The 20th International Conference on Hadron Spectroscopy and Structure (HADRON 2023),
Genova, Italy, June 5th – 9th 2023.

Pion-nucleon sigma term by the pion deep bound states

Satoru Hirenzaki (Nara Women's Univ.)
Natsumi Ikeno (Tottori Univ., TEXAS,)

based on

N. Ikeno, T. Nishi, K. Itahashi, N. Nose-Togawa,
A. Tani, S. Hirenzaki
PTEP 2023 (2023) 3, 033D03 (arXiv:2204.09211v2)

Time/Place : 17:30-17:50, 5 June (Mon.) 2023, DAD Room 1A

Meson bound states: Interest and Motivation

1. Exotic Many Body Physics:

Interaction, Structure, Formation

Extension of the research area of nuclear physics

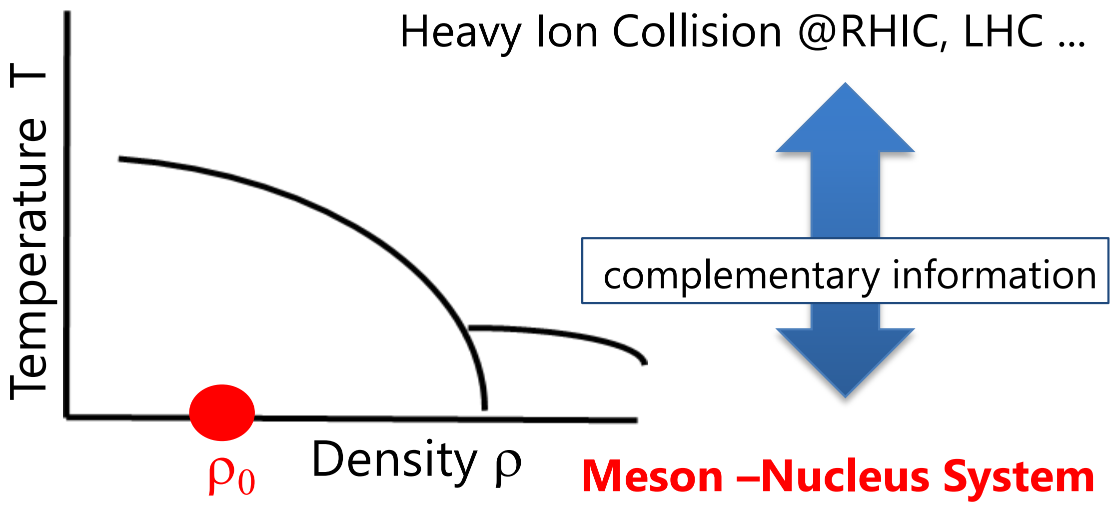
2. Meson properties at finite density:

Aspect of QCD symmetries

<u>Chiral symmetry</u>

Spontaneous, Explicit breaking@Vacuum

Partial restoration @Nuclear density



How do the hadron properties change in the nucleus from the vacuum?

Meson bound states: Interest and Motivation

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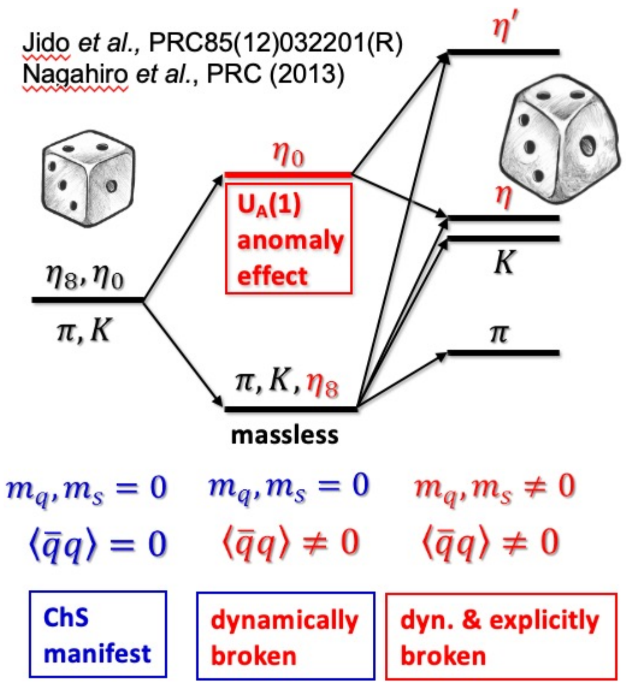


Structure of the pionic atoms

- Klein-Gordon equation:

$$[-\nabla^2+\mu^2+2\mu V_{\mathrm{opt}}(r)]\phi(\mathbf{r}) = [E-V_{\mathrm{coul}}(r)]^2\phi(\mathbf{r})$$

- Pion-Nucleus Optical Potential ：

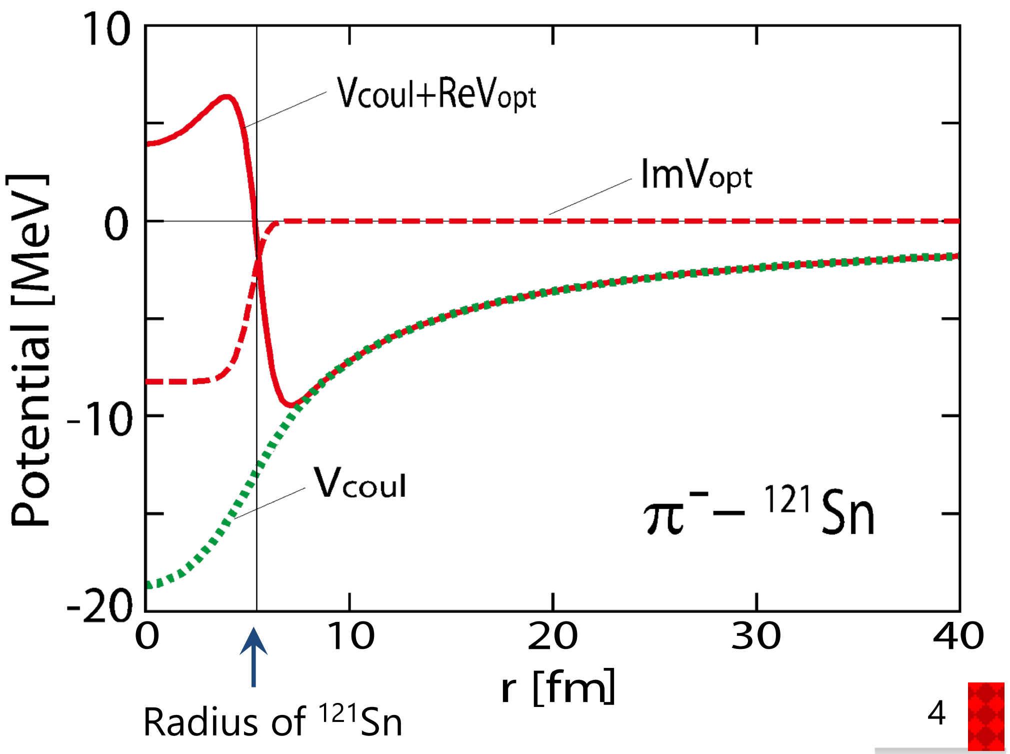
$$2\mu V_{\mathrm{opt}}(r) = \underbrace{-4\pi[b(r)+\varepsilon_2 B_0\rho^2(r)]}_{\text{s-wave term}} + \underbrace{4\pi\nabla\cdot[c(r)+\varepsilon_2^{-1}C_0\rho^2(r)]L(r)\nabla}_{\text{p-wave term}}$$

$$b(r) = \varepsilon_1\{b_0\rho(r)+b_1\ [\rho_n(r)-\rho_p(r)]\}$$

$$c(r) = \varepsilon_1^{-1}\{c_0\rho(r)+c_1[\rho_n(r)-\rho_p(r)]\}$$

$$L(r) = \{1+\tfrac{4}{3}\pi\lambda[c(r)+\varepsilon_2^{-1}C_0\rho^2(r)]\}^{-1}$$

M. Ericson, T. E. O Ericson, Ann. Phys.36(66)496
R. Seki, K. Masutani, PRC27(83)2799

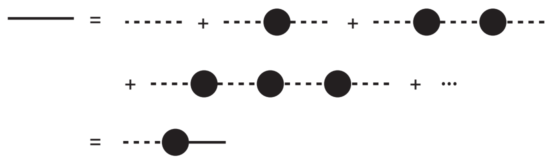


- ✓ Strong interaction s-wave terms are repulsive
- ✓ Pocket structure near the nuclear surface

Optical pot. by Dyson eq.

$$iD^0(q^\mu) = \frac{i}{q^2 - m_\pi^2 + i\varepsilon}$$

$$\begin{aligned} iD(q^\mu) &= iD^0 + iD^0(-i\Pi)iD^0 + iD^0(-i\Pi)iD^0(-i\Pi)iD^0 + \ldots \\ &= i(D^0 + D^0\Pi D^0 + D^0\Pi D^0\Pi D^0 + \ldots) \\ &= iD^0(q^\mu) + iD^0(q^\mu)\Pi(q^\mu)D(q^\mu), \end{aligned}$$



$$iD(q^\mu) = \frac{iD^0(q^\mu)}{1 - D^0(q^\mu)\Pi(q^\mu)} = \frac{i}{(D^0(q^\mu))^{-1} - \Pi(q^\mu)} = \frac{i}{q^2 - m_\pi^2 - \Pi(q^\mu)}$$

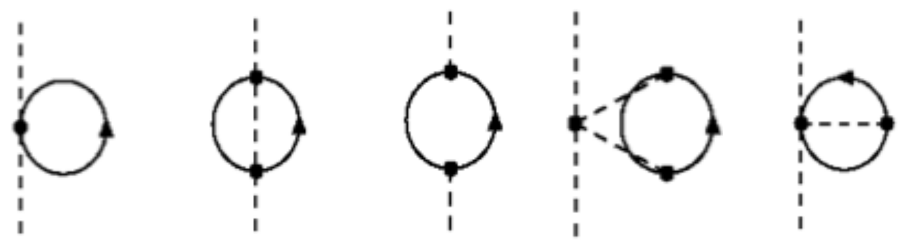
$$H = D^{-1} = q^2 - m_\pi^2 - \Pi(q^\mu)$$

$$\Pi(q^\mu) = \Sigma\, \Pi^{\text{irreducible}}$$ ➔ Optical Potential

Ex.) Kolomeitsev, Kaiser, & W. Weise, Phys.Rev.Lett. 90 ('03)

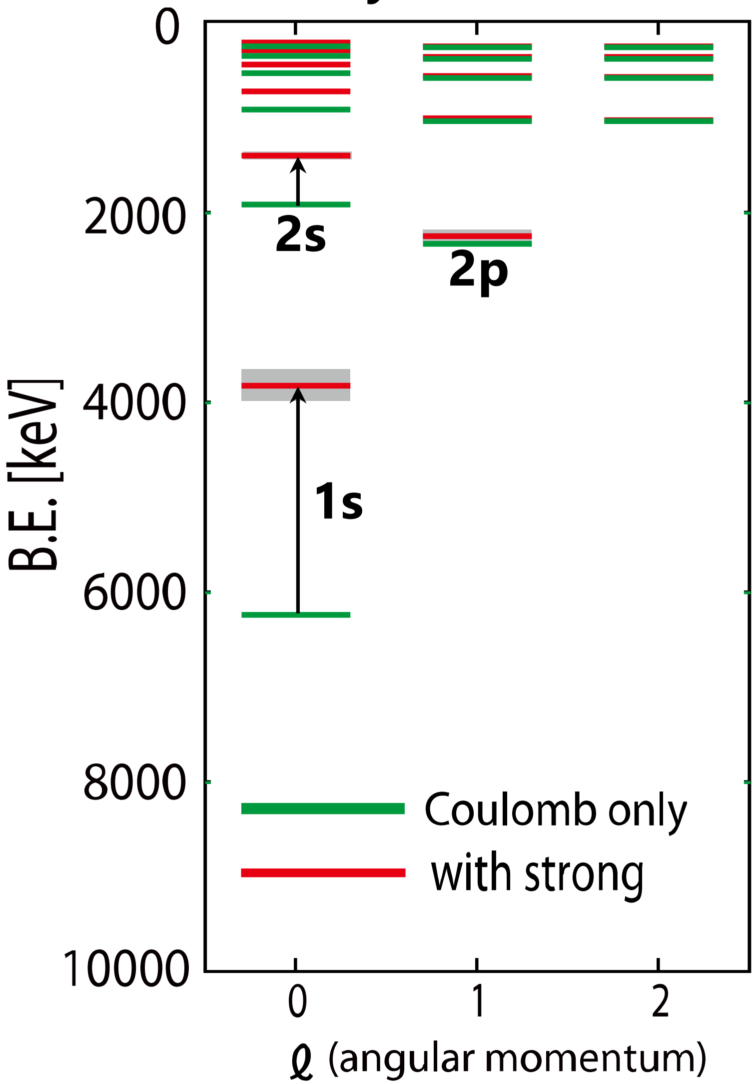
$$\begin{aligned} \Pi_{\text{tot}}(\omega;\rho_p,\rho_n) &= \Pi(\omega) + \Delta\Pi_{\text{S}}(\omega;\rho_p,\rho_n) \\ &\quad + \Pi_{\text{P}}(\omega;\rho_p,\rho_n), \end{aligned}$$

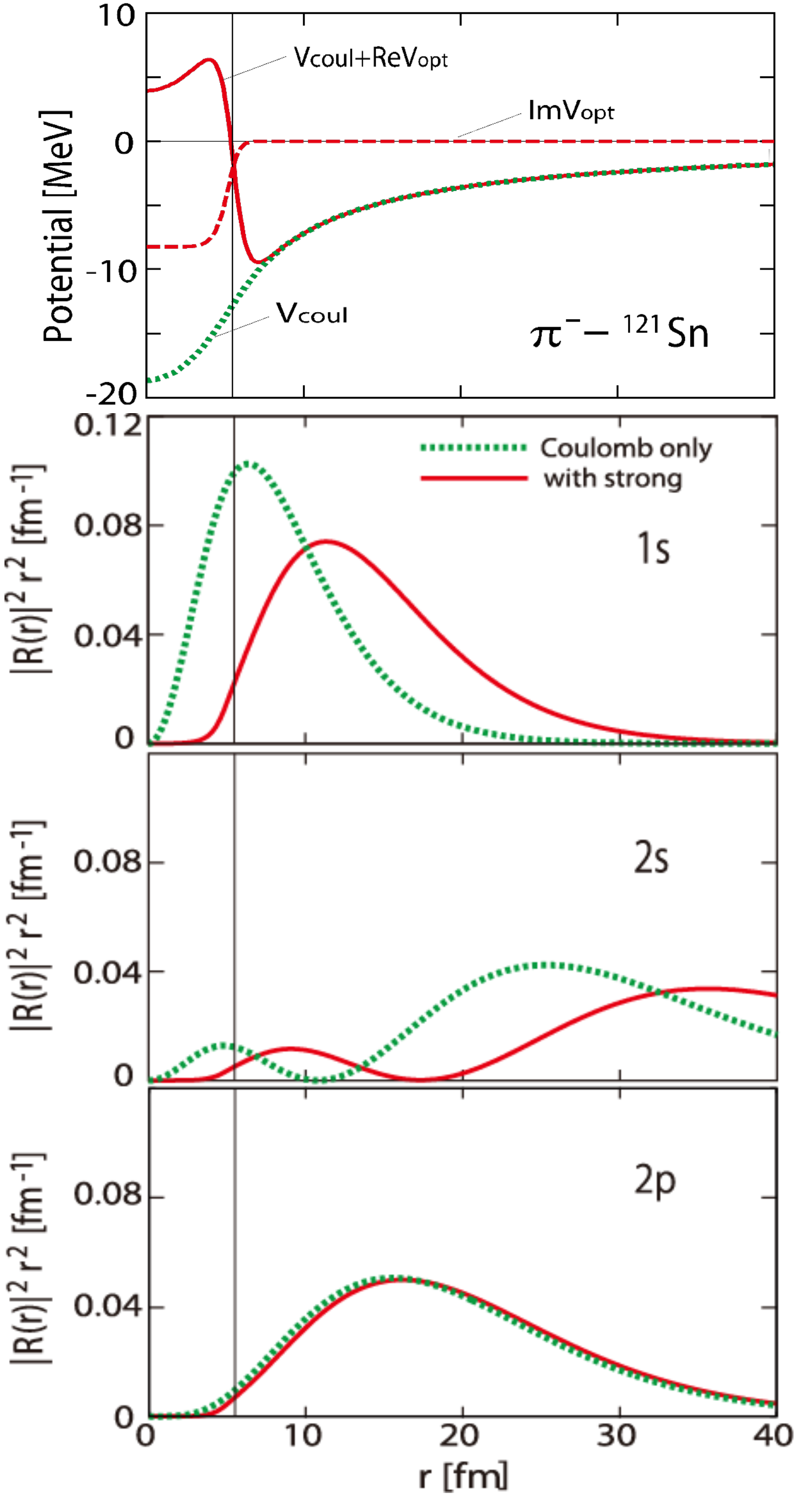
(ChPT up to NNLO) + (Pheno. 2−body abs.)
+(Pheno. Pwave)

$\Pi(\omega) =$ 

Structure of the pionic atoms

➤ π-^{121}Sn system

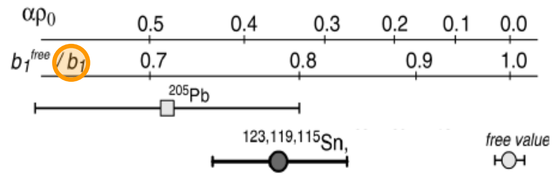




Deeply bound pionic states **1s, 2s, 2p** :
Strong interaction effects are large

EXAMPLE Experimental Results

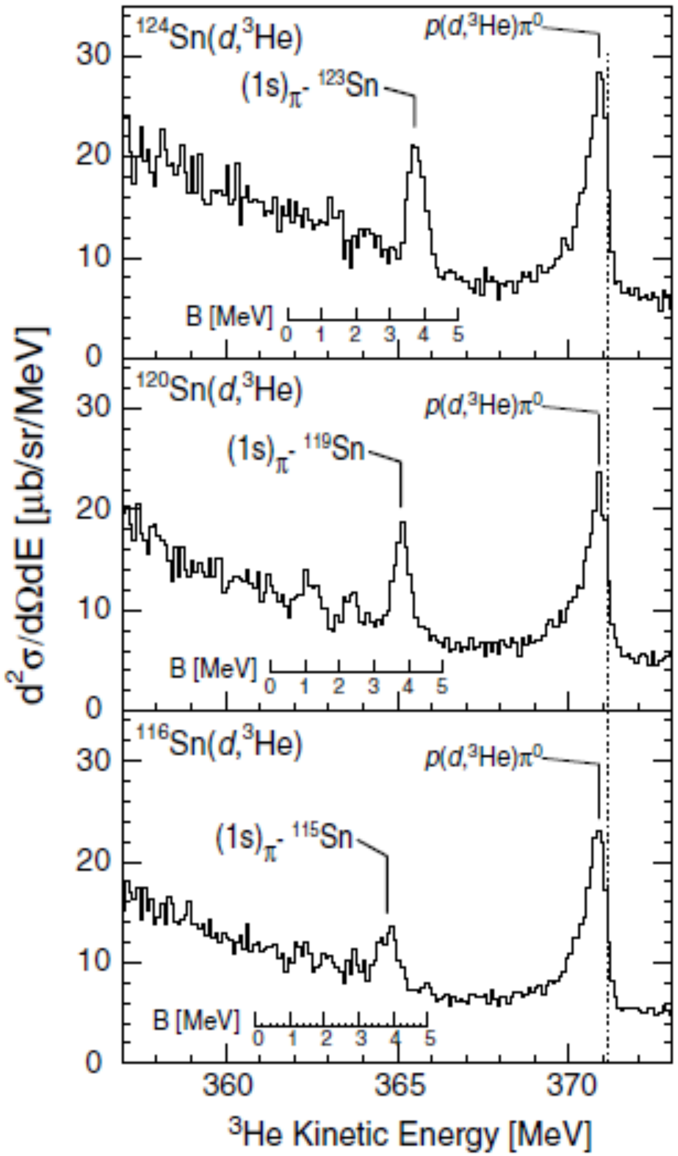
In-medium property of pion



K. Suzuki *et al.*, PRL92 (04) 072302:

Missing mass spectroscopy by (d,3He) reaction

Exp@GSI



➢ Pion-Nucleus optical potential

$$2\mu V^{s}_{\text{opt}} = -4\pi[\varepsilon_1\{b_0\rho(r) + b_1\delta\rho(r)\} + \varepsilon_2 B_0\rho^2(r)]$$

✓ b1 determination, Comparison with value in vacuum

✓ Relation between b_1 ⇔ f_π ⇔ $<\bar{q}q>$

- Tomozawa – Weinberg (TW) relation

$$T^{(-)}_{\pi A} = -4\pi\varepsilon_1 b_1 = \frac{\omega}{2f_\pi^{*2}}$$

- Gell-Mann – Oakes – Renner (GOR) relation

$$m_\pi^2 f_\pi^{*2} = -2m_q\langle\bar{q}q\rangle_\rho$$

$$\frac{\langle\bar{q}q\rangle_\rho}{\langle\bar{q}q\rangle_0} \simeq \frac{f_\pi^{*2}}{f_\pi^{\;2}} \simeq \frac{b_1^{\text{free}}}{b_1^*(\rho)} = 0.78 \;@\; \rho \simeq 0.6\rho_0$$

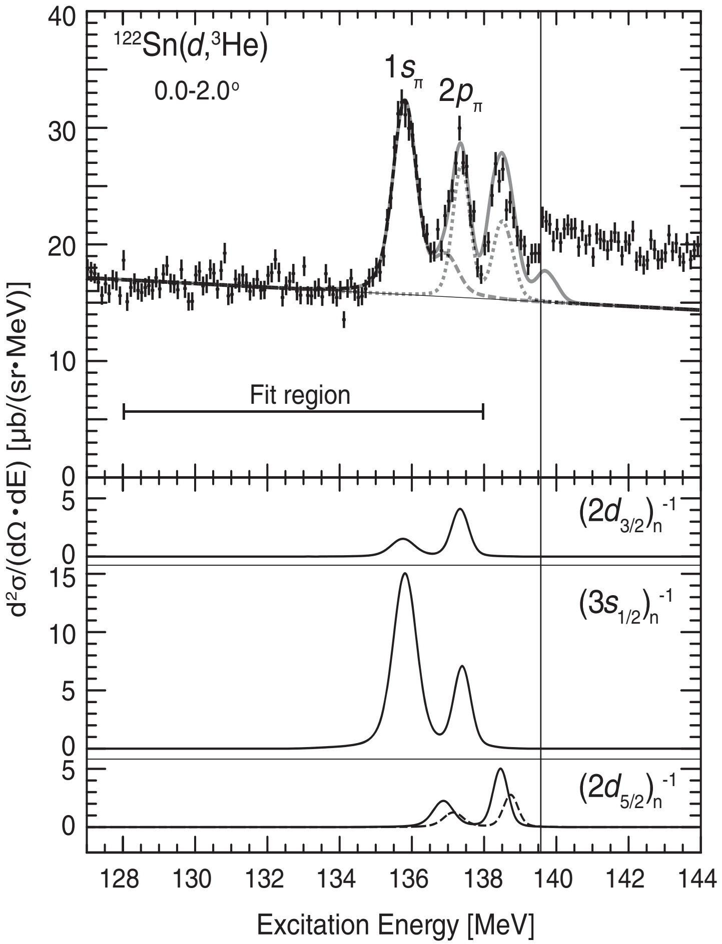
$$\sim 0.67 \;@\; \rho = \rho_0$$

- Data corresponds to info. at Effective Density

EXAMPLE Experimental Results @ RIKEN/RIBF

Spectroscopy of Pionic Atoms in ^{122}Sn$(d,{}^3$He$)$ Reaction and Angular Dependence of the Formation Cross Sections

T. Nishi, K. Itahashi et al., (piAF Collaboration) PRL120, 152505 (2018)



Observation of Pionic 1s and 2p states in Sn

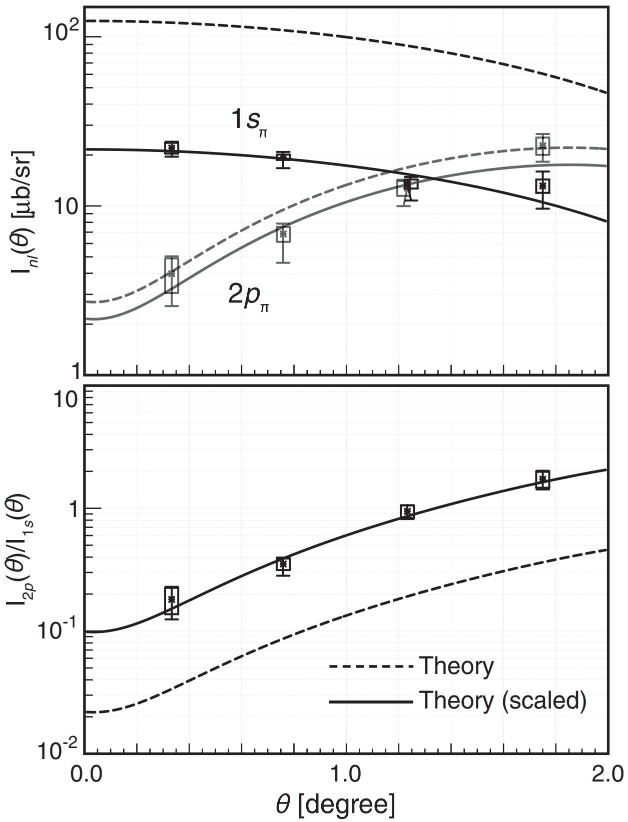


FIG. 4. (Top panel) Determined pionic-nl-state formation cross sections $I_{nl}(\theta)$ for different θ ranges. Statistical errors are shown by the boxes and systematic errors in addition by the bars. The deduced cross sections are compared with the theoretical calculations [19,28]. (Bottom panel) $I_{2p}(\theta)/I_{1s}(\theta)$. Systematic errors are canceled by taking the ratios.

First observation of the angular dependence of (d,3He) reaction in experiment

Theory:
Qualitative agreement

Quantitative No...
Further improvement is needed

Deeply Bound Pionic Atom data by (d,3He)

＊ Multi-states (1s, 2p, , ,) observation for each nuclei

＊Cross section data (Formation spectra)

＊High sensitivity (expected) to the sigma term

（＊Long isotope chain information for Sn ）

For pion-nucleon sigma term

$$\sigma_{\pi N} = \frac{\bar{m}_q}{2m_N}\sum_{u,d}\langle N|\bar{q}q|N\rangle$$

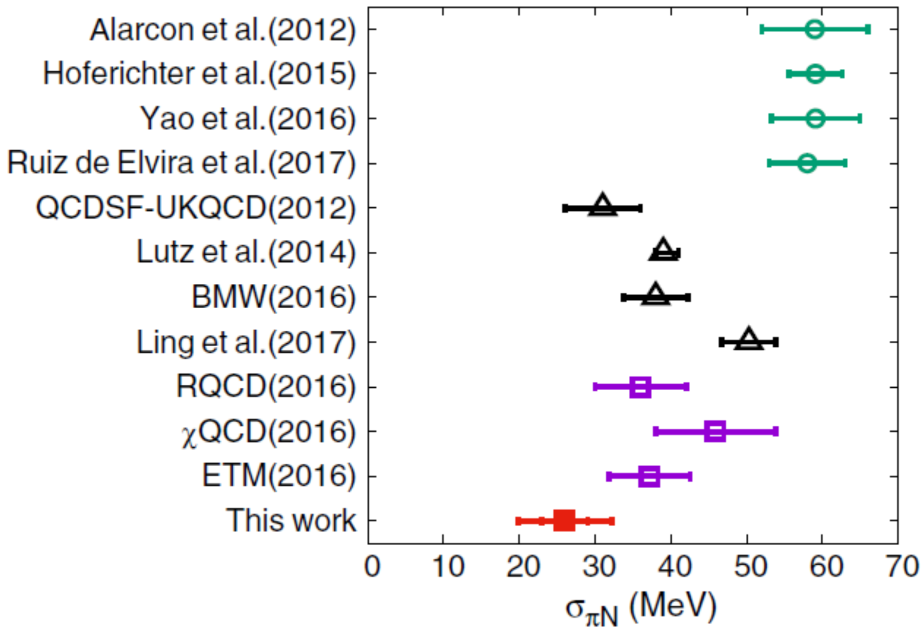
Pion-nucleon sigma term $\sigma_{\pi N}$ ``distribution''

$$\bar{m}_q = \frac{m_u + m_d}{2}$$

Nucleon charges with dynamical overlap fermions

PHYSICAL REVIEW D **98**, 054516 (2018)

N. Yamanaka* et al., (JLQCD Collaboration)

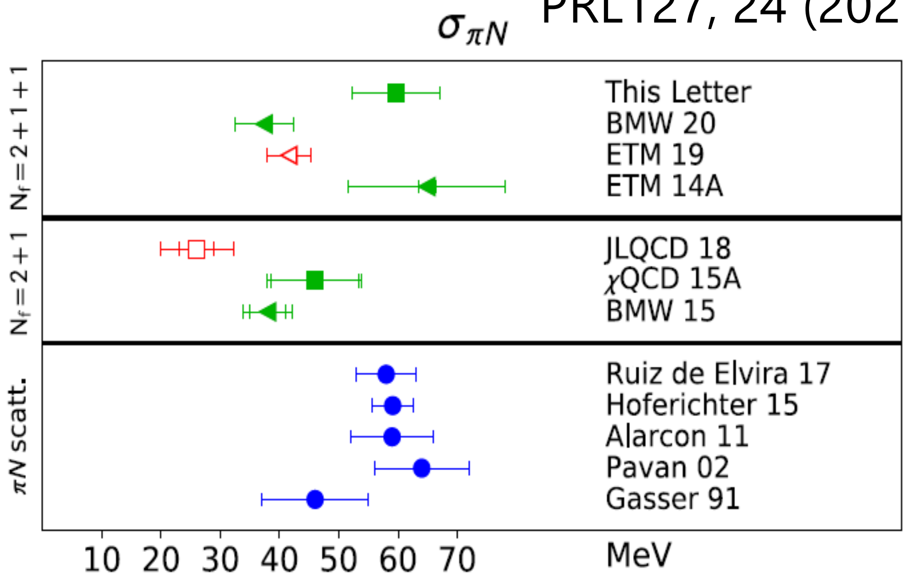


The nucleon sigma term from lattice QCD

R. Gupta, S. Park, M. Hoferichter, E. Mereghetti,]

B. Yoon and T. Bhattacharya, [arXiv:2105.12095 [hep-lat]].

PRL127, 24 (2021)



The value of $\sigma_{\pi N}$ has not been determined accurately enough: **$\sigma_{\pi N}$ = 25~60 MeV**

=> It seems to be very interesting to determine the $\sigma_{\pi N}$ value by the deeply bound pionic atoms.

$\sigma_{\pi N}$ term in the optical potential

➢ Pion-Nucleus optical potential

$$2\mu V_{\rm opt}(r) = -4\pi[b(r) + \varepsilon_2 B_0 \rho^2(r)] + 4\pi\nabla\cdot[c(r) + \varepsilon_2^{-1} C_0 \rho^2(r)]L(r)\nabla$$

$$b(r) = \varepsilon_1\{b_0\rho(r) + b_1[\rho_n(r) - \rho_p(r)]\}$$

$$c(r) = \varepsilon_1^{-1}\{c_0\rho(r) + c_1[\rho_n(r) - \rho_p(r)]\}$$

$$L(r) = \{1 + \tfrac{4}{3}\pi\lambda[c(r) + \varepsilon_2^{-1} C_0 \rho^2(r)]\}^{-1}$$

$$\boxed{b_1(\rho) = b_1^{\rm free}\left(1 - \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2}\rho\right)^{-1},}$$

$$b_0(\rho) = b_0^{\rm free} - \varepsilon_1 \frac{3}{2\pi}(b_0^{\rm free2} + 2b_1^2(\rho))\left(\frac{3\pi^2}{2}\rho\right)^{1/3}$$

- The $\sigma_{\pi N}$ value determined by the existing pionic atom data was reported:

  χ^2 fitting for (all) atomic data (BE, Width)

➡ $\sigma_{\pi N}^{\rm FG} = 57 \pm 7$ MeV, E. Friedman and A. Gal, Phys. Lett. B **792**, 340 (2019).
E. Friedman and A. Gal, Acta Phys. Polon. B **51**, 45-54 (2020).

We especially focus on the observables of the high-precision deeply bound pionic states

Sensitivity of the deeply bound pionic atom observables to the pion-nucleon sigma term $\sigma_{\pi N}$

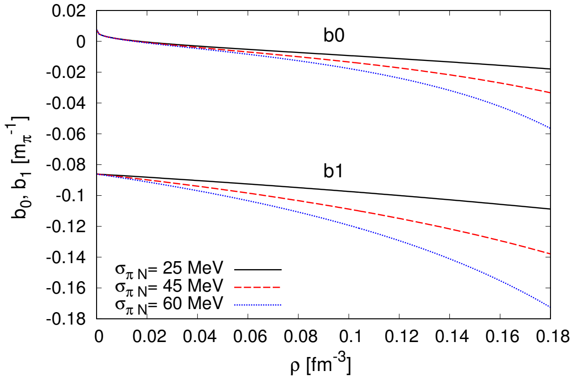


FIG. 1: Density dependence of the $b_0(\rho)$ and $b_1(\rho)$ for the different $\sigma_{\pi N}$ term.

$$b_1(\rho) = b_1^{\text{free}} \left(1 - \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2}\rho\right)^{-1},$$

$$b_0(\rho) = b_0^{\text{free}} - \varepsilon_1 \frac{3}{2\pi}(b_0^{\text{free}2} + 2b_1^2(\rho)) \left(\frac{3\pi^2}{2}\rho\right)^{1/3}$$

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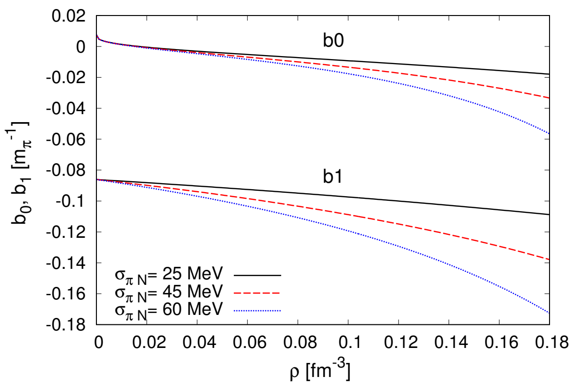

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$$\frac{\Delta BE(1s)}{\Delta\sigma_{\pi N}} \sim \frac{6keV}{1MeV}$$

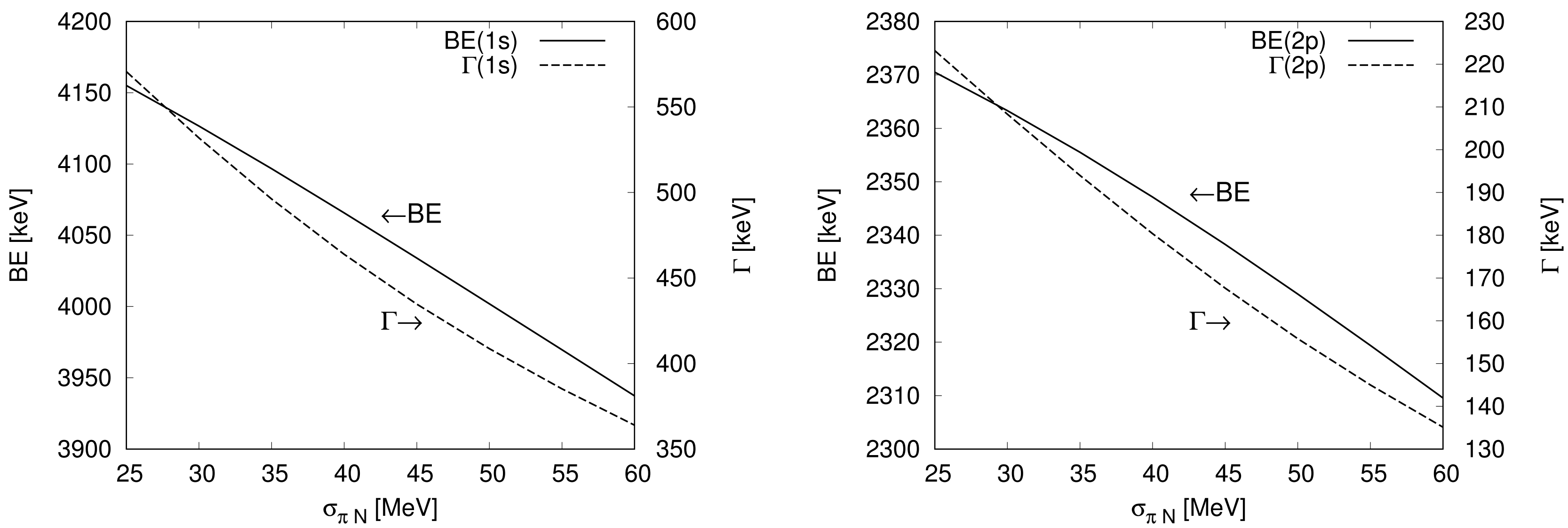


FIG. 2: Binding energies (B.E.) and widths (Γ) of the pionic $1s$ and $2p$ states in ^{123}Sn as a function of $\sigma_{\pi N}$. The results correspond to the calculation with the parameter set (I).

Sensitivity of the deeply bound pionic atom observables to the pion-nucleon sigma term $\sigma_{\pi N}$

Table 3. The calculated average shifts of the observables of the deeply bound pionic states are shown in keV for the 1 MeV change of the $\sigma_{\pi N}$ value $\Delta\sigma_{\pi N} = 1$ MeV. $\Delta(B_\pi(1s) - B_\pi(2p))$ and $\Delta(\Gamma_\pi(1s) - \Gamma_\pi(2p))$ indicate the average shifts of the differences of the binding energies and widths between the 1s and 2p states for the $\sigma_{\pi N}$ change $\Delta\sigma_{\pi N} = 1$ MeV, respectively.

Shifts of the observables for 1 MeV change of sigma term.

[keV]	^{123}Sn	^{111}Sn
$\lvert\Delta B_\pi(1s)\rvert$	6.2	7.5
$\lvert\Delta \Gamma_\pi(1s)\rvert$	5.9	12.9
$\lvert\Delta B_\pi(2p)\rvert$	1.7	1.7
$\lvert\Delta \Gamma_\pi(2p)\rvert$	2.5	3.6
$\lvert\Delta(B_\pi(1s) - B_\pi(2p))\rvert$	4.5	5.8
$\lvert\Delta(\Gamma_\pi(1s) - \Gamma_\pi(2p))\rvert$	3.4	9.3

Interesting Observables

* B(1s)-B(2p)
* Width (1s)

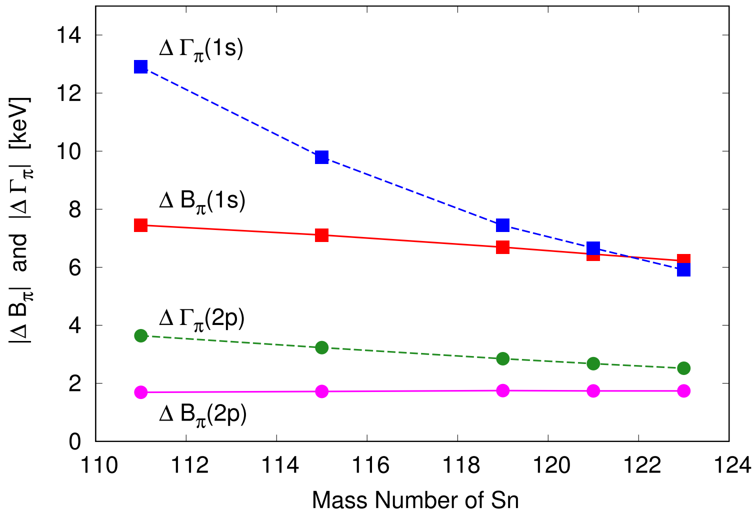


Fig. 4. The mass number dependence of the calculated shifts of the observables of the deeply bound pionic states is shown for the 1 MeV change of the $\sigma_{\pi N}$ value $\Delta\sigma_{\pi N} = 1$ MeV in Sn isotopes.

Sensitivities of Deeply Bound pionic atom observables
-- Calculated senseitivity vs. Experimental Errors --

＊<u>Typical error of data</u>

80 keV for B(1S)

10-15keV BE(1s) – BE(2p)

40 keV for Width (1s)

Sensitivities of Deeply Bound pionic atom observables
-- Calculated senseitivity vs. Experimental Errors --

＊ Typical error of data

80 keV for B(1S)

10-15keV BE(1s) – BE(2p)

40 keV for Width (1s)

＊ Expected uncertainties for sigma term are

B(1s) ~ 10 MeV

B(1s) – B(2p) ~ 2.5 MeV

Width(1s) (^{111}Sn) ~ 3MeV

Formation spectra

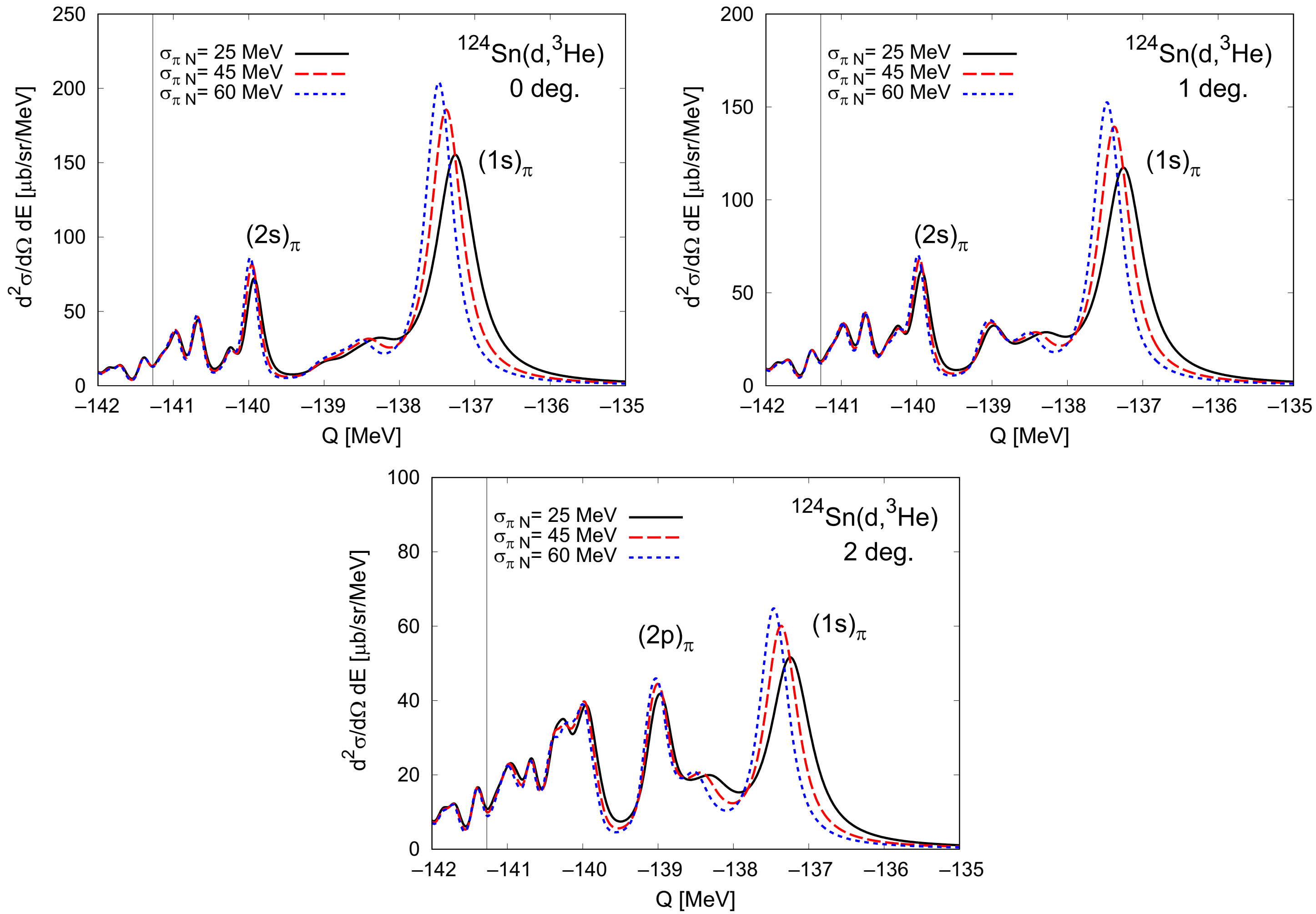
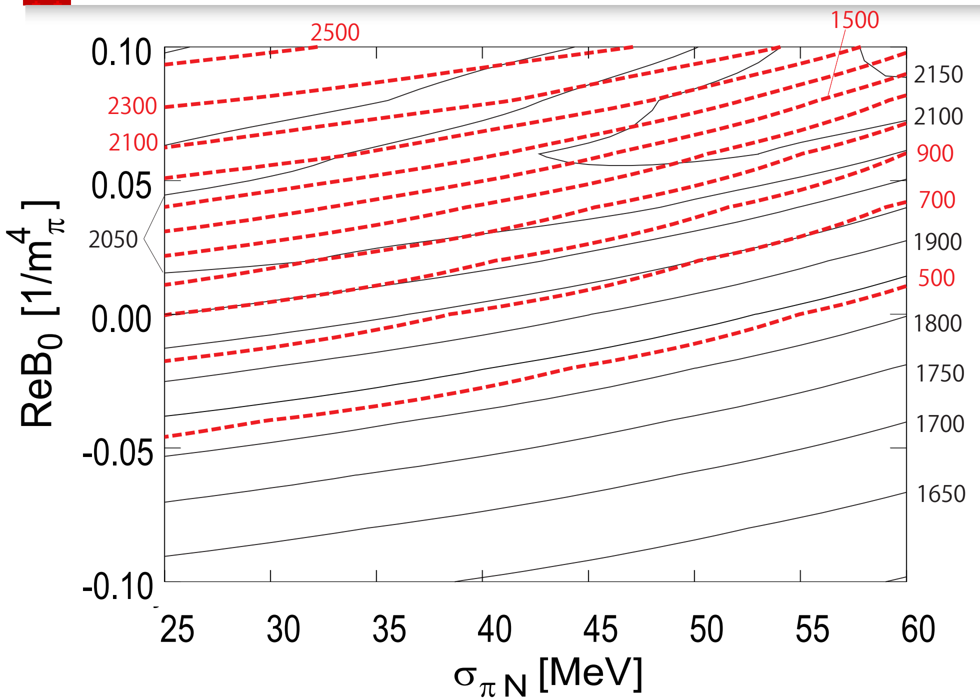


Fig. 5. Formation cross sections of the deeply bound pionic atoms in ^{123}Sn by the ^{124}Sn(d, ^{3}He) reactions are shown at different scattering angles of the emitted ^{3}He nucleus in the laboratory frame as $\theta^{\rm Lab}_{d\rm He} = 0^\circ$, 1°, 2°, respectively. The results are obtained with the density-dependent $b_0(\rho)$ and $b_1(\rho)$ parameters with three different $\sigma_{\pi N}$ values as indicated in the figure. The experimental energy resolution is assumed to be $\Delta E = 150$ keV. The contributions from the quasi-free pion production are not included in the theoretical spectra.

Difficulty: Parameter Correlations



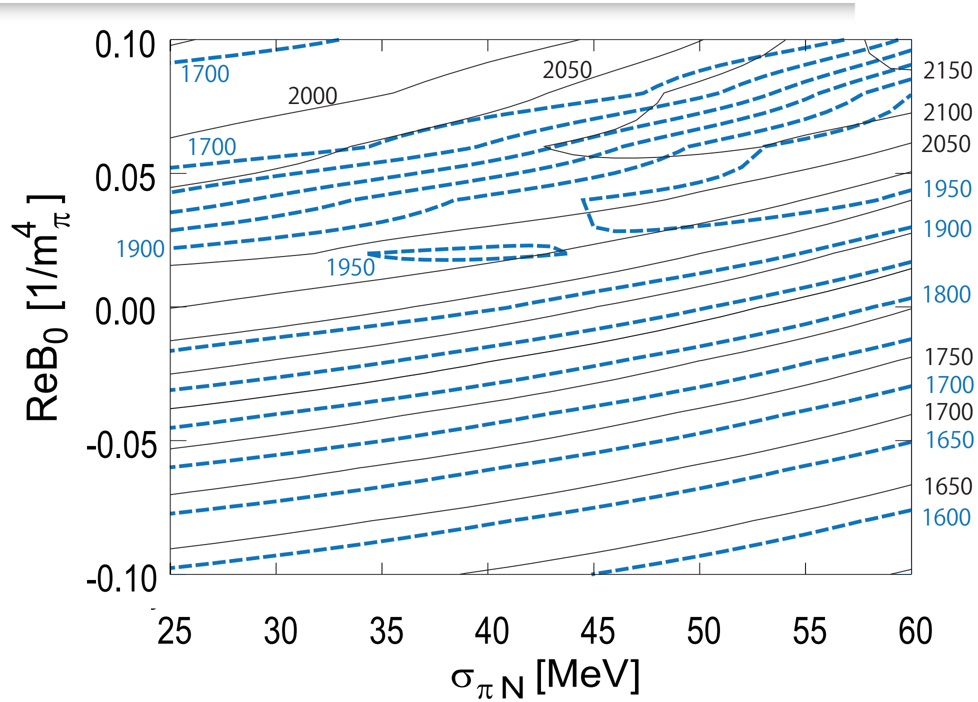


Fig. 8 The contour plots of the observables of the deeply bound pionic atoms in ^{111}Sn in the $\sigma_{\pi N} - \mathrm{Re}B_0$ plane. The numbers written in the contour plots are in the unit of keV. (Upper) The solid and the dashed lines show the contour plots of the gap of the binding energies of the $1s$ and $2p$ states $B_\pi(1s) - B_\pi(2p)$ and the width of the $1s$ state $\Gamma_\pi(1s)$ calculated with the same shape of the proton (ρ_p) and the neutron (ρ_n) distributions with the appropriate normalization as described in the text. (Lower) The solid lines are the same as those in the upper figure. The dashed lines show $B_\pi(1s) - B_\pi(2p)$ for the nuclear distributions with $R_p = R$ and $R_n = R + 0.2$ fm where R_p and R_n are the radius parameters of ρ_p and ρ_n written in the Woods-Saxon form. The parameter R is same as in Eq. (9) in the text. Between two dashed lines of 1700 keV in the positive $\mathrm{Re}B_0$ region, there exists the shallow valley structure around 1675 keV.

$$b_0(\rho) = b_0^{\mathrm{free}} - \varepsilon_1 \frac{3}{2\pi}(b_0^{\mathrm{free}2} + 2b_1^2(\rho)) \left(\frac{3\pi^2}{2}\rho\right)^{1/3} \quad b_1(\rho) = b_1^{\mathrm{free}} \left(1 - \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2}\rho\right)^{-1},$$

Summary

Deeply bound pionic atoms:

- Exotic many-body systems
- Useful system to study the pion properties in nuclei and partial restoration of chiral symmetry
- Precise information is expected for pion-nucleon sigma term

- Progress on the systematic and precise studies (e.g., Sn isotope target at RIKEN/RIBF)

- Theoretical calculations need to be improved for more precise discussions based on experimental data in future.

Some memos for pi atom

Basic Story (Prediction, Observation, <u>Feedback</u>)

- Observe meson in nucleus (B.E., Width, , , ,)
- Deduce in-medium meson properties (b1, , ,)
- Relate them to fundamental parameters
(Condensate, , ,)

Some points

∗ States with <u>well-defined quantum numbers</u>
(something like "selection rule")

∗ <u>Exclusive information</u> (s-wave isovector int., , ,)

∗ <u>Reliable connection</u> between Theoretical formula
and Exp. Result

∗ <u>Model independent</u> theoretical treatment
(... for feedback/fitting)

In reality, we need some phenomenological pieces.

Green's function method

Ref. O.Morimatsu and K. Yazaki, NPA435(1985)727-737

where $\varepsilon_\alpha = E_\alpha - E_i$ is the nucleon separation energy for the state $|\alpha\rangle$, and G is the Green function for the optical potential U, satisfying the equation

$$G = G_0 + G_0 U G \tag{10}$$

with G_0 denoting the free Green function for Σ.

Taking the imaginary part of eq. (10), we obtain the following identity:

$$\mathrm{Im}\, G = (1 + G^{+} U^{+})\, \mathrm{Im}\, G_0 (1 + UG) + G^{+}\, \mathrm{Im}\, UG\,. \tag{11}$$

The first term on the r.h.s. of eq. (11) represents the contribution from the escape of the Σ from the nucleus, while the second term is due to the conversion of the Σ into Λ because the imaginary part of U is due to this conversion effect. Let us define the following quantities:

$$\begin{aligned} S_{\mathrm{tot}}(E) &= -\tilde{f}\, \mathrm{Im}\, Gf \\ &= -\sum_\alpha \mathrm{Im} \int \mathrm{d}\boldsymbol{r}\, \mathrm{d}\boldsymbol{r}'\, f_\alpha^*(\boldsymbol{r}') G(E - \varepsilon_\alpha; \boldsymbol{r}', \boldsymbol{r}) f_\alpha(\boldsymbol{r})\,, \end{aligned} \tag{12}$$

➔ $$S_{\mathrm{esc}}(E) = -\tilde{f}(1 + G^{+} U^{+})\, \mathrm{Im}\, G_0 (1 + UG) f\,, \tag{13}$$

➔ $$S_{\mathrm{con}}(E) = -\tilde{f} G^{+}\, \mathrm{Im}\, UGf\,. \tag{14}$$

Model independent analysis (here low density expressions)

In-medium pion and partial restoration of chiral symmetry

D. Jido[a,*], T. Hatsuda[b], T. Kunihiro[a,c]

Physics Letters B 670 (2008) 109–113

$$\frac{\langle\bar{q}q\rangle^*}{\langle\bar{q}q\rangle} \simeq \left(\frac{b_1}{b_1^*}\right)^{1/2}\left(1-\gamma\frac{\rho}{\rho_0}\right)$$, where $$Z_\pi^{*1/2} \equiv \left(\frac{G_\pi^*}{G_\pi}\right)^{1/2} = 1-\gamma\frac{\rho}{\rho_0}$$

* Model independent (low density expression)
* Z_π : wave function renormalization
* Equivalent to GOR
* m_π* not necessary (but scattering length)

In-medium GOR

$$\left(F_\pi^t\right)^2 m_\pi^{*2} = -2m_q\langle\bar{q}q\rangle^*, \quad \Rightarrow \quad \left(\frac{F_\pi^t}{F_\pi}\right)^2\left(\frac{m_\pi^*}{m_\pi}\right)^2 = \frac{\langle\bar{q}q\rangle^*}{\langle\bar{q}q\rangle}$$

➔ Adopt these theoretical relations at the effective density

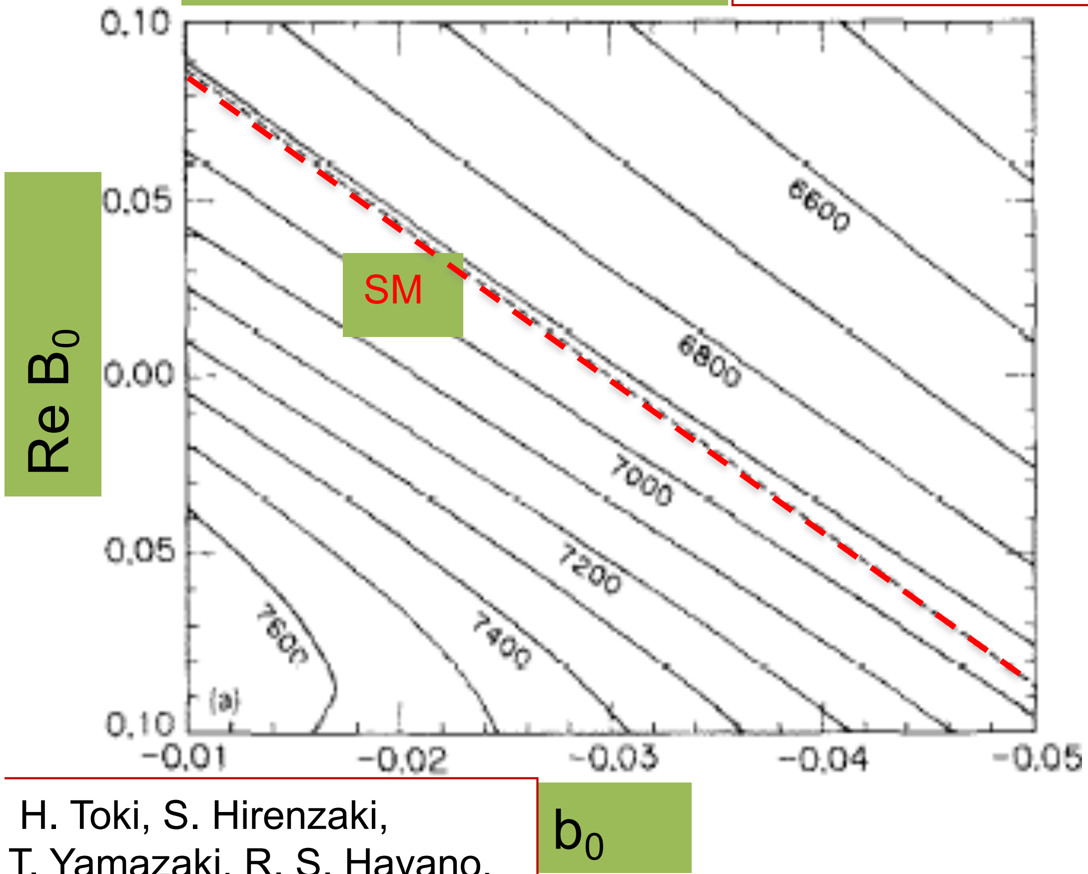
Parameter correlation and Effective density

R. Seki, K. Masutani,
Phys. Rev. C27(1983)2799

$$b_0 + \alpha_s B_0 = \beta_s = (0.003 + 0.01i)\mathrm{m}_\pi^{-1}$$
$$\alpha_s \simeq 0.23\mathrm{m}_\pi^3$$

All potentials which satisfies the SM relation between potential parameters
reproduce the experimental data.

Contour plot of B.E. ^{208}Pb 1s state



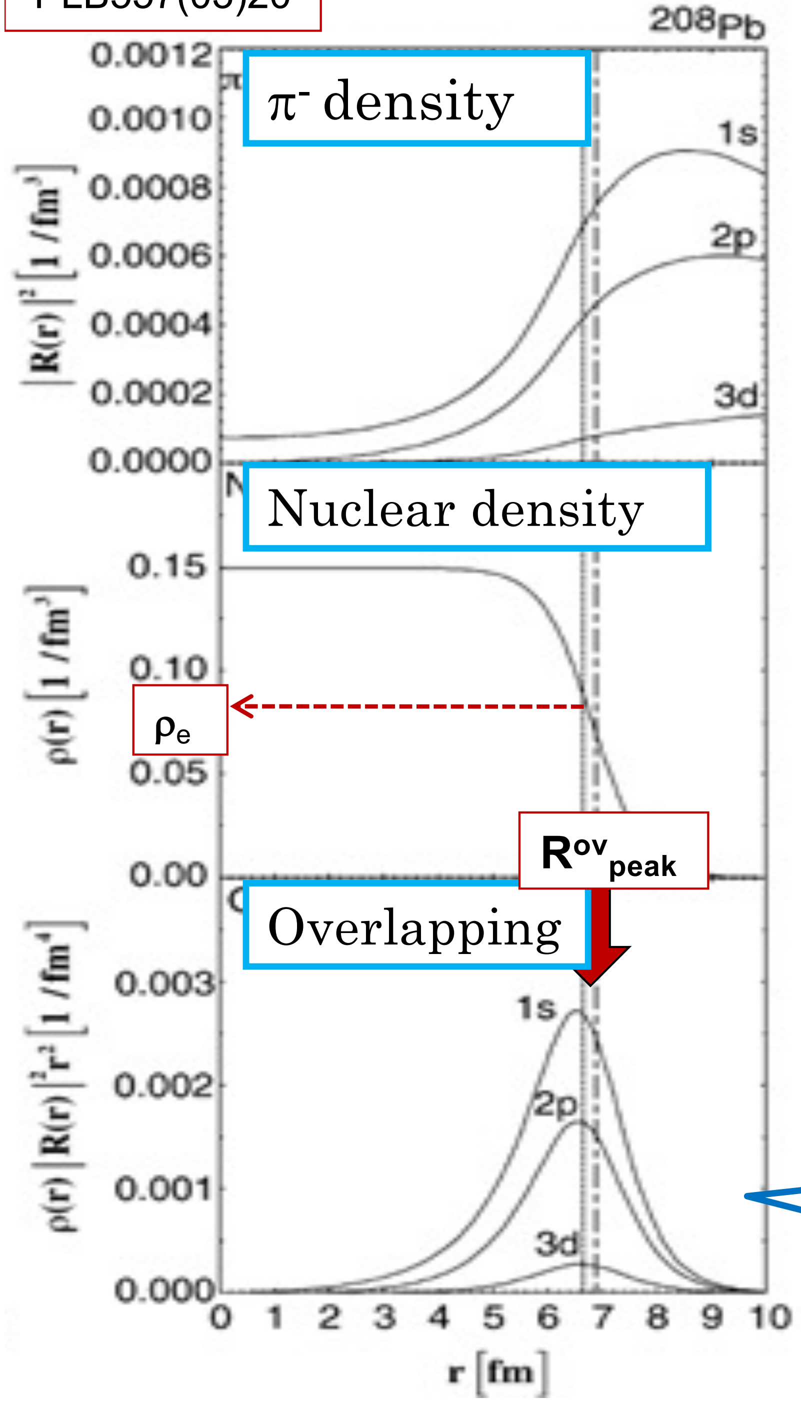
H. Toki, S. Hirenzaki,
T. Yamazaki, R. S. Hayano,
Nucl. Phys. 501(1989)653

Binding energies are almost unchanged along the line of SM relation.

=> Pionic atom properties are determined by potential strength at $\rho_e \sim 1/2\rho_0$

T. Yamazaki,
S. Hirenzaki
PLB557(03)20

Parameter correlation and Effective density



◆Peak positions of the overlapping density are almost same for all states.

◆The effective nuclear density ρ_e is almost same, $\rho_e \sim 1/2\rho_0$ for all states.

=> consistent with the expectation from the contour plot

$$S(r) = \underbrace{\rho(r)}_{N}\underbrace{|R_{nl}(r)|^2}_{\pi} r^2$$: Overlapping density