Glueballs from DSEs and BSEs

Markus Q. Huber

Institute of Theoretical Physics
Giessen University

20th International Conference on Hadron Spectroscopy and Structure (HADRON2023)
Genoa, Italy
June 6, 2023

In collaboration with:
Christian S. Fischer
Hèlios Sanchis-Alepuz

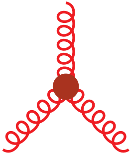
Eur.Phys.J.C 80, arXiv:2004.00415 → J=0
Eur.Phys.J.C 80, arXiv:2110.09180 → J=0,2,3,4
vConf21, arXiv:2111.10197 → +higher terms
HADRON2021, arXiv:2201.05163 → +higher terms

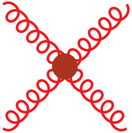
JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

DFG Deutsche Forschungsgemeinschaft

Glueballs

Non-Abelian nature of QCD $\rightarrow$ self-interaction of force fields.





Mass dynamically created from massless (due to gauge invariance) gluons.

Theory:
Glueballs from gauge inv. operators, e.g., $F_{\mu\nu}F^{\mu\nu}$.
$\rightarrow$ Mixing of operators with equal quantum numbers.

Experiment:
Production in glue-rich environments, e.g., $p\bar{p}$ annihilation (PANDA), pomeron exchange in pp (central exclusive production), radiative J/ψ decays

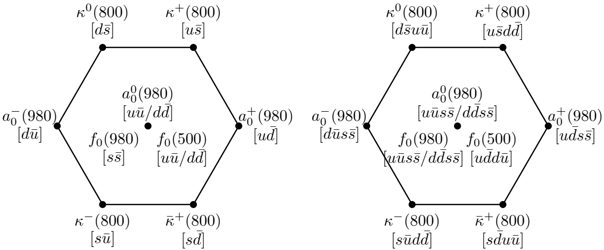
Reviews on glueballs: [Klempt, Zaitsev, Phys.Rept.454 (2007); Mathieu, Kochelev, Vento, Int.J.Mod.Phys.18 (2009); Crede, Meyer, Prog.Part.Nucl.Phys.63 (2009); Ochs, J.Phys.G40 (2013); Llanes-Estrada, EPJST 230 (2021); Vadacchino, 2305.04869]

Scalar sector

Classification not always easy, e.g., scalar sector $J^{PC} = 0^{++}$:

- $q\bar{q}$ mesons, tetraquarks: (inverted) mass hierarchy?

[Jaffe, Phys. Rev. D 15 (1977)]



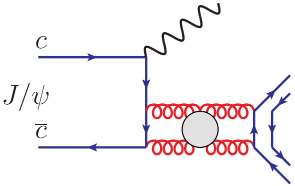
Functional review:
[Eichmann, Fischer, Santowsky, Wallbott, Few-Body Syst.61 (2020)]

- Glueballs?

$f_0(500)$
$f_0(980)$
$f_0(1370)$
$f_0(1500)$
$f_0(1710)$

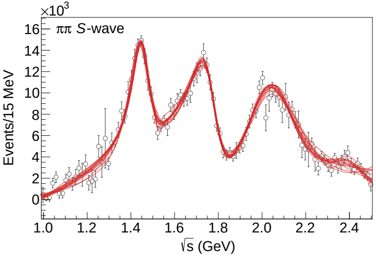
glueball candidates

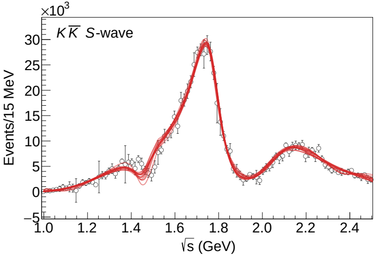
Scalar glueballs from J/ψ decay



Coupled-channel analyses of exp. data (BESIII):

- +add. data, largest overlap with $f_0(1770)$ [Sarantsev, Denisenko, Thoma, Klempt, Phys. Lett. B 816 (2021)]
- largest overlap with $f_0(1710)$ [JPAC Coll., Rodas et al., Eur.Phys.J.C 82 (2022)]





Glueball studies

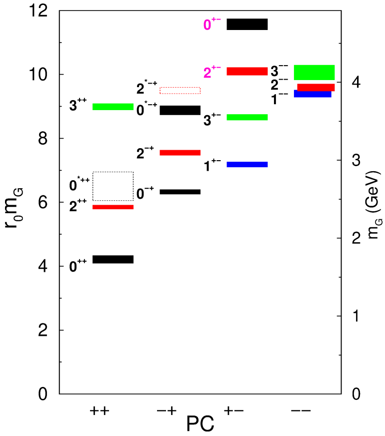
- Reviews on glueballs: [Klempt, Zaitsev, Phys.Rept.454 (2007); Mathieu, Kochelev, Vento, Int.J.Mod.Phys.18 (2009); Crede, Meyer, Prog.Part.Nucl.Phys.63 (2009); Ochs, J.Phys.G40 (2013); Llanes-Estrada, EPJST 230 (2021); Vadacchino, 2305.04869]
- Lattice: [Morningstar, Peardon, Phys. Rev. D60 (1999); Athenodorou, Teper, JHEP11 (2020); Gregory et al., JHEP10 (2012); Brett et al., AIP Conf.Proc. 2249 (2020); Chen et al., 2111.11929; . . .]
- Hamiltonian many body methods: [Szczepaniak, Swanson, Ji, Cotanch, PRL 76 (1996); Szczepaniak, Swanson, Phys. Lett. B 577 (2003); . . .]
- Chiral Lagrangians: [Janowski, Parganlija, Giacosa, Rischke, Phys. Rev. D 84 (2011); Eshraim, Janowski, Giacosa, Rischke, Phys. Rev. D 87 (2013); . . .]
- Holographic QCD: [Brower, Mathur, Tan, Nucl. Phys. B 587 (2000); Colangelo, De Fazio, Jugeau, Nicotri, Phys. Lett. B 652 (2007); Brünner, Parganlija, Rebhan, Phys. REv. D 93 (2016);. . .]
- Gribov-Zwanziger framework: [Dudal, Guimaraes, Sorella, Phys. Lett. B 732 (2014)]
- Functional studies: [Meyers, Swanson, Phys.Rev.D87 (2013); Sanchis-Alepuz, Fischer, Kellermann, von Smekal, Phys.Rev.D92, (2015); Souza et al., Eur.Phys.J.A56 (2020); Kaptari, Kämpfer, Few Body Syst.61 (2020); MQH, Phys.Rev.D 101 (2020); MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020); Pawlowski et al., 2212.01113]

Glueball calculations: Lattice

Lattice methods

Pure gauge theory:
No dynamic quarks.
→ "Pure" glueballs

- [Morningstar, Peardon, Phys. Rev. D60 (1999)]: standard reference
- [Athenodorou, Teper, JHEP11 (2020)]: improved statistics, more states



[Morningstar, Peardon, Phys. Rev. D60 (1999)]

Glueball calculations: Lattice

Lattice methods

Pure gauge theory:
No dynamic quarks.
$\rightarrow$ "Pure" glueballs

- [Morningstar, Peardon, Phys. Rev. D60 (1999)]: standard reference
- [Athenodorou, Teper, JHEP11 (2020)]: improved statistics, more states

"Real QCD":

- [Gregory et al., JHEP10 (2012)]
- [Brett et al., AIP Conf.Proc. 2249 (2020)]
- [Chen et al., 2111.11929]
- [Vadacchino, Lattice2022, 2305.04869]
- ...

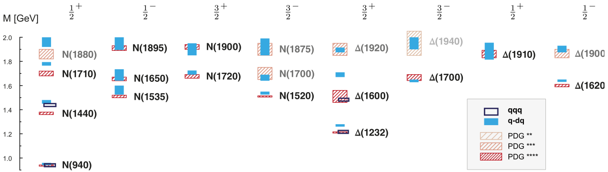
Challenging:

- Much higher statistics required (poor signal-to-noise ratio)
- Continuum extrapolation and inclusion of fermionic operators still to be done
- Mixing with $\bar{q}q$ challenging
- $m_\pi = 360\,\text{MeV}$
- Small unquenching effects found

No quantitative results yet.

Functional spectrum calculations

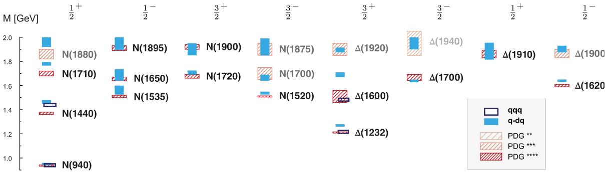
Functional methods successful in describing many aspects of the hadron spectrum qualitatively and quantitatively!



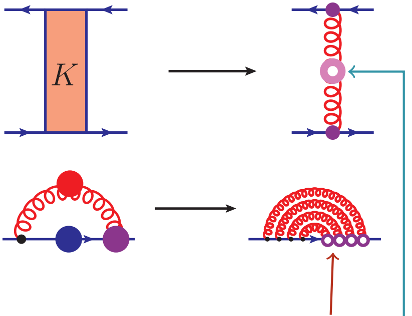
[Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, Prog.Part.Nucl.Phys. 91 (2016); Eichmann, Few Body Syst. 63 (2022)]

Functional spectrum calculations

Functional methods successful in describing many aspects of the hadron spectrum qualitatively and quantitatively!



[Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, Prog.Part.Nucl.Phys. 91 (2016); Eichmann, Few Body Syst. 63 (2022)]



Workhorse for more than 20 years: Rainbow-ladder truncation with an effective interaction, e.g., Maris-Tandy (or similar).

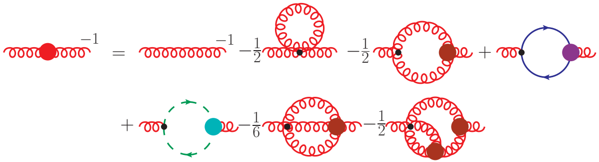
restricted structure of equations ($\Gamma_\mu \to \gamma_\mu$)

IR strength + perturbative UV

Results for mesons beyond rainbow-ladder, e.g., [Williams, Fischer, Heupel, Phys.Rev.D 93 (2016)].

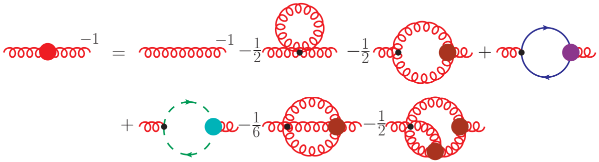
Functional glueball calculations

Glueballs? Rainbow-ladder?



Functional glueball calculations

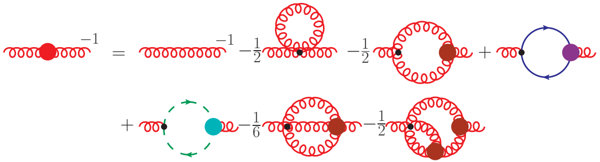
Glueballs? Rainbow-ladder?



There is no rainbow for gluons!

Functional glueball calculations

Glueballs? Rainbow-ladder?



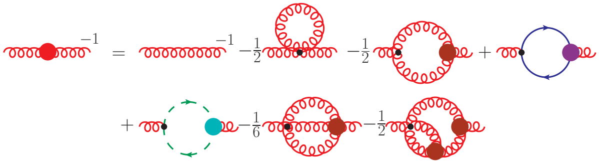
There is no rainbow for gluons!

Model based BSE calculations ($J = 0$):

- [Meyers, Swanson, Phys.Rev.D87 (2013)]
- [Sanchis-Alepuz, Fischer, Kellermann, von Smekal, Phys.Rev.D92, (2015)]
- [Souza et al., Eur.Phys.J.A56 (2020)]
- [Kaptari, Kämpfer, Few Body Syst.61 (2020)]

Functional glueball calculations

Glueballs? Rainbow-ladder?



There is no rainbow for gluons!

Model based BSE calculations ($J = 0$):

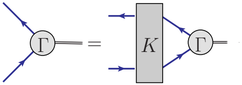
- [Meyers, Swanson, Phys.Rev.D87 (2013)]
- [Sanchis-Alepuz, Fischer, Kellermann, von Smekal, Phys.Rev.D92, (2015)]
- [Souza et al., Eur.Phys.J.A56 (2020)]
- [Kaptari, Kämpfer, Few Body Syst.61 (2020)]

Alternative: Calculated input [MQH, Phys.Rev.D 101 (2020)]

- $J = 0$: [MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020)]
- $J = 0, 2, 3, 4$: [MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

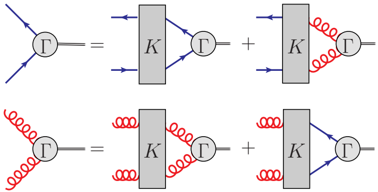
Extreme sensitivity on input!

Bound state equations for QCD



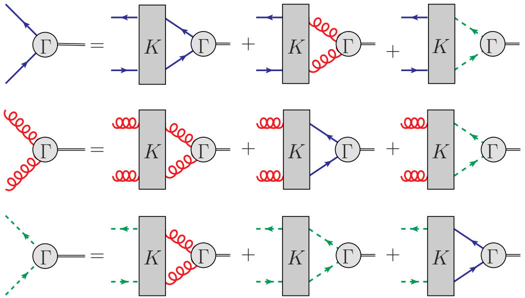
- Require scattering kernel K and propagator.

Bound state equations for QCD



- Require scattering kernels K and propagators.
- Quantum numbers determine which amplitudes Γ couple.

Bound state equations for QCD



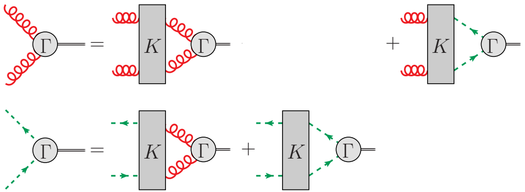
- Require scattering kernels K and propagators.
- Quantum numbers determine which amplitudes Γ couple.
- Ghosts from gauge fixing

One framework

- Natural description of mixing.
- Similar equations for hadrons with more than two constituents

Bound state equations for QCD

Focus on pure glueballs.



- Require scattering kernels K and propagators.
- Quantum numbers determine which amplitudes Γ couple.
- Ghosts from gauge fixing

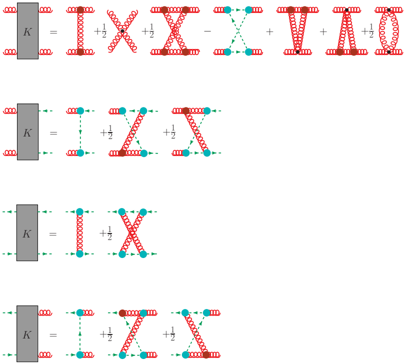
One framework

- Natural description of mixing.
- Similar equations for hadrons with more than two constituents

Kernels

Systematic derivation from 3PI effective action: [Berges, Phys. Rev. D 70 (2004); Carrington, Gao, Phys. Rev. D 83 (2011)]

Self-consistent treatment of 3-point functions requires 3-loop expansion.

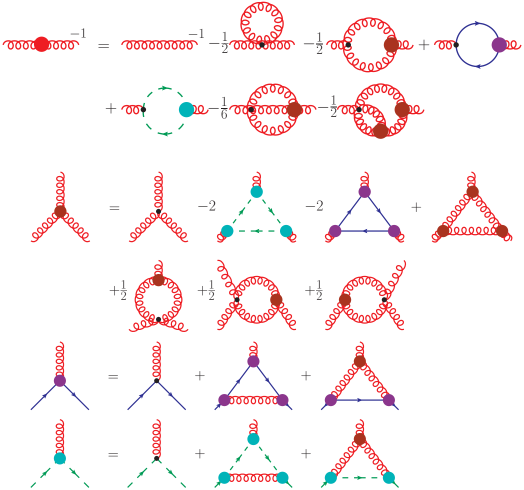


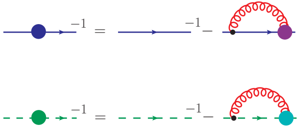
[Fukuda, Prog. Theor. Phys 78 (1987); McKay, Munczek, Phys. Rev. D 40 (1989); Sanchis-Alepuz, Williams, J. Phys: Conf. Ser. 631 (2015); MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020)]

Correlation functions of quarks and gluons

Equations of motion: 3-loop 3PI effective action

→ [Review: MQH, Phys.Rept. 879 (2020)]



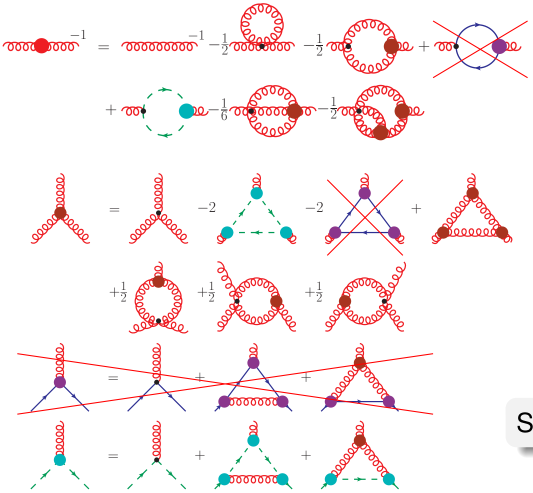


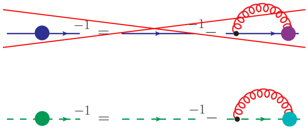
- Conceptual and technical challenges: nonperturbative renormalization, two-loop diagrams, convergence, size of kernels, . . .
- Self-contained: Only parameters are the strong coupling and the quark masses!
- Long way, e.g., ghost-gluon vertex, three-gluon vertex, four-gluon vertex, . . .
- → MQH, Phys.Rev.D 101 (2020)

Correlation functions of quarks and gluons

Equations of motion: 3-loop 3PI effective action

→ [Review: MQH, Phys.Rept. 879 (2020)]





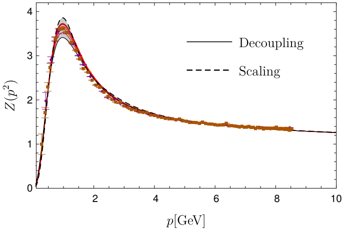
- Conceptual and technical challenges: nonperturbative renormalization, two-loop diagrams, convergence, size of kernels, ...
- Self-contained: Only parameters are the strong coupling and the quark masses!
- Long way, e.g., ghost-gluon vertex, three-gluon vertex, four-gluon vertex, ...
- → MQH, Phys.Rev.D 101 (2020)

Start with pure gauge theory.

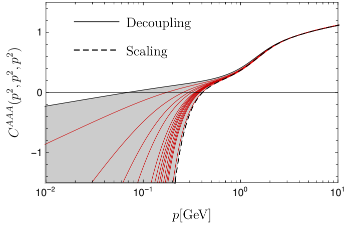
Landau gauge propagators

Self-contained: Only external input is the coupling! [MQH, Phys.Rev.D 101 (2020)]

Gluon dressing function:



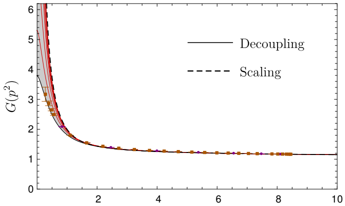
Three-gluon vertex:



Family of solutions [von Smekal, Alkofer, Hauck, PRL79 (1997); Aguilar, Binosi, Papavassiliou, Phys.Rev.D 78 (2008); Boucaud et al., JHEP06 (2008); Fischer, Maas, Pawlowski, Ann.Phys. 324 (2008); Alkofer, MQH, Schwenzer, Phys. Rev. D 81 (2010)]

Nonperturbative completions of Landau gauge [Maas, Phys. Lett. B 689 (2010)]?

Ghost dressing function:



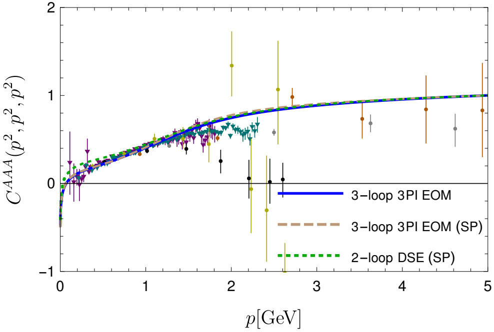
Stability of the solution

- Agreement with lattice results. ✓

Stability of the solution

- Agreement with lattice results. ✓
- Concurrence between functional methods:

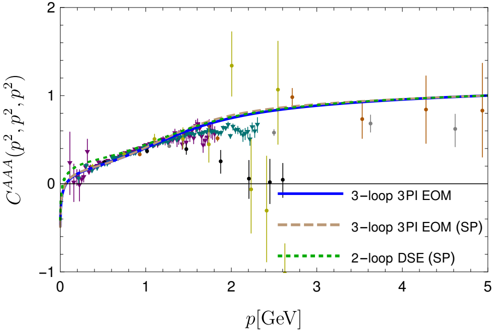
3PI vs. 2-loop DSE:



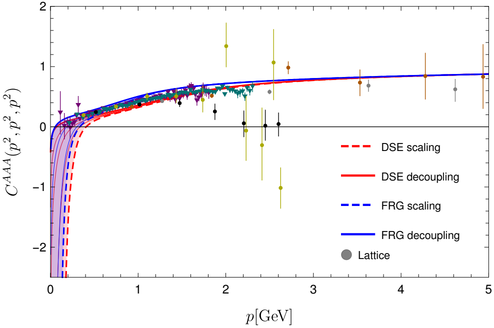
Stability of the solution

- Agreement with lattice results. ✓
- Concurrence between functional methods: ✓

3PI vs. 2-loop DSE:



DSE vs. FRG:



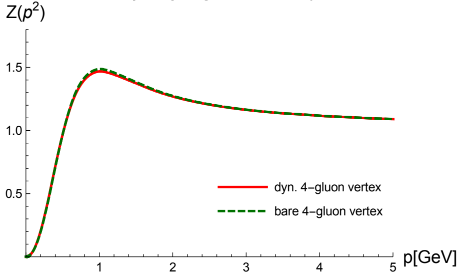
[Cucchieri, Maas, Mendes, Phys.Rev.D77 (2008); Sternbeck et al., Proc.Sci. LATTICE2016 (2017); Cyrol et al., Phys.Rev.D 94 (2016); MQH, Phys.Ref.D101 (2020)]

Stability of the solution: Extensions

- Three-gluon vertex: Tree-level dressing dominant, others subleading [Eichmann, Williams, Alkofer, Vujinovic, Phys.Rev.D89 (2014); Pinto-Gómez et al., 2208.01020] ✓

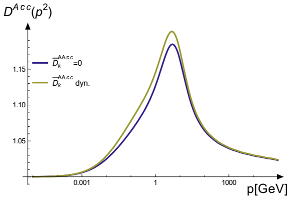
Stability of the solution: Extensions

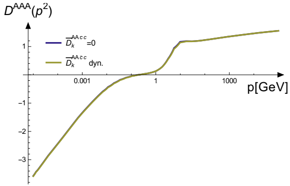
- Three-gluon vertex: Tree-level dressing dominant, others subleading [Eichmann, Williams, Alkofer, Vujinovic, Phys.Rev.D89 (2014); Pinto-Gómez et al., 2208.01020] ✓
- Four-gluon vertex: Influence on propagators tiny for $d = 3$ [MQH, Phys.Rev.D93 (2016)]

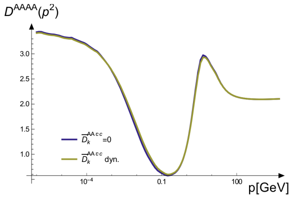


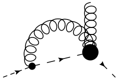
Stability of the solution: Extensions

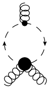
- Three-gluon vertex: Tree-level dressing dominant, others subleading [Eichmann, Williams, Alkofer, Vujinovic, Phys.Rev.D89 (2014); Pinto-Gómez et al., 2208.01020] ✓
- Four-gluon vertex: Influence on propagators tiny for $d = 3$ [MQH, Phys.Rev.D93 (2016)]
- Two-ghost-two-gluon vertex [MQH, Eur. Phys.J.C77 (2017)]: ✓
  (FRG: [Corell, SciPost Phys. 5 (2018)])

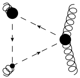




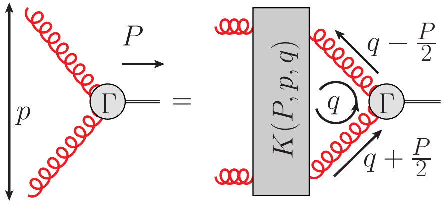








Correlation functions for complex momenta



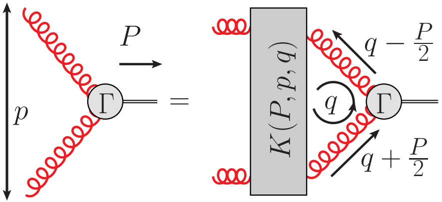
(pseudoscalar glueball)

$$\boldsymbol{\lambda(P)}\Gamma(P) = \mathcal{K} \cdot \Gamma(P)$$

$\rightarrow$ Eigenvalue problem for $\Gamma(P)$:

1. Solve for $\lambda(P)$.
2. Find P with $\lambda(P) = 1$.
   $\Rightarrow M^2 = -P^2$

Correlation functions for complex momenta



(pseudoscalar glueball)

$$\lambda(P)\Gamma(P) = \mathcal{K} \cdot \Gamma(P)$$

$\to$ Eigenvalue problem for $\Gamma(P)$:

1. Solve for $\lambda(P)$.
2. Find P with $\lambda(P) = 1$.
   $\Rightarrow M^2 = -P^2$

However:

Propagators are probed at $\left(q \pm \frac{P}{2}\right)^2 = \frac{P^2}{4} + q^2 \pm \sqrt{P^2\, q^2} \cos\theta = -\frac{M^2}{4} + q^2 \pm i\, M \sqrt{q^2} \cos\theta$

$\to$ Complex for $P^2 < 0$!

Time-like quantities ($P^2 < 0$) $\to$ Correlation functions for complex arguments.

Extrapolation of $\lambda(P^2)$

Extrapolation method

- Extrapolation to time-like P^2 using Schlessinger's continued fraction method (proven superior to default Padé approximants) [Schlessinger, Phys.Rev.167 (1968)]
- Average over extrapolations using subsets of points for error estimate

$$f(x) = \frac{f(x_1)}{1 + \frac{a_1(x-x_1)}{1+\frac{a_2(x-x_2)}{1+\frac{a_3(x-x_3)}{\dots}}}}$$

Coefficients a_i can determined such that $f(x)$ exact at x_i.

Extrapolation of $\lambda(P^2)$

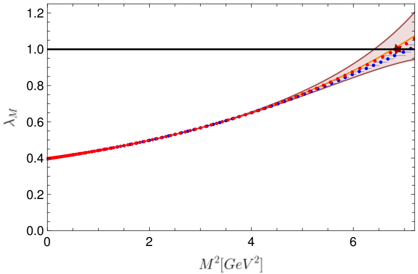
Extrapolation method

- Extrapolation to time-like P^2 using Schlessinger's continued fraction method (proven superior to default Padé approximants) [Schlessinger, Phys.Rev.167 (1968)]
- Average over extrapolations using subsets of points for error estimate

$$f(x) = \cfrac{f(x_1)}{1 + \cfrac{a_1(x-x_1)}{1+\cfrac{a_2(x-x_2)}{1+\cfrac{a_3(x-x_3)}{\cdots}}}}$$

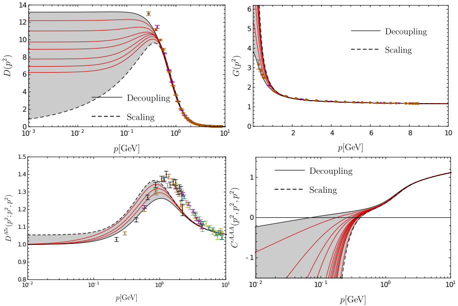
Coefficients a_i can determined such that $f(x)$ exact at x_i.

Test extrapolation for solvable system:
Heavy meson [MQH, Sanchis-Alepuz, Fischer, Eur.Phys.J.C 80 (2020)]



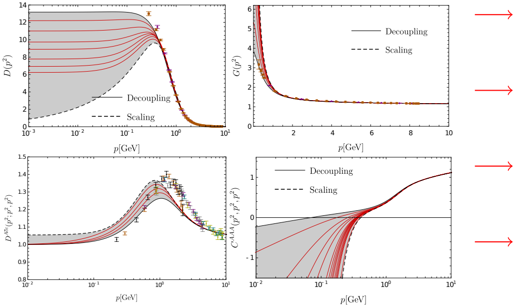
Glueball results J=0

Gauge-variant correlation functions:

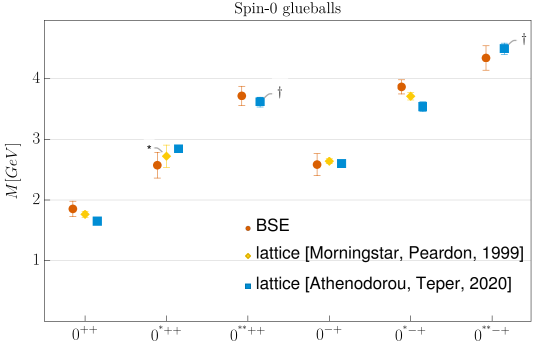


Glueball results J=0

Gauge-variant correlation functions:

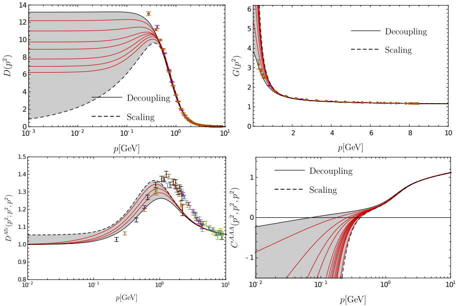


Unique physical spectrum:

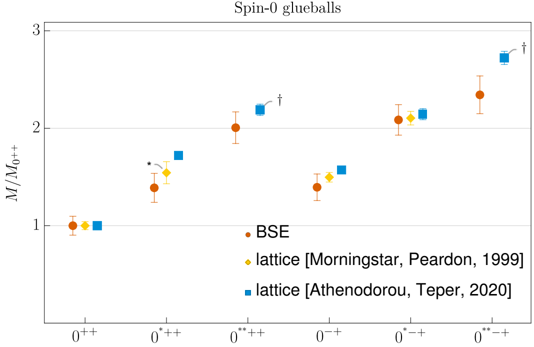


Glueball results J=0

Gauge-variant correlation functions:



Unique physical spectrum:

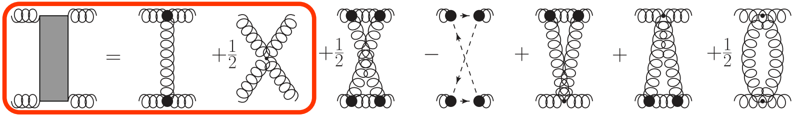


Spectrum independent! $\rightarrow$ Family of solutions yields the same physics.

All results for $r_0 = 1/418(5)$ MeV.

[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020)]

Higher order diagrams



One-loop diagrams only:

[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020); MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

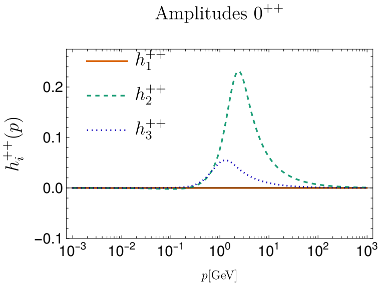
Two-loop diagrams: subleading effects

- 0^{-+}: none
  [MQH, Fischer, Sanchis-Alepuz, EPJ Web Conf. 258 (2022)]
- 0^{++}: $< 2\%$
  [MQH, Fischer, Sanchis-Alepuz, HADRON2021, arXiv:2201.05163]

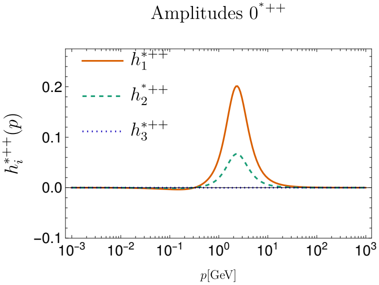
Amplitudes

Information about significance of single parts.

Ground state scalar glueball:



Excited scalar glueball:



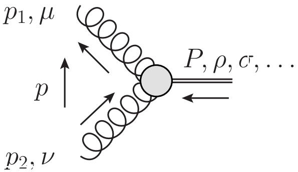
→ Amplitudes have different behavior for ground state and excited state. Useful guide for future developments.

→ Meson/glueball amplitudes: Information about mixing.

Glueball amplitudes for spin J

[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

$$\Gamma_{\mu\nu\rho\sigma\ldots}(p_1,p_2)=\sum\tau^i_{\mu\nu\rho\sigma\ldots}(p_1,p_2)h_i(p_1,p_2)$$



Increase in complexity:

- 2 gluon indices (transverse)
- J spin indices (symmetric, traceless, transverse to P)

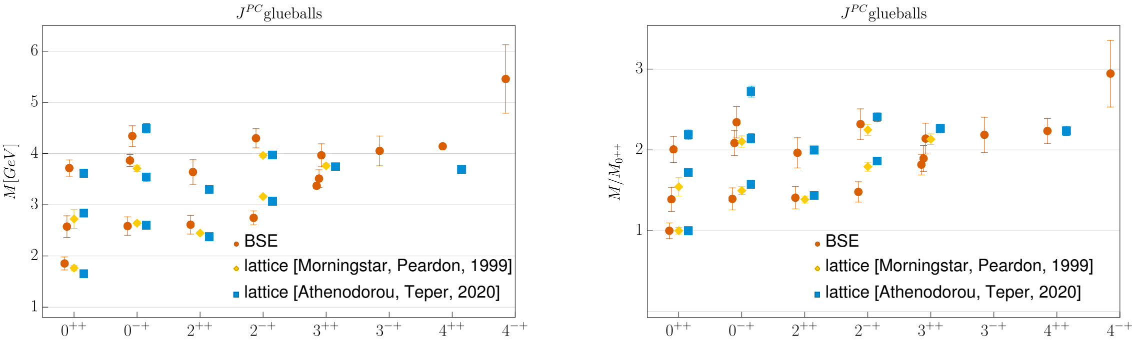
Numbers of tensors:

J	P $=+$	P $=-$
0	2	1
1	4	3
>2	5	4

Low number of tensors, but high-dimensional tensors!

$\rightarrow$ Computational cost increases with J.

Glueball results



[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

- Agreement with lattice results
- New states: 0^{**++}, 0^{**-+}, 3^{-+}, 4^{-+}

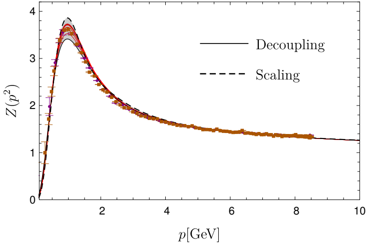
Summary and outlook

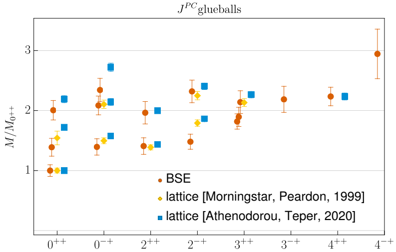
- Alternative to models in bound state equations: Direct calculation of input.
- Large system of equations may be necessary.
- Independent tests:
  - Agreement with other methods: lattice + continuum
  - Extensions

Pure glueball spectrum from first principles.

Future:

- +quarks $\rightarrow$ QCD
- three-body bound state eq. $\rightarrow C = -1$

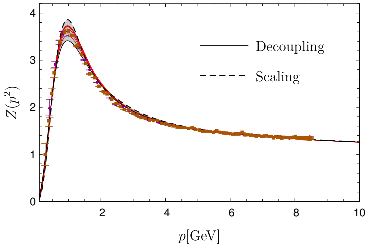


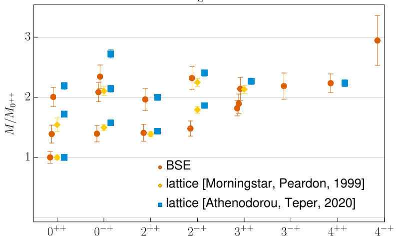


Summary and outlook

- Alternative to models in bound state equations: Direct calculation of input.
- Large system of equations may be necessary.
- Independent tests:
  - Agreement with other methods: lattice + continuum
  - Extensions

Pure glueball spectrum from first principles.





Future:

- +quarks → QCD
- three-body bound state eq. → $C = -1$

Thank you for your attention.

$J = 1$ glueballs

Landau-Yang theorem

Two-photon states cannot couple to $J^P = \mathbf{1}^{\pm}$ or $(2n+1)^-$

[Landau, Dokl.Akad.Nauk SSSR 60 (1948); Yang, Phys. Rev. 77 (1950)].

($\rightarrow$ Exclusion of $J = 1$ for Higgs because of $h \rightarrow \gamma\gamma$.)

Applicable to glueballs?

$\rightarrow$ Not in this framework, since gluons are not on-shell.

$\rightarrow$ Presence of $J = 1$ states is a dynamical question.

$J = 1$ not found here.

Glueballs as bound states

Hadron masses from correlation functions of color singlet operators.

Glueballs as bound states

Hadron masses from correlation functions of color singlet operators.

Example: For $J^{\mathrm{PC}} = 0^{++}$ glueball take $O(x) = F_{\mu\nu}(x)F^{\mu\nu}(x)$:

$$\boldsymbol{D}(\boldsymbol{x}-\boldsymbol{y}) = \langle O(x)O(y)\rangle$$

Glueballs as bound states

Hadron masses from correlation functions of color singlet operators.

Example: For $J^{\mathrm{PC}} = 0^{++}$ glueball take $O(x) = F_{\mu\nu}(x)F^{\mu\nu}(x)$:

$$\boldsymbol{D}(\boldsymbol{x} - \boldsymbol{y}) = \langle O(x)O(y)\rangle$$

Lattice: Mass exponential Euclidean time decay:

$$\lim_{t\to\infty}\langle O(x)O(0)\rangle \sim e^{-t\,M}$$

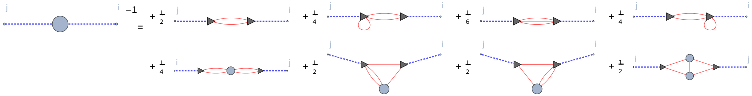
Glueballs as bound states

Hadron masses from correlation functions of color singlet operators.

Example: For $J^{PC} = 0^{++}$ glueball take $O(x) = F_{\mu\nu}(x)F^{\mu\nu}(x)$:

$$\boldsymbol{D}(\boldsymbol{x}-\boldsymbol{y}) = \langle O(x)O(y)\rangle$$

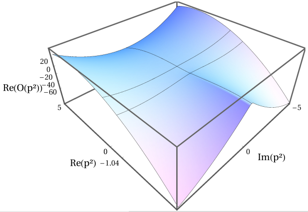
Functional approach: Complicated object in a diagrammatic language, 2-, 3- and 4-gluon contributions [MQH, Cyrol, Pawlowski, Comput.Phys.Commun. 248 (2020)]



+ 3-loop diagrams

Leading order:

[Windisch, MQH, Alkofer, Phys.Rev.D87 (2013)]



Glueballs as bound states

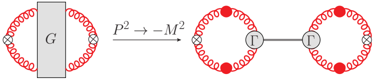
Hadron masses from correlation functions of color singlet operators.

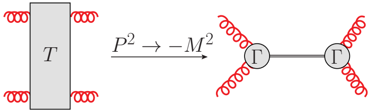
Example: For $J^{PC} = 0^{++}$ glueball take $O(x) = F_{\mu\nu}(x)F^{\mu\nu}(x)$:

$$\boldsymbol{D}(\boldsymbol{x} - \boldsymbol{y}) = \langle O(x)O(y)\rangle$$

Put total momentum on-shell and consider individual 2-, 3- and 4-gluon contributions. $\rightarrow$ Each can have a pole at the glueball mass.

A^4-part of $D(x-y)$, total momentum on-shell:



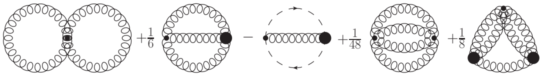


Kernel construction

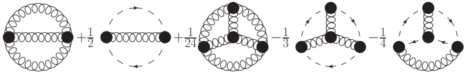
From 3PI effective action truncated to three-loops: [Berges, Phys. Rev. D 70 (2004); Carrington, Gao, Phys. Rev. D 83 (2011)]

$$\Gamma^{3l}[\Phi, D, \Gamma^{(3)}] = \Gamma^{0,3l}[\Phi, D, \Gamma^{(3)}] + \Gamma^{\text{int},3l}[\Phi, D, \Gamma^{(3)}]$$

$\Gamma^{0,3l}[\Phi, D, \Gamma^{(3)}] = \frac{1}{8}$



$\Gamma^{\text{int},3l}[D, \Gamma^{(3)}] = -\frac{1}{12}$



Kernels constructed by cutting two legs:
gluon/gluon,ghost/gluon, gluon/ghost, ghost/ghost

[Fukuda, Prog. Theor. Phys 78 (1987); McKay, Munczek, Phys. Rev. D 40 (1989); Sanchis-Alepuz, Williams, J. Phys: Conf. Ser. 631 (2015); MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020)]

Charge parity

Transformation of gluon field under charge conjugation:

$$A^a_\mu \to -\eta(a) A^a_\mu$$

where

$$\eta(a) = \begin{cases} +1 & a = 1,3,4,6,8 \\ -1 & a = 2,5,7 \end{cases}$$

Color neutral operator with two gluon fields:

$$A^a_\mu A^a_\nu \to \eta(a)^2 A^a_\mu A^a_\nu = A^a_\mu A^a_\nu.$$

$\Rightarrow C = +1$

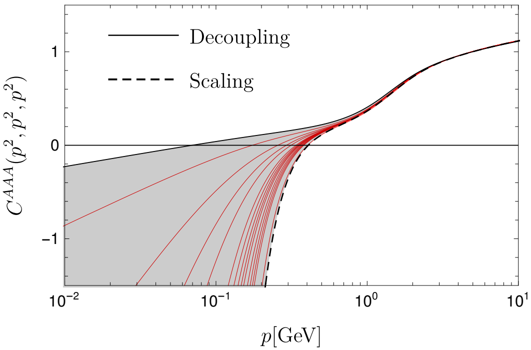
Negative charge parity, e.g.:

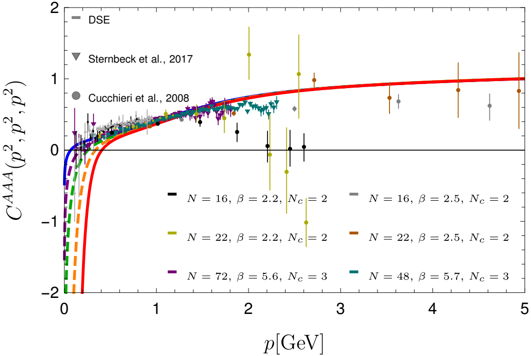
$$d^{abc} A^a_\mu A^b_\nu A^c_\rho \to -\, d^{abc} \eta(a)\eta(b)\eta(c) A^a_\mu A^b_\nu A^c_\rho = \\ -\, d^{abc} A^a_\mu A^b_\nu A^c_\rho.$$

Only nonvanishing elements of the symmetric structure constant d^{abc}: zero or two indices equal to 2, 5 or 7.

Three-gluon vertex

[Cucchieri, Maas, Mendes, Phys. Rev. D 77 (2008); Sternbeck et al., 1702.00612; MQH, Phys. Rev. D 101 (2020)]

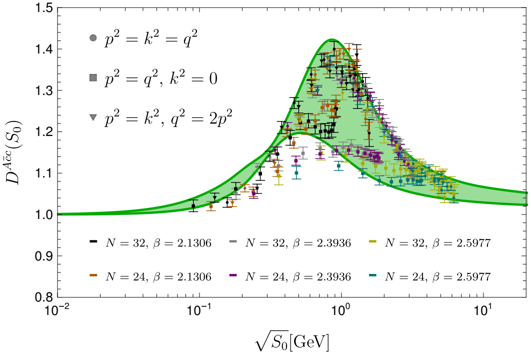




- Simple kinematic dependence of three-gluon vertex (only singlet variable of S_3)
- Large cancellations between diagrams

Ghost-gluon vertex

Ghost-gluon vertex:



[Maas, SciPost Phys. 8 (2019);

MQH, Phys. Rev. D 101 (2020)]

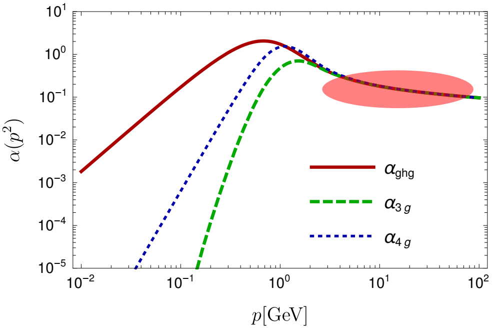
- Nontrivial kinematic dependence of ghost-gluon vertex
- Qualitative agreement with lattice results, though some quantitative differences (position of peak!).

Gauge invariance

[MQH, Phys. Rev. D 101 (2020)]

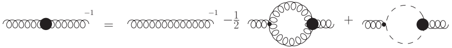
Couplings can be extracted from each vertex.

- Slavnov-Taylor identities (gauge invariance): Agreement perturbatively (UV) necessary.
  [Cyrol et al., Phys.Rev.D 94 (2016)]
- Difficult to realize: Small deviations → Couplings cross and do not agree.
- Here: Vertex couplings agree down to GeV regime (IR can be different).



Landau gauge propagators in the complex plane

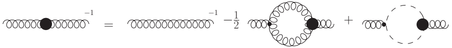
Simpler truncation:



[Fischer, MQH, Phys.Rev.D 102 (2020)]

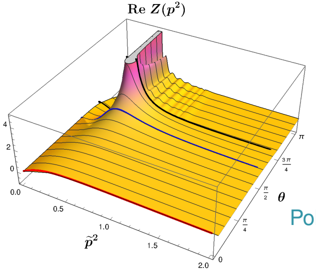
Landau gauge propagators in the complex plane

Simpler truncation:

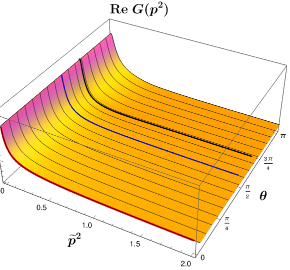


[Fischer, MQH, Phys.Rev.D 102 (2020)]

Ray technique for self-consistent solution of a DSE:



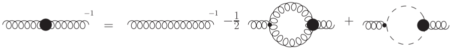
Polar coordinates: $p^2 = \tilde{p}^2 e^{i\,\theta}$



- Current truncation leads to a pole-like structure in the gluon propagator.
- Analyticity up to 'pole' confirmed by various tests (Cauchy-Riemann, Schlessinger, reconstruction)

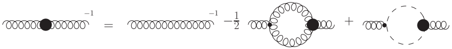
Landau gauge propagators in the complex plane

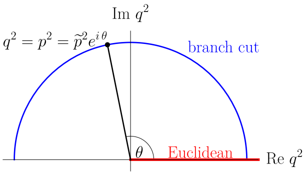
Simpler truncation:



Landau gauge propagators in the complex plane

Simpler truncation:

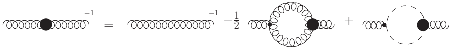


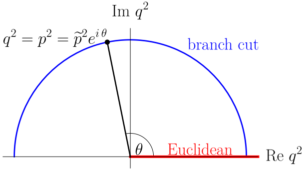


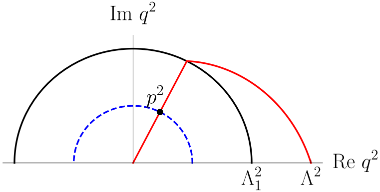
$\rightarrow$ Opening at $q^2 = p^2$.

Landau gauge propagators in the complex plane

Simpler truncation:





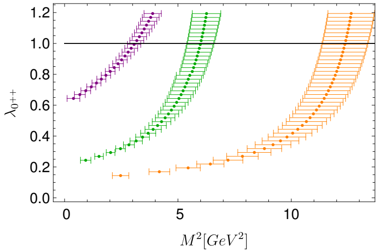


$\rightarrow$ Opening at $q^2 = p^2$.

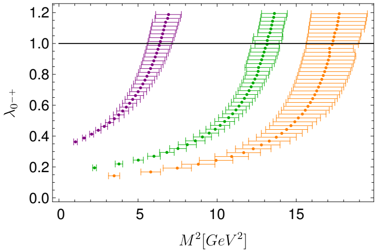
Appearance of branch cuts for complex momenta forbids integration directly to cutoff.

Extrapolation for glueball eigenvalue curves

0^{++}:



0^{-+}:



Several curves: ground state and excited states.