Revisiting Exotic Decays with XEFT

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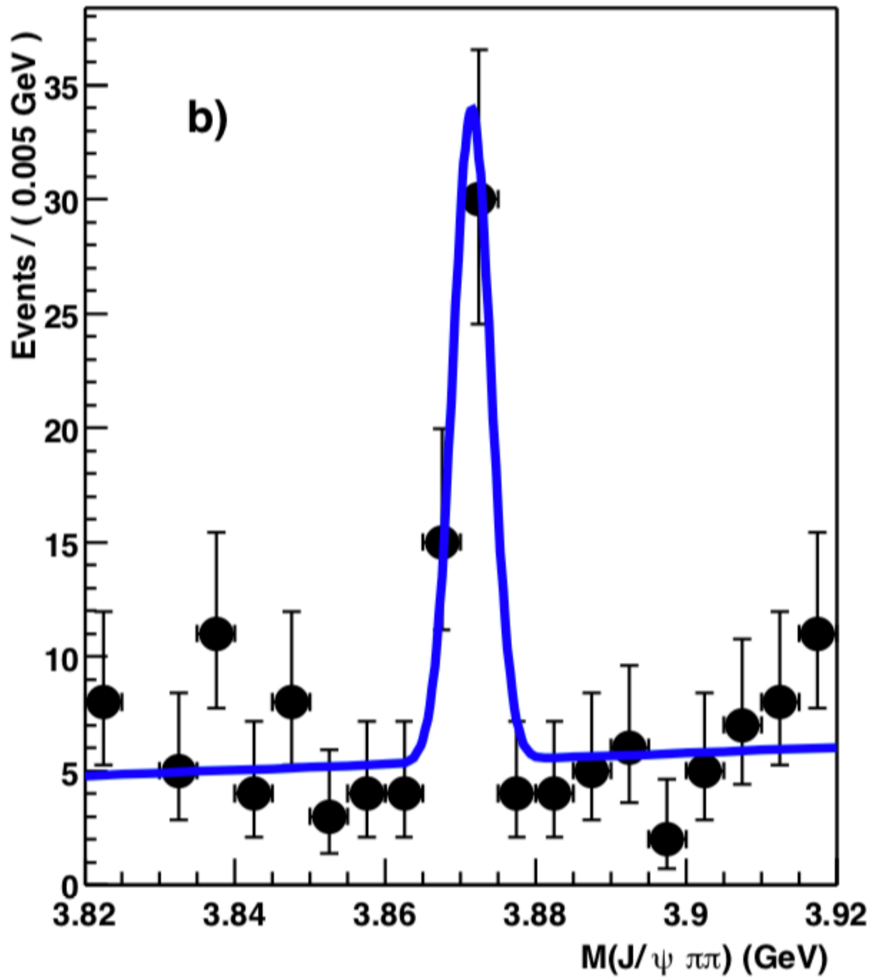
(Phys. Rev. D 107 (2023) 7, 076001; Phys. Rev. D 101 (2020) 5, 054024)

(HADRON2023, GENOVA, 05-JUN-23)

Outline

- Review XEFT
- XEFT and Power Counting
- NLO Calculations of X and Tcc decay width and Numerical Results
- Conclusion

X(3872) threshold



Belle Collaboration, Phys.Rev.Lett. 91, 262001 (2003)

$$B^{\pm} \to K^{\pm} X(3872)$$

$$J/\psi \pi^+ \pi^-$$

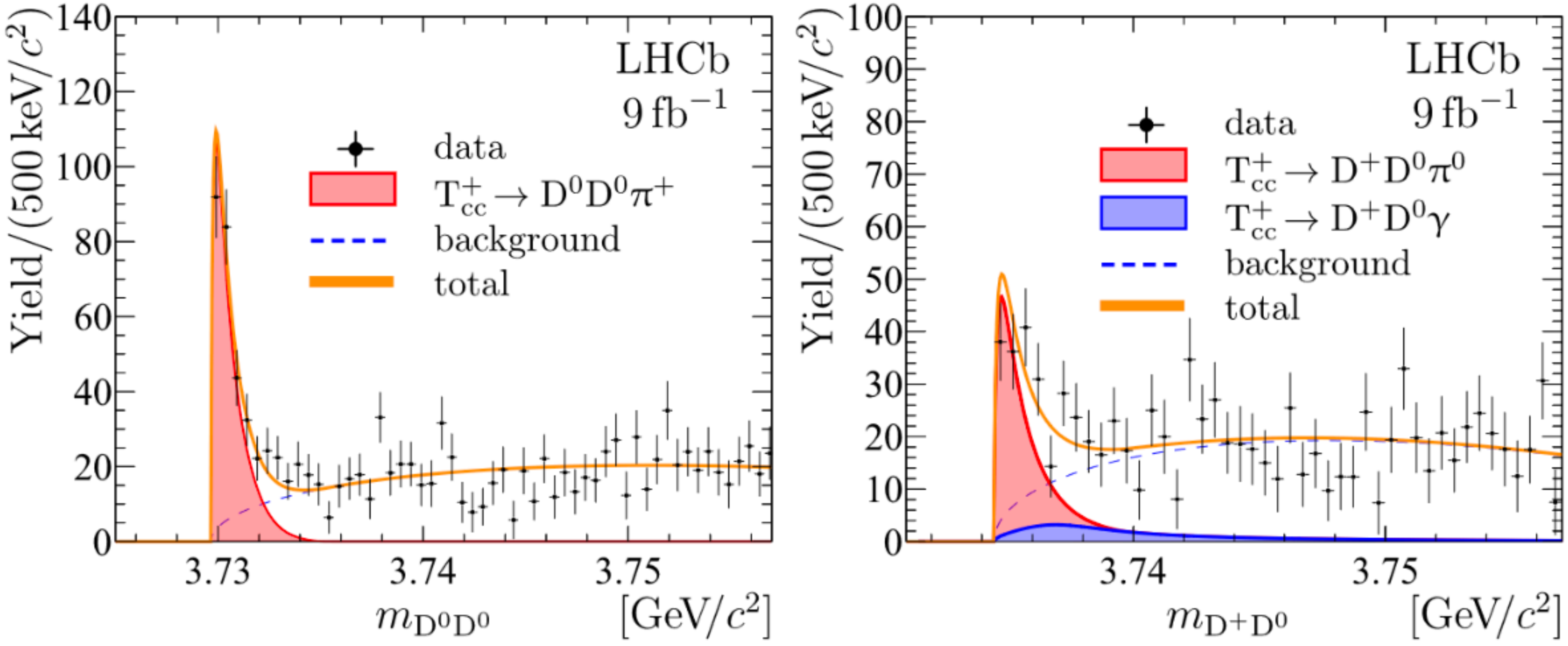
$$B_X = M_{D^0} + M_{D^{*0}} - M_X = (0.00 \pm 0.18)\mathrm{MeV}$$

- Assume X(3872) a DD* bound state in our analysis

$$X \to D^0 \overline{D}^0 \pi^0$$

$$\frac{B\left(X(3872) \to D^0 \bar{D}^0 \pi^0\right)}{B\left(X(3872) \to \pi^+ \pi^- J/\psi\right)} = 9.4^{+3.6}_{-4.3}$$

T_{cc}^{+} threshold



LHCb, *Nature Commun.* 13 (2022) 1, 3351

$D^{*+}D^0$

$\delta m \sim -0.36\text{MeV}$

$D^{*0}D^+$

$\delta m \sim 1.7\text{MeV}$

XEFT Lagrangian

$$\mathcal{L}_{\rm XEFT} = \sum_{\phi=\boldsymbol{D},\bar{\boldsymbol{D}}} \boldsymbol{\phi}^\dagger \left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2M_{D^{*0}}}\right)\boldsymbol{\phi} + \sum_{\phi=D,\bar{D}} D^\dagger \left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2M_{D^0}}\right) D + \pi^\dagger \left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2M_{\pi^0}} + \delta\right)\pi$$

$$+ \left[\frac{\bar{g}}{F_\pi}\frac{1}{\sqrt{2M_{\pi^0}}}\left(D\boldsymbol{D}^\dagger \cdot \boldsymbol{\nabla}\pi + \bar{D}^\dagger \bar{\boldsymbol{D}} \cdot \boldsymbol{\nabla}\pi^\dagger\right) + \text{H.c.}\right]$$

HHχPT

$$- \frac{C_0}{2}\left(\bar{\boldsymbol{D}}D + \boldsymbol{D}\bar{D}\right)^\dagger \cdot \left(\bar{\boldsymbol{D}}D + \boldsymbol{D}\bar{D}\right)$$

Non-Pert.

$$+ \left[\frac{C_2}{16}\left(\bar{\boldsymbol{D}}D + \boldsymbol{D}\bar{D}\right)^\dagger \cdot \left(\bar{\boldsymbol{D}}(\overleftrightarrow{\nabla})^2 D + \boldsymbol{D}(\overleftrightarrow{\nabla})^2\bar{D}\right) + \text{H.c.}\right]$$

$$+ \left[\frac{B_1}{\sqrt{2}}\frac{1}{\sqrt{2M_{\pi^0}}}\left(\bar{\boldsymbol{D}}D + \boldsymbol{D}\bar{D}\right)^\dagger \cdot D\bar{D}\boldsymbol{\nabla}\pi + \text{H.c.}\right]$$

$$+ \frac{C_\pi}{2M_{\pi^0}}\left(D^\dagger\pi^\dagger D\pi + \bar{D}^\dagger\pi^\dagger\bar{D}\pi\right) + C_{0D}D^\dagger\bar{D}^\dagger D\bar{D}\,,$$

NLO Pert.

S. Fleming, M. Kusunoki,
T. Mehen, Phys. Rev. D **76**, 034006 (2007)

L. Dai, FK. Guo, T. Mehen, PhysRevD.101.054024 (2020)

XEFT Power Counting

- P.C. parameter Q (dynamical mom of X(3872)):

$$\{p_D, p_{D^*}, p_\pi, \mu, \gamma_0\} = \mathcal{O}(Q)$$

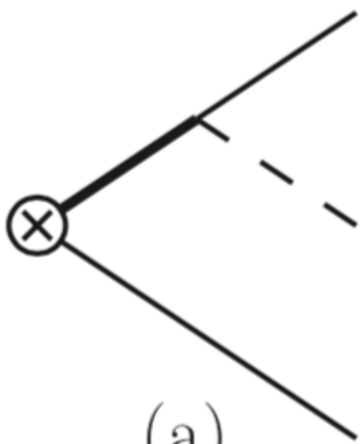
$$\mu = \sqrt{\Delta^2 - M_{\pi^0}^2} \simeq 44\text{MeV} \quad \text{with} \quad \Delta \equiv M_{D^*} - M_D$$

$$\gamma_0 = \sqrt{2\mu_0 B_X}$$

LOOP: $\int d^4p \sim Q^5$ Propagator: $\frac{1}{p^2-\gamma^2} \sim Q^{-2}$

- Dynamic pion: NR & Perturbative

$$D^* \to D + \pi \quad \text{with} \quad M_{D*} - M_D - M_\pi \sim 7\text{MeV}$$



(a)

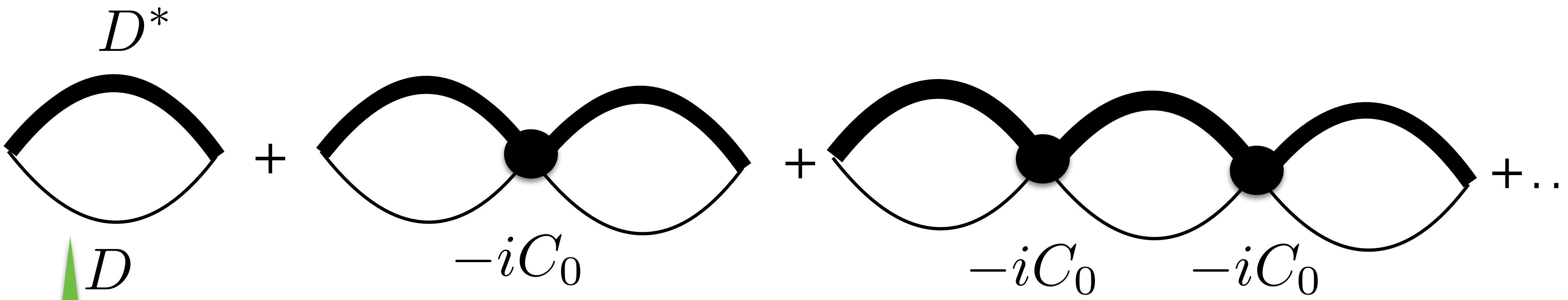
$$\frac{g^2\mu_0\mu}{8\pi F_\pi^2} \simeq \frac{1}{20} \cdots \frac{1}{10}$$

S. Fleming, M. Kusunoki,
T. Mehen, Phys. Rev. D **76**, 034006 (2007)

XEFT Power Counting

- C_0 treated non-perterbatively, generate X(3872) bound state

$$-\frac{C_0}{2}(\overline{\boldsymbol{D}}D + \boldsymbol{D}\overline{D})^\dagger \cdot (\overline{\boldsymbol{D}}D + \boldsymbol{D}\overline{D})$$



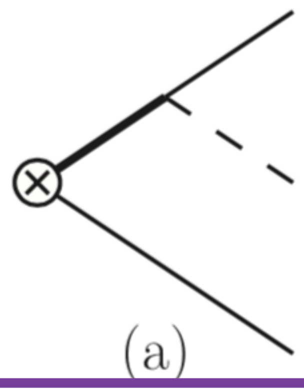
$$\Sigma_0(-B_X) \sim Q^5 Q^{-4} = Q^1$$

- Pole at B_X

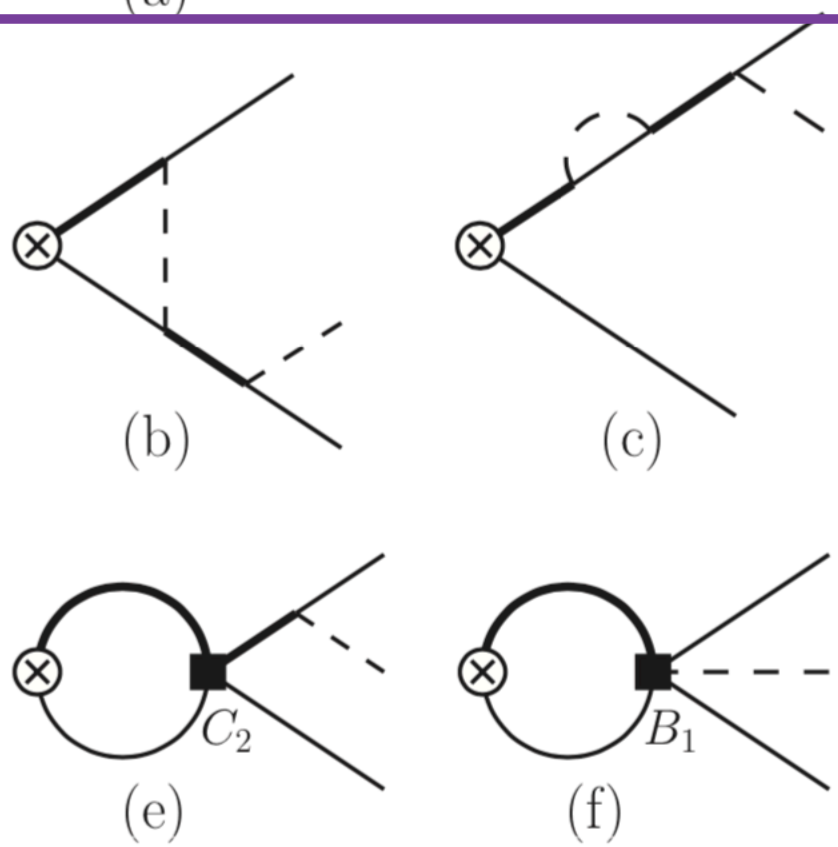
$$1 + C_0 \Sigma_0(-B_X) = 0 \to C_0 \sim Q^{-1}$$

XEFT Power Counting

$\mathbf{D}D^\dagger \cdot \nabla\pi$



$$\mathcal{O}\left(Q/Q^2\right) = \mathcal{O}\left(Q^{-1}\right)$$



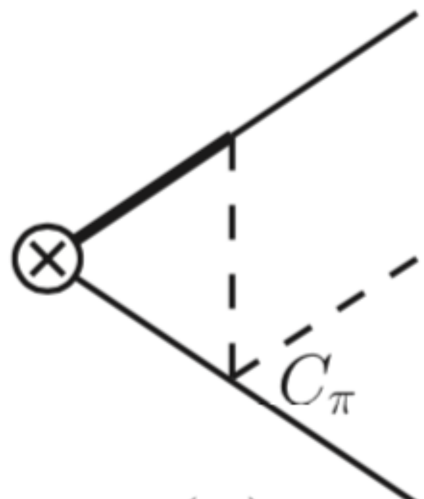
(b): $\mathcal{O}(Q^3 Q^{-8} Q^5) = \mathcal{O}(Q^0)$

Similar for others

S. Fleming, M. Kusunoki, T. Mehen, Phys. Rev. D **76**, 034006 (2007)

XEFT Power Counting

The diagram: $\mathcal{O}(QQ^{-6}Q^5) = \mathcal{O}(Q^0)$

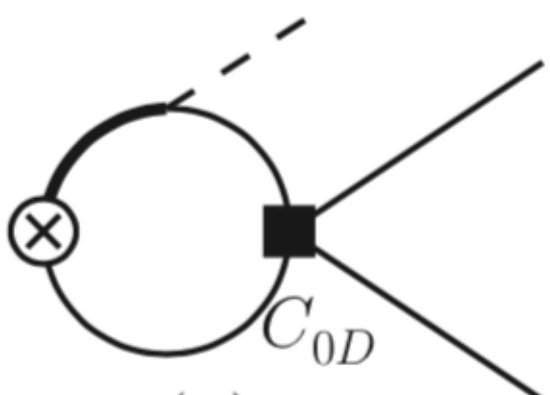


(d)

Num. HHχPT: $i\mathcal{A}_{h_0,h_1} = i\frac{2}{3}(6h_0 + h_1)\frac{M_\pi^2}{F_\pi^2} \sim 0.65$

Match to $\frac{C_\pi}{2M_{\pi^0}}\left(D^\dagger\pi^\dagger D\pi + \overline{D}^\dagger\pi^\dagger\overline{D}\pi\right) \to C_\pi = \mathcal{O}(Q^0)$

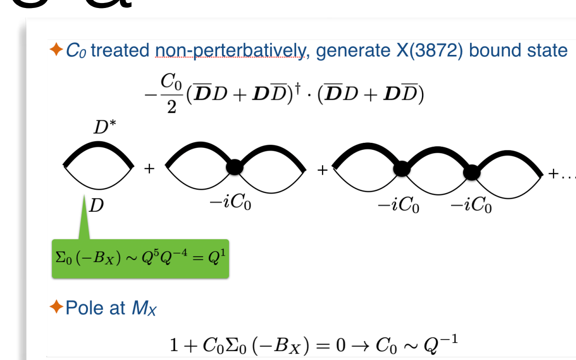
FK. Guo, C. Hanhart, S. Kreward, Phys. Lett. B666, 251–255 (2008)
FK. Guo, C. Hanhart, Ulf-G. Meissner Eur. Phys. J. A40, 171–179 (2009)



(g)

The diagram: $\mathcal{O}(QQ^{-6}Q^5) = \mathcal{O}(Q^0)$

To estimate C_{0D}, assume there is a bound state near $D\bar{D}$ threshold:



$a \sim -\frac{m_D C_{0D}}{4\pi} = -\frac{1}{262}\ \mathrm{MeV}^{-1}$ $\qquad C_{0D} \sim 1\ \mathrm{fm}^2$

FK. Guo, C.Hidalgo-Duque, J. Nieves, A. Ozpineci, Manuel Pavón Valderrama, Eur. Phys. J. C (2014) 74:2885

L. Dai, FK. Guo, T. Mehen, PhysRevD.101.054024 (2020)

Decay Rate of X ➡ $D\bar{D}\pi$

$$\left(\frac{\bar{g}\mu_0}{F_\pi}C_2\left(\Lambda_{\rm PDS}\right)+B_1\left(\Lambda_{\rm PDS}\right)\right)\left(\Lambda_{\rm PDS}-\gamma\right)=\pm r_0^3$$

$$C_2\left(\Lambda_{\rm PDS}\right)=\frac{2\pi}{\mu_0}\frac{r_0}{2}\frac{1}{\left(\Lambda_{\rm PDS}-\gamma\right)^2}$$

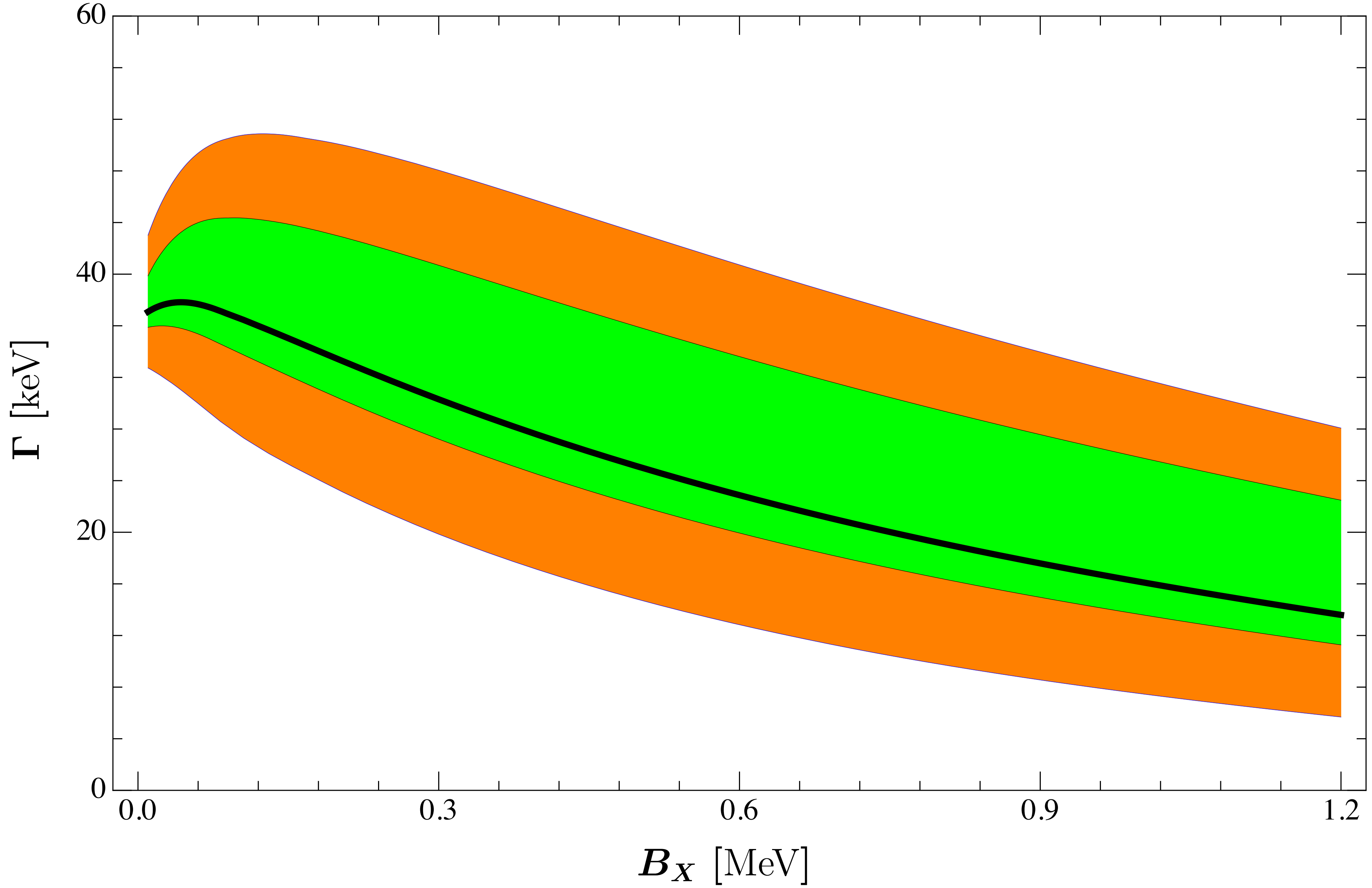
$$\begin{aligned}\frac{d\Gamma_{\rm NLO}}{dp_D^2dp_{\bar{D}}^2}=&\frac{d\Gamma_{\rm LO}}{dp_D^2dp_{\bar{D}}^2}\left(1+\frac{\bar{g}^2\mu_0\gamma}{3\pi F_\pi^2}\left(\frac{4\gamma^2-\mu^2}{4\gamma^2+\mu^2}\right)+C_2\left(\Lambda_{\rm PDS}\right)\frac{\mu_0\gamma\left(\gamma-\Lambda_{\rm PDS}\right)^2}{\pi}\right)\\
&-\frac{\bar{g}\gamma}{8\sqrt{2}\pi^3F_\pi}\left(\sqrt{2}\frac{\bar{g}\mu_0}{F_\pi}C_2\left(\Lambda_{\rm PDS}\right)-B_1\left(\Lambda_{\rm PDS}\right)\right)\left(\Lambda_{\rm PDS}-\gamma\right)\left(\vec{p}_\pi\cdot\vec{\epsilon}_X\right)^2\left(\frac{1}{p_D^2+\gamma^2}+\frac{1}{p_{\bar{D}}^2+\gamma^2}\right)\\
&+\frac{1}{8\pi^2}\frac{\bar{g}^4}{F_\pi^4}\frac{\gamma}{M_{\pi^0}}\left[\left(\vec{p}_\pi\cdot\vec{\epsilon}_X\right)^2\left(\frac{1}{p_D^2+\gamma^2}+\frac{1}{p_{\bar{D}}^2+\gamma^2}\right)\left(\frac{p_D^2\tilde{I}_1^{(2)}\left(p_D\right)}{P_D^2+\gamma^2}+\frac{p_{\bar{D}}^2\tilde{I}_1^{(2)}\left(p_{\bar{D}}\right)}{P_{\bar{D}}^2+\gamma^2}\right)\right]\\
&+\frac{1}{8\pi^2}\frac{\bar{g}^4}{F_\pi^4}\frac{\gamma}{M_{\pi^0}}\left[\vec{p}_\pi\cdot\vec{\epsilon}_X\left(\frac{1}{p_D^2+\gamma^2}+\frac{1}{p_{\bar{D}}^2+\gamma^2}\right)\left(\frac{\vec{p}_D\cdot\vec{\epsilon}_X\vec{p}_D\cdot\vec{p}_\pi}{p_D^2+\gamma^2}\left(I(p_D)-2I^{(1)}(p_D)+I_0^{(2)}(p_D)\right)\right.\right.\\
&\left.\left.+\frac{\vec{p}_{\bar{D}}\cdot\vec{\epsilon}_X\vec{p}_{\bar{D}}\cdot\vec{p}_\pi}{p_{\bar{D}}^2+\gamma^2}\left(I(p_{\bar{D}})-2I^{(1)}(p_{\bar{D}})+I_0^{(2)}(p_{\bar{D}})\right)\right)\right]\\
&+\frac{C_\pi\bar{g}^2\gamma}{16\pi^2F_\pi^2M_{\pi^0}\mu_0}\vec{p}_\pi\cdot\vec{\epsilon}_X\left(\frac{1}{p_D^2+\gamma^2}+\frac{1}{p_{\bar{D}}^2+\gamma^2}\right)\left(\vec{p}_D\cdot\vec{\epsilon}_X\left(I^{(1)}(p_D)-I(p_D)\right)+\vec{p}_{\bar{D}}\cdot\vec{\epsilon}_X\left(I^{(1)}(p_{\bar{D}})-I(p_{\bar{D}})\right)\right)\\
&+\frac{C_{0D}\bar{g}^2\gamma}{4\pi^2F_\pi^2\mu_0}\left(\vec{p}_\pi\cdot\vec{\epsilon}_X\right)^2I(p_\pi)\left(\frac{1}{p_D^2+\gamma^2}+\frac{1}{p_{\bar{D}}^2+\gamma^2}\right).\end{aligned}$$

DD re-scattering large

S. Fleming, M. Kusunoki, T. Mehen, Phys. Rev. D **76**, 034006 (2007)

L. Dai, FK. Guo, T. Mehen, PhysRevD.101.054024 (2020)

Decay Rate of X ➟ $D\bar{D}\pi$



Parameters: $0 \le r_0 \le \frac{1}{100}\text{MeV}^{-1}, C_\pi = (4.1 \pm 0.7) \times 10^{-3}\text{MeV}^{-1}, C_{0D} = \pm 1\text{fm}^2$

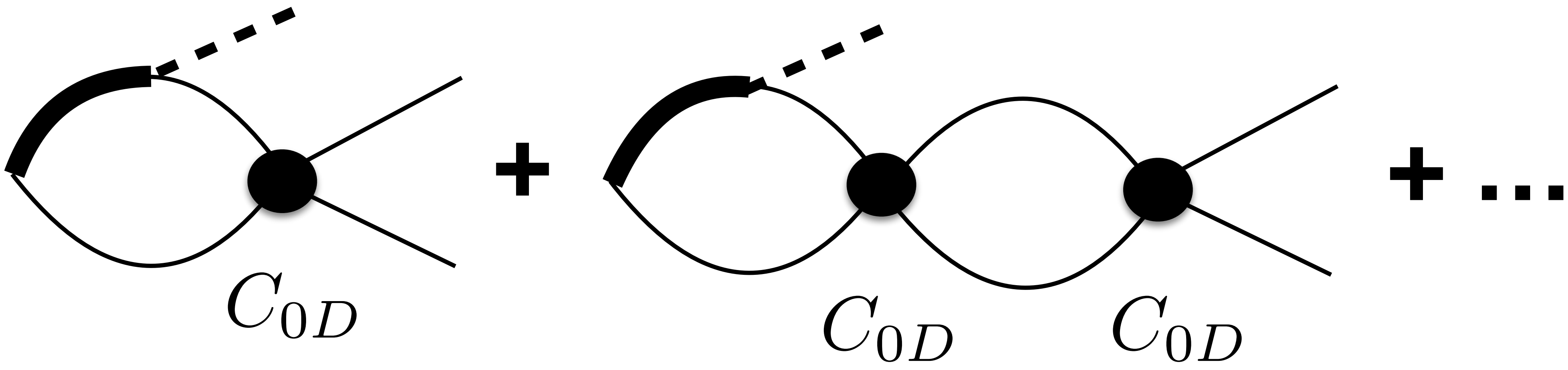
NLO Correction: $C_\pi \to \le 1\%, \quad C_{0D} \to 20\%$

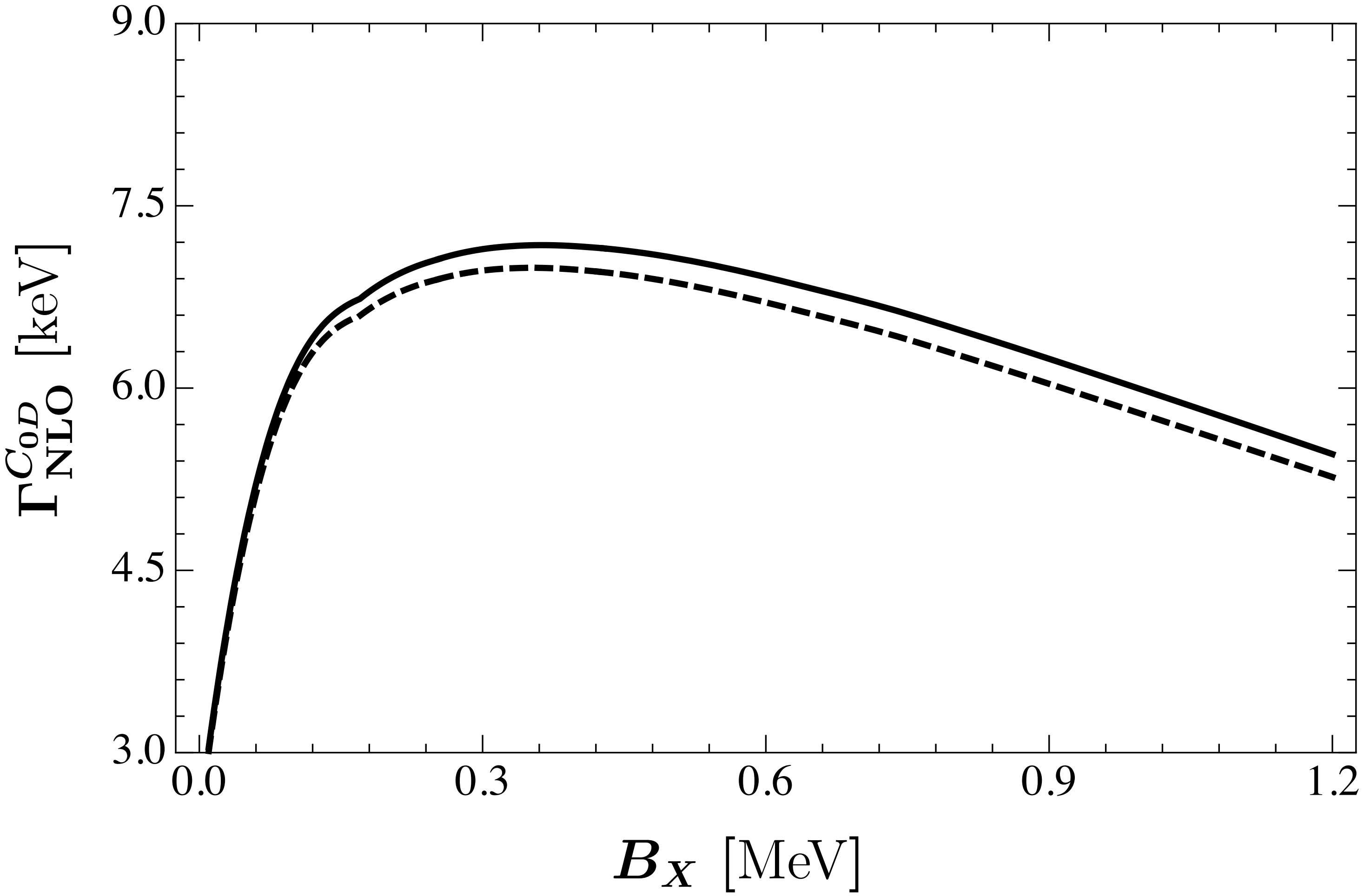
S. Fleming, M. Kusunoki, T. Mehen, Phys. Rev. D **76**, 034006 (2007)

L. Dai, FK. Guo, T. Mehen, PhysRevD.101.054024 (2020)

Final States Rescattering

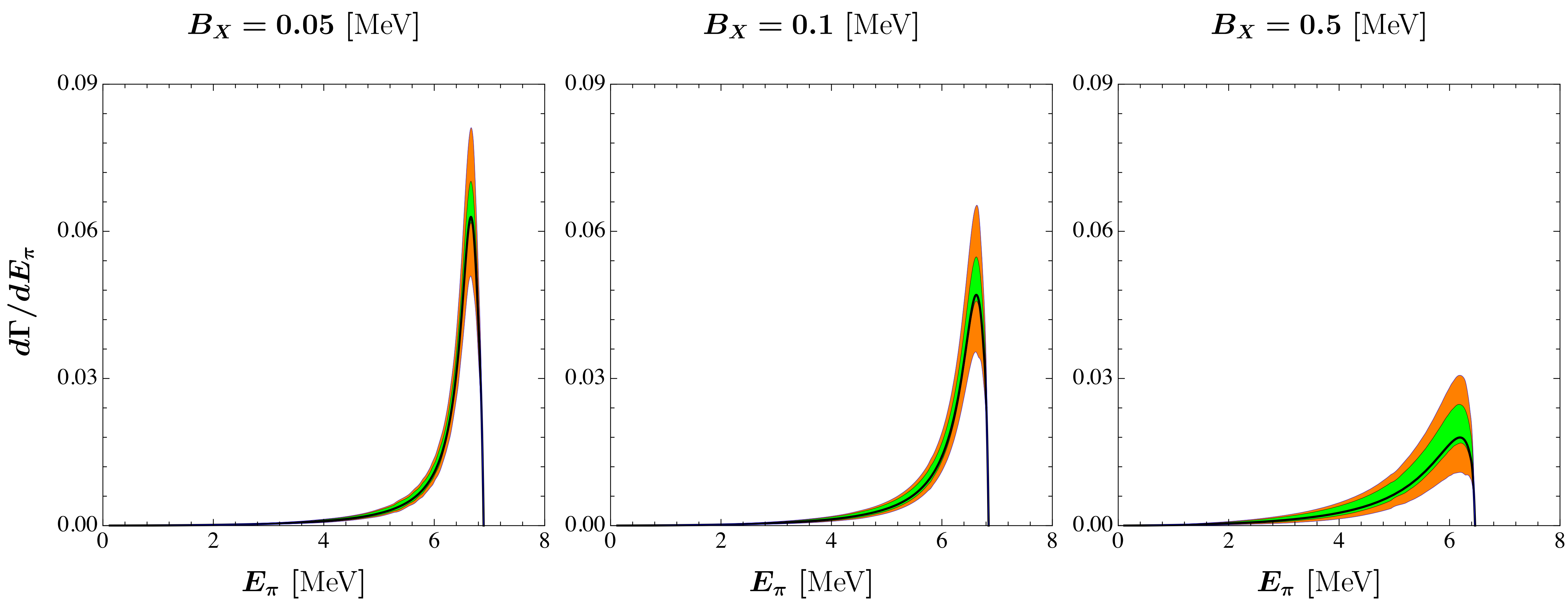
✦ Final State Rescattering





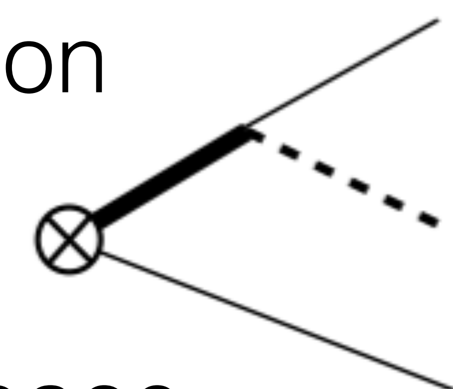
L. Dai, FK. Guo, T. Mehen,
PhysRevD.101.054024 (2020)

π distribution



Location of peaks less sensitive to NLO correction

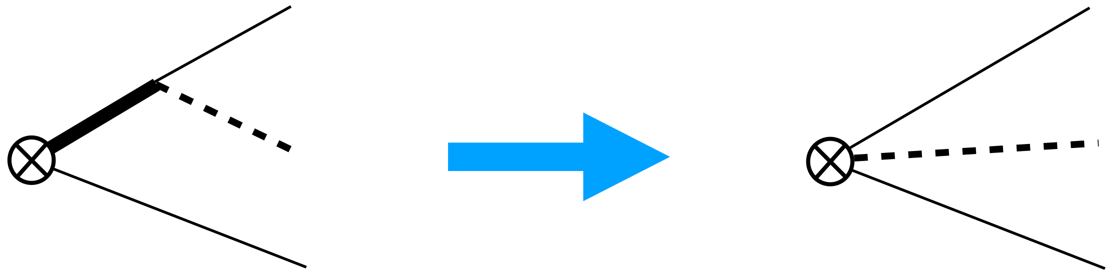
$$M_{D*} - M_D - M_\pi \sim 7\text{MeV}$$



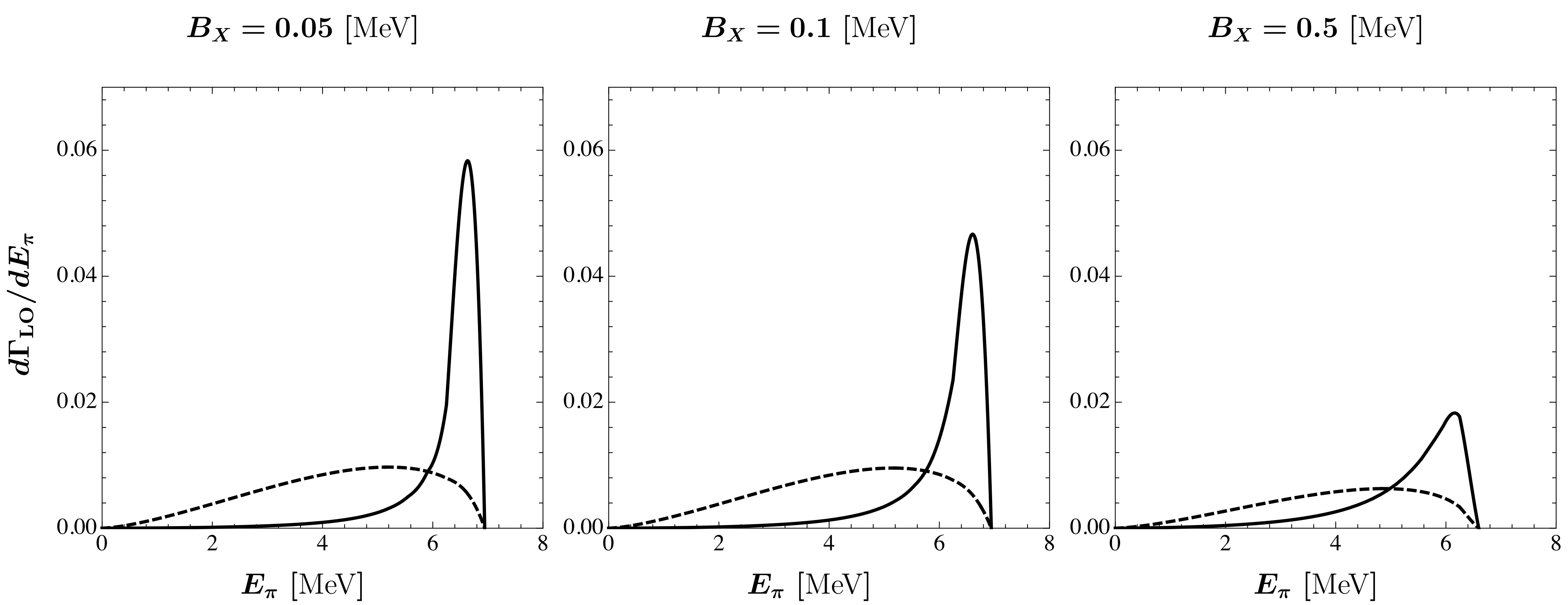
Locations mostly controlled by 3-body phase space, however…

L. Dai, FK. Guo, T. Mehen, PhysRevD.101.054024 (2020)

D* propagator



$$iA_{\mathrm{LO}} = \frac{\bar{g}\mu_0}{F_\pi\sqrt{M_{\pi^0}}}\vec{p}_\pi\cdot\vec{\epsilon}_X\left(\frac{1}{\vec{p}_D^{\,2}+\gamma^2}+\frac{1}{\vec{p}_{\bar{D}}^{\,2}+\gamma^2}\right) \rightarrow \vec{p}_\pi\cdot\vec{\epsilon}_X$$



D* propagator effects, reflection of Molecular nature?

L. Dai, FK. Guo, T. Mehen, PhysRevD.101.054024 (2020)

T_{cc}^{+} Lagrangian

$$\mathcal{L}_{\rm LO} = H^{*i\dagger}\left(i\partial^0 + \frac{\nabla^2}{2m_{H^*}} - \delta^*\right)H^{*i} + H^\dagger\left(i\partial^0 + \frac{\nabla^2}{2m_H} - \delta\right)H + \frac{g}{f_\pi}H^\dagger\partial^i\pi H^{*i} + \text{H.c.} - C_0^{(0)}(H^{*T}\tau_2 H)^\dagger(H^{*T}\tau_2 H) - C_0^{(1)}(H^{*T}\tau_2\tau_a H)^\dagger(H^{*T}\tau_2\tau_a H)$$

$$H = \begin{pmatrix} D^0 \\ D^+ \end{pmatrix} \qquad H^{*i} = \begin{pmatrix} D^{*0i} \\ D^{*+i} \end{pmatrix}$$

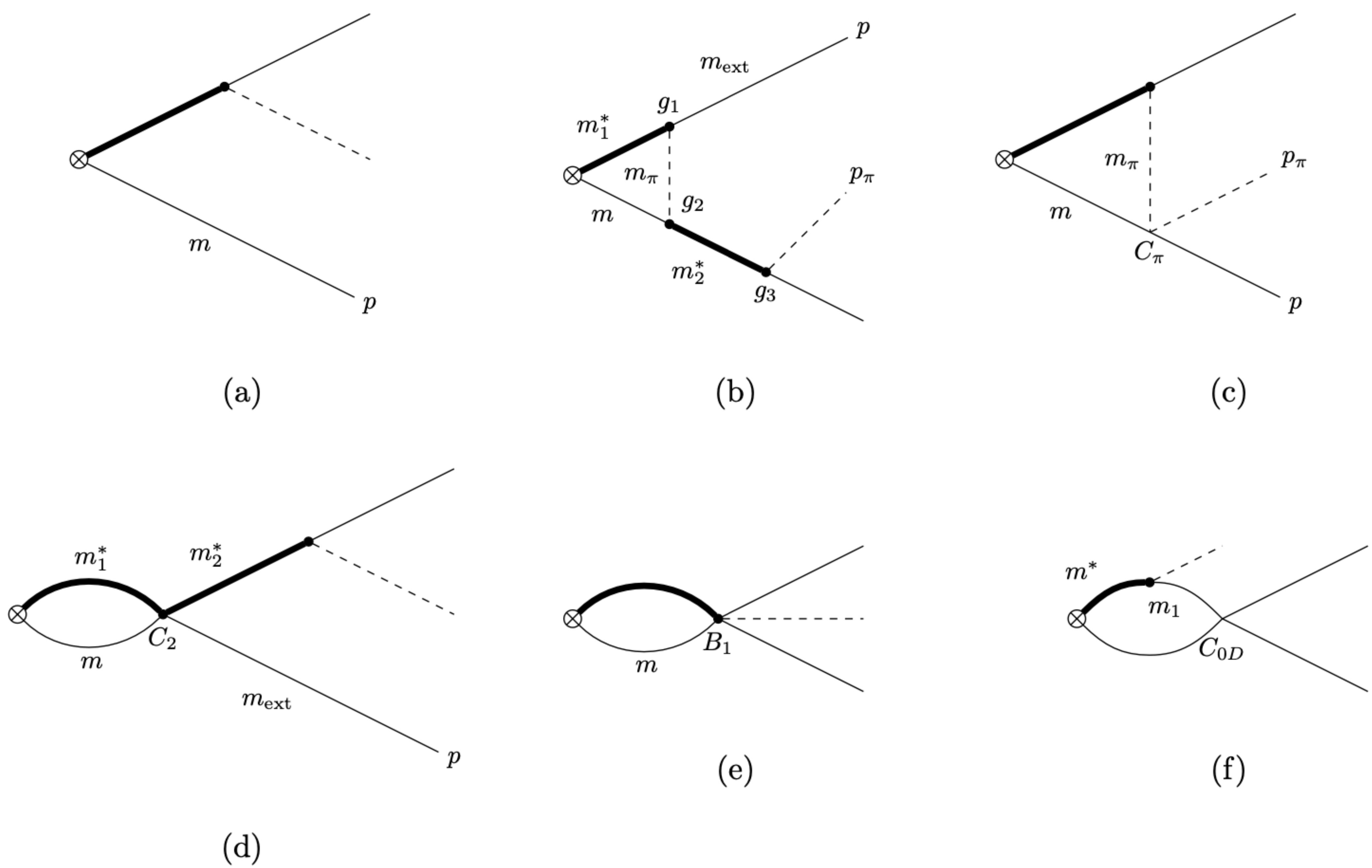
$$\mathcal{L}_{B_1} \to B_1^{(1)}(D^+D^{*0})^\dagger(D^+D^0\nabla\pi^0) + B_1^{(2)}(D^0D^{*+})^\dagger(D^+D^0\nabla\pi^0) + \frac{B_1^{(3)}}{2}(D^0D^{*+})^\dagger(D^0D^0\nabla\pi^+) + \frac{B_1^{(4)}}{2}(D^+D^{*0})^\dagger(D^0D^0\nabla\pi^+) + \text{H.c.}\,.$$

$$\mathcal{L}_{C_{0D}} = C_{0D}^{(1)}(H\tau_2\tau_a H)^\dagger(H\tau_2\tau_a H) \to \frac{C_{0D}^{(1)}}{2}(D^0D^0)^\dagger(D^0D^0) + C_{0D}^{(1)}(D^+D^0)^\dagger(D^+D^0)\,,$$

Other contact terms

L. Dai, S. Fleming R. Hodges, T. Mehen, *Phys. Rev. D* 107 (2023) 7, 076001

T_{cc}^+ Strong Decay Diagrams



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T_{cc}^{+} Decay rate

$$
\begin{aligned}
\frac{d\Gamma_0^{NLO}(T_{cc}^+ \to D^+ D^0 \pi^0)}{d\mathbf{p}_0^2 d\mathbf{p}_+^2} = \frac{2}{\mathrm{Re}\,\mathrm{tr}\,\Sigma'^{LO}(-E_T)} \mathrm{Re}\Big[& \mathcal{A}_{(2a)}(\mathbf{p}_+, m_+, m_0^*, -g/\sqrt{2}f_\pi) \\
& \times \Big(\mathcal{A}_{(2b)}(\mathbf{p}_0, m_+, m_0, m_{\pi^0}, m_0^*, m_+^*, -g/\sqrt{2}f_\pi, g/\sqrt{2}f_\pi, g/\sqrt{2}f_\pi) \\
& + \mathcal{A}_{(2b)}(\mathbf{p}_+, m_+, m_+, m_{\pi^-}, m_0^*, m_0^*, g/f_\pi, g/f_\pi, -g/\sqrt{2}f_\pi) \\
& - \mathcal{A}_{(2b)}(\mathbf{p}_0, m_0, m_0, m_{\pi^+}, m_+^*, m_+^*, g/f_\pi, g/f_\pi, g/\sqrt{2}f_\pi) \\
& - \mathcal{A}_{(2b)}(\mathbf{p}_+, m_0, m_+, m_{\pi^0}, m_+^*, m_0^*, g/\sqrt{2}f_\pi, -g/\sqrt{2}f_\pi, -g/\sqrt{2}f_\pi) \\
& + \mathcal{A}_{(2c)}(\mathbf{p}_0, m_+, m_0, m_{\pi^0}, m_0^*, -g/\sqrt{2}f_\pi, C_\pi^{(2)}) \\
& - \mathcal{A}_{(2c)}(\mathbf{p}_0, m_0, m_0, m_{\pi^+}, m_+^*, g/f_\pi, C_\pi^{(1)}) \\
& + \mathcal{A}_{(2f)}(m_0, m_+, m_0^*, -g/\sqrt{2}f_\pi, C_{0D}^{(1)}) \\
& - \mathcal{A}_{(2f)}(m_+, m_0, m_+^*, g/\sqrt{2}f_\pi, C_{0D}^{(1)}) \Big)^* + (D^0 \leftrightarrow D^+, \pi^+ \leftrightarrow \pi^-) \Big] \\
& - \frac{1}{\mathrm{Re}\,\mathrm{tr}\,\Sigma'^{LO}(-E_T)} \Big[[\beta_1(\mathbf{p}_+^2 + \gamma_+^2) + \beta_2] \big(\big|\mathcal{A}_{(2a)}(\mathbf{p}_+, m_+, m_0^*, -g/\sqrt{2}f_\pi)\big|^2 \\
& - \mathcal{A}_{(2a)}(\mathbf{p}_0, m_0, m_+^*, g/\sqrt{2}f_\pi) \mathcal{A}_{(2a)}^*(\mathbf{p}_+, m_+, m_0^*, -g/\sqrt{2}f_\pi) \big) \\
& + [\beta_3(\mathbf{p}_0^2 + \gamma_0^2) + \beta_4] \big(\big|\mathcal{A}_{(2a)}(\mathbf{p}_0, m_0, m_+^*, g/\sqrt{2}f_\pi)\big|^2 \\
& - \mathcal{A}_{(2a)}(\mathbf{p}_+, m_+, m_0^*, -g/\sqrt{2}f_\pi) \mathcal{A}_{(2a)}^*(\mathbf{p}_0, m_0, m_+^*, g/\sqrt{2}f_\pi) \big) \Big] \\
& - \frac{d\Gamma_0^{LO}(T_{cc}^+ \to D^+ D^0 \pi^0)}{d\mathbf{p}_0^2 d\mathbf{p}_+^2} \frac{\mathrm{Re}\,\Sigma_0'^{NLO}}{\mathrm{Re}\,\mathrm{tr}\,\Sigma'^{LO}} \Big|_{C_2 \to 0, E = -E_T}
\end{aligned}
$$

L. Dai, S. Fleming R. Hodges, T. Mehen,
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T_{cc}^+ Decay Width

$$
\begin{aligned}
\frac{d\Gamma_0^{NLO}(T_{cc}^+ \to D^0 D^0 \pi^+)}{d\mathbf{p}_1^2 d\mathbf{p}_2^2} = \frac{1}{\mathrm{Re\,tr}\,\Sigma'^{LO}(-E_T)} \mathrm{Re}\Big[& \mathcal{A}_{(2a)}(\mathbf{p}_2, m_0, m_+^*, g/f_\pi) \\
& \times \Big(\mathcal{A}_{(2b)}(\mathbf{p}_1, m_0, m_0, m_{\pi^+}, m_+^*, m_+^*, g/f_\pi, g/f_\pi, g/f_\pi) \\
& + \mathcal{A}_{(2b)}(\mathbf{p}_2, m_0, m_0, m_{\pi^+}, m_+^*, m_+^*, g/f_\pi, g/f_\pi, g/f_\pi) \\
& - \mathcal{A}_{(2b)}(\mathbf{p}_1, m_+, m_0, m_{\pi^0}, m_0^*, m_+^*, -g/\sqrt{2}f_\pi, g/\sqrt{2}f_\pi, g/f_\pi) \\
& - \mathcal{A}_{(2b)}(\mathbf{p}_2, m_+, m_0, m_{\pi^0}, m_0^*, m_+^*, -g/\sqrt{2}f_\pi, g/\sqrt{2}f_\pi, g/f_\pi) \\
& + \mathcal{A}_{(2c)}(\mathbf{p}_1, m_0, m_0, m_{\pi^+}, m_+^*, g/f_\pi, C_\pi^{(3)}) \\
& - \mathcal{A}_{(2c)}(\mathbf{p}_1, m_+, m_0, m_{\pi^0}, m_0^*, -g/\sqrt{2}f_\pi, C_\pi^{(1)}) \\
& + \mathcal{A}_{(2f)}(m_0, m_0, m_+^*, g/f_\pi, C_{0D}^{(1)}/2) \Big)^* + (\mathbf{p}_1 \leftrightarrow \mathbf{p}_2) \\
& - \left(\frac{2g\mu_0}{f_\pi}\right)^2 \frac{\mathbf{p}_\pi^2}{3} \beta_5 \left(\frac{1}{\mathbf{p}_1^2 + \gamma_0^2} + \frac{1}{\mathbf{p}_2^2 + \gamma_0^2} \right) \Big] \\
& - \frac{d\Gamma_0^{LO}(T_{cc}^+ \to D^0 D^0 \pi^+)}{d\mathbf{p}_1^2 d\mathbf{p}_2^2} \left(\beta_4 + \frac{\mathrm{Re}\,\Sigma_0'^{NLO}}{\mathrm{Re\,tr}\,\Sigma'^{LO}} \Big|_{C_2 \to 0, E = -E_T} \right)
\end{aligned}
$$

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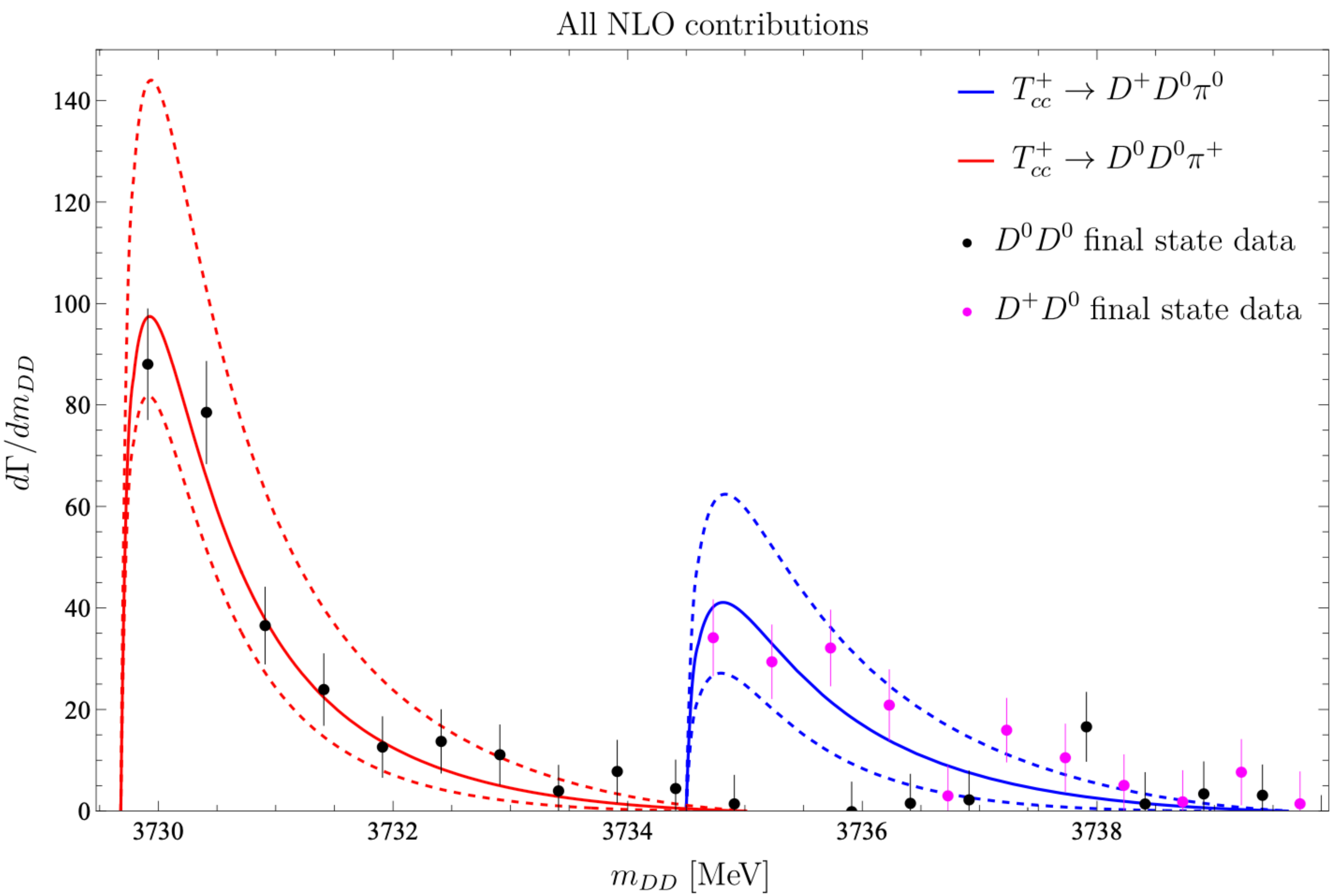
T_{cc}^{+} Decay Width



$$-1 \text{ fm}^2 \leq C_{0D} \leq 0.25 \text{ fm}^2 \text{ and } -0.26 \leq \beta_{2/4} \leq 0.$$

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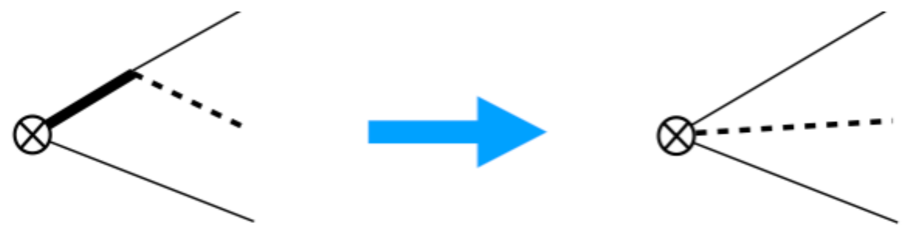
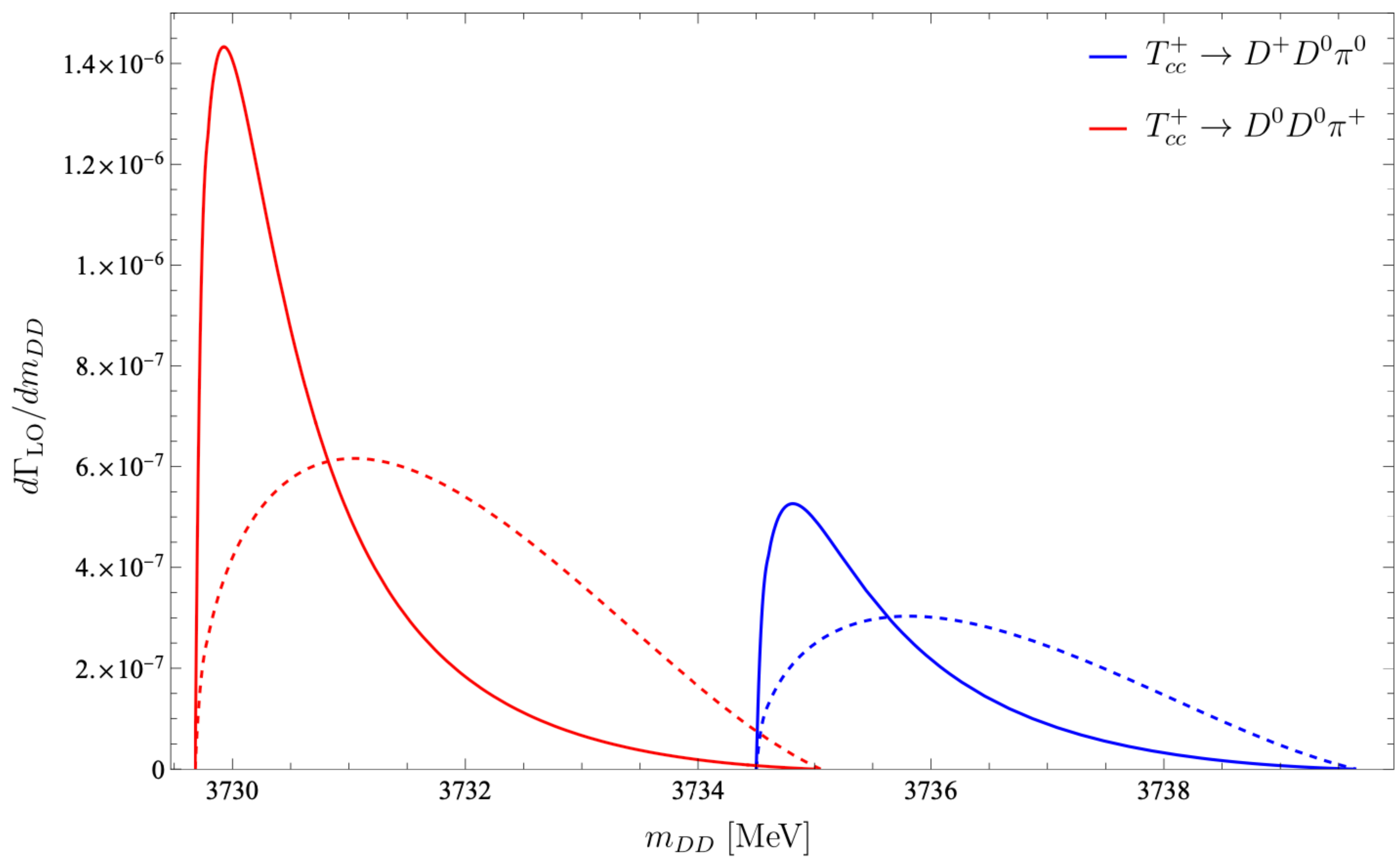
D* propagator





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Phys. Rev. D 107 (2023) 7, 076001

Summary

- Did NLO calculations X and Tcc decays; Added πD and DD re-scattering terms, which are needed by symmetry and power counting
- C_π correction is small, C_{0D} can be very large
- π distribution (m_{DD} distribution for Tcc) reflecting molecular nature of X and Tcc

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