# Reaction of $T_{c c}$ states of $D^{*} D^{*}$ and $D_{s}^{*} D^{*}$ molecular nature 

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## Outline

## 1. Motivation

## 2. Formalism

## 3. Results

4. Summary

## 1. Motivation

Recent experimental observation of $T_{c c}$ state close to the $D^{*} D$ threshold and the width is very small ([LHCb Collaboration, PRL125(2020)242001

- this $T_{c c}$ state can be explained as a molecular state of $D^{*} D$
[Feijoo, Liang, Oset, PRD104 (2021) 114015]


FIG. 5. $\left|T_{D^{++} D^{0}, D^{++} D^{\circ}}\right|^{2}$ as a function of $\sqrt{s}$. Dashed vertical line, $D^{*+} D^{0}$ threshold. Continuous vertical line, $D^{* 0} D^{+}$ threshold.

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$D^{0} D^{0} \pi^{+}$mass distribution in the production of the $T_{c c}$ exotic state

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We perform a unitary coupledchannel study of the interaction of the $D^{++} D^{0}, D^{* 0} D^{+}$channels and find a state barely bound, very close to isospin $I=0$. We take the experimental mass as input and obtain the width of the state and the $D^{0} D^{0} \pi^{+}$mass distribution. When the mass of the $T_{c c}$ state quoted in the expenmental paper from raw data is used, the width obtained is of the order of the 80 keV , small compared to the value given in that work. Yet, when the mass obtained in an analysis of the data considering the experimental resolution is taken, the width obtained is about 43 keV and both the width and the $D^{\circ} D^{0} \pi^{+}$mass distribution are in remarkable agreement with the results obtained in that latter analysis.
both the width and the $D^{0} D^{0} \pi^{+}$mass distribution are in remarkable agreement with the $T_{c c}$ experiment $\Longrightarrow$ Encouraged by this $D^{*} D$ work

## [Feijoo, Liang, Oset, PRD104 (2021) 114015] <br> Theoretical framework of $D^{*} D$ system

1) Unitary coupled channel approach $\left(D^{*+} D^{0}, D^{* 0} D^{+}\right)$
2)Interaction obtained from exchange of vector mesons in a straight extrapolation of the local hidden gauge approach

Bando, Kugo, Yamawaki, Phys Rep 164,217;
Harada and Yamawaki, Phys Rep 381, 1;
Meissner, Phys. Rep. 161, 213;
Nagahiro, Roca, Hosaka, Oset, PRD79, 014015
3) successfully to the charm sector [Wu, Molina, Oset, Zou, PRL105 (2010) 232001]
4) The only parameter was a cutoff regulator in the Bethe-Salpeter equation

# Extend the above theoretical framework to $D^{*} D^{*}$ and $D_{s}^{*} D^{*}$ systems with $J^{P}=1^{+}$ 

[Dai, Molina, Oset, PRD105 (2022) 016029; PRD106 (2022) 099902(E)]
1)Heavy quark spin symmetry allows to relate the $D$ and $D^{*}$ sectors

Isgur and Wise, PLB 232 (1989)113;
Neubert, Phys. Rept. 245 (1994) 259;
Manohar and Wise, Heavy Quark Physics, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology, vol. 10
2) Before the recent experimental finding on the $D^{*} D$ system It was found that the $D^{*} D^{*}$ system in isospin $I=0$, and $J^{P}=1^{+}$and the $D_{s}^{*} D^{*}$ system in $I=\frac{1}{2}, J^{P}=1^{+}$had an attractive potential, strong enough to support a bound state
more details in [Molina, Branz and Oset, PRD82 (2010) 014010]
3) New experimental information from the $T_{c c}$ state

LHCb Collaboration, PRL125 (2020) 242001
LHCb Collaboration, Nature Communicatitions, 13 (2022) 3351
valuable information from the $T_{c c}$ state to fix regulator of the meson-meson loop function and the width can be obtained from pseudoscalar-vector decay channel

Feijoo, Liang, Oset, PRD104 (2021) $114015 \Longrightarrow$ to fix the cutoff

1) we will use the cutoff from above $T_{c c}$ work
2) the width can be obtained from pseudoscalar-vector decay channel (due to spin-parity conservation) which has a much larger phase space
[Dai, Molina, Oset, PRD105 (2022) 016029; PRD106 (2022) 099902(E)]

## 2. Formalism

## - Direct interaction

[Molina, Nicmorus, and Oset, PRD78(2008)114018;
Geng, and Oset, PRD79(2009)074009]

(a)

(b)

It contains a contact term (a) and the exchange of vector mesons (b) $\Longrightarrow$ producing bound states or resonances

Extrapolated to the charm sector, it predicted the pentaquark states with hidden charm and hidden charm and strangeness
[Wu, Molina, Oset, Zou, PRL105(2010)232001]; PRC84(2011)115202
which were found later by the LHCb collaboration [PRL115(2015)072001 122(2019)222001; Sci Bull 66(2021)1278

$$
\begin{aligned}
\mathcal{L}^{(c)} & =\frac{g^{2}}{2}\left\langle V_{\mu} V_{\nu} V^{\mu} V^{\nu}-V_{\nu} V_{\mu} V^{\mu} V^{\nu}\right\rangle \\
\mathcal{L}_{V V V} & =i g\left\langle V^{\mu} \partial_{\nu} V_{\mu}-\partial_{\nu} V_{\mu} V^{\mu}\right\rangle
\end{aligned}
$$

$V_{\mu}$ the $q \bar{q}$ matrix written in terms of vector mesons

$$
V_{\mu}=\left(\begin{array}{cccc}
\frac{\omega}{\sqrt{2}}+\frac{\rho^{0}}{\sqrt{2}} & \rho^{+} & K^{*+} & \bar{D}^{* 0} \\
\rho^{-} & \frac{\omega}{\sqrt{2}}-\frac{\rho^{0}}{\sqrt{2}} & K^{* 0} & D^{*-} \\
K^{*-} & \bar{K}^{* 0} & \phi & D_{s}^{*-} \\
D^{* 0} & D^{*+} & D_{s}^{*+} & J / \psi
\end{array}\right)_{\mu}
$$

with $g=\frac{M_{V}}{2 f}\left(M_{V}=800 \mathrm{MeV}, f=93 \mathrm{MeV}\right)$

$$
\begin{aligned}
V_{D^{*} D^{*} \rightarrow D^{*} D^{*}} & =\frac{g^{2}}{4}\left(\frac{2}{m_{J / \psi}^{2}}+\frac{1}{m_{\omega}^{2}}-\frac{3}{m_{\rho}^{2}}\right) \\
& \times\left\{\left(p_{1}+p_{4}\right) \cdot\left(p_{2}+p_{3}\right)+\left(p_{1}+p_{3}\right) \cdot\left(p_{2}+p_{4}\right)\right\} \\
V_{D_{s}^{*} D^{*} \rightarrow D_{s}^{*} D^{*}} & =-\frac{g^{2}\left(p_{1}+p_{4}\right) \cdot\left(p_{2}+p_{3}\right)}{m_{K^{*}}^{2}}+\frac{g^{2}\left(p_{1}+p_{3}\right) \cdot\left(p_{2}+p_{4}\right)}{m_{J / \psi^{2}}}
\end{aligned}
$$

Note that $\left(p_{1}+p_{3}\right) \cdot\left(p_{2}+p_{4}\right)$ projected in $s$-wave can be written as

$$
\frac{1}{2}\left\{3 s-\left(M_{1}^{2}+M_{2}^{2}+M_{3}^{2}+M_{4}^{2}\right)-\frac{1}{s}\left(M_{1}^{2}-M_{2}^{2}\right)\left(M_{3}^{2}-M_{4}^{2}\right)\right\}
$$

and the T-matrix was obtained using the Bethe-Salpeter equation

$$
T=[1-V G]^{-1} V
$$

[more details in Molina, Branz, and Oset, PRD82(2010)014010]

- Vector-pseudoscalar decay channels

The $V V$ states with $1^{+}$cannot decay to $P P$ if we want to conserve spin and parity.

Thus we consider instead the decay into vector-pseudoscalar (VP) channel which will give a width to the bound states that we find.

## 1) $D^{*} D^{*} \rightarrow D^{*} D(I=0)$ decay

1) the isospin doublets $\left(D^{+},-D^{0}\right)$ and $\left(D^{*+},-D^{* 0}\right)$,

$$
\left|D^{*} D^{*}, I=0\right\rangle=-\frac{1}{\sqrt{2}}\left|D^{*+} D^{* 0}-D^{* 0} D^{*+}\right\rangle
$$

2) This system can decay into $D^{*+} D^{0}$ or $D^{* 0} D^{+}$
3) consider these decay box diagrams $\Longleftarrow$ imaginary part
4) same structure and only the isospin coefficients are different the sum of the 32 diagrams $(8 \times 4=32)$, the total weight of each kind of diagrams is $\frac{1}{4}(1+2+2+4+4+2+2+1)=\frac{18}{4}=\frac{9}{2}$


## Two new vertices for the box diagrams

a) the ordinary $V P P$ coupling
[in the local hidden gauge approach]

$$
\mathcal{L}_{V P P}=-i g\left\langle\left[P, \partial_{\mu} P\right] V^{\mu}\right\rangle
$$

b) the anomalous $V V P$ coupling
[Bramon,Grau, Pancheri, PLB344 (1995) 240;

Oset, Pelaez, Roca, PRD67 (2003) 073013]

$$
\begin{aligned}
\mathcal{L}_{V V P} & =\frac{G^{\prime}}{\sqrt{2}} \epsilon^{\mu \nu \alpha \beta}\left\langle\partial_{\mu} V_{\nu} \partial_{\alpha} V_{\beta} P\right\rangle \\
G^{\prime} & =\frac{3 g^{\prime 2}}{4 \pi^{2} f} ; \quad g^{\prime}=-\frac{G_{V} m_{\rho}}{\sqrt{2} f^{2}} \\
G_{V} & =55 \mathrm{MeV} ; \quad f=93 \mathrm{MeV}
\end{aligned}
$$

we evaluate the amplitudes at the $D^{*} D^{*}$ threshhold

$$
\text { 1) } \begin{aligned}
& D^{* 0} \pi^{0} \rightarrow D^{0}, \\
& -i t=-2 i \operatorname{g} \boldsymbol{q} \epsilon\left(D^{* 0}\right) \frac{1}{\sqrt{2}}
\end{aligned}
$$

$$
\text { 2) } D^{0} \rightarrow \pi^{0} D^{* 0}
$$

$$
-i t=-2 i g \boldsymbol{q} \boldsymbol{\epsilon}\left(D^{* 0}\right) \frac{1}{\sqrt{2}}
$$

$$
\text { 3) } D^{*+} \rightarrow \pi^{0} D^{*+}
$$

$$
-i t=
$$

$$
-i \frac{G^{\prime}}{\sqrt{2}} \epsilon^{i j k} E\left(D_{\mathrm{ext}}^{*+}\right) \epsilon_{i}\left(D_{\mathrm{ext}}^{*+}\right) q_{j} \epsilon_{k}(\mathrm{int}) \frac{1}{\sqrt{2}}
$$

$$
\text { 4) } D^{*+} \pi^{0} \rightarrow D^{*+}
$$

$$
-i t=
$$

$$
-i \frac{G^{\prime}}{\sqrt{2}} \epsilon^{i j k} E\left(D_{\mathrm{ext}}^{*+}\right) \epsilon_{i}\left(D_{\mathrm{ext}}^{*+}\right) q_{j} \epsilon_{k}(\text { int }) \frac{1}{\sqrt{2}}
$$

$E\left(D^{*}\right)$ stands for the energy of the $D^{*}$. The indices ext or int stand for the external or internal vectors of the diagrams.

## The product of all four vertices

$(\sqrt{2} g)^{2}\left(\frac{G^{\prime}}{2}\right)^{2} E(1) E(3)\left\{\epsilon_{i}(1) \epsilon_{l}(2) \epsilon_{i}(3) \epsilon_{m}(4) \boldsymbol{q}^{2} q_{l} q_{m}-\epsilon_{j}(1) \epsilon_{l}(2) \epsilon_{i}(3) \epsilon_{m}(4) q_{i} q_{j} q_{l} q_{m}\right\}$
where the indices $1,2,3,4$ refer to the particles on the order of decay box diagrams. at threshold all the propagators in the loop depend only on $\boldsymbol{q}^{2}$

$$
\begin{aligned}
\int d^{3} q f\left(\boldsymbol{q}^{2}\right) q_{l} q_{m} & =\int d^{3} q f\left(\boldsymbol{q}^{2}\right) \frac{1}{3} \boldsymbol{q}^{2} \delta_{l m} \\
\int d^{3} q f\left(\boldsymbol{q}^{2}\right) q_{i} q_{j} q_{l} q_{m} & =\int d^{3} q f\left(\boldsymbol{q}^{2}\right) \frac{1}{15} \boldsymbol{q}^{4}\left(\delta_{i j} \delta_{l m}+\delta_{i l} \delta_{j m}+\delta_{i m} \delta_{j l}\right)
\end{aligned}
$$

and using the projectors into the spin states of $J=1,2,3, \mathcal{P}^{(0)}, \mathcal{P}^{(1)}, \mathcal{P}^{(2)}$ from [PLB811(2020)135870; PRD78(2008)114018]

$$
\begin{aligned}
\epsilon_{j} \epsilon_{j} \epsilon_{i} \epsilon_{i} & =3 \mathcal{P}^{(0)} \\
\epsilon_{j} \epsilon_{l} \epsilon_{j} \epsilon_{l} & =\mathcal{P}^{(0)}+\mathcal{P}^{(1)}+\mathcal{P}^{(2)} \\
\epsilon_{j} \epsilon_{i} \epsilon_{i} \epsilon_{j} & =\mathcal{P}^{(0)}-\mathcal{P}^{(1)}+\mathcal{P}^{(2)}
\end{aligned}
$$

Altogether for the $J^{P}=1^{+}$state the contribution for the four diagrams, keeping the positive energy part of the propagators of the heavy particles

$$
\begin{aligned}
-i t & =4 \frac{9}{2} \frac{1}{3} \int \frac{d^{4} q}{(2 \pi)^{4}} \frac{1}{2 E_{D^{*}}(\boldsymbol{q})} \frac{i}{p_{1}^{0}-q^{0}-E_{D^{*}}(\boldsymbol{q})+i \epsilon} \frac{1}{2 E_{D}(\boldsymbol{q})} \\
& \times \frac{i}{p_{2}^{0}+q^{0}-E_{D}(\boldsymbol{q})+i \epsilon} \frac{i}{q^{2}-m_{\pi}^{2}+i \epsilon} \frac{i}{\left(p_{2}-p_{4}+q\right)^{2}-m_{\pi}^{2}+i \epsilon} \boldsymbol{q}^{4}
\end{aligned}
$$

by performing the $q^{0}$ analytically and then use $\operatorname{Im} \frac{1}{x+i \epsilon}=-i \pi \delta(x)$,

$$
\operatorname{Im} V_{\mathrm{box}}=-\frac{6}{8 \pi} \frac{q^{5}}{\sqrt{s}} E_{D^{*}}^{2}(\sqrt{2} g)^{2}\left(\frac{G^{\prime}}{2}\right)^{2}\left[\frac{1}{\left(p_{2}^{0}-E_{D}(\boldsymbol{q})\right)^{2}-\boldsymbol{q}^{2}-m_{\pi}^{2}}\right]^{2} F^{4}(q) F_{H Q}
$$

where

$$
q=\frac{\lambda^{1 / 2}\left(s, m_{D^{*}}^{2}, m_{D}^{2}\right)}{2 \sqrt{s}} ; \quad E_{D^{*}}=\frac{\sqrt{s}}{2}
$$

a)form factor $F(q)$ used in [PRD82(2010)014010; PLB811(2020)135870] b) $F_{H Q}$ to correct the $V P P$ vertex for heavy particles [PRD89(2014)054023]

$$
F(q)=e^{\left(\left(q^{0}\right)^{2}-q^{2}\right) / \Lambda^{2}} ; \quad q^{0}=p_{1}^{0}-E_{D^{*}}(\boldsymbol{q}) ; \quad F_{H Q}=\left(\frac{m_{D^{*}}}{m_{K^{*}}}\right)^{2}
$$

## 2) $D_{s}^{*} D^{*} \rightarrow D_{s}^{*} D+D_{s} D^{*}$

$$
\begin{gathered}
\operatorname{Im} V_{\mathrm{box}}=-\frac{1}{3} \frac{1}{8 \pi} \frac{1}{\sqrt{s}}(2 g)^{2}\left(\frac{G^{\prime}}{\sqrt{2}}\right)^{2}\left(E_{1} E_{3}+E_{2} E_{4}\right) \\
\times q^{5}\left(\frac{1}{\left(p_{2}^{0}-E_{D_{s}}(\boldsymbol{q})\right)^{2}-\boldsymbol{q}^{2}-m_{K}^{2}}\right)^{2} F^{4}(q) F_{H Q} \\
q^{0}=p_{2}^{0}-E_{D_{s}}(\boldsymbol{q}) ; \quad q=\frac{\lambda^{1 / 2}\left(s, m_{D^{*}}^{2}, m_{D_{s}}^{2}\right)}{2 \sqrt{s}} ; \quad p_{2}^{0}=\frac{s+m_{D^{*}}^{2}-m_{D_{s}^{*}}^{2}}{2 \sqrt{s}}
\end{gathered}
$$

## 3. Results

For the two cases with $J^{P}=1^{+}$

$$
\begin{aligned}
& D^{*} D^{*}, I=0 \\
& D_{s}^{*} D^{*}, I=\frac{1}{2}
\end{aligned}
$$

- we solve the Bethe-Salpeter equation with

$$
V \rightarrow V+i \operatorname{Im} V_{\mathrm{box}}
$$

and obtain the T-matrix.

- By plotting $|T|^{2}$ we find the mass of the state and its width


## Squared amplitude of $D^{*} D^{*} \rightarrow D^{*} D^{*}$

- taking $q_{\text {max }}$ to regularize the $G$ function [ $T_{c c}$ state in PRD104,114015]
- form factor $\Lambda$ to regularize the box diagram [studying the $D^{*} \bar{K}^{*}$ molecule $X_{0}(2866)$ decaying to $D \bar{K}$ in Molina, Branz, Oset, PRD82, 014010; Molina and E. Oset, PLB 811, 135870]


heavy quark symmetry phenomenological information
a) changing the value of $\Lambda$
results do not change much a bound state around 3960 MeV
b) changing the value of $q_{\text {max }}$ there is always a bound state the width becomes smaller as we get closer to the threshold

PRD103(2021)116019; PR130(1963)776;
PLB 586 (2004)53; PRD81(2010)014029;
PRD77(2008) 034001

## Squared amplitude of $D_{s}^{*} D^{*} \rightarrow D_{s}^{*} D^{*}$



a) changing the value of $\Lambda$
we get a bound state and the width does not change much with the value of $\Lambda$
b) changing the value of $q_{\text {max }}$
with three values of $q_{\max }$, we see a similar trend as $D^{*} D^{*}$, but the bindings are smaller as a consequence of the smaller strength of the potential
[Dai, Molina \& Oset, PRD105 (2022) 016029; PRD106 (2022) 099902(E)]

## The predictions for $D^{*} D^{*}$ system



- at threshold $D^{*} D, 4017.1 \mathrm{MeV}$
- finding bound states in the $J^{P}=1^{+}$channel
with bindings of the order of the MeV
Similar widths for $D^{*} D^{*}$ system

|  | $q_{\max }=450 \mathrm{MeV}$ | $q_{\max }=420 \mathrm{MeV}$ |
| :---: | :---: | :---: |
| $M_{D^{*} D^{*}}$ | 4010.6 MeV | 4013.0 MeV |
| $B_{D^{*} D^{*}}$ | 6.5 MeV | 4.1 MeV |
| $\Gamma_{D^{*} D^{*}}$ | 29 MeV | 20 MeV |

The width of the $D^{*} D^{*}$ system is much larger than the one of the $T_{c c}$ state (40-50 keV , due to the very little phase space of decay into $D^{*}$ into $D \pi$ ), we have the decay channel $D^{*} D$ and there is a much larger phase space for the decay.

## The predictions for $D_{s}^{*} D^{*}$ system



- at threshold $D_{s}^{*} D^{*}, 4122.46 \mathrm{MeV}$
- no bound states
instead we find pronounced cusps at the $D_{s}^{*} D^{*}$ threshold.
$\Uparrow$ This is a consequence of the weaker potential $V$ compared to $D^{*} D^{*}$

|  | $q_{\max }=450 \mathrm{MeV}$ | $q_{\max }=420 \mathrm{MeV}$ |
| :---: | :---: | :---: |
| $M_{D_{s}^{*} D^{*}}$ | 4122.46 MeV (cusp) | 4122.46 MeV (cusp) |
| $\Gamma_{D_{s}^{*} D^{*}}$ | 4 MeV | 2 MeV |

[Dai, Molina \& Oset, PRD105 (2022) 016029; PRD106 (2022) 099902(E)]

## 4. Summary

Encouraged by the experiment of the $T_{c c}$ state close to the $D^{*} D$ threshold, and this can be explained as a molecular state of $D^{*} D$.

- An extension to $D^{*} D^{*}$ with $I=0$ and $D_{s}^{*} D^{*}$ with $I=\frac{1}{2}$ systems

Exp. binding $T_{c c}$ state $\Longrightarrow$ fix the cutoff
decay into the $D^{*} D$ system $\Longrightarrow$ get the width
$\Longrightarrow$ We find that

1) bound states of $D^{*} D^{*}$ system with binding of the MeV order, and similar widths
the width of the $D^{*} D^{*}$ system is much larger than the one of the $T_{c c}$ state due to a much larger phase space
2)while the $D_{s}^{*} D^{*}$ system develops a strong cusp around threshold the width of the $D_{s}^{*} D^{*}$ is much smaller than the $D^{*} D^{*}$ state due to the different factors of $\operatorname{Im} V_{\text {box }}$ from $\pi$ exchange and kaon exchange, respectively

## Thank you

