Is $f_2(1950)$ the tensor glueball?

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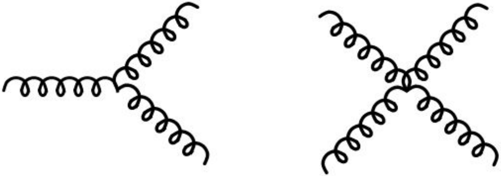
Overview

Introduction

The QCD lagrangian contains gluon self-interaction due to its non-abelian SU(3) symmetry

$$\mathcal{L}_{QCD} = \bar{\psi}_i (i\gamma^\mu (D_\mu)_{ij} - m_i \delta_{ij})\psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A_{a\mu} + g f^{abc} A^b_\mu A^c_\nu$$



This begs the question: is there a bound state made of only gluons, a particle that does not contain any matter?

Lattice QCD spectrum

$n\ J^{PC}$	M[MeV]			$n\ J^{PC}$	M[MeV]		
	Chen et.al.	Meyer	A & T		Chen et.al.	Meyer	A & T
$1\ 0^{++}$	1710(50)(80)	1475(30)(65)	1653(26)	$1\ 1^{--}$	3830(40)(190)	3240(330)(150)	4030(70)
$2\ 0^{++}$		2755(30)(120)	2842(40)	$1\ 2^{--}$	4010(45)(200)	3660(130)(170)	3920(90)
$3\ 0^{++}$		3370(100)(150)		$2\ 2^{--}$		3740(200)(170)	
$4\ 0^{++}$		3990(210)(180)		$1\ 3^{--}$	4200(45)(200)	4330(260)(200)	
$1\ 2^{++}$	2390(30)(120)	2150(30)(100)	2376(32)	$1\ 0^{+-}$	4780(60)(230)		
$2\ 2^{++}$		2880(100)(130)	3300(50)	$1\ 1^{+-}$	2980(30)(140)	2670(65)(120)	2944(42)
$1\ 3^{++}$	3670(50)(180)	3385(90)(150)	3740(70)	$2\ 1^{+-}$			3800(60)
$1\ 4^{++}$		3640(90)(160)	3690(80)	$1\ 2^{+-}$	4230(50)(200)		4240(80)
$1\ 6^{++}$		4360(260)(200)		$1\ 3^{+-}$	3600(40)(170)	3270(90)(150)	3530(80)
$1\ 0^{-+}$	2560(35)(120)	2250(60)(100)	2561(40)	$2\ 3^{+-}$		3630(140)(160)	
$2\ 0^{-+}$		3370(150)(150)	3540(80)	$1\ 4^{+-}$			4380(80)
$1\ 2^{-+}$	3040(40)(150)	2780(50)(130)	3070(60)	$1\ 5^{+-}$		4110(170)(190)	
$2\ 2^{-+}$		3480(140)(160)	3970(70)				
$1\ 5^{-+}$		3942(160)(180)					
$1\ 1^{-+}$			4120(80)				
$2\ 1^{-+}$			4160(80)				
$3\ 1^{-+}$			4200(90)				

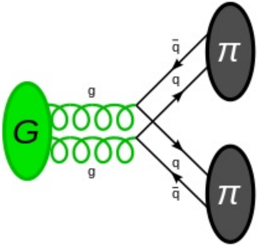
Glueball width

Glueballs are expected to have relatively small decay widths, from large N_c scaling:

$$A_{gg\to\bar{q}q+\bar{q}q} \propto N_c^{-1}$$

$$A_{\bar{q}q\to\bar{q}q+\bar{q}q} \propto N_c^{-\frac{1}{2}}$$

All processes glueball $\to$ hadrons are also suppressed because of the OZI rule



Experimental Search

Numerous experiments are working on data related to glueballs

- BESIII
- LHCb
- GlueX
- Compass
- Clas 12
- PANDA

Experimentally J/ψ decays are one of the best places to search for glueballs.

Linear Sigma Model

The most important symmetry breaking patterns for the eLSM are:

- Breaking of dilatation symmetry by dilaton field G (scalar glueball), leading to gluon condensate

$$\mathcal{L}_{dil} = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda_G^2}\left[G^4 \log(\frac{G}{\Lambda_G}) - \frac{G^4}{4}\right]$$

- Spontaneous chiral symmetry breaking; QCD Lagrangian is (almost) invariant under chiral transformations, but the vacuum is not. This leads to a chiral condensate and pions as massless scalars
- The condensates lead to shifts e.g. $G \to G + G_0, \Phi \to \Phi + \Phi_0$ which leads to mass terms similarly to the Higgs mechanism.
- Explicit chiral symmetry breaking gives pions a small mass compared to the other mesons

Linear Sigma Model

The LSM was previously extended for tensor and axial tensor mesons and its decay products of vectors, axial vectors, etc.

$$\mathcal{L}_{\text{eLSM}} = \mathcal{L}_{\text{dil}} + \text{Tr}\left[\left(D_\mu \Phi\right)^\dagger \left(D_\mu \Phi\right)\right] - m_0^2 \left(\frac{G}{G_0}\right)^2 \text{Tr}\left[\Phi^\dagger \Phi\right]$$
$$- \frac{1}{4}\text{Tr}\left[\left(L_{\mu\nu}^2 + R_{\mu\nu}^2\right)\right] + \cdots ,$$

Gave us decent results for tensor mesons:

Decay process (in model)	eLSM (MeV)	PDG-2020 (MeV)
$a_2(1320) \longrightarrow \rho(770)\,\pi$	71.0 ± 2.6	$73.61 \pm 3.35 \leftrightarrow (70.1 \pm 2.7)\%$
$K_2^*(1430) \longrightarrow \bar{K}^*(892)\,\pi$	27.9 ± 1.0	$26.92 \pm 2.14 \leftrightarrow (24.7 \pm 1.6)\%$
$K_2^*(1430) \longrightarrow \rho(770)\,K$	10.3 ± 0.4	$9.48 \pm 0.97 \leftrightarrow (8.7 \pm 0.8)\%$
$K_2^*(1430) \longrightarrow \omega(782)\,\bar{K}$	3.5 ± 0.1	$3.16 \pm 0.88 \leftrightarrow (2.9 \pm 0.8)\%$
$f_2'(1525) \longrightarrow \bar{K}^*(892)\,K + \text{c.c.}$	19.89 ± 0.73	

Glueball chiral interactions

Compared to the work on tensor mesons, we need to replace the tensors to realize flavour blindness:

$$T_{\mu\nu} \longrightarrow G_{2,\mu\nu} \cdot \mathbf{1}$$

The lagrangian leading to tensor glueball decays involves solely left- and right-handed chiral fields:

$$\mathcal{L} = \lambda G_{\mu\nu} \Big(\mathrm{Tr}\Big[\{ L^{\mu}, L^{\nu} \} \Big] + \mathrm{Tr}\Big[\{ R^{\mu}, R^{\nu} \} \Big] \Big)$$

Left- and right-handed fields are in terms of the vector and axial vector nonets

$$L^{\mu} := V^{\mu} + A_1^{\mu} \ , \ R^{\mu} := V^{\mu} - A_1^{\mu} \ .$$

Tensor glueball decays

The Lagrangian leads to three kinematically allowed decay channels

- Decaying of the tensor glueball to the two pseudoscalar mesons have the following decay rate formula

$$\Gamma_{G_2 \longrightarrow P^{(1)} P^{(2)}} = \frac{\kappa_{gpp\,,i}\, \lambda^2\, |\vec{k}_{p^{(1)},p^{(2)}}|^5}{60\,\pi\, m_{g_2}^2} \; ;$$

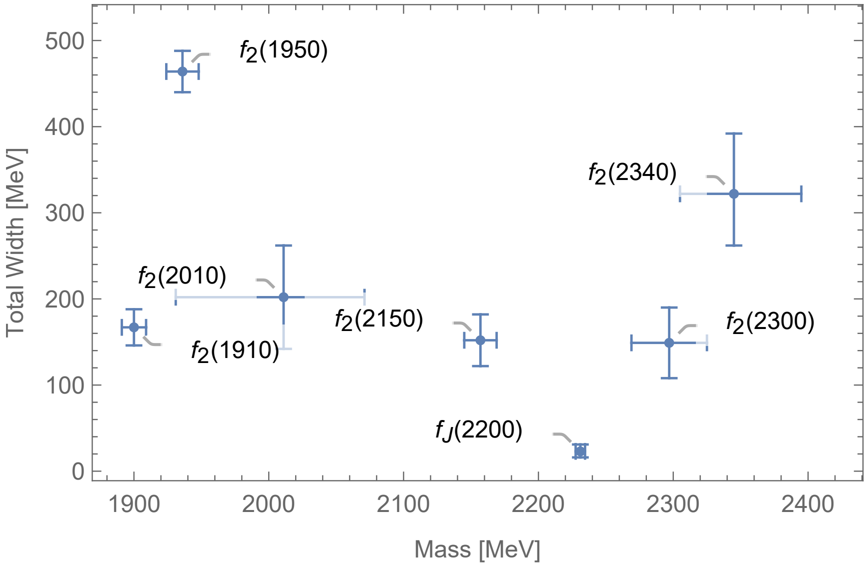
- while for the vector and pseudoscalar mesons

$$\Gamma_{G_2 \to V^{(1)} V^{(2)}} = \frac{\kappa_{gvv\,,i} \lambda^2 |\vec{k}_{v^{(1)},v^{(2)}}|}{120\,\pi\, m_{g_2}^2} \Bigg(15 + \frac{5|\vec{k}_{v^{(1)},v^{(2)}}|^2}{m_{v^{(1)}}^2} + \frac{5|\vec{k}_{v^{(1)},v^{(2)}}|^2}{m_{v^{(2)}}^2} + \frac{2|\vec{k}_{v^{(1)},v^{(2)}}|^4}{m_{v^{(1)}}^2\, m_{v^{(2)}}^2} \Bigg) \; ;$$

- and for the axial-vector and pseudoscalar mesons

$$\Gamma_{G_2 \longrightarrow A_1 P} = \frac{\kappa_{gap\,,i}\, \lambda^2\, |\vec{k}_{a_1,p}|^3}{120\,\pi\, m_{g_2}^2} \left(5 + \frac{2\,|\vec{k}_{a_1,p}|^2}{m_{a_1}^2} \right) \; .$$

Isoscalar-tensor resonances



Decay ratios

- Coupling constant is not known so we can only compute decay ratios
- Computation is done for a tensor glueball mass of 2210 MeV
- Vector channels are dominant, in particular $\rho\rho$ and $K^* K^*$
- Serves as a qualitative baseline, we can input different masses when comparing to specific resonances

Decay Ratio	theory
$\frac{G_2(2210) \longrightarrow \bar{K}\, K}{G_2(2210) \longrightarrow \pi\, \pi}$	0.4
$\frac{G_2(2210) \longrightarrow \eta\, \eta}{G_2(2210) \longrightarrow \pi\, \pi}$	0.1
$\frac{G_2(2210) \longrightarrow \eta\, \eta'}{G_2(2210) \longrightarrow \pi\, \pi}$	0.004
$\frac{G_2(2210) \longrightarrow \eta'\, \eta'}{G_2(2210) \longrightarrow \pi\, \pi}$	0.006
$\frac{G_2(2210) \longrightarrow \rho(770)\, \rho(770)}{G_2(2210) \longrightarrow \pi\, \pi}$	55
$\frac{G_2(2210) \longrightarrow \bar{K}^*(892)\, \bar{K}^*(892)}{G_2(2210) \longrightarrow \pi\, \pi}$	46
$\frac{G_2(2210) \longrightarrow \omega(782)\, \omega(782)}{G_2(2210) \longrightarrow \pi\, \pi}$	18
$\frac{G_2(2210) \longrightarrow \phi(1020)\, \phi(1020)}{G_2(2210) \longrightarrow \pi\, \pi}$	6
$\frac{G_2(2210) \longrightarrow a_1(1260)\, \pi}{G_2(2210) \longrightarrow \pi\, \pi}$	0.24
$\frac{G_2(2210) \longrightarrow K_{1\,,A}\, K}{G_2(2210) \longrightarrow \pi\, \pi}$	0.08
$\frac{G_2(2210) \longrightarrow f_1(1285)\, \eta}{G_2(2210) \longrightarrow \pi\, \pi}$	0.02
$\frac{G_2(2210) \longrightarrow f_1(1420)\, \eta}{G_2(2210) \longrightarrow \pi\, \pi}$	0.01

Data Comparison

Resonances	Decay Ratios	PDG	Model Prediction
$f_2(1910)$	$\rho\rho/\omega\omega$	2.6 ± 0.4	3.1
$f_2(1910)$	$f_2(1270)\eta/a_2(1320)\pi$	0.09 ± 0.05	0.07
$f_2(1910)$	$\eta\eta/\eta\eta'$	< 0.05	~ 8
$f_2(1910)$	$\omega\omega/\eta\eta\prime$	2.6 ± 0.6	~ 200
$f_2(1950)$	$\eta\eta/\pi\pi$	0.14 ± 0.05	0.081
$f_2(1950)$	$K\bar{K}/\pi\pi$	~ 0.8	0.32
$f_2(1950)$	$4\pi/\eta\eta$	> 200	> 700
$f_2(2150)$	$f_2(1270)\eta/a_2(1320)\pi$	0.79 ± 0.11	0.1
$f_2(2150)$	$K\bar{K}/\eta\eta$	1.28 ± 0.23	~ 4
$f_2(2150)$	$\pi\pi/\eta\eta$	< 0.33	~ 10

Table: Decay ratios for the decay channels with available data.

For $f_J(2220)$ PDG lists $\pi\pi/\bar{K}K$ ratio, but only $\eta\eta\prime$ is regarded as "seen".

Estimating glueball width

- A rough guess on the tensor glueball width can be made.
- Consider $f_2 \equiv f_2(1270) \simeq \sqrt{1/2}(\bar{u}u + \bar{d}d)$ and $f_2' \equiv f_2'(1525) \simeq \bar{s}s$, with $\Gamma_{f_2 \to \pi\pi} = 157.2$ MeV and $\Gamma_{f_2' \to \pi\pi} = 0.71$ MeV.
- The amplitude for $f_2 \to \pi\pi$ requires the creation of a single $\bar{q}q$ pair from the vacuum and scales as $1/\sqrt{N_c}$, where N_c is the number of colors. On the other hand, the amplitude for $f_2' \to \pi\pi$ scales as $1/N_c^{3/2}$ and goes schematically like

$$\bar{s}s \to gg \to \sqrt{1/2}(\bar{u}u + \bar{d}d)$$

Estimating glueball width

- Consider a transition Hamiltonian

$$H_{int} = \lambda \left(|\bar{u}u\rangle \langle gg| + \left|\bar{d}d\right\rangle \langle gg| + |\bar{s}s\rangle \langle gg| + h.c. \right), \ \ \lambda \propto 1/\sqrt{N_c}.$$

  Then: $A_{f_2' \to \pi\pi} \simeq \sqrt{2}\lambda^2 A_{f_2 \to \pi\pi}$, hence $\Gamma_{f_2' \to \pi\pi} \simeq 2\lambda^4 \Gamma_{f_2 \to \pi\pi}$,
- Tensor glueball decay into $\pi\pi$ intuitively speaking, is at an 'intermediate stage', since it starts with a *gg* pair. One has:

$$A_{G_2 \to \pi\pi} \simeq \sqrt{2}\lambda A_{f_2 \to \pi\pi},$$

$$\Gamma_{G_2 \to \pi\pi} \simeq 2\lambda^2 \Gamma_{f_2 \to \pi\pi} \simeq \sqrt{2}\sqrt{\Gamma_{f_2 \to \pi\pi}\Gamma_{f_2' \to \pi\pi}} \simeq 15\text{MeV}.$$

- We emphasize that this is a **rough estimate**, based on large N_c scaling.
- Similar results to some holographic models: very large decay widths in vector modes.

Glueball candidates

Resonances	Interpretation status
$f_2(1910)$	Agreement with some data, but large discrepancies in $\eta\eta\prime$ mode
$\mathbf{f_2(1950)}$	$\eta\eta/\pi\pi$ agrees with data, no contradictions with data, but broad tensor glueball **Best fit as predominantly glueball**
$f_2(2010)$	Likely primarily strange-antistrange content
$f_2(2150)$	All available data contradicts theoretical prediction
$f_J(2220)$	Data on $\pi\pi/K\bar{K}$ disagrees with theory largest predicted decay channels are not seen
$f_2(2300)$	Likely primarily strange-antistrange content
$f_2(2340)$	Likely primarily strange-antistrange content would also imply a broad glueball

Table: Spin 2 resonances and their status as the tensor glueball.

Summary

- Glueballs are a yet undiscovered prediction of QCD and an active research topic of both theoretical models and experimental efforts
- We have adapted the eLSM for tensor mesons to describe the tensor glueball
- We obtain decay ratios; vector channels are dominant, in particular $\rho\rho$ and K^*K^*
- The $f_2(1950)$ is clearly favored as a candidate by the eLSM.
- Sometimes data is limited, in particular, the analysis for the states $f_J(2220)$, $f_2(2300)$, and $f_2(2340)$ would benefit from more experimental data.
- Preliminary estimate for the decay widths gives 15 MeV for the $\pi\pi$ channel, which implies a very broad glueball in the vector channels.

Thank you for your attention