Equations of state for Neutron Stars, Supernovae and Neutron Star Mergers

Adriana R. Raduta

HADRON 2023, Genova, Italy, June 5th to 9th 2023

Executive Agency for Higher Education, Research, Development and Innovation Funding

PHAROS
THE MULTI-MESSENGER PHYSICS AND ASTROPHYSICS OF NEUTRON STARS

Overview

- Neutron Stars (NS): formation, properties, composition, observables
  - Equations of State (EoSs)
  - NS as laboratories for cold dense matter physics

- Core-collapse supernovae, binary neutron star mergers
  - temperature, density, charge fraction domains
  - Equations of State
  - Observables

Neutron stars

- NSs are residues of supernovas
  - ▷ NS are born hot $T \approx 10 - 100$ MeV $\approx 10^{11} - 10^{12}$ K $\qquad T_{M_\odot} \approx 1.57 \cdot 10^7$ K
  - ▷ $t(1\ \text{h}) \approx 10^9$ K ≈ 100 keV; cooling by ν and γ emission
- mass range: $1M_\odot \lesssim M \lesssim 2M_\odot$
  - ▷ $M_{\min}$, $M_{\max}$ inform on formation, EoS and composition
- radii $R \approx 10 - 15$ km

$R_{M_\odot} = 6.96 \cdot 10^5$ km

- average density $\approx 2 \cdot 10^{14}$ g/cm$^3 \approx \rho_0$ $\qquad \rho_{M_\odot} \approx 1.4$ g/cm^3
- highly non-uniform $0 \lesssim \rho \lesssim 5 - 10\rho_0$ $\qquad$ what are NS made of?
- compactness $0.1 \lesssim GM/c^2R \lesssim 0.35$ $\qquad C_{BH} = 0.5$
- surface gravity is $7 \cdot 10^{12}$ m/s^2 $\qquad g_{Earth} = 9.8$ m/s^2
- fast spinning: $\nu = 716$ Hz (PSR J1748-2446)
- huge magnetic fields: $B = 10^{15}$ G $\qquad B_{Earth;core} = 25$ G, $B_{RMN} = 10^5$ G

NS are labs for dense matter, General Relativity, physics of magnetic fields ...

Structure and Composition

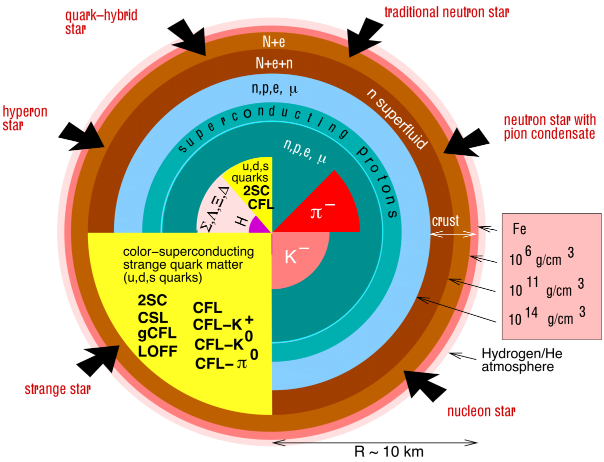


Image credit: F. Weber

CRUST

inhomogeneous (crystal)

nuclei, neutrons, electrons

uncertainties: inner crust, due to $E_{\rm sym}(n)$

CORE - This talk

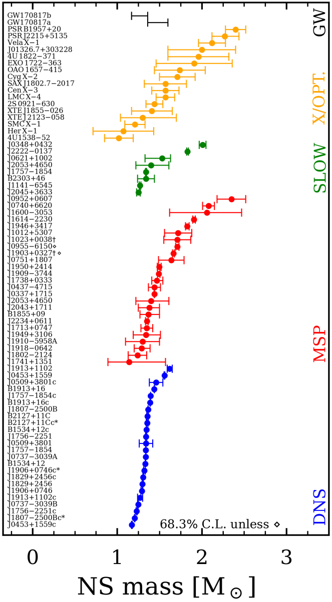
homogeneous struct.

uncertain composition: nucleons, hyperons, pions, quarks, electrons, muons, due to $E(n, Y_e)$

Equation of State $P(e)$

key for structure

Observables: Masses



[Suleiman et al., PRC104, 015801]

DNS: binaries with two neutron stars,

MSP: millisecond pulsars with $f > 50$ Hz ,

SLOW: slowly rotating pulsars with $f < 50$ Hz ,

X/OPT measurement via X-ray or optical obs. ,

GW: measurements using detection of GW

Massive NS

PSR J1614-2230 ($M = 1.908 \pm 0.016 \mathrm{M}_\odot$) [Demorest+, 2010; Arzoumanian+, 2018]; PSR J0348+0432 ($M = 2.01 \pm 0.04 \mathrm{M}_\odot$) [Antoniadis+, 2013]; MSP J0740+6620 ($M = 2.08^{+0.07}_{-0.07} \mathrm{M}_\odot$) [Fonseca+, 2021]; PSR J1810+1744 ($M = 2.13 \pm 0.04 \mathrm{M}_\odot$ [Romani+, 2021]

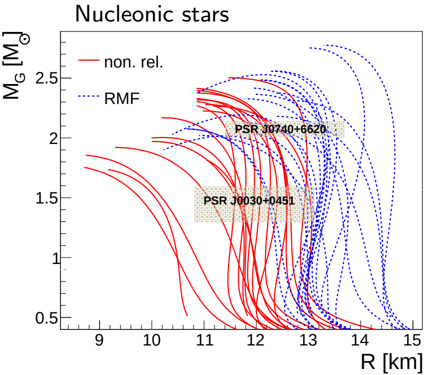
Relevant for the composition of the core

Observables: Radii

Two measurements:

▷ **PSR J0030+0451** by NICER
$R(1.44^{0.15}_{-0.14}\mathrm{M}_\odot) = 13.02^{+1.24}_{-1.06}$ km [Miller+, 2019]
$R(1.34^{+0.15}_{-0.16}\mathrm{M}_\odot) = 12.71^{+1.14}_{-1.19}$ km [Riley+, 2019]

▷ **J0740+6620** by NICER+XMM Newton
$R(2.08 \pm 0.07\mathrm{M}_\odot) = 13.7^{+2.6}_{-1.5}$ km [Miller+, 2021
$R(2.072^{+0.067}_{-0.066}\mathrm{M}_\odot) = 12.39^{+1.30}_{-0.98}$ km [Riley+, 2021



• uncertainties still large but enough to rule out a number of EoS

• more measurements from NICER and future LOFT, Athena missions

Data from CompOSE,
`https://compose.obspm.fr/`

Observables: Tidal deformabilities

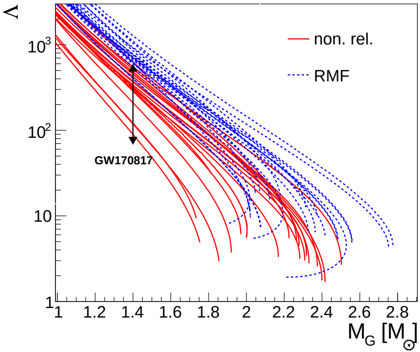
▷ the tidal deformability, Λ, describes how much a body is deformed by tidal forces, which arise when two massive bodies orbit each other;

▷ GW170817 - detection of GW emitted by the merging of two NS with $M_T = 2.73^{+0.04}_{-0.01}\ \mathrm{M}_\odot$ and $0.72 \leq q = M_2/M_1 \leq 1$

▷ tidal deformability $70 < \Lambda_{1.4} \leq 580$ [Abbott+, PRL 2018], constraint on the NS EoS over $2n_{\rm sat} \lesssim n \lesssim 3n_{\rm sat}$

• enough to rule out a number of realistic EoS

Nucleonic stars



Data from CompOSE,
https://compose.obspm.fr/

Equations of state: Cold nuclear matter

$E/A(n,\delta)$ is Taylor expanded in terms of deviation from isospin asymmetry, $\delta = (n_n - n_p)/n$, and saturation density, $\chi = (n - n_{\rm sat})/3n_{\rm sat}$, with $n = n_n + n_p$.

$$
\begin{aligned}
E/A(n,\delta) &= E/A(n,0) + S(n)\,\delta^2 + \ldots \\
&= \underbrace{\sum_{i\geq 0} \frac{1}{i!} X_{\rm sat}^{(i)} \chi^i}_{\text{energy SNM}} + \underbrace{\sum_{j\geq 0} \frac{1}{j!} X_{\rm sym}^{(j)} \chi^j \delta^2}_{\text{symmetry energy}} + \ldots
\end{aligned}
$$

$$
X_{\rm sat}^{(i)} = 3^i n_{\rm sat}^i \left(\frac{\partial^i (E/A)}{\partial n^i}\right)_{n=n_{\rm sat},\delta=0} ; \quad X_{\rm sym}^{(j)} = 3^j n_{\rm sat}^j \left(\frac{\partial^j S(n)}{\partial n^j}\right)_{n=n_{\rm sat},\delta=0}
$$

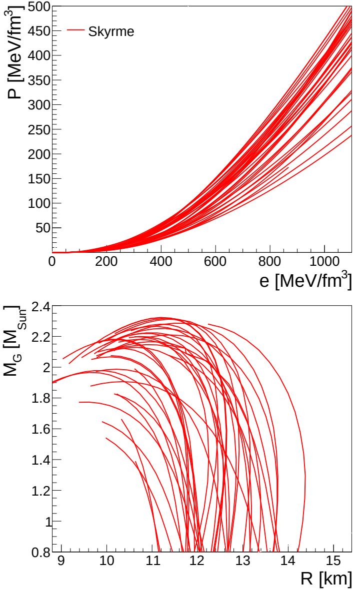
i=0, 2, ... binding energy per nucleon $E_{\rm sat}$, incompressibility $K_{\rm sat}$, etc. at $n_{\rm sat}$
j=0, 1, 2, ... symmetry energy $J_{\rm sym}$ and its slope $L_{\rm sym}$, curvature $K_{\rm sym}$, etc. at $n_{\rm sat}$

▷ EoS exist for phenomenological and microscopic models

▷ **large uncertainties away from** $(n_{\rm sat}, \delta \approx 0)$

How to build a Neutron Star? - I. Nucleonic stars

- a model of eff. interaction is needed
- for any n_B, composition is determined by solving for
  $n_B = \sum_{i \in B} n_i, \qquad \sum_{i \in B} n_i + \sum_{\alpha \in L} n_\alpha = 0,$
  $\mu_n = \mu_p + \mu_e$
- one obtains $n_i(n_B)$, $e(n_B)$, $P(n_B)$
  $\rightarrow P(e)$ equation of state
- $P(e)$ enters the hydrostatic eqs.
  NS structure and composition
- **mapping between $P(e)$ and $M - R$**
- uncertainties in potentials $\rightarrow$ uncertainties in $P(e)$, properties of NS
- dominated by $E_{\rm sym}(n)$
- correlations among properties of NS and NM



Exotic particles: Why? Which? How?

- Why? To minimize the energy.
- Which?
  - heavy baryons: Λ, $\Sigma^{-,0,+}$, $\Xi^{-,0}$ hyperons, $\Delta(1232)$-resonances [Glendenning, PLB, 1982; Sedrakian+, PPNP (2023)]
  - mesons: π, K [Glendenning, PLB, 1982]
  - d^* hexaquark [Mantziris+, A&A, 2020]
  - other? please, suggest!
  - onset depends on interactions, e.g., NY, $N\Delta$, YY, $N\pi$
- scarce experimental info:
  - few hundreds scattering events for $N\Lambda$ and $N\Sigma$;
  - spectroscopic data of single- and double-hypernuclei;
  - pion-nucleus scattering and pion photo-production, electron scattering on nuclei and electromagnetic excitations
- $U_{\Lambda}^{(N)} \approx -28\mathrm{MeV}$, $U_{\Xi}^{(N)} \approx -18\mathrm{MeV}$ $U_{\Sigma}^{(N)} \approx 30\mathrm{MeV}$, [Millener et al., 1998]
- $-30\ \mathrm{MeV} + U_N^{(N)} \leq U_{\Delta}^{(N)} \leq U_N^{(N)}$ [Drago+, 2014; Kolomeitsev+, 2017]
- onset densities $n \approx 2-3n_{\mathrm{sat}}$
- not every species is present

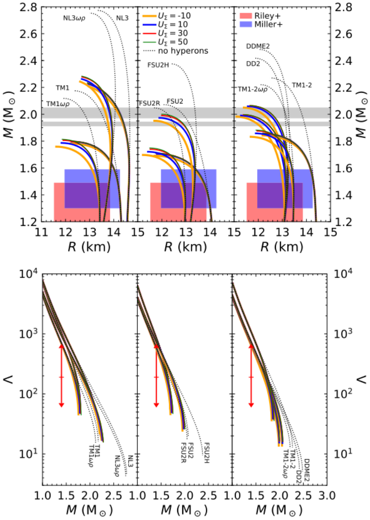
How to build a Neutron Star? - II. Stars with exotic cores

- a model of eff. interaction
- the hadronic blend: $N\Lambda$, NY, $NY\Delta$, $NY\pi$, NK; add leptons (e^-, μ^-)
- tune the coupling constants to get $U_Y^{(N)}$, $U_\Delta^{(N)}$, etc.;
  use flavor sym. group arguments,
- for any n_B composition is determined by solving for $n_B = \sum_{i \in B} n_i$,
  $$\sum_{i \in B} n_i + \sum_{\alpha \in L} n_\alpha = 0,$$
  $$\mu_i = Q_B \mu_B + Q_Q \mu_Q + Q_S \mu_s,$$
  $$\mu_\alpha = Q_Q \mu_Q + Q_L \mu_L,$$
  $$[\mu_S = 0;\ \mu_L = 0 \text{ or } \mu_L \neq 0]$$
- "switch-on" particles with $\mu_i > m_i c^2$, $\mu_\alpha > m_\alpha c^2$
- one gets $n_i(n_B)$, $n_L(n_B)$, $P(n_B)$, $e(n_B)$, etc.

  $\rightarrow P(e)$ equation of state
- solve Tolman-Oppenheimer-Volkoff (TOV) eqs.; NS structure and composition
- how $M - R$ gets modified? tidal deformabilities? moments of inertia?

NS with exotic cores: Structure

- exotic species soften the EoS
  - exotic NS have lower M_{max}
  - exotic NS have smaller R, Λ
- best studied case: onset of hyperons
- onset of Δs, π, K also studied; various blends;
- agreement with all astrophys. observations $M_{max} \gtrsim 2\mathrm{M}_\odot$; joint mass and radii from NICER; tidal deformability from GW170817
- none species is confirmed nor ruled out
- degeneracy in $P(e)$

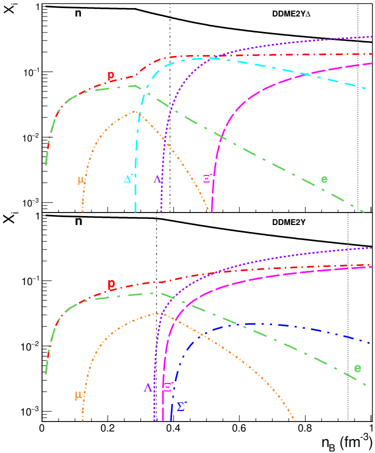


Hyperon admixed NS

[Fortin+, PRD (2020)]

NS with exotic cores: Composition

- onset and abundances are decided upon rest mass, interaction potential, charge
- uncertainties in NN, NY, $N\Delta$ $\rightarrow$ uncertainties in composition, especially at high n_B
- not every "allowed" species is present
  - hyperonic NS: only Λ, Ξ^- and Σ^-
    $n_\Lambda \approx 2n_{\mathrm{sat}}$, $M_\Lambda \approx 1.5\mathrm{M}_\odot$
    $n_{\Xi^-} \approx 2.5n_{\mathrm{sat}}$, $M_{\Xi^-} \approx 1.6-1.8\mathrm{M}_\odot$
  - Δ-admixed hyperonic NS: only Δ^-, Λ, Ξ^-
    $n_{\Delta^-} \approx 1.7n_{\mathrm{sat}}$, $M_{\Delta^-} \approx 1\mathrm{M}_\odot$
    $n_\Lambda \approx 2n_{\mathrm{sat}}$, $M_\Lambda \approx 1.3\mathrm{M}_\odot$



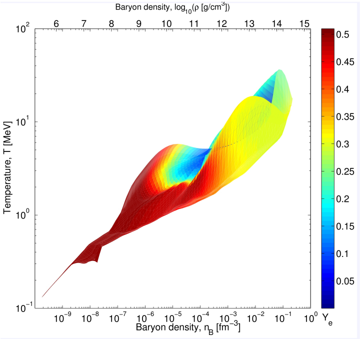
[Raduta+, MNRAS (2020)]

Hot astrophysical environments

- in core-collapse supernovae, proto-neutron stars, binary NS mergers wide ranges of baryonic densities $[10^{-10} \leq n_B \leq 1 - 10 \text{ fm}^{-3}]$, temperature $[0 \leq T \leq 100 \text{ MeV}]$, charge fraction $[0 \leq Y_q \leq 0.6]$ are populated

- numerical simulations require EoS tables; thermodyn. and composition are stored in 2D tables

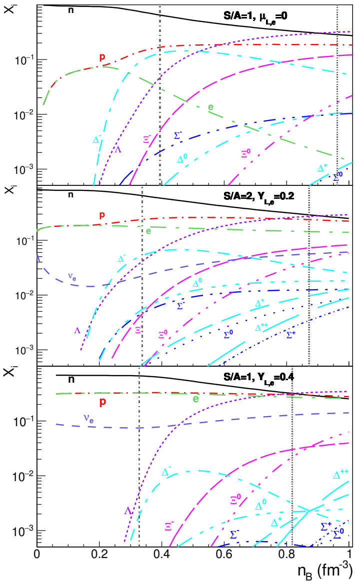
[Pons+, ApJ 667, 282; Janka+, Phys Rep 442, 38; Fischer+, AA 499, 1; Shibata+, Living Rev. Rel.14, 6; O'Connor+, ApJ 730, 70; Hempel+, ApJ 48, 70; Mezzacappa+, 1507.05680; Rosswog, Int J Mod Phys D24, 1530012; Baiotti+, Rep Prog Phys 80, 096901; O'Connor+, ApJ 865, 81; Burrows+, MNRAS 491, 2715; Ruiz+, PRD101, 064042; Janka, Ann Rev Nucl Part Phys 62, 407; Bauswein+, PRD86, 063001; Koppel+, ApJ872, L16; Bauswein+, PRL125]



Fischer et al., Eur. Phys. J A50 (2014)

[Fischer+, EPJA (2014)]

Heavy baryons in hot and dense matter



[Raduta+, MNRAS (2020)]

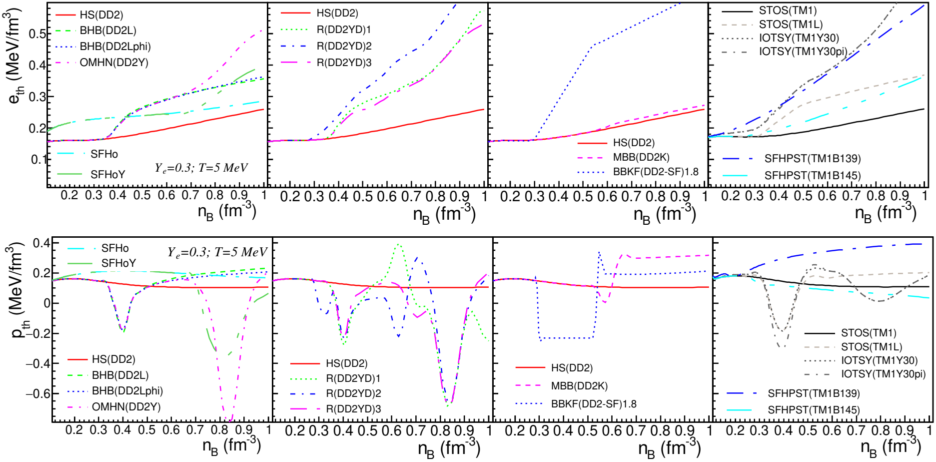
- thermal excitation of new d.o.f.
- ν_e trapping modifies the composition
- high T: hyperons and Δs appear at $n_B < n_{sat}$
- high T favor exotic species
- Λ and Δ^- dominate

- thermodyn. potentials, microscopic quantities will depend on T, $Y_{p/L}$, particle d.o.f. and nucleonic EOS

- effects on properties and stability of hot stars

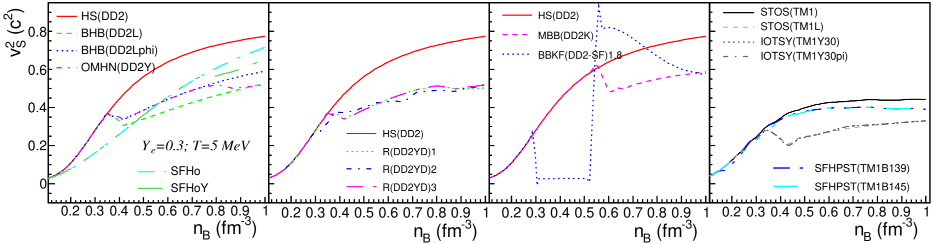
Thermal properties

Thermal contrib.: $X_{th} = X(n_B, Y_e, T) - X(n_B, Y_e, 0)$



- the larger the number of particle d.o.f. the larger $e_{\rm th}$; [Raduta, EPJA (2022)]
- nucleation of exotic d.o.f. diminishes $p_{\rm th}$; under specific conditions $p_{\rm th} < 0$

Speed of sound: $c_S^2 = dP/de|_{S,N_b,N_q}$



[Raduta, EPJA (2022)]

$$c_S^2 = 1/hk_S = p\gamma/e;\ k_S = -1/V \cdot dV/dp|_{S,N_b,N_q}; \quad \gamma = \partial \ln p/\partial \ln e|_S$$

- strong n_B- and EOS- dependence;
- for Gibbs treatment of phase coex., $c_S^2 = 0$
- heavy baryons, mesons: c_S^2 decreases over a narrow n_B domain
- transition to quarks: c_S^2 decreases over large n_B domain
- signatures of exotica are seen in numerical simulations

CompOSE

online repository for EOS (https://compose.obspm.fr/)

- stores thermodyn., composition, microscopic, transport properties in standardized format
- tabulation with respect to temperature (T), charge fraction (Y_Q), part. number density (n_B)
- wide ranges: $0.1 \leq T$ [MeV] ≤ 100; $0.01 \leq Y_Q \leq 0.6$; $10^{-10} \leq n_B$ [fm^{-3}] $\leq 1-2$
- fine mesh; allows interpolation
- various types of EOS: cold neutron stars; neutron matter; "general purpose", ready for input in simulations; from microscopic, phenomenological, schematic models; various particle d.o.f.

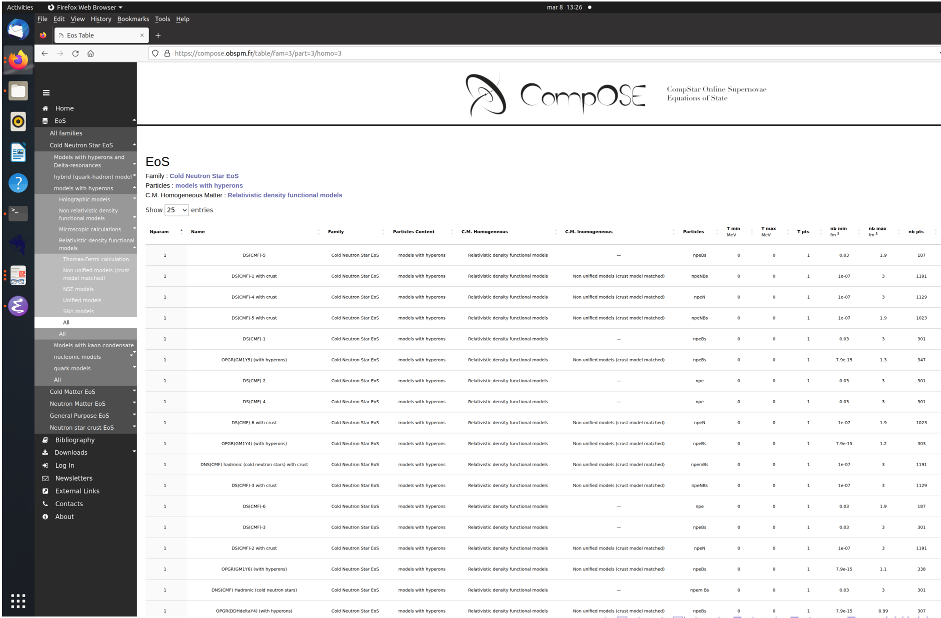
provides tools

- to sort by type; approach; particle composition; prop. of NM; group of authors
- to compute thermodyn. quantities, thermal coefficients,
- to extract information for arbitrary thermodyn. conditions,

modular; constantly upgrading

[Typel, Oertel, Klaehn, Phys.Part.Nucl. (2015); Typel et al., Eur.Phys.J.A (2022)]

CompOSE



CompStar Online Supernovae Equations of State

https://compose.obspm.fr/table/fam=3/part=3/homo=3

EoS

Family : Cold Neutron Star EoS
Particles : models with hyperons
C.M. Homogeneous Matter : Relativistic density functional models

Show 25 entries

Nparam	Name	Family	Particles Content	C.M. Homogeneous	C.M. Inomogeneous	Particles	T min MeV	T max MeV	T pts	nb min fm^{-3}	nb max fm^{-3}	nb pts
1	DS(CMF)-5	Cold Neutron Star EoS	models with hyperons	Relativistic density functional models	—	npeBs	0	0	1	0.03	1.9	187
1	DS(CMF)-1 with crust	Cold Neutron Star EoS	models with hyperons	Relativistic density functional models	Non unified models (crust model matched)	npeNBs	0	0	1	1e-07	3	1191
1	DS(CMF)-4 with crust	Cold Neutron Star EoS	models with hyperons	Relativistic density functional models	Non unified models (crust model matched)	npeN	0	0	1	1e-07	3	1129
1	DS(CMF)-5 with crust	Cold Neutron Star EoS	models with hyperons	Relativistic density functional models	Non unified models (crust model matched)	npeNBs	0	0	1	1e-07	1.9	1023
1	DS(CMF)-1	Cold Neutron Star EoS	models with hyperons	Relativistic density functional models	—	npeBs	0	0	1	0.03	3	301
1	OPGR(GM1Y5) (with hyperons)	Cold Neutron Star EoS	models with hyperons	Relativistic density functional models	Non unified models (crust model matched)	npeBs	0	0	1	7.9e-15	1.3	347
1	DS(CMF)-2	Cold Neutron Star EoS	models with hyperons	Relativistic density functional models	—	npe	0	0	1	0.03	3	301
1	DS(CMF)-4	Cold Neutron Star EoS	models with hyperons	Relativistic density functional models	—	npe	0	0	1	0.03	3	301
1	DS(CMF)-6 with crust	Cold Neutron Star EoS	models with hyperons	Relativistic density functional models	Non unified models (crust model matched)	npeN	0	0	1	1e-07	1.9	1023
1	OPGR(GM1Y4) (with hyperons)	Cold Neutron Star EoS	models with hyperons	Relativistic density functional models	Non unified models (crust model matched)	npeBs	0	0	1	7.9e-15	1.2	303
1	DNS(CMF) hadronic (cold neutron stars) with crust	Cold Neutron Star EoS	models with hyperons	Relativistic density functional models	Non unified models (crust model matched)	npemBs	0	0	1	1e-07	3	1191
1	DS(CMF)-3 with crust	Cold Neutron Star EoS	models with hyperons	Relativistic density functional models	Non unified models (crust model matched)	npeNBs	0	0	1	1e-07	3	1129
1	DS(CMF)-6	Cold Neutron Star EoS	models with hyperons	Relativistic density functional models	—	npe	0	0	1	0.03	1.9	187
1	DS(CMF)-3	Cold Neutron Star EoS	models with hyperons	Relativistic density functional models	—	npeBs	0	0	1	0.03	3	301
1	DS(CMF)-2 with crust	Cold Neutron Star EoS	models with hyperons	Relativistic density functional models	Non unified models (crust model matched)	npeN	0	0	1	1e-07	3	1191
1	OPGR(GM1Y6) (with hyperons)	Cold Neutron Star EoS	models with hyperons	Relativistic density functional models	Non unified models (crust model matched)	npeBs	0	0	1	7.9e-15	1.1	338
1	DNS(CMF) Hadronic (cold neutron stars)	Cold Neutron Star EoS	models with hyperons	Relativistic density functional models	—	npem Bs	0	0	1	0.03	3	301
1	OPGR(DDHdeltaY4) (with hyperons)	Cold Neutron Star EoS	models with hyperons	Relativistic density functional models	Non unified models (crust model matched)	npeBs	0	0	1	7.9e-15	0.99	307

Conclusions

- Neutron Stars (NS): formation, properties, composition, observables
  - Equations of State (EoSs)
  - NS as laboratories for cold dense matter physics

- Core-collapse supernovae, binary neutron star mergers
  - temperature, density, charge fraction domains
  - Equations of State
  - Observables

Heavy baryons

Baryon	B	Q	S	I_3	J^Π	rest mass (MeV)		mean life (s)
n	1	0	0	1/2	$1/2^+$	939.565	udd	879.4(6)
p	1	1	0	1/2	$1/2^+$	938.272	uud	$> 3.6 \cdot 10^{29}$ years
Λ	1	0	-1	0	$1/2^+$	1115.683	uds	$2.60 \cdot 10^{-10}$
Σ^+	1	1	-1	-1	$1/2^+$	1189.37	uus	$8.02 \cdot 10^{-11}$
Σ^0	1	0	-1	0	$1/2^+$	1192.642	uds	$7.4 \cdot 10^{-20}$
Σ^-	1	-1	-1	1	$1/2^+$	1197.449	dds	$1.48 \cdot 10^{-10}$
Ξ^0	1	0	-2	-1/2	$1/2^+$	1314.83	uss	$2.90 \cdot 10^{-10}$
Ξ^-	1	-1	-2	1/2	$1/2^+$	1321.31	dss	$1.64 \cdot 10^{-10}$
Δ^{++}	1	2	0	-3/2	$3/2^+$	1232	uuu	$5.63 \cdot 10^{-24}$
Δ^+	1	1	0	-1/2	$3/2^+$	1232	uud	$5.63 \cdot 10^{-24}$
Δ^0	1	0	0	1/2	$3/2^+$	1232	udd	$5.63 \cdot 10^{-24}$
Δ^-	1	-1	0	3/2	$3/2^+$	1232	ddd	$5.63 \cdot 10^{-24}$

Phenomenological RMF (I)

$$
\begin{aligned}
\mathcal{L} \;=\; & \sum_{j\in\mathcal{B}} \bar{\psi}_j \left(i\gamma_\mu \partial^\mu - m_j + g_{\sigma j}\sigma + g_{\sigma^* j}\sigma^* \right. \\
& \left. + g_{\delta j}\vec{\delta}\cdot\vec{I}_j - g_{\omega j}\gamma_\mu\omega^\mu - g_{\phi j}\gamma_\mu\phi^\mu - g_{\rho j}\gamma_\mu\vec{\rho}^\mu\cdot\vec{I}_j \right) \psi_j \\
& + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 \\
& + \frac{1}{2}(\partial_\mu\sigma^*\partial^\mu\sigma^* - m_{\sigma^*}^2\sigma^{*2}) + \frac{1}{2}(\partial_\mu\vec{\delta}\partial^\mu\vec{\delta} - m_\delta^2\vec{\delta}^2) \\
& - \frac{1}{4}W_{\mu\nu}^\dagger W^{\mu\nu} - \frac{1}{4}P_{\mu\nu}^\dagger P^{\mu\nu} - \frac{1}{4}\vec{R}_{\mu\nu}^\dagger\vec{R}^{\mu\nu} \\
& + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}c_3(\omega_\mu\omega^\mu)^2 + \frac{1}{2}m_\phi^2\phi_\mu\phi^\mu + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu \;,
\end{aligned}
$$

where ψ_j denotes the field of baryon j, and $W_{\mu\nu}, P_{\mu\nu}, \vec{R}_{\mu\nu}$ are the vector meson field tensors of the form

$$V^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu \;.$$

Phenomenological RMF (II)

In the mean field approximation, the meson fields are replaced by their respective mean-field expectation values, which are given in uniform matter as

$$m_\sigma^2 \bar\sigma \quad + \quad g_2 \bar\sigma^2 + g_3 \bar\sigma^3 = \sum_{i \in B} g_{\sigma i} n_i^s; \quad m_{\sigma^*}^2 \bar\sigma^* = \sum_{i \in B} g_{\sigma^* i} n_i^s; \quad m_\delta^2 \bar\delta = \sum_{i \in B} g_{\delta i} t_{3i} n_i^s$$

$$m_\omega^2 \bar\omega \quad + \quad c_3 \bar\omega^3 = \sum_{i \in B} g_{\omega i} n_i; \quad m_\phi^2 \bar\phi = \sum_{i \in B} g_{\phi i} n_i; \quad m_\rho^2 \bar\rho = \sum_{i \in B} g_{\rho i} t_{3i} n_i \ ,$$

$$n_i^s = \langle \bar\psi_i \psi_i \rangle = \frac{1}{\pi^2} \int_0^{k_{Fi}} k^2 \frac{M_i^*}{\sqrt{k^2 + M_i^{*2}}} dk \ ,$$

and the number density by

$$n_i = \langle \bar\psi_i \gamma^0 \psi_i \rangle = \frac{1}{\pi^2} \int_0^{k_{Fi}} k^2 dk = \frac{k_{Fi}^3}{3\pi^2} \ .$$

The effective baryon mass M_i^* depends on the scalar mean fields as

$$M_i^* = M_i - g_{\sigma i} \bar\sigma - g_{\sigma^* i} \bar\sigma^* - g_{\delta i} t_{3i} \bar\delta \ ,$$

and the effective chemical potentials, $(\mu_i^*)^2 = (M_i^*)^2 + k_{Fi}^2$, are related to the chemical potentials via

$$\mu_i^* = \mu_i - g_{\omega i} \bar\omega - g_{\rho i} \, t_{3i} \bar\rho - g_{\phi i} \bar\phi - \Sigma_0^R \ .$$