In-Medium Properties of Light Mesons in Magnetized Matter - Effects of (Inverse) Magnetic Catalysis

Pallabi Parui

Department of Physics, Indian Institute of Technology Delhi (IITD), New Delhi

June 9, 2023

20th International Conference on Hadron Spectroscopy and Structure
Genoa, Italy

Contents of the Presentation

HADRON 2023

Motivation & Relevance

HADRON 2023

In the high energy heavy-ion collision experiments

- Matter at high density, temperature, or, magnetic field can be produced.
- Study of in-medium spectral properties (mass, decay widths) of hadrons
- In peripheral heavy-ion collision experiments,

  $\Rightarrow$ Strong magnetic fields have been estimated of the order of

  $\Rightarrow |eB| \sim 10^{18}$ Gauss $\approx m_\pi^2$ at RHIC, $|eB| \sim 10^{19}$ Gauss $\approx 15 m_\pi^2$ at LHC *
- Emerging phenomena from the magnetized QCD vacuum
- Effects on the strong interacting physics $\longrightarrow$ Hadron physics

*V. Skokov, A. Y. Illarionov and V. Toneev, Int. J. Mod. Phys. A **24**, 5925 (2009); D. Kharzeev, L. McLerran and H. Warringa, Nucl. Phys. A **803**, 227 (2008); K. Tuchin, Adv. High Energy Phys. **2013**, 490495 (2013).

HADRON 2023

Light vector and axial-vector mesons with constituent u, d quark flavors

1. Vector meson with $J^{PC} = 1^{--}$

$$\Rightarrow \rho : J^{\rho}_{\mu} = \frac{1}{2}(\bar{u}\gamma_{\mu}u - \bar{d}\gamma_{\mu}d) \qquad [m_{vacuum} = 770 \text{ MeV}]$$

$$\Rightarrow \omega : J^{\omega}_{\mu} = \frac{1}{2}(\bar{u}\gamma_{\mu}u + \bar{d}\gamma_{\mu}d) \qquad [m_{vacuum} = 783 \text{ MeV}]$$

2. Axial-vector meson with $J^{PC} = 1^{++}$

$$\Rightarrow A_1 : J^{A_1}_{\mu} = \frac{1}{2}(\bar{u}\gamma_{\mu}\gamma_5 u - \bar{d}\gamma_{\mu}\gamma_5 d) \qquad [m_{vacuum} = 1230 \text{ MeV}]$$

- Hadron decay width: $A_1 \rightarrow \rho\pi$

R.L. Workman et al. (Particle Data Group), Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).

Current-current correlator:-

$$\Pi_{\mu\nu}(q) = i \int d^4x \, e^{iqx} \left\langle T J_\mu(x) J_\nu(0) \right\rangle \quad (1)$$

✓ $\Pi_{\mu\nu}(q) = q_\mu q_\nu R(q^2) - g_{\mu\nu} K(q^2)$ ✓ $q_\mu \Pi^{\mu\nu} = 0 \rightarrow K(q^2) = q^2 R(q^2)$ | for $J^{PC} = 1^{--}$

- Dispersion relation for $R(q^2) \rightarrow$

$$R_{phen.}(q^2) = \frac{1}{\pi} \int_0^\infty ds \, \frac{Im R^{phen.}(s)}{(s - q^2)} \quad (2)$$

- Wilson's operator product expansion (OPE) for $(Q^2 = -q^2) >> 1 \; GeV^2 \rightarrow$

$$R_{OPE}(q^2 = -Q^2) = \left(-c_0 \ln\left(\frac{Q^2}{\mu^2}\right) + \frac{c_1}{Q^2} + \frac{c_2}{Q^4} + \frac{c_3}{Q^6} + \ldots \right) \quad (3)$$

$\rightarrow$ at a scale $\mu^2 = 1 \; GeV^2$

$\rightarrow$ c_i's contain the contribution of QCD condensates and parameters

F. Klingl, N. Kaiser, W. Weise, Nuclear Physics A **624** 527 (1997); L Govaerts, L. J. Reinders, F. De Viron and J. Weyers, Nucl. Phys. B **283** (1987) 706-722; T. Hatsuda, Y. Koike and S. H. Lee, Nuclear Physics B **394** 221 (1993); T. Hatsuda, S. H. Lee, and H. Shiomi, Phys. Rev. C **52**, 3364 (1995); S. Leupold, Phys. Rev. C **64**, 015202 (2001).

- $\left[\alpha_s(\mu^2) = 4\pi/(b\ ln(\mu^2/\Lambda^2_{QCD}))\right]$, $\Lambda_{QCD} = 140\text{MeV}$ $b = 11 - (2/3)N_f = 9$
- $c_0 = \frac{1}{8\pi^2}\left(1 + \frac{\alpha_s}{\pi}\right)$, $c_1 = -\frac{3}{8\pi^2}(m_u^2 + m_d^2)$
- $c_2^{A_1,\rho} = \frac{1}{24}\langle\frac{\alpha_s}{\pi}G^{\mu\nu}G_{\mu\nu}\rangle \mp \frac{1}{2}(m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle)$
- $c_3 = -\frac{\pi\alpha_s}{81}x_j\kappa\left(\langle\bar{u}u\rangle^2 + \langle\bar{d}d\rangle^2\right)$; $x_{j=A_1^0(\rho^0)} = -88(56)$;

$$R_{phen.}(q^2) = R_{OPE}(q^2 = -Q^2)$$

Applying Borel transform ($\hat{B}$:) on eqs.(2) and (3) -

$$\frac{1}{\pi}\int_0^\infty ds\ e^{-s/M^2}\ ImR^{phen.}(s) = \left[c_0M^2 + c_1 + \frac{c_2}{M^2} + \frac{c_3}{2M^4}\right] \quad (4)$$

$$g\ (Q^2) \xrightarrow{\hat{B}} \tilde{g}\ (M^2)$$

$$\hat{B} := \lim_{Q^2\to\infty,\ n\to\infty} \frac{1}{\Gamma(n)}(-Q^2)^n\left(\frac{d}{dQ^2}\right)^n$$

$Q^2/n = M^2$ =fixed, M is the Borel mass.

M.A. Shifman et. al., Nuclear Physics B **147** 448 (1979); T. Hatsuda, Y. Koike and S. H. Lee, Nuclear Physics B **394** 221 (1993).

Borel Transform

HADRON 2023

Some relations applied on eqs.(2)-(3)

1. $\hat{B}[(Q^2)^{-p}] = \frac{1}{(p-1)!}\frac{1}{(M^2)^p}$
2. $\hat{B}[(Q^2)^p ln(Q^2)] = -p!(-M^2)^p$
3. $\hat{B}[(Q^2+A)^{-p}] = \frac{1}{(p-1)!}\frac{1}{(M^2)^p}e^{-A/M^2}$

Usefulness of Borel transform

From eq.(4)

- The exponential function on the l.h.s enhances the contribution of the lowest lying resonance by suppressing the continuum part at large s.
- In the r.h.s, the higher dimensional operators of the R_{OPE} are suppressed by an additional factor of $1/(n-1)!$, which leads to the better convergence of the operator product expansion.

T. Song, T. Hatsuda and S. H. Lee, Phys. Lett. B **792** 160 (2019) .

Finite Energy Sum Rules

HADRON 2023

$$\underbrace{ImR^{phen.}(s)/\pi}_{R^{spec.}(s)} = \underbrace{R_{res}(s)\ \Theta(s_0 - s)}_{s \le s_0} + \underbrace{c_0\ \Theta(s - s_0)}_{s > s_0} \tag{5}$$

Spectral parametrization:

$$R_A^{spec.}(s) = F_A \delta(s - m_A^2) + f_\pi^2 \delta(s - m_\pi^2) + c_0 \Theta(s - s_0^A) \tag{6}$$

$$R_V^{spec.}(s) = F_V \delta(s - m_V^2) + c_0 \Theta(s - s_0^V) \tag{7}$$

Eq.(5) $\rightarrow$ eq.(4) and $\Rightarrow$ comparing powers of $\frac{1}{M^2}$ on l.h.s & r.h.s

1. $\int_0^{s_0} ds\ R_{res}(s) = (c_0 s_0 + c_1)$
2. $\int_0^{s_0} ds\ sR_{res}(s) = \left(\frac{c_0 s_0^2}{2} - c_2\right)$
3. $\int_0^{s_0} ds\ s^2 R_{res}(s) = \left(\frac{c_0 s_0^3}{3} + c_3\right)$

A. Mishra, Phys. Rev. C **91**, 035201 (2015); A. Mishra, A. Kumar, P. Parui, S. De, Phys. Rev. C **100**, 015207 (2019); P. Parui, A. Mishra, arXiv: 2209.02455 [hep-ph] (2022).

Finite Energy Sum Rules

HADRON 2023

Effect of meson-nucleon scattering:-

$$\int_0^\infty ds\, e^{-s/M^2}\, \frac{ImR^{had.}(s)}{\pi} + \underbrace{\rho_{sc}}_{\text{damping term}} = \left[c_0 M^2 + c_1 + \frac{c_2'}{M^2} + \frac{c_3'}{2M^4}\right] \quad (8)$$

- **FESRs for A_1 meson:—**
- $F_A = \left(c_0 s_0^A + c_1 - f_\pi^2\right) \quad \longleftarrow \frac{\rho_B}{4M_N} \longrightarrow$
- $F_A m_A^2 = \left(\frac{c_0 (s_0^A)^2}{2} - c_2 - f_\pi^2 m_\pi^2\right)$
- $F_A m_A^4 = \left(\frac{c_0 (s_0^A)^3}{3} + c_3^A - f_\pi^2 m_\pi^4\right)$

- **FESRs for ρ and ω mesons:—**
- $F_V = \left(c_0 s_0^V + c_1\right)$
- $F_V m_V^2 = \left(\frac{c_0 (s_0^V)^2}{2} - c_2\right)$
- $F_V m_V^4 = \left(\frac{c_0 (s_0^V)^3}{3} + c_3^V\right)$

✓ $m_u \langle \bar{u}u \rangle = \frac{1}{2} m_\pi^2 f_\pi (\sigma + \delta)$; $m_d \langle \bar{d}d \rangle = \frac{1}{2} m_\pi^2 f_\pi (\sigma - \delta) \rightarrow$ explicit chiral symmetry breaking

✓ $\left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \right\rangle = \frac{8}{9}\left[(1-d)\chi^4 + \left(m_\pi^2 f_\pi \sigma + \left(\sqrt{2} m_k^2 f_k - \frac{1}{\sqrt{2}} m_\pi^2 f_\pi\right)\zeta\right)\right] \rightarrow$ comparing the trace of the energy momentum tensor in the QCD and in chiral model Lagrangian

Effective Chiral $SU(3)$ Model

HADRON 2023

- $G \equiv SU(3)_L \times SU(3)_R$ symmetry
- $g = exp[i\sum_{a=1}^{8} \theta_L^a \frac{\lambda^a}{2}] \times exp[i\sum_{a=1}^{8} \theta_R^a \frac{\lambda^a}{2}] \equiv g_L \times g_R;\ g \in G$
- $SU(3)_L \times SU(3)_R \rightarrow$ *Spontaneously broken down to* $SU(3)_V \equiv H$
- A non-linear realization of G is defined on the elements $u(\phi')$ of G/H as

  $\longrightarrow u(\phi') \xrightarrow{G} g_L u(\phi') h(\phi')^\dagger = h(\phi') u(\phi') g_R^\dagger;\ h(\phi') \in SU(3)_V$
- ϕ' are the Goldstone boson fields and $\phi = \frac{1}{\sqrt{2}} \sum_{i=1}^{8} \lambda_i \phi_i'$.
- The elements of the coset space are defined as, $u(\phi') = exp(-\frac{i}{\sqrt{2}} \frac{\phi}{f})$.
- Quarks in this representation transform with the vectorial subgroup $SU(3)_V$
- The $SU(3)$ multiplets of baryons and mesons transform according to,
  $S \xrightarrow{G} h(\phi') S h(\phi')^\dagger, \quad S = B, X, V_\mu, A_\mu$

$$\mathscr{L} = \mathscr{L}_{kin} + \sum_{W=X,V,A,u} \underbrace{\mathscr{L}_{BW}}_{\downarrow} + \mathscr{L}_{vec} + \mathscr{L}_0 + \underbrace{\mathscr{L}_{scale-break}}_{\downarrow} + \underbrace{\mathscr{L}_{SB}}_{\downarrow} + \underbrace{\mathscr{L}_{mag}}_{\downarrow}$$

Terms in the Lagrangian

HADRON 2023

∗ Baryon-scalar meson interaction:-

$$\mathcal{L}_{BX} = -\sum_{i=p,n} \bar{\psi}_i m_i^* \psi_i \qquad \rightarrow m_i^* = -(g_{\sigma i}\sigma + g_{\zeta i}\zeta + g_{\delta i}\delta)$$

∗ Explicit chiral symmetry breaking term, $\mathcal{L}_{SB}$

In chiral model: $Tr\Big[diag\Big(-m_\pi^2 f_\pi(\sigma+\delta)/2,\ -m_\pi^2 f_\pi(\sigma-\delta)/2,\ (\sqrt{2}m_k^2 f_k - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi)\zeta\Big)\Big]$

In QCD: $-Tr\Big[diag(m_u\langle\bar{u}u\rangle, m_d\langle\bar{d}d\rangle, m_s\langle\bar{s}s\rangle)\Big]$

$$\Rightarrow \sigma \sim (\langle\bar{u}u\rangle + \langle\bar{d}d\rangle), \quad \zeta \sim \langle\bar{s}s\rangle, \quad \delta \sim (\langle\bar{u}u\rangle - \langle\bar{d}d\rangle)$$

∗ Scale-invariance breaking effect → trace anomaly

- In QCD, $\theta_\mu^\mu = \frac{\beta_{QCD}}{2g} G_{\mu\nu}^a G^{\mu\nu,a} + \sum_i m_i \bar{q}_i q_i$
- Simulated in the effective Lagrangian (at tree level) by

$$\mathcal{L}_{scale-break} = -\frac{1}{4}\chi^4 ln\left(\frac{\chi^4}{\chi_0^4}\right) + \frac{d}{3}\chi^4 ln\left(\frac{(\sigma^2-\delta^2)\zeta}{\sigma_0^2\zeta_0}\frac{\chi^3}{\chi_0^3}\right)$$

$$^*\mathcal{L}_{mag} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - e_i\bar{\psi}_i\gamma_\mu A^\mu\psi_i - \frac{1}{4}\kappa_i\mu_N\bar{\psi}_i\sigma^{\mu\nu}F_{\mu\nu}\psi_i; \qquad \kappa_{p\ (n)} = 3.5856\ (-3.8263)$$

P. Papazoglou, D. Zschiesche, S. Schramm, J. Schaffner-Bielich, H. Stöcker, and W. Greiner, Phys. Rev. C **59**, 411 (1999);
A. Mishra, K. Balazs, D. Zschiesche, S. Schramm, H. Stöcker, and W. Greiner, Phys. Rev. C **69**, 024903 (2004).

Inputs from the chiral effective model

HADRON 2023

- Classical fields approximation $\rightarrow$

$$\phi(x) \rightarrow \langle\phi\rangle; \quad V^{\mu}(x) \rightarrow \delta^{\mu 0}\langle V^{\mu}\rangle$$

$$\bar{\psi}_i\gamma^{\mu}\psi_j \rightarrow \delta_{ij}\delta^0_{\mu}\langle\bar{\psi}_i\gamma^{\mu}\psi_j\rangle \equiv \delta_{ij}\rho_i = \int_0^{k_{f,i}} \frac{2d^3k}{(2\pi)^3}$$

$$\bar{\psi}_i\psi_j \rightarrow \delta_{ij}\langle\bar{\psi}_i\psi_j\rangle \equiv \delta_{ij}\rho^s_i = \frac{2}{(2\pi)^3}\int_0^{k_{f,i}} d^3k \, \frac{m_i^*}{E_i^*(k)}$$

$$E_i^*(\vec{k}) = \sqrt{\vec{k}^2 + m_i^{*2}}$$

- Solutions of the scalar fields σ, ζ, δ and χ from their equations of motion
- Effects of magnetized matter through $\rho_{p,n}$ and $\rho^s_{p,n}$
- Solved for given values of

  (i) Baryon density, $\rho_B = \rho_p + \rho_n$

  (ii) Isospin asymmetry parameter, $\eta = \frac{\rho_n - \rho_p}{2\rho_B}$

  (iii) Magnetic field strength, $|eB|$ (in units of m_π^2)

A. Jahan C.S., A. Mishra, Int.J.Mod.Phys.E **31** 2250083 (2022); A. Mishra, K. Balazs, D. Zschiesche, S. Schramm, H. Stöcker, and W. Greiner, Phys. Rev. C **69**, 024903 (2004).

Variation of $\langle \bar{q}q \rangle$ with magnetic field

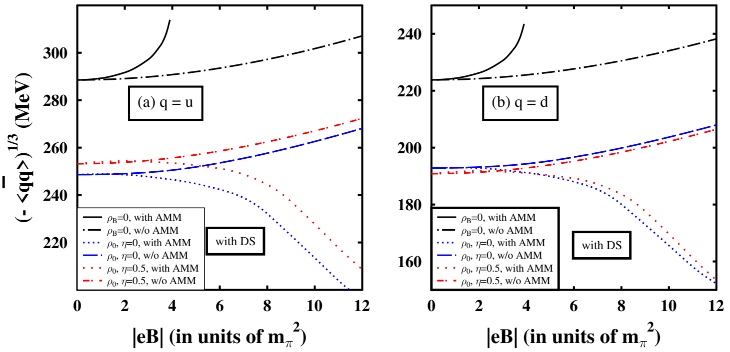

Figure 1: The light quark condensates $(-\langle \bar{q}q \rangle)^{1/3}$ in units of MeV for (a) $q = u$ and (b) $q = d$, are shown as functions of magnetic field $|eB|$ in units of m_π^2, at $\rho_B = 0$, and ρ_0 for $\eta = 0$ (symmetric) and 0.5 (asymmetric) nuclear matter.

Show the phenomena of Magnetic catalysis and Inverse Catalysis

P. Parui, S. De, A. Kumar, and A. Mishra, Phys. Rev. D **106** 114033 (2022).

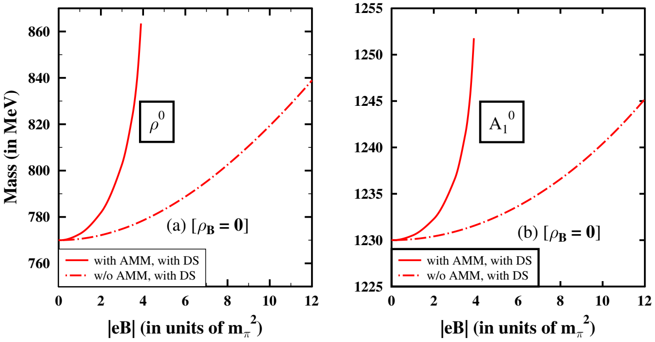

Figure 2: In-medium masses of the light vector ρ^0 and axial-vector $A_1^{(0)}$ mesons states as a function of the magnetic field $|eB|$ (in units of m_π^2), accounting for the magnetized Dirac sea (DS) effects at zero baryon density $\rho_B = 0$.

Pallabi Parui, Amruta Mishra, arXiv:2209.02455 [hep-ph] (2022).

Variation of ρ meson Mass with $|eB|$

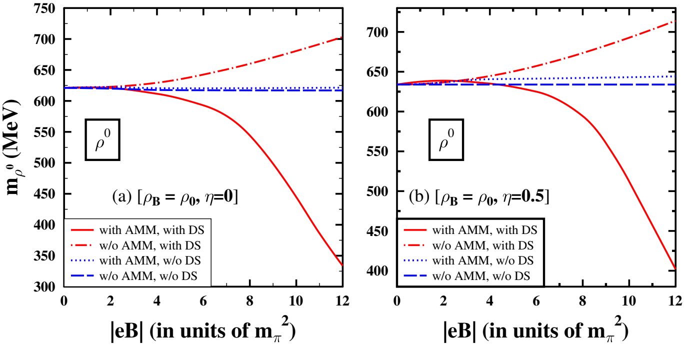

Figure 3: In-medium mass of the light vector ρ mesons as a function of the magnetic field $|eB|$ (in units of m_π^2), accounting for the magnetized DS effects at the nuclear matter saturation density, $\rho_B = \rho_0$. The isospin asymmetry $\eta = 0$ and 0.5 (asymmetric).

Pallabi Parui, Amruta Mishra, arXiv:2209.02455 [hep-ph] (2022).

Mass variation of ω meson with $|eB|$

HADRON 2023

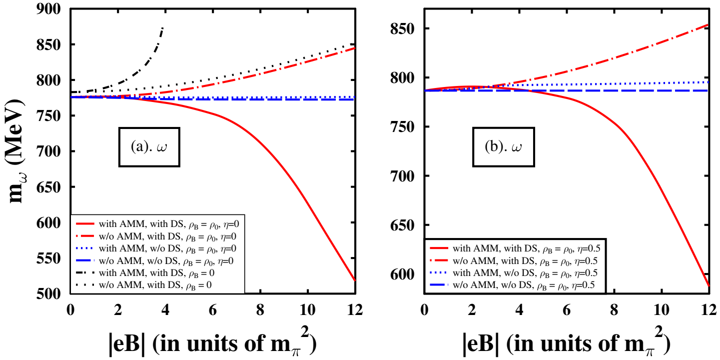

Figure 4: In-medium mass of the light vector ω mesons as a function of the magnetic field $|eB|$ (in units of m_π^2), accounting for the magnetized DS effects at $\rho_B = 0,\ \rho_0$ for $\eta = 0,\ 0.5$.

Pallabi Parui, Amruta Mishra, arXiv:2209.02455 [hep-ph] (2022).

Mass variation of A_1 meson with $|eB|$

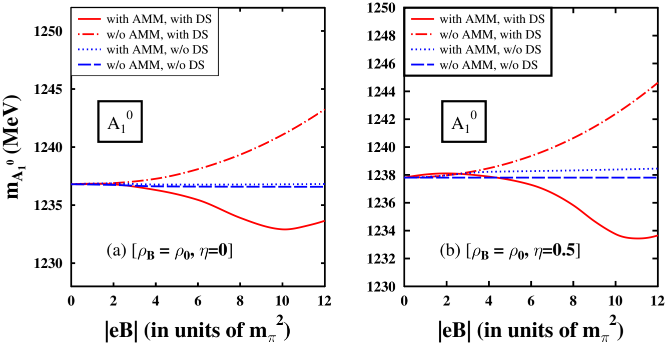

Figure 5: In-medium mass of the light axial-vector A_1 meson as a function of the magnetic field $|eB|$ (in units of m_π^2), accounting for the magnetized DS effects at $\rho_B = \rho_0$ for $\eta = 0,\ 0.5$

Pallabi Parui, Amruta Mishra, arXiv:2209.02455 [hep-ph] (2022).

In-medium decay widths of $A_1 \to \rho\pi$

HADRON 2023

$$\Gamma_{A_1\to\rho\pi} = \frac{|\lambda_{av\phi}|^2}{2\pi m_{A_1}^2} q\left(1+\frac{2q^2}{3m_\rho^2}\right); \quad q = \frac{1}{2m_A}\left([m_A^2-(m_V+m_\pi)^2]\times[m_A^2-(m_V-m_\pi)^2]\right)^{1/2}$$

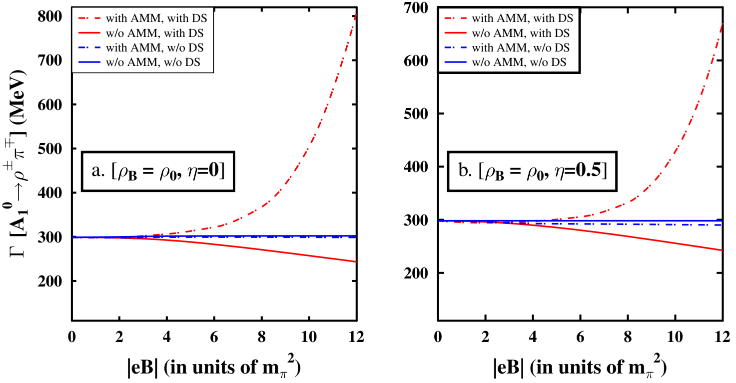

Figure 6: In-medium decay widths of $A_1^0 \to \rho^\pm\pi^\mp$ channels, as functions of the magnetic field $|eB|/m_\pi^2$, accounting for the magnetized DS effects at $\rho_B = \rho_0$ for $\eta = 0,\ 0.5$.

Breit-Wigner Spectral function of A_1 meson

HADRON 2023

$$A(M) = \frac{2}{\pi}\frac{M^2\Gamma_A^*}{(M^2-m_A^{*2})^2+(M\Gamma_A^*)^2} \quad \text{with} \quad \int_0^\infty A(M)dM = 1$$

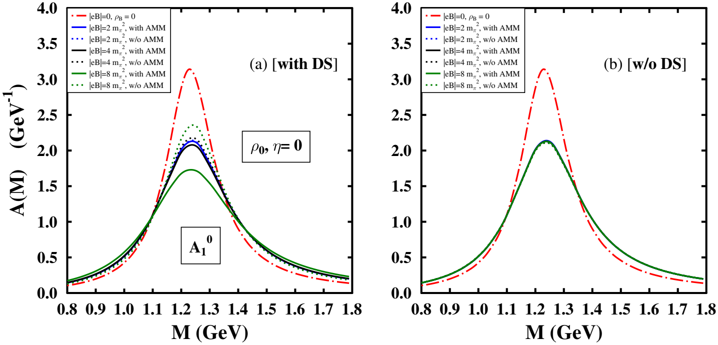

Figure 7: Spectral function of A_1^0 meson at $\rho_B = \rho_0$, $\eta = 0$, accounting for the magnetized Dirac sea contribution on l.h.s plot. It is compared to the case on r.h.s where the DS effect is not incorporated. The vacuum case is shown by dot-dashed line.

Production cross-section for A_1 meson

$$\sigma(M) = \frac{6\pi^2\Gamma_A A(M)}{q(m_A,m_V,m_\pi)^2} \text{ with } q = \frac{1}{2M}\left([m_A^2-(m_V+m_\pi)^2]\times[m_A^2-(m_V-m_\pi)^2]\right)^{1/2}$$

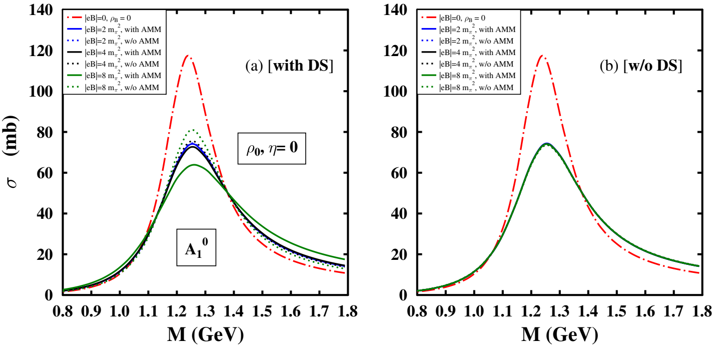

Figure 8: Production cross-section for A_1^0 meson at $\rho_B = \rho_0$ and $\eta = 0$. The vacuum case is shown by dot-dashed line.

Conclusion

HADRON 2023

- The effects of magnetic field on the in-medium masses are observed to be appreciable through the Dirac sea contribution as compared to the case when this effect is not considered.
- The effects of anomalous magnetic moments of the nucleons are seen to be important through the magnetized vacuum contribution, due to the Dirac sea effects.
- The mass modifications of the light vector and axial-vector mesons, may lead to the change in the yields of these mesons due to the modified in-medium partial decay width of light axial meson (A_1) going to a vector (ρ) and pseudoscalar meson (π).

THANK YOU!

BACKUP SLIDES

Number and Scalar Densities of Nucleons

HADRON 2023

- $\rho_p = \frac{|eB|}{2\pi^2}\left[\sum_{\nu=0}^{\nu_{max}} k^p_{f,\nu,s=1} + \sum_{\nu=1}^{\nu_{max}} k^p_{f,\nu,s=-1}\right]$
- $\rho^s_p = \frac{|eB|m^*_p}{2\pi^2}\left[\sum_{\nu=0,s=1}^{\nu_{max}} \frac{\sqrt{m^{*2}_p+2eB\nu}+\Delta_p}{\sqrt{m^{*2}_p+2eB\nu}} \times ln\left|\frac{k^p_{f,\nu,1}+E^p_f}{\sqrt{m^{*2}_p+2eB\nu}+\Delta_p}\right| + \sum_{\nu=1,s=-1}^{\nu_{max}} \frac{\sqrt{m^{*2}_p+2eB\nu}-\Delta_p}{\sqrt{m^{*2}_p+2eB\nu}} \times ln\left|\frac{k^p_{f,\nu,-1}+E^p_f}{\sqrt{m^{*2}_p+2eB\nu}-\Delta_p}\right|\right]; \quad \nu^{max} = \left\lfloor \frac{(E_f-s\Delta_p)^2-m^{*2}_p}{2eB}\right\rfloor$
- $\rho_n = \frac{1}{4\pi^2}\sum_{s=\pm1}\left[\frac{2}{3}k^{(n)3}_{f,s} + s\Delta_n\left[(m^*_n+s\Delta_n)k^n_{f,s} + E^{(n)2}_f\left(sin^{-1}\left(\frac{m^*_n+s\Delta_n}{E^{(n)}_f}\right)-\frac{\pi}{2}\right)\right]\right]$
- $\rho^s_n = \frac{m^*_n}{4\pi^2}\sum_{s=\pm1}\left[k^{(n)}_{f,s}E^n_f - (m^*_n+s\Delta_n)^2 ln\left|\frac{k_{f,s}+E^n_f}{m^*_n+s\Delta_n}\right|\right]$
- $k^p_{f,\nu,s} = \sqrt{E^{(p)2}_f - (\sqrt{m^{*2}_p+2eB\nu}+s\Delta_p)^2}; \quad k^n_{f,s} = \sqrt{E^{(n)2}_f-(m^*_n+s\Delta_n)^2}$
- $\Delta\rho^s_i \sim \left[\frac{(qB)^2}{3m^*_i} + ((-\Delta_p)^2 m^*_i + (-\Delta_n)^2 m^*_i - qB\times\Delta_p)\left(\frac{1}{2}+2ln\left(\frac{m^*_i}{m_i}\right)\right)\right]$

for protons, $q \to |e|$, $\Delta_p = -1.79\frac{|e|B}{2m_N}$; for neutrons $q \to 0$, $\Delta_n = 1.91\frac{|e|B}{2m_N}$

A. Mukherjee, S. Ghosh, M. Mandal, S. Sarkar, and P. Roy, Phys. Rev. D **98**, 056024 (2018); A.E. Broderick, M. Prakash and J. M. Lattimer, Phys. Lett. B **531**, 167 (2002); M. Strickland, V. Dexheimer, and D. P. Menezes, Phys. Rev. D **86**, 125032 (2012); Sushruth Reddy P, Amal Jahan CS, Nikhil Dhale, Amruta Mishra, J. Schaffner-Bielich, Phys. Rev. C **97**, 065208 (2018).

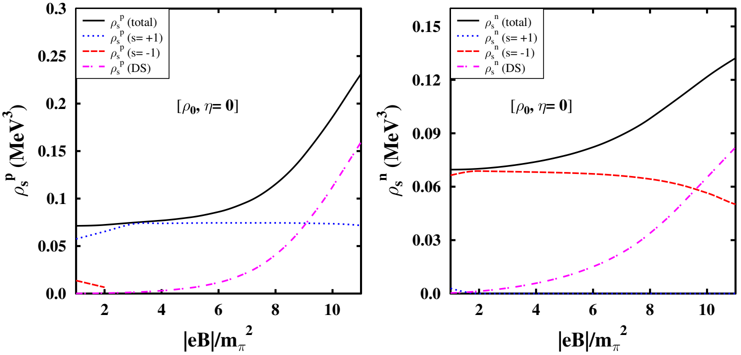

Figure 9: The scalar density of nucleons as functions of magnetic field in units of $|eB|/m_\pi^2$, at ρ_0 $(\eta = 0)$, are plotted for (a) protons and (b) neutrons. The contributions of both spin projections of protons and neutrons $s = \pm 1$ are taken in the magnetized Fermi sea. The effects of the magnetized Dirac sea are shown as dash-dot lines.

Energy Momentum Tensor

HADRON 2023

- The energy momentum tensor of QCD,

$$T_{\mu\nu} = -\frac{\pi}{\alpha_s}\underbrace{\left(u_\mu u_\nu - \frac{g_{\mu\nu}}{4}\right)G_2}_{G^a_{\mu\sigma}G^{a\sigma}_\nu} + \frac{g_{\mu\nu}}{4}\left(\sum_i m_i\bar{q}_i q_i + \frac{\beta_{QCD}}{2g}G^a_{\sigma k}G^{a\sigma k}\right) \quad (9)$$

- In chiral effective model,

$$\mathcal{L}_\chi = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - k_4\chi^4 - \frac{\chi^4}{4}\ln\left(\frac{\chi^4}{\chi_0^4}\right) + \frac{d}{3}\chi^4\ln\left(\left(\frac{(\sigma^2-\delta^2)\zeta}{\sigma_0^2\zeta_0}\right)\left(\frac{\chi}{\chi_0}\right)^3\right)$$

$$T_{\mu\nu} = (\partial_\mu\chi)\left(\frac{\partial\mathcal{L}_\chi}{\partial(\partial^\nu\chi)}\right) - g_{\mu\nu}\mathcal{L}_\chi \quad (10)$$

- Comparing trace of energy momentum tensor in QCD and in model →

$$\left\langle\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^{a\mu\nu}\right\rangle = \frac{8}{9}\left[(1-d)\chi^4 + \left(\frac{\chi}{\chi_0}\right)^2\left(m_\pi^2 f_\pi\sigma + \left(\sqrt{2}m_k^2 f_k - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi\right)\zeta\right)\right]$$

χ_0: vacuum expectation value of χ

Arvind Kumar and Amruta Mishra, Phys. Rev. C **82**, 045207 (2010);

Dirac sea contribution in the weak field limit

HADRON 2023

$$Re\Sigma_S = -\Big(\frac{g^2_{\sigma NN}}{m^2_\sigma}\Big) Rei \int \frac{d^4p}{(2\pi)^4} Tr[\hat{F}^{(p)}(p, m^*_N, m_1) + \hat{F}^{(n)}(n, m^*_N, m_1)]\Delta_F(p, m_1)$$

$$\Sigma_s^{(vac)} = \frac{g^2_{\sigma NN}}{m^2_\sigma} \; Rei \int \frac{d^dp}{(2\pi)^d} \hat{T}(p, m^*_N, m_1) \frac{1}{p^2 - m_1^2 + i\epsilon}$$

Using

$$\int \frac{d^dp}{(2\pi)^d}\Big(\frac{1}{p^2 - m_1^2}\Big) = \frac{-i}{(4\pi)^{d/2}}\Gamma\Big(1 - \frac{d}{2}\Big)\Big(\frac{1}{m_1^2}\Big)^{1-d/2}$$

$$\int \frac{d^dp}{(2\pi)^d}\Big(\frac{p^2}{p^2 - m_1^2}\Big) = \frac{i}{(4\pi)^{d/2}}\Big(\frac{d}{2}\Big)\Gamma\Big(-\frac{d}{2}\Big)\Big(\frac{1}{m_1^2}\Big)^{-d/2} \quad etc.$$

$$\begin{aligned}\hat{T}(p, m^*_N, m_1) &= Tr[\hat{F}^{(p)}(p, m^*_N, m_1) + \hat{F}^{(n)}(n, m^*_N, m_1)] \\ &= 8m^*_N - 8m^*_N(eB)^2 p_\perp^2 \hat{A}_3 + 4m^*_N\{(\kappa_p B)^2 + (\kappa_n B)^2\}\{(m^*_N)^2 + p^2 - 2p_\perp^2 + 2p_\parallel^2\}\hat{A}_2 \\ &\quad + 4(|eB|)(\kappa_p B)\{(m^*_N)^2 - p^2 + 4p_\parallel^2\}\hat{A}_2\end{aligned}$$

$$\Delta_F(p, m) = \Big(\frac{-1}{p^2 - m^2 + i\epsilon}\Big), \quad \hat{A}_n = \frac{(-1)^n}{n!}\frac{\partial^n}{\partial(m_1^2)^n}, \quad n = 2, 3$$

A. Mukherjee, S. Ghosh, M. Mandal, S. Sarkar, and P. Roy, Phys. Rev. D **98**, 056024 (2018).

Continued

$$\Sigma_s^{(vac)} = Re\Sigma_s^{(pure \quad vac)} + \Sigma_s^{(divergent)} + \Sigma_s^{(regular)}$$

$Re\Sigma_s^{(pure\ vac)}$ is ultraviolet divergent, neglected using MFT

$$\Sigma_s^{(divergent)} = -\Big(\frac{g_{\sigma NN}^2}{4\pi^2 m_\sigma^2}\Big)\Big[\{(\kappa_p B)^2 m_N^* + (\kappa_n B)^2 m_N^* + (|eB|\kappa_p B)\}\Big]\Gamma\Big(2-\frac{d}{2}\Big)\Big(\frac{1}{m_N^{*2}}\Big)$$

$$\Sigma_s^{(regular)} = \Big(\frac{g_{\sigma NN}^2}{4\pi^2 m_\sigma^2}\Big)\Big[\frac{(eB)^2}{3m_N^*} + \frac{1}{2}\{(\kappa_p B)^2 m_N^* + (\kappa_n B)^2 m_N^* + (|eB|\kappa_p B)\}\Big]$$

Using the $\overline{MS}$ scheme,

$$\Sigma_s^{(divergent)} = \Big(\frac{g_{\sigma NN}^2}{4\pi^2 m_\sigma^2}\Big)\{(\kappa_p B)^2 m_N^* + (\kappa_n B)^2 m_N^* + (|eB|\kappa_p B)\}\ln\frac{m_N^{*2}}{\Lambda}$$

Λ is of the scale of GeV^2, $\Sigma_s^{(divergent)}(m_N^* = m_N) = 0 \to \Lambda = m_N^2$.

$$\Sigma_s^{(Vac)} = \Big(\frac{g_{\sigma NN}^2}{4\pi^2 m_\sigma^2}\Big)\Big[\frac{(eB)^2}{3m_N^*} + \{(\kappa_p B)^2 m_N^* + (\kappa_n B)^2 m_N^* + (|eB|\kappa_p B)\}\Big\{\frac{1}{2} + 2\ln\frac{m_N^*}{m_N}\Big\}\Big]$$

Scalar four quark condensates

HADRON 2023

$$c_3^{\rho^0,\omega} = -\frac{\pi}{2}\alpha_s\Bigg[\left\langle\left(\bar{u}\gamma_\mu\gamma_5\lambda^a u \mp \bar{d}\gamma_\mu\gamma_5\lambda^a d\right)^2\right\rangle + \frac{2}{9}\left\langle\left(\bar{u}\gamma_\mu\lambda^a u + \bar{d}\gamma_\mu\lambda^a d\right)\times\left(\sum_{q=u,d,s}\bar{q}\gamma^\mu\lambda^a q\right)\right\rangle\Bigg] = -\pi\alpha_s \times \frac{56}{81}\kappa_1\left(\langle\bar{u}u\rangle^2 + \langle\bar{d}d\rangle^2\right) \quad (11)$$

$$c_3^{A_1^0} = -\frac{\pi}{2}\alpha_s\Bigg[\left\langle\left(\bar{u}\gamma_\mu\lambda^a u - \bar{d}\gamma_\mu\lambda^a d\right)^2\right\rangle + \frac{2}{9}\left\langle\left(\bar{u}\gamma_\mu\lambda^a u + \bar{d}\gamma_\mu\lambda^a d\right)\times\left(\sum_{q=u,d,s}\bar{q}\gamma^\mu\lambda^a q\right)\right\rangle\Bigg] = \pi\alpha_s \times \frac{88}{81}\kappa_1\left(\langle\bar{u}u\rangle^2 + \langle\bar{d}d\rangle^2\right) \quad (12)$$

To simplify the expression for the scalar four-quark condensates, factorization method is used

$$\langle(\bar{q}_i\gamma_\mu\lambda^a q_j)^2\rangle = -\langle(\bar{q}_i\gamma_\mu\gamma_5\lambda^a q_j)^2\rangle = -\delta_{ij}\frac{16}{9}\kappa\langle\bar{q}_i q_j\rangle^2 \quad (13)$$

Continued

HADRON 2023

$$c_3^{A_1^\pm} = -\frac{\pi}{2}\alpha_s\Bigg[2\left\langle\left(\bar{u}\gamma_\mu\lambda^a d\right)\left(\bar{d}\gamma_\mu\lambda^a u\right)\right\rangle + \frac{2}{9}\Bigg\langle\left(\bar{u}\gamma_\mu\lambda^a u + \bar{d}\gamma_\mu\lambda^a d\right)\times \left(\sum_{q=u,d,s}\bar{q}\gamma^\mu\lambda^a q\right)\Bigg\rangle\Bigg] = \pi\alpha_s\times\kappa_2\left[\frac{16}{81}\left(\langle\bar{u}u\rangle^2+\langle\bar{d}d\rangle^2\right)+\frac{16}{9}\langle\bar{u}u\rangle\langle\bar{d}d\rangle\right] \quad (14)$$

$$c_3^{\rho^\pm} = -\frac{\pi}{2}\alpha_s\Bigg[2\left\langle\left(\bar{u}\gamma_\mu\gamma^5\lambda^a d\right)\left(\bar{d}\gamma_\mu\gamma^5\lambda^a u\right)\right\rangle + \frac{2}{9}\Bigg\langle\left(\bar{u}\gamma_\mu\lambda^a u + \bar{d}\gamma_\mu\lambda^a d\right)\times \left(\sum_{q=u,d,s}\bar{q}\gamma^\mu\lambda^a q\right)\Bigg\rangle\Bigg] = \pi\alpha_s\times\kappa_3\left[\frac{16}{81}\left(\langle\bar{u}u\rangle^2+\langle\bar{d}d\rangle^2\right)-\frac{16}{9}\langle\bar{u}u\rangle\langle\bar{d}d\rangle\right] \quad (15)$$

⇒ *Dirac sea effects are taken into account through summation over the nucleonic tadpole diagrams –

$$\mathcal{L}_{BV} + \mathcal{L}_{BX} = -\bar{\psi}_i(g_{i\omega}\gamma_\mu\omega^\mu + g_{i\rho}\gamma_\mu\tau_3\rho^\mu + m_i^*)\psi_i$$

$$m_i^* = -(g_{\sigma i}\sigma + g_{\zeta i}\zeta + g_{\delta i}\delta),\ E_i^*(k) = \sqrt{k^2 + m_i^{*2}}; i = p, n$$

$$\left(D_i^H(p)\right)^{-1} = \gamma.\bar{p} - m_i^*;\ \bar{p} = p + \Sigma_i^V,\ m_i^* = m_i + \Sigma_i^S$$

$$\Sigma_i^S = -\underbrace{(g_{\sigma i}\tilde{\sigma} + g_{\zeta i}\tilde{\zeta} + g_{\delta i}\tilde{\delta})}_{\tilde{x}=x-x_0;\ x=\sigma,\zeta,\delta} = i\left(\sum_x \frac{g_{xi}^2}{m_x^2}\right)\int \frac{d^4p}{(2\pi)^4} Tr[D_i^F(p) + D_i^D(p)]e^{ip^0\eta} \equiv \underbrace{(\Sigma_i^S)^F}_{\text{F. part}} + \underbrace{(\Sigma_i^S)^D}_{\text{Density part}}$$

- * $(\Sigma_i^S)^D = -\left(\sum_x \frac{g_{xi}^2}{m_x^2}\right)\rho_i^s$
- * $\rho_i^s = \gamma_i \int \frac{d^3k}{(2\pi)^3}\frac{m_i^*}{E_i^*}\Theta(\mu - E)$; μ: Fermi energy

F. part: Feynman part

- * $(\Sigma_i^S)^F = -(\sum_x \frac{g_{xi}^2}{m_x^2}) Rei \int \frac{d^4p}{(2\pi)^4} Tr[\hat{S}_i]\ \hat{S}_i$: the magnetic field expansion of the fermion propagator up to quadratic in **B**, incorporating the AMMs of nucleons.

A. Mukherjee, S. Ghosh, M. Mandal, S. Sarkar, and P. Roy, Phys. Rev. D **98**, 056024 (2018); A. Mishra, K. Balazs, D. Zschiesche, S. Schramm, H. Stöcker, and W. Greiner, Phys. Rev. C **69**, 024903 (2004).

Effects of magnetic field on the Fermi sea

HADRON 2023

$$\mathcal{L}_{mag} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - q_i\bar{\psi}_i\gamma_\mu A^\mu\psi_i - \frac{1}{4}\kappa_i\mu_N\bar{\psi}_i\sigma^{\mu\nu}F_{\mu\nu}\psi_i;\quad i = p, n,\quad \kappa_{p,n} = 3.5856(-3.8263)$$

Effects from Landau energy levels of protons and anomalous magnetic moments (AMMs) of neutrons and protons as

- For the charged particles with electric charge q and $\vec{B} = B\hat{z}$

  $\Rightarrow$ transverse momenta, $k_\perp^2 = 2\nu|q|B$ ($\nu \geq 0$), and $\int_k \rightarrow \frac{|q|B}{(2\pi)^2}\sum_n \int_{-\infty}^{\infty} dk_z$;
  $k_\perp^2 = k_x^2 + k_y^2$, n: orbital angular momentum of the particle.

  $\Rightarrow$ For spin-$\frac{1}{2}$ protons, $\nu = n + \frac{1}{2} - \frac{s}{2}\frac{q}{|q|}$; $s = \pm 1$, spin-projection along $\vec{B}$.

  $\Rightarrow$ Energy $E = \sqrt{k_z^2 + (\sqrt{m^2 + 2\nu|q|B} + s\Delta_p)^2}$

- For uncharged particles,

  $\Rightarrow \int_k \rightarrow \int \frac{d^3k}{(2\pi)^3}$; and $E = \sqrt{k_z^2 + (\sqrt{m^2 + k_\perp^2} + s\Delta_n)^2}$

  terms corresponding to AMMs of protons, $\Delta_p = -1.79\frac{eB}{2m_p}$; neutrons $\Delta_n = 1.91\frac{eB}{2m_n}$

A.E. Broderick, M. Prakash and J. M. Lattimer, Phys. Lett. B **531**, 167 (2002); M. Strickland, V. Dexheimer, and D. P. Menezes, Phys. Rev. D **86**, 125032 (2012); Sushruth Reddy P, Amal Jahan CS, Nikhil Dhale, Amruta Mishra, J. Schaffner-Bielich, Phys. Rev. C **97**, 065208

Nature of mass variation with density

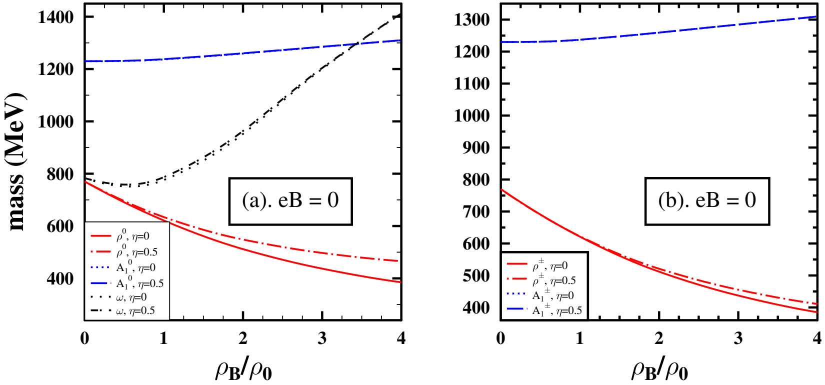

Figure 10: Variation of mass (in MeV) of the light vector and axial-vector mesons (ρ, ω and A_1) as a function of nuclear matter density, in units of ρ_0, at $\eta = 0$ and 0.5 for zero magnetic field $|eB| = 0$

Hadronic Lagrangian for $AV\phi$ interaction

$$\mathcal{L}_{av\phi} = i\tilde{f}\langle a_{\mu\nu}[v^{\mu\nu}, \phi]\rangle.$$

For tree level calculations, $(\partial_\mu X_\nu - \partial_\nu X_\mu)$ instead of $X_{\mu\nu}$ for $X = a, v$.

Normalization of these fields:

$$\langle 0|X_{\mu\nu}|X; p, \epsilon\rangle = \frac{i}{m_X}[p_\mu \epsilon_\nu(X) - p_\nu \epsilon_\mu(X)].$$

$$\Gamma_{A_1 \to \rho\pi} = \frac{q}{8\pi m_{A_1}^2}\overline{|\mathcal{M}|^2}$$

$$\mathcal{M} = \frac{-2\lambda_{av\phi}}{m_{A_1} m_\rho}\left(p'.p \quad \epsilon'.\epsilon - \epsilon'.p \quad \epsilon.p'\right)$$

$\lambda_{av\phi} = \sqrt{2}i\tilde{f};\ p^2 = m_\rho^2,\ p'^2 = m_{A_1}^2$ and $2p.p' = (m_{A_1}^2 + m_\rho^2 - m_\pi^2)$

$$\Gamma_{A_1 \to \rho\pi} = \frac{|\lambda_{av\phi}|^2}{2\pi m_{A_1}^2} q \left(1 + \frac{2q^2}{3m_\rho^2}\right) \tag{16}$$

L. Roca, J. E. Palomar, and E. Oset, Phys. Rev. D 70, 094006 (2004); G. Ecker, J. Gasser, A. Pich, and E. de Rafael, Nucl. Phys. B **321**, 311 (1989).

HADRON 2023

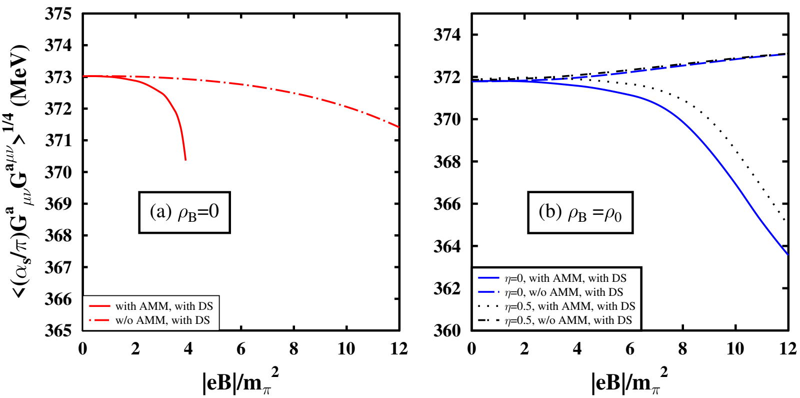

Figure 11: The scalar gluon condensate $\langle(\alpha_s/\pi)G^a_{\mu\nu}G^{a\mu\nu}\rangle^{1/4}$ in MeV at (a) $\rho_B = 0$ and (b) $\rho_B = \rho_0$ (for $\eta = 0,\ 0.5$), are shown as functions of $|eB|$ (in units of m_π^2). The condensates are calculated in terms of the in-medium scalar fields, accounting for the Dirac sea (DS) effects.

P. Parui, S. De, A. Kumar, and A. Mishra, Phys. Rev. D **106** 114033 (2022)..