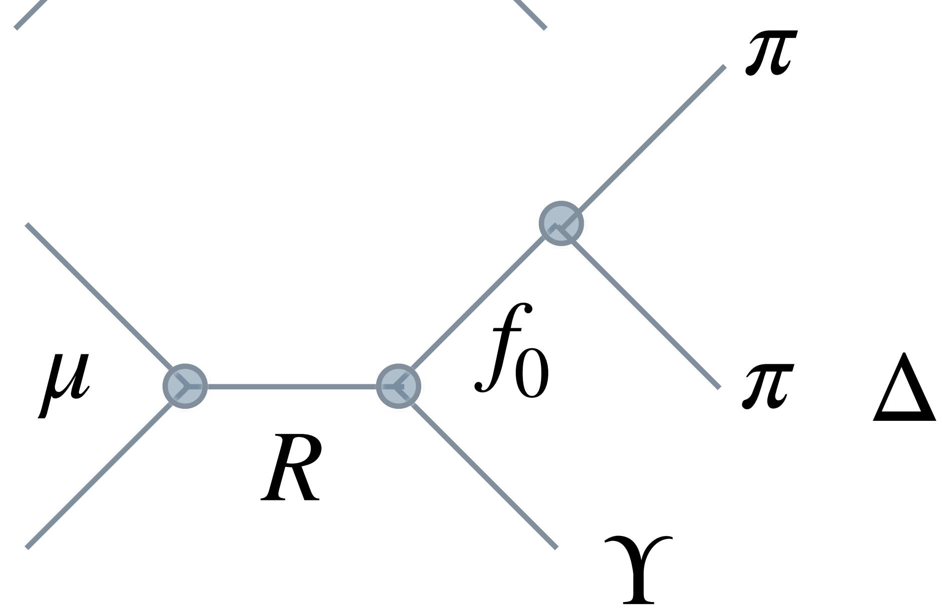
$$F_{\boldsymbol{\mu}}^{(\Delta)} = f_{\Delta:\mu}$$



$$F_{\boldsymbol{\mu}}^{(\Delta)} = \sum_R \frac{g_{R:\Delta} g_{R:\mu}}{m_R^2 - s}$$



$$F_{\boldsymbol{\mu}}^{(\Delta)} = \frac{g_{R:\mu} \cdot g_{R:f_0\Upsilon} \cdot g_{f_0:\pi\pi}}{(m_R^2 - s)(m_{f_0}^2 - s_{\pi\pi})}$$



Formalism

Coupling model

$$g_{R:\mu}(s) = \hat{g}_{R:\mu}\left(\frac{k_\mu(s)}{\beta}\right)^{\ell_\mu} \cdot \exp[-k_\mu^2(s)/\beta^2]$$

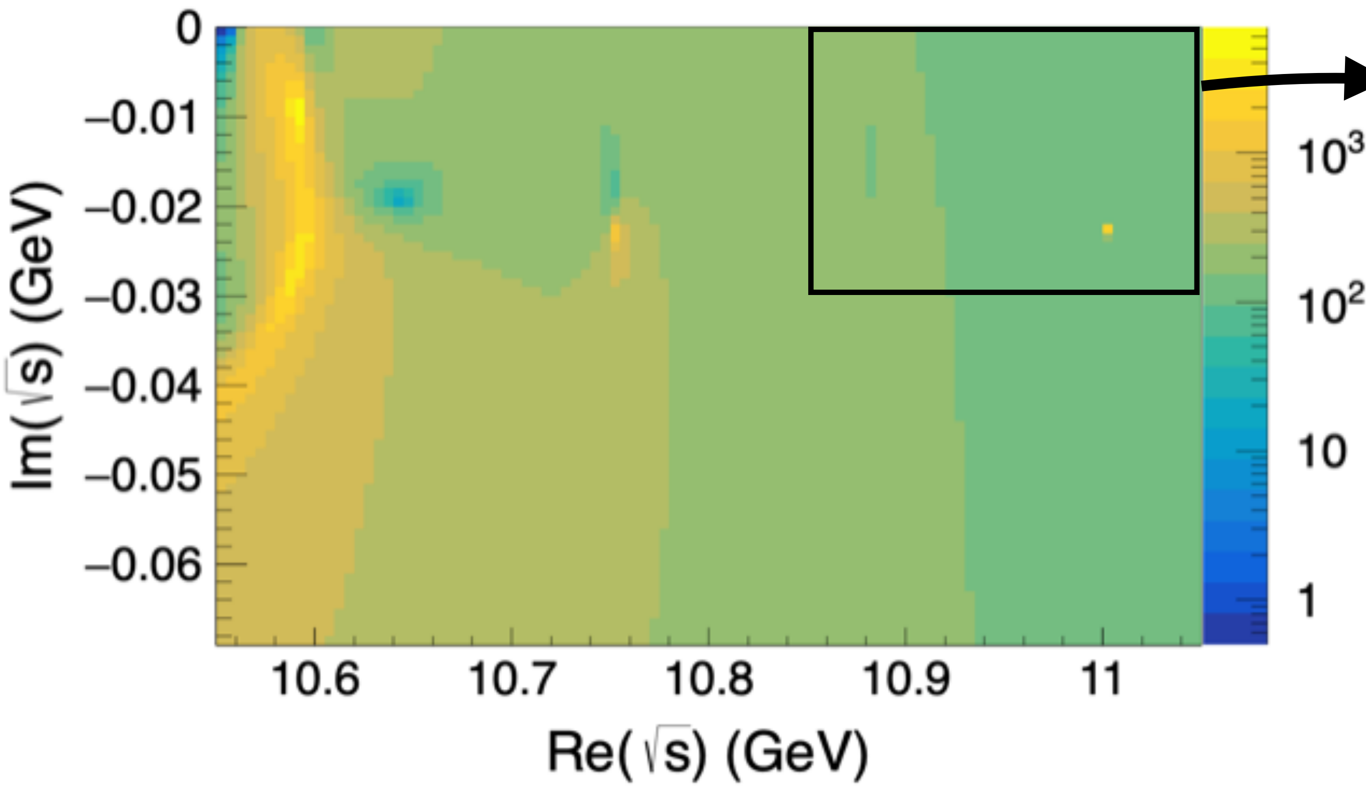
$$f_{\mu,\nu} = \hat{f}_{\mu,\nu} \cdot \left(\frac{k_\mu(s)}{\beta}\right)^{\ell_\mu} \cdot \left(\frac{k_\nu(s)}{\beta}\right)^{\ell_\nu} \cdot \exp\left[-\left(\frac{k_\nu^2(s)+k_\mu^2(s)}{\beta^2}\right)\right]$$

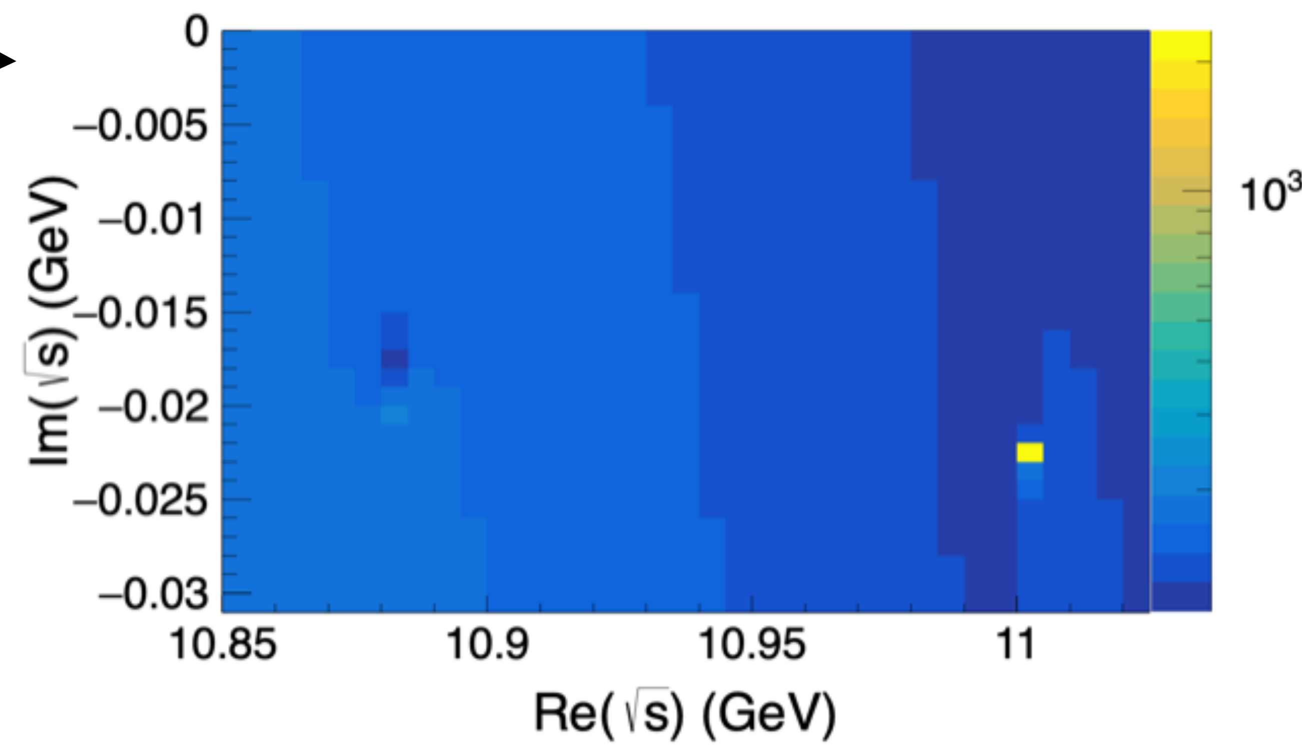
High energy damping in the 'background' terms helps fit robustness.

Formalism

Poles and Residues

Search for poles on various sheets





Residues from Cauchy's theorem

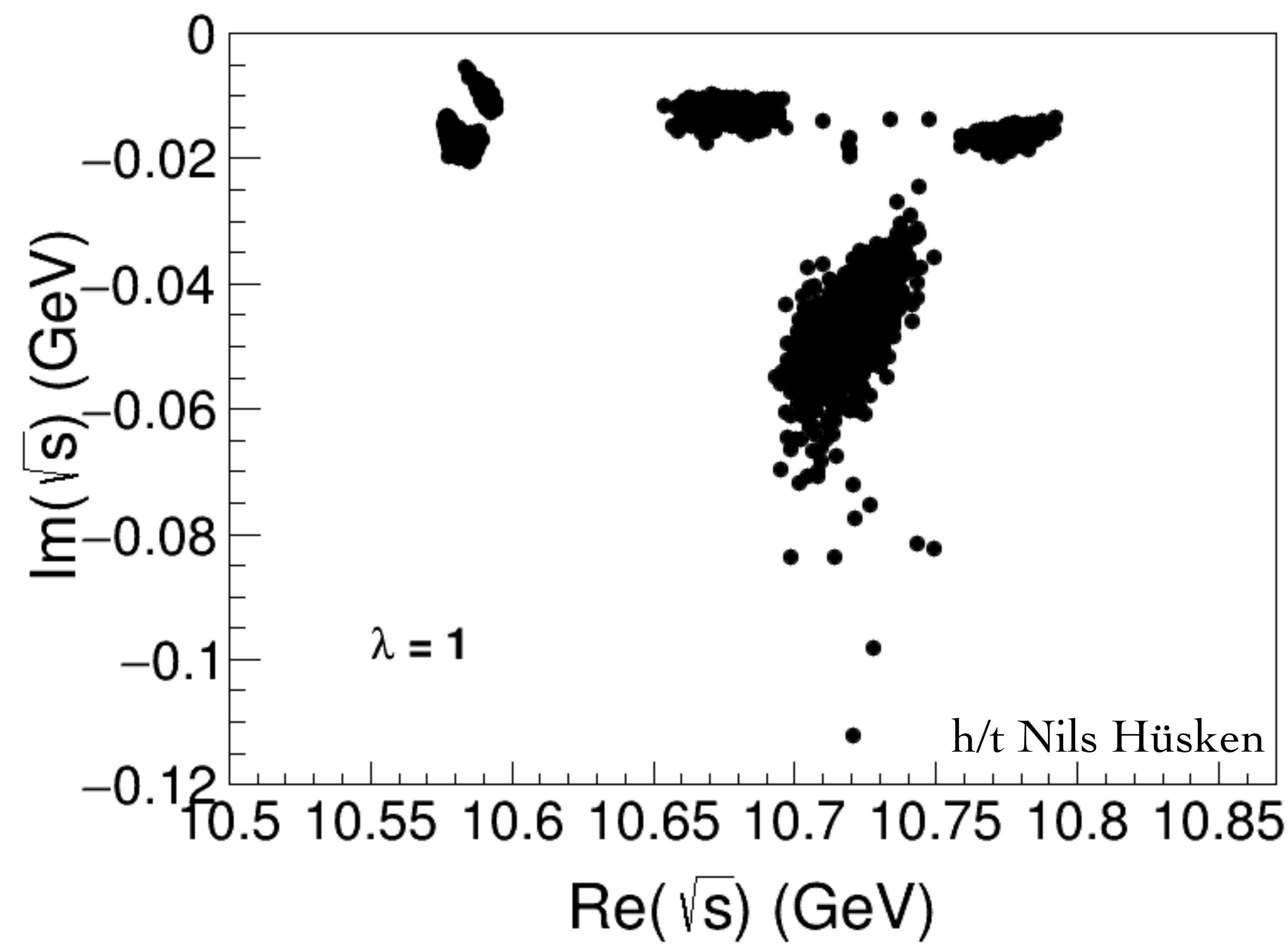
$$\mathrm{Res}_\mu = \frac{i}{N}\sum_{n=1}^{N}\mathcal{M}_{\mu,\mu}(s_n)\cdot\frac{\rho_\mu(s_n)}{\sqrt{s_n}}\cdot\left(s_n - m_{\mathrm{pole}}^2\right)$$

Formalism

Ghost Poles

Can arise due to spurious non-invertibility of (1+CK).

Identify by anomalous residues and behaviour under rescaling the couplings $\hat{g} \to \lambda \hat{g}$.



Results

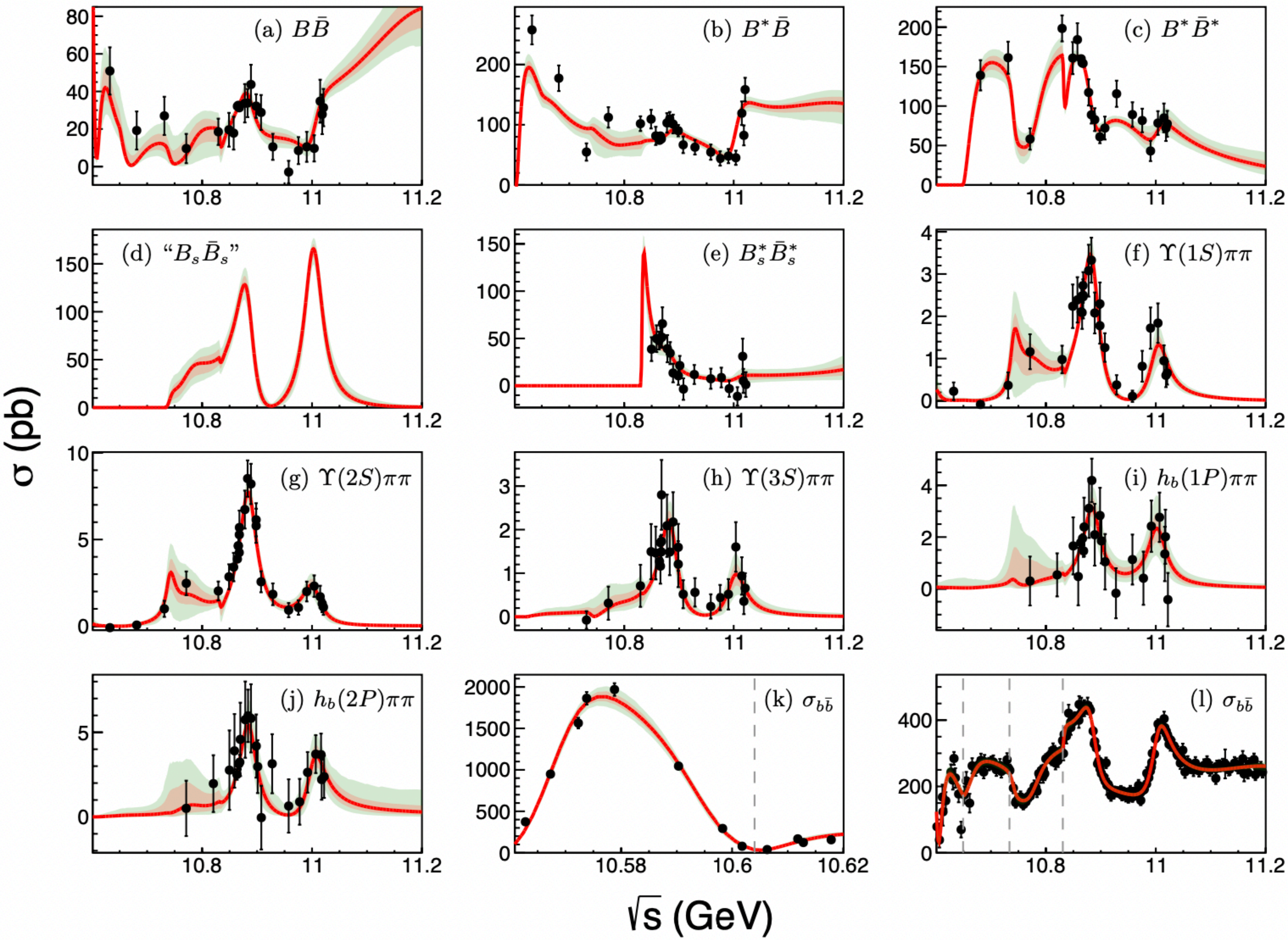
fitting procedure:

fit total xsec below BB* with BB channel and a single resonance
add three additional resonances
iterate:
  add a new channel to absorb difference between inclusive and exclusive data
  refit
add dummy channel
iterate:
  add a three body channel
  refit

[LASSO was a failure]

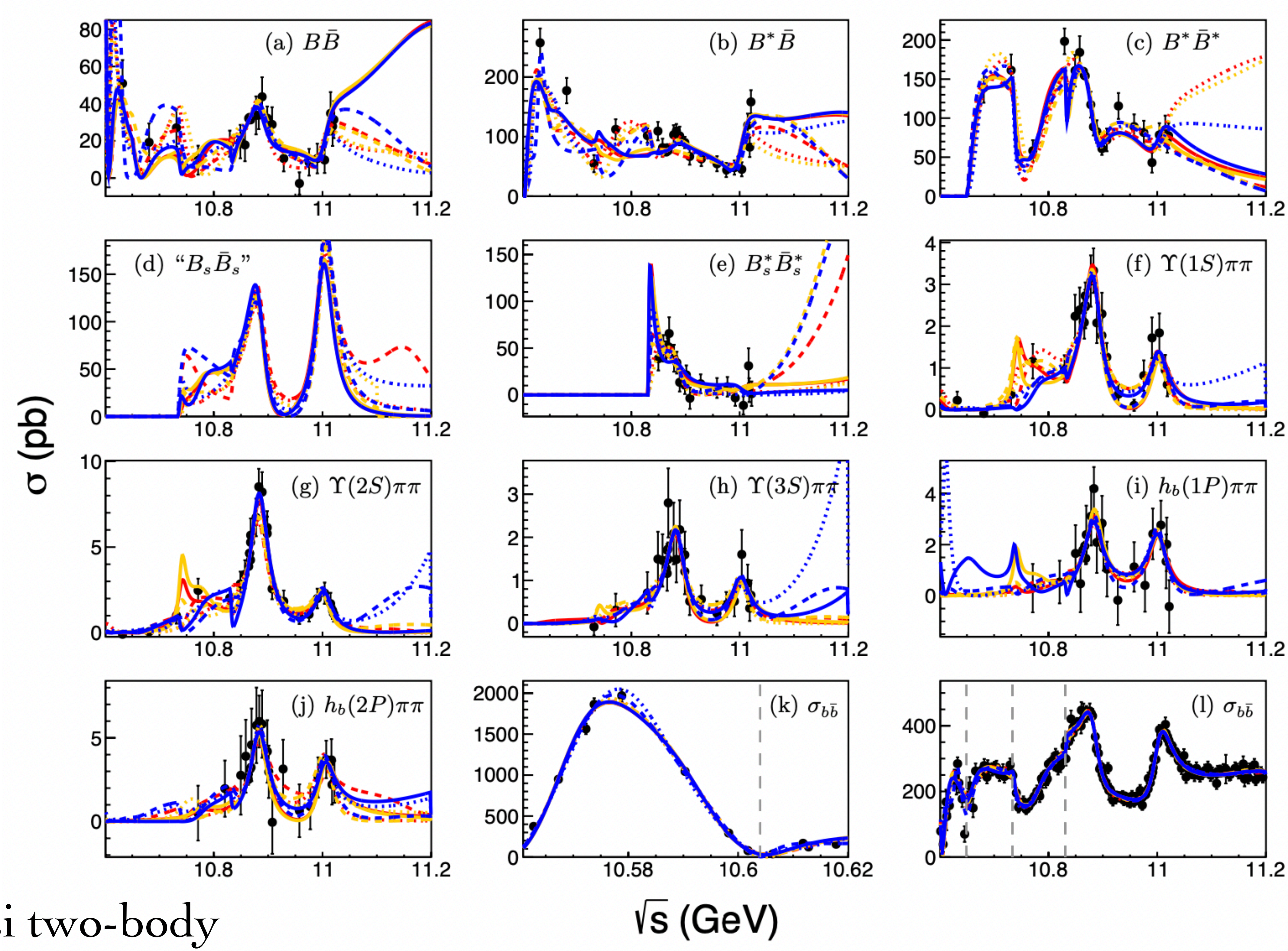
Results

quasi two-body, beta=1, fit+68% and 90% CL regions



Results

nine models

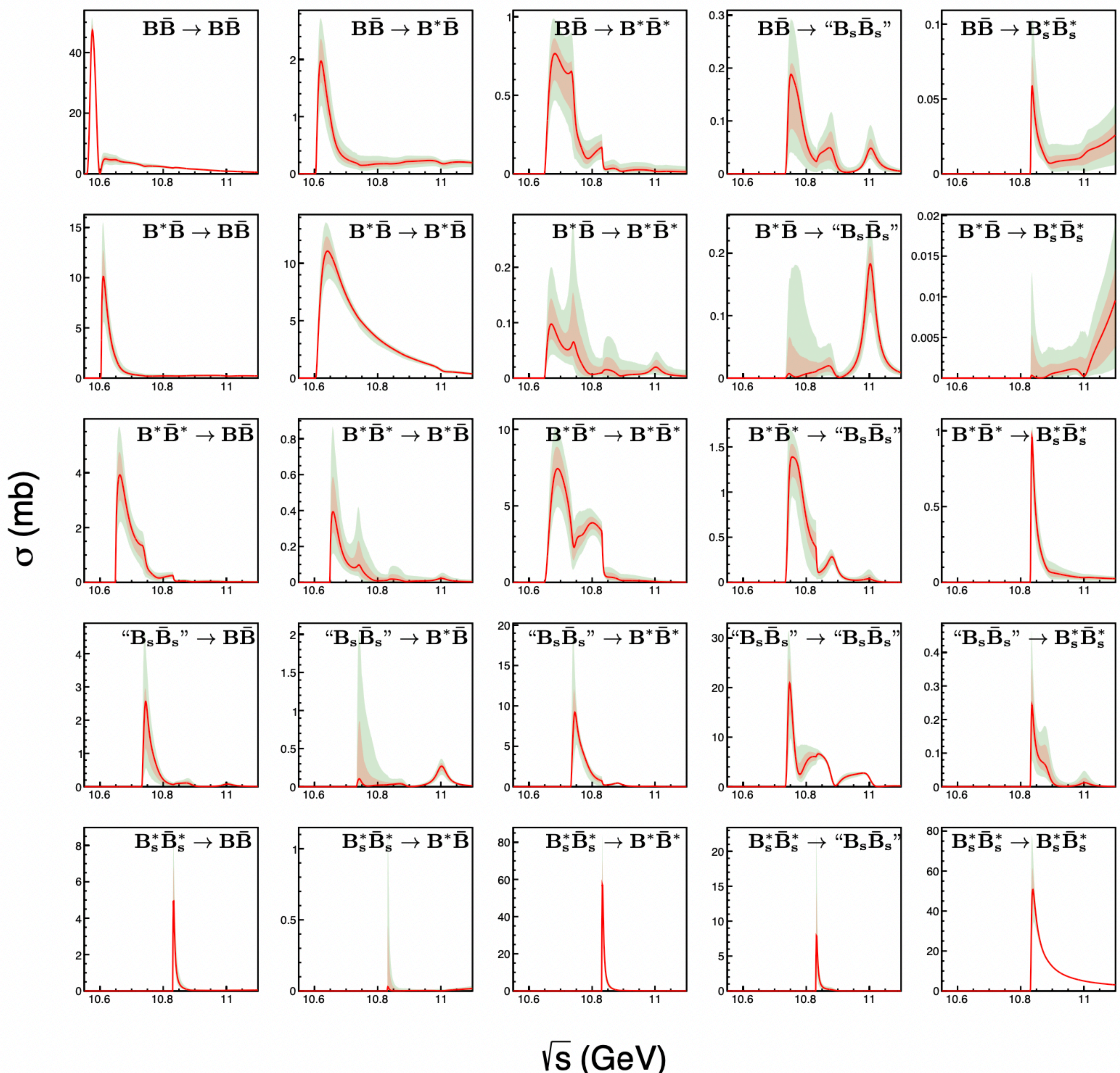


red=quasi two-body
yellow = three-body+res coupling
blue = three-body + f

model variation recapitulates bootstrap variation

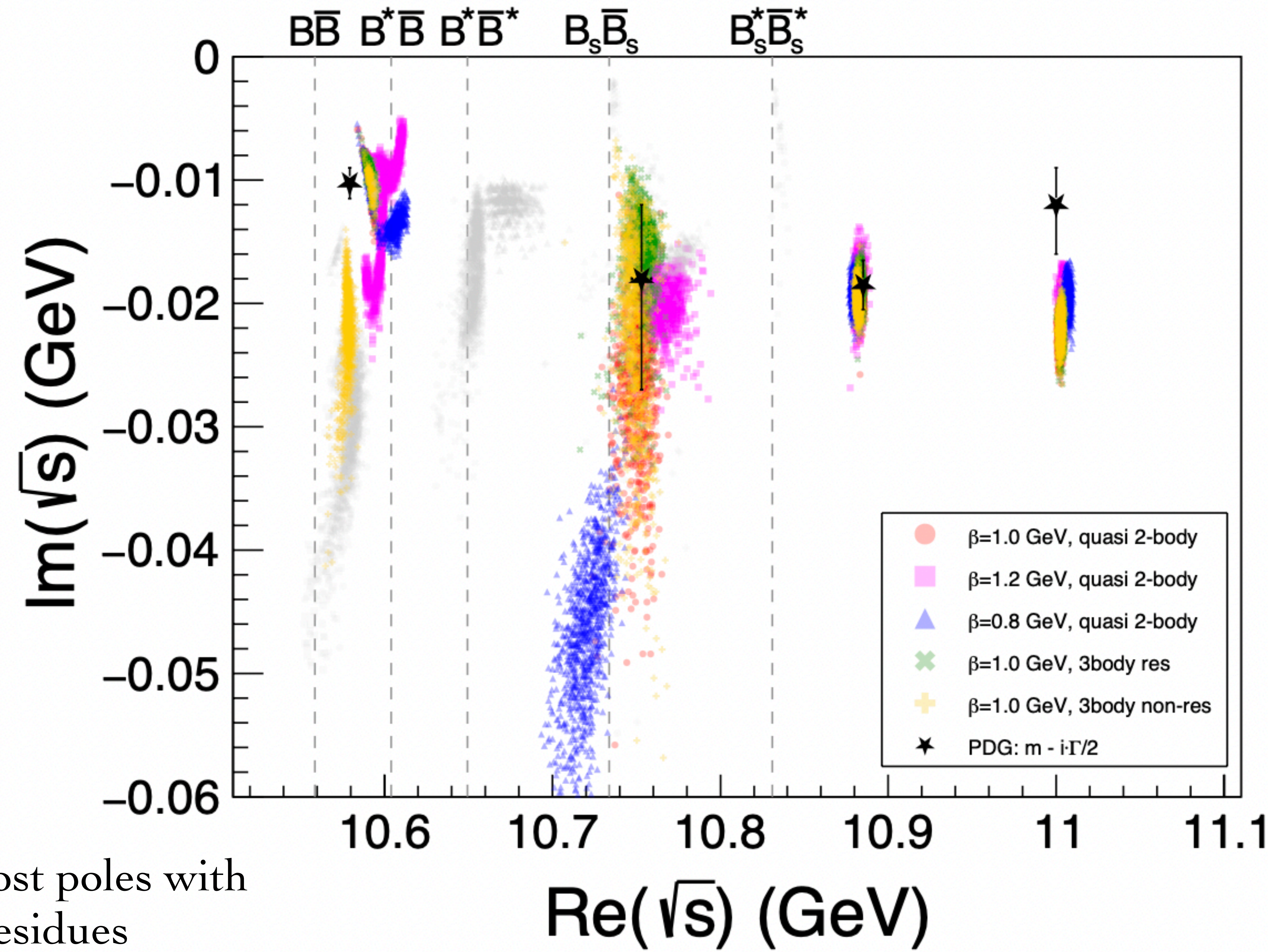
Results

hadronic scattering cross sections



Results

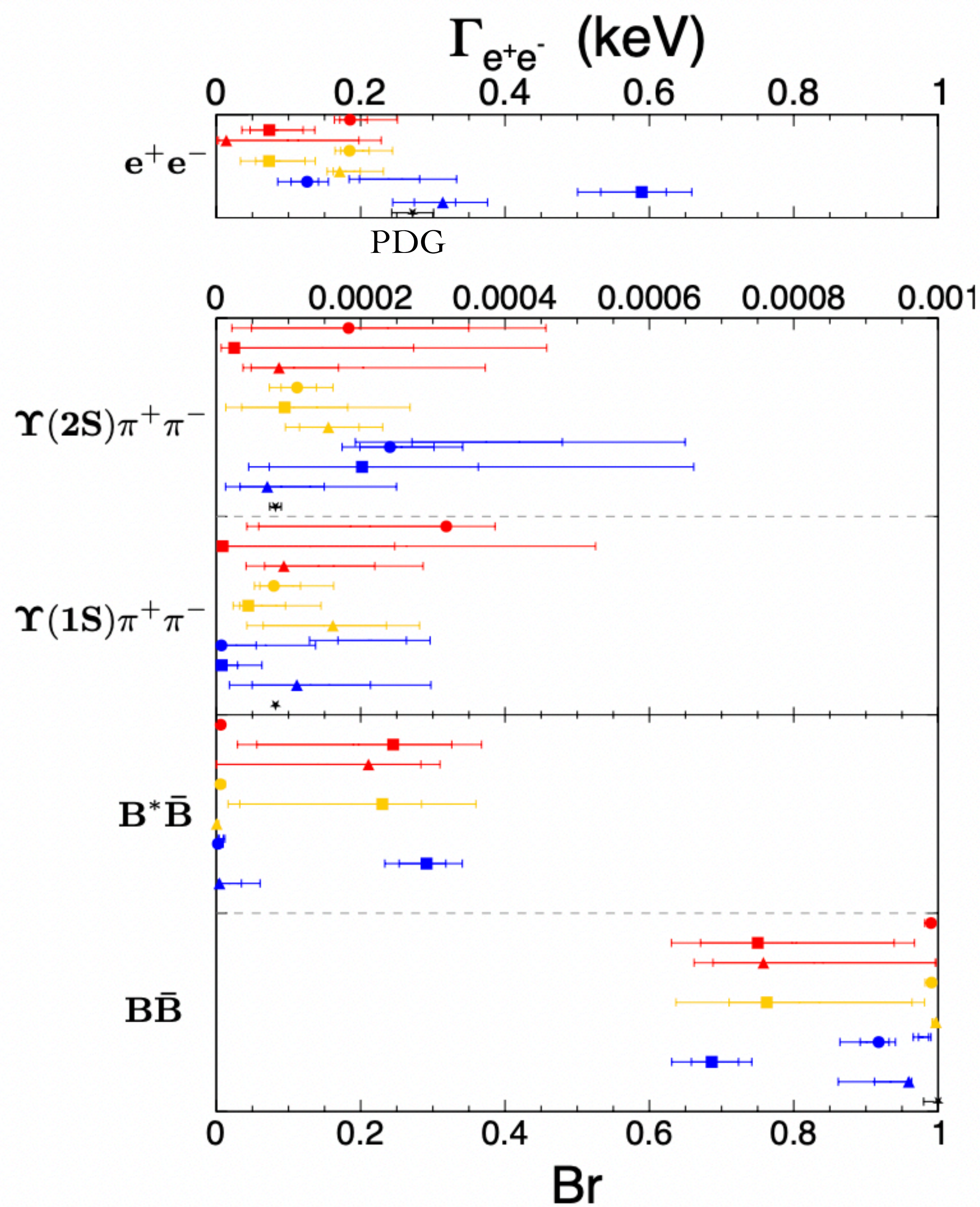
pole positions (+ 1000 bootstrap datasets)



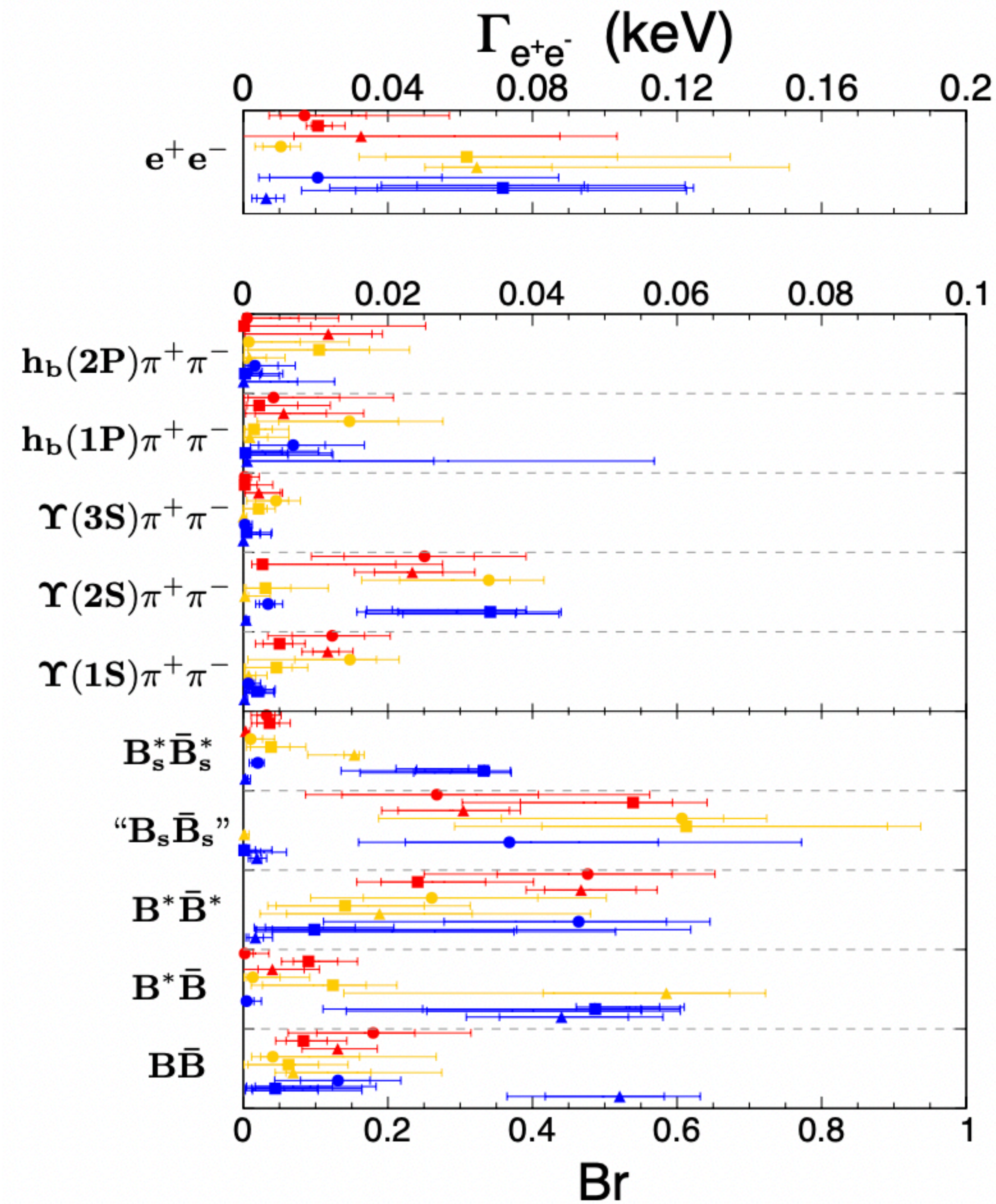
grey=ghost poles with sizable residues

Results

couplings and branching fractions



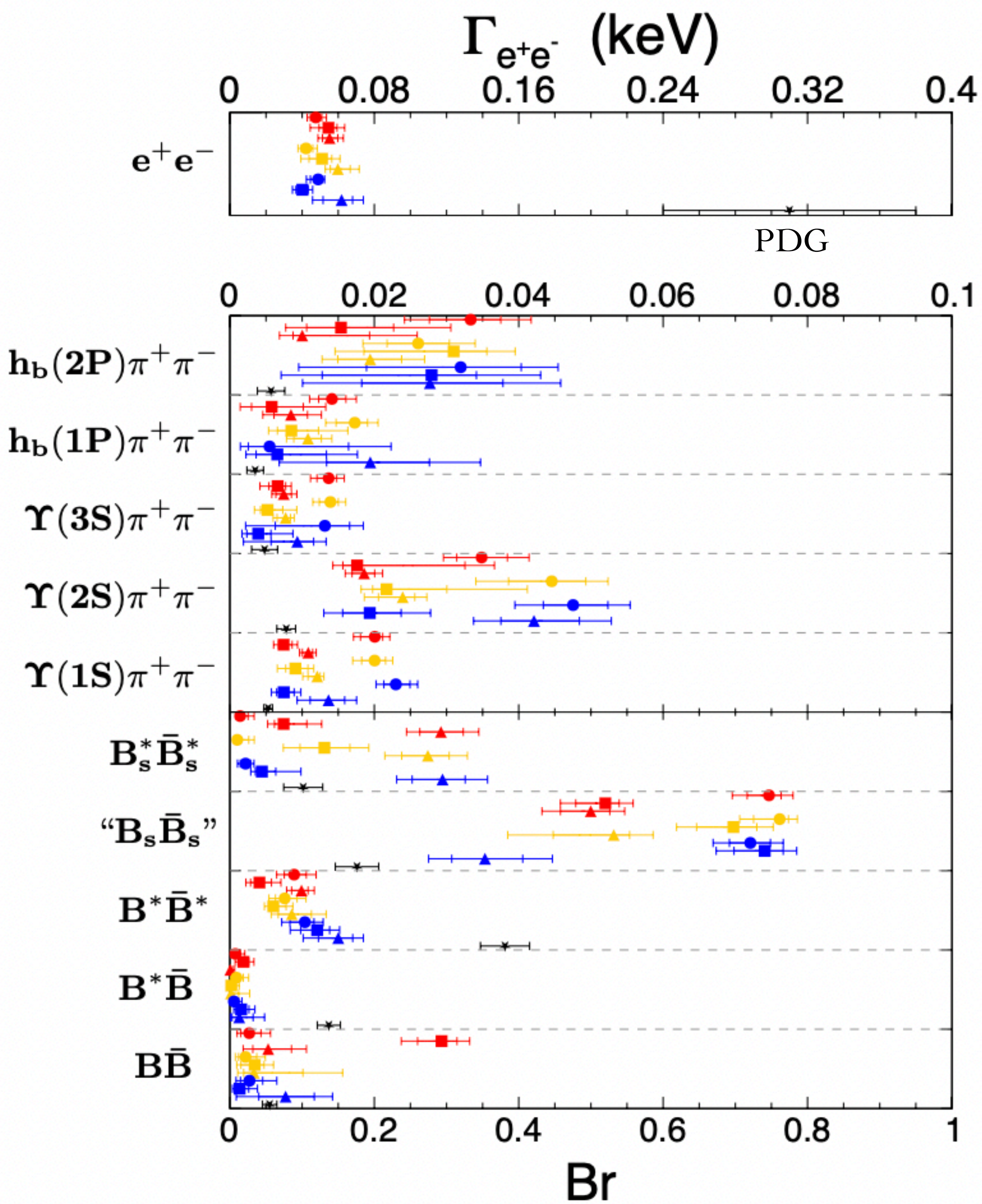
Y(4S)



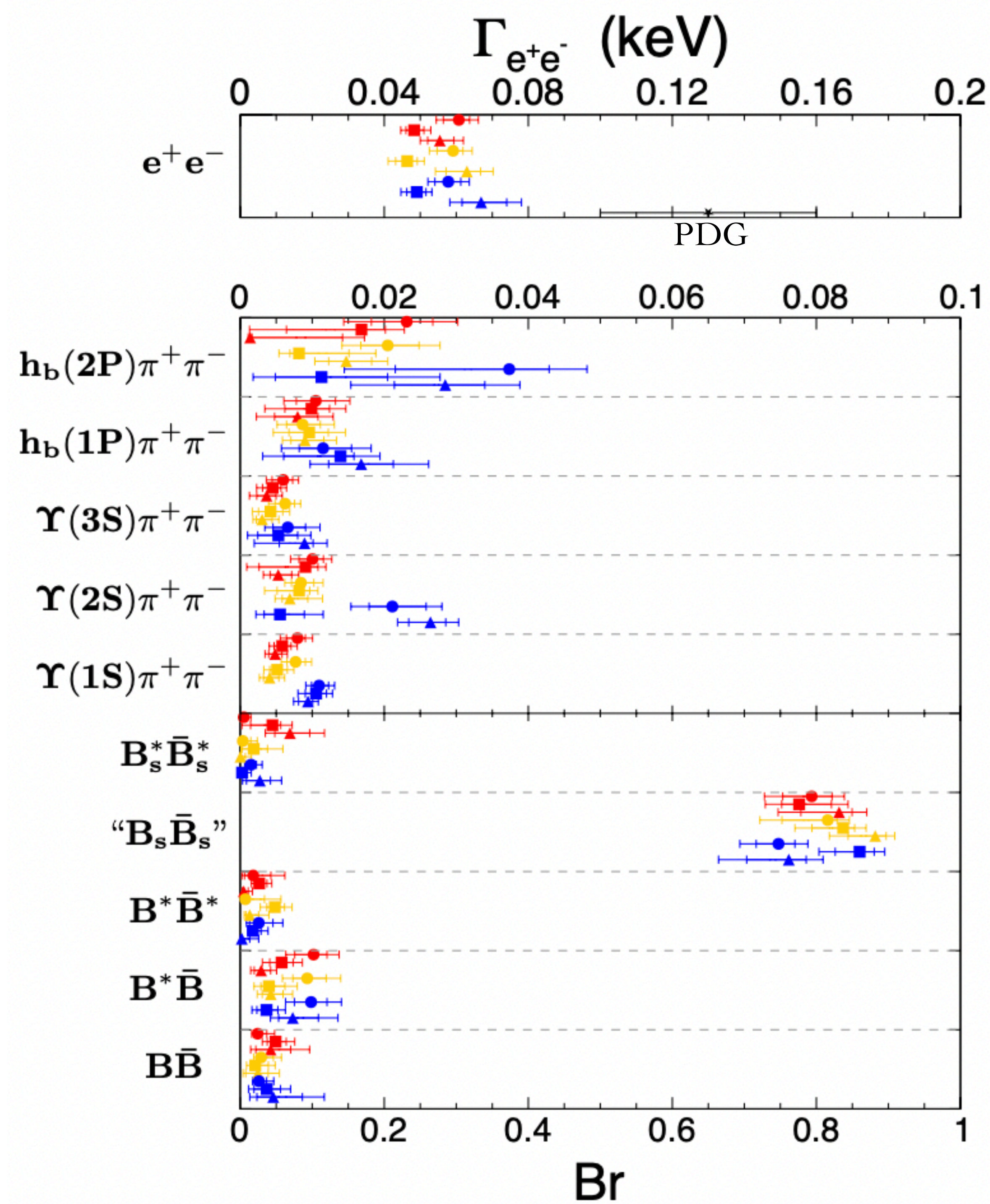
Y(10750)

Results

couplings and branching fractions



Υ(5S)

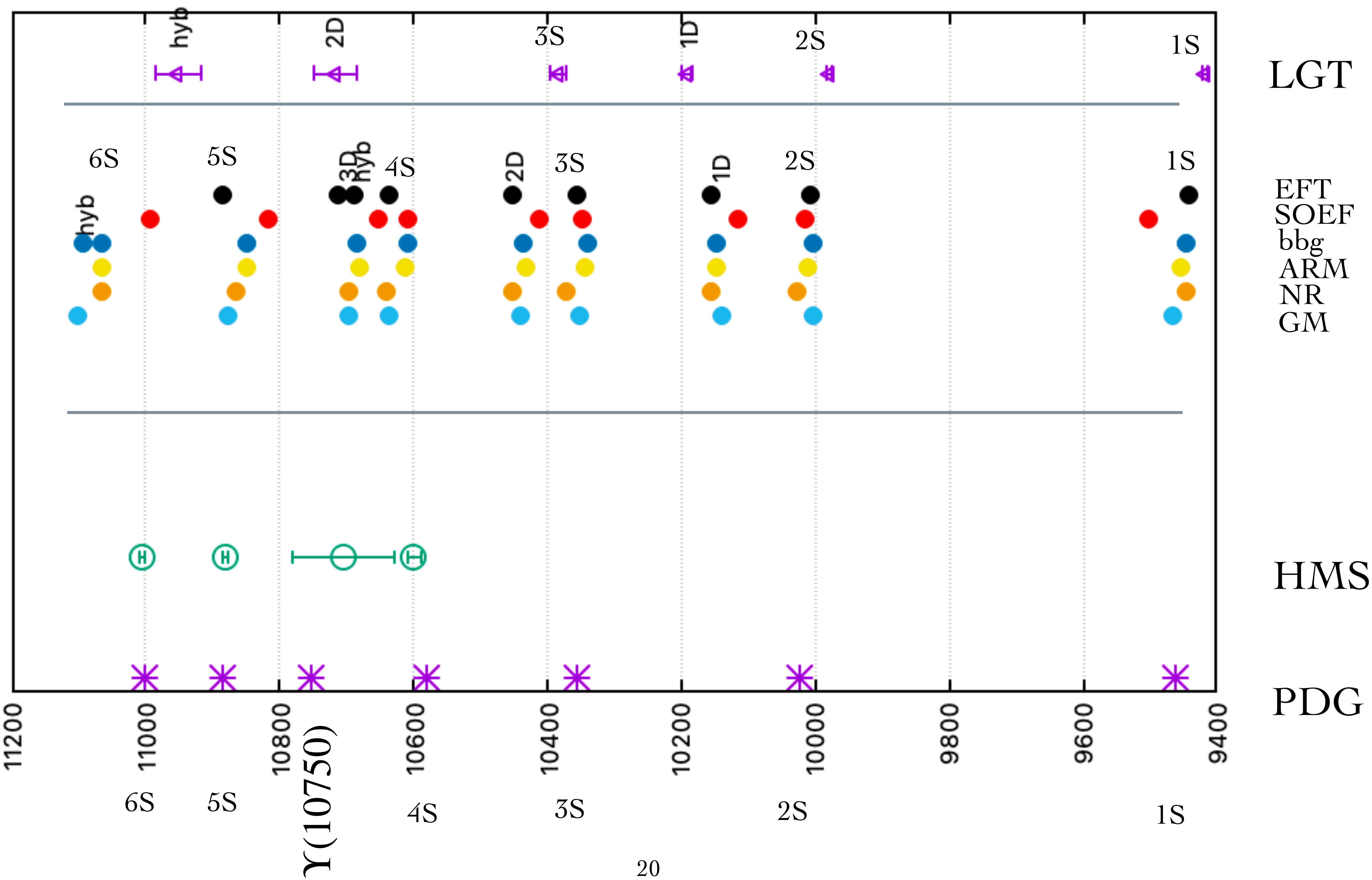


Υ(6S)

Discussion

masses

[BW params, poles, quark model eigenvalues, LGT plateaus]



Discussion

$\Gamma(ee)$ (keV)

state	RPP	our estimate	LS	GM	SOEF
$\Upsilon(4S)$	0.272	(0.003 - 0.62)	0.31	0.39	0.21
$\Upsilon(5S)$	0.31	(0.037 - 0.068)	0.28	0.33	0.18
$\Upsilon(6S)$	0.13	(0.043 - 0.074)	0.26	0.27	0.15
$\Upsilon(10750)$	(0.01 - 0.40)[a]	(0.004 - 0.10)		2.38 eV [b]	

[a] from ambiguous solutions in Ref. [5]
[b] assuming a $3D$ state

Conclusions & Observations

First comprehensive and unitary analysis of ee to bottom.

First determination of absolute branching ratios of all four states.

$\Upsilon(10750)$ is confirmed.

$\Gamma_{ee}(10750) = 0.06(0.06)$ keV.

$\Upsilon(4S)$ is 10-20 MeV higher in mass than RPP.

$\Upsilon(6S)$ width is twice RPP value.

Γ_{ee} are significantly smaller than RPP and quark models for higher n.

Indications that open bottom strong decays of higher N states are not well modelled

Missing channels for 5S and 6S are likely dominated by $Z_b^{(\prime)}$.

Data near 10750 and above 11000 MeV would be very helpful!

~thank you~