RUHR UNIVERSITÄT BOCHUM RUB

Towards nature of the Tcc(3875)$^+$

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Hadron 2023, Genoa

in collaboration with

X. Dong, M. Du, E. Epelbaum, A. Filin, F.-K. Guo, C. Hanhart, A. Nefediev, J. Nieves and Q. Wang

PRD 105, 014024 (2022) and 2303.09441 [hep-ph]

Outline

— Intro: near-threshold exotic states

— Chiral EFT for Tcc and its applications

- Analysis of LHCb data and the role of 3-body cut

  Du et al., PRD 105, 014024 (2022)

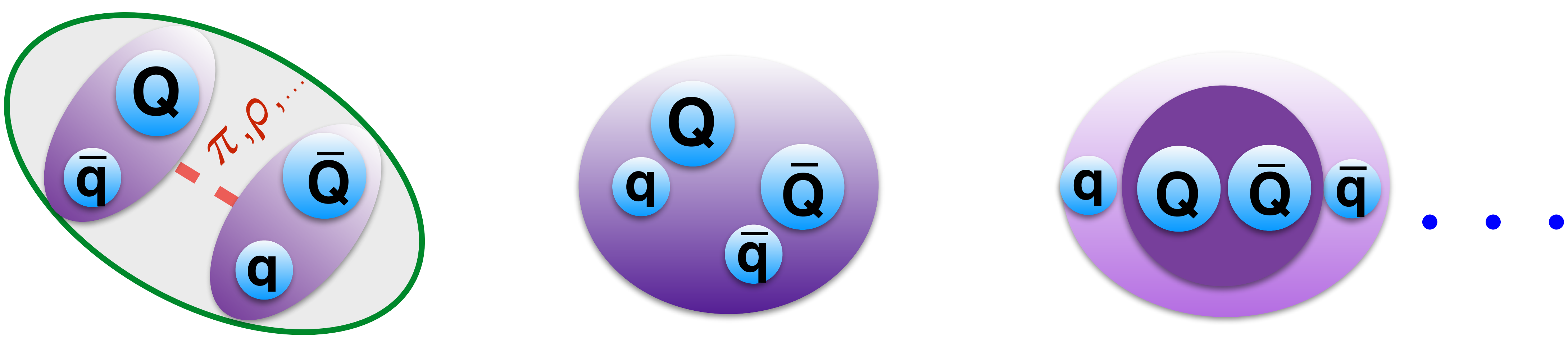
- Analysis of lattice data and the role of left-hand cuts

  Du et al., arXiv:2303.09441 [hep-ph]

— Conclusions

Evidence for Exotic States near thresholds

- Heavy-light sector $D_{s0}(2317)$, $D_{s1}(2460)$, $X_{0/1}(2900)$,… $cqq\bar{q}$
- XYZ
  - X(3872), … / $Z_c(3900)$, $Z_c(4020)$, $Z_{cs}(3982)$ … / Y(4230), Y(4360), Y(4660), … $c\bar{c}q\bar{q}$
  - $Z_b(10610)$, $Z_b(10650)$ $b\bar{b}q\bar{q}$
  - X(6900) $cc\bar{c}\bar{c}$
- Pentaquarks $P_c(4312)$, $P_c(4440)$, $P_c(4457)$, $P_{cs}(4459)$ $c\bar{c}qqq$
- double c-quark Tcc $ccq\bar{q}$



molecule tetraquark hadroquarkonium

Size: $\sim 1/\sqrt{2\mu E_B} >> 1\text{fm}$ $\sim 1/\Lambda_{\rm QCD} \sim 1\text{fm}$

Weinberg compositeness

Weinberg 1963-65

Physical coupling and ERE parameters via probability of a molecular component X

$$a = -2\,\frac{X}{1-X}\frac{1}{\gamma} + \mathcal{O}(1/\beta) \qquad r = -\frac{1-X}{X}\frac{1}{\gamma} + \mathcal{O}(1/\beta) \qquad g_R^2 = \frac{2\pi\gamma}{\mu^2}X + \mathcal{O}(1/\beta)$$

a < 0 — bound state

VB et al. 2004

If $|a| \gg |r|,\ r \sim 1/\beta$ $\quad\Longrightarrow\quad X \to 1$ $\Longrightarrow$ Molecule

If $|a| \ll |r|,\ r < 0$ $\quad\Longrightarrow\quad X \to 0$ $\Longrightarrow$ Compact state

- Insights on range effects — Albaladejo, Nieves 2022, Li et al. 2022, Song et al 2022, Kinogawa, Hyodo 2022
- Extensions mostly for resonances by Jido, Kamai, Nieves, Oller, Oset, Sekihara,… — review Kamai and Hyodo 2017
- Recent generalisations to virtual states, coupled-channels, … — Matuschek et al. EPJA 57 (2021), VB et al., PLB 833 (2022)

$T_{cc}(3875)^+$ $cc\bar{u}\bar{d}$

see also Talks related with Tcc at this Conference by

E. Spadaro Norella on Monday

L. Dai on Monday

Y. Yamaguchi on Tuesday

Simon Eidelman's prize Winner: I. Polyakov on Thursday

M. Sarpis on Thursday

E. Oset on Thursday

L. Dai on Thursday

V. Montesinos Llacer on Friday

What do we know about Tcc+?

Aaij et al [LHCb] Nature Physics (2022)

- LHCb reports a clear peak in $D^0 D^0 \pi^+$ spectrum right below the $D^{*+} D^0$ threshold

- Unitarized Breit-Wigner fit

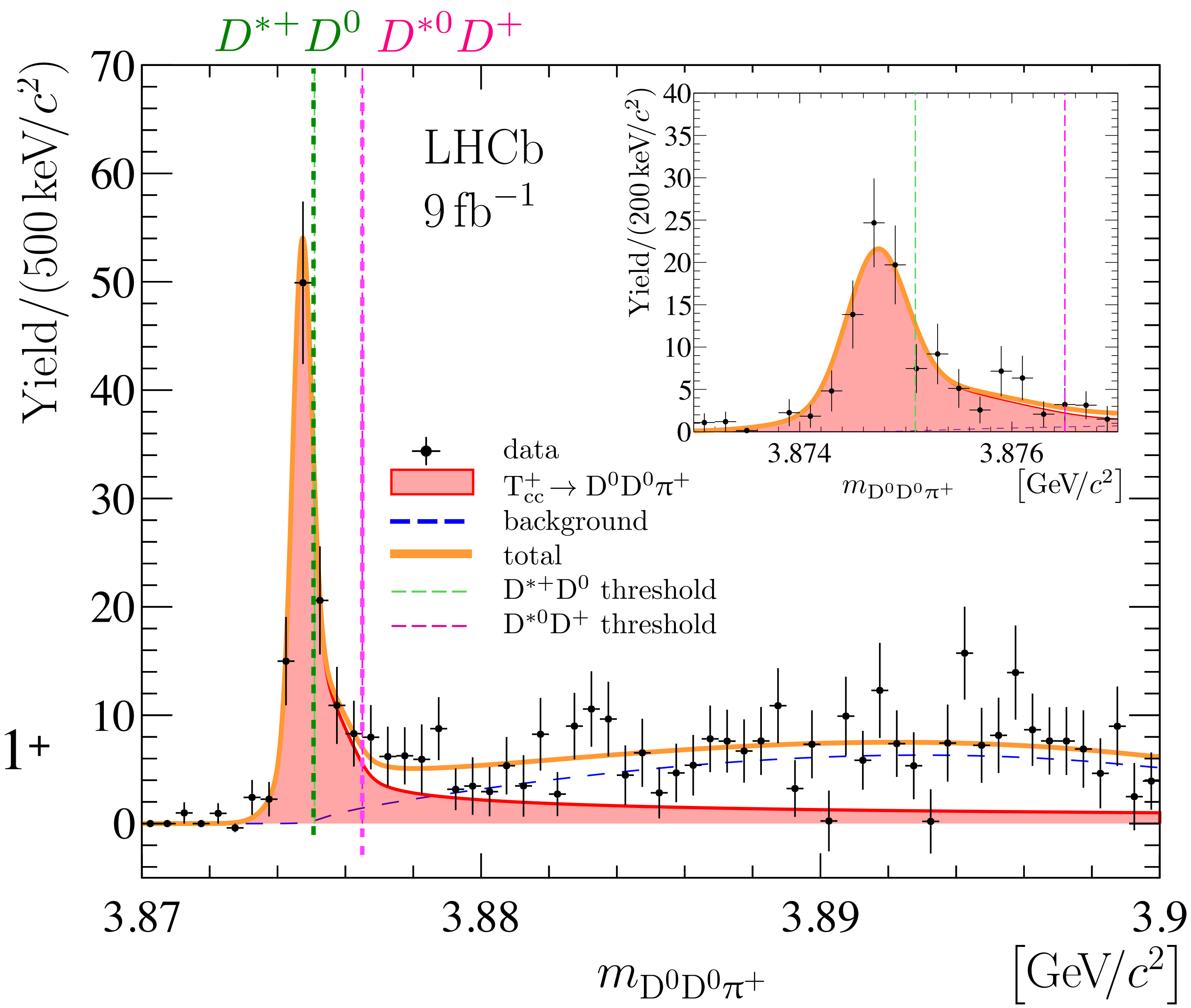
$$\delta m_{\rm pole} = -360 \pm 40^{+4}_{-0} \ {\rm keV}$$
$$\Gamma_{\rm pole} = 48 \pm 2^{+0}_{-14} \ {\rm keV}$$

$$\Gamma_{\rm pole} \ll \Gamma_{D^{*+}}$$

LHCb: Nature Comm.(2022)

- Proceed via D^* $DD^* \to D[D\pi]$ 90%

- No signal in $D^{*+} D^+$— isoscalar; $J^P = 1^+$



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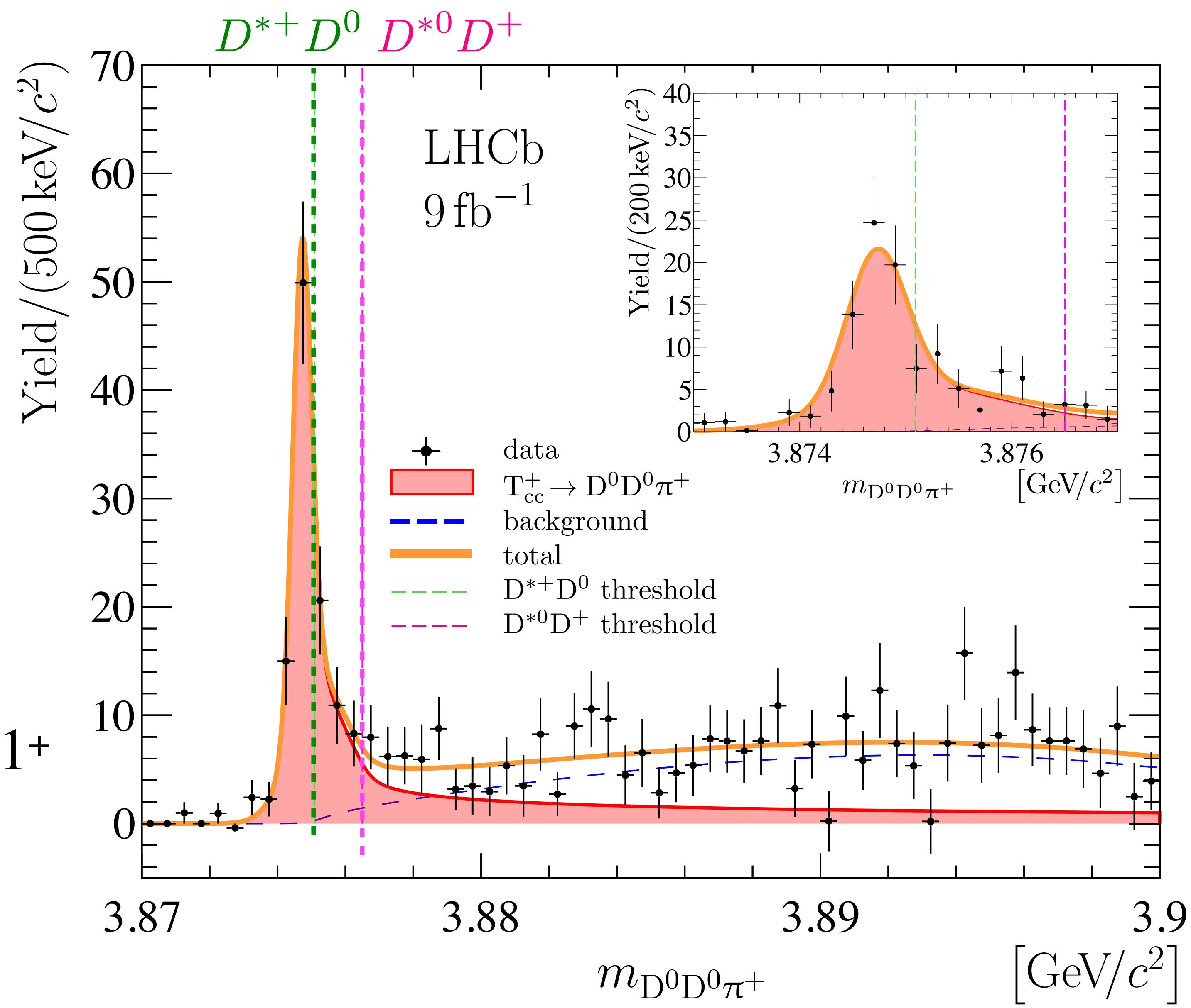

- Several studies on lattice at different m_π. Conclusion: Tcc is probably a virtual state

Padmanath and Prelovsek, PRL 129, 032002 (2022), Chen et al. , PLB 833, 137391 (2022), Lyu et al [HAL QCD]: 2302.04505 [hep-lat]

- Plenty of theoretical studies; in particular, the Tcc width is addressed in

Meng et al (2021), Fleming et al (2021), Ling et al (2022), Feijoo et al. (2021), Yan et al. (2022), Albaladejo (2022), Dai et al. (2023),…

Tcc in EFT's

- T_{cc} is an excellent case for a low-energy EFT

Expansion in χEFT : $\chi = \frac{\sqrt{2\mu\Delta_M}}{\Lambda_\chi} < 0.1$

$\Delta_M = m(D^+D^{*0}) - m(D^0D^{*+})$

— extremely close to the DD* threshold

— no admixture of inelastic channels

— 98% of the width from the only strong decay channel: $T_{cc}^+ \to D^0D^{*+} \to D^0D^0\pi^+/D^0D^+\pi^0$

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Varieties of EFT's

(Only initial work is cited)

— Contact EFT	No pions		AlFiky et al. PLB640 (2006), …
— X-EFT	Perturbative pions	see Talk by Dai on Monday	Fleming et al, PRD76 (2007), …
— χEFT	Non-perturbative pions + coupled-channels		VB et al., PRD84 (2011)

⇒ Larger applicability range allows for direct fits to experimental line shapes

⇒ Analytic structure of the amplitude is more complete

⇒ We study the role of pions and various cuts (3-body, left-hand cuts) for Tcc in χEFT

χEFT for Tcc

- LO effective Lag consistent with chiral and heavy-quark spin symmetries (HQSS)

Mehen and Powell, PRD 84,114013(2011)
AlFiky et al., PLB640,238(2006)

$$\mathcal{L}_{\rm LO} = -\frac{D_{10}}{8}\mathrm{Tr}\left(\tau^A_{aa'}H^\dagger_{a'}H_b\tau^A_{bb'}H^\dagger_{b'}H_a\right) - \frac{D_{11}}{8}\mathrm{Tr}\left(\tau^A_{aa'}\sigma^i H^\dagger_{a'}H_b\tau^A_{bb'}\sigma^i H^\dagger_{b'}H_a\right)$$
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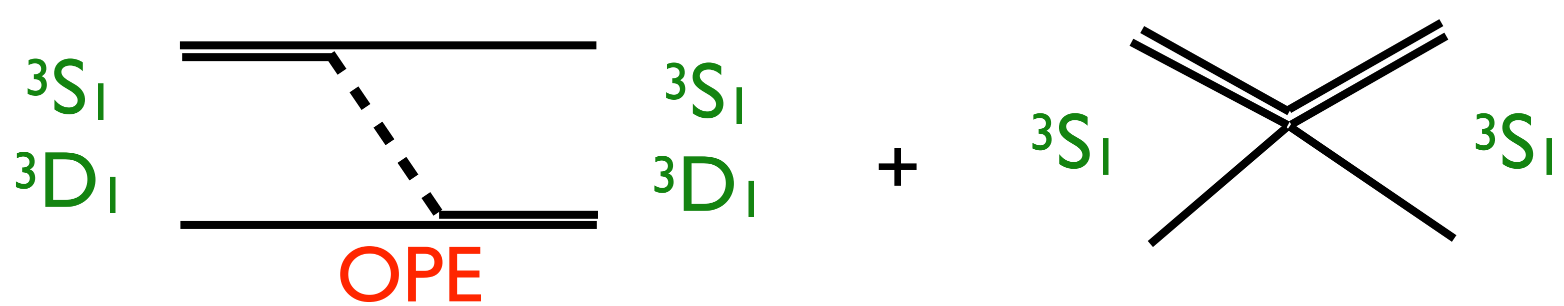
$$H_a = P_a + \boldsymbol{V}_a\cdot\boldsymbol{\sigma}, \qquad P_a = \begin{pmatrix} D^0 \\ D^+ \end{pmatrix}_a, \quad \boldsymbol{V}_a = \begin{pmatrix} \boldsymbol{D}^{*0} \\ \boldsymbol{D}^{*+} \end{pmatrix}_a \qquad \boldsymbol{u} = -\nabla\Phi/f_\pi \qquad \Phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$

— HQSS and isospin constrain the # of param's to just one: $v_0 \equiv -2(D_{01} - 3D_{11})$

— g is known from D* → Dπ

- LO coupled-channel isoscalar potential in the particle basis $\{D^{*+}D^0, D^{*0}D^+\}$

$V_{\rm LO}$ =



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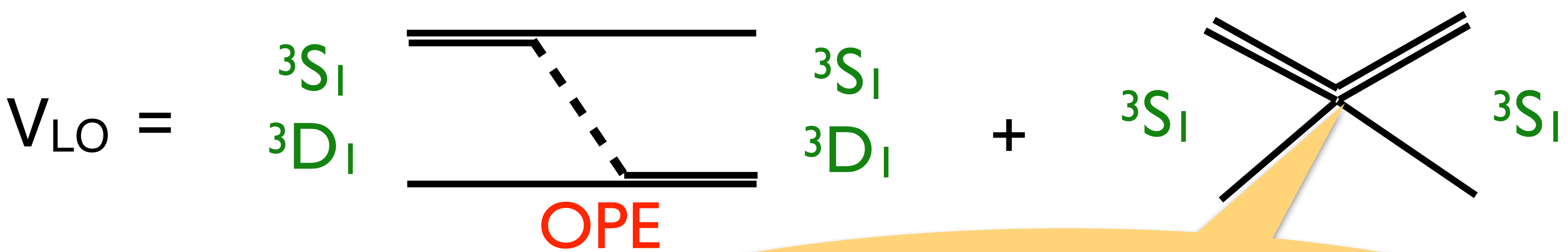

$$V_{\rm CT}(D^*D \to D^*D; 1^+) = \frac{1}{2}\begin{pmatrix} v_0 & -v_0 \\ -v_0 & v_0 \end{pmatrix}$$

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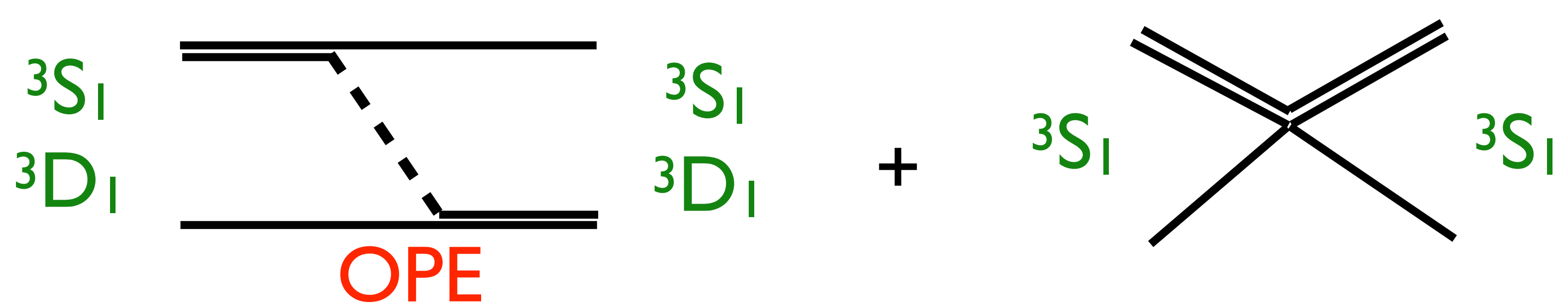

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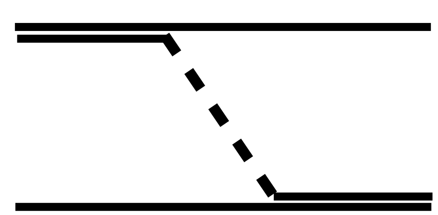
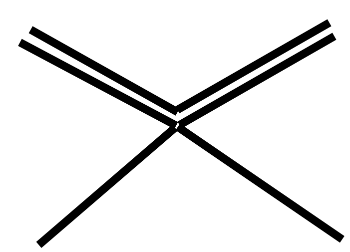
nonperturbative

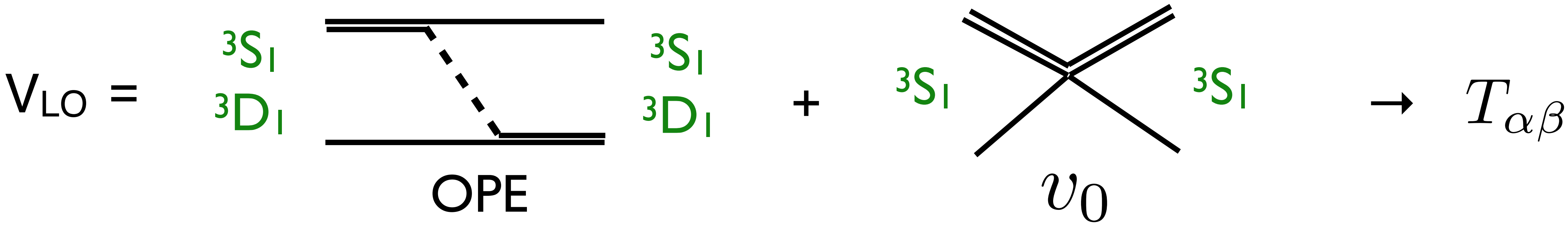
$V_{\rm LO}$ = $^3S_1$ $^3D_1$  $^3S_1$ $^3D_1$ + $^3S_1$  $^3S_1$ $\to T_{\alpha\beta}$

OPE

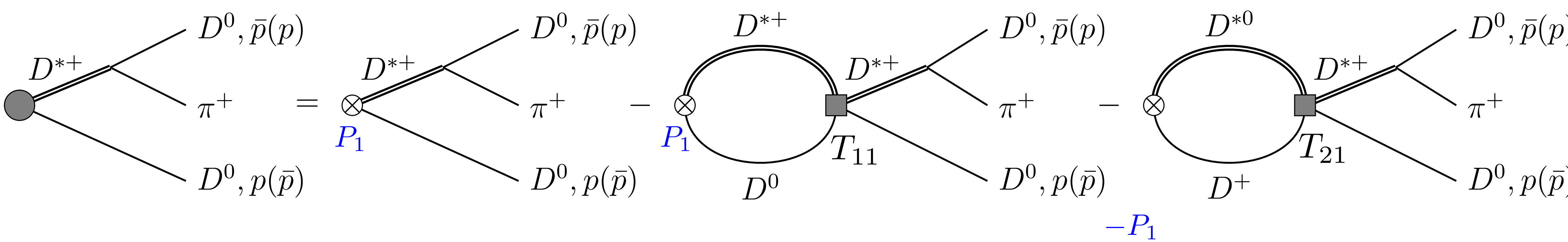
$$T_{\alpha\beta} = V^{\rm eff}_{\alpha\beta} - \sum_\gamma \int \frac{d^3q}{(2\pi)^3} V^{\rm eff}_{\alpha\gamma} G_\gamma T_{\gamma\beta}$$

Pointlike production source

- LO isoscalar potential:



- Production amplitude:



- Only two parameters to be fitted to the $D^0D^0\pi^+$ spectrum: v_0 and overall Norm ~ P_1^2

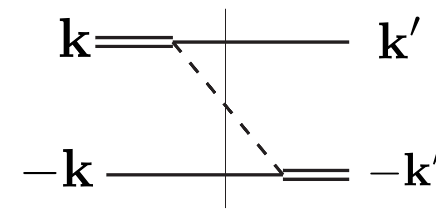
One pion exchange and 3-body cut

- OPE potential:

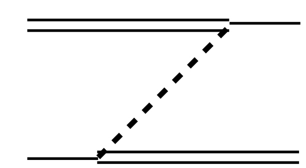
$$V_{\mathrm{DD^*\to DD^*}}(\mathbf{k},\mathbf{k}') \propto \frac{g_c^2}{(4\pi f_\pi)^2}\,\tau_1\cdot\tau_2\,\frac{(\epsilon_1\cdot\vec{q})\,(\epsilon_2'^{\,*}\cdot\vec{q})}{2E_\pi(\mathbf{k}-\mathbf{k}')}\left(\frac{1}{D_{DD\pi}(\mathbf{k},\mathbf{k}')}+\frac{1}{D_{D^*D^*\pi}(\mathbf{k},\mathbf{k}')}\right)$$

☛ TOPT propagators with NR heavy mesons and relativistic pions $E_\pi = \sqrt{m_\pi^2 + (\mathbf{k}-\mathbf{k}')^2}$

$$D_{DD\pi}(\mathbf{k},\mathbf{k}') = m + m + \frac{k^2}{2m} + \frac{k'^2}{2m} + E_\pi(\mathbf{k}-\mathbf{k}') - E \quad \Rightarrow$$



$$D_{D^*D^*\pi}(\mathbf{k},\mathbf{k}') = m_* + m_* + \frac{k^2}{2m_*} + \frac{k'^2}{2m_*} + E_\pi(\mathbf{k}-\mathbf{k}') - E \quad \Rightarrow$$



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goes on shell!

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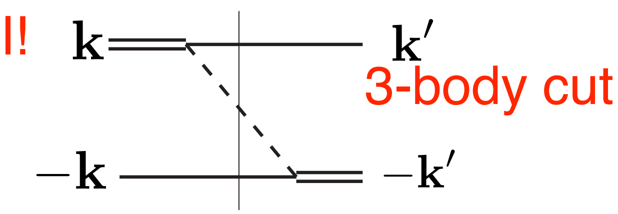

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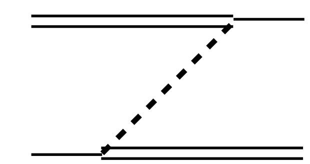
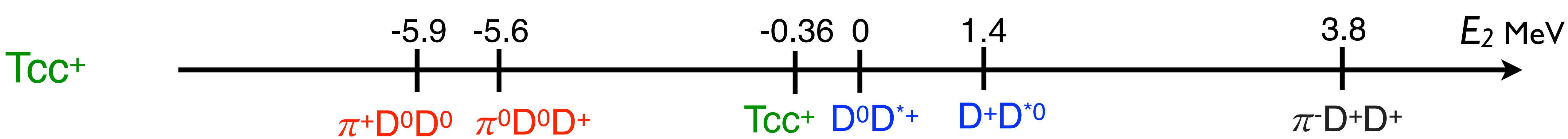


3-body cut condition

☛ For each $E > E_{\mathrm{thr}} \equiv 2m + m_\pi,$ there are real values of k and k′ such that $D_{DD\pi}(k,k',E)=0$

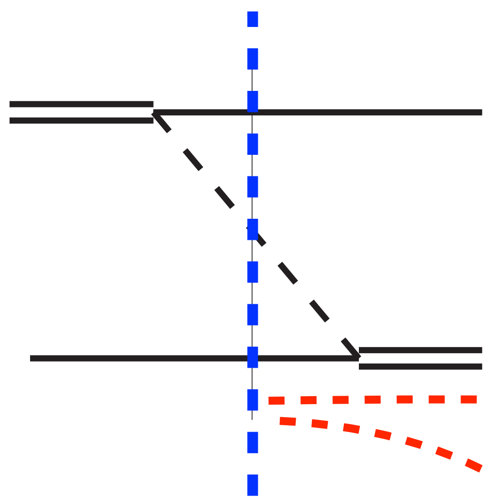
☛ If we count energy relative to 2-body DD* threshold $E = m + m_* + E_2$

3-body branch point reads (k=k′=0) $E_2 = m_\pi - \Delta M = m_\pi + m - m_*$

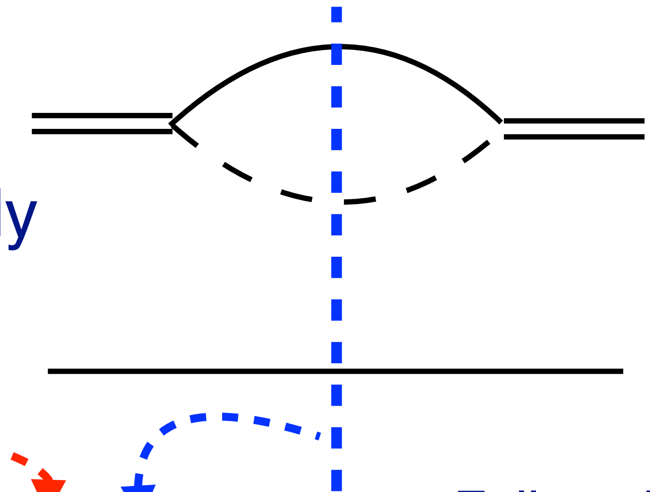


Green function

3-body cut stems from one-pion exchange (OPE) and self energies in the Green funct.



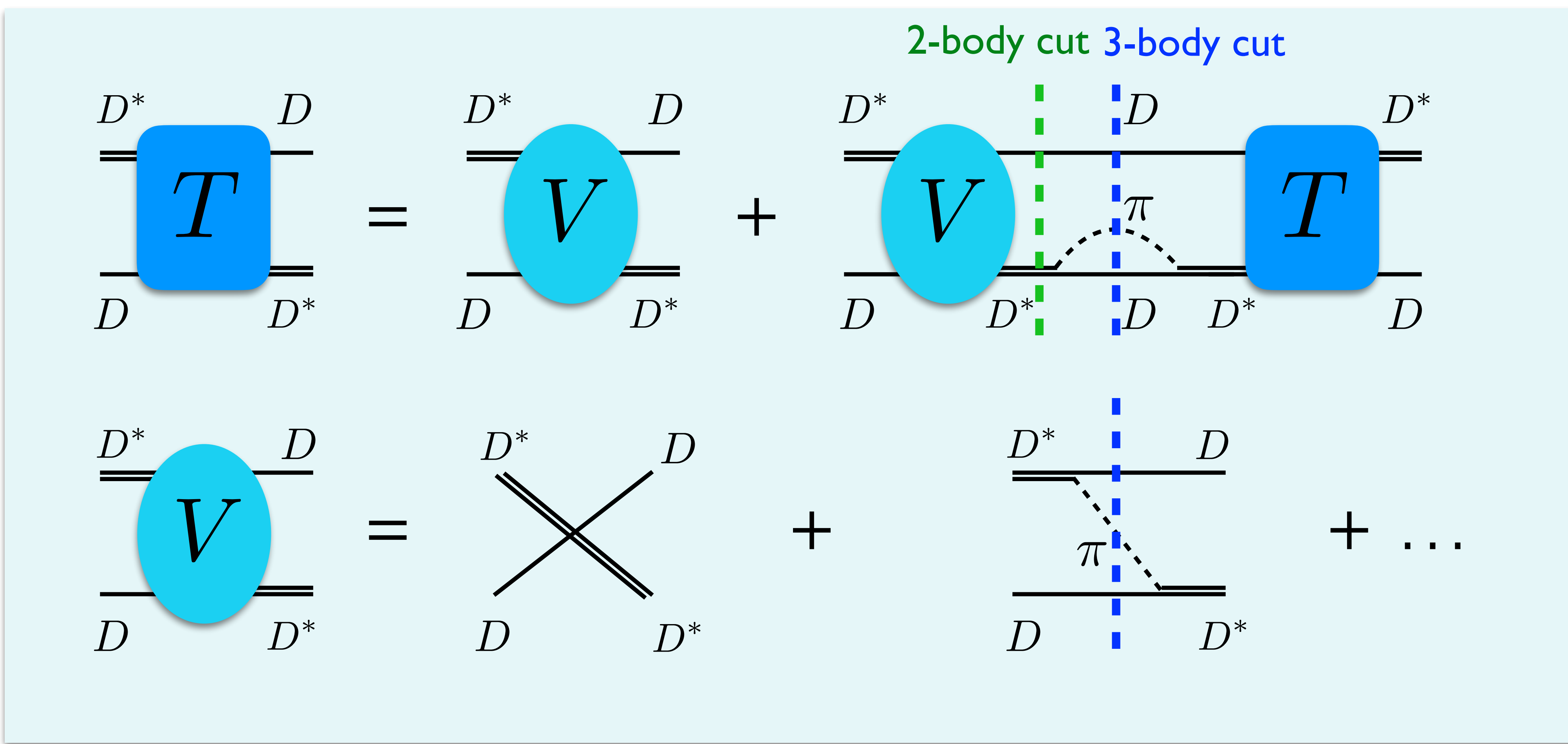
on shell ⇒ 3-body



$$T_{\alpha\beta} = V^{\text{eff}}_{\alpha\beta} - \sum_{\gamma} \int \frac{d^3q}{(2\pi)^3} V^{\text{eff}}_{\alpha\gamma} G_\gamma T_{\gamma\beta}$$

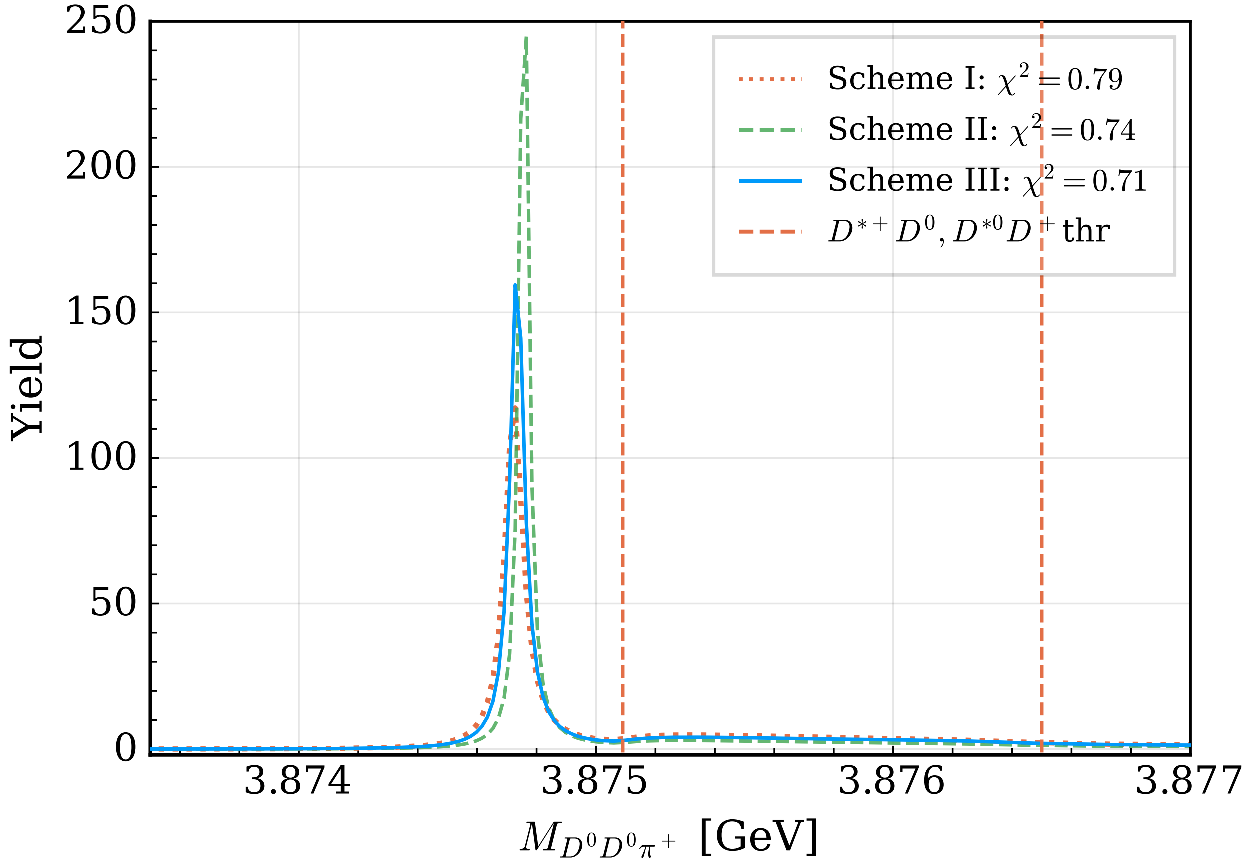
Full analogy to the X(3872) from Faddeev-type 3-body Eqs.

VB et al. PRD84 2011

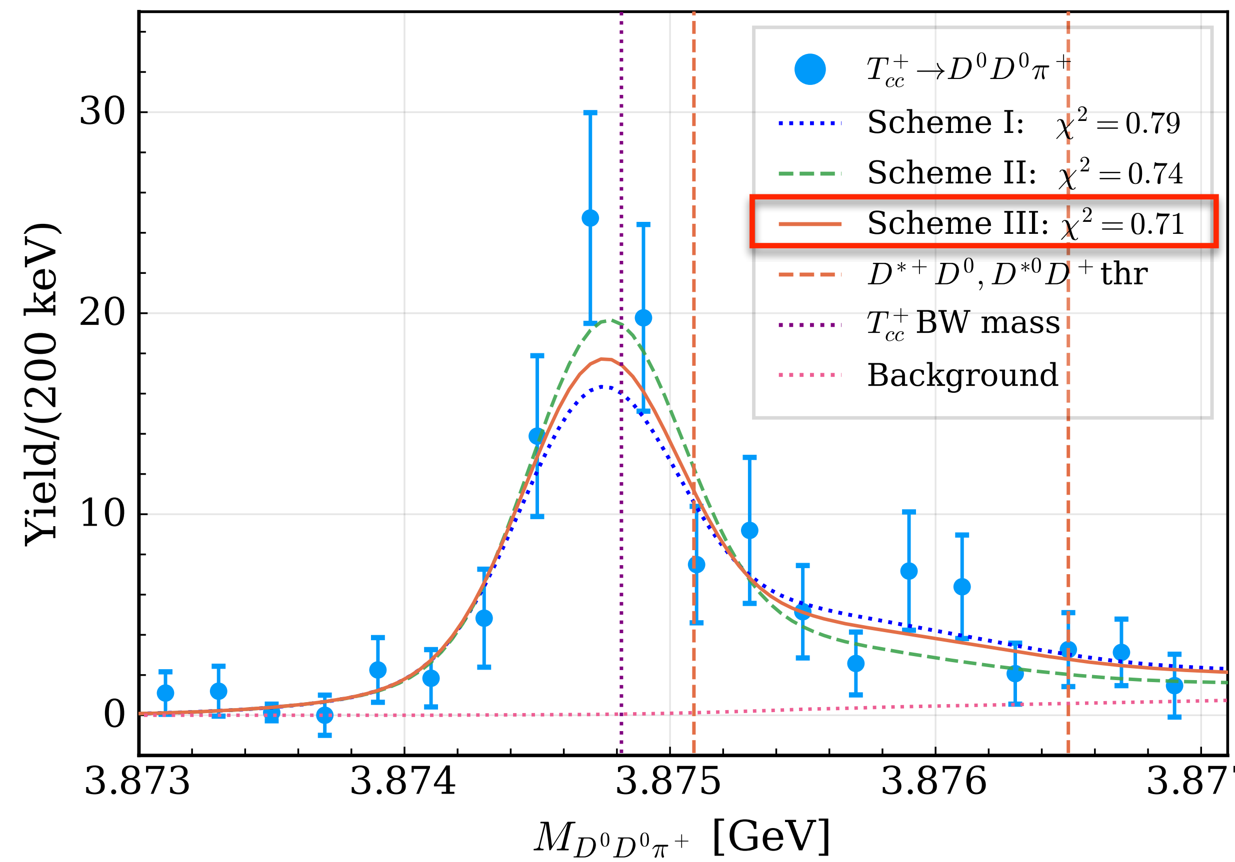


Fits to the $D^0D^0\pi^+$ mass spectrum

w/o resolution



with resolution



Scheme	I	II	III
Description	2-body unitarity: No OPE, static D^* width	Incomplete 3-body unitarity: No OPE, dynamical D^* width	full 3-body unitarity: OPE + dynamical D^* width
Pole [keV]	$-368^{+43}_{-42} - i(37 \pm 0)$	$-333^{+41}_{-36} - i(18 \pm 1)$	$-356^{+39}_{-38} - i(28 \pm 1)$
χ^2	0.79	0.74	0.71

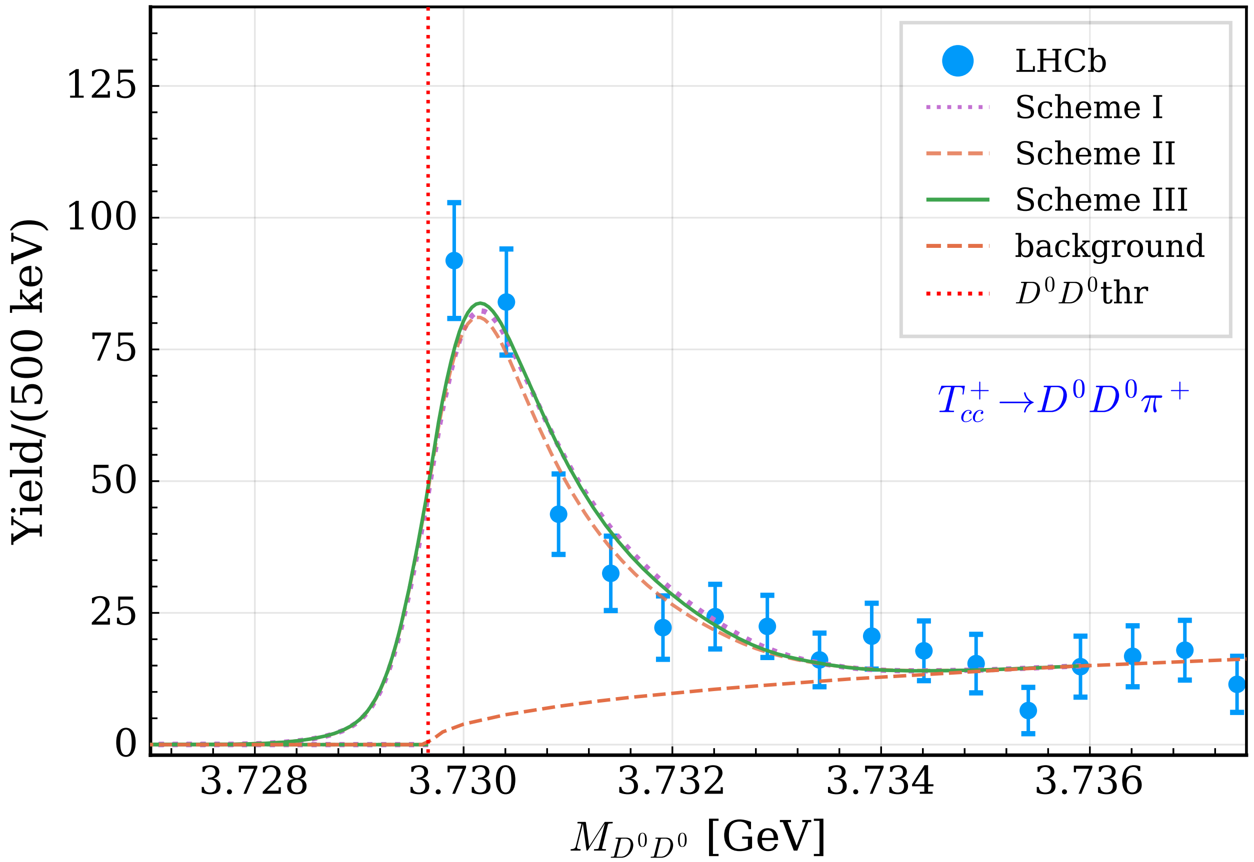
Real part of the pole: all Fits are consistent within 1σ —more precise data are needed

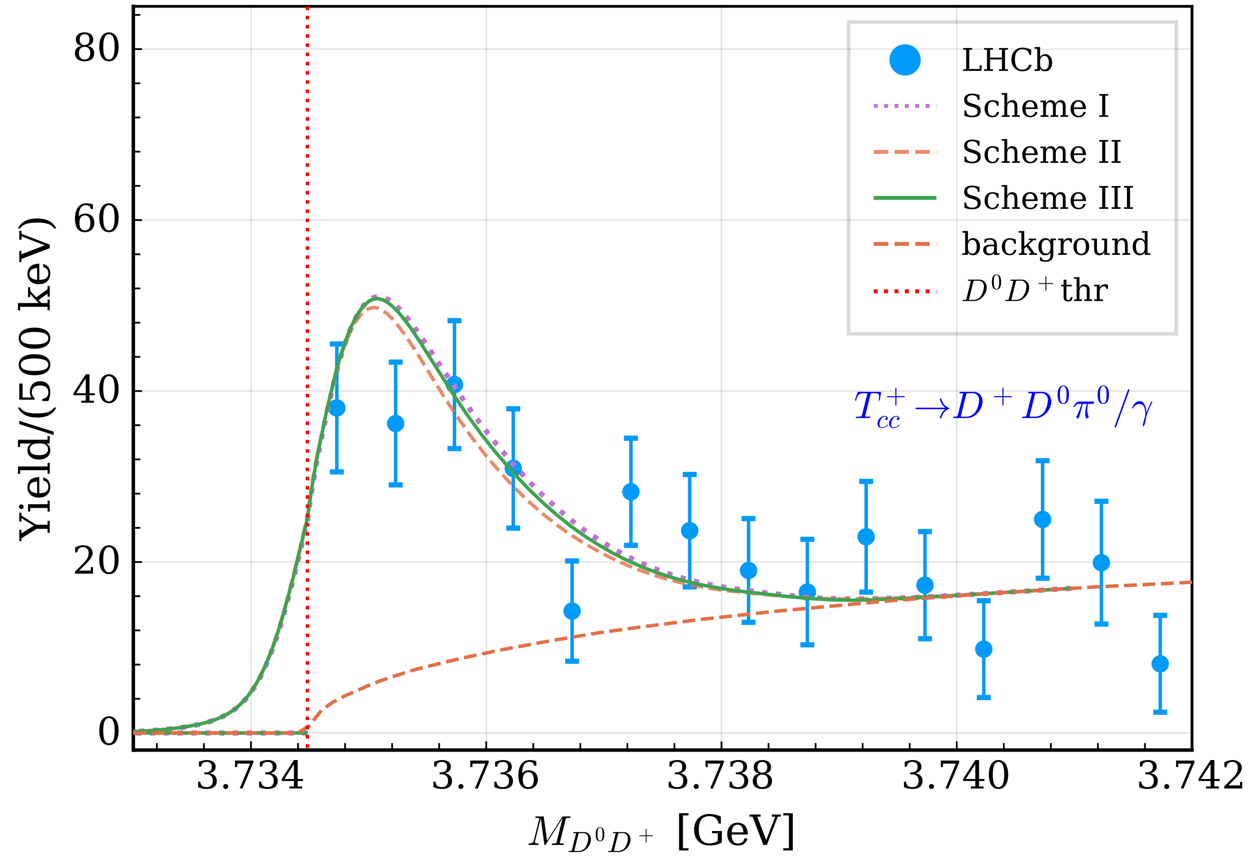
Width of Tcc+ : Accuracy requires 3-body effects

$$\Gamma^{3-\text{body}}_{T_{cc}} = 56 \pm 2\,\text{keV} \xrightarrow[\text{OPE}]{\text{remove}} 36\text{ keV} \xrightarrow[\text{dynam.width}]{\text{remove}} 74\text{ keV}$$

Predictions for D^0D^0 and D^0D^+ spectra

with resolution





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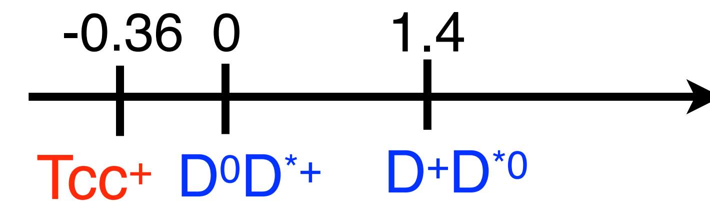
Low-energy parameters

Du et al. PRD 105, 014024 (2022)

Scattering amplitude in the 1st (close to the pole) channel :

$$T_{D^{*+}D^0\to D^{*+}D^0}(k) = -\frac{2\pi}{\mu_{c0}}\left(\frac{1}{a_0}+\frac{1}{2}r_0k^2 - ik + \mathcal{O}(k^4)\right)^{-1}$$



$$r'_0 = r_0 - \Delta r$$

Eff. range in the 1st channel

$$\Delta r = -\sqrt{\frac{\mu_2}{2\mu_1^2\delta_2}} \simeq -3.8 \text{ fm}$$

$$\delta_2 = m_{\text{thr2}} - m_{\text{thr1}}$$

VB et al., PLB 833 (2022)

Negative "correction" from 2nd $D^{*0}D^+$ channel caused by isospin breaking δ_2

see talk by Polosa on Wednesday

a_0 [fm]	r_0 [fm]	r'_0 [fm]
$\left(-6.72^{+0.36}_{-0.45} \pm 0.27\right) - i\left(0.10^{+0.03}_{-0.03} \pm 0.03\right)$	$-2.40 \pm 0.01 \pm 0.85$	$1.38 \pm 0.01 \pm 0.85$

– r'_0 positive and is of natural size

– Contrib. to r'_0 from OPE is ~ 0.4 fm

– $r'_0 \ll |a_0|$

X_1	X_2
$0.73\,{}^{\pm 0.01}_{\pm 0.11}$	$0.27\,{}^{\pm 0.01}_{\pm 0.02}$

Tcc+ is consistent with a pure isoscalar molecule!

Tcc on lattice

- HAL QCD Collaboration at m_π = 146 MeV: 2302.04505 [hep-lat]

  — calculate the DD* scattering potential — see talk by Yamaguchi on Wednesday

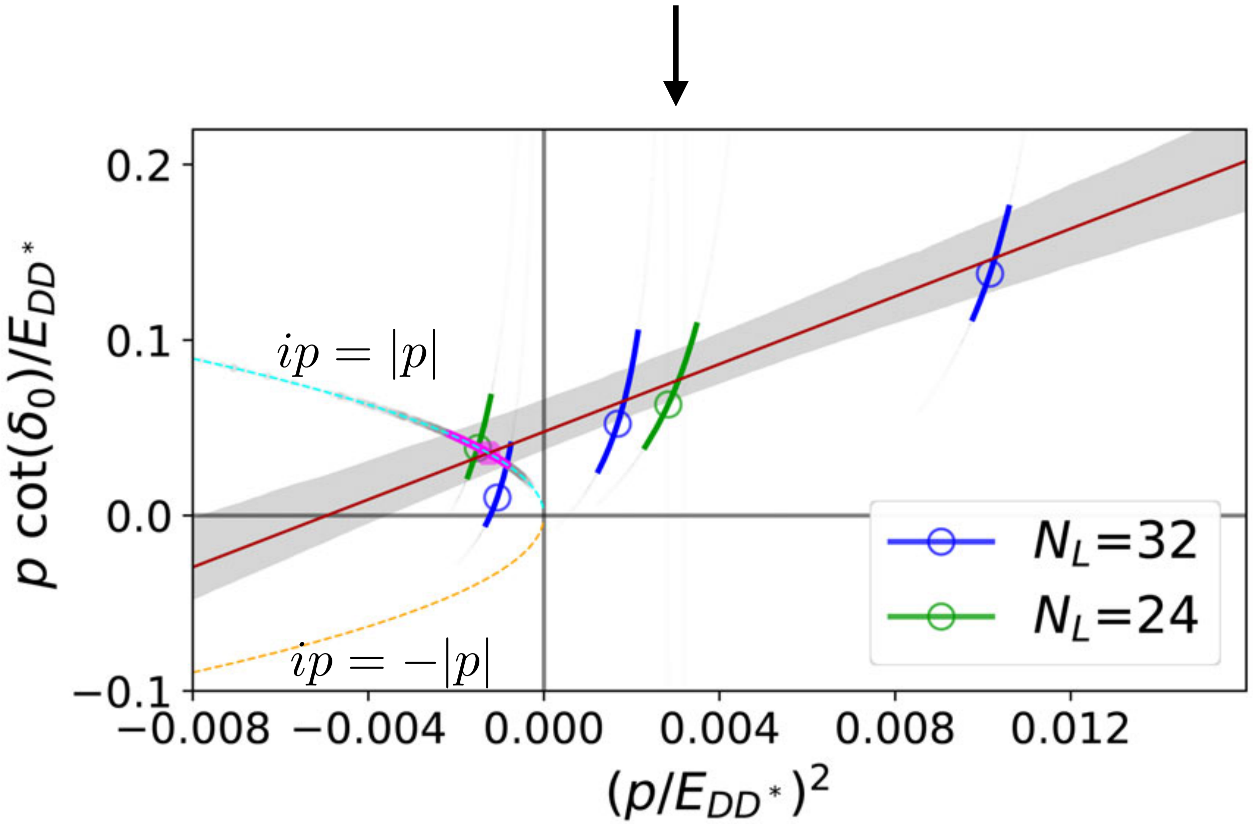
  — use it to calculate the phase shifts above the two-body threshold ⇒ virtual state

- DD* phase shifts δ(E) are extracted using the Lüscher method

  — m_π = 391 MeV, one volume L=16 — Cheung et al. (Hadron Spectrum collaboration), JHEP 11, 033 (2017)

  — m_π = 350 MeV, one volume L=16 — Chen et al. , PLB 833, 137391 (2022).

  — m_π = 280 MeV, two volumes L = 24 and 32 — Padmanath and Prelovsek, PRL 129, 032002 (2022)



— phase shifts parameterised using the ERE:

$$p\cot\delta = \frac{1}{a} + \frac{1}{2} r p^2 + \mathcal{O}(p^4)$$

— Intersection with

$ip = -|p|$ gives the bound state pole

$ip = \ |p|$ gives the virtual state pole

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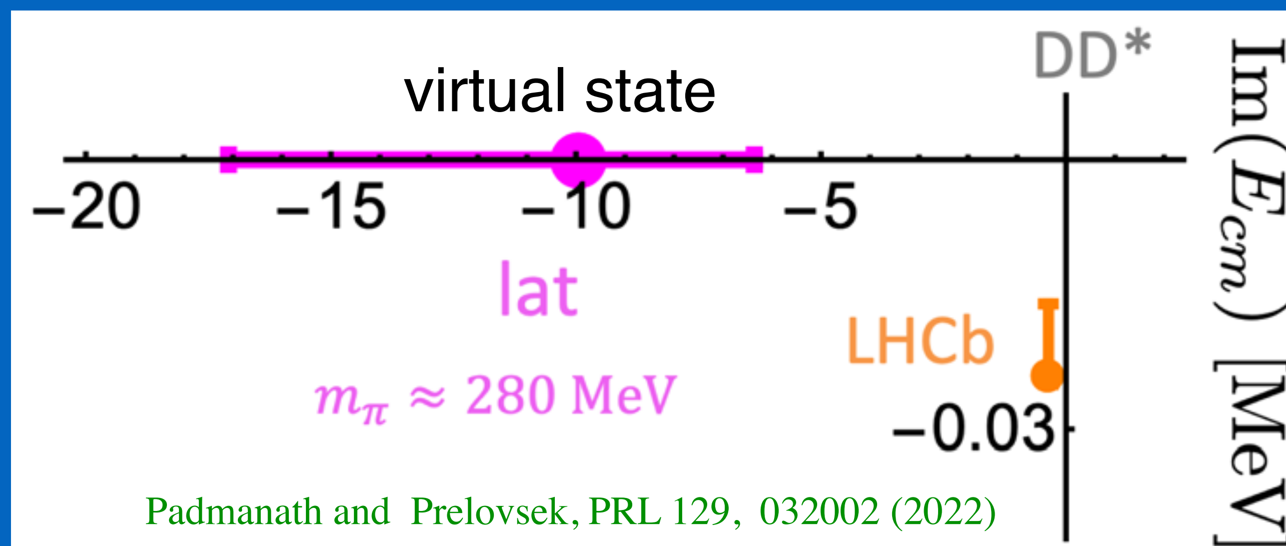
  — … hold ⇒ virtual state

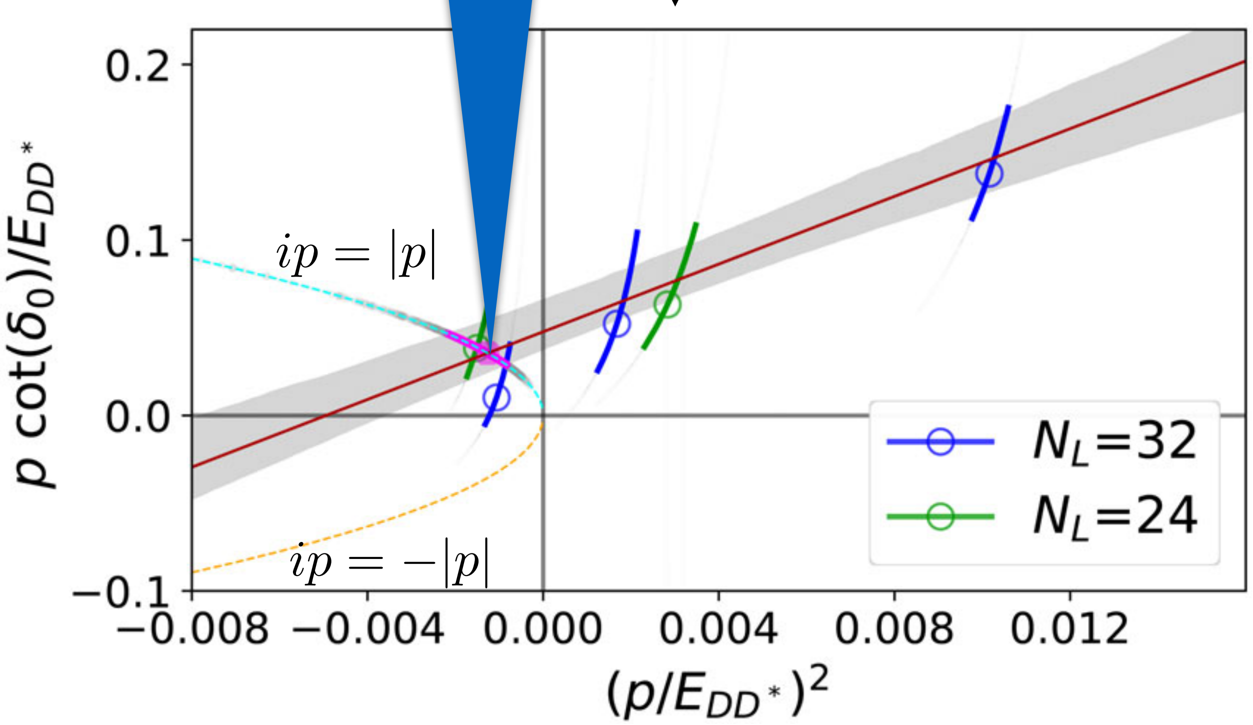
- DD … hod

  — … on Spectrum collaboration), JHEP 11, 033 (2017)

  — … 3, 137391 (2022).

  — … lovsek, PRL 129, 032002 (2022)





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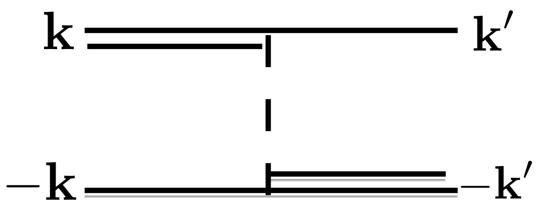
ERE applicability range

- Convergence Radius of the ERE is set by the nearest singularity irrespective of its origin

— For masses from Padmanath and Prelovsek (2022) 3-body cut starts at $E_2 = m_\pi - \Delta M = 158\,\mathrm{MeV}$

$\Rightarrow$ 3-body effects are small, static OPE is justified



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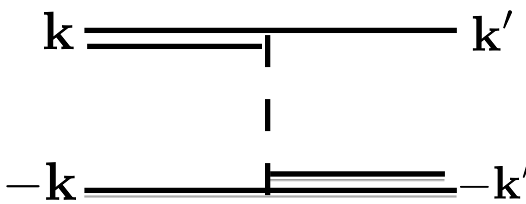

— Partial-wave scattering amplitude may also have left hand cuts (lhc)

$$\int dz \frac{1}{(\mathbf{k}-\mathbf{k}')^2 + m_\pi^2 - \Delta M^2} = \frac{1}{2kk'} \log \frac{(k+k')^2 + m_\pi^2 - \Delta M^2}{(k-k')^2 + m_\pi^2 - \Delta M^2} \xrightarrow[k=k'=p]{\text{on shell}} \frac{1}{2p^2} \log \frac{4p^2 + m_\pi^2 - \Delta M^2}{m_\pi^2 - \Delta M^2}$$

$\Rightarrow$ left-hand cut branch point is at

$$(p_{\mathrm{lhc}}^{1\pi})^2 = \frac{\Delta M^2 - m_\pi^2}{4}$$

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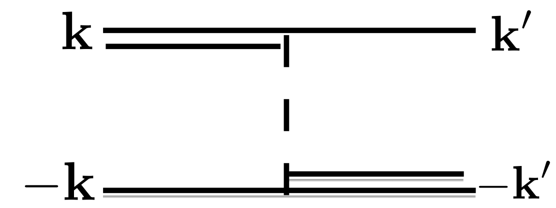

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$\Rightarrow$ left-hand cut branch point is at $(p_{\text{lhc}}^{1\pi})^2 = \dfrac{\Delta M^2 - m_\pi^2}{4}$

Numerically $(p_{\text{lhc}}^{1\pi})^2 = -(126\,\text{MeV})^2 \quad \Rightarrow E_{\text{lhc}}^{1\pi} = \dfrac{(p_{\text{lhc}}^{1\pi})^2}{2\mu} = -8\,\text{MeV}$

$\Rightarrow$ $E_{\text{lhc}}^{1\pi}$ sets the range of convergence of the ERE: $E \ll |E_{\text{lhc}}^{1\pi}|$

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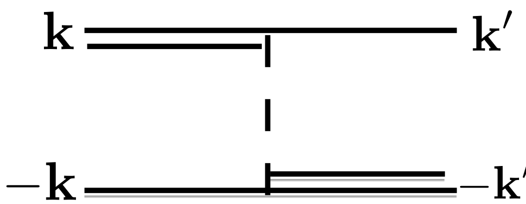

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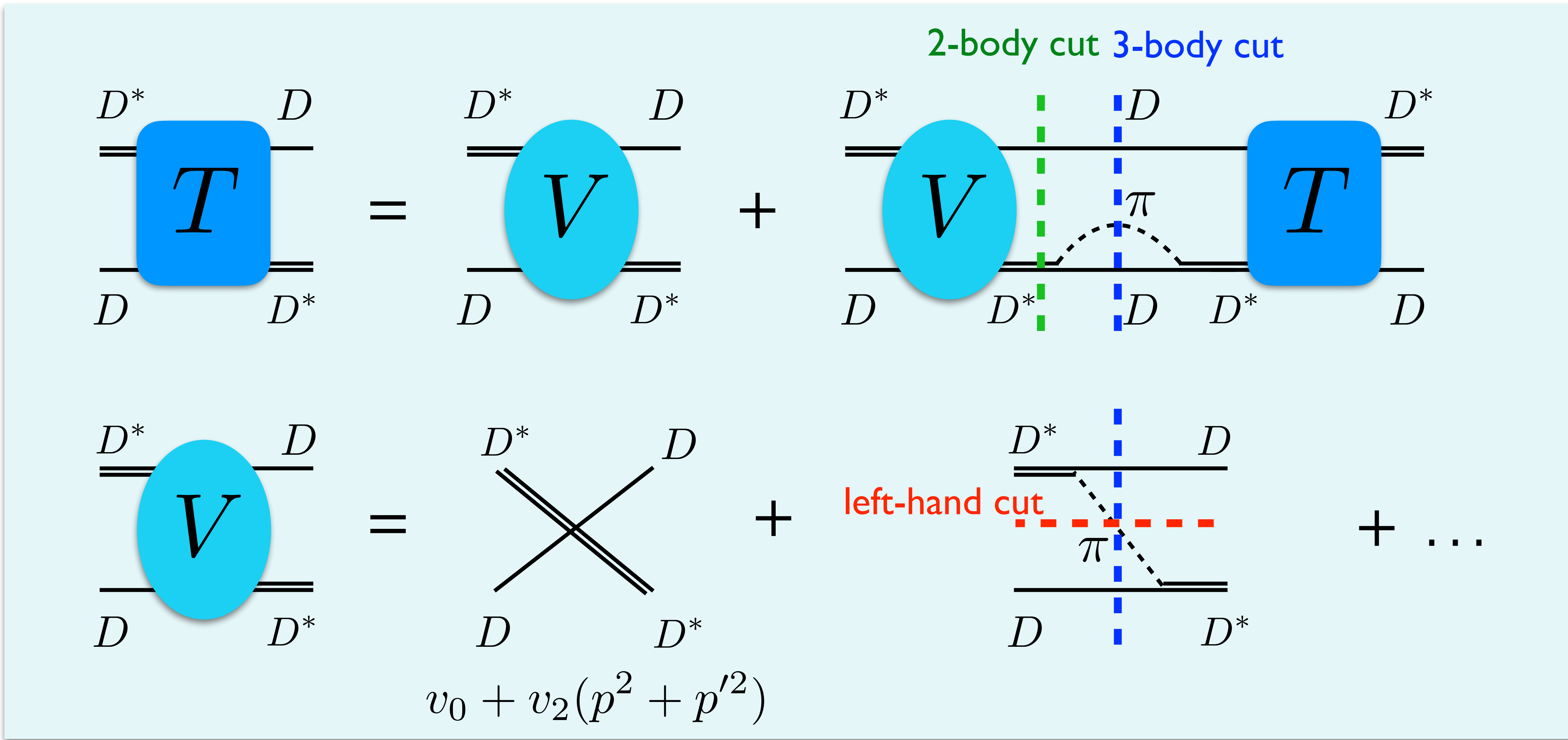
If $|E_{\rm lhc}^{1\pi}| \leq |E_{\rm Tcc}|$ $\Rightarrow$ ERE is not applicable

To extract Tcc pole accurately $\Rightarrow$ Calculate $p \cot\delta$ including the scale $E_{\rm lhc}^{1\pi}$ explicitly!

Analysis of lattice data including the left-hand cut

Du, Filin, VB, Epelbaum, Dong, Guo, Hanhart, Nefediev, Nieves and Wang 2303.09441 [hep-ph]

- $p \cot \delta$ from scattering T matrix including all relevant cuts

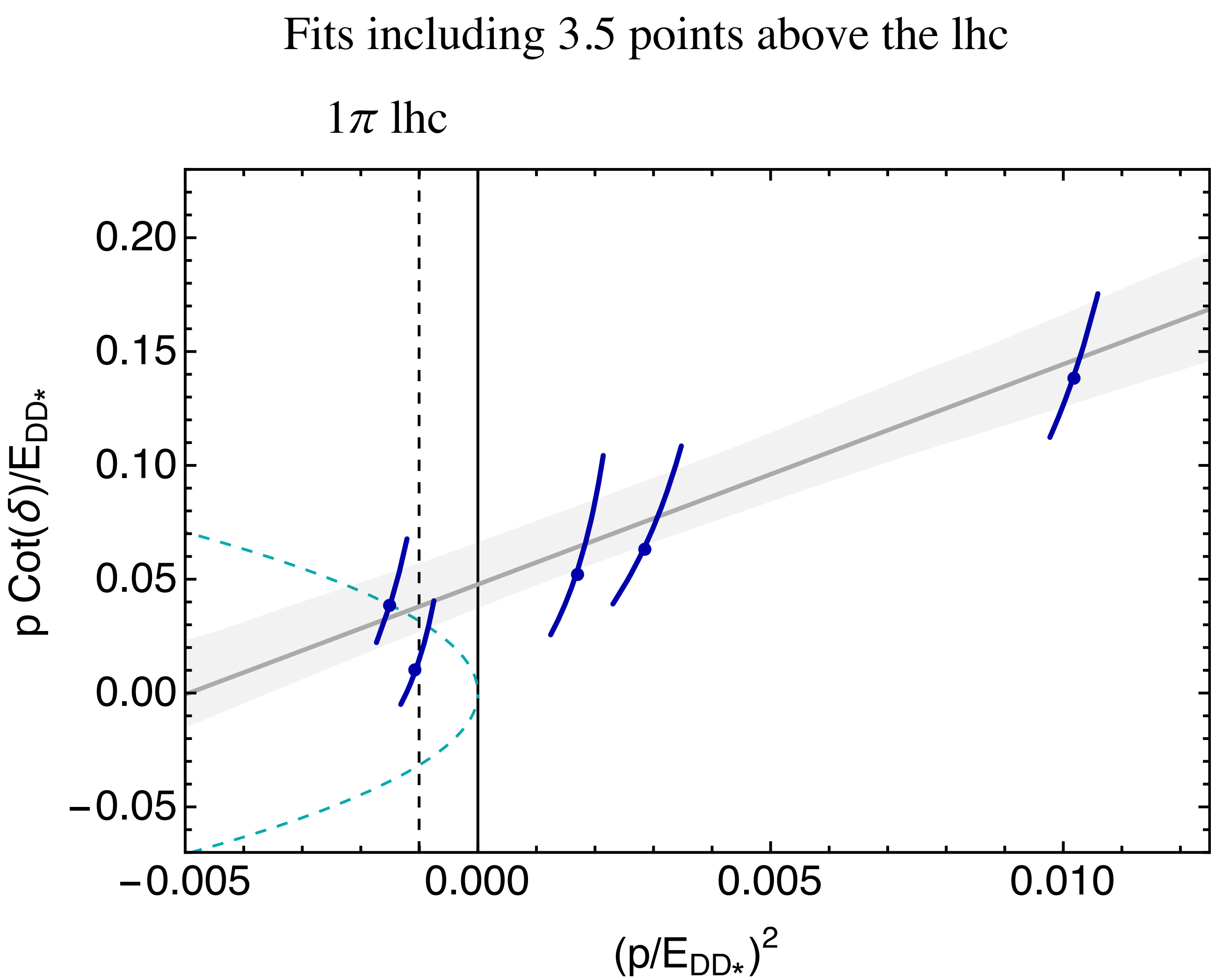


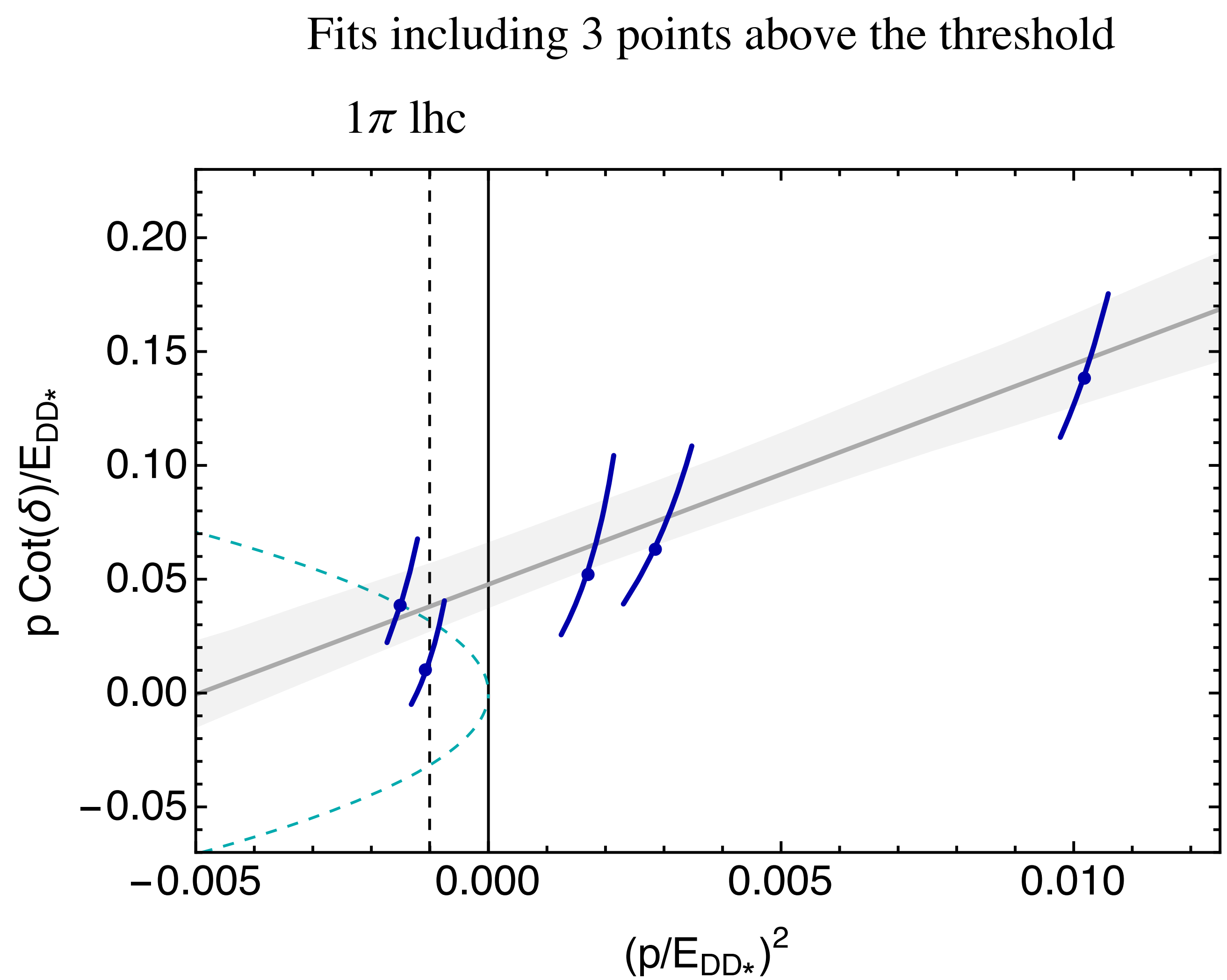
- 2 contact terms to account for additional range corrections

- Chiral extrapolation of D*Dπ coupling is included along the lines of
  Becirevic and Sanfilippo, PLB 721, 94 (2013) VB et al PLB 726, 537 (2013)

- Similar in spirit to the analysis of NN scattering at unphysical m_π
  VB, Epelbaum, Filin, Gegelia PRC92 014001(2015), PRC94 014001(2016)

Results: Analytic structure of the amplitude

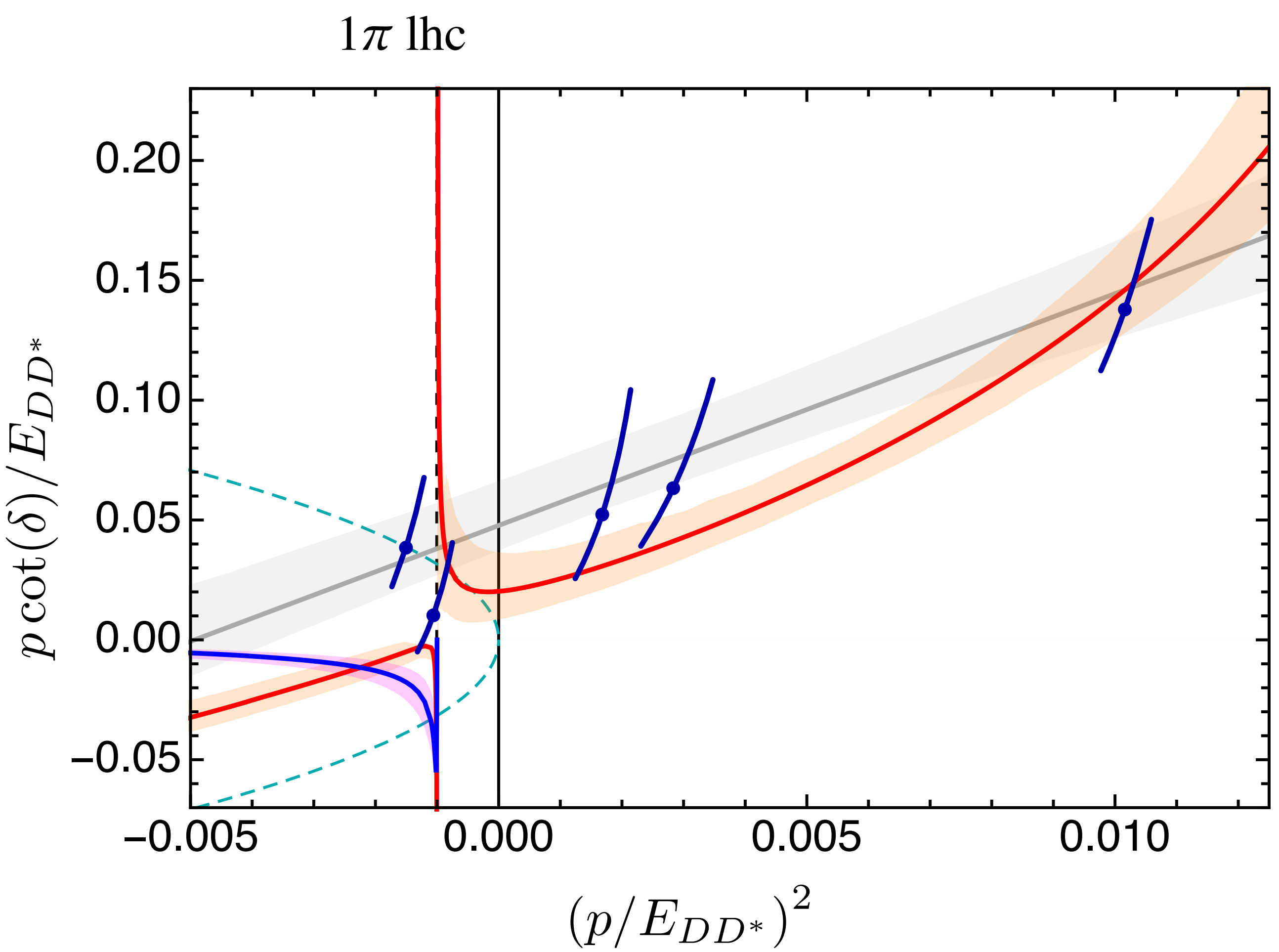




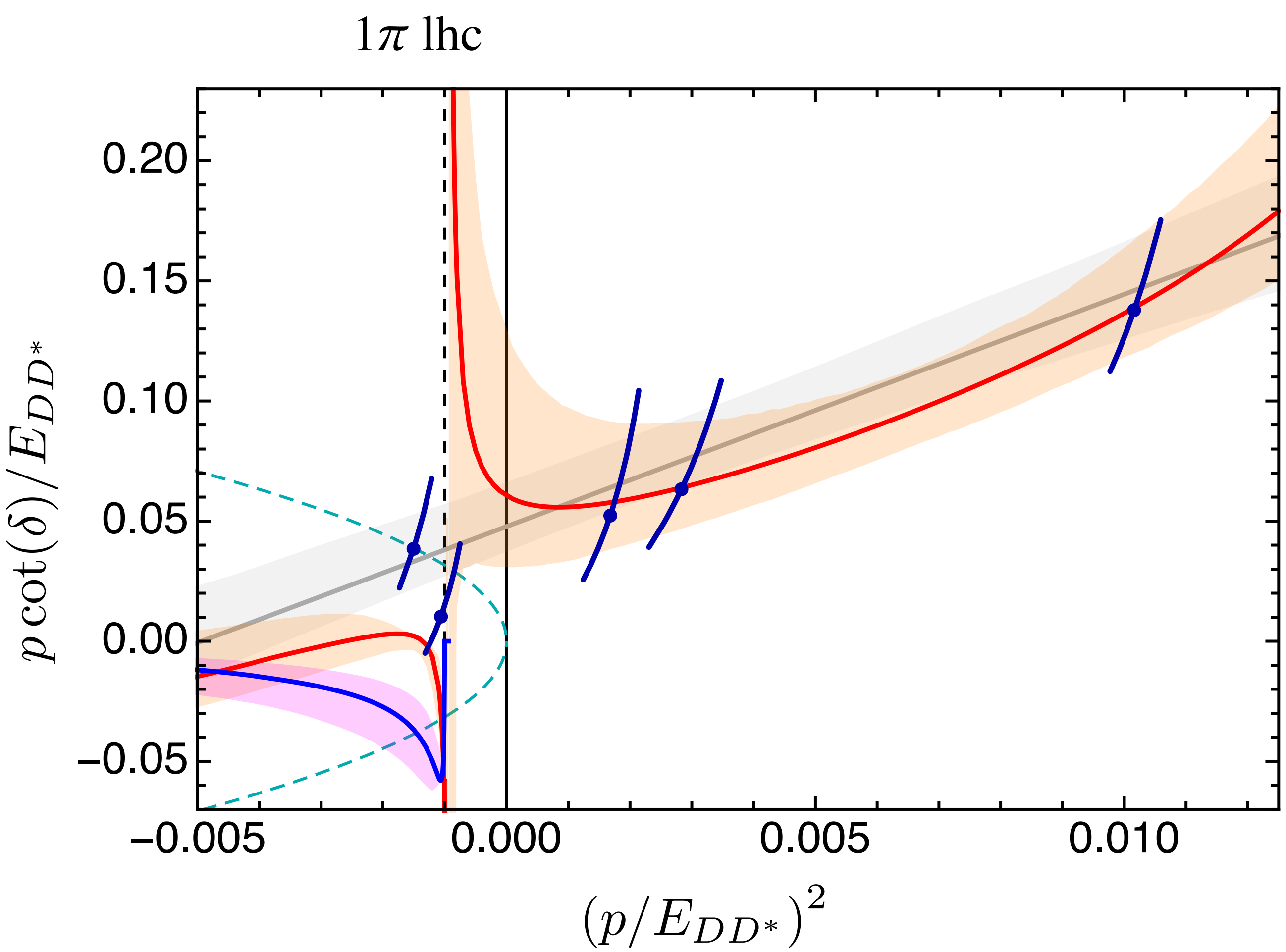
- Lowest energy point is not included in the fit — phase shift must be complex below the lhc!

Results: Analytic structure of the amplitude

Fits including 3.5 points above the lhc



Fits including 3 points above the threshold



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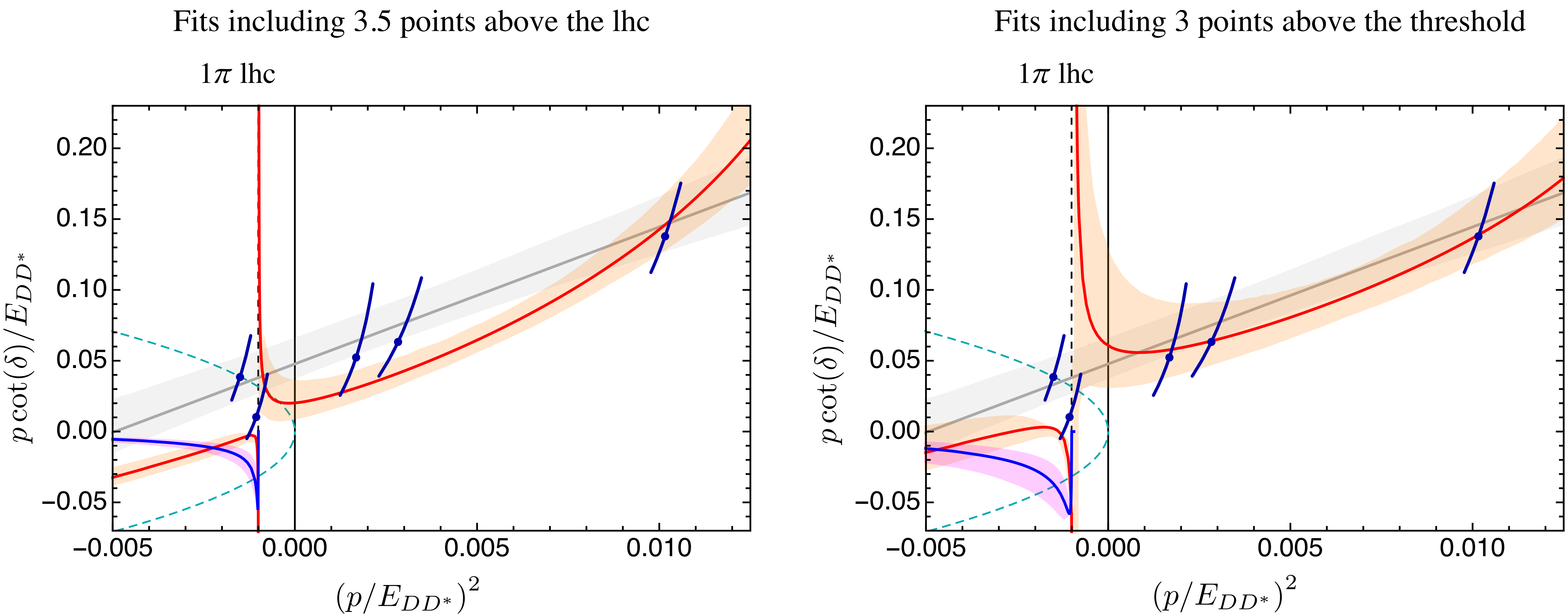

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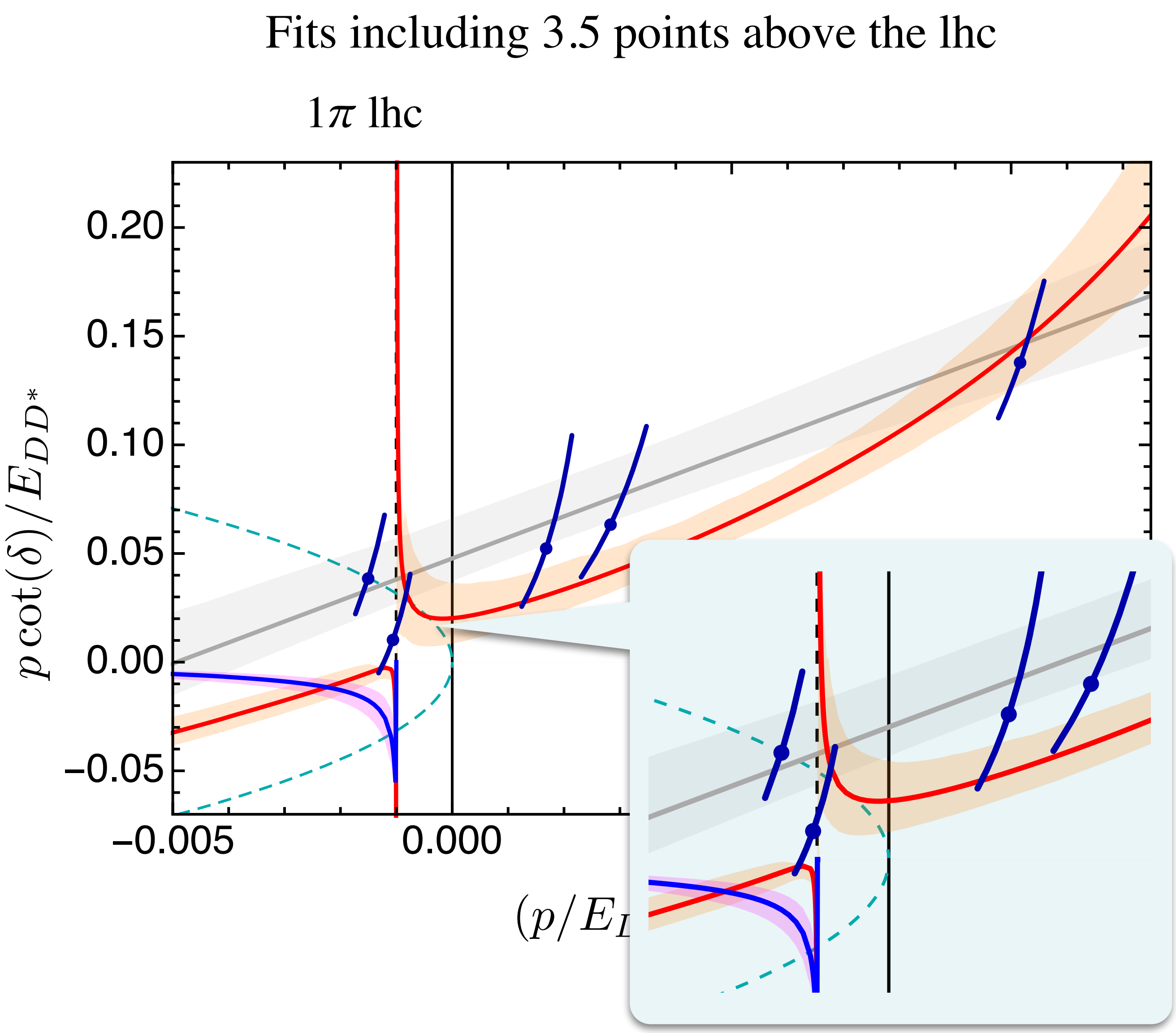

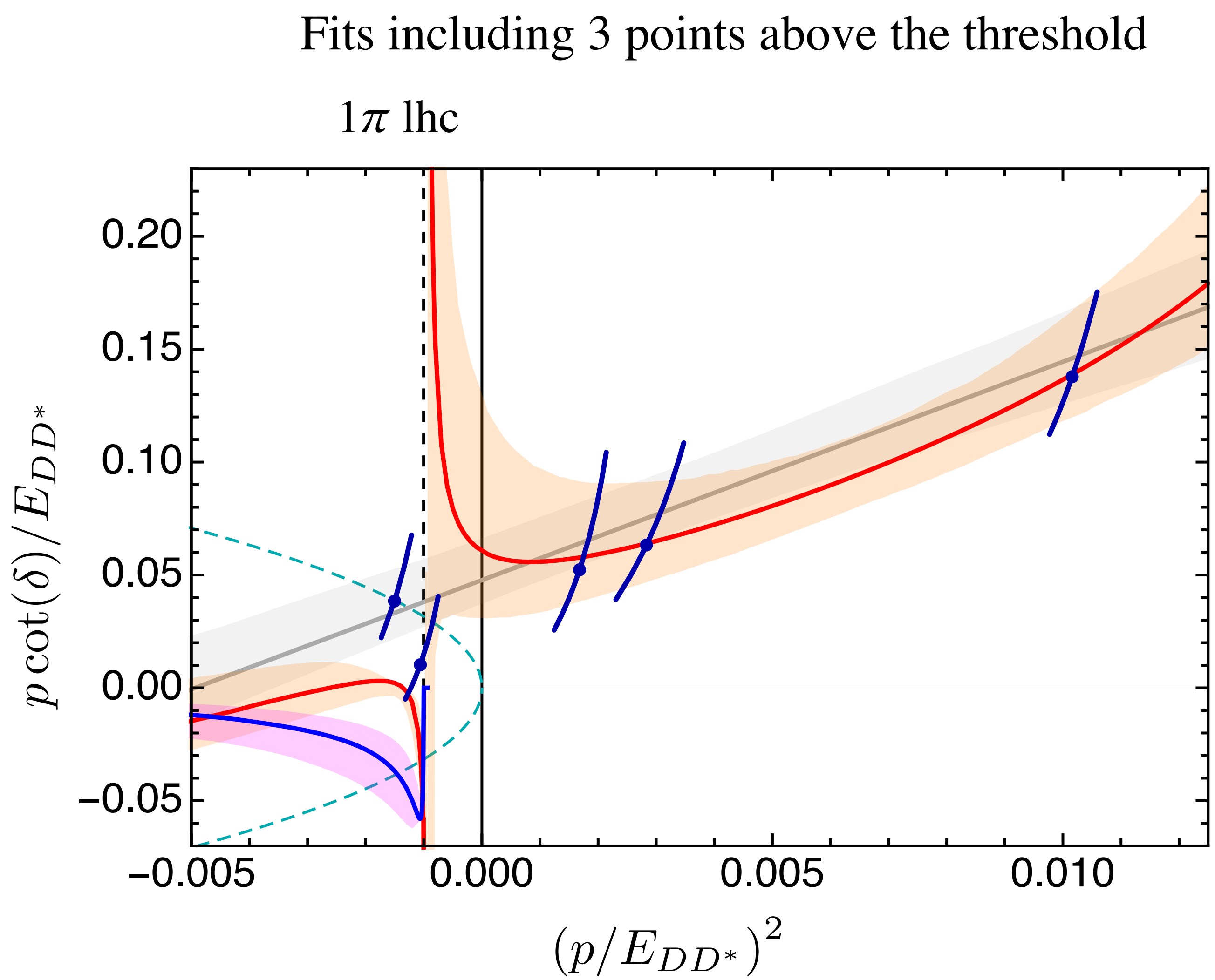

$$-\frac{2\pi}{\mu}T^{-1}(E) = p\cot\delta - ip$$

<u>*T_{CC} poles:*</u>

— 3.5 point fit: 2 virtual states; both much closer to the threshold than in the lattice paper

— 3 point fit: a resonance scenario is preferred

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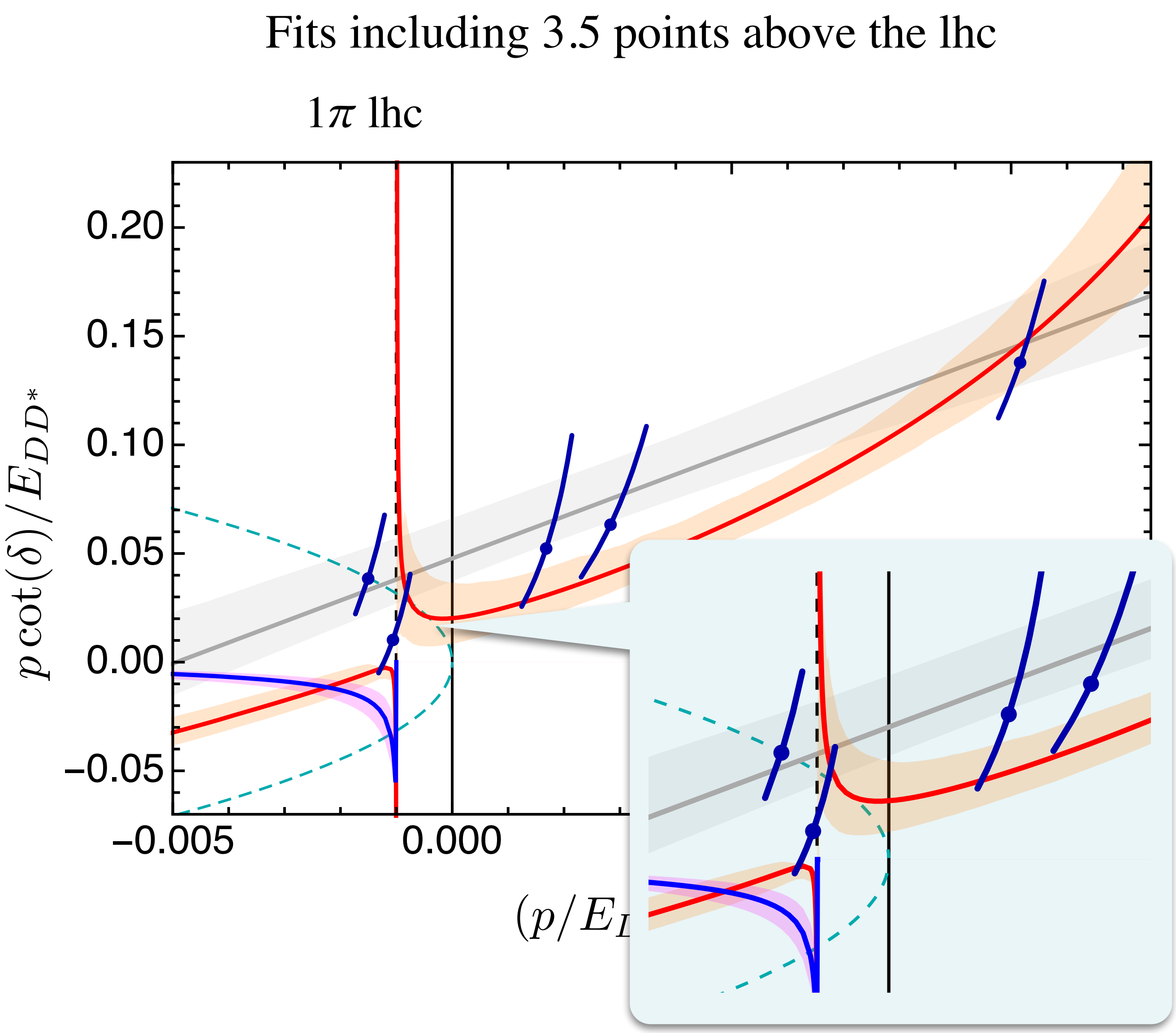

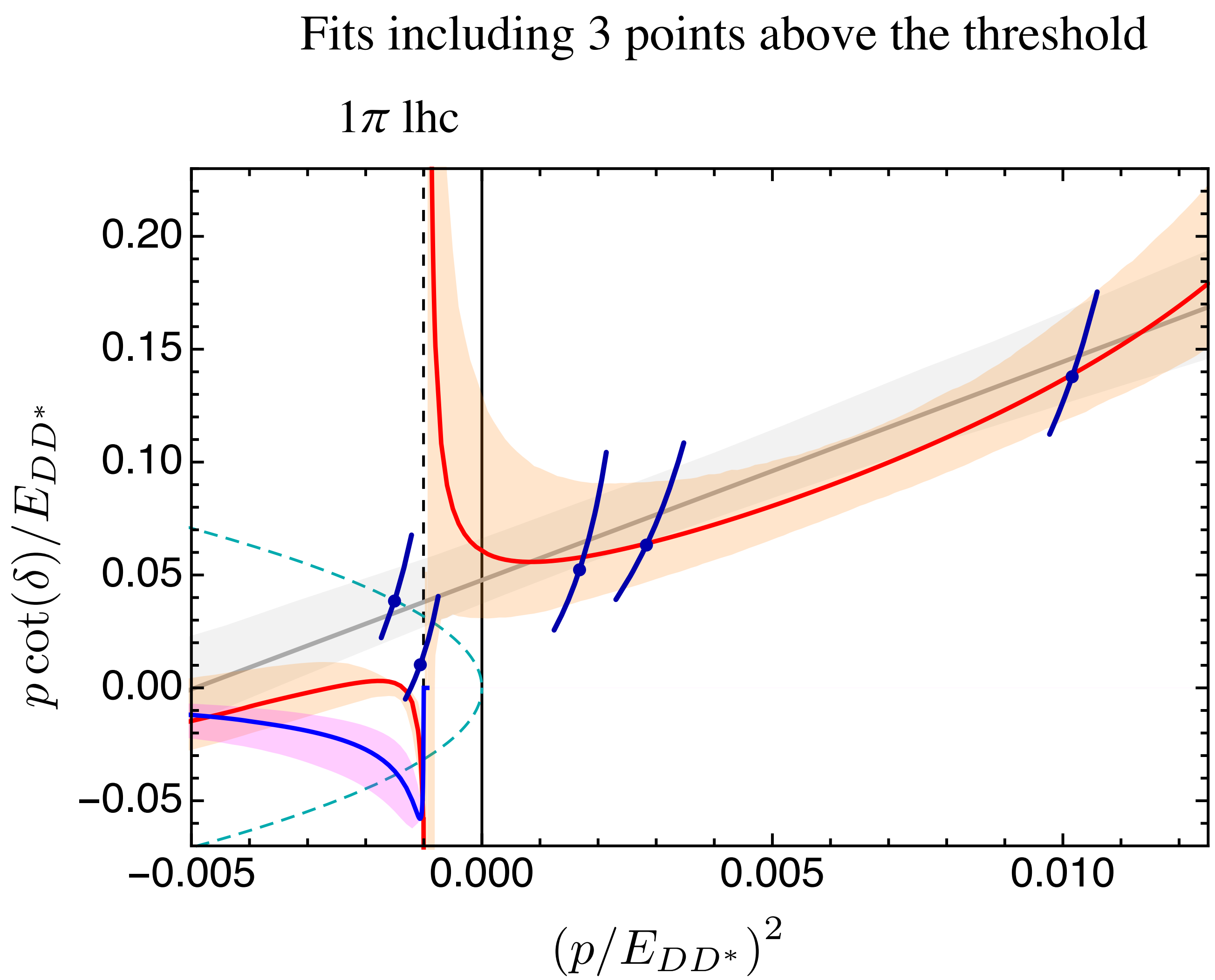

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T_{CC} poles:

— 3.5 point fit: 2 virtual states; both much closer to the threshold than in the lattice paper

— 3 point fit: a resonance scenario is preferred

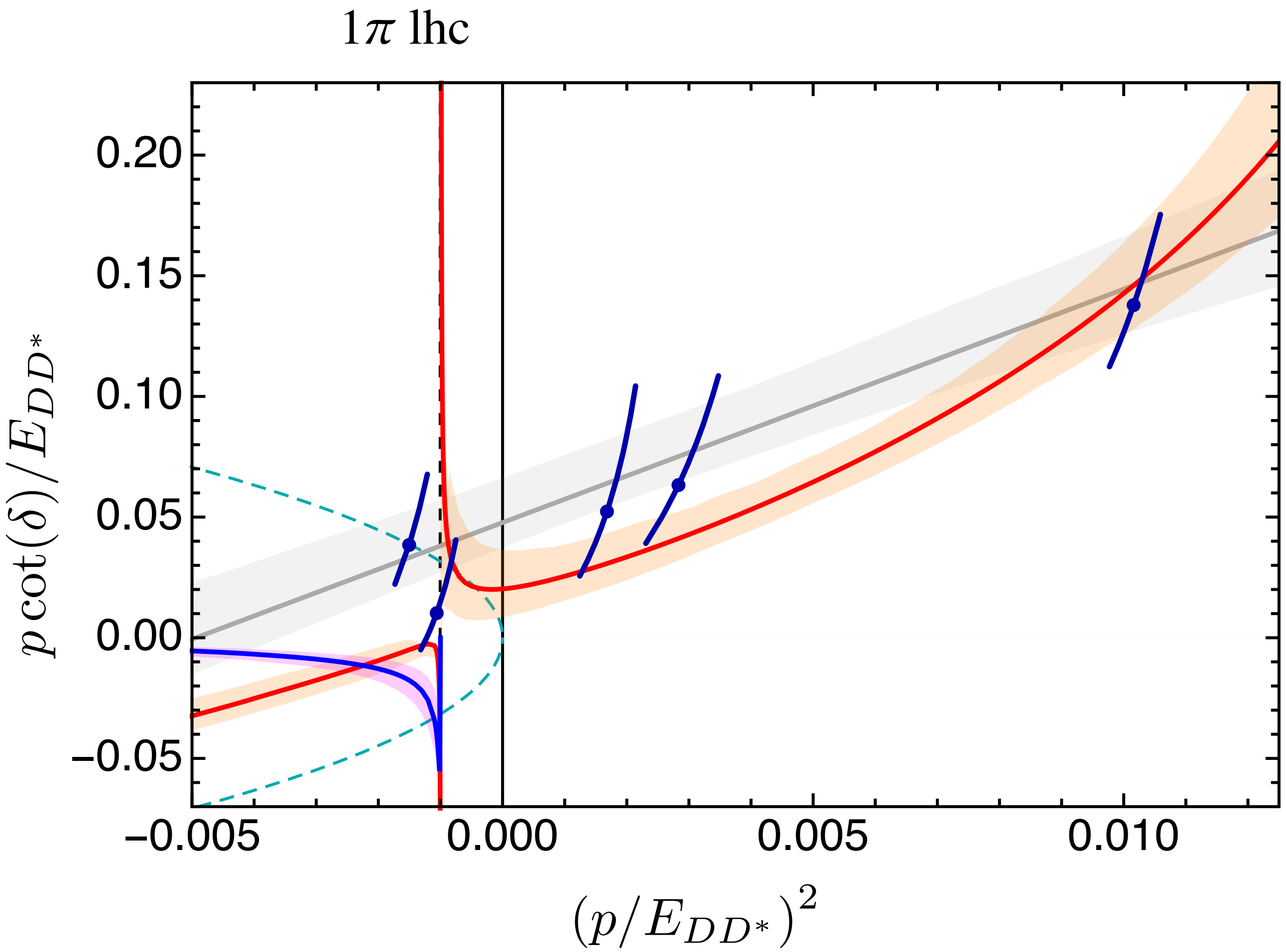
Zeros:

— Subtle interplay of repulsive OPE and attractive short range interactions

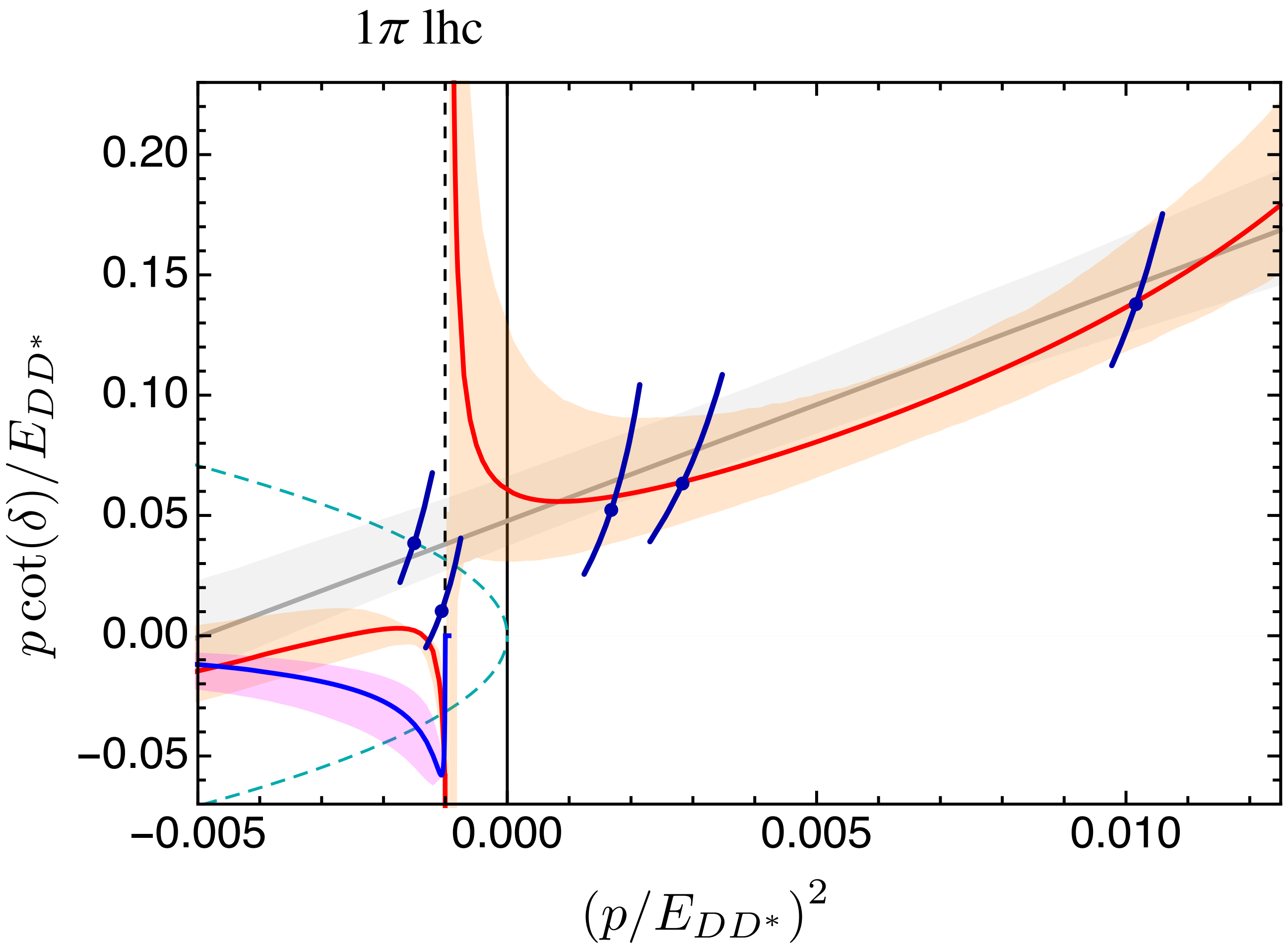
⇒ $p\cot\delta$ has a pole near the lhc ⇒ amplitude has a zero.

Disclaimer

Fits including 3.5 points above the lhc



Fits including 3 points above the threshold



- Phase shifts used here for granted may need to be revisited in the future:

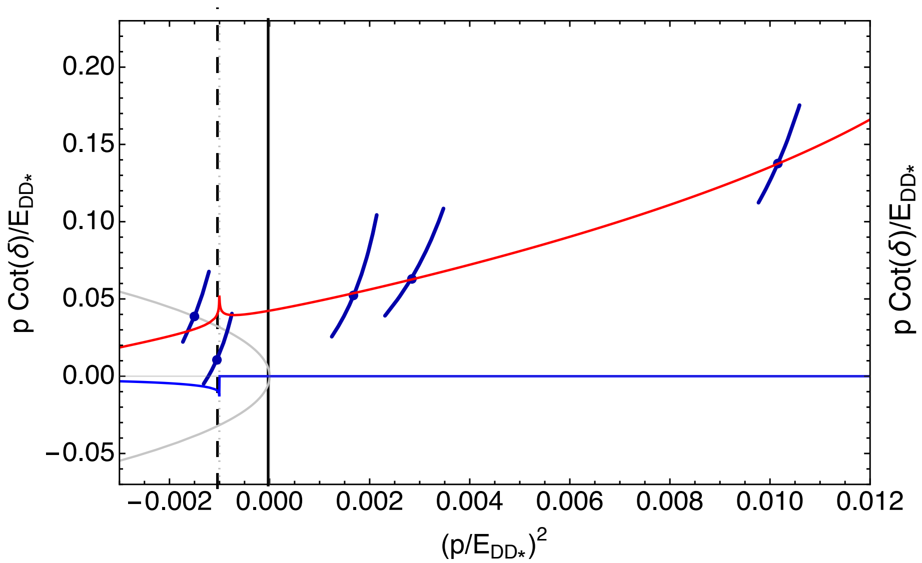
— lhc requires a modification of the Lüscher method Raposo and Hansen, PoS LATTICE2022, 051 (2023)

— lhc may induce partial-wave mixing effects Meng and Epelbaum, JHEP 10, 051 (2021)

Dependence on the pion coupling

- Importance of lhc is controlled by its position and strength (discontinuity)

$\frac{1}{10} V^{\mathrm{OPE}}_{\mathrm{DD}^* \to \mathrm{DD}^*}(m_\pi = 280\mathrm{MeV})$



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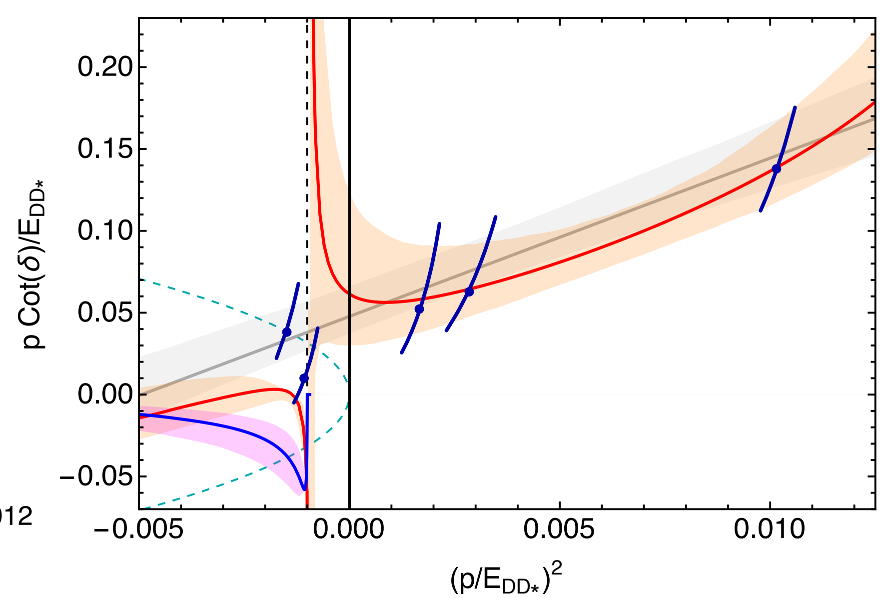
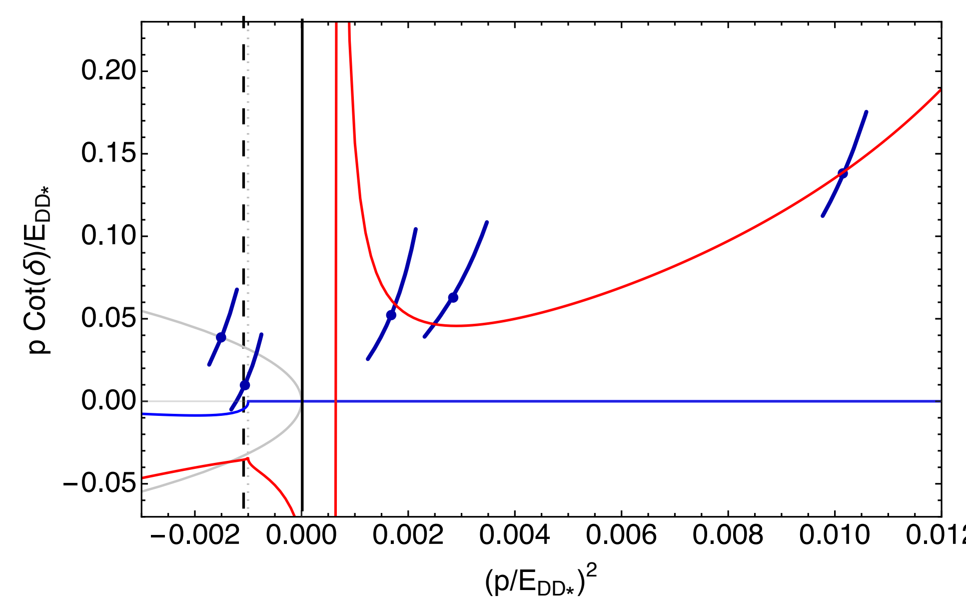
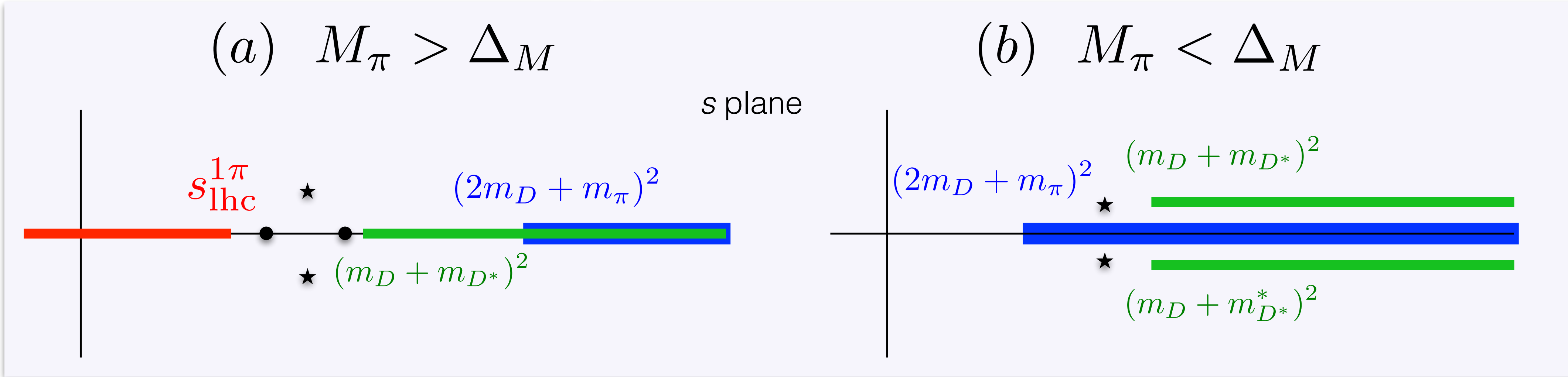


$10\ V^{\mathrm{OPE}}_{\mathrm{DD}^* \to \mathrm{DD}^*}(m_\pi = 280\mathrm{MeV})$



- The smaller the coupling the closer the fit is to the ERE

Summary and conclusions



χEFT: correct analytic structure of the LO DD* scatt. amplitude including relevant cuts

Real world: $m_\pi^{\rm ph} < \Delta M$

— 3-body πDD cuts ⇒ prominent role for understanding the width of the T_{cc}

— LHCb data are consistent with T_{cc} being a pure isoscalar molecule

⇒ If so, the D*D* I=0 (J^P= 1+) HQ spin partner should exist: $\delta_{cc}^{*+} = -503(40)$ keV

First pointed out by Albaladejo PLB 829 (2022) 137052 in contact EFT

Lattice $m_\pi > \Delta M$

— Left-hand cuts

⇒ constraints on the applicability range of the ERE

⇒ needed for an accurate extraction of the Tcc pole from lattice data

Thanks for your attention!

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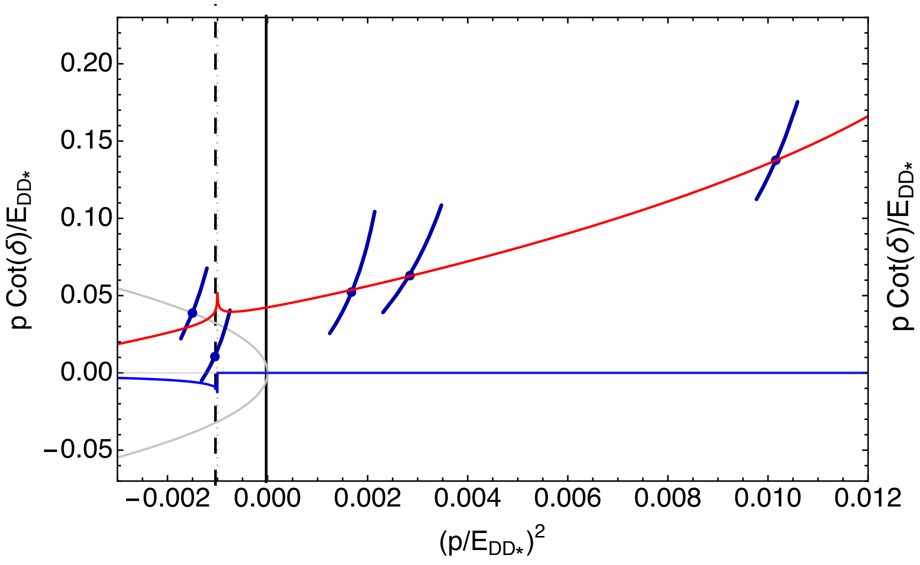

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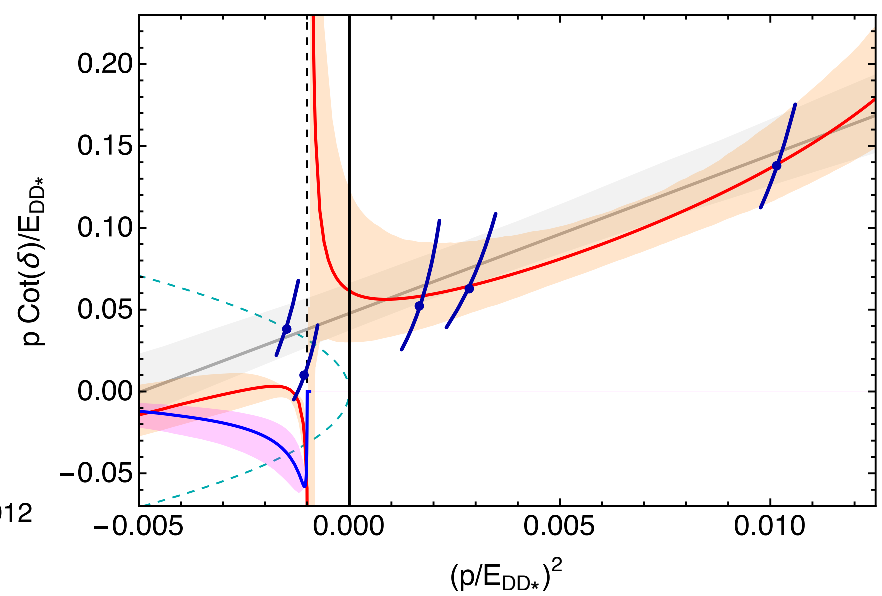

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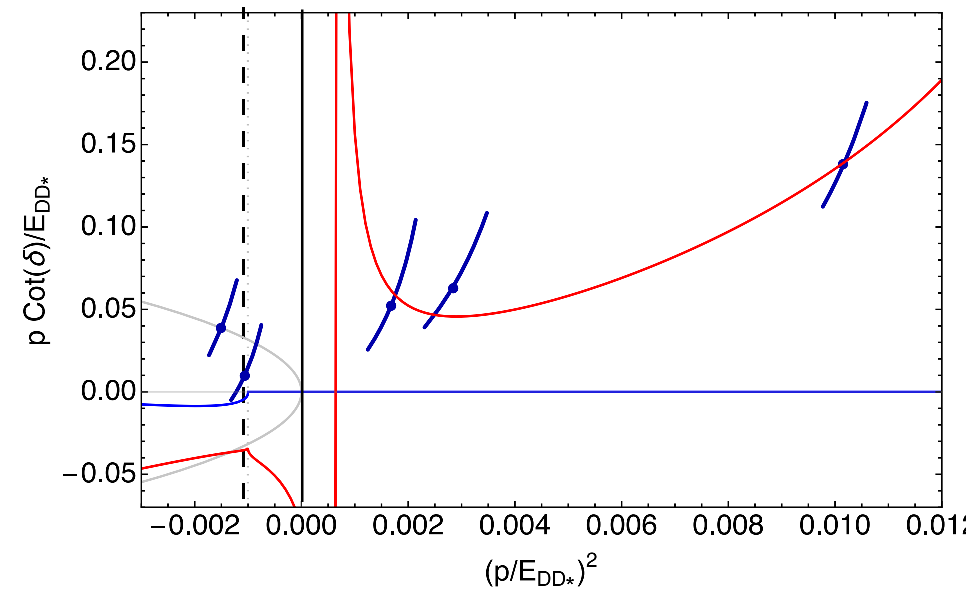

Qualitative behaviour of poles vs v_0 for different values of the pion coupling

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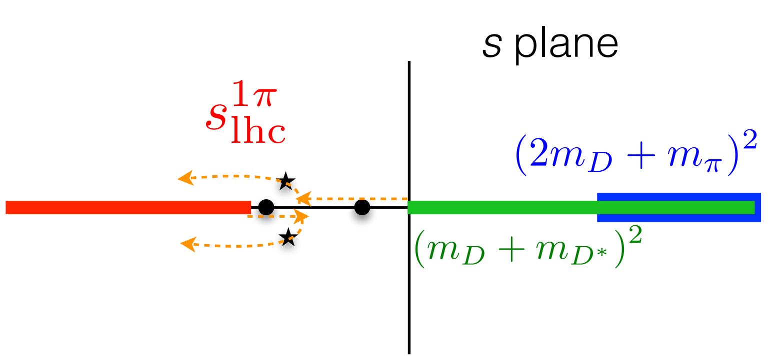

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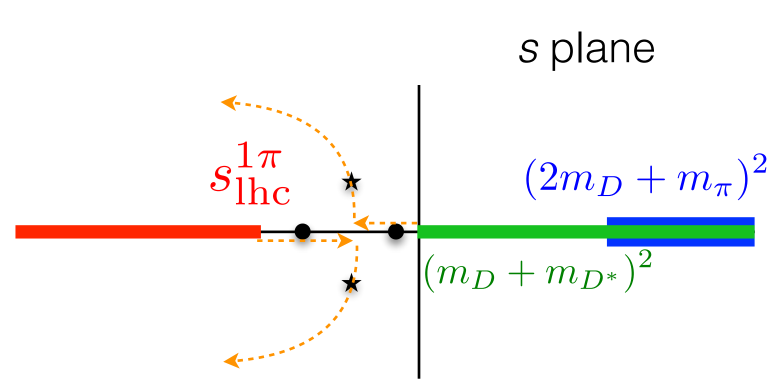

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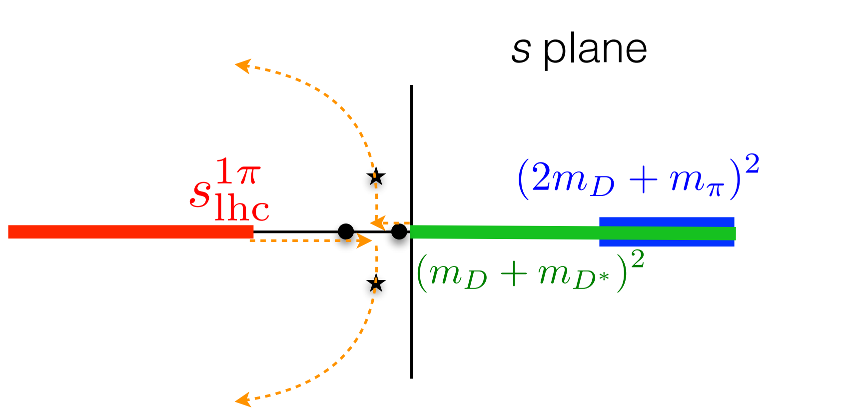

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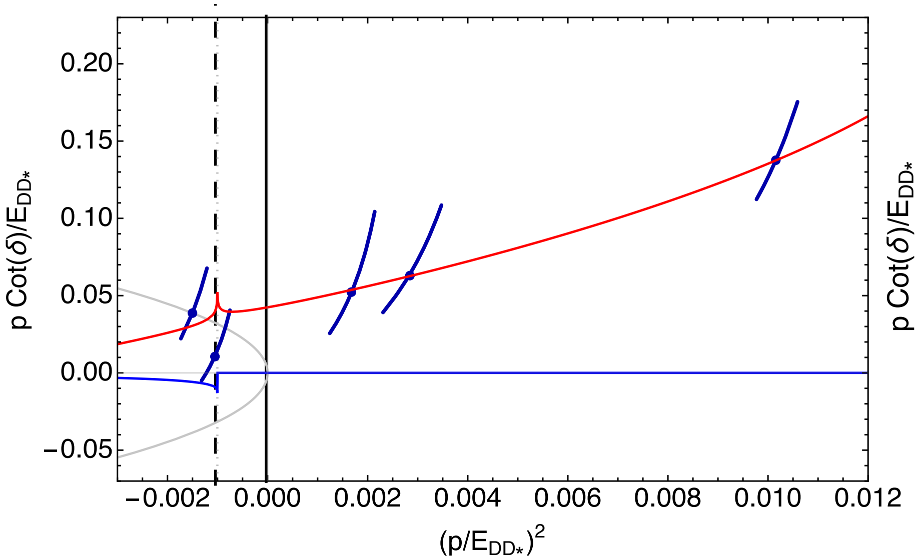

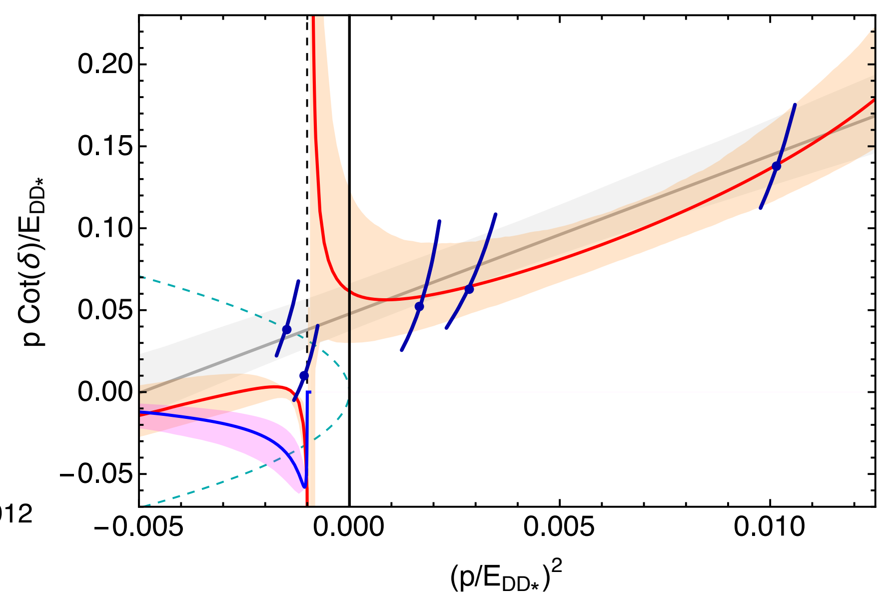

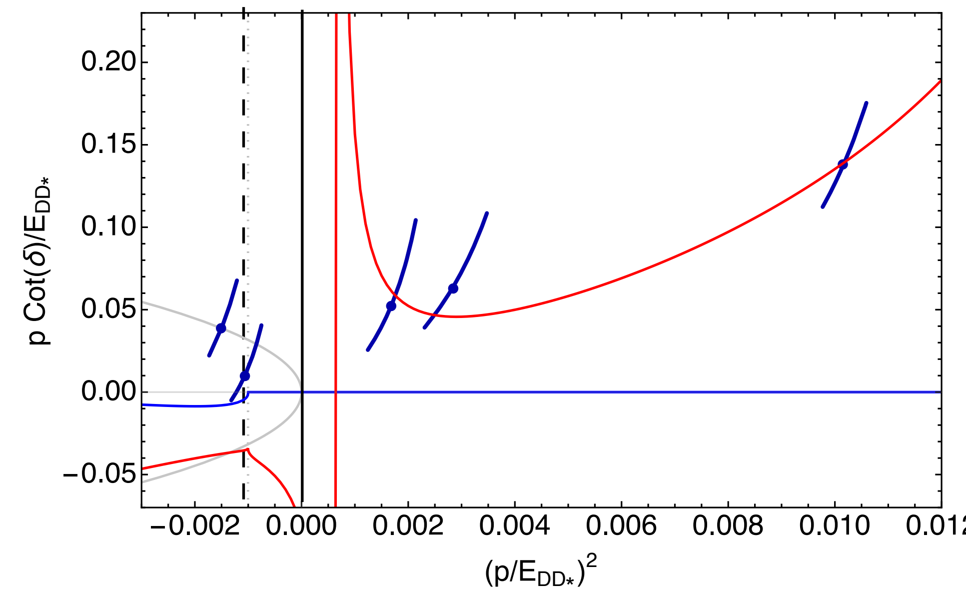

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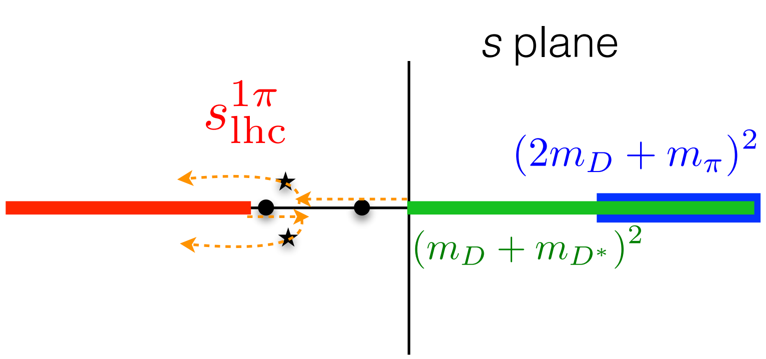
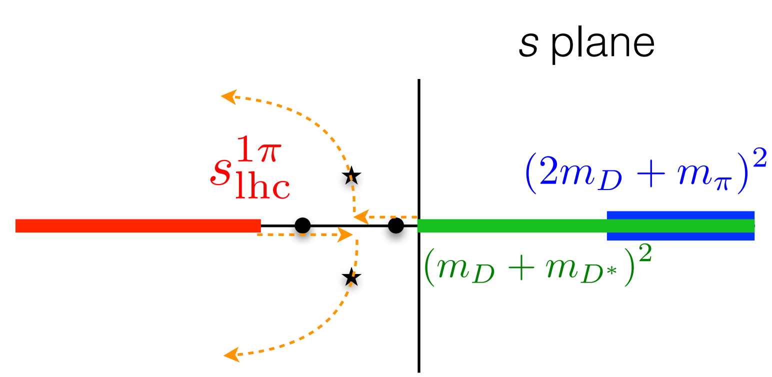
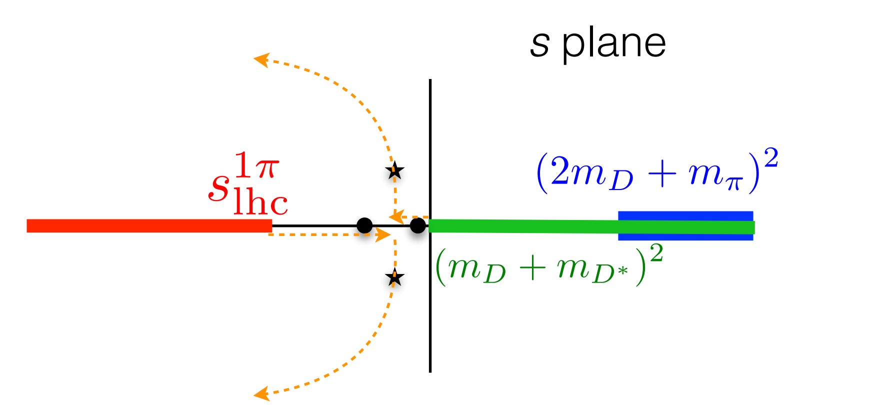
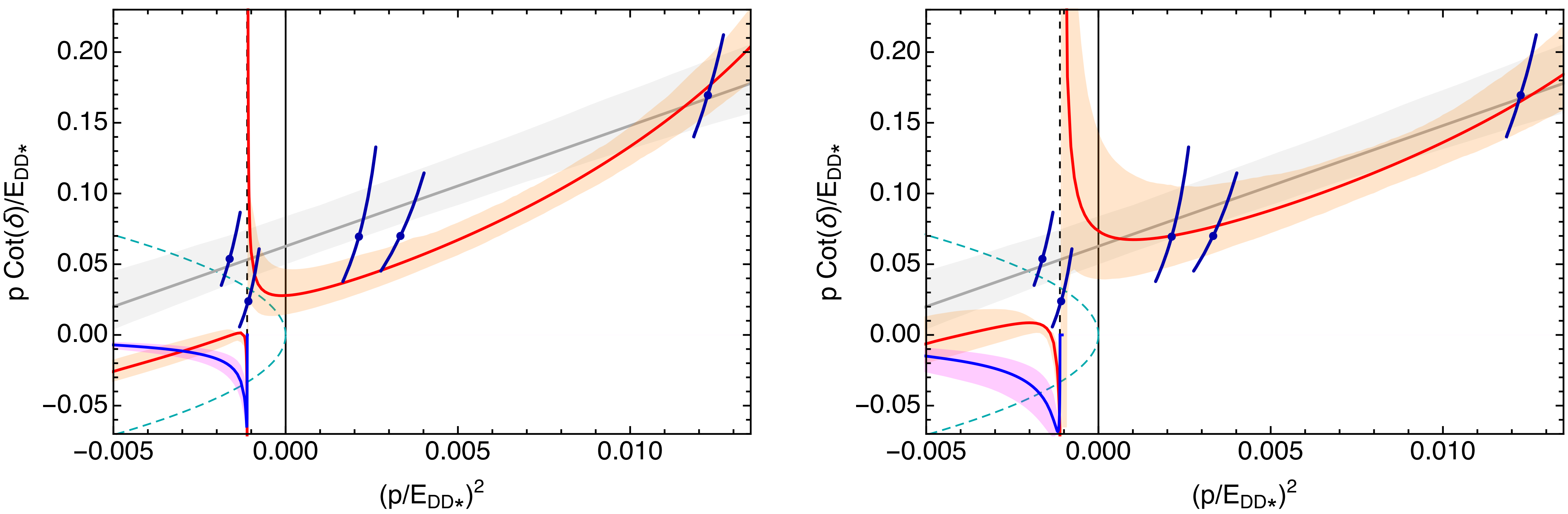






Fits to data for smaller c-quark masses



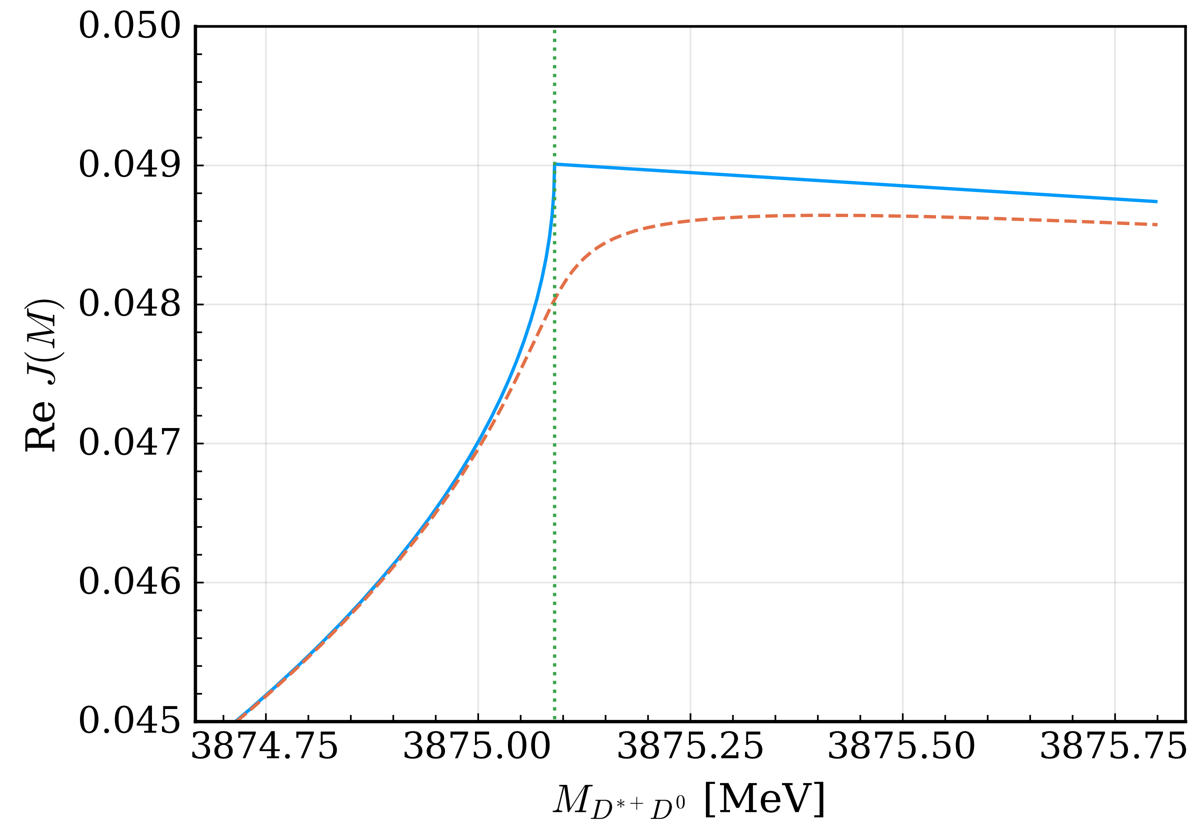
Conclusions are the same as for the other fit

Extraction of the effective range

Scattering amplitude in the 1st channel:

$$T_{D^{*+}D^0\to D^{*+}D^0}(k) = -\frac{2\pi}{\mu_{c0}}\left(\frac{1}{a_0}+\frac{1}{2}r_0k^2 - ik + \mathcal{O}(k^4)\right)^{-1}$$

$$T^{-1}(M) = V_{\rm CT}^{-1} + J(M), \quad J(M) = \int \frac{{\rm d}^3\boldsymbol{p}}{(2\pi)^3} G(M,p)$$



no width:

$$r_0 \propto -\mathfrak{Re}\frac{{\rm d}J(M)}{{\rm d}M}\bigg|_{M=M_{\rm thr}+0^+}$$

finite width: ERE has a small radius of convergence

$$k \leqslant \sqrt{\mu_{c0}\Gamma_{D^{*+}}} \approx 9\ {\rm MeV}$$

Approximate Solution: expand around the pole of the Green function

Braten and Stapleton (2010)

$$M = m_c^* - i\Gamma_c/2 + m_0 + \frac{k^2}{2\mu_{c0}}$$

Corrections scale as $\frac{1}{2}\frac{\Gamma_{D^*}}{M_{\rm thr2} - M_{\rm thr3}}$ ⟶ tiny for the problem at hand

Du et al. 2110.13765

Hanhart et al (2010)