北京航空航天大學
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$D_{s0}^{*}(2317)$ and $D_{s1}(2460)$ as hadronic molecules from the perspective of their productions

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HADRON 2023

M. Altenbuchinger, **LSG** and W. Weise, Phys. Rev. D 89(2014)014026
M. Z. Liu, X. Z. Ling, **LSG**, E. Wang and J. J. Xie, Phys. Rev. D 106 (2022) 114011
T. C. Wu and **LSG**, arXiv:2211.01846 [hep-ph]

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- 2003—the beginning of a new era in hadron spectroscopy
- $D_{s0}^{*}(2317)$ and $D_{s1}(2460)$ as DK/D*K molecules: masses
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  - Inclusive electron-positron annihilations
- Summary and outlook

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Beginning of a new era： 2003

LEPS, 0301020
1174 citations

$\boldsymbol{\Theta^{+}}$ **(1540)**

BaBar, 0304021
973 citations

$\boldsymbol{D_{s0}^{*}(2317)}$

BABAR

Belle, 0309032
2317 citations

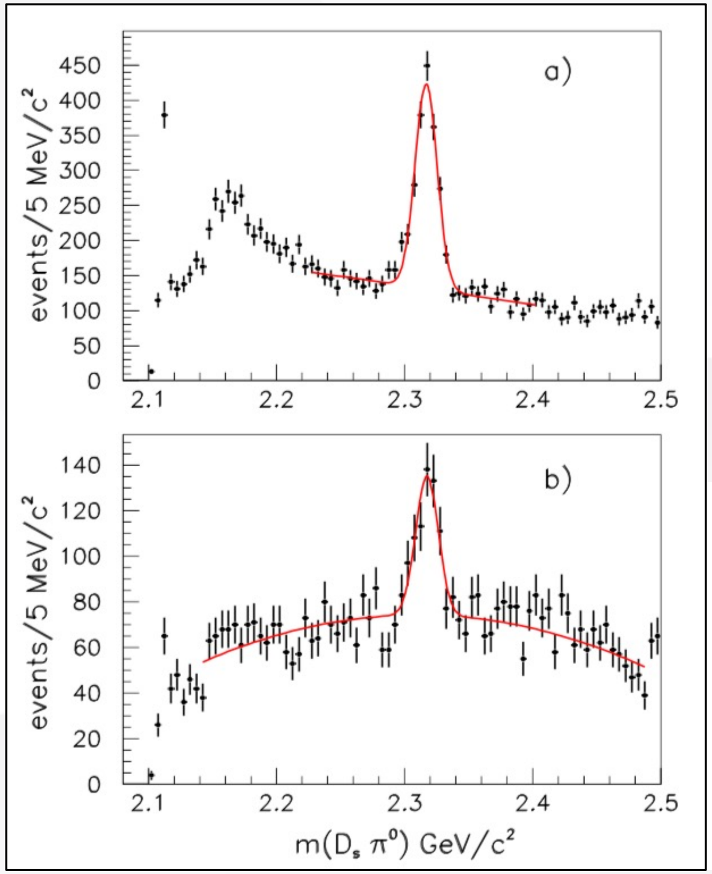
$\mathbf{X(3872)}$

BELLE

as of June 3rd, 2023

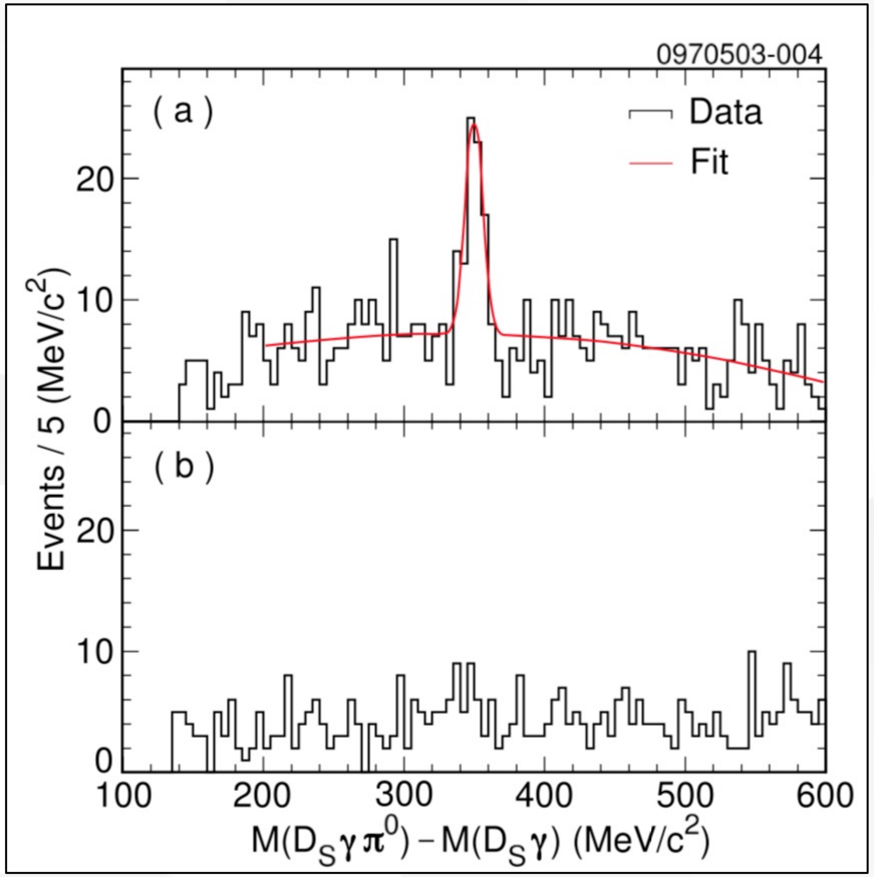
$D_{s0}^*(2317)$ and $D_{s1}(2460)$

$D_{s0}^*(2317)$



BaBar PRL90,242001(2003)

$D_{s1}(2460)$



CLEO PRD68,032002(2003)

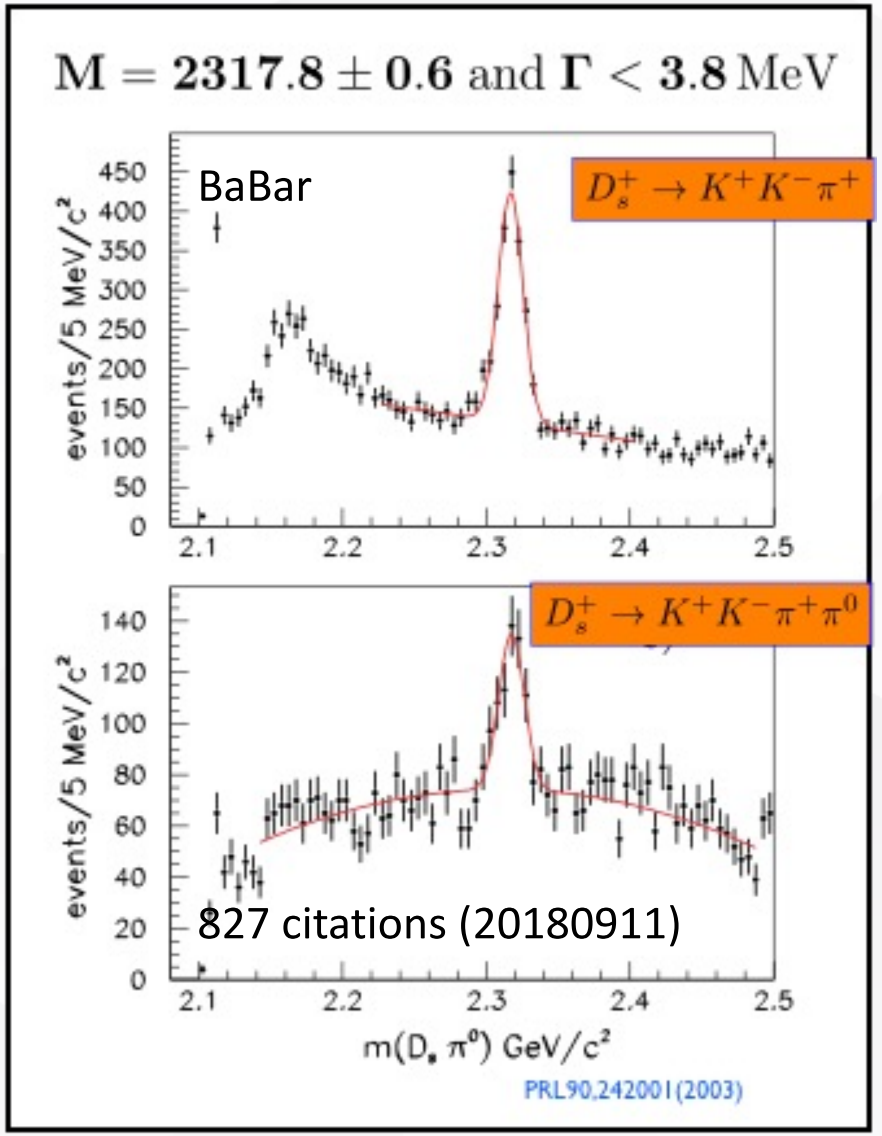
What are special about these two states

$\mathbf{D^*_{s0}(2317),\ D_{s1}(2460)}$

- 160/70 MeV lower than the GI quark model predictions—difficult to be understood as conventional csbar states.

- **"Dynamically generated" from attractive DK/D*K interaction**
  - ✓ E. E. Kolomeitsev 2004,
  - ✓ F. K. Guo 2006,
  - ✓ D. Gamermann 2007

$$\boxed{m_{D_{s1}(2460)} - m_{D^*_{s0}(2317)} \approx m_{D^*} - m_D}$$



Feng-Kun Guo, EPJ Web of Conferences **202**, 02001 (2019)

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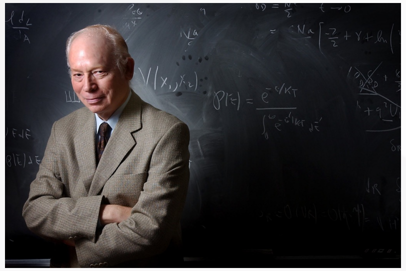
UChPT in Bethe-Salpeter equation

- **Model independent DK/D*K interaction from ChPT**

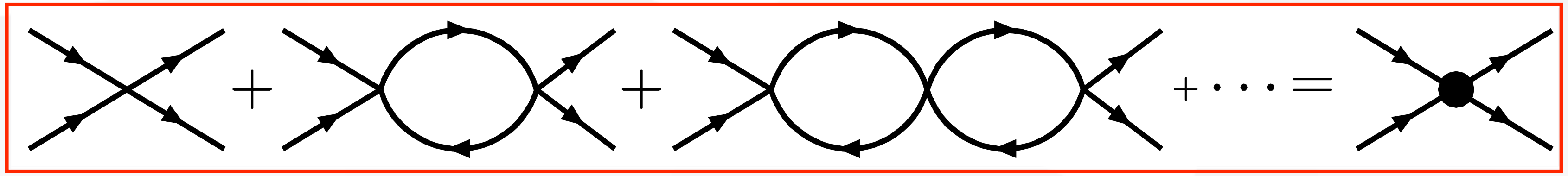
$$\mathcal{V}_{\mathrm{WT}}(P(p_1)\phi(p_2) \to P(p_3)\phi(p_4)) = \frac{1}{4f_0^2}\mathcal{C}_{\mathrm{LO}}\,(s-u)$$

Weinberg-Tomozawa



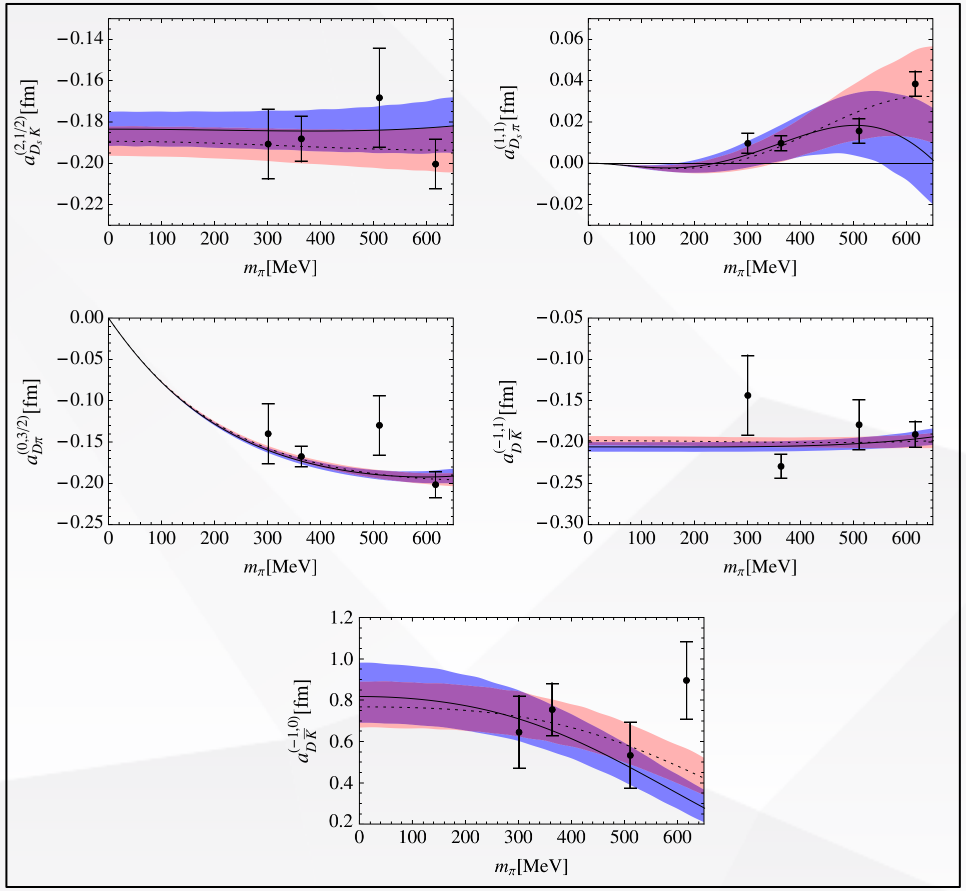
$$\begin{aligned}\mathcal{V}_{\mathrm{NLO}}(P(p_1)\phi(p_2) \to P(p_3)\phi(p_4)) = &-\frac{8}{f_0^2}C_{24}\left(c_2\, p_2\cdot p_4 - \frac{c_4}{m_P^2}\,(p_1\cdot p_4\; p_2\cdot p_3 + p_1\cdot p_2\; p_3\cdot p_4)\right)\\ &-\frac{4}{f_0^2}\mathcal{C}_{35}\left(c_3\, p_2\cdot p_4 - \frac{c_5}{m_P^2}\,(p_1\cdot p_4\; p_2\cdot p_3 + p_1\cdot p_2\; p_3\cdot p_4)\right)\\ &-\frac{4}{f_0^2}\mathcal{C}_6\,\frac{c_6}{m_P^2}\,(p_1\cdot p_4\; p_2\cdot p_3 - p_1\cdot p_2\; p_3\cdot p_4)\\ &-\frac{8}{f_0^2}\mathcal{C}_0\, c_0 + \frac{4}{f_0^2}\mathcal{C}_1\, c_1\,, \qquad (11)\end{aligned}$$

- **Resumed in the Bethe-Salpeter equation (two-body elastic unitarity)**

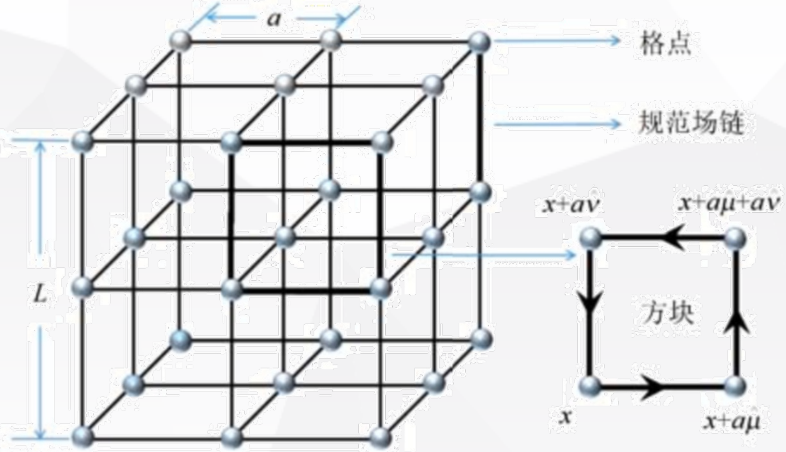


*M. Altenbuchinger, **LSG** and W. Weise, Phys. Rev. D 89(2014)014026*

Fixing the LECs using latest LQCD* data



- NLO ChPT kernel: 5 LECs
- A quite good description of the 20 Lattice **scattering lengths of pseudoscalar mesons and D mesons （I=0 DK excluded）** can be achieved.



*M. Altenbuchinger, **LSG** and W. Weise, Phys. Rev. D 89(2014)014026*

$D_{s0}^*(2317)$ and $D_{s1}(2460)$ dynamically generated

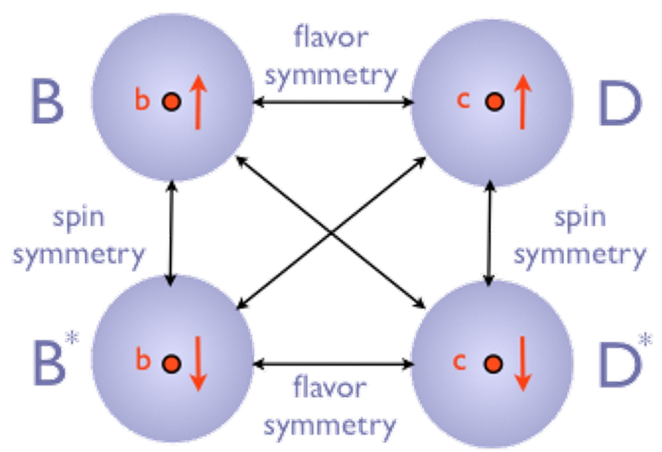
"Post-diction"

- **Charm sector**

$\mathbf{D_{s0}^*(2317), D_{s1}(2460)}$

TABLE V. Pole positions $\sqrt{s} = M - i\frac{\Gamma}{2}$ (in units of MeV) of charm mesons dynamically generated in the HQS UChPT.

(S, I)	$J^P = 0^+$	$J^P = 1^+$
(1, 0)	2317 ± 10	2457 ± 17
(0, 1/2)	$(2105 \pm 4) - i(103 \pm 7)$	$(2248 \pm 6) - i(106 \pm 13)$



- **Bottom Sector**

TABLE VI. Pole positions $\sqrt{s} = M - i\frac{\Gamma}{2}$ (in units of MeV) of bottom mesons dynamically generated in the HQS UChPT.

(S, I)	$J^P = 0^+$	$J^P = 1^+$
(1, 0)	5726 ± 28	5778 ± 26
(0, 1/2)	$(5537 \pm 14) - i(118 \pm 22)$	$(5586 \pm 16) - i(124 \pm 25)$

*M. Altenbuchinger, **LSG** and W. Weise, Phys. Rev. D 89(2014)014026*

Predicted $B_{s0}^{*}(5726)$ and $B_{s1}(5778)$ states

Physics Letters B 750 (2015) 17–21

Contents lists available at ScienceDirect

Physics Letters B

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Predicting positive parity B_s mesons from lattice QCD

C.B. Lang [a], Daniel Mohler [b,*], Sasa Prelovsek [c,d], R.M. Woloshyn [e]

[a] Institute of Physics, University of Graz, A-8010 Graz, Austria
[b] Fermi National Accelerator Laboratory, Batavia, IL 60510-5011, USA
[c] Department of Physics, University of Ljubljana, 1000 Ljubljana, Slovenia
[d] Jozef Stefan Institute, 1000 Ljubljana, Slovenia
[e] TRIUMF, 4004 Wesbrook Mall, Vancouver, BC V6T 2A3, Canada

Table 5
Comparison of masses from this work to results from various model based calculations; all masses in MeV.

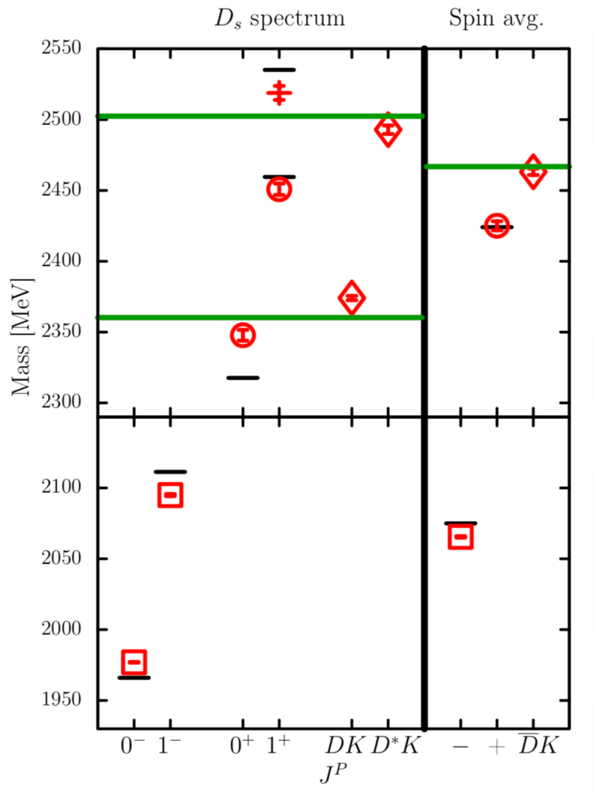
J^P	0^+	1^+
Covariant (U)ChPT [24]	5726(28)	5778(26)
NLO UHMChPT [19]	5696(20)(30)	5742(20)(30)
LO UChPT [17,18]	5725(39)	5778(7)
LO χ-SU(3) [16]	5643	5690
HQET + ChPT [20]	5706.6(1.2)	5765.6(1.2)
Bardeen, Eichten, Hill [15]	5718(35)	5765(35)
rel. quark model [5]	5804	5842
rel. quark model [22]	5833	5865
rel. quark model [23]	5830	5858
HPQCD [30]	5752(16)(5)(25)	5806(15)(5)(25)
this work	5713(11)(19)	5750(17)(19)

In agreement with lQCD

More support from recent lQCD studies

- G.K.C. Cheung et al., arXiv:2008.06432[hep-lat].
- G. S. Bali et al., arXiv:1706.01247 [hep-lat].

C. B. Lang et al., arXiv:1403.8103 [hep-lat].
D. Mohler et al., arXiv:1308.3175 [hep-lat].

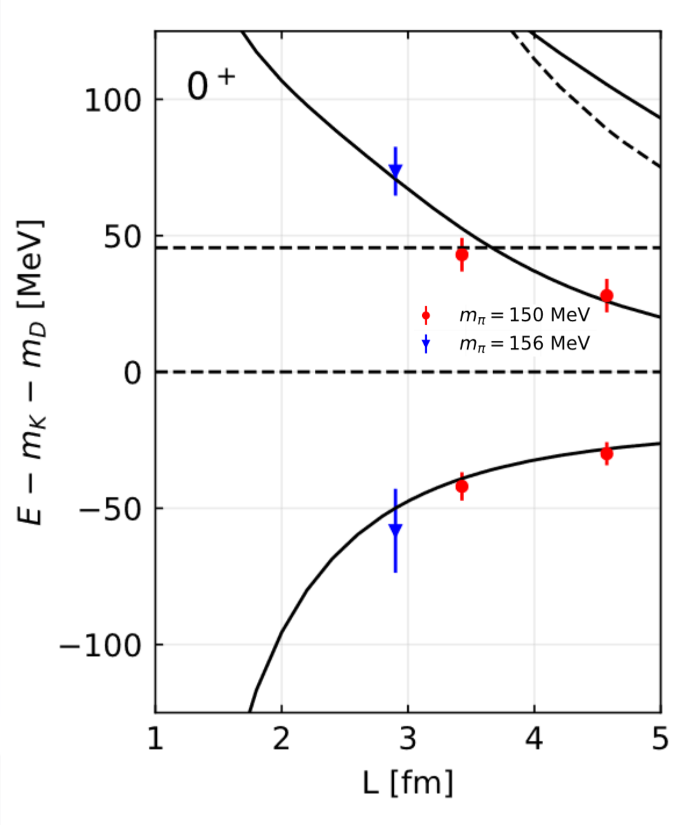


"DK components substantial"

FIG. 12. On the left, our final results for the lower lying D_s spectrum as detailed in Table VII. The short horizontal black lines indicate the corrected experimental values (see Section II) while the green horizontal lines give the positions of the DK and D^*K non-interacting thresholds. Our lattice results for the finite volume thresholds are labelled DK and D^*K, respectively. The errors indicated are statistical only. On the right, the negative parity spin-averaged $1S$ mass $m_- = \frac{1}{4}(m_{0^-} + 3m_{1^-})$ is shown and denoted $-$, while the same spin-average of the positive parity 0^+ and 1^+ states is labelled with $+$ and the weighted average of the threshold is labelled as $\overline{DK}$.

See as well Miguel Albaladejo et al. arXiv:1805.07104

Support from LQCD+an unquenched QM



DK about 70%

Novel Coupled Channel Framework Connecting the Quark Model and Lattice QCD for the Near-threshold D_s States

Zhi Yang, Guang-Juan Wang, Jia-Jun Wu, Makoto Oka, and Shi-Lin Zhu
Phys. Rev. Lett. **128**, 112001 – Published 14 March 2022

TABLE II. The comparison of D_s pole masses (MeV) (Ours) with the experimental results. The script $P(c\bar{s})$ represents the content of the bare $c\bar{s}$ cores in the D_s states at $L = 4.57$ fm.

	$P(c\bar{s})(\%)$	Ours	Experimental results
$D_{s0}^*(2317)$	$32.0^{+5.2}_{-3.9}$	$2338.9^{+2.1}_{-2.7}$	2317.8 ± 0.5
$D_{s1}^*(2460)$	$52.4^{+5.1}_{-3.8}$	$2459.4^{+2.9}_{-3.0}$	2459.5 ± 0.6
$D_{s1}^*(2536)$	$98.2^{+0.1}_{-0.2}$	$2536.6^{+0.3}_{-0.5}$	2535.11 ± 0.06
$D_{s2}^*(2573)$	$95.9^{+1.0}_{-1.5}$	$2570.2^{+0.4}_{-0.8}$	2569.1 ± 0.8

$M(D_{s1}(2460)) - M(D_{s0}(2317)) = 120$!!!

Latest work from Raquel Molina

Quark mass dependence of the $D_{s0}(2317)$ resonance in DK

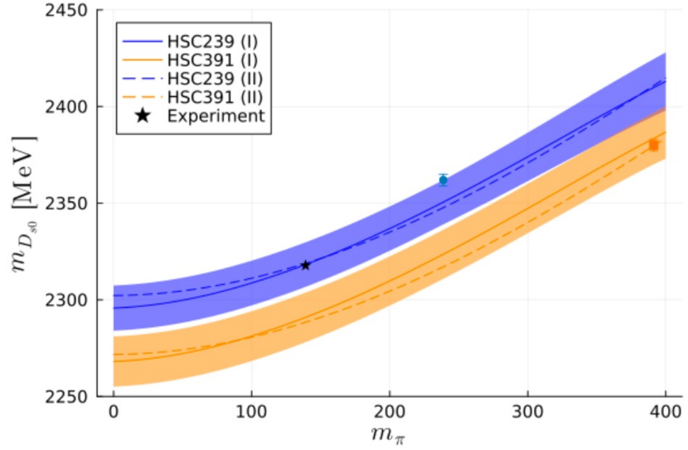
Potential $V(s)$ consistent with HQSS, See Fernando's talk at 2.30pm

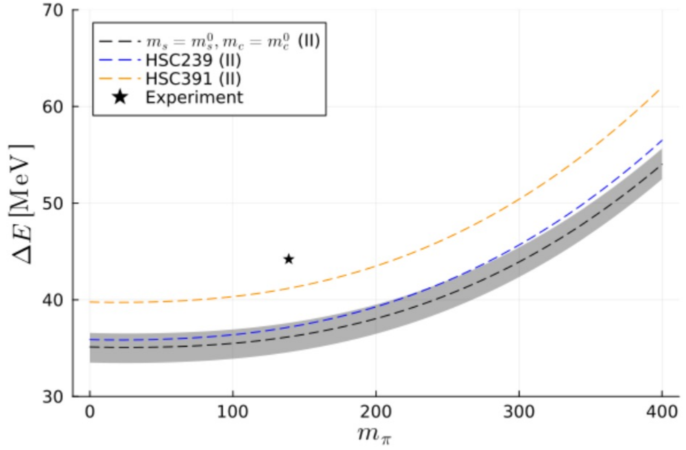
See also L.S. Geng and Albaladejo's talk about $D_{s0}(2317)$ (Femtoscopy)

$$V(s) = V_{DK}(s) + V_{\rm ex}(s); \qquad \boxed{1 - Z \simeq 0.7 - 0.8}$$

2306.01848

$$V_{DK} = -\frac{s-u}{2f^2} \quad ; V_{\rm ex} = \frac{V_{c\bar{s}}^2}{s - m_{c\bar{s}}^2} \tag{5}$$





$m_\pi = 236$ MeV; $a_t^{-1} = 5.667$ GeV; $a_t M_{\eta_c} = 0.2412$, $M_{\eta_c} = 2986$ MeV;

$m_\pi = 391$ MeV; $a_t^{-1} = 6.079$ GeV; $a_t M_{\eta_c} = 0.2735$; $M_{\eta_c} = 2963$ MeV;

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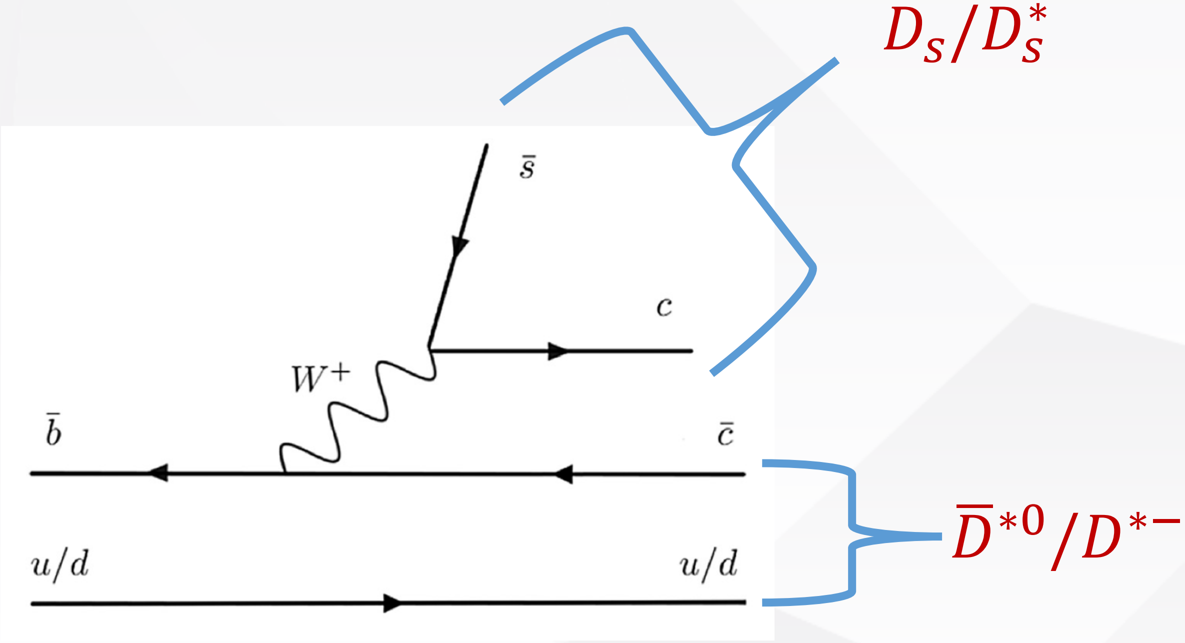
Can the productions be explained in the Molecular picture?

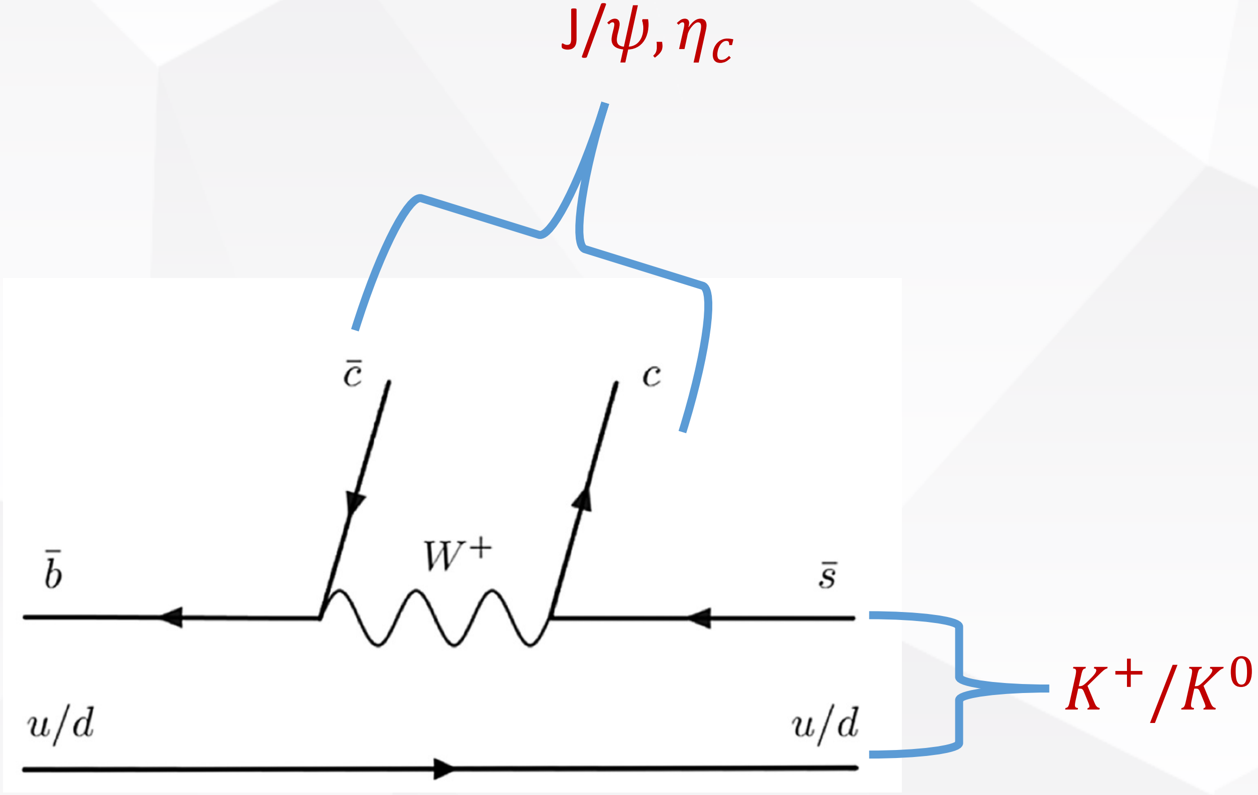
*M. Z. Liu, X. Z. Ling, **LSG**, E. Wang and J. J. Xie, 2209.01103*

Decay modes	PDG 10^{-3}	BarBar 10^{-3}
$B^+ \to \bar{D}^0 D_{s0}^{*+}(2317)$	$0.80^{+0.16}_{-0.13}$	$1.0 \pm 0.3 \pm 0.1$
$B^0 \to D^- D_{s0}^{*+}(2317)$	$1.06^{+0.16}_{-0.16}$	$1.8 \pm 0.4 \pm 0.3$
$B^+ \to \bar{D}^{*0} D_{s0}^{*+}(2317)$	$0.90^{+0.70}_{-0.70}$	$0.9 \pm 0.6 \pm 0.2$
$B^0 \to D^{*-} D_{s0}^{*+}(2317)$	$1.50^{+0.60}_{-0.60}$	$1.5 \pm 0.4 \pm 0.2$
$B^+ \to \bar{D}^0 D_{s1}^{+}(2460)$	$3.1^{+1.0}_{-0.9}$	$2.7 \pm 0.7 \pm 0.5$
$B^0 \to D^- D_{s1}^{+}(2460)$	3.5 ± 1.1	$2.8 \pm 0.8 \pm 0.5$
$B^+ \to \bar{D}^{*0} D_{s1}^{+}(2460)$	12.0 ± 3.0	$7.6 \pm 1.7 \pm 1.8$
$B^0 \to D^{*-} D_{s1}^{+}(2460)$	9.3 ± 2.2	$5.5 \pm 1.2 \pm 1.0$

Quark-level diagrams

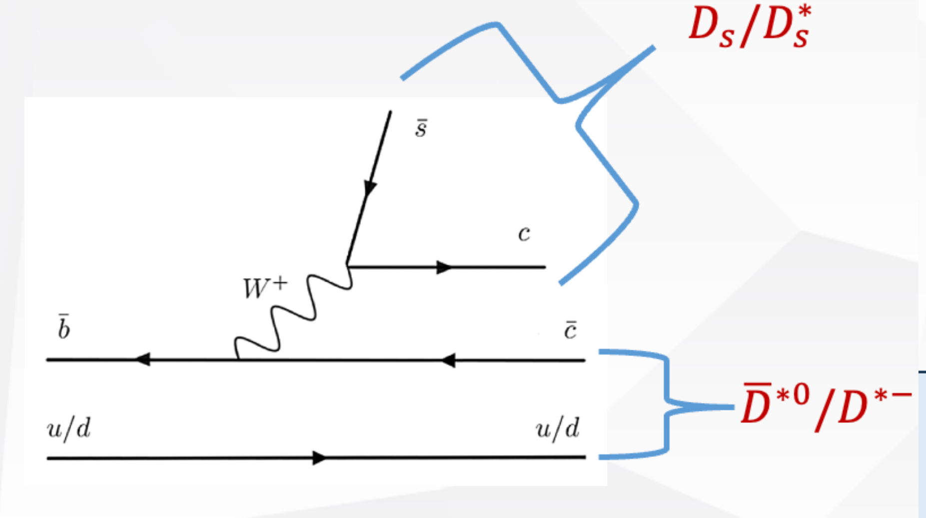


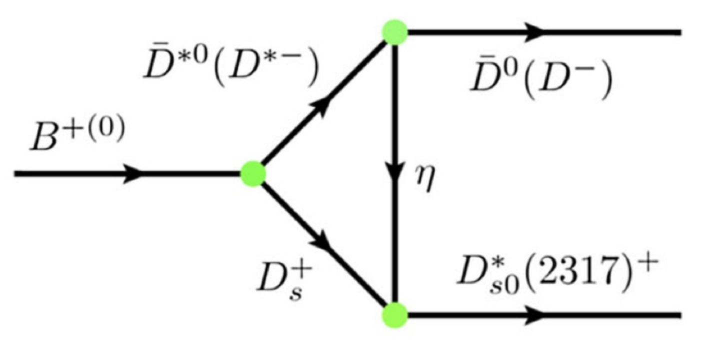
External W-emission for $B^{+(0)} \to D_s^{(*)+}\bar{D}^{*0}(D^{*-})$

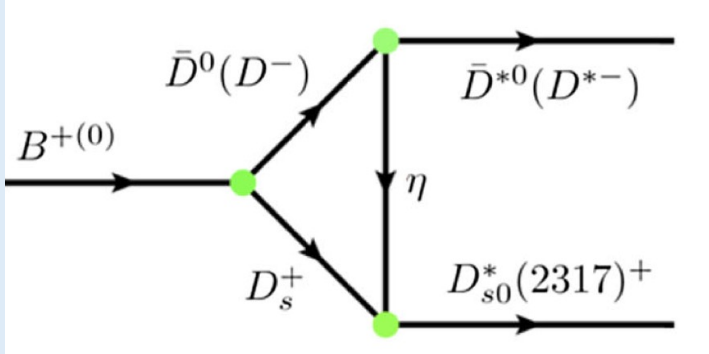


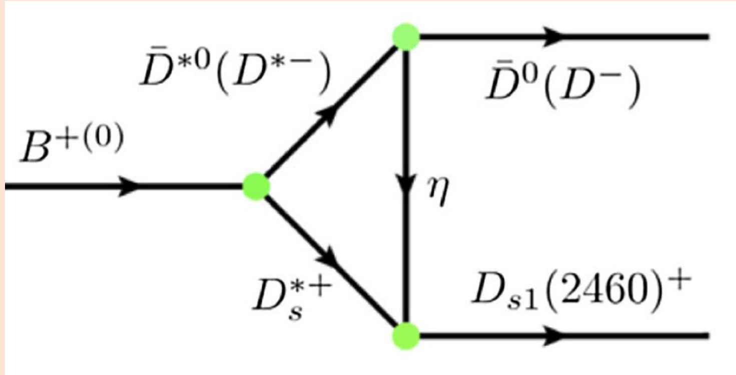
Internal W-conversion for $B^{+(0)} \to J/\psi/\eta_c K^{+(0)}$

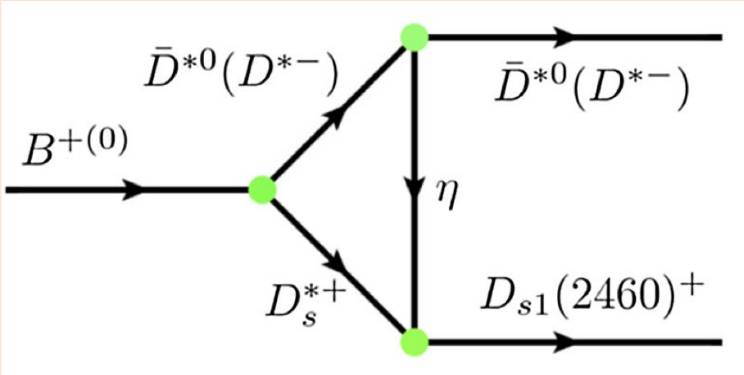
Hadron-level diagrams

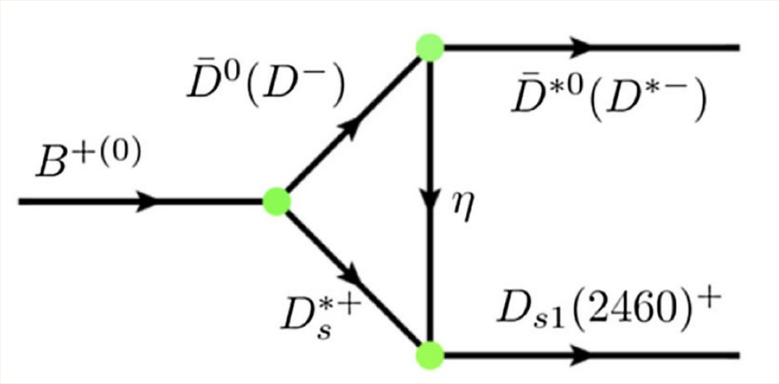




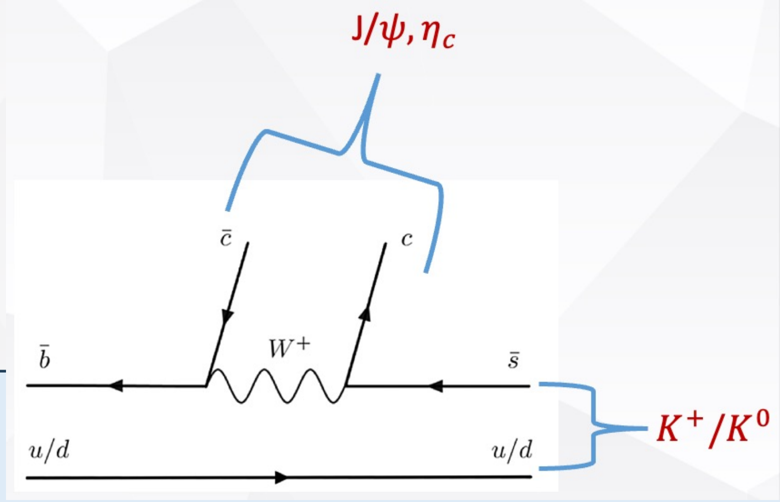


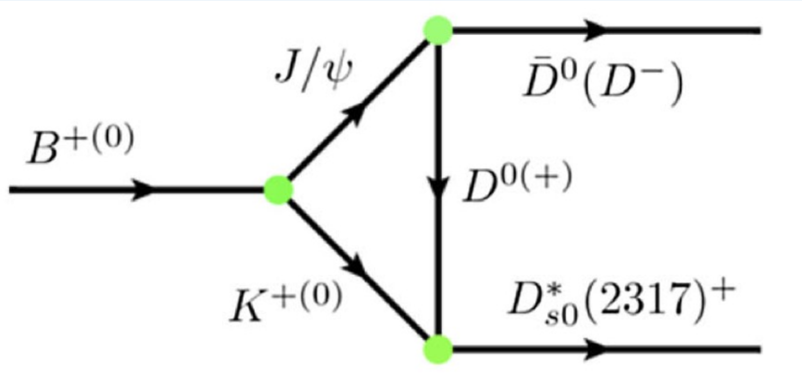


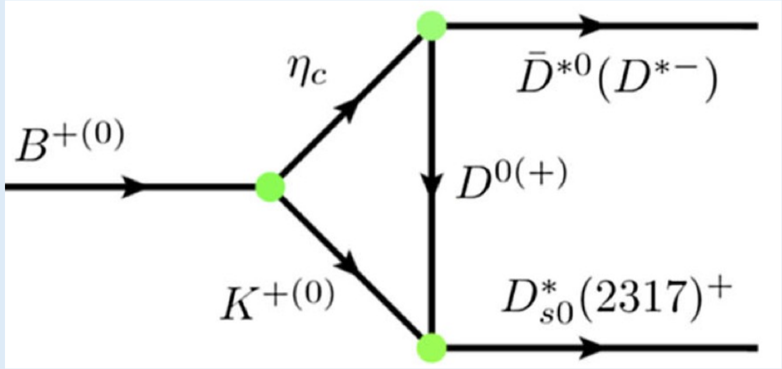


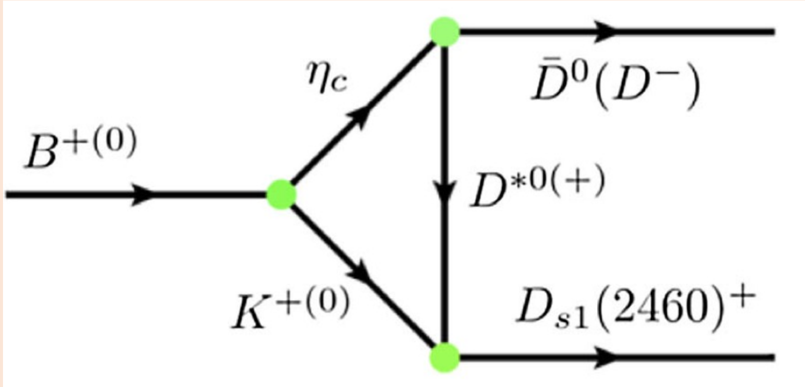


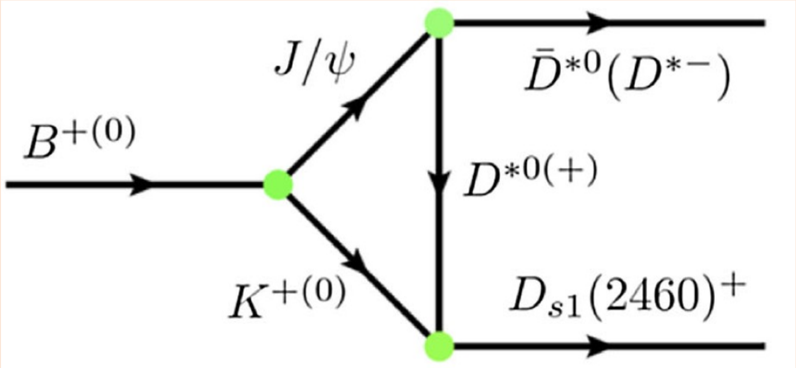
Hadron-level diagrams

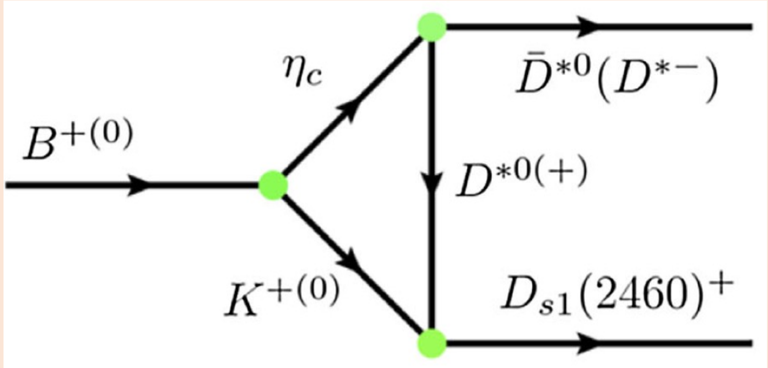




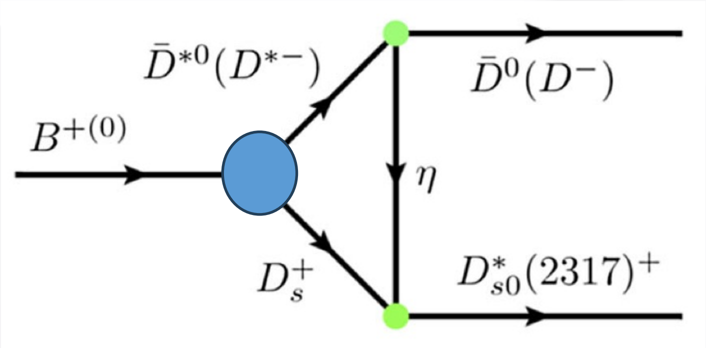








How to calculate the diagram in a model independent way



$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs} [c_1^{\rm eff} \mathcal{O}_1 + c_2^{\rm eff} \mathcal{O}_2] + {\rm H.c.},$$

$$\mathcal{O}_1 = (s\bar{c})_{V-A}(c\bar{b})_{V-A}, \mathcal{O}_2 = (\bar{c}c)_{V-A}(s\bar{b})_{V-A}$$

$$\mathcal{A}(B^+ \to D_s^+ \bar{D}^{*0}) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs} \boxed{a_1} \boxed{\langle D_s^+ | (s\bar{c}) | 0 \rangle} \boxed{\langle \bar{D}^{0*} | (c\bar{b}) | B^+ \rangle},$$

$$\boxed{\begin{aligned}\langle \bar{D}^{*0} | (c\bar{b}) | B^+ \rangle = \epsilon_\alpha^* \Big\{ & -g^{\mu\alpha}(m_{\bar{D}^{*0}} + m_{B^+}) A_1(q^2) + P^\mu P^\alpha \frac{A_2(q^2)}{m_{\bar{D}^{*0}} + m_{B^+}} \\ & + i\varepsilon^{\mu\alpha\beta\gamma} P_\beta q_\gamma \frac{V(q^2)}{m_{\bar{D}^{*0}} + m_{B^+}} + q^\mu P^\alpha \left[\frac{m_{\bar{D}^{*0}} + m_{B^+}}{q^2} A_1(q^2) - \frac{m_{B^+} - m_{\bar{D}^{*0}}}{q^2} A_2(q^2) - \frac{2m_{\bar{D}^{*0}}}{q^2} A_0(q^2) \right] \Big\}\end{aligned}}$$

Decay constant

$$\boxed{\langle D_s^+ | (s\bar{c}) | 0 \rangle = i f_{D_s^+} p_{D_s^+}^\mu}$$

$$f_{D_s} = 250 \text{ MeV}$$

Parameter a1

$$\boxed{a_1 = c_1^{\rm eff} + c_2^{\rm eff}/N_c}$$

TABLE I. Branching ratios (10^{-3}) of $B^{+(0)} \to D_s^{(*)+} \bar{D}^{(*)0} \times (D^{(*)-})$ and $B^{+(0)} \to J/\psi(\eta_c) K^{+(0)}$.

Decay mode	RPP [27]	Decay mode	RPP [27]
$B^+ \to \bar{D}^0 D_s^+$	9.0 ± 0.9	$\bar{B}^0 \to D^- D_s^+$	7.2 ± 0.8
$B^+ \to \bar{D}^0 D_s^{*+}$	7.6 ± 1.6	$\bar{B}^0 \to D^- D_s^{*+}$	7.4 ± 1.6
$B^+ \to \bar{D}^{*0} D_s^+$	8.2 ± 1.7	$\bar{B}^0 \to D^{*-} D_s^+$	8.0 ± 1.1
$B^+ \to \bar{D}^{*0} D_s^{*+}$	17.1 ± 2.4	$\bar{B}^0 \to D^{*-} D_s^{*+}$	17.7 ± 1.4
$B^+ \to J/\psi K^+$	1.010 ± 0.029	$\bar{B}^0 \to J/\psi K^0$	0.873 ± 0.032
$B^+ \to \eta_c K^+$	1.09 ± 0.09	$\bar{B}^0 \to \eta_c K^0$	0.79 ± 0.12

$$a_1 = 0.93$$

Form Factors

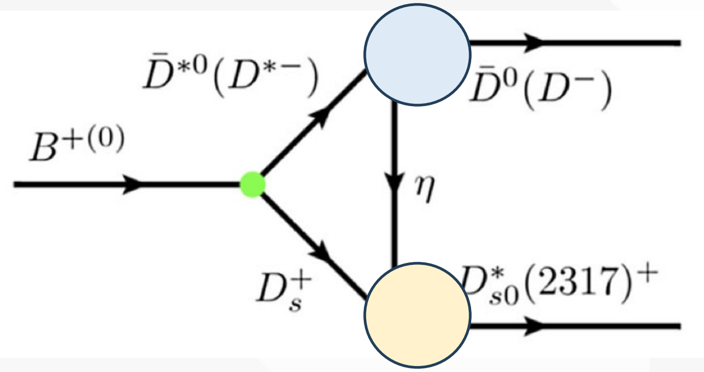
$$F(q^2) = \frac{F(0)}{1 - a\,(q^2/m_B^2) + b\,(q^4/m_B^4)}$$

parameterisation

$$F(q^2) = F(0) \frac{\Lambda_1^2}{q^2 - \Lambda_1^2} \frac{\Lambda_2^2}{q^2 - \Lambda_2^2}$$

R. C. Verma, J. Phys. G 39, 025005 (2012).

How to calculate the diagram in a model independent way



$$\mathcal{L}_{D_{s0}^* D_s \eta} = g_{D_{s0}^* D_s \eta} D_{s0}^* D_s \eta,$$

$$g_{D_{s0}^* D_s \eta} = 7.4 \text{ GeV}$$

H.-L. Fu, H. W. Grießhammer, F.-K. Guo, C. Hanhart, and U.-G. Meißner, Eur. Phys. J. A **58**, 70 (2022).

$$\mathcal{L}_{DD^*\eta} = i g_{DD^*\eta}(D_\mu^* \partial^\mu \eta \bar{D} - D \partial^\mu \eta \bar{D}_\mu^*)$$

$$g_{D^{*0}D^0\eta} = g_{D^{*-}D^-\eta} = \frac{g_{D^{*0}D^0\pi^0}}{\sqrt{3}}$$ SU(3)

$$g_{D^{*0}D^0\pi^0} = 11.7$$ Exp.

Final results

M. Z. Liu, X. Z. Ling, **LSG**, E. Wang and J. J. Xie, 2209.01103

Decay modes	Our results 10^{-3}	PDG 10^{-3}	BarBar 10^{-3}
$B^+ \to \bar{D}^0 D_{s0}^{*+}(2317)$	0.677 ± 0.190	$0.80^{+0.16}_{-0.13}$	$1.0 \pm 0.3 \pm 0.1$
$B^0 \to D^- D_{s0}^{*+}(2317)$	0.637 ± 0.178	$1.06^{+0.16}_{-0.16}$	$1.8 \pm 0.4 \pm 0.3$
$B^+ \to \bar{D}^{*0} D_{s0}^{*+}(2317)$	1.210 ± 0.339	$0.90^{+0.70}_{-0.70}$	$0.9 \pm 0.6 \pm 0.2$
$B^0 \to D^{*-} D_{s0}^{*+}(2317)$	0.889 ± 0.249	$1.50^{+0.60}_{-0.60}$	$3.5 \pm 0.4 \pm 0.2$
$B^+ \to \bar{D}^0 D_{s1}^{+}(2460)$	1.255 ± 0.351	$3.7 \pm 0.7 \pm 0.5$	
$B^0 \to D^- D_{s1}^{+}(2460)$	1.158 ± 0.324	3.5 ± 1.1	$2.8 \pm 0.8 \pm 0.5$
$B^+ \to \bar{D}^{*0} D_{s1}^{+}(2460)$	3.065 ± 0.858	$7.6 \pm 1.7 \pm 1.8$	
$B^0 \to D^{*-} D_{s1}^{+}(2460)$	2.709 ± 0.759	9.3 ± 2.2	$5.5 \pm 1.2 \pm 1.0$

- Overall the data **can be explained** in the molecular picture
- We also reproduce the pattern that $D_{s1}^{+}(2460)$ **is more abundantly** produced than $D_{s0}^{*}(2317)$ (due to the first vertex)

Contents

- ☐ 2003—the beginning of a new era in hadron spectroscopy
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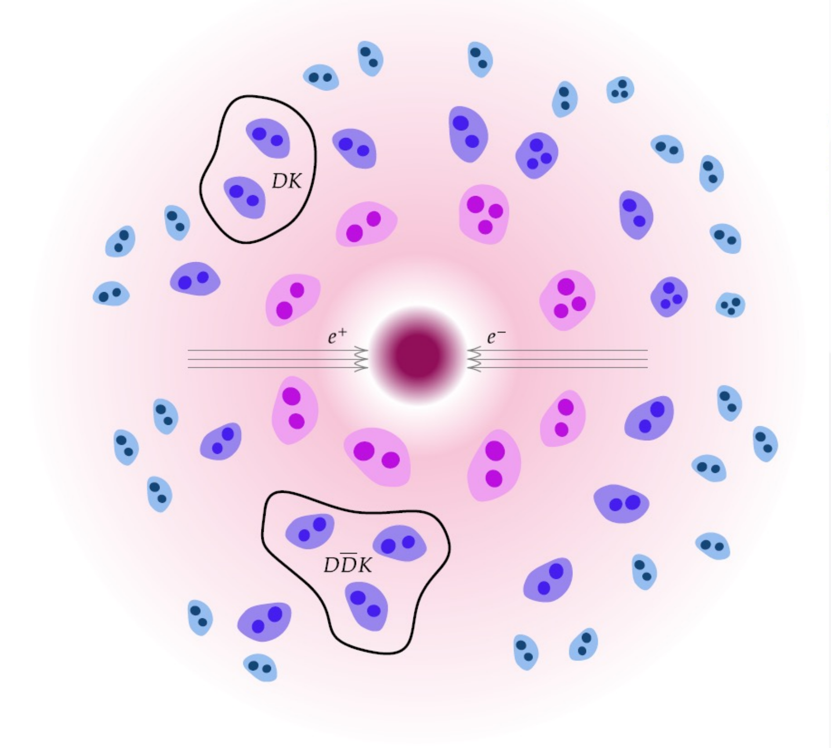
  - ➢ B decays
  - ➢ Inclusive electron-positron annihilations
- ☐ Summary and outlook

Productions in e^+e^- annihilations

- Widely used in studied of light (anti)nuclei-hypernuclei
  Talk by Malgorzata Janik on Moday
- Already used to study exotic hadrons in hadron colliders
  S. Cho et al., Phys. Rev. Lett. 106 (2011) 212001
  H. Zhang et al., Phys. Rev. Lett. 126 (2021) 012301

Coalescence Method

1. Productions of D/D* and K in e^+e^- collisions—transport model
2. Coalescence of D/D* and K to form $\boldsymbol{D_{s0}^*(2317)}$ and $\boldsymbol{D_{s1}(2460)}$—wigner functional approach

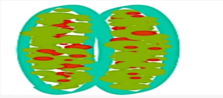
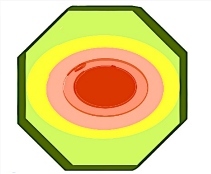
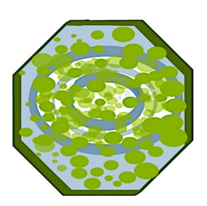
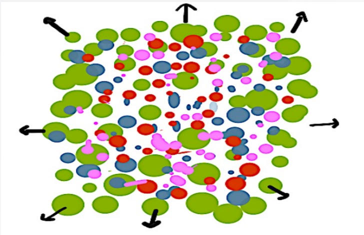


T. C. Wu and **LSG**, arXiv:2211.01846 [hep-ph]

Transport process—PACIAE

/pa:sia/ PA: Parton; CIAE: China Institute of Atomic Energy

Productions of hadrons in four steps:

(PACIAE) Parton initiation → Parton rescattering → Hadronization → Hadron rescattering → Decay & freeze-out

- From PYTHIA
- Additional Transport Process

Reminder:

- PACIAE is a transport model based on the event generator PYTHIA
- PACIAE can simulate e^+e^- , pp , nuclei-nuclei collisions
- PACIAE can provide phase-space distributions of final-states

B.-H. Sa et al., Comput. Phys. Commun. 183 (2012) 333–346.

Coalescence—Wigner function approach

Coalescence of n constituents into a compound particle

- Information of constituents is encoded in the multi-constituents source density $\boldsymbol{\rho_S(t)}$ (provided by the transport model)
- Information of compound particles is contained in their wave functions

$$\boldsymbol{\rho_C = |\Psi_C\rangle\langle\Psi_C|}$$

- The yield forming a compound particle from its constituents:

$$\mathbf{Yield} = \lim_{t\to\infty} Tr\,[\rho_C\,\rho_S(t)]$$

- $\rho_S(t)$ is semi-classical , while ρ_C is quantum mechanical. As a result we need to work in the Wigner representation, i.e, we need to transfrom $\rho_S(t), \rho_C$ into $\rho_S^W(t)$与ρ_C^W.

M. Gyulassy et al., Nucl. Phys. A 402 (1983) 596–611.

Coalescence—Wigner function approach

Yield

$$\frac{dN_X}{dP} = \mathrm{g}\int \delta(P-(p_1+p_2))\rho_c^{\mathrm{W}}\rho_s^{\mathrm{W}}\frac{dr_1 dp_1}{(2\pi)^3}\cdot\frac{dr_2 dp_2}{(2\pi)^3}$$

$$\rho_s^{\mathrm{W}} = (2\pi)^6\delta^3(r_1-\tilde{r}_1)\delta^3(p_1-\tilde{p}_1)\delta^3(r_2-\tilde{r}_2)\delta^3(p_2-\tilde{p}_2)$$

$$\rho_X^{\mathrm{W}}(r,q) = \int \phi\left(r+\frac{R}{2}\right)\phi\left(r-\frac{R}{2}\right)\exp(-iq\cdot R)dR^3$$

$$N = g\left\langle\sum_{(c)}\rho_C^W(\tilde{r}_1,\tilde{q}_1,\cdots,\tilde{r}_{n-1},\tilde{q}_{n-1})\right\rangle$$

M. Gyulassy et al., Nucl. Phys. A 402 (1983) 596–611.

T. W. Wu, M. Z. Liu, and LSG Phys.Rev.D 103 (2021) L031501

ρ_S^W is the Wigner source density ($\tilde{r}_{n-1}$ and $\tilde{q}_{n-1}$ are from the transport model) , ρ_C^W is the Wigner density of the composite particle (from the wave function) , (c) is the number of combinations , g is the statistical factor.

Results – constituent mesons

Yields of D mesons in $e^+e^- \to c\bar{c}$ processes and K mesons in $e^+e^- \to$ hadrons processes near 10.5 GeV

Particle	Experimental Data	Simulation
D^+	0.2639±0.0139	0.2386
D^0	0.5772±0.0241	0.5276
D^{*+}	0.2470±0.0137	0.2100
D^{*0}	0.2241±0.0304	0.2026
$K^\pm$	0.972±0.012±0.016	1.153

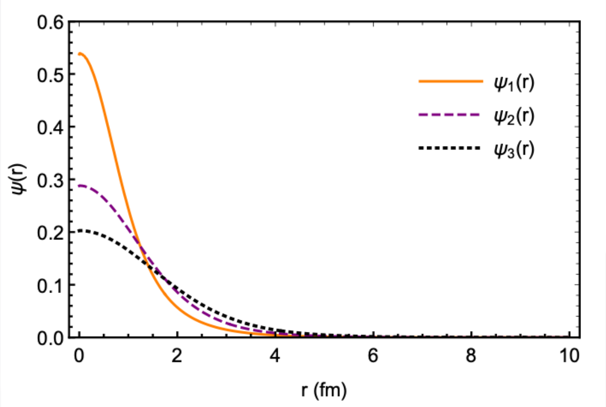
M. Lisovyi, A. Verbytskyi, O. Zenaiev, Eur. Phys. J. C 76 (7) (2016) 397.

[Belle Collaboration] M. Leitgab et al. Phys.Rev.Lett. 111 (2013) 062002

parj(13), the probability that a charm or heavier meson has spin 1 , is set at 0.54 according to the measured ratios of $D^+$and D^0, D^{*+} and D^{*0}, $D^+$and D^{*+}, D^0 and D^{*0}, instead of its default value of 0.75 .

Results: $D_{s0}(2317)$ & $D_{s1}(2460)$

r=1.28, 1.74, and 2.13 fm



Yields of $D_{s0}(2317)$ & $D_{s1}(2460)$ $in\ e^+e^- \to c\bar{c}$ process and their ratio (containing charge-congugated states , and $p^* > 3.2$ GeV/c) near $\sqrt{s} = 10.58$ GeV

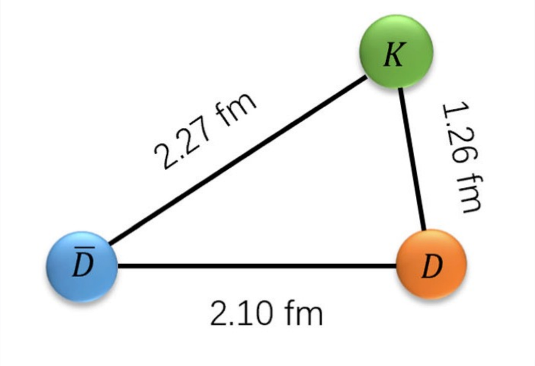
	Case 1	Case 2	Case 3	Data [58]
$Y_{D_{s0}(2317)}$	$5.87^{+3.54}_{-2.27} \times 10^{-3}$	$4.72^{+1.60}_{-1.44} \times 10^{-3}$	$3.43^{+0.75}_{-0.84} \times 10^{-3}$	$5.37^{+1.39}_{-1.75} \times 10^{-3}$
$Y_{D_{s1}(2460)}$	$3.68^{+2.43}_{-1.32} \times 10^{-3}$	$2.98^{+1.10}_{-0.73} \times 10^{-3}$	$2.15^{+0.56}_{-0.33} \times 10^{-3}$	$3.91^{+1.43}_{-1.43} \times 10^{-3}$
$Y_{D_{s0}(2317)}/Y_{D_{s1}(2460)}$	1.56 ± 0.04	1.53 ± 0.07	1.53 ± 0.09	$1.37^{+0.39}_{-0.47}$ a

[58] B. Aubert, et al., BaBar Collaboration, Phys. Rev. D 74 (2006) 032007.

- The agreement for the branching fractions is reasonable
- **The ratio is larger than 1,** consistent with data, which should be smaller 1 in the naïve $c\bar{s}$ picture

Results: $K_{c\bar{c}}(4180)$

- Two recent works showed **the existence of a $D\bar{D}K$ bound state** with isospin ½ and spin-parity 0^- and a mass of about 4180 MeV.



T. W. Wu, M. Z. Liu and **LSG***, Phys. Rev. D 103 (2021) L031501
X. Wei, Q. H. Shen and J. J. Xie, Eur. Phys. J. C 82 (2022) 718

- With the same approach, we estimated the production yield of this state is **of the order of 10^{-6}** and within the reach of BelleII

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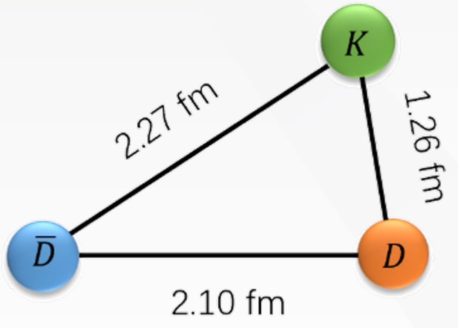
- Summary and outlook

Summary and outlook

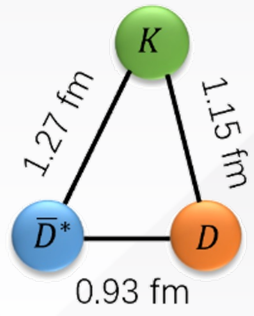
- $D_{s0}^*(2317)$ **and** $D_{s1}(2460)$ can be understood as **hadronic molecules from their masses** **and decays (not covered here)**
- We showed that the hadronic molecular picture can also naturally **explain their productions** in both B decays and inclusive electron-positron annihilations
- We further showed the **three-body** $D\bar{D}K$ molecule **can be observed** in the Belle-II experiment

Summary and outlook

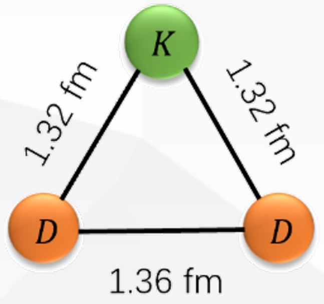
- In the molecular picture, we anticipate **the existence of other multi-hadron** molecules and encourage dedicate experimental searches for them



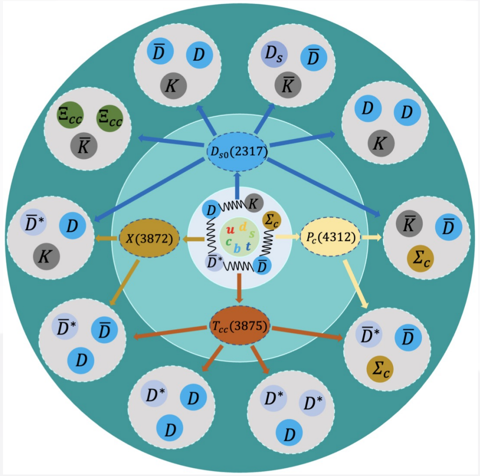
Kcc(4180)



K*(4307)



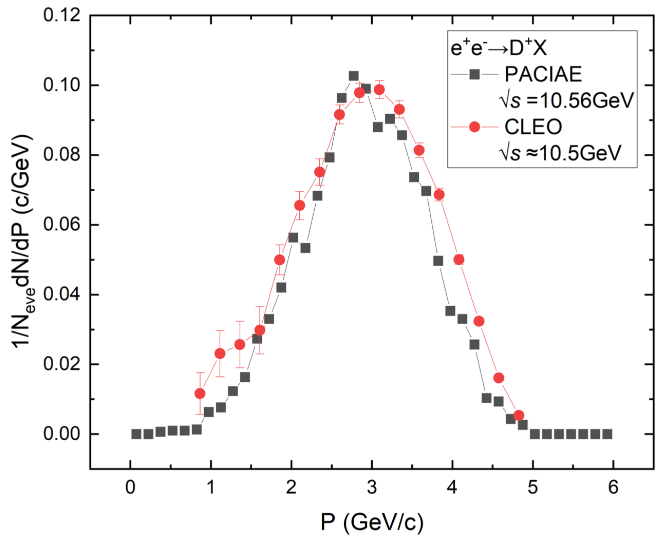
R(4140)

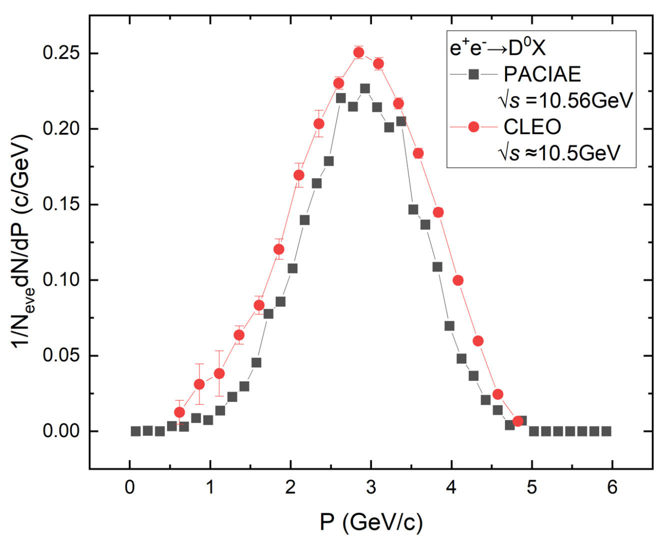


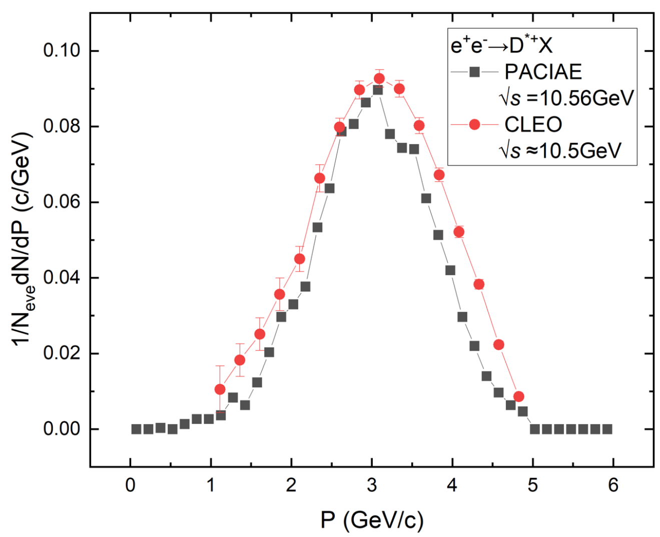
T. W. Wu, M. Z. Liu and **LSG***,
Phys. Rev. D 103 (2021) L031501

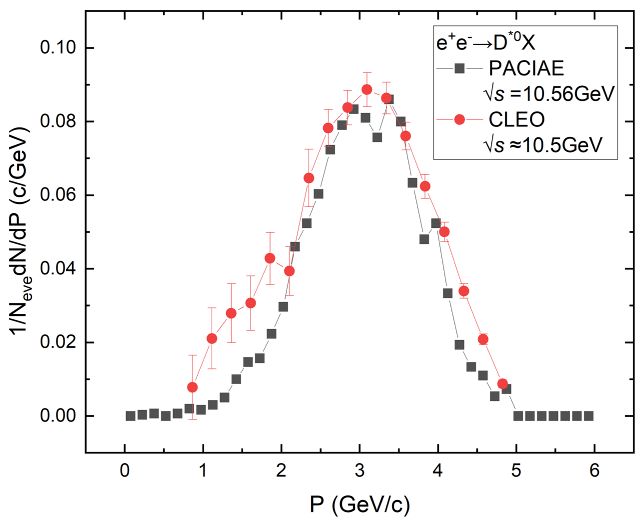
T. W. Wu and LSG*,
Science Bulletin 67 (2022) 1735–1738

Thanks a lot for your attention!









Charm meson spectra
in e^+e^- annihilation at 10.5 GeV
center of mass energy
M. Artuso *et al.* (CLEO)
Phys. Rev. D **70**, 112001