The 20th Interaction Conference
on Hadron Spectroscopy and Structure
(HADRON 2023)

Jun. 6, 2023

p- and sd-shell Λ-hypernuclei with shell model approach

Atsushi UMEYA (Nippon Inst. of Tech.)

collaborated with

Toshio MOTOBA (RCNP, Osaka Univ. / Osaka E-C Univ.)

Kazunori ITONAGA (Miyazaki Univ. / Gifu Univ.)

Introduction (1)

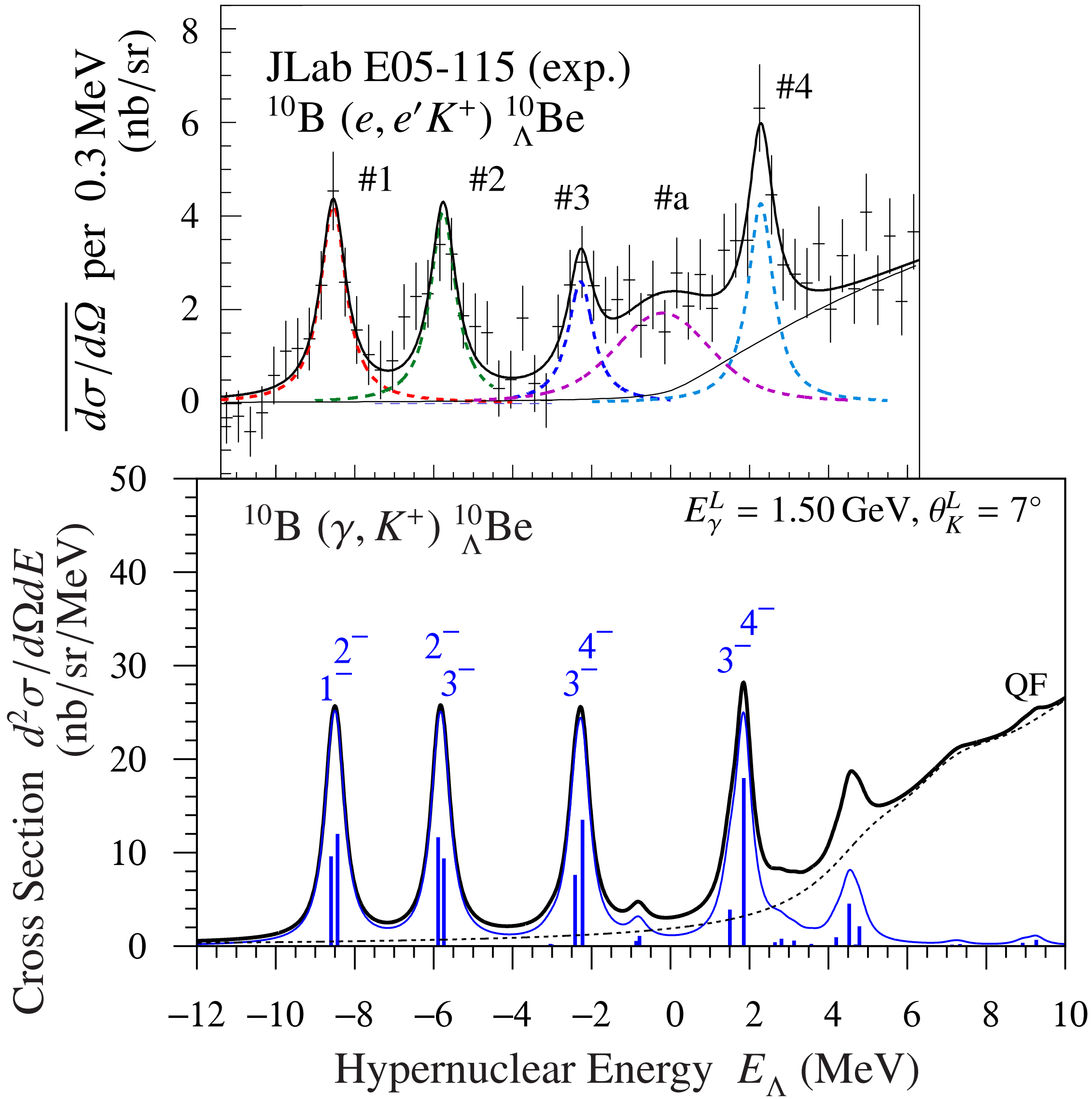
- **Hypernuclear structure studies have been progressing steadily through the K- and π-induced production reaction experiments, especially by the recent γ-ray coincidence measurements with the large volume Ge detector. Moreover a series of recent $(e, e'K^+)$ reaction experiments from the Jefferson Laboratory provide high-resolution data of the low-lying energy levels for p-shell hypernuclei. These data are quite helpful in better understanding of hyperon-nucleon interactions, though the data are still limited to about ten hypernuclear species.**
- **As the next stage of hypernuclear studies, new projects of high-intensity and high-resolution $(K^-, \pi^-\gamma)$ and $(\pi^+, K^+\gamma)$ reaction experiments are being scheduled at the J-PARC facility. New experiments are also planned at the Jefferson Laboratory.**
- **In order to meet these experimental projects, updated theoretical studies are needed for prediction and/or comparison with the coming quality data.**

Introduction (2)

- **In this talk we focus our attention on the interplay between the hyperon motion and the nuclear core states.**
- **First, we discuss that the extended shell-model calculation is successful in explaining the new peak observed in the ^{10}B $(e, e'K^+)$ $^{10}_{\Lambda}$Be experiment. It is attributed to the lowering of p_Λ (parallel) state due to the strong coupling with α-α like nuclear core deformation as already known in the case of $^{9}_{\Lambda}$Be.**
- **Second, we will show the results of new calculations for an sd-shell hypernuclear structure of $^{27}_{\Lambda}$Mg, in which the even-even core nucleus ^{26}Mg is shown to have rotational bands. Thus we see coupling of the p_Λ orbital and the core deformation. For the ^{27}Al (γ, K^+) $^{27}_{\Lambda}$Mg reaction, we also discuss the DWIA cross-section spectra that are calculated with the microscopic shell-model wave functions.**

Recent ^{10}B $(e, e'K^+)$ reaction experiment done at the Jefferson Lab

The cross sections have been obtained with high resolution.



Recent experimental result

T. Gogami *et al.*, PRC93, 034314 (2016)

Shell-model prediction

T. Motoba *et al.*, PTPS117, 123 (1994)

- Core nucleus calculated with conventional *p*-shell model
- Λ in *s*-orbit

This experiment has confirmed the major peaks (#1, #2, #3, #4) predicted by the DWIA calculations based on the normal-parity nuclear core wave functions coupled with a Λ-hyperon in *s*-orbit.

At the same time, the data also show an extra subpeak (#a) which seem difficult to be explained within the *p*-shell nuclear normal parity configurations employed so far.

Extension of the model space in the shell model ($^{10}_{\Lambda}$Be case)

Model space for ^{9}Be core

(A) conventional $\rightarrow J^{-}_{\rm core}$ $\boxed{(0s)^4\,(0p)^5}$ $(0p\text{-}0h,\ 0\hbar\omega)$

(B) extended $\rightarrow J^{+}_{\rm core}$ $\boxed{(0s)^3\,(0p)^6} \oplus \boxed{(0s)^4\,(0p)^4\,(sd)^1}$ $(1p\text{-}1h,\ 1\hbar\omega)$

Conventional model space for $^{10}_{\Lambda}$Be

(I) $\boxed{J^{-}_{\rm core} \otimes 0s^{\Lambda}} \Rightarrow {}^{10}_{\Lambda}\mathrm{Be}(J^{-})$ **(II)** $\boxed{J^{-}_{\rm core} \otimes 0p^{\Lambda}} \Rightarrow {}^{10}_{\Lambda}\mathrm{Be}(J^{+})$

Extension (1) 1*p*-1*h* (1$\hbar\omega$) core excitation is taken into account

(a) $\boxed{J^{-}_{\rm core} \otimes 0s^{\Lambda}} \Rightarrow {}^{10}_{\Lambda}\mathrm{Be}(J^{-})$ **(b)** $\boxed{J^{-}_{\rm core} \otimes 0p^{\Lambda}} \Rightarrow {}^{10}_{\Lambda}\mathrm{Be}(J^{+})$

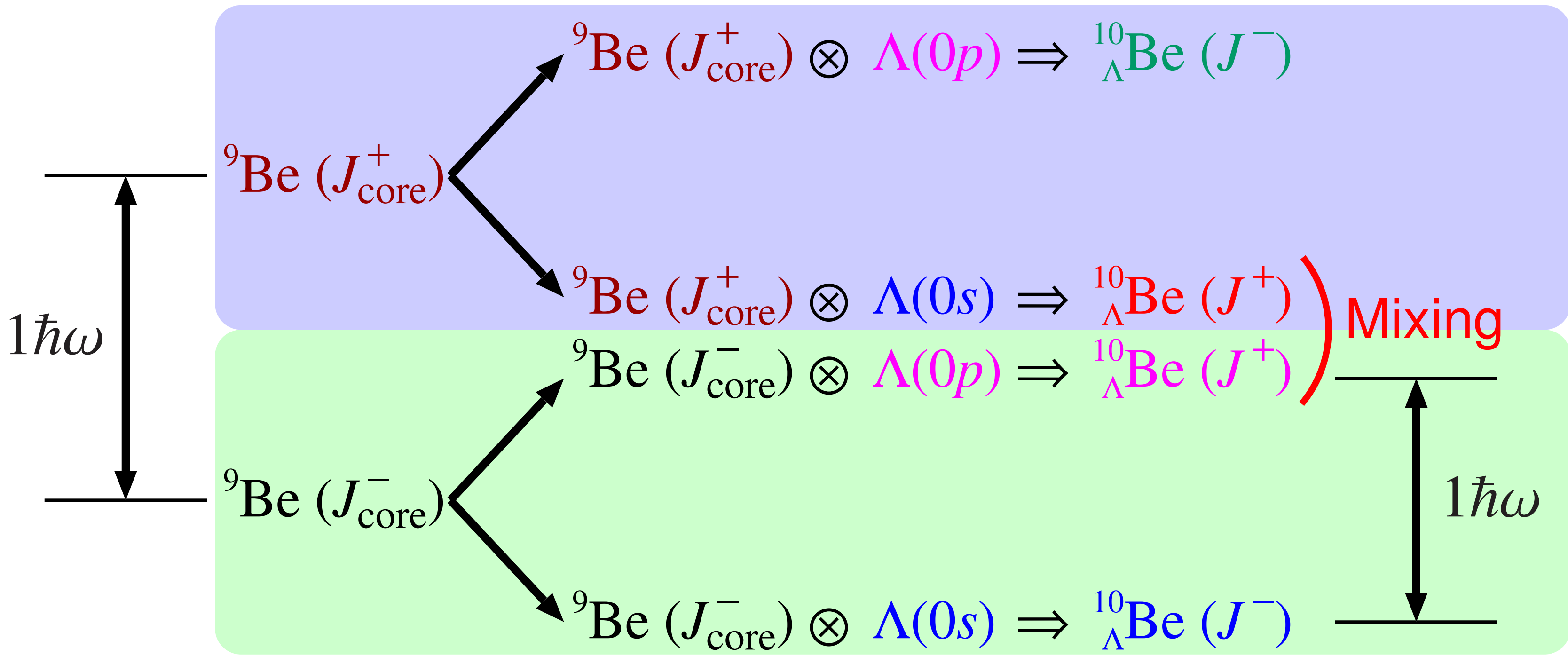
(c) $\boxed{J^{+}_{\rm core} \otimes 0s^{\Lambda}} \Rightarrow {}^{10}_{\Lambda}\mathrm{Be}(J^{+})$ **(d)** $\boxed{J^{+}_{\rm core} \otimes 0p^{\Lambda}} \Rightarrow {}^{10}_{\Lambda}\mathrm{Be}(J^{-})$

Extension (2) Configrations mixed by ΛN interaction

$$\boxed{J^{-}_{\rm core} \otimes 0s^{\Lambda}} \oplus \boxed{J^{+}_{\rm core} \otimes 0p^{\Lambda}} \Rightarrow {}^{10}_{\Lambda}\mathrm{Be}(J^{-})$$

$$\boxed{J^{-}_{\rm core} \otimes 0p^{\Lambda}} \oplus \boxed{J^{+}_{\rm core} \otimes 0s^{\Lambda}} \Rightarrow {}^{10}_{\Lambda}\mathrm{Be}(J^{+})$$

Configration mixing in ${}^{10}_{\Lambda}\mathrm{Be}$ unnatural parity states

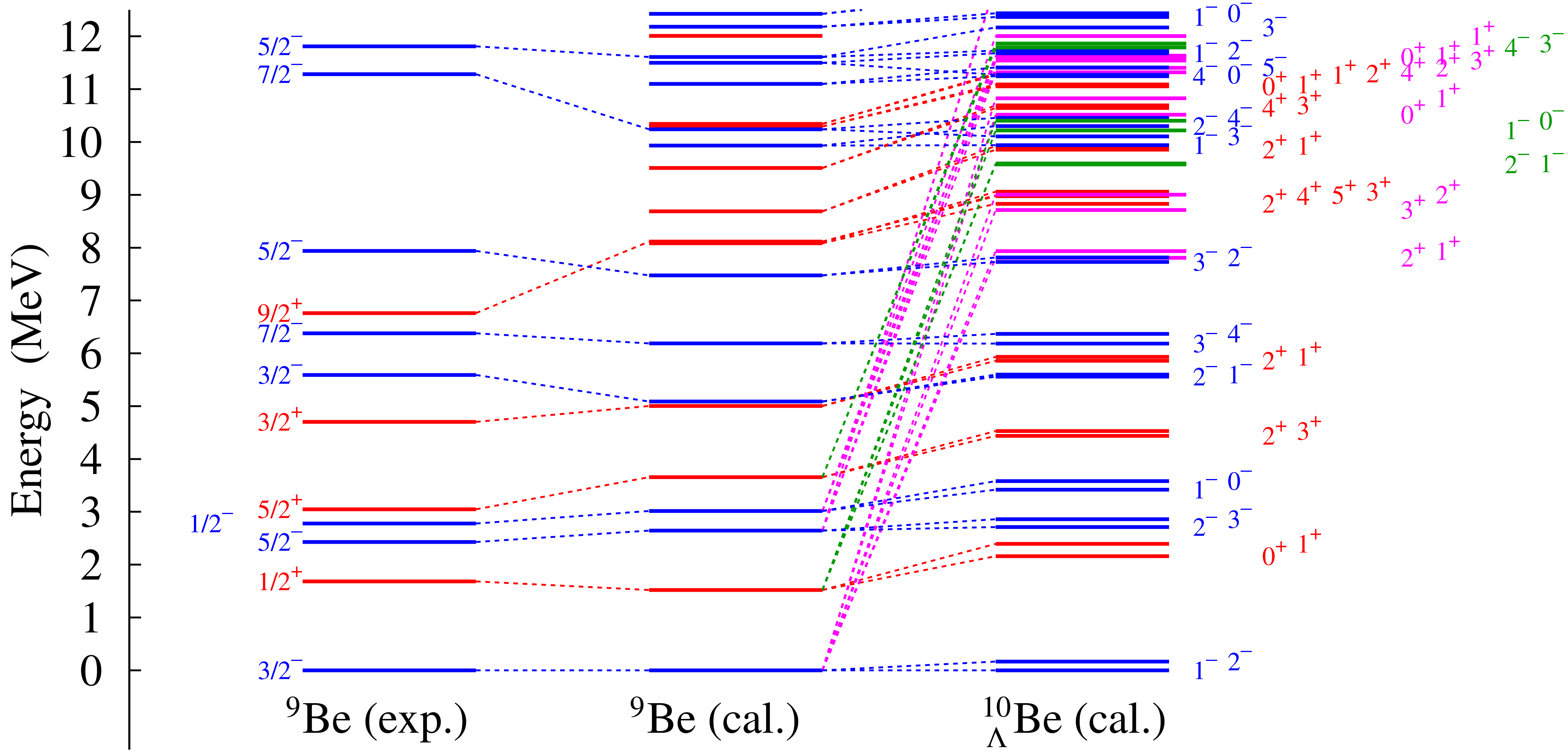


In the conventional shell model, only natural-parity nuclaer-core states (J^{-}_{core}) are taken into account. Λ particle is in the 0*s* orbit in ${}^{10}_{\Lambda}\mathrm{Be}(J^{-})$.

In ${}^{10}_{\Lambda}\mathrm{Be}(J^{+})$, the energy difference between Λ(0*s*) and Λ(0*p*) is $1\hbar\omega$, and the energy difference between ${}^{9}\mathrm{Be}(J^{-}_{\mathrm{core}})$ and ${}^{9}\mathrm{Be}(J^{+}_{\mathrm{core}})$ is $1\hbar\omega$.

By Λ*N* interaction, natural-parity nuclaer-core configurations and unnatural-parity nuclaer-core configurations can be mixed.

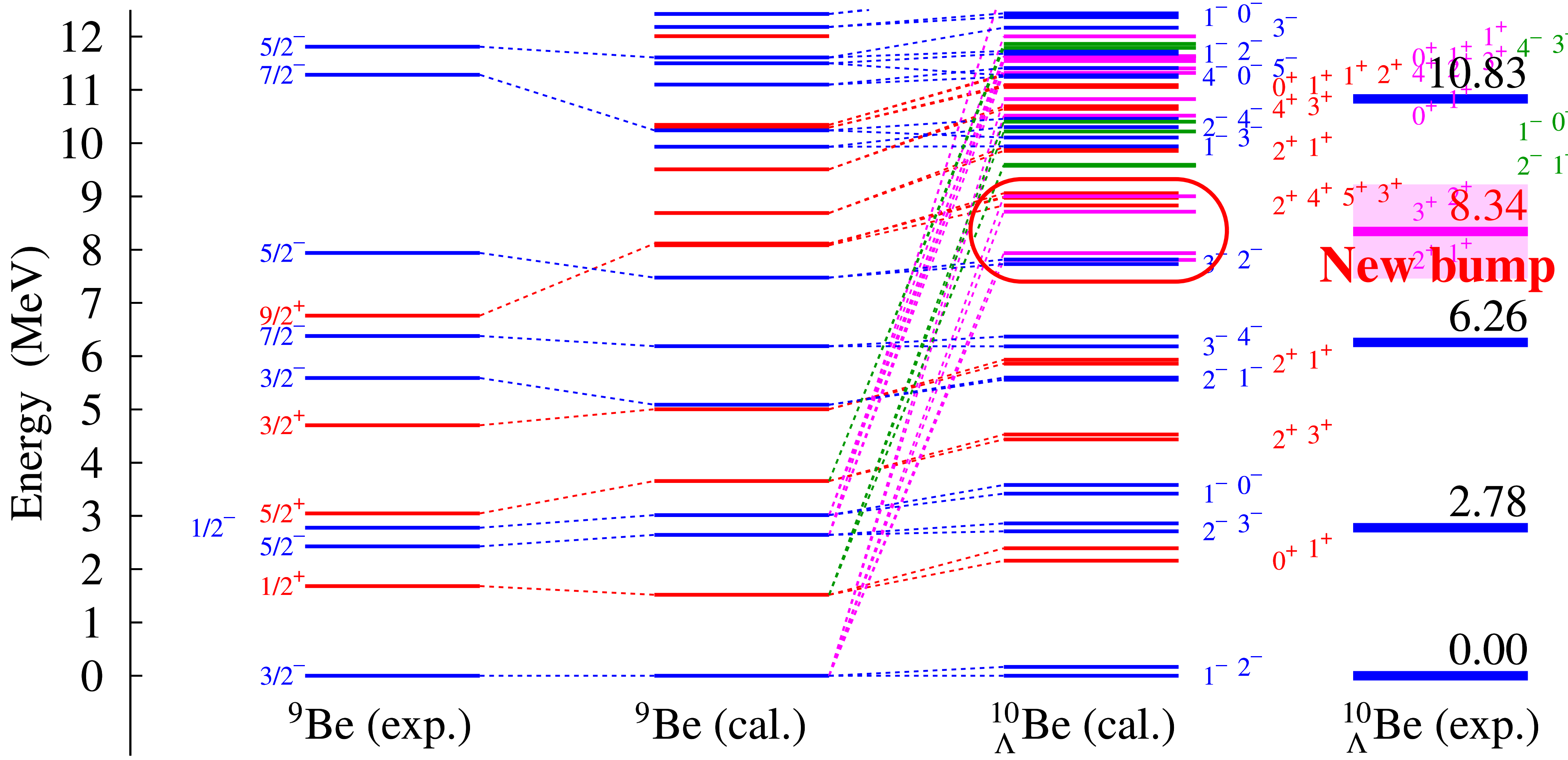
Results : Energy levels of ^{9}Be and $^{10}_{\Lambda}$Be



dominant configurations

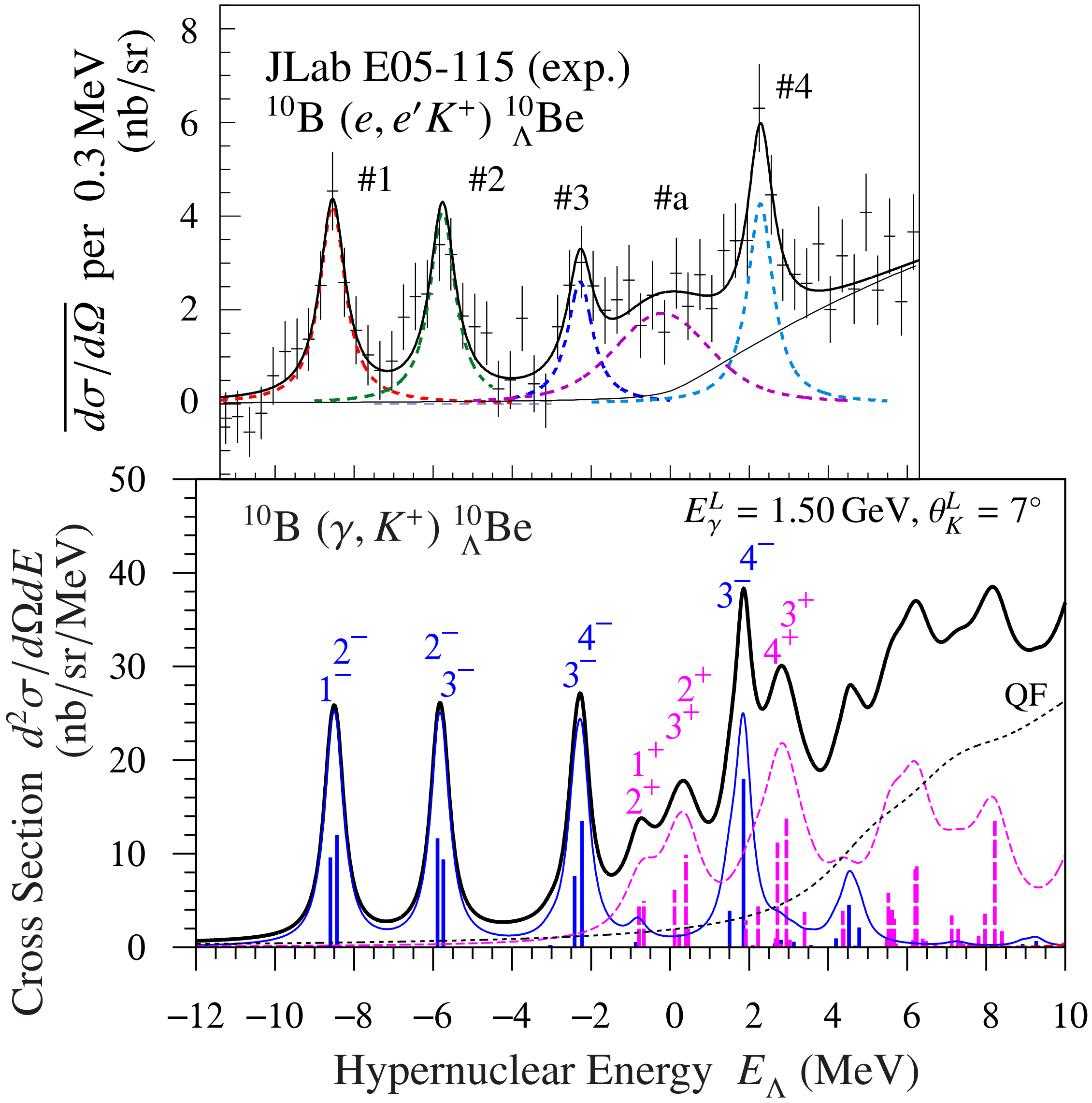
blue	J^-; $^{9}\mathrm{Be}(J^-_{\mathrm{core}}) \otimes \Lambda(0s)$	**green**	J^+; $^{9}\mathrm{Be}(J^+_{\mathrm{core}}) \otimes \Lambda(0p)$
magenta	J^+; $^{9}\mathrm{Be}(J^-_{\mathrm{core}}) \otimes \Lambda(0p)$	**red**	J^+; $^{9}\mathrm{Be}(J^+_{\mathrm{core}}) \otimes \Lambda(0s)$

Results : Energy levels of $^{10}_{\Lambda}$Be (comparison with JLab experiments)



T. Gogami *et al.*, PRC93, 034314 (2016)

Results : Cross sections of the $^{10}\mathrm{B}\ (\gamma, K^+)\ ^{10}_{\Lambda}\mathrm{Be}$ reaction



Recent experimental result

T. Gogami *et al.*, PRC93, 034314 (2016)

DWIA calculation by using Saclay-Lyon model A

Our calculation reproduces the four major peaks (#1, #2, #3, #4).

Our new calculation explains the new bump (a) as a sum of cross sections of some J^+ states.

The present calculation by using the extended shell model configurations can provide us with new positive-parity-states at right position.

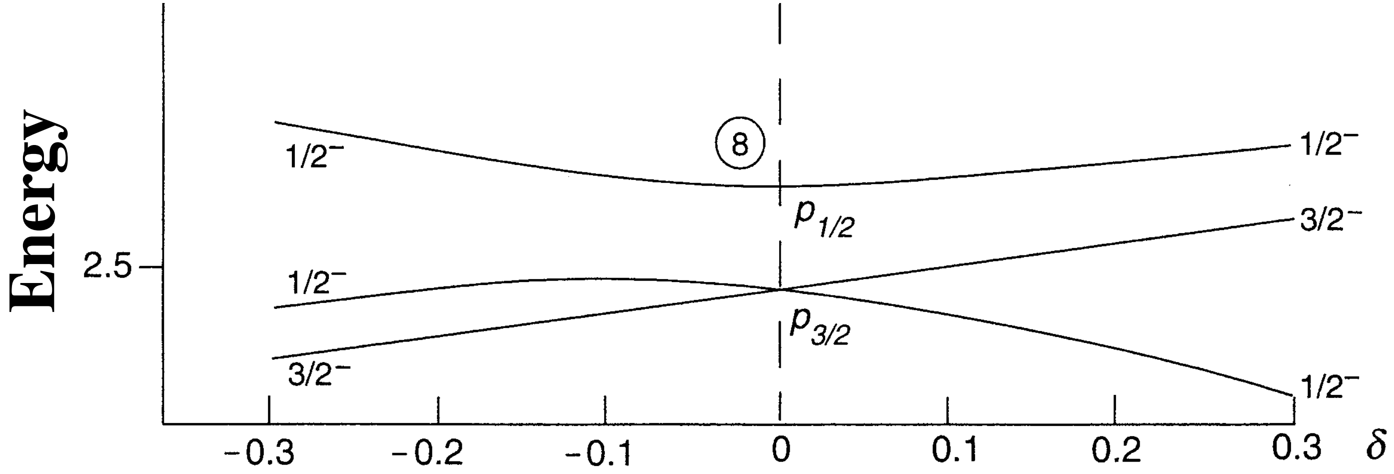
Results : Configrations of J^+ states corresponding to the new bump

$J^\pi_n(-B_\Lambda[\mathrm{MeV}])$ XS [nb/sr]	$[J^\pi_{\mathrm{core}}]j^\Lambda$	$[J^\pi_{\mathrm{core}}]j^\Lambda$	$[J^\pi_{\mathrm{core}}]j^\Lambda$
$2^+_3(-0.739)$ 4.49		$[3/2^-_1](p_{3/2}p_{1/2})^\Lambda$ 82.5%	$[5/2^-_1](p_{3/2}p_{1/2})^\Lambda$ 15.8%
$1^+_3(-0.665)$ 4.97		$[3/2^-_1](p_{3/2}p_{1/2})^\Lambda$ 79.5%	$[5/2^-_1]p^\Lambda_{3/2}$ 17.9%
$2^+_4(0.228)$ 1.43	$[5/2^+_2]s^\Lambda_{1/2}$ 87.5%	$[3/2^-_1](p_{3/2}p_{1/2})^\Lambda$ 9.4%	$[5/2^-_1](p_{3/2}p_{1/2})^\Lambda$ 2.4%
$2^+_5(0.402)$ 9.89	$[5/2^+_2]s^\Lambda_{1/2}$ 11.3%	$[3/2^-_1](p_{3/2}p_{1/2})^\Lambda$ 70.9%	$[5/2^-_1](p_{3/2}p_{1/2})^\Lambda$ 10.8%
$3^+_2(0.112)$ 6.15	$[5/2^+_2]s^\Lambda_{1/2}$ 31.6%	$[3/2^-_1]p^\Lambda_{3/2}$ 55.4%	$[5/2^-_1](p_{3/2}p_{1/2})^\Lambda$ 9.7%
$3^+_3(0.459)$ 2.43	$[5/2^+_2]s^\Lambda_{1/2}$ 67.5%	$[3/2^-_1]p^\Lambda_{3/2}$ 27.1%	$[5/2^-_1](p_{3/2}p_{1/2})^\Lambda$ 2.7%

The large mixing in four positive-parity states

Splitting of p-state in the deformed nuclei

The bump in the cross sections of $^{10}_{\Lambda}\mathrm{Be}$ will be explained by the splitting of p^{Λ}-state in the deformed core-nucleus.



Deformation parameter δ

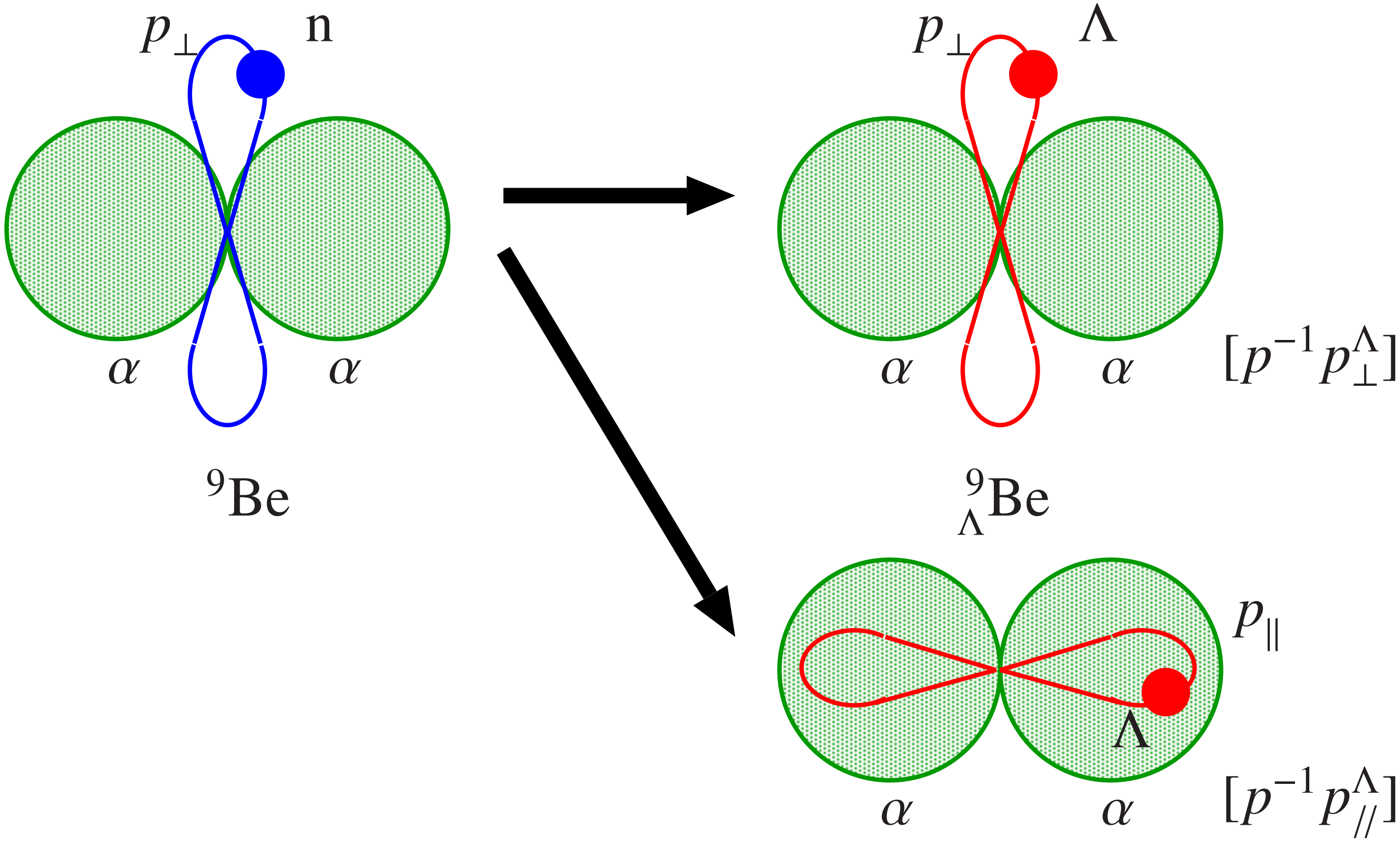
The nucleon $p_{3/2}$-state splits into two orbital states in the deformed nuclei described by the Nilsson model.

S. G. Nilsson, Mat. Fis. Medd. Dan. Vid. Selsk. 29 (1955) No. 16

Eigenvalues Ω of z-component of angular momentum operator and parities are good quantum numbers in the Nilsson diagram.

$$p_{3/2} \rightarrow \Omega^{\pi} = 1/2^{-}, 3/2^{-}$$

Splitting of p_Λ-state in the ${}^{9}_{\Lambda}$Be hypernucleus

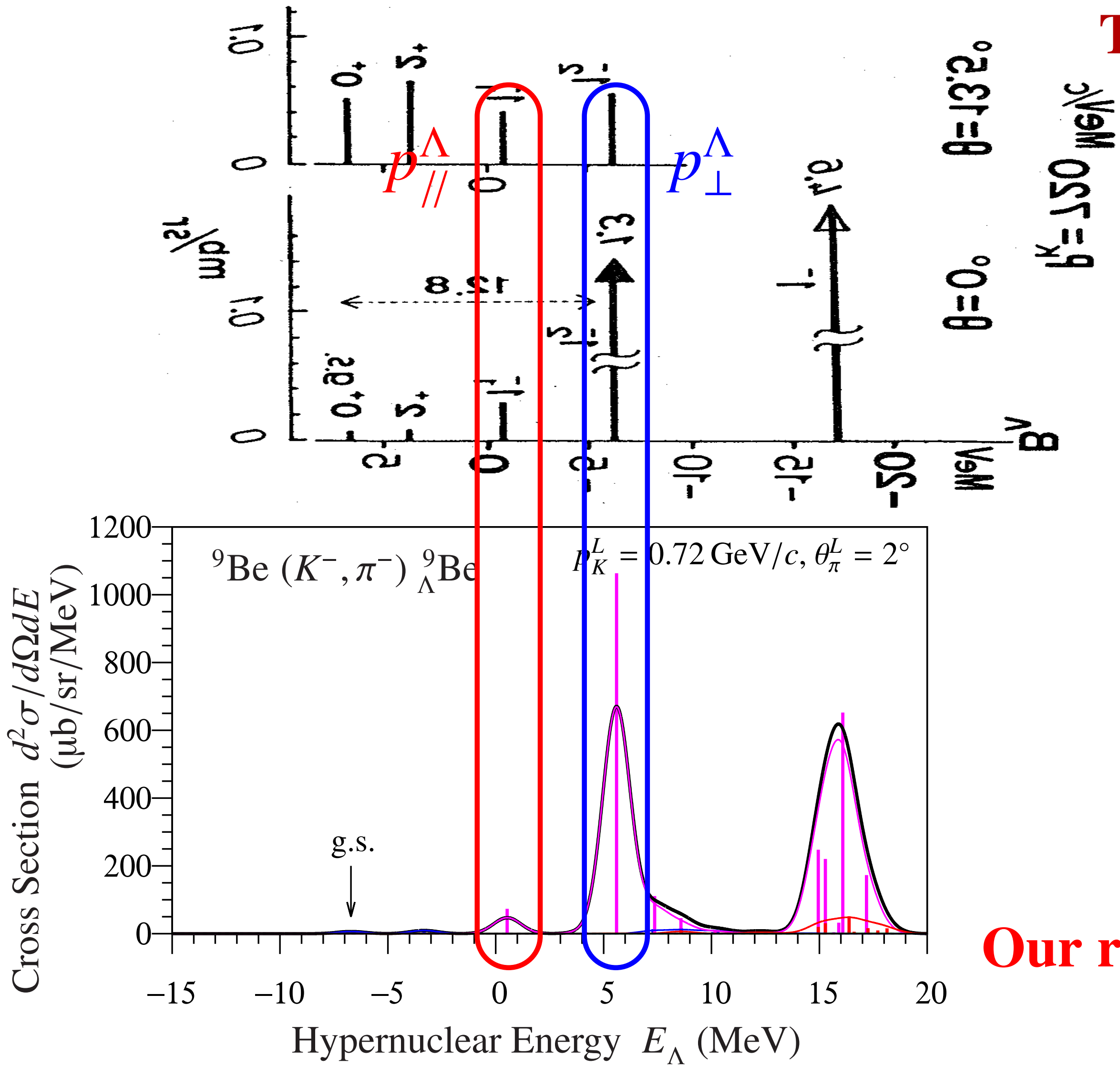


In ${}^{9}_{\Lambda}$Be, it is well known that the p_Λ-state splits into two orbital states expressed by $p_\perp$ and $p_{//}$, which is due to the strong coupling with nuclear core deformation having the α-α structure.

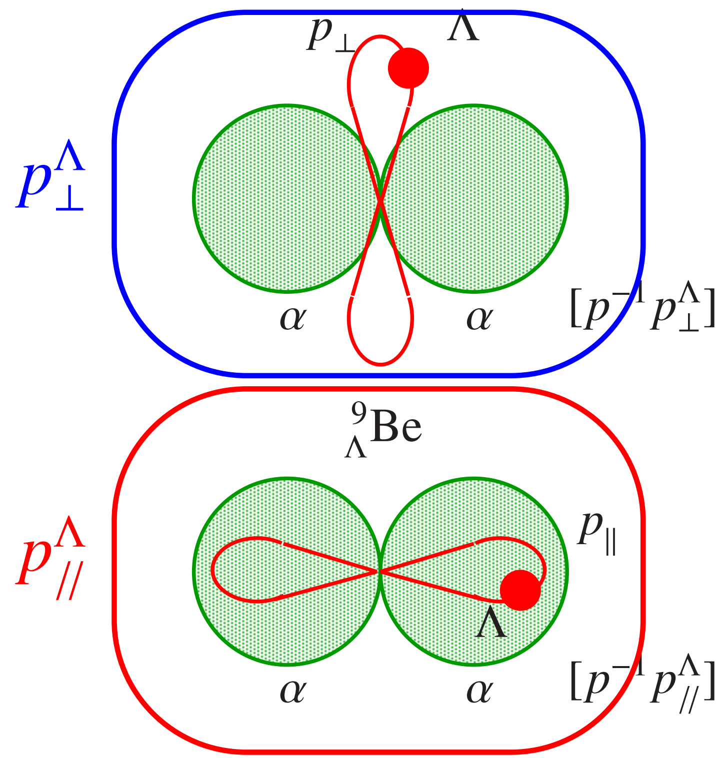
T. Motoba *et al.*, PTPS81, 42 (1985)
R. H. Dalitz, A. Gal, PRL36, 362 (1976); AP131, 314 (1981)

The DWIA cross-sections of the $^{9}\mathrm{Be}\ (K^{-}, \pi^{-})\ ^{9}_{\Lambda}\mathrm{Be}$ reaction

T. Motoba *et al.*, PTPS81, 42 (1985)



Our results

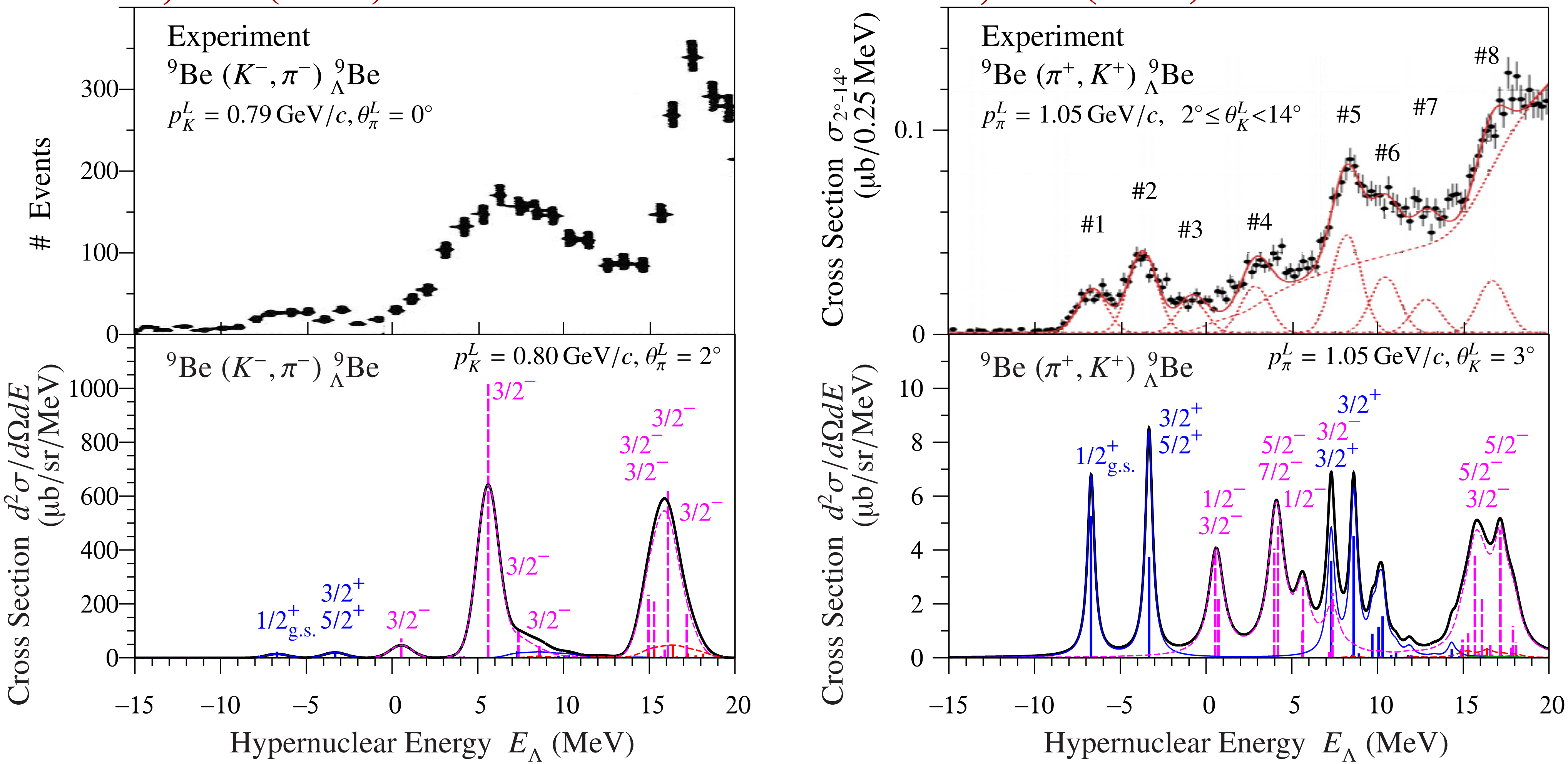


Our new results are good agreement with the cluster-model calculation, and show the $p^{\Lambda}_{//}$- and $p^{\Lambda}_{\perp}$-states.

Cross sections of $^{9}\mathrm{Be}\ (K^-, \pi^-)\ ^{9}_{\Lambda}\mathrm{Be}$ and $^{9}\mathrm{Be}\ (\pi^+, K^+)\ ^{9}_{\Lambda}\mathrm{Be}$ reactions

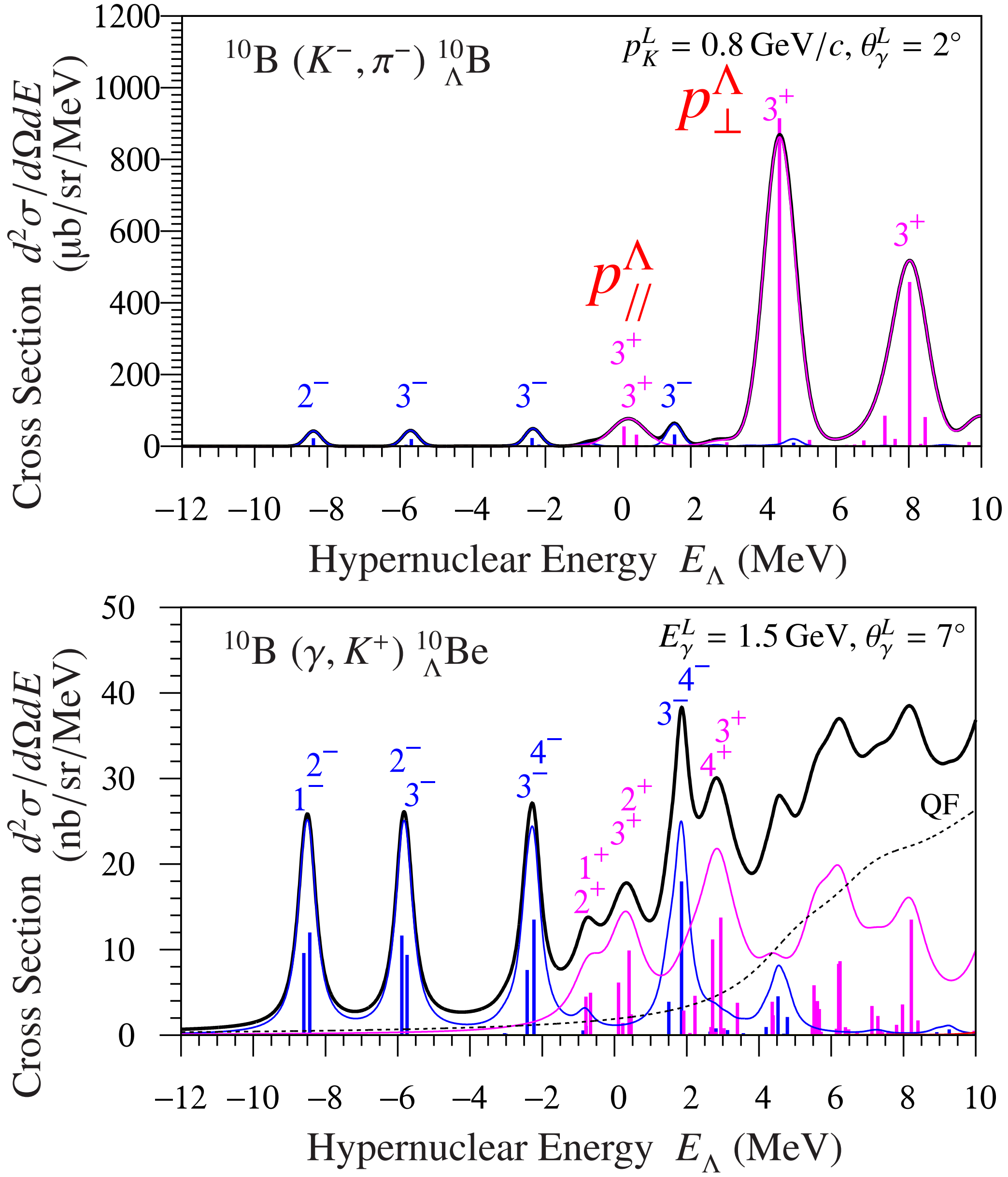
R. Bertini et al. (H-S-S Collaboration), NPA368, 365 (1981)

O. Hashimoto, H. Tamura, PPNP57, 564 (2006)



In the (π^+, K^+) production, the peak #3 is explained by the $p^{\Lambda}_{//}$-state in the present extended shell model.

Splitting of p_Λ-state in the ${}^{10}_{\Lambda}$Be and ${}^{10}_{\Lambda}$B hypernuclei (1)



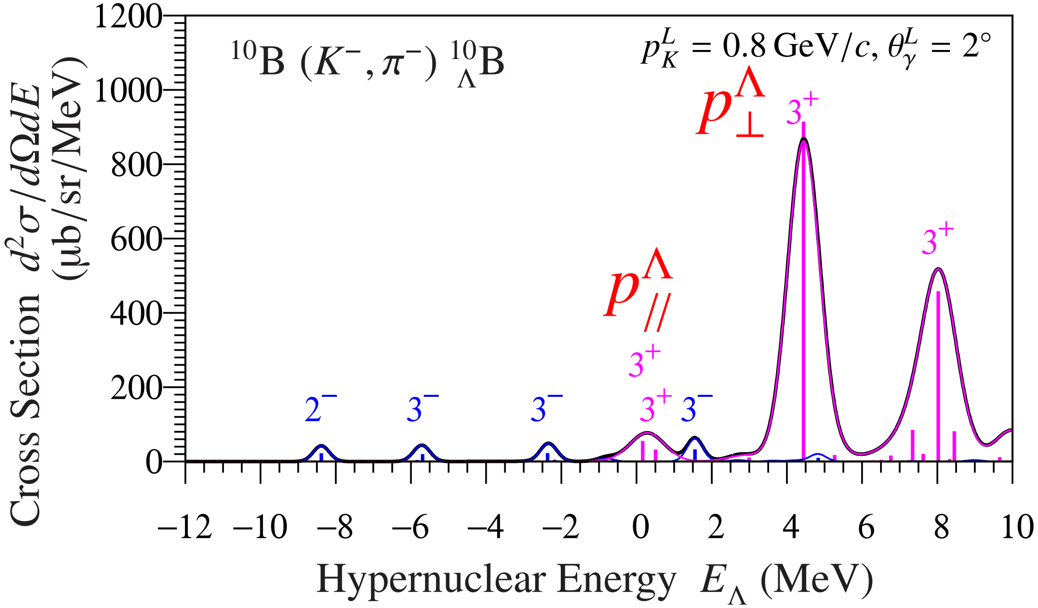
In the (K^-,π^-) reaction, the large peak at $E_\Lambda = 4.4$ MeV is a p-substitutional state via the $p^N_{3/2} \rightarrow p^\Lambda_{3/2}$, which is strongly excited by recoilless reaction.

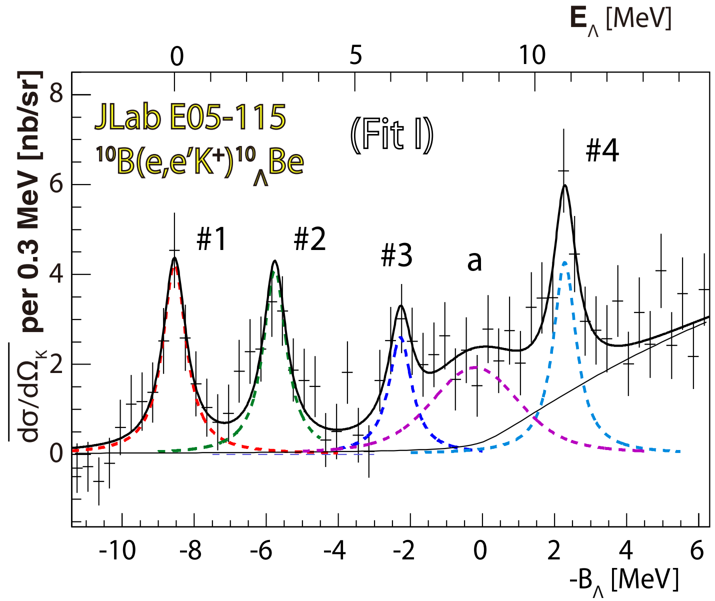
The small peak at $E_\Lambda = 0$ MeV corresponds to the new bump and is explained as a mixture of s^Λ and p^Λ states.

The large peak at $E_\Lambda = 4.4$ MeV in ${}^{10}_{\Lambda}$Be corresponds to the $[p^{-1}p^\Lambda_\perp]$ state in ${}^{9}_{\Lambda}$Be (^{9}Be analog state).

The small peak at $E_\Lambda = 0$ MeV in ${}^{10}_{\Lambda}$Be corresponds to the $[p^{-1}p^\Lambda_{//}]$ state in ${}^{9}_{\Lambda}$Be.

Splitting of p_Λ-state in the ${}^{10}_{\Lambda}\mathrm{Be}$ and ${}^{10}_{\Lambda}\mathrm{B}$ hypernuclei (2)





CONCLUDE:

$\alpha\alpha$-like core deformation causes splitting of p^Λ-states, then low-energy $p^\Lambda_{//}$ can mix with s^Λ-states.

$$[^9\mathrm{Be}(J^-) \times \Lambda(p_{//})] + [^9\mathrm{Be}(J^+) \times \Lambda(s)]$$

These parity-mixed wave functions at $E_\Lambda = 0\,\mathrm{MeV}$ can explain the extra peak #a.

Model space for sd-shell hypernuclei

We applied this extended model to sd-shell hypernuclei.

Core nucleus $^{26}\mathrm{Mg}$

$$J^{+}_{\mathrm{core}}\ (0\hbar\omega) = (^{16}\mathrm{O})(sd)^{10},\ J^{-}_{\mathrm{core}}\ (1\hbar\omega) = (^{16}\mathrm{O})(0p)^{-1}(sd)^{11}$$

Λ hyperon

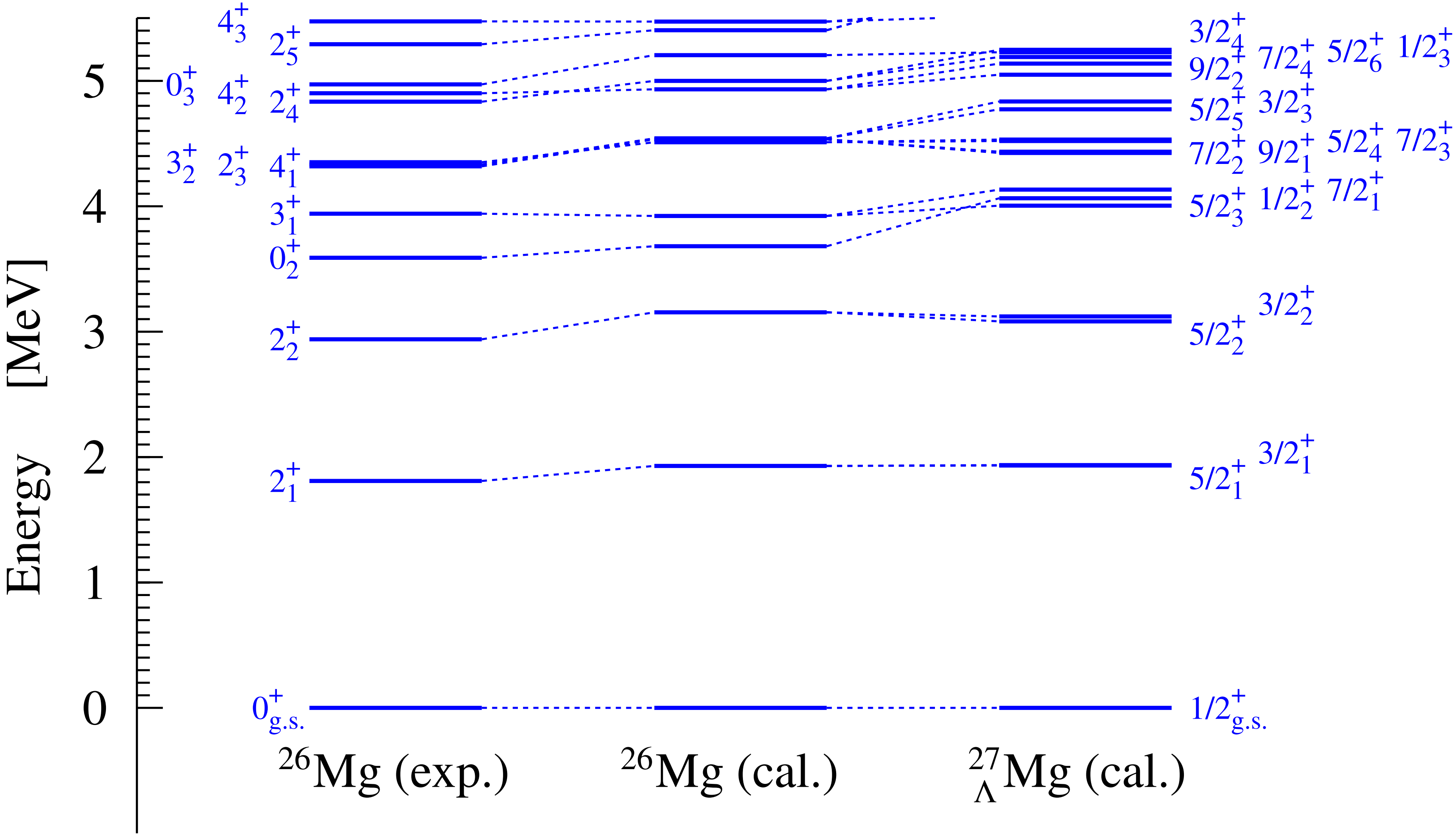
$$0s(0\hbar\omega),\ 0p(1\hbar\omega),\ sd(2\hbar\omega)$$

6 type configuration sets for $^{27}_{\Lambda}\mathrm{Mg}(J)$

$$J^{+}_{\mathrm{core}} \otimes \Lambda(0s) \to J^{+}\ (0\hbar\omega) \qquad J^{-}_{\mathrm{core}} \otimes \Lambda(0s) \to J^{-}\ (1\hbar\omega)$$
$$J^{+}_{\mathrm{core}} \otimes \Lambda(0p) \to J^{-}\ (1\hbar\omega) \qquad J^{-}_{\mathrm{core}} \otimes \Lambda(0p) \to J^{+}\ (2\hbar\omega)$$
$$J^{+}_{\mathrm{core}} \otimes \Lambda(sd) \to J^{+}\ (2\hbar\omega) \qquad J^{-}_{\mathrm{core}} \otimes \Lambda(sd) \to J^{-}\ (3\hbar\omega)$$

For sd-shell hypernucleus $^{27}_{\Lambda}\mathrm{Mg}$, we will show the first stage calculation within each of the configuration-diagonal spaces for the positive-parity core states.

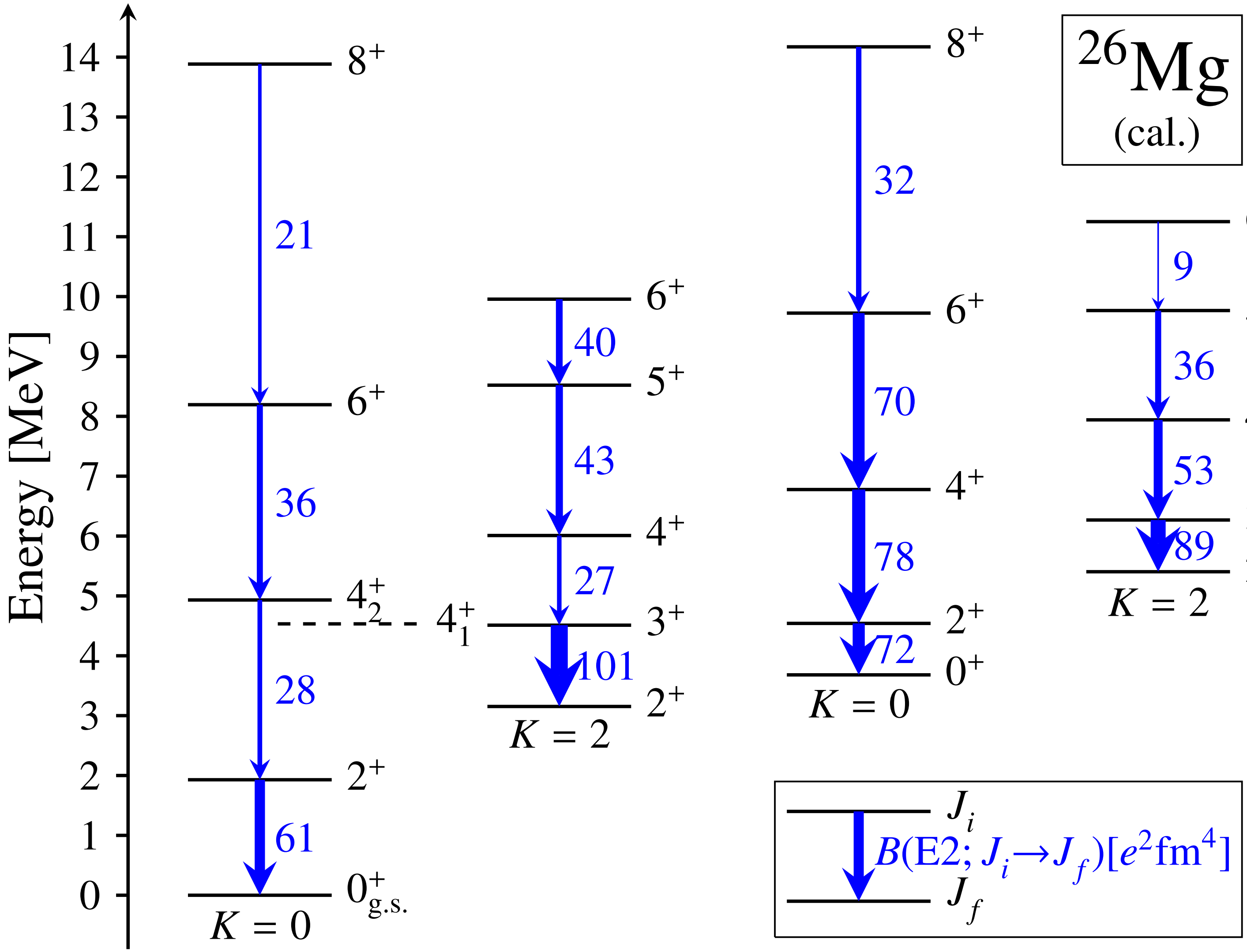
Results : Energy levels of ^{26}Mg and $^{27}_{\Lambda}$Mg



The energy spacings of doublets with the 2_1^+ and 2_2^+ core nuclei are narrow.

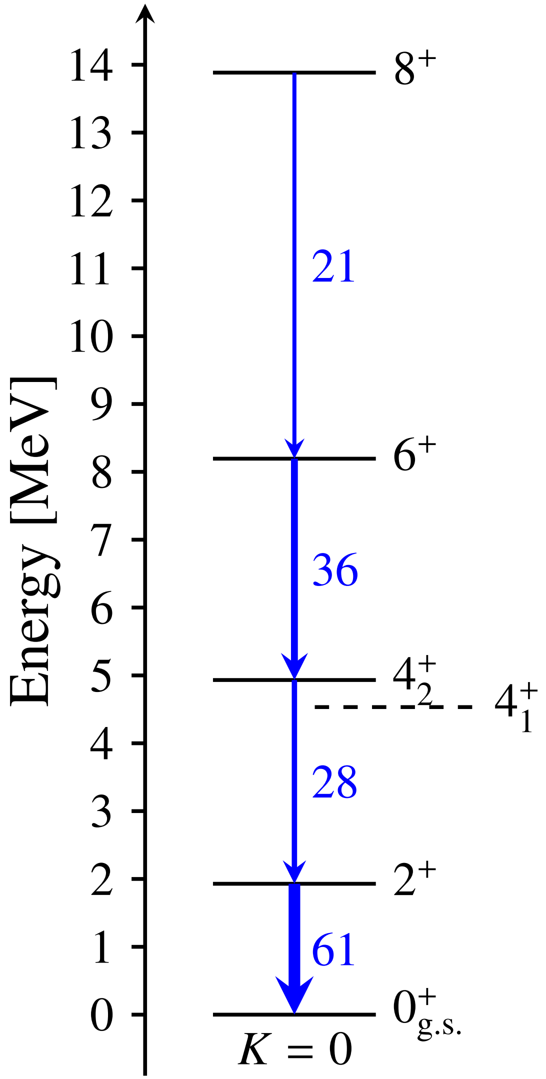
← The 2_1^+ and 2_2^+ cores are states with spin $S = 0$ of rotational bands.

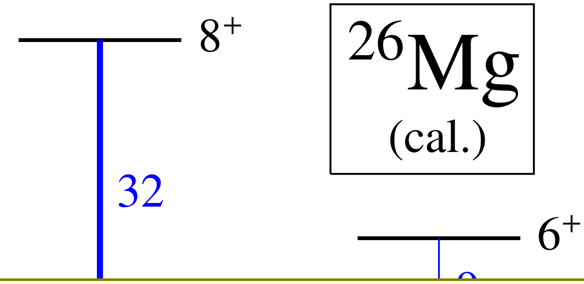
Results : Rotational bands in ^{26}Mg (1)

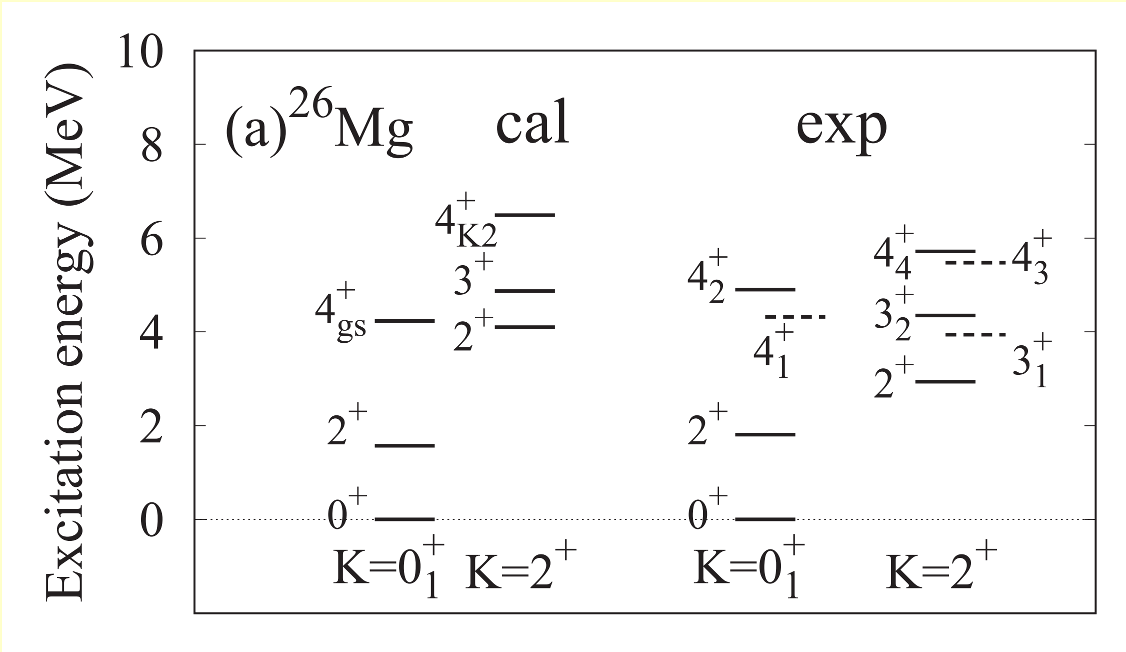


The effective charges are used to reproduce the experimental value $B(E2; 2^+_1 \rightarrow 0^+_{\mathrm{g.s.}})_{\mathrm{exp.}} = 61.3\, e^2\mathrm{fm}^4$

Results : Rotational bands in ^{26}Mg (2)







Results in the AMD calculation and the experiment

Y. Kanada-En'yo, Y. Shikata, Y. Chiba, K. Ogata,

Phys. Rev. C 102, 014607 (2020)

The effective charges are used to reproduce

Λ coupling with rotational bands

In the hypernuclear states consisting of a rotor core with $S = 0$ and a $\Lambda(0s)$ hyperon, a spin-spin ΛN interaction cannot contribute to energy.

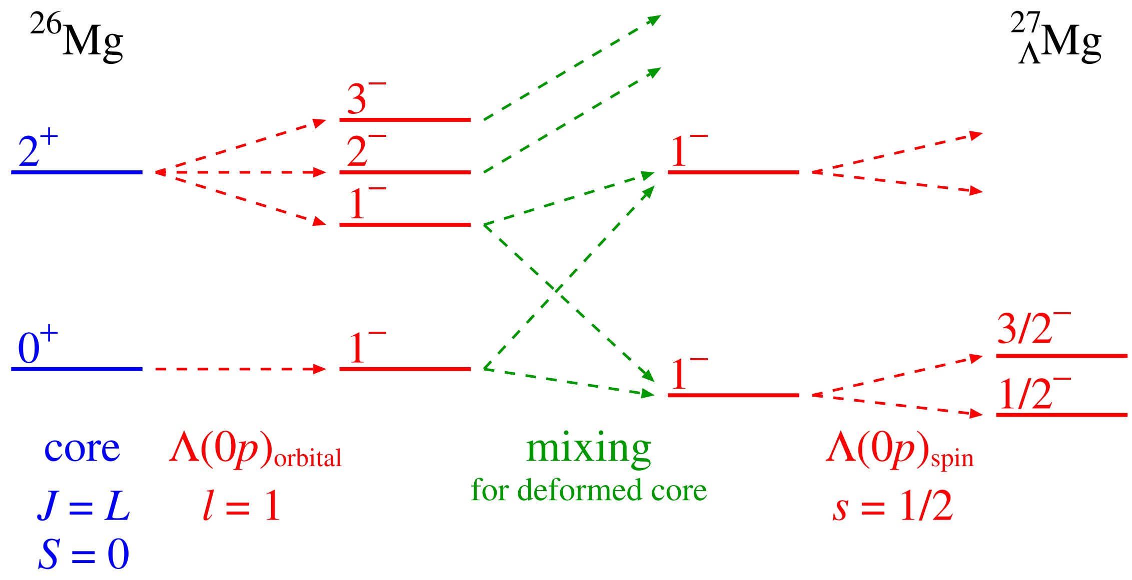
$$\langle\, (LS{=}0)J_{\text{core}} \otimes \Lambda(0s) \,|\, V_\sigma(\sigma_N \cdot \sigma_\Lambda) \,|\, (LS{=}0)J_{\text{core}} \otimes \Lambda(0s) \,\rangle = 0$$

Thus, the doublet states with the pure rotor core are degenerate.

The low-lying negative-parity states show an admixture of the $\Lambda(0p)$ configurations coupled with nuclear core states having J_{core} and $J_{\text{core}} \pm 2$. The mixing amplitude is large for the deformed core.

← It is suggested by the study for $^{145\text{–}155}_{\Lambda}$Sm by using a covariant density functional theory.

H. Mei, K. Hagino, J.M. Yao, T. Motoba, Phys. Rev. C 96, 014308 (2017)

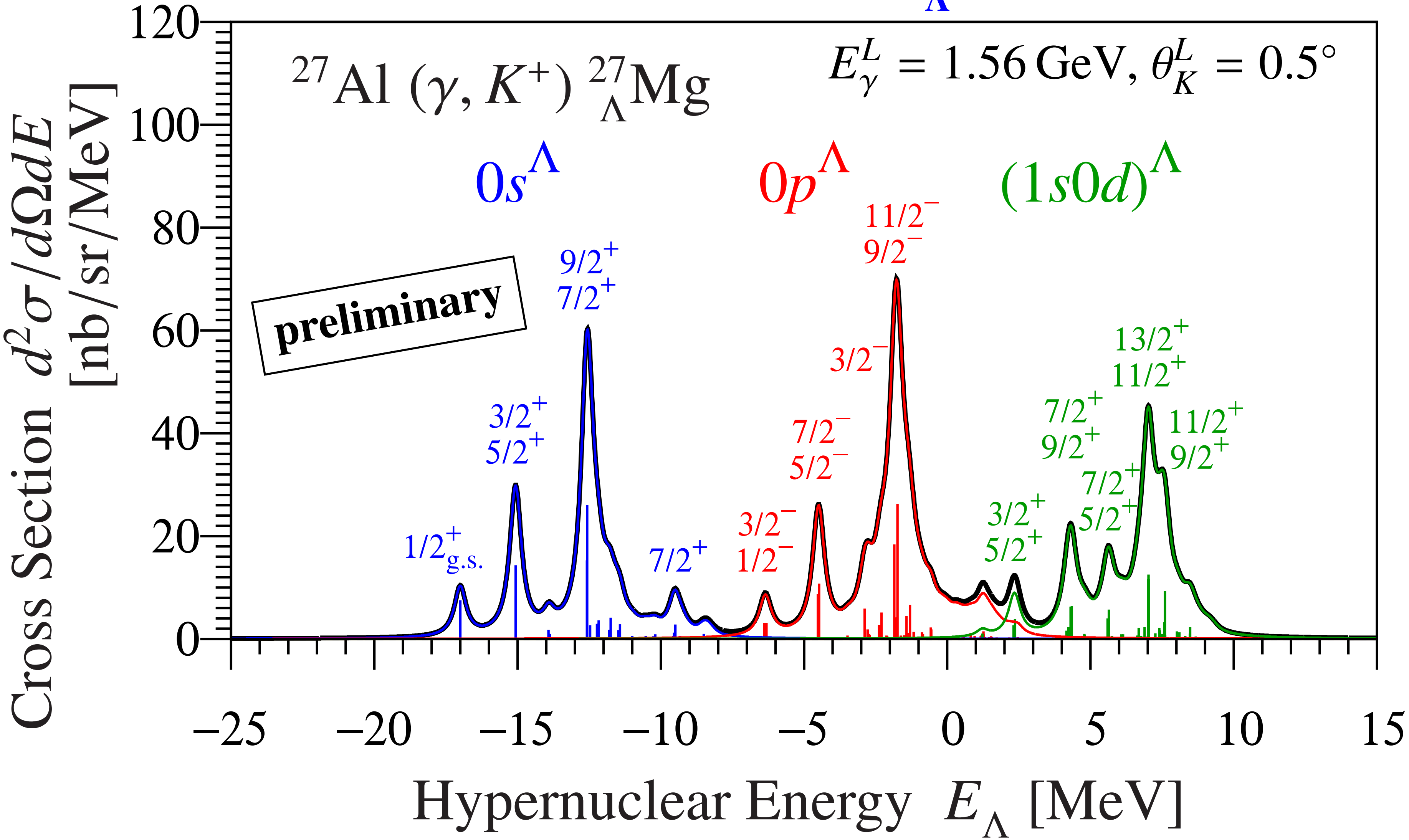


Results : Configurations of the several states of ${}^{27}_{\Lambda}\mathrm{Mg}$

state	E_x [MeV]	E_Λ [MeV]	configuration ${}^{26}\mathrm{Mg} \otimes \Lambda$	percentage [%]
$1/2^+_{\mathrm{g.s.}}$	0.000	−17.000	$0^+_{\mathrm{g.s.}} \otimes 0s^{\Lambda}_{1/2}$	99
$5/2^+_1$	1.932	−15.068	$2^+_1 \otimes 0s^{\Lambda}_{1/2}$	99
$3/2^+_1$	1.935	−15.065	$2^+_1 \otimes 0s^{\Lambda}_{1/2}$	99
$1/2^-$	10.615	−6.385	$0^+_{\mathrm{g.s.}} \otimes 0p^{\Lambda}_{1/2}$	70
			$2^+_1 \otimes 0p^{\Lambda}_{3/2}$	28
$3/2^-$	10.685	−6.315	$0^+_{\mathrm{g.s.}} \otimes 0p^{\Lambda}_{3/2}$	68
			$2^+_1 \otimes 0p^{\Lambda}_{3/2}$	15
			$2^+_1 \otimes 0p^{\Lambda}_{1/2}$	15

The mixing amplitudes are large for the negative parity states, which have the deformed core.

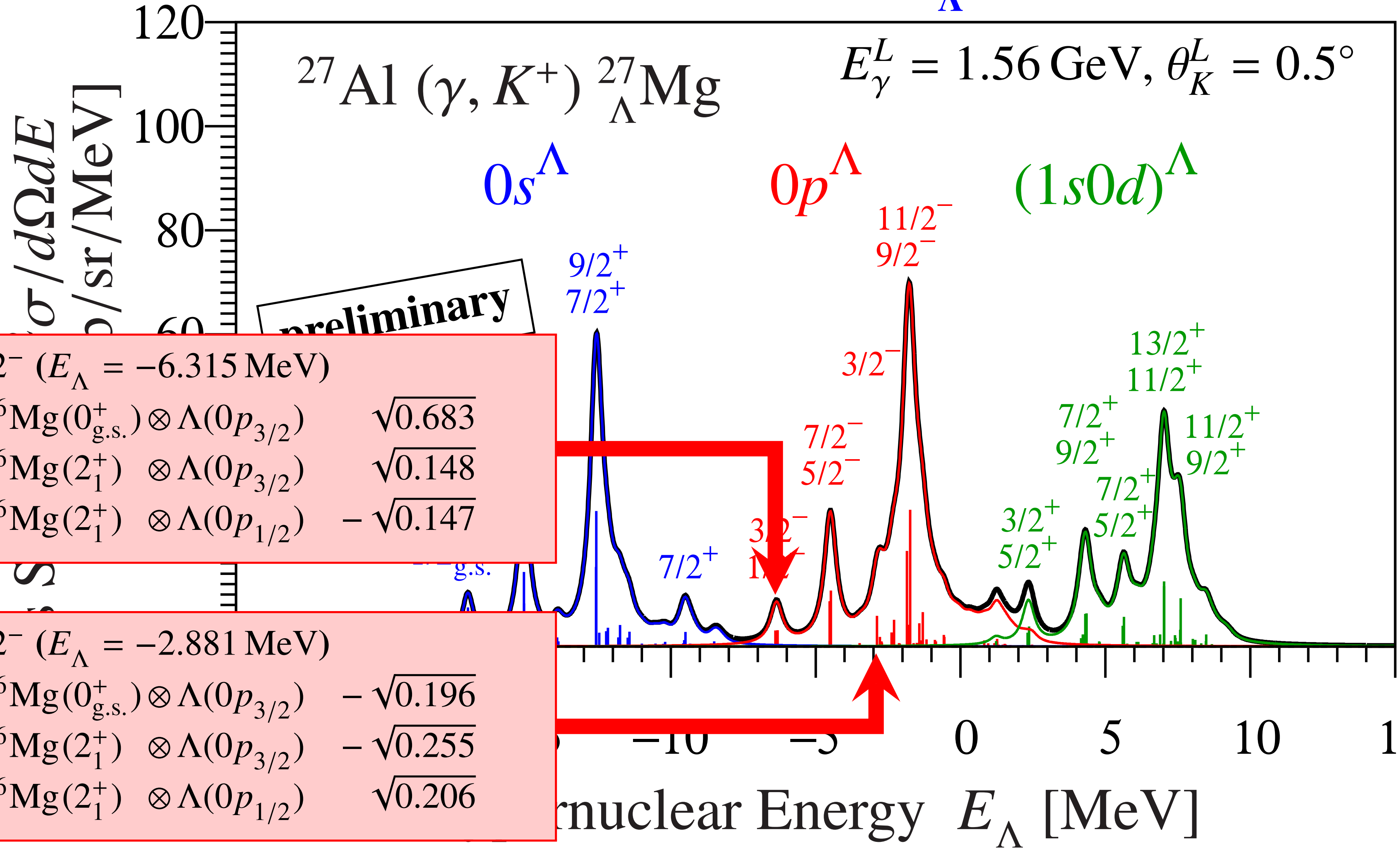
Results : Cross sections of the ^{27}Al (γ, K^+) $^{27}_{\Lambda}$Mg reaction (1)



FWHM = 0.50 MeV

blue $J^+;\ J^+_{\mathrm{core}} \otimes 0s^\Lambda$ **red** $J^-;\ J^+_{\mathrm{core}} \otimes 0p^\Lambda$ **green** $J^+;\ J^+_{\mathrm{core}} \otimes (1s0d)^\Lambda$

Results : Cross sections of the ^{27}Al (γ, K^+) $^{27}_{\Lambda}$Mg reaction (2)



$3/2^-$ ($E_\Lambda = -6.315$ MeV)

$^{26}\mathrm{Mg}(0^+_{\mathrm{g.s.}}) \otimes \Lambda(0p_{3/2})$	$\sqrt{0.683}$
$^{26}\mathrm{Mg}(2^+_1) \otimes \Lambda(0p_{3/2})$	$\sqrt{0.148}$
$^{26}\mathrm{Mg}(2^+_1) \otimes \Lambda(0p_{1/2})$	$-\sqrt{0.147}$

$3/2^-$ ($E_\Lambda = -2.881$ MeV)

$^{26}\mathrm{Mg}(0^+_{\mathrm{g.s.}}) \otimes \Lambda(0p_{3/2})$	$-\sqrt{0.196}$
$^{26}\mathrm{Mg}(2^+_1) \otimes \Lambda(0p_{3/2})$	$-\sqrt{0.255}$
$^{26}\mathrm{Mg}(2^+_1) \otimes \Lambda(0p_{1/2})$	$\sqrt{0.206}$

FWHM = 0.50 MeV

blue $J^+;\ J^+_{\mathrm{core}} \otimes 0s^\Lambda$ **red** $J^-;\ J^+_{\mathrm{core}} \otimes 0p^\Lambda$ **green** $J^+;\ J^+_{\mathrm{core}} \otimes (1s0d)^\Lambda$

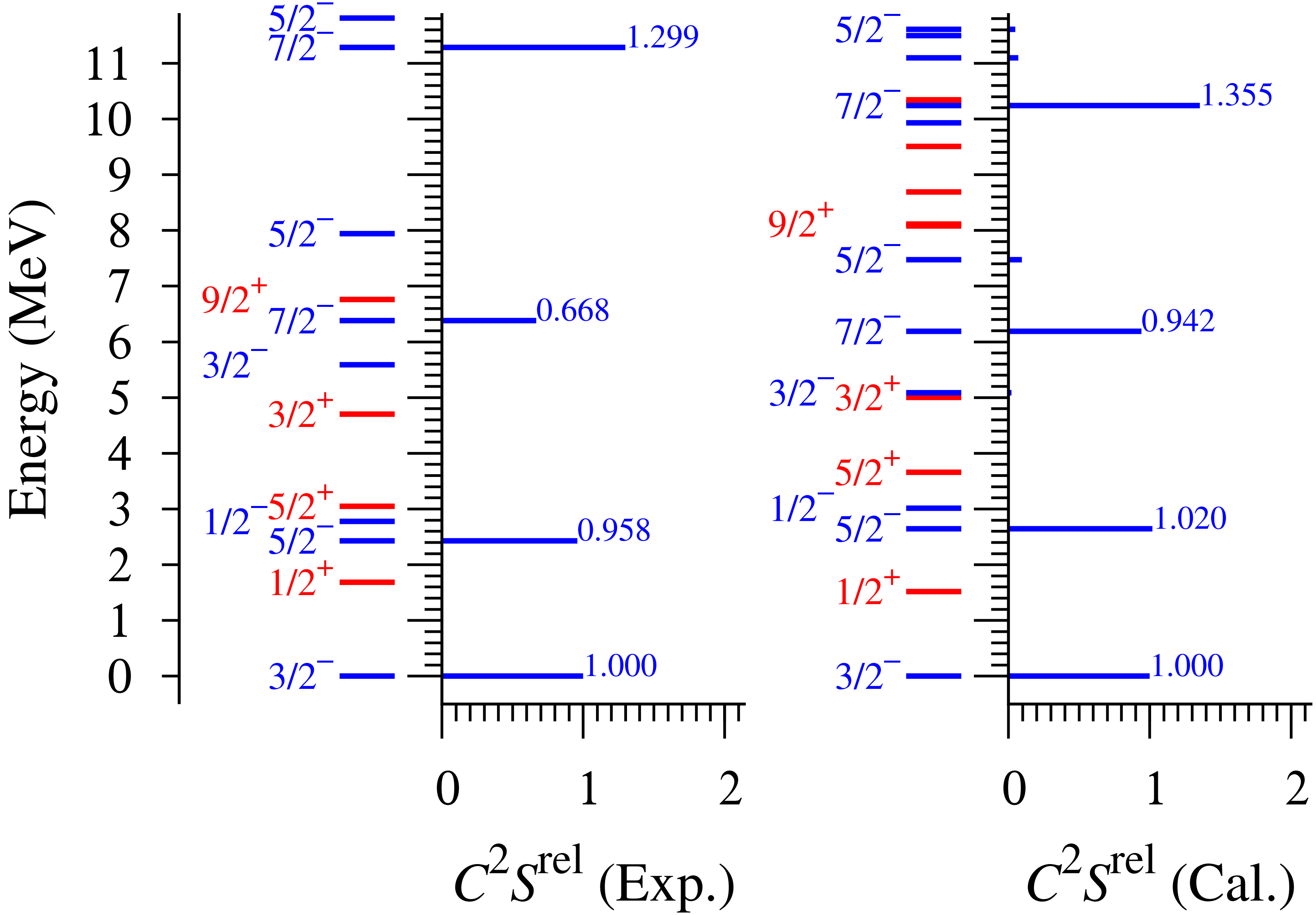
Summary

We have calculated the energy levels and the production cross sections for the $^{10}_{\Lambda}\mathrm{Be}$ and $^{27}_{\Lambda}\mathrm{Mg}$ hypernuclei by using the shell-model wave functions.

- **Strong coupling between p-state Λ and core deformation is realized in $^{10}_{\Lambda}\mathrm{Be}$ and $^{27}_{\Lambda}\mathrm{Mg}$.**
- **For deformed core, p^{Λ}-state splits into $p^{\Lambda}_{//}$ and $p^{\Lambda}_{\perp}$.**
- **In $^{10}_{\Lambda}\mathrm{Be}$, the lower $p^{\Lambda}_{//}$ comes down in energy and $[{}^{9}\mathrm{Be}(J^{-})\times\Lambda(p_{//})]$ couples easily with $[{}^{9}\mathrm{Be}(J^{+})\times\Lambda(s)]$.**
- **In the energy levels of $^{27}_{\Lambda}\mathrm{Mg}$, the energy spacings of doublets with the 2^{+}_{1} and 2^{+}_{2} core nuclei are narrow because these cores are states with spin $S=0$ of rotational bands.**
- **In $^{27}_{\Lambda}\mathrm{Mg}$, the low-lying negative-parity states show an admixture of the $\Lambda(0p)$ configurations coupled with nuclear core states having 0^{+} and 2^{+}, which are deformed.**

Backup

Results : Spectroscopic factors of the pickup reaction, $^{10}\mathrm{B} \rightarrow {}^{9}\mathrm{Be}$



Results : Cross sections of the $^{10}\mathrm{B}\ (\gamma, K^{+})\ ^{10}_{\Lambda}\mathrm{Be}$ reaction

E_γ = 1.5 GeV
θ = 7 deg

EXP = T. Gogami et al, PRC93 (2016)

^{9}Be (J_i)

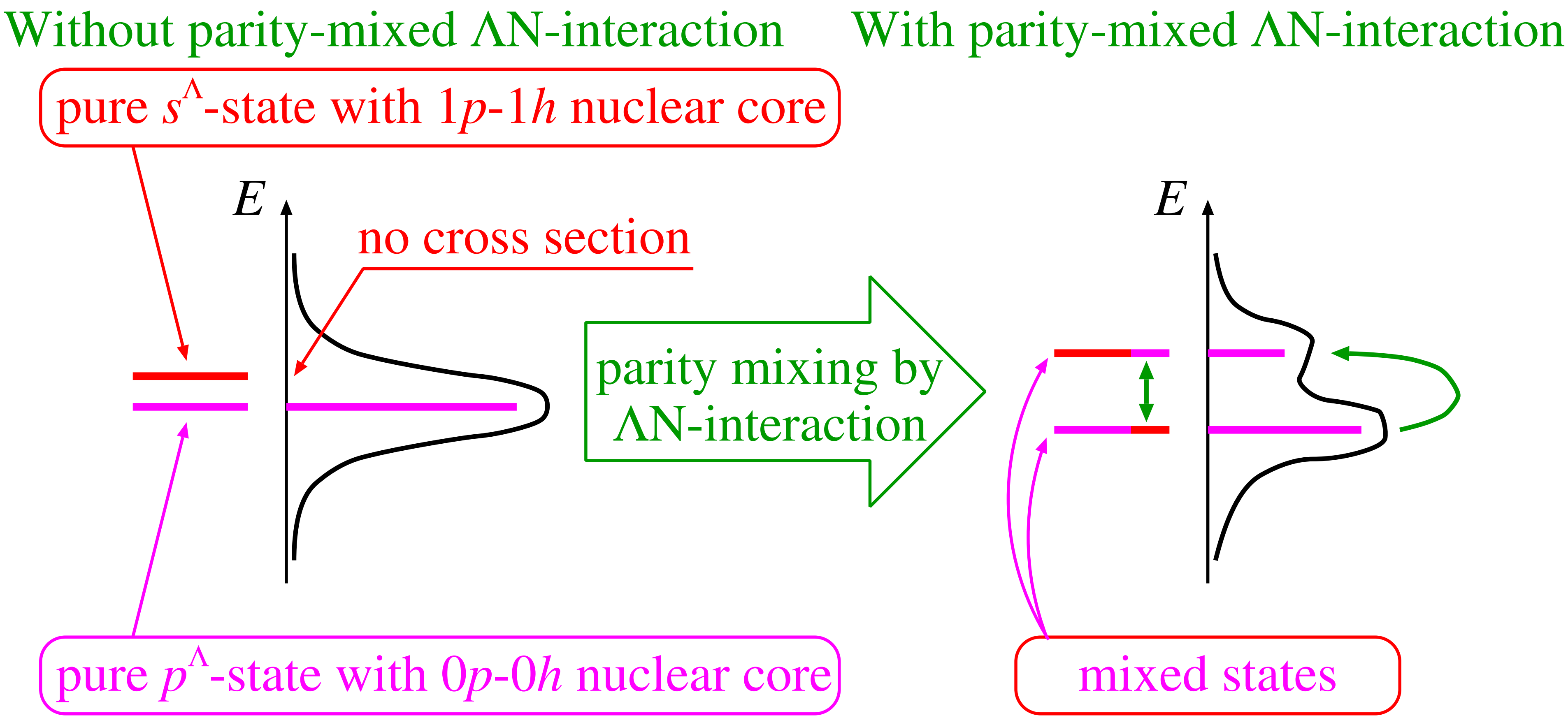
J_i	E_i (exp) C2S	E_i (cal) C2S
3/2⁻	0.000 *1.0(rel)*	0.000 *1.0(rel)*
5/2⁻	2.429 *0.958*	2.644 *1.020*
7/2⁻	6.380 *0.668*	6.189 *0.942*
7/2⁻	11.283 *1.299*	10.241 *1.355*

$_{\Lambda}^{10}$Be (J_k) CAL

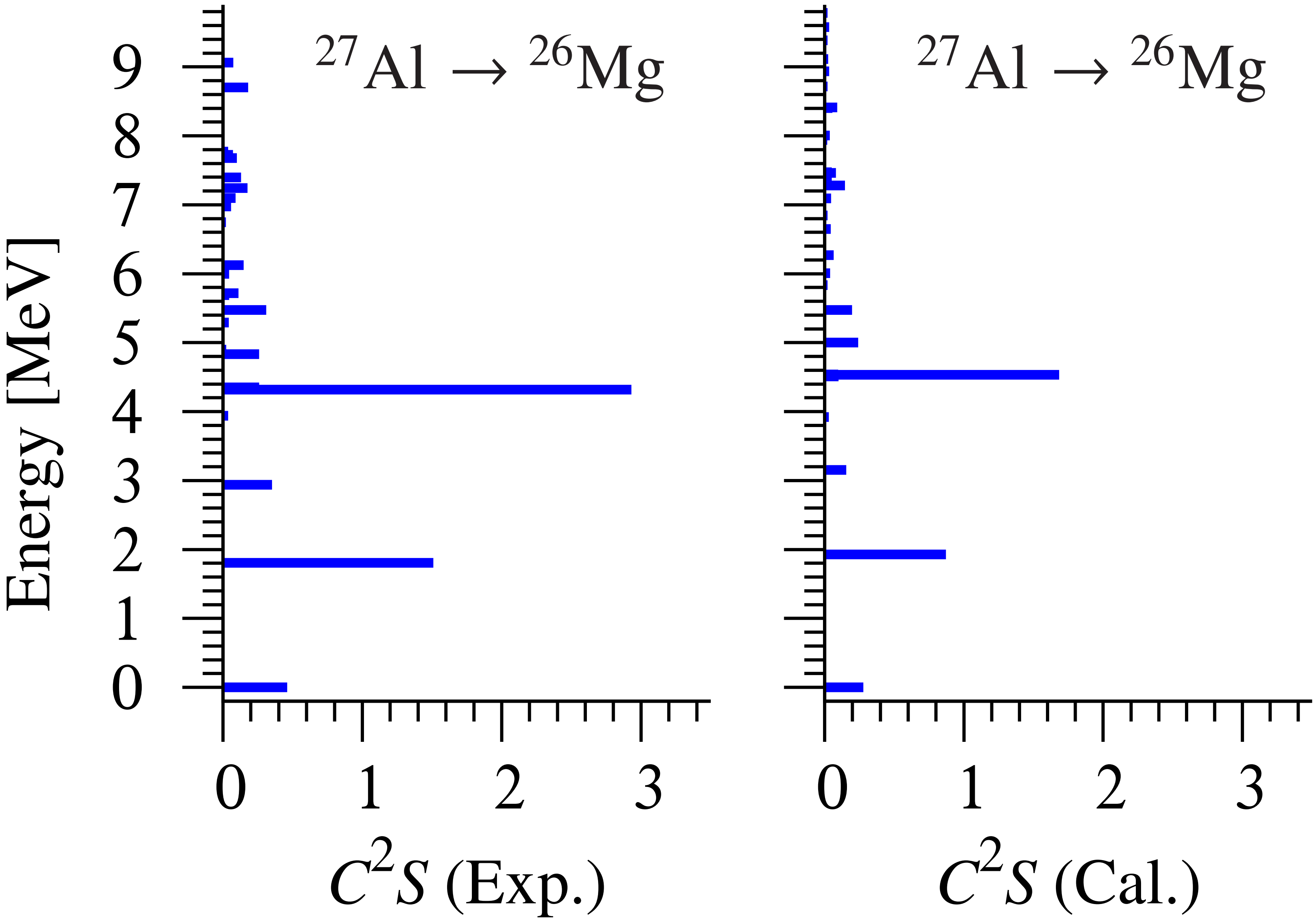
J_k	E_x [MeV]	$-B_\Lambda$ [MeV]	$d\sigma/d\Omega$ [nb/sr]	
1⁻	0.000	−8.600	9.609	**21.62**
2⁻	0.165	−8.435	12.008	
2⁻	2.712	−5.888	11.654	**21.05**
3⁻	2.860	−5.740	9.391	
3⁻	6.183	−2.417	7.625	**21.13**
4⁻	6.370	−2.230	13.505	
2⁺(3)	7.807	−0.793	4.495	**9.46**
1⁺(3)	7.935	−0.665	4.968	
3⁺(2)	8.712	0.112	6.150	**19.91 (29.37)**
2⁺(4)	8.828	0.228	1.431	
2⁺(5)	9.002	0.402	9.893	
3⁺(3)	9.059	0.459	2.434	
3⁻	10.105	1.505	3.913	**21.90**
4⁻	10.455	1.855	17.985	
1⁺(5)	10.828	2.228	4.598	**29.54 (51.44)**
4⁺(3)	11.318	2.718	11.185	
3⁺(5)	11.543	2.943	13.759	

exp peak	EXP E_x [MeV]	EXP $-B_\Lambda$ [MeV]	Fit I $d\sigma/d\Omega$ [nb/sr]
#1	0.00	−8.55±0.07	**17.0±0.5**
#2	2.78±0.11	−5.76±0.09	**16.5±0.5**
#3	6.26±0.16	−2.28±0.14	**10.5±0.3**
#a	8.34±0.41	−0.20±0.40	**23.2±0.7**
#4	10.83±0.10	2.28±0.07	**17.2±0.5**

Parity-mixing and the new bump



Results : Spectroscopic factors of proton pickup reaction from ^{27}Al



(Exp.) **J. Vernotte et al., Phys. Rev. C 48, 205 (1993)**