# Structure of heavy mesons in light-front quark model

**Ahmad Jafar Arifi**

A. J. A, H-M. Choi, C-R. Ji, and Y. Oh[†], Phys. Rev. D **106**, 014009 (2022)

HADRON 2023, Genova, Italy
June 5-9, 2023

# Contents



- Introduction
- Light-front quark model
- Radially excited states

  —> Mass gaps

  —> Decay constants

  —> Charge radii

  —> Global analysis
- Summary

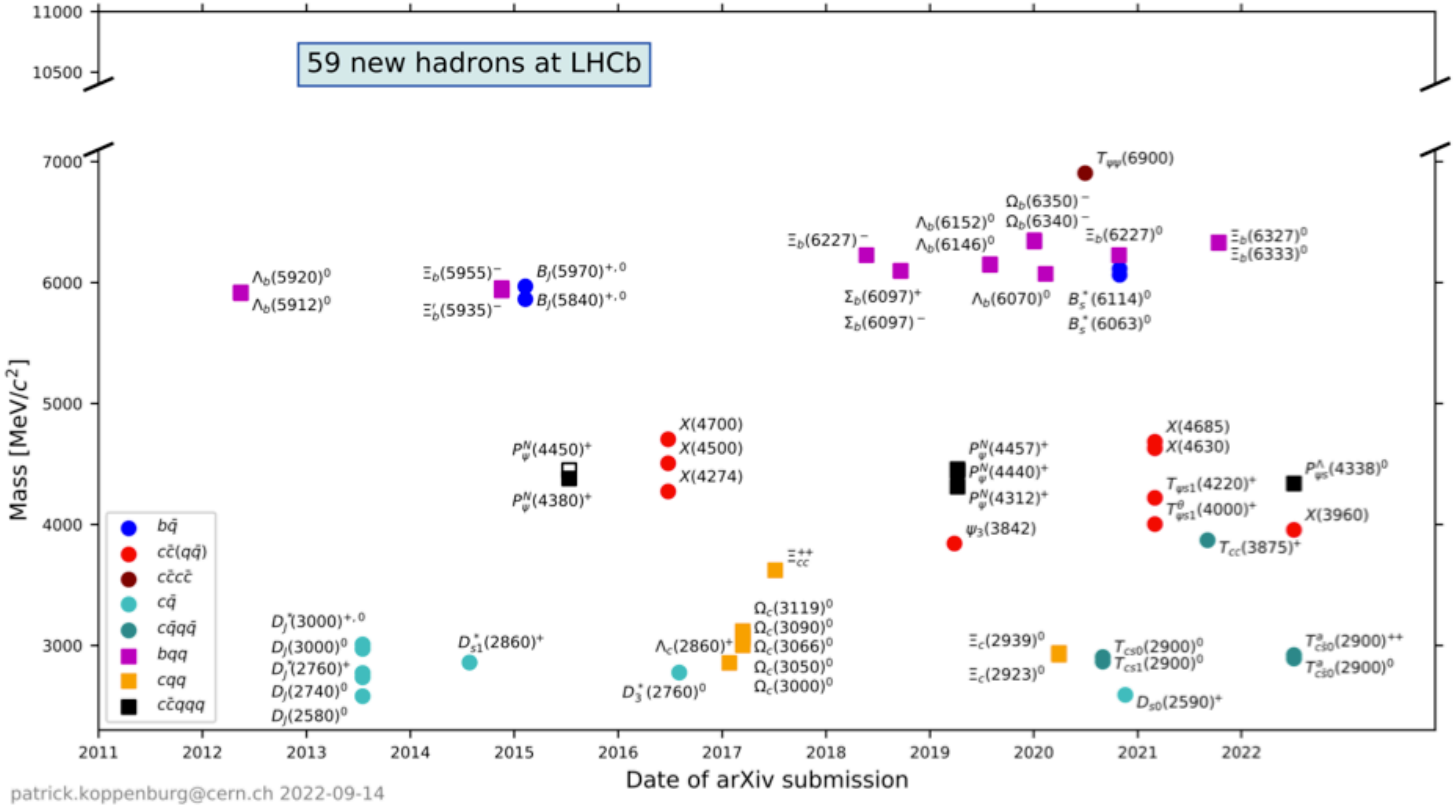
**Recent works on LFQM**

—> Phys. Rev. D **106**, 014009 (2022)

—> Phys. Rev. D **107**, 053003 (2023)

—> Arxiv: 2302.12382 (2023)

# Introduction




Experiments:

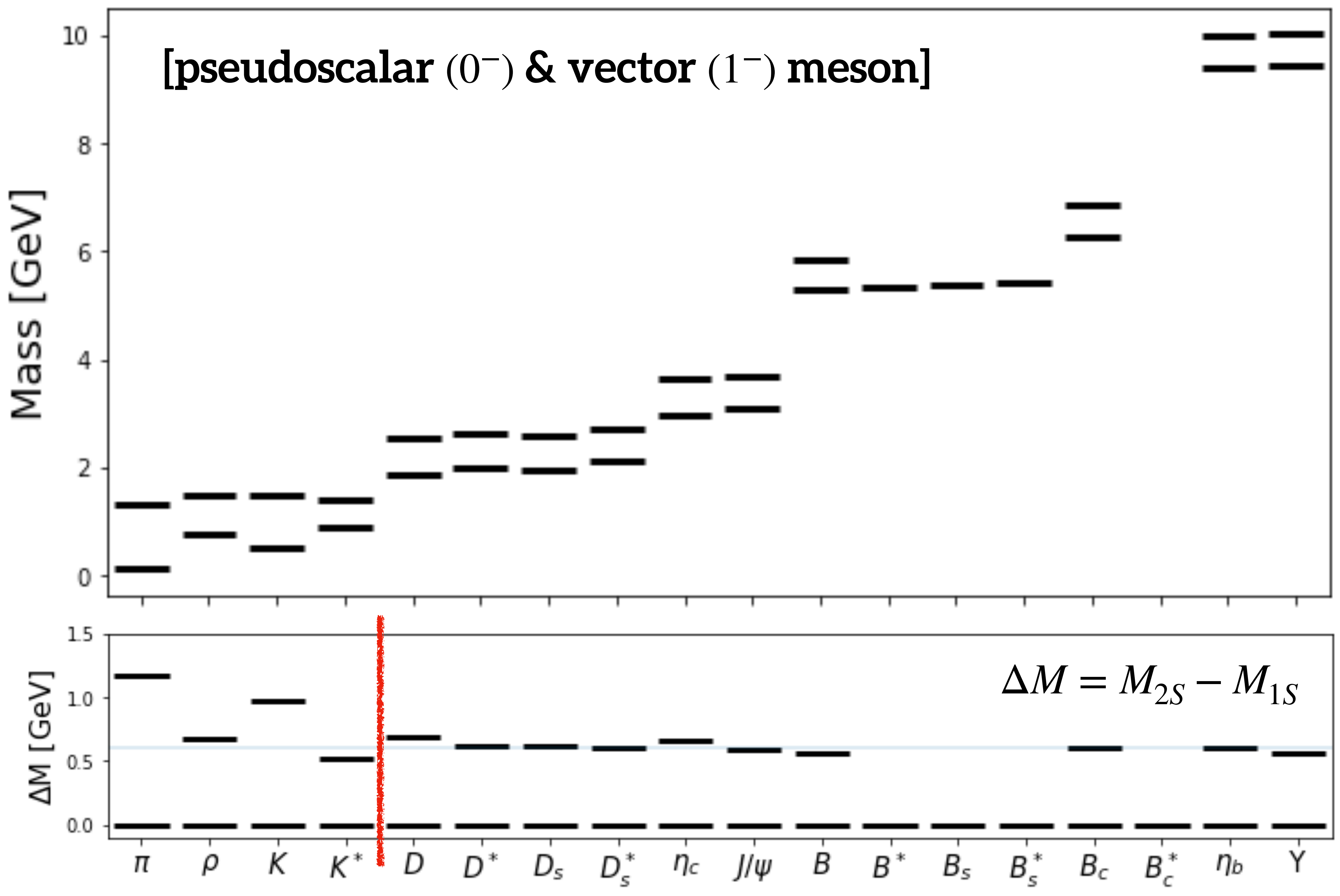
LHC, KEK, J-PARC, BESIII, J-Lab, EIC, etc…

- Hadrons
  —> Meson & Baryon
  —> Exotics
- Lots of new hadrons
  —> Mass spectra,
  —> Transitions,
  —> Productions, etc
- Internal structure?
  —> Lattice QCD
  —> Effective models

# Our focus: 1S and 2S state Mesons



## Mass spectra and splitting




## Other properties

- Decay constants
- Electromagnetic form factors
- Transition form factors

  —> Expt and Lattice data

## Model

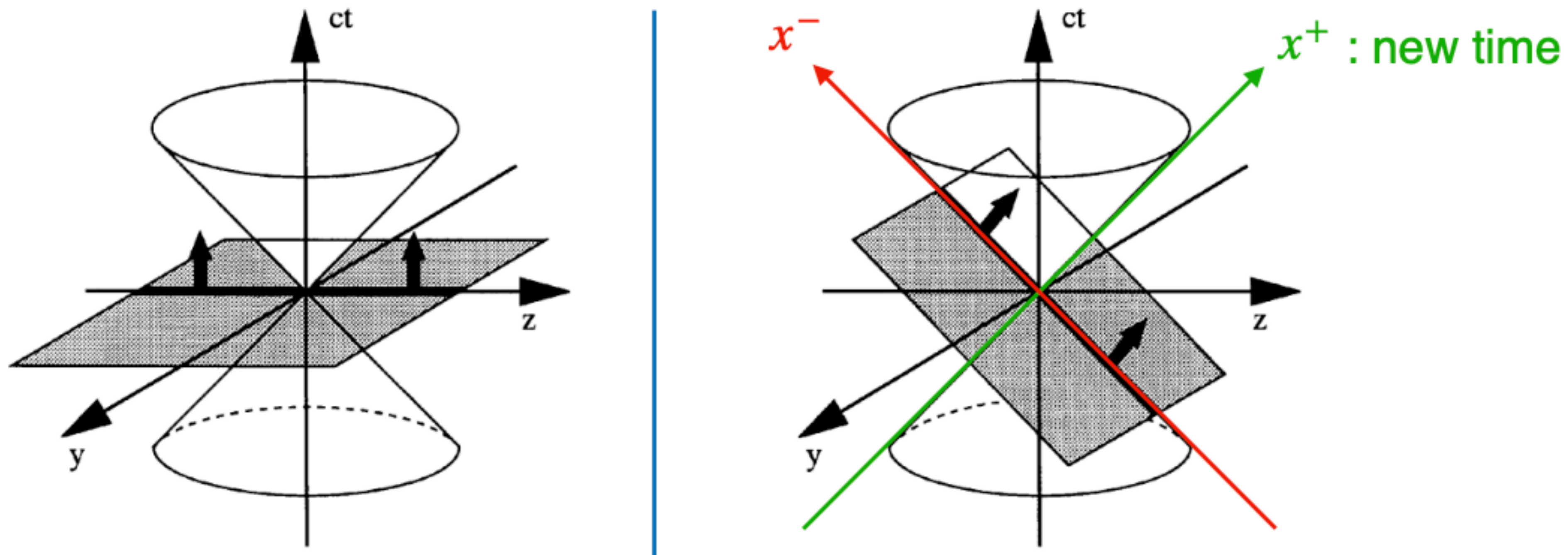
- Explain the mass spectra
- Explain other properties
- Include both light & heavy sectors

# Light-front quark model (LFQM)

- **LFQM**
  —> Constituent quark picture
  => various quark flavors
  —> Light-front dynamics
  => Handle relativistic effect

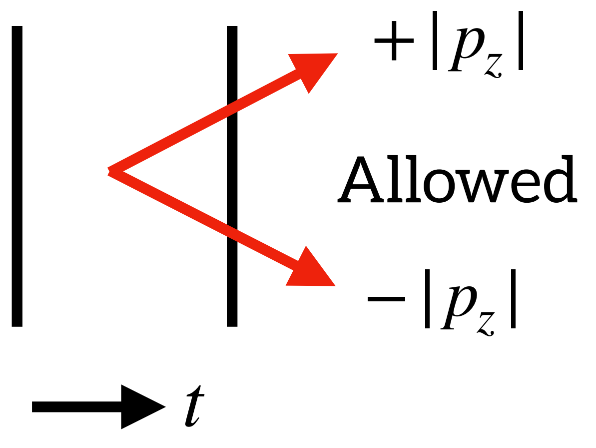
- **LFWF**
  —> boost invariant, frame independent
  —> Spin-orbit WF
  => Melosh transformation

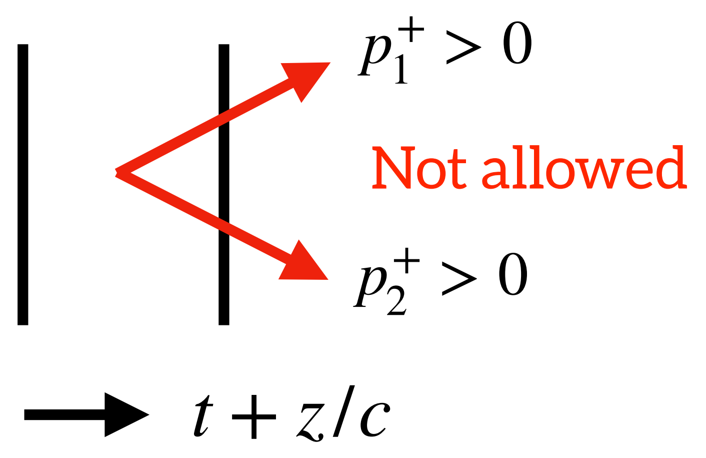
$$\Psi_{nS}^{JJ_z}(x,\mathbf{k}_\perp,\lambda_i) = \Phi_{nS}(x,\mathbf{k}_\perp)\,\mathcal{R}_{\lambda_q\lambda_{\bar q}}^{JJ_z}(x,\mathbf{k}_\perp),$$




| | Instant form | Light-Front form |
|---|---|---|
| Time | $x^0$ | $x^+ = x^0 + x^3$ |
| Space | $x^1,\, x^2,\, x^3$ | $x^- = x^0 - x^3,\ \ \mathbf{x}_\perp = (x^1,\, x^2)$ |
| Hamiltonian | $p^0$ | $p^- = p^0 - p^3$ |
| Momentum | $p^1,\, p^2,\, p^3$ | $p^+ = p^0 + p^3,\ \ \mathbf{p}_\perp = (p^1,\, p^2)$ |
| Product | $x \cdot p = x^0 p^0 - \mathbf{x} \cdot \mathbf{p}$ | $x \cdot p = (x^+ p^- + x^- p^+)/2 - \mathbf{x}_\perp \cdot \mathbf{p}_\perp$ |
| Vacuum | very complex | can only contain zero-mode excitations |

$$E = \sqrt{\mathbf{p}^2 + m^2}$$




$$p^- = \frac{m^2 + \mathbf{p}_\perp^2}{p^+}$$

# Light-front quark model (LFQM)



- Mainly focus on ground states
  => Various processes have been studied

  Phys. Rev. D 59, 074015 (1999),

  Phys. Lett. B 460, 461 (1999), etc

- Assuming the form of wave function
  => Harmonic oscillator, Power-law, etc.

  Phys. Rev. D 95, 056002 (2017), etc

- BLFQ (Basis light-front quantization)
  —> Solving eigenvalue problem

  Phys. Rev. D 96, 016022 (2017), etc

- Studies for excited states are still limited
  => More studies are required.
  => Need more data.

- Some studies
  => Modifying the HO basis function
  —> mimicking the Hydrogen WF

  $$\exp(-p^2/2\beta^2) \rightarrow \exp(-n^\delta p^2/2\beta^2)$$

  Phys. Rev. D 82, 034023 (2010)

  => Fixing beta from decay constant
  —> violate the orthogonality

  Eur. Phys. J. A 48, 66 (2012)

# Variational analysis

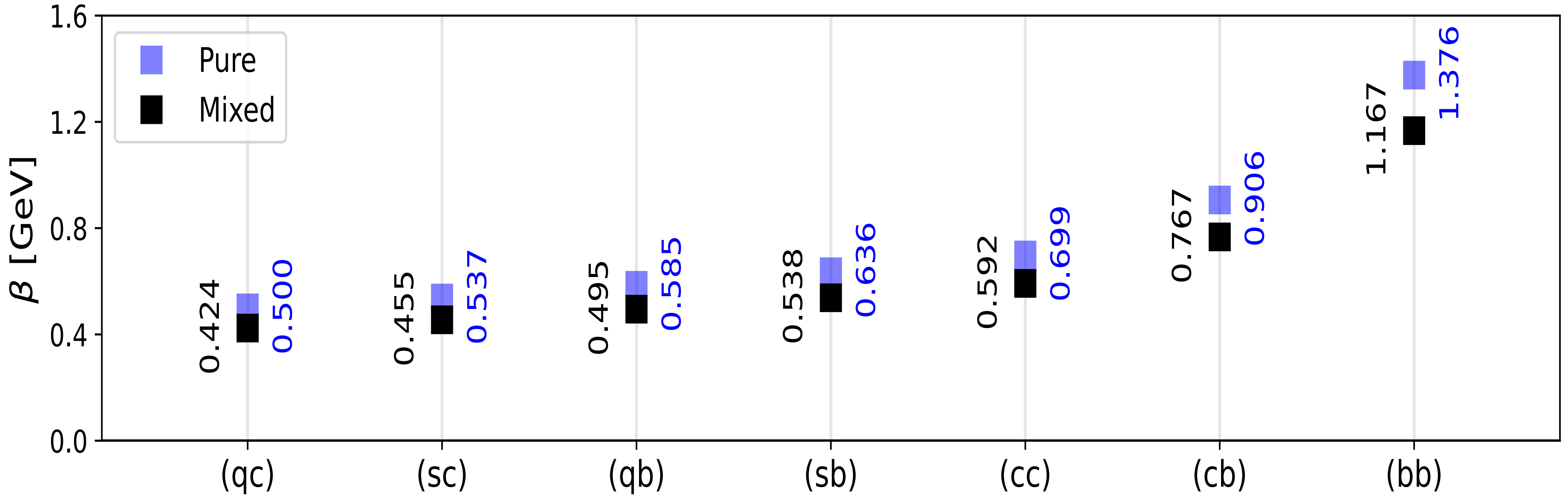


- HO basis functions

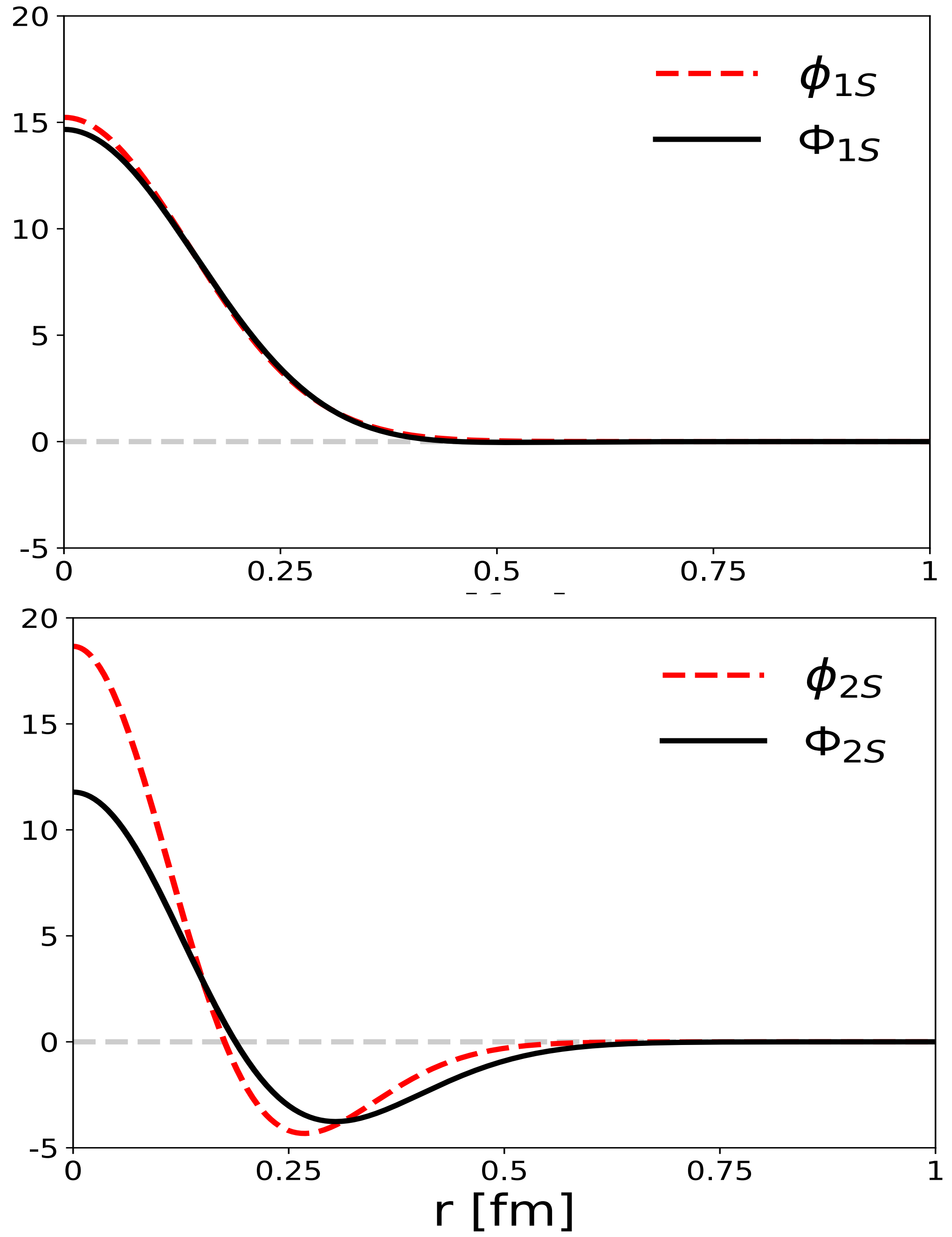
$$\Phi_{1S} = \phi_{1S}^{H0} \qquad \Phi_{2S} = \phi_{2S}^{H0}$$

$$\Phi_{1S} = \cos\theta \; \phi_{1S}^{H0} + \sin\theta \; \phi_{2S}^{H0}$$

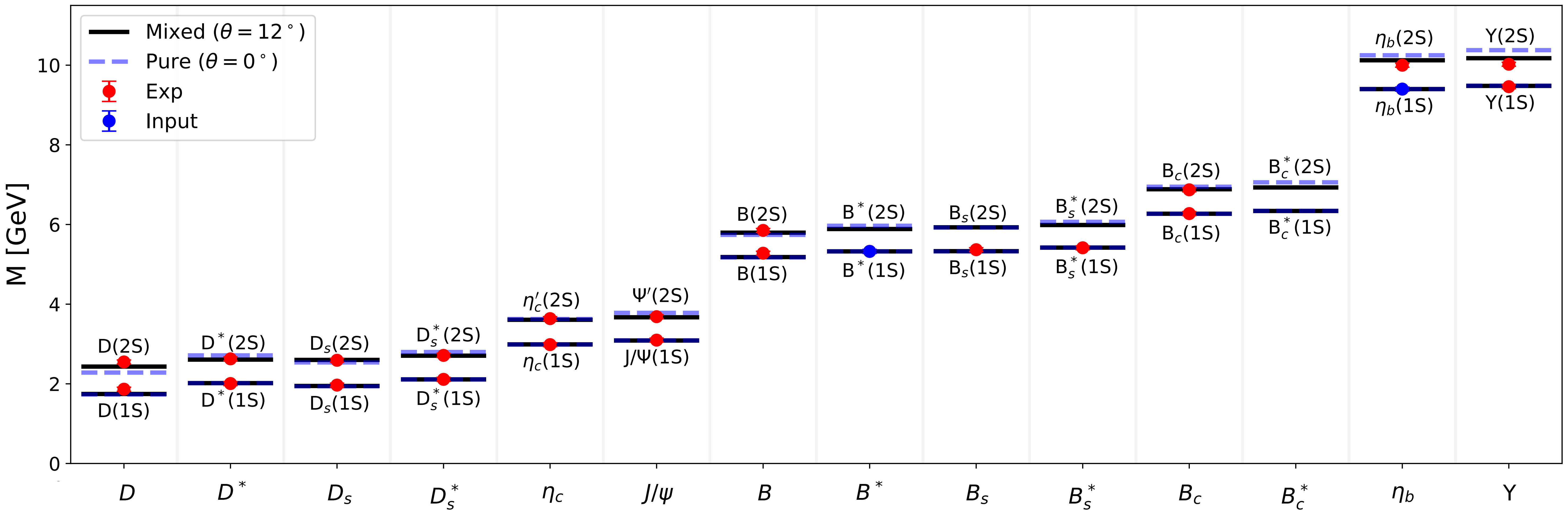
$$\Phi_{2S} = -\sin\theta \; \phi_{1S}^{H0} + \cos\theta \; \phi_{2S}^{H0}$$




- The universal $\theta$ parameter —> good approximation

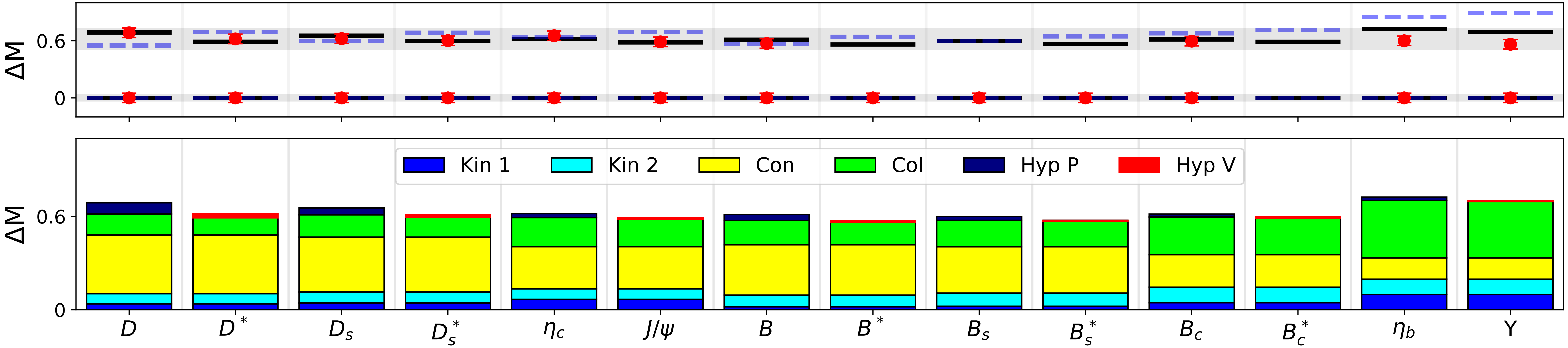



- Mass spectra $V_{q\bar{q}} = a + br - \dfrac{4\alpha_s}{3r} + \dfrac{2}{3}\dfrac{\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}}}{m_q m_{\bar{q}}} \nabla^2 V_{\text{Coul}}$



# Mass splitting $\Delta M = M_{2S} - M_{1S}$






<> Competing contribution:

—> Confinement int

$$\Delta M_{conf} \propto \frac{1}{\beta}$$

—> Coulomb-like int

$$\Delta M_{colmb} \propto \beta$$

<> Hyperfine int

—> Small, but very important

—> Mixing is needed

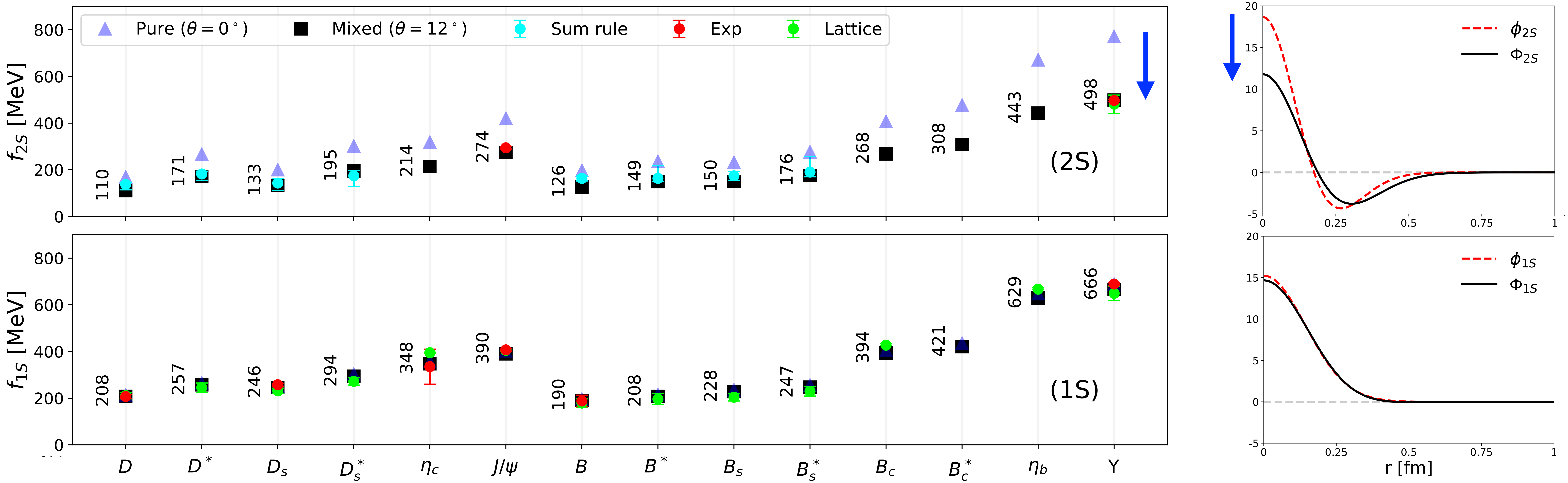
—> $\Delta M_P > \Delta M_V$

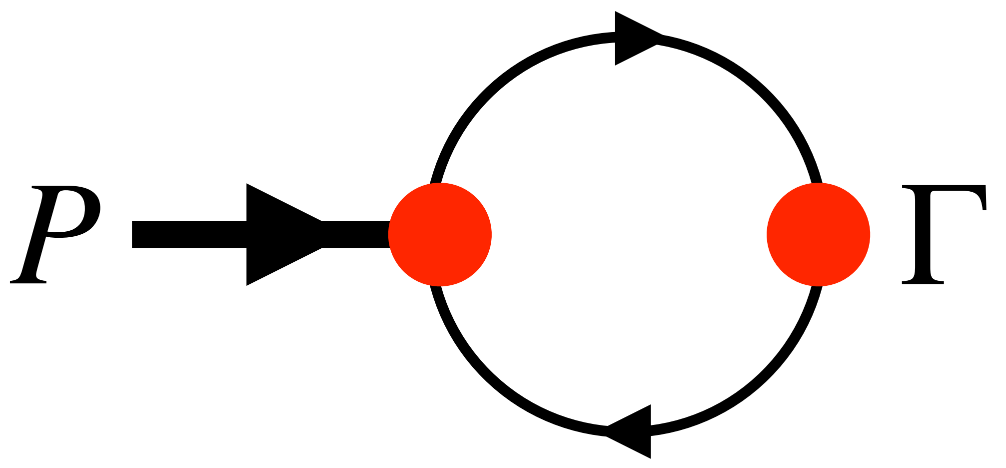
$$\Delta M_{hyp} \propto (S_q \cdot S_{\bar{q}})(\cos 2\theta - 2\sqrt{6}\sin 2\theta)$$

—> $\theta_c \approx 6^\circ$

# Weak decay constant





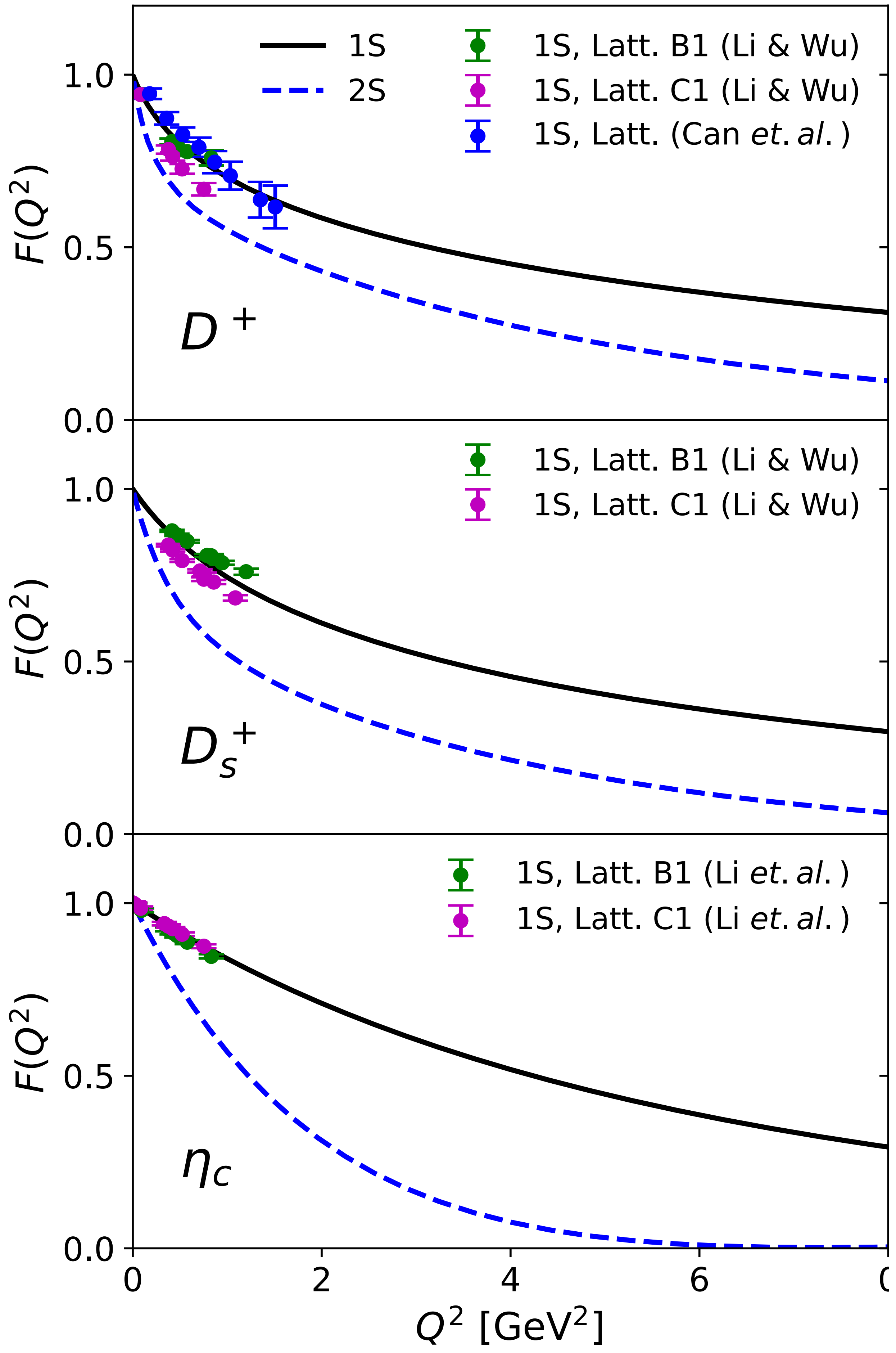



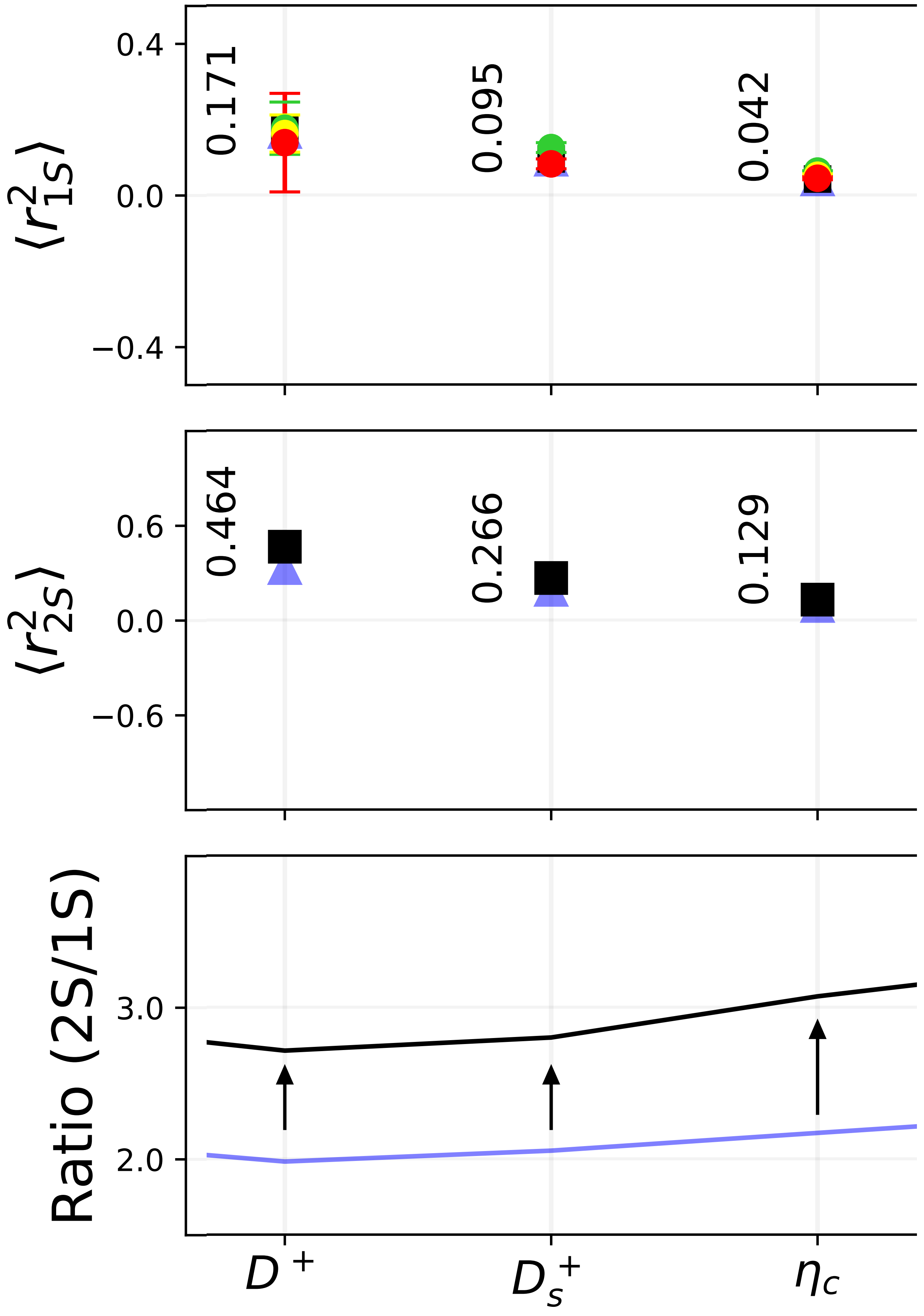

$$\langle 0|\,\bar{q}\gamma^\mu\gamma_5 q\,|\mathrm{P}(P)\rangle = i f_\mathrm{P} P^\mu,$$
$$\langle 0|\,\bar{q}\gamma^\mu q\,|\mathrm{V}(P,J_z)\rangle = f_\mathrm{V} M \epsilon^\mu(J_z),$$

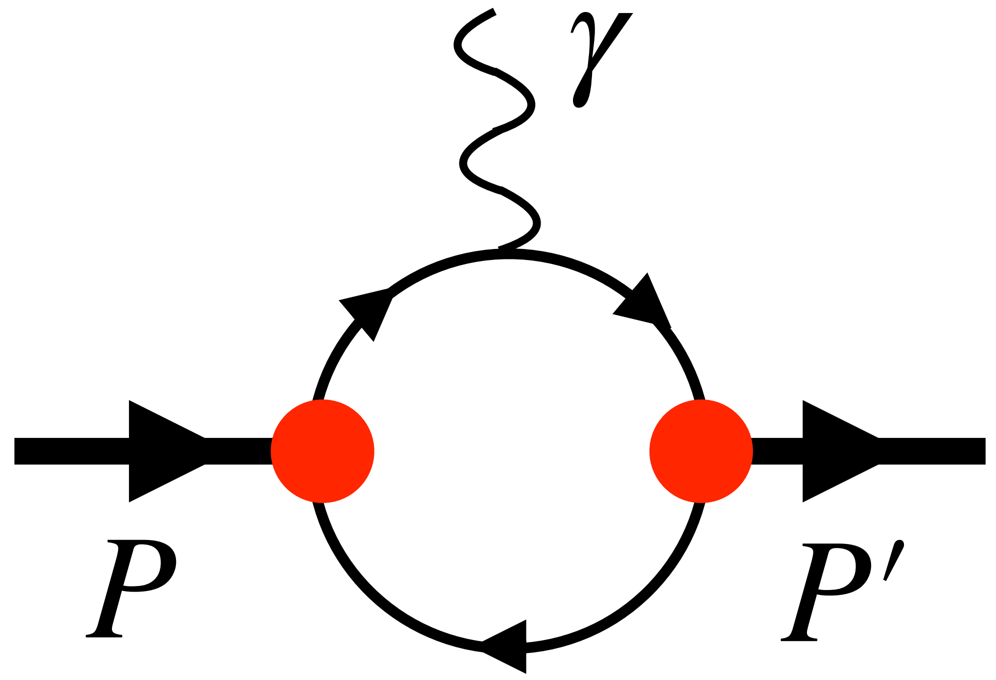
- Exp. data (2S State)
  => $V \to e^+e^-$
- Lattice data?
  –> only for $\Upsilon(2S)$

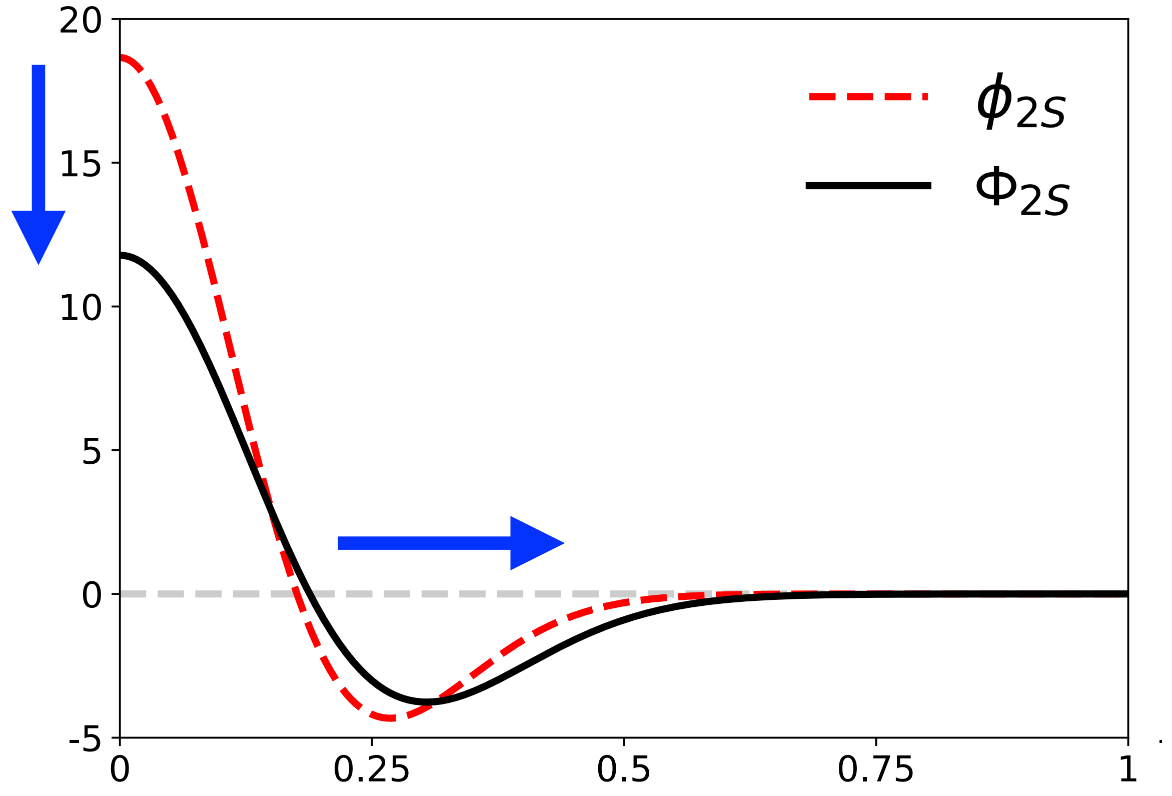
# Electromagnetic Form factor






- Radius
  —> Larger
- Lattice Data
  —> 2S state??

# Global fit: light and heavy mesons



$\sigma^b_{model} = 0.7\ \%$
$\theta = 12.9^\circ (5^\circ)$

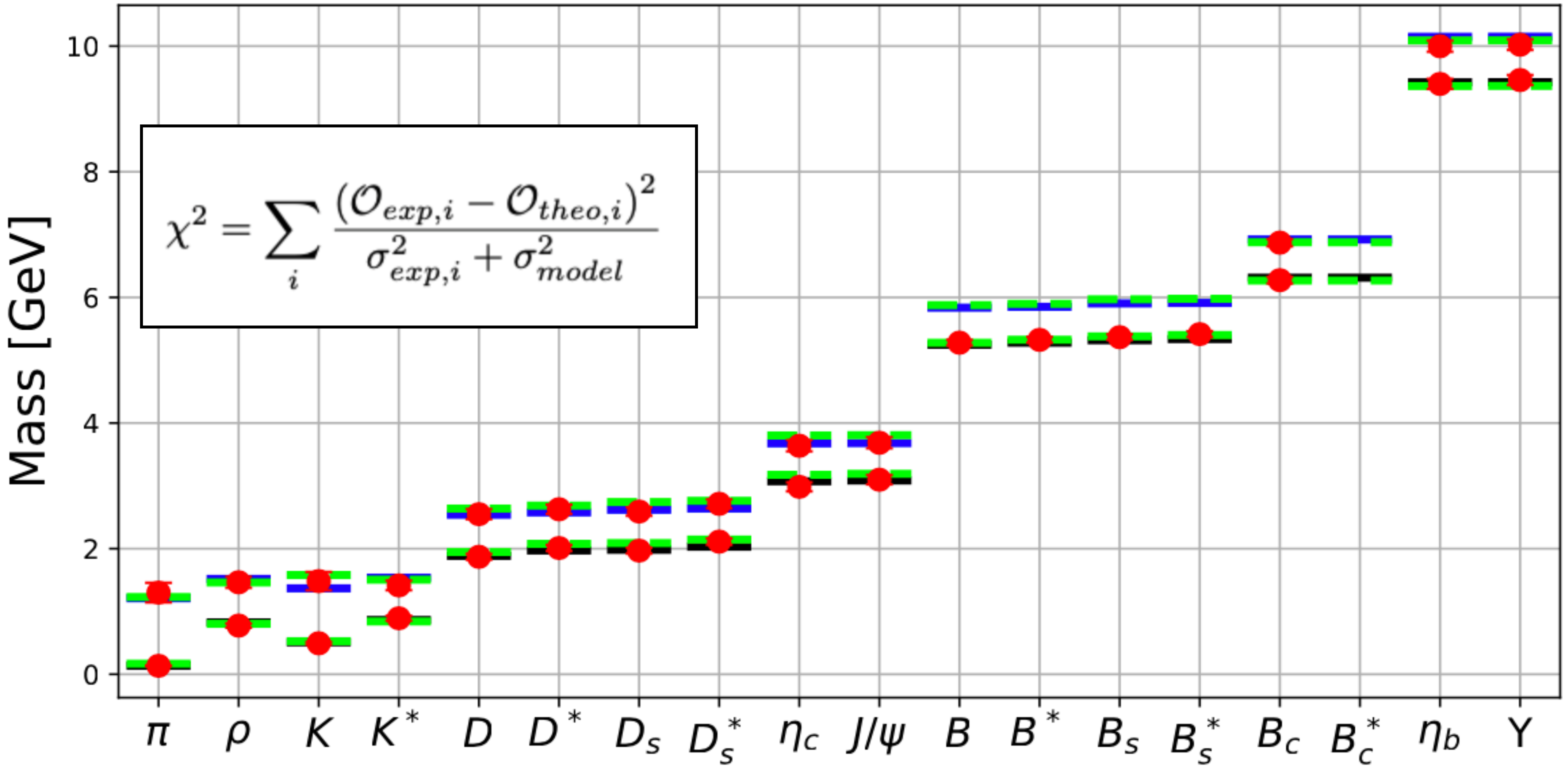
$\sigma^c_{model} = 2.0\ \%$
$\theta = 10^\circ (1^\circ)$

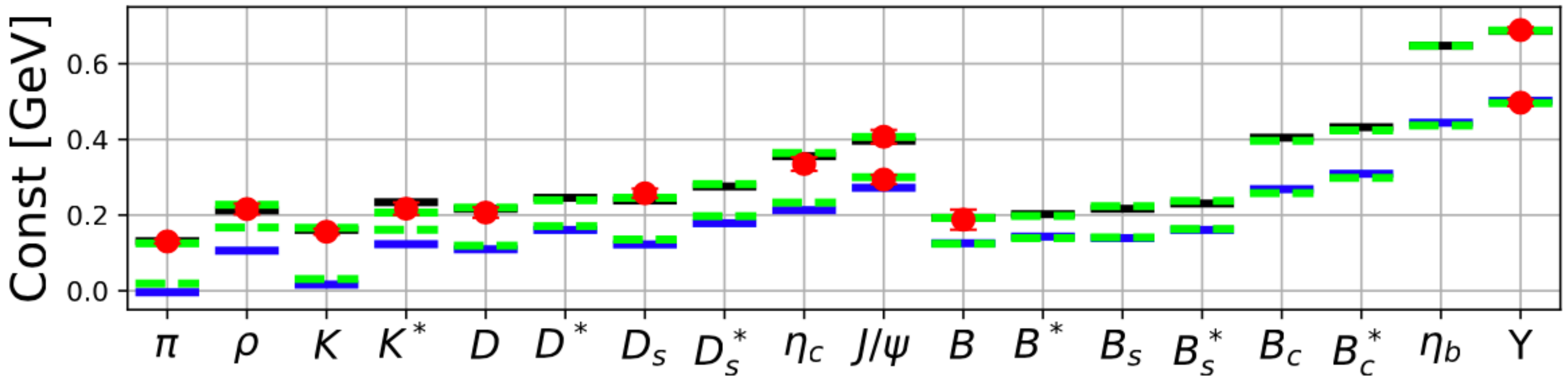
$\sigma^q_{model} = 3.6\ \%$
$\theta = 2^\circ (8^\circ)$

$\sigma_{model} = \lambda \sigma^i_{model}$

$\lambda = 1.2$

$\theta = 12.6^\circ (5^\circ)$




- Global fit
  —> iMinuit (python)
- Trial WF
  —> $\theta = 12.6^\circ (5^\circ)$
- Model error
  —> less than 5%
- LFWF
  —> other properties

# Bc & Heavy quarkonia: 3S states

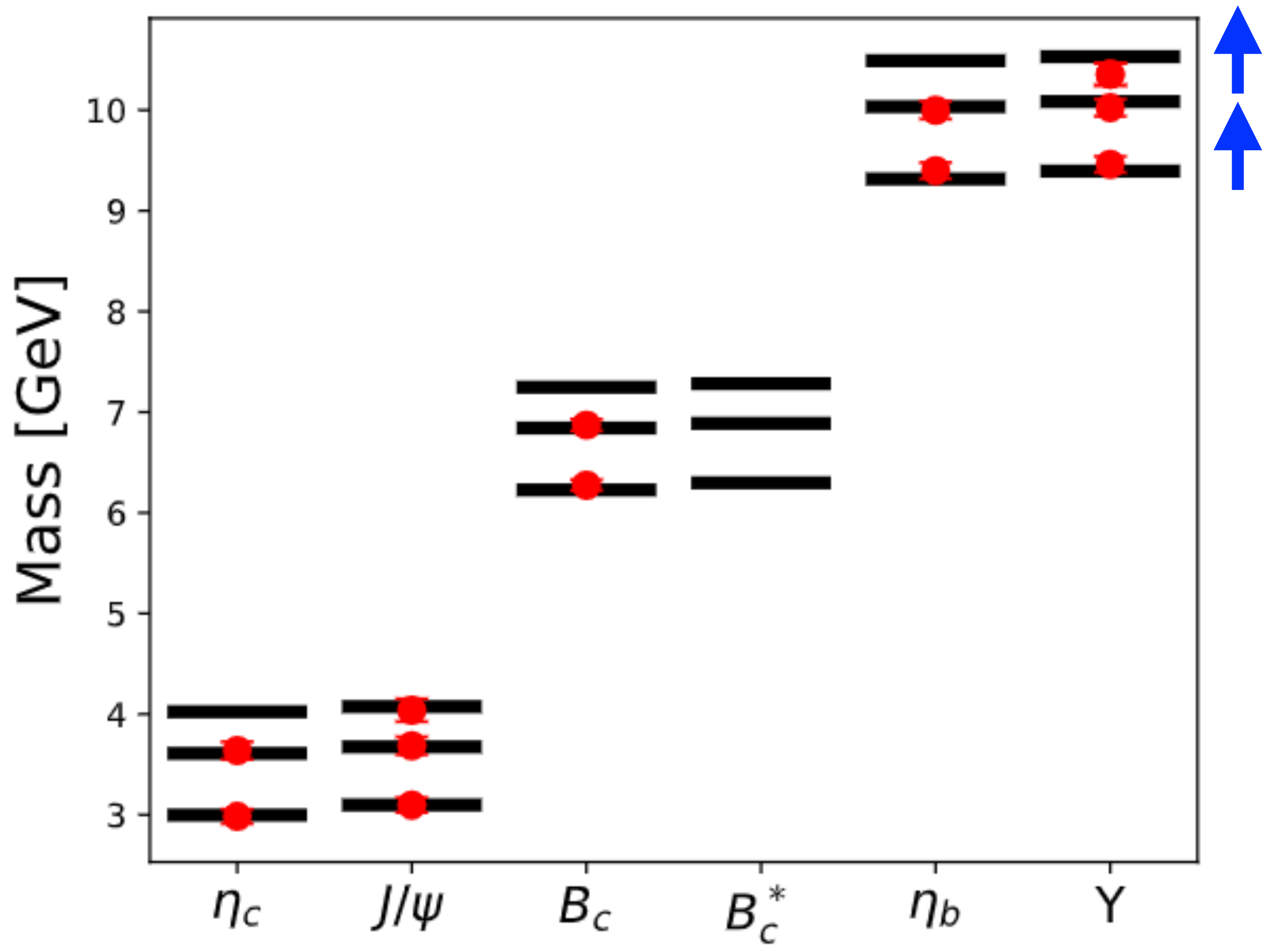


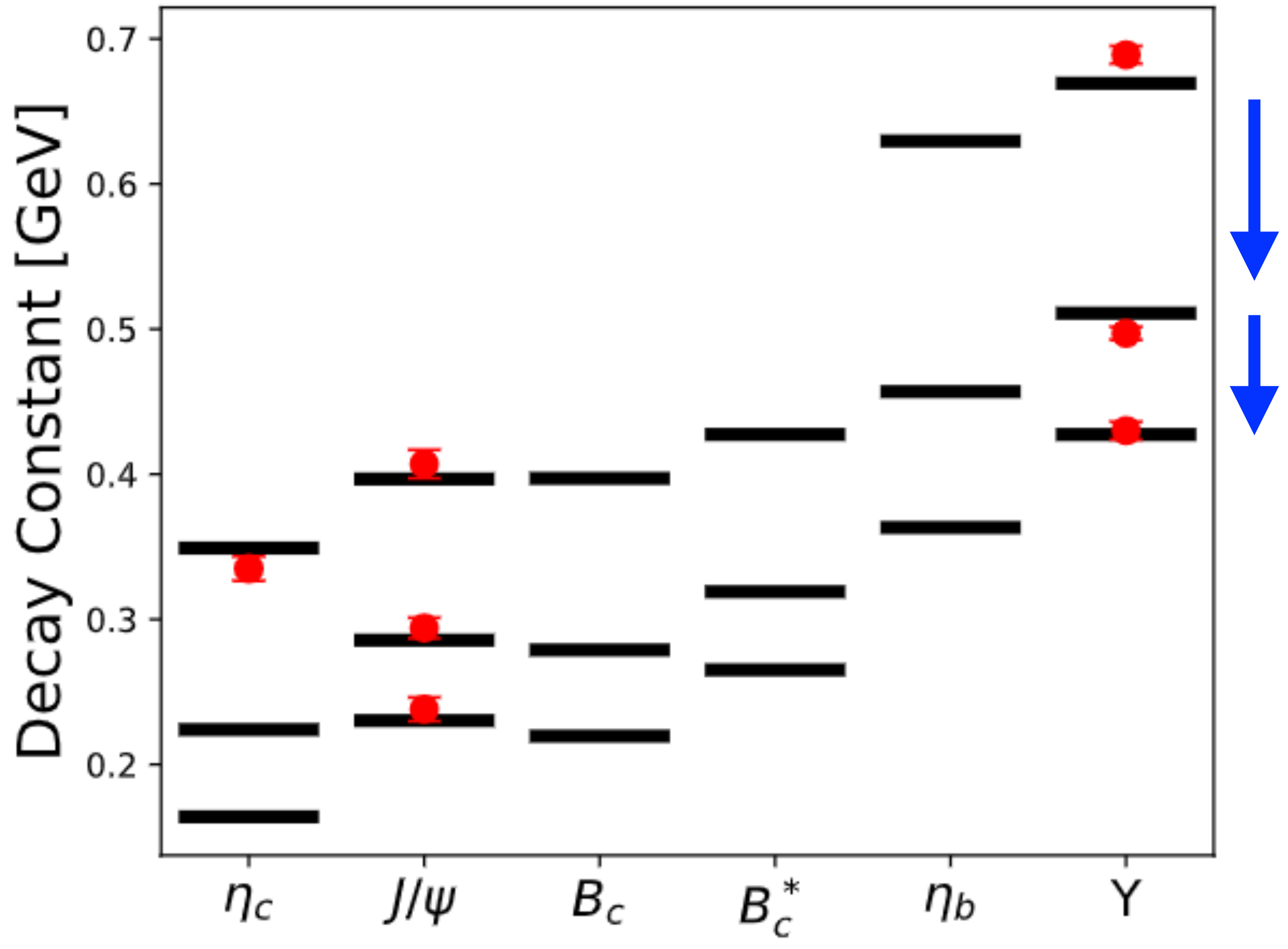
- Since there are some data
  —> Extension to the 3S states

- Following a similar trial WF
  —> Including 3S WF

$$R_2 = \begin{pmatrix} c_{12} & s_{12} \\ -s_{12} & c_{12} \end{pmatrix}.$$

$$R_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}$$




- Trial LFWF
  —> $\theta_{12} = 12^\circ$
  —> $\theta_{31} = \theta_{32} = 8^\circ$

- The splitting
  —> becomes smaller

- Transition
  —> E. m. decays

- Can be extended to
  —> 4S state, etc.

# Summary



- Several discoveries of **radial excitations** of hadrons.
  —> need to understand their
  internal structure

- Similar mass gap: baryon & mesons
  —> Competing behavior:
  confinement vs coulomb
  —> Role of hyperfine int.

- Wave function of radial excitation:
  —> HO basis expansion
  —> Huge impact to the prediction

- Future works of LFQM
  —> Gaussian expansion method
  —> Extension to baryon system
  —> Various transition process
  —> Self-consistent study

**Thank you!**

https://ajarifi.github.io