Hybrid decays into Quarkonia in Born-Oppenheimer EFT (BOEFT)

Hadron Spectroscopy and Structure (HADRON 2023)
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Phys. Rev. D 107, 054034 (2023)

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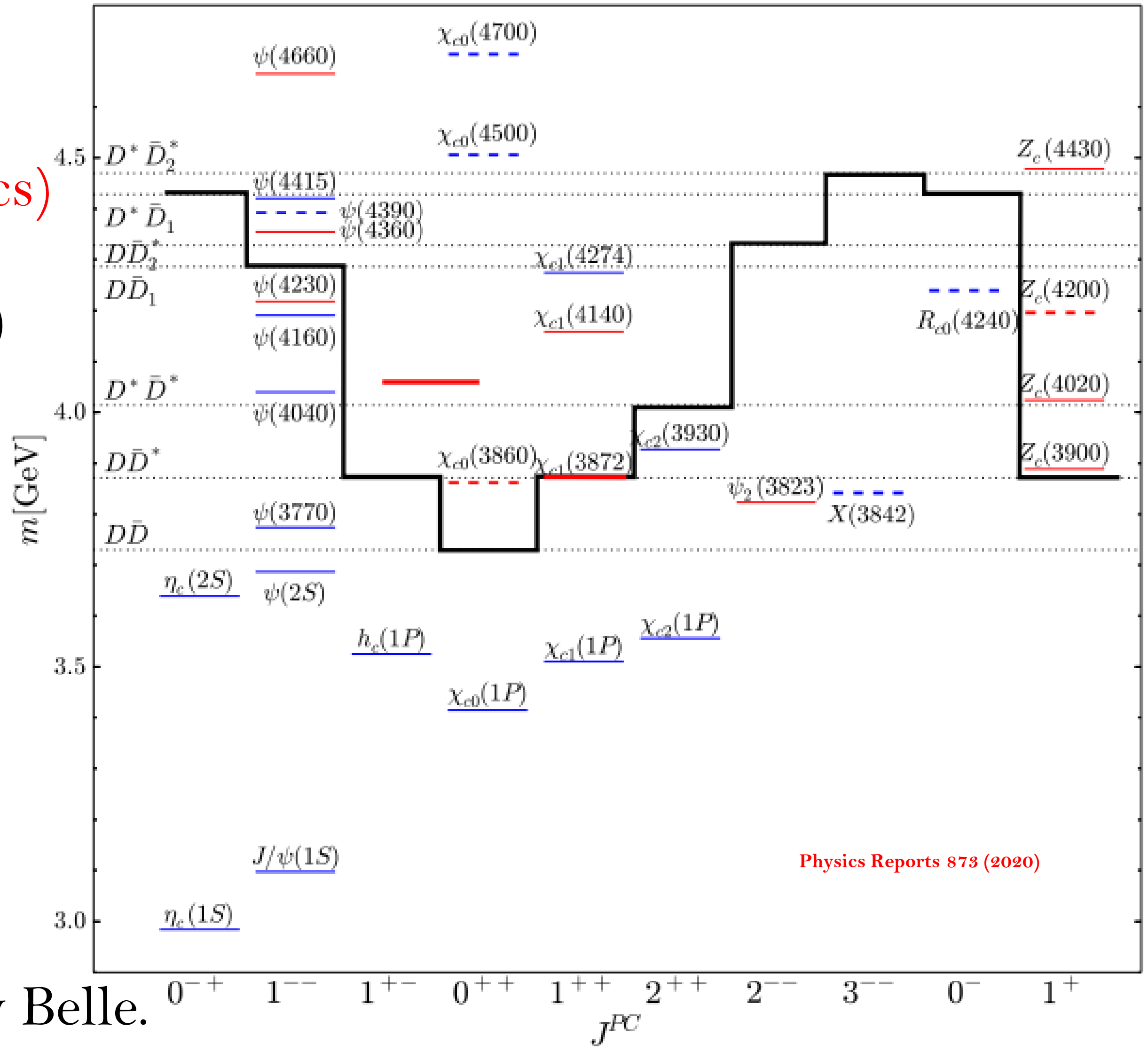
DFG Deutsche Forschungsgemeinschaft

Exotic Hadron

- Quark Model: Mesons or Baryons
- QCD spectrum: more complex structures (Exotics)
- Exotic states: XYZ mesons (heavy-quark sector)
  - ✓ States that don't fit traditional $Q\bar{Q}$ spectrum.
  - ✓ Exotic quantum numbers:
    - $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$ etc. are exotic
    - Charged Z_c and Z_b states: minimal 4-quark state:

      $Z_c(4430)^{\pm}$ $Z_b(10650)^{\pm}$ **Tetraquarks**

For review see Brambilla et al. Phys. Reports. 873 (2020)



- X(3872): First exotic state discovered in 2003 by Belle.

  Phys. Rev. Lett. 91, 262001 (2003)

- Dozens of XYZ mesons discovered since 2003.

PDG 2022

Exotic Hadron

- Multiple Models for Exotics:

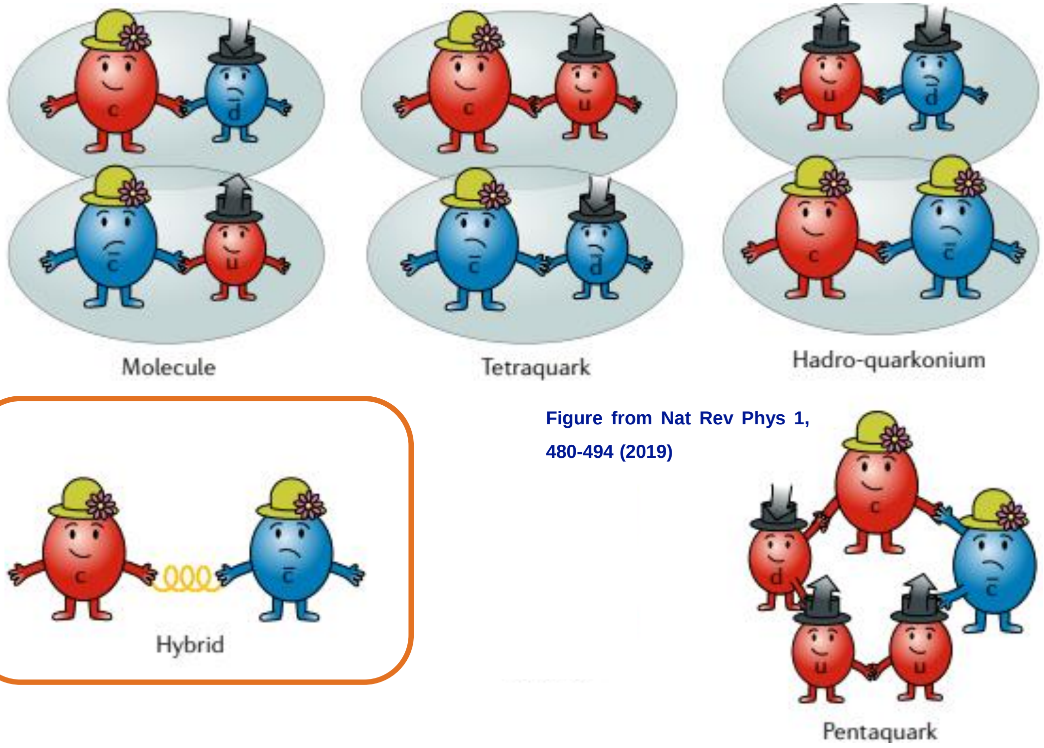


Figure from Nat Rev Phys 1, 480-494 (2019)

Major **challenge** to understanding of **QCD spectrum!!!**

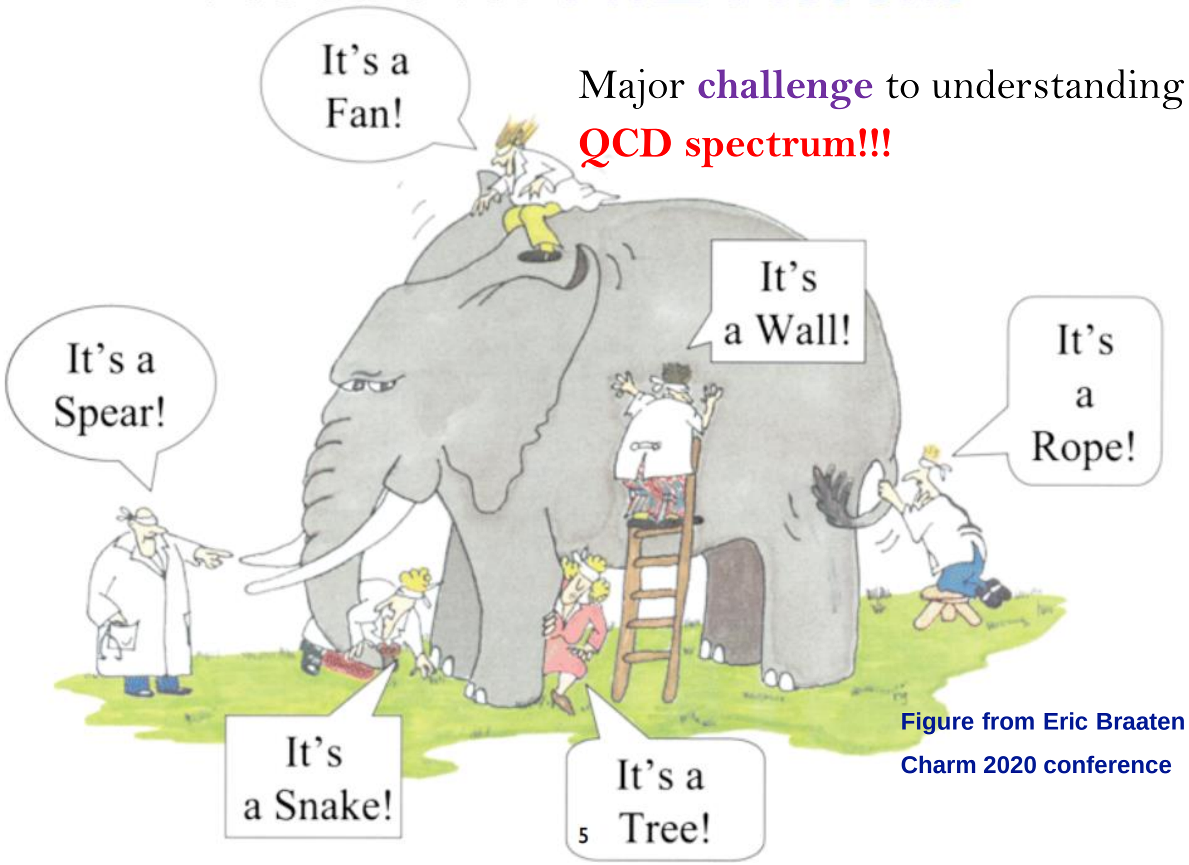


Figure from Eric Braaten talk: Charm 2020 conference

- Individual success in describing some XYZ hadrons. No success in revealing general pattern.

Hybrids ($Q\bar{Q}g$): Extension of quarkonium. Isospin scalar exotic state. Focus of this work.

Use EFT + lattice for describing hybrid

Exotic: Hybrid candidates

State (PDG)	State (Former)	M (MeV)	Γ (MeV)	J^{PC}	Decay modes
$\chi_{c1}(4140)$	$X(4140)$	4146.5 ± 3.0	19^{+7}_{-5}	1^{++}	$\phi J/\psi$
$X(4160)$		4153^{+23}_{-21}	136^{+60}_{-35}	$?^{??}$	$\phi J/\psi$, $D^* \bar{D}^*$
$\psi(4230)$	$Y(4230)$ $Y(4260)$	4222.7 ± 2.6	49 ± 8	1^{--}	$\pi^+\pi^- J/\psi$, $\omega \chi_{c0}(1P)$, $\pi^+\pi^- h_c(1P)$
$\chi_{c1}(4274)$	$Y(4274)$	4286^{+8}_{-9}	51 ± 7	1^{++}	$\phi J/\psi$
$X(4350)$		$4350.6^{+4.7}_{-5.1}$	13^{+18}_{-10}	$(0/2)^{++}$	$\phi J/\psi$
$\psi(4360)$	$Y(4360)$ $Y(4320)$	4372 ± 9	115 ± 13	1^{--}	$\pi^+\pi^- J/\psi$, $\pi^+\pi^- \psi(2S)$
$\psi(4390)$[a]	$Y(4390)$	4390 ± 6	139^{+16}_{-20}	1^{--}	$\eta J/\psi$, $\pi^+\pi^- h_c(1P)$
$\chi_{c0}(4500)$	$X(4500)$	4474 ± 4	77^{+12}_{-10}	0^{++}	$\phi J/\psi$
$Y(4500)$[b]		4484.7 ± 27.5	111 ± 34	1^{--}	
$X(4630)$[c]		4626^{+24}_{-111}	174^{+137}_{-78}	$?^{?+}$	$\phi J/\psi$
$\psi(4660)$	$Y(4660)$ $X(4660)$	4630 ± 6	72^{+14}_{-12}	1^{--}	$\pi^+\pi^- \psi(2S)$, $\Lambda_c^+ \bar{\Lambda}_c^-$, $D_s^+ D_{s1}(2536)$
$\chi_{c1}(4685)$[d]		4684^{+15}_{-17}	126^{+40}_{-44}	1^{++}	$\phi J/\psi$
$\chi_{c0}(4700)$	$X(4700)$	4694^{+17}_{-5}	87^{+18}_{-10}	0^{++}	$\phi J/\psi$
$Y(4710)$[e]		4704 ± 87	183 ± 146	1^{--}	
$\Upsilon(10753)$		$10752.7^{+5.9}_{-6.0}$	36^{+18}_{-12}	1^{--}	$\pi\pi\Upsilon(1S, 2S, 3S)$
$\Upsilon(10860)$	$\Upsilon(5S)$	$10885.2^{+2.6}_{-1.6}$	37 ± 4	1^{--}	$\pi\pi\Upsilon(1S, 2S, 3S)$, $\pi^+\pi^- h_b(1P, 2P)$, $\eta\Upsilon(1S, 2S)$, $\pi^+\pi^-\Upsilon(1D)$ (see PDG listings)
$\Upsilon(11020)$	$\Upsilon(6S)$	11000 ± 4	24^{+8}_{-6}	1^{--}	$\pi\pi\Upsilon(1S, 2S, 3S)$, $\pi^+\pi^- h_b(1P, 2P)$, (see PDG listings)

PDG 2022

- ✓ **Candidates based on mass and quantum numbers.**
- ✓ **Isoscalar neutral meson states** above the open-flavor thresholds
- ✓ Y(4500): New state recently seen by BESIII experiment.

  M. Ablikim et al, Chin.Phys.C,46,111002(2022).
- ✓ X(4630): New state recently seen by LHCb experiment.
- ✓ $\chi_{c1}(4685)$: New state recently seen by LHCb experiment.

  R. Aaji et al, Phys. Rev. Lett. 127, 082001 (2021)
- ✓ Y(4710): New state recently seen by BESIII experiment.

  M. Ablikim et al, arXiv: 2211.08561.

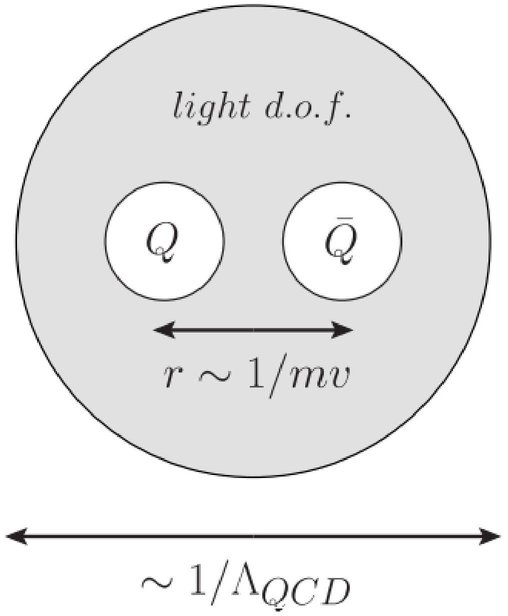
Brambilla, Lai, AM, Vairo arXiv:2212.09187

Hybrids: BOEFT

- Hybrids ($Q\bar{Q}g$): Color singlet combination of color octet $Q\bar{Q}$ + gluonic excitations.
- Hierarchy of scales in hybrids:

$$m \gg mv \gtrsim \Lambda_{QCD} \gg mv^2$$

- ❖ Mass of heavy quark: m
- ❖ Energy scale for light d.o.f: Λ_{QCD}
- ❖ Relative separation between heavy quarks: $r \sim 1/mv$
- ❖ Hybrids are extended objects: $< r > \gtrsim 0.7\ fm$
- ❖ Heavy Quark K.E scale: mv^2



R. Oncala, J. Soto, Phys. Rev. D96 (2017) Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015) Brambilla, Krein, Castellà , Vairo Phys. Rev. D. 97, (2018)

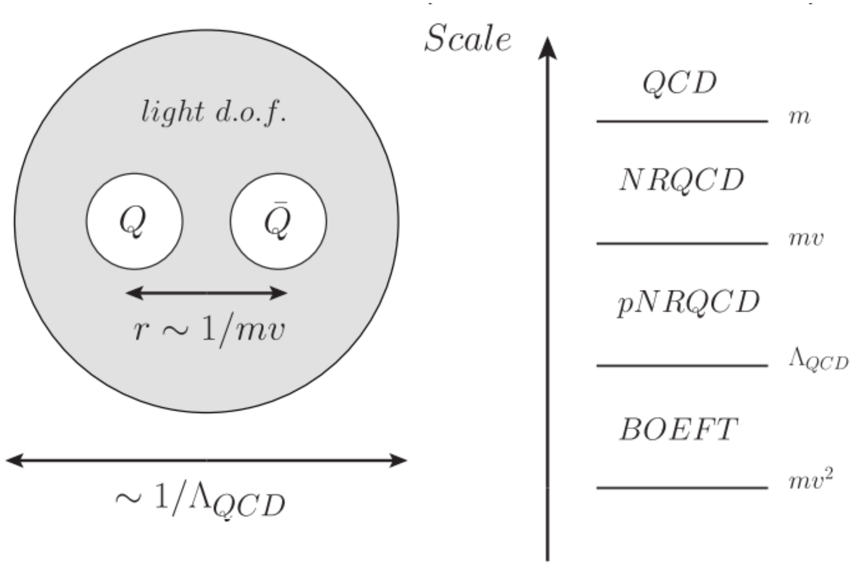
Hybrids: BOEFT

- Hierarchy of scales in hybrids:

$$m \gg mv \gtrsim \Lambda_{QCD} \gg mv^2$$

- Time-scale for dynamics of $Q\bar{Q}$: $\sim \frac{1}{mv^2} \gg \frac{1}{\Lambda_{QCD}}$

Born-Oppenheimer Approximation



Braaten, Langmack, Smith Phys. Rev. D. 90, 014044 (2014)

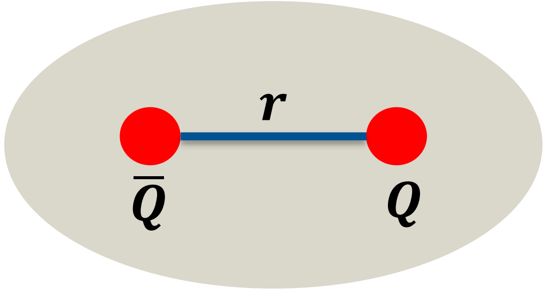
- **Born-Oppenheimer EFT (BOEFT):** EFT for hybrids. **Describes physics at the scale mv^2.**

$$\text{QCD} \to \text{NRQCD} \to \text{pNRQCD/BOEFT}$$

- BOEFT: Extension of pNRQCD for hybrid states.

R. Oncala, J. Soto, Phys. Rev. D96 (2017) Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015) Brambilla, Krein, Castellà , Vairo Phys. Rev. D. 97, (2018)

BOEFT: Quantum #'s



- **Static limit (m→ ∞):** heavy quarks are fixed in position.

- **BOEFT potentials ($V_\Gamma(r)$):** Potential between 2 heavy quarks given by energy of LDF (light quarks, gluons) known as **static energies**

- $V_\Gamma(r)$: **Γ** labelled by cylindrical symmetry ($D_{\infty h}$) representation (diatomic molecules):

  ✓ Absolute value of component of angular momentum of light d.o.f

  $|\boldsymbol{r}\cdot \boldsymbol{K}_{\text{light}}| \equiv \Lambda = 0, 1, 2, \ldots\ldots..(\text{or } \Sigma, \Pi, \Delta, \Phi, \ldots..)$

  ✓ Product of charge conjugation and parity ($\boldsymbol{CP}$):

  $\eta = +1\ (\mathbf{g}), -1\ (\boldsymbol{u})$

  ✓ $\boldsymbol{\sigma}$: Eigenvalue of reflection about a plane containing static sources.

  $\sigma = P\,(-1)^{K_{\text{light}}} = \pm 1$

$$\Gamma \equiv \Lambda_\eta^\sigma$$

Braaten, Langmack, Smith
Phys. Rev. D. 90, 014044 (2014)

- **r→ 0**: dynamics independent of relative coordinate **r.** Characterized by **spherical symmetry**: Labelled by gluon quantum #'s $\kappa = K^{PC}$.

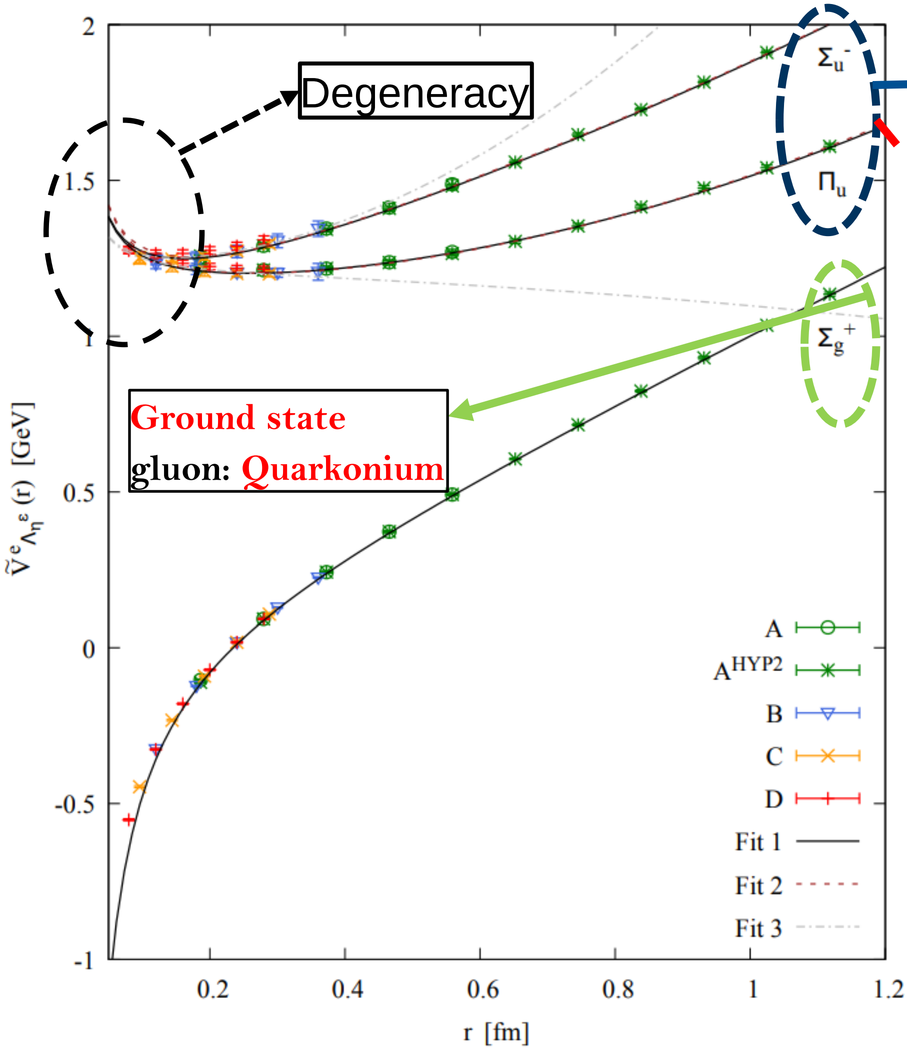
Brambilla, Pineda , Soto, and Vairo
Nucl. Phys. B566 (2000)

Berwein, Brambilla, Castellà , Vairo
Phys. Rev. D. 92, (2015), 114019

$Q\bar{Q}$ pair: Static Energies

- **Static limit ($m \to \infty$):** heavy quarks are fixed in position. Interquark potential given by LDF energy.

Schlosser and Wagner Phys. Rev. D. 105, (2022)



First excited state gluon configuration: Hybrid

Gluonic operators characterizing hybrids

Λ_η^σ	K^{PC}	O_n
Σ_u^-	1^{+-}	$\hat{\boldsymbol{r}}\cdot\boldsymbol{B},\ \hat{\boldsymbol{r}}\cdot(\boldsymbol{D}\times\boldsymbol{E})$
Π_u	1^{+-}	$\hat{\boldsymbol{r}}\times\boldsymbol{B},\ \hat{\boldsymbol{r}}\times(\boldsymbol{D}\times\boldsymbol{E})$
$\Sigma_g^{+\prime}$	1^{--}	$\hat{\boldsymbol{r}}\cdot\boldsymbol{E},\ \hat{\boldsymbol{r}}\cdot(\boldsymbol{D}\times\boldsymbol{B})$
Π_g	1^{--}	$\hat{\boldsymbol{r}}\times\boldsymbol{E},\ \hat{\boldsymbol{r}}\times(\boldsymbol{D}\times\boldsymbol{B})$
Σ_g^-	2^{--}	$(\hat{\boldsymbol{r}}\cdot\boldsymbol{D})(\hat{\boldsymbol{r}}\cdot\boldsymbol{B})$
Π_g'	2^{--}	$\hat{\boldsymbol{r}}\times((\hat{\boldsymbol{r}}\cdot\boldsymbol{D})\boldsymbol{B}+\boldsymbol{D}(\hat{\boldsymbol{r}}\cdot\boldsymbol{B}))$
Δ_g	2^{--}	$(\hat{\boldsymbol{r}}\times\boldsymbol{D})^i(\hat{\boldsymbol{r}}\times\boldsymbol{B})^j+(\hat{\boldsymbol{r}}\times\boldsymbol{D})^j(\hat{\boldsymbol{r}}\times\boldsymbol{B})^i$
Σ_u^+	2^{+-}	$(\hat{\boldsymbol{r}}\cdot\boldsymbol{D})(\hat{\boldsymbol{r}}\cdot\boldsymbol{E})$
Π_u'	2^{+-}	$\hat{\boldsymbol{r}}\times((\hat{\boldsymbol{r}}\cdot\boldsymbol{D})\boldsymbol{E}+\boldsymbol{D}(\hat{\boldsymbol{r}}\cdot\boldsymbol{E}))$
Δ_u	2^{+-}	$(\hat{\boldsymbol{r}}\times\boldsymbol{D})^i(\hat{\boldsymbol{r}}\times\boldsymbol{E})^j+(\hat{\boldsymbol{r}}\times\boldsymbol{D})^j(\hat{\boldsymbol{r}}\times\boldsymbol{E})^i$

Foster and Micheal (UKQCD collaboration), Phys. Rev. D 59, 094509 (1999)

Brambilla, Pineda, Soto and Vairo, Rev. Mod. Phys 77, (2005)

Focus on these two for low lying hybrids

BOEFT

Brambilla, Lai, AM, Vairo arXiv:2212.09187

Brambilla, Krein, Castellà , Vairo Phys. Rev. D. 97, (2018)

- BOEFT Lagrangian:

$$L_{\text{BOEFT}} = L_{\Psi} + L_{\Psi_{\kappa\lambda}} + L_{\text{mixing}},$$

Quarkonium:

$$L_{\Psi} = \int d^3\mathbf{R} \int d^3\mathbf{r}\, \text{Tr}\left[\Psi^\dagger(\mathbf{r},\,\mathbf{R},\,t)\left(i\partial_t + \frac{\nabla_r^2}{m_Q} - V_\Psi(r)\right)\Psi(\mathbf{r},\,\mathbf{R},\,t)\right]$$

Trace over spin indices.

Hybrid:

$$L_{\Psi_{\kappa\lambda}} = \int d^3\mathbf{R} \int d^3\mathbf{r} \sum_{\kappa\lambda\lambda'} \text{Tr}\left\{\Psi^\dagger_{\kappa\lambda}(\mathbf{r},\,\mathbf{R},\,t)\left[i\partial_t - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda}\frac{\nabla_r^2}{m_Q}P^i_{\kappa\lambda'}\right]\Psi_{\kappa\lambda'}(\mathbf{r},\,\mathbf{R},\,t)\right\}$$

Hybrid-Quarkonium mixing:

$$L_{\text{mixing}} = -\int d^3\mathbf{R} \int d^3\mathbf{r} \sum_{\kappa\lambda} \text{Tr}\left[\Psi^\dagger\, V^{\text{mix}}_{\kappa\lambda}\, \Psi_{\kappa\lambda} + \text{h.c.}\right]$$

r: relative coordinate

R: COM coordinate

- Hybrid-Quarkonium mixing: Hybrid states in the same energy range as quarkonium can mix (same quantum #'s). O (1/m) term in BOEFT.
- No lattice calculations on mixing potential. Current work, ignore mixing, $V^{\text{mix}}_{\kappa\lambda} = 0$
- More details on mixing, see Oncala & Soto, PRD (2017).

BOEFT

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Hybrid: $$L_{\Psi_{\kappa\lambda}} = \int d^3\mathbf{R} \int d^3\mathbf{r} \sum_{\kappa\lambda\lambda'} \mathrm{Tr}\left\{\Psi^{\dagger}_{\kappa\lambda}(\mathbf{r},\mathbf{R},t)\left[i\partial_t - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda}\frac{\nabla_r^2}{m_Q}P^{i}_{\kappa\lambda'}\right]\Psi_{\kappa\lambda'}(\mathbf{r},\mathbf{R},t)\right\}$$

Hybrid-Quarkonium mixing: $$L_{\mathrm{mixing}} = -\int d^3\mathbf{R} \int d^3\mathbf{r} \sum_{\kappa\lambda} \mathrm{Tr}\left[\Psi^{\dagger} V^{\mathrm{mix}}_{\kappa\lambda}\Psi_{\kappa\lambda} + \mathrm{h.c.}\right]$$

$P^i_{\kappa\lambda}$: Projection operators of light d.o.f along heavy quark-antiquark axis. For quarkonium it is Identity.

$$\kappa = 1^{+-}$$
$$\lambda = 0, \pm 1$$

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- More details on mixing, see Oncala & Soto, PRD 96 (2017).

BOEFT

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Trace over spin indices.

Hybrid:

$$L_{\Psi_{\kappa\lambda}} = \int d^3\mathbf{R} \int d^3\mathbf{r} \sum_{\kappa\lambda\lambda'} \mathrm{Tr}\left\{\Psi^\dagger_{\kappa\lambda}(\mathbf{r},\,\mathbf{R},\,t)\left[i\partial_t - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda}\frac{\nabla_r^2}{m_Q}P^i_{\kappa\lambda'}\right]\Psi_{\kappa\lambda'}(\mathbf{r},\,\mathbf{R},\,t)\right\}$$

r: relative coordinate

R: COM coordinate

Hybrid potential: $V_{\kappa\lambda\lambda'}(r) \equiv P^{i\dagger}_{\kappa\lambda} V^{ij}_{\kappa}(r) P^{j}_{\kappa\lambda'} = V^{(0)}_{\kappa\lambda}(r)\delta_{\lambda\lambda'} + \frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m_Q} + \ldots$

Static potential

Incudes spin-dependent potentials

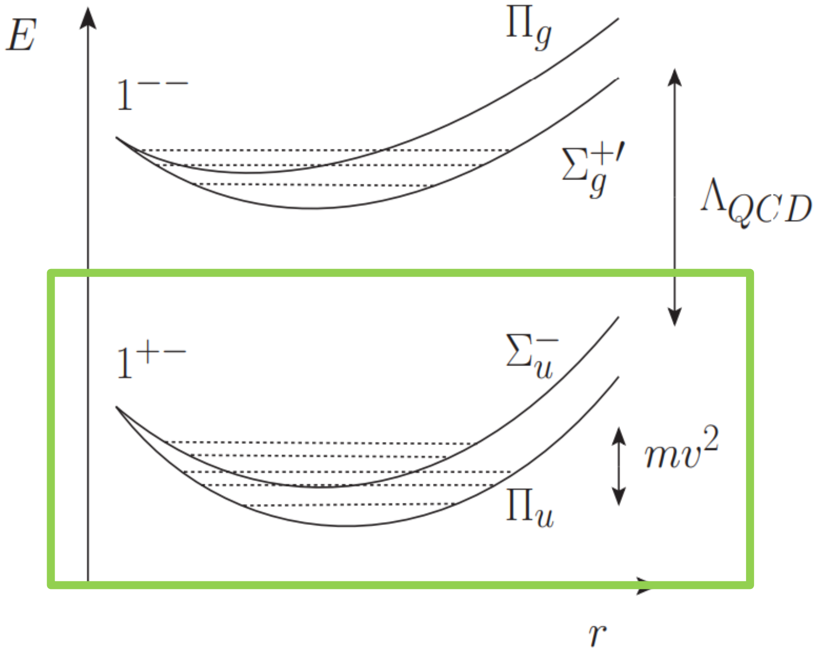
- Hybrid spin-dependent potentials: at order $1/m_Q$ (contrary to quarkonium $O(1/m_Q^2)$)

Brambilla, Lai, Segovia, Castellà, Vairo Phys. Rev. D. 101, (2020)

Brambilla, Lai, Segovia, Castellà, Vairo Phys. Rev. D. 99, (2019)

Soto, Valls, arXiv 2302.01765

BOEFT: Hybrids



- Degeneracy at short distances $r \to 0$, mixes hybrid states corresponding to Σ_u^- and Π_u potential
- Coupled Schrödinger Eq: Dynamics of $Q\bar{Q}$ at scale $mv^2 \ll \Lambda_{QCD}$

Schrödinger equation

$$\left[-P^{i\dagger}_{\kappa\lambda}\frac{\nabla_r^2}{m}P^i_{\kappa\lambda'} + V_{\kappa\lambda\lambda'}(r)\right]\Psi^n_{\kappa\lambda'}(\boldsymbol{r}) = E_n\Psi^n_{\kappa\lambda}(\boldsymbol{r})$$

$\kappa = 1^{+-}$

$\lambda = 0, \pm 1$

Hybrid Spectrum:

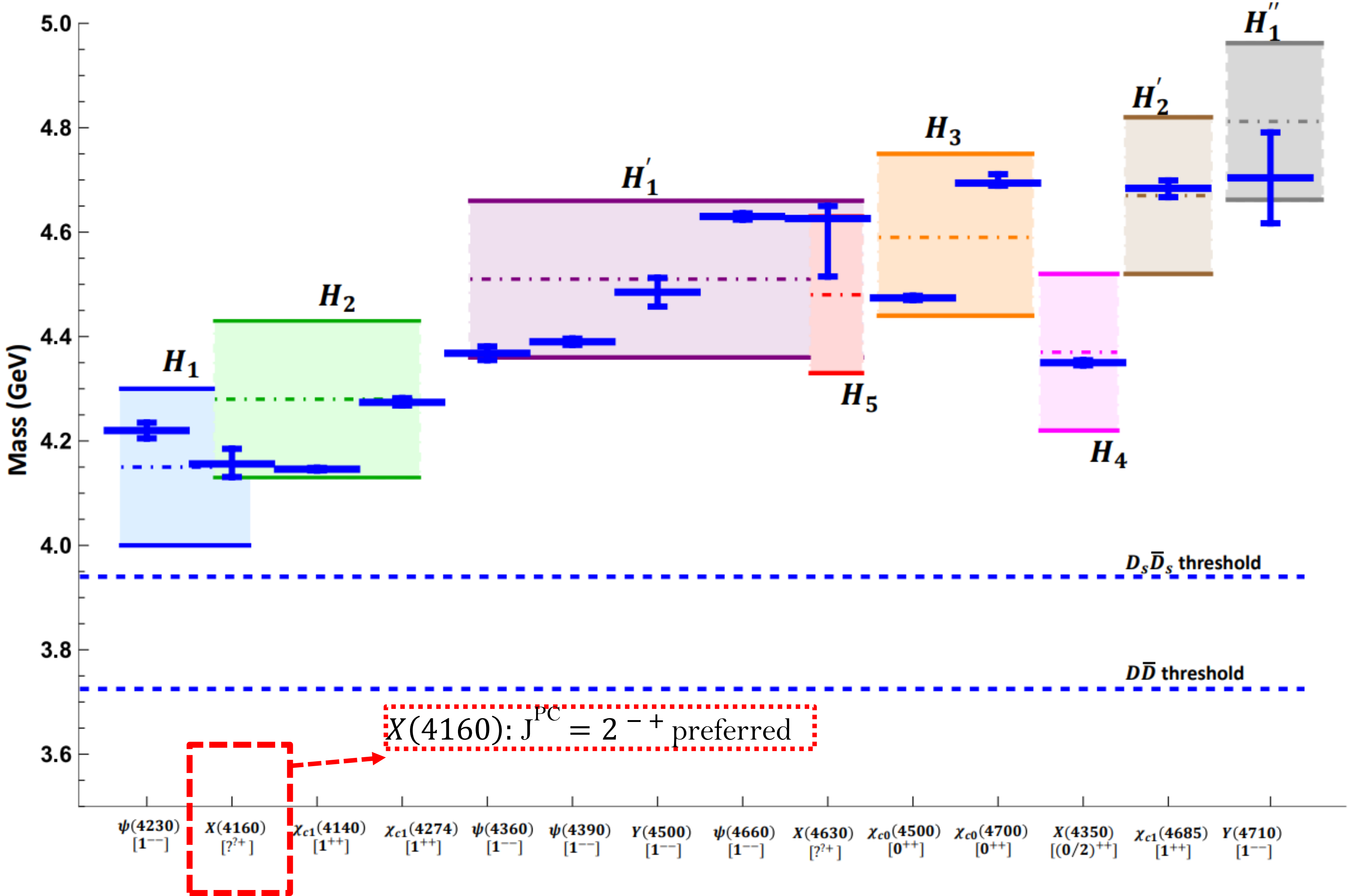
Multiplet	J^{PC}	$M_{c\bar{c}g}$	$M_{b\bar{b}g}$
H_1	$\{1^{--}, (0,1,2)^{-+}\}$	4155	10786
H_1'		4507	10976
H_1''		4812	11172
H_2	$\{1^{++}, (0,1,2)^{+-}\}$	4286	10846
H_2'		4667	11060
H_2''		5035	11270
H_3	$\{0^{++}, 1^{+-}\}$	4590	11065
H_3'		5054	11352
H_3''		5473	11616
H_4	$\{2^{++}, (1,2,3)^{+-}\}$	4367	10897
H_5	$\{2^{--}, (1,2,3)^{-+}\}$	4476	10948

Λ- doubling: opposite parity states non-degenerate.

Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015)   Brambilla, Lai, AM, Vairo arXiv:2212.09187

BOEFT: Hybrids

- **Charmonium hybrids**: comparison with experimental results:



	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

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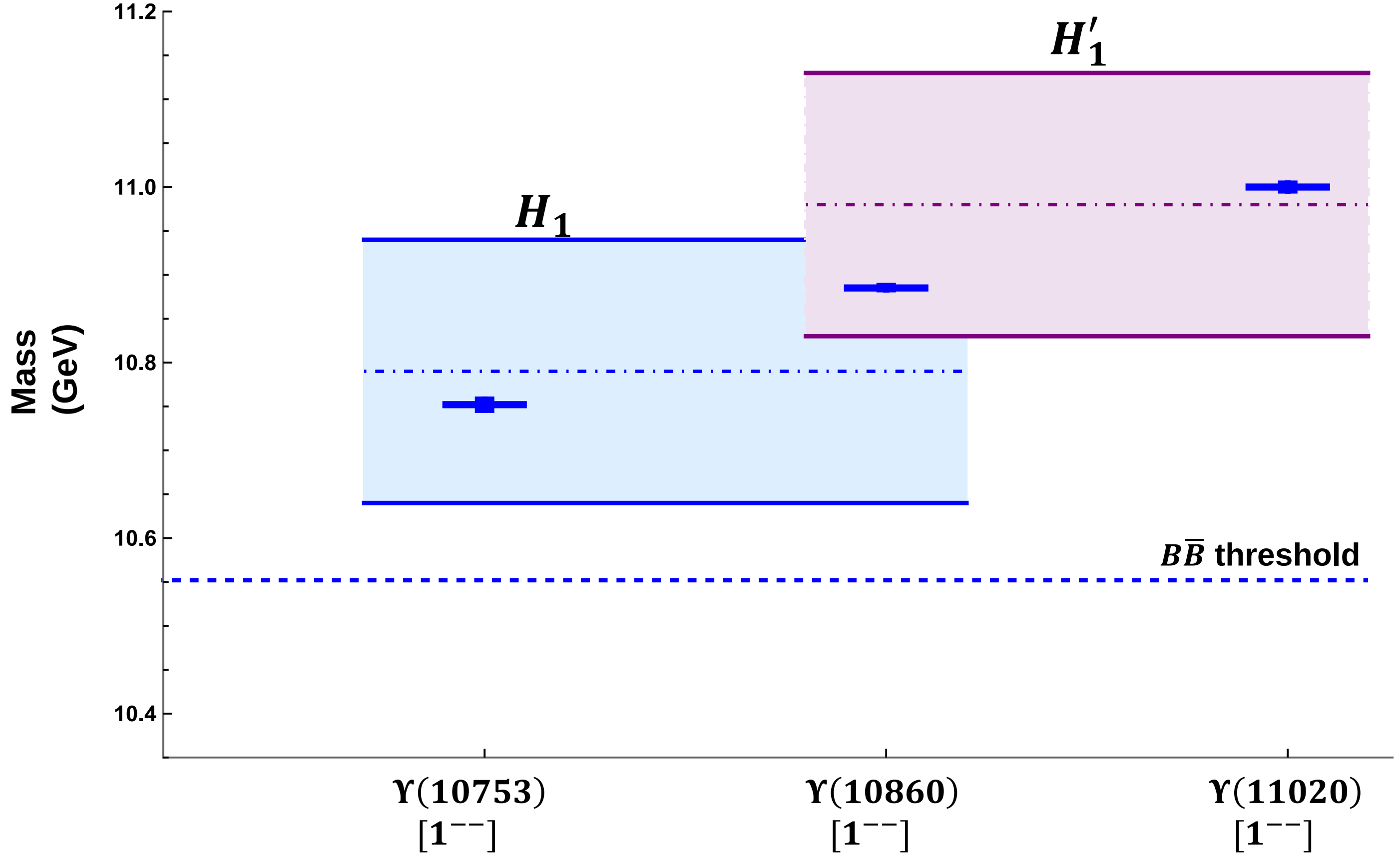
Brambilla, Lai, AM, Vairo arXiv:2212.09187

BOEFT: Hybrids

- **Bottomonium hybrids**: comparison with experimental results:



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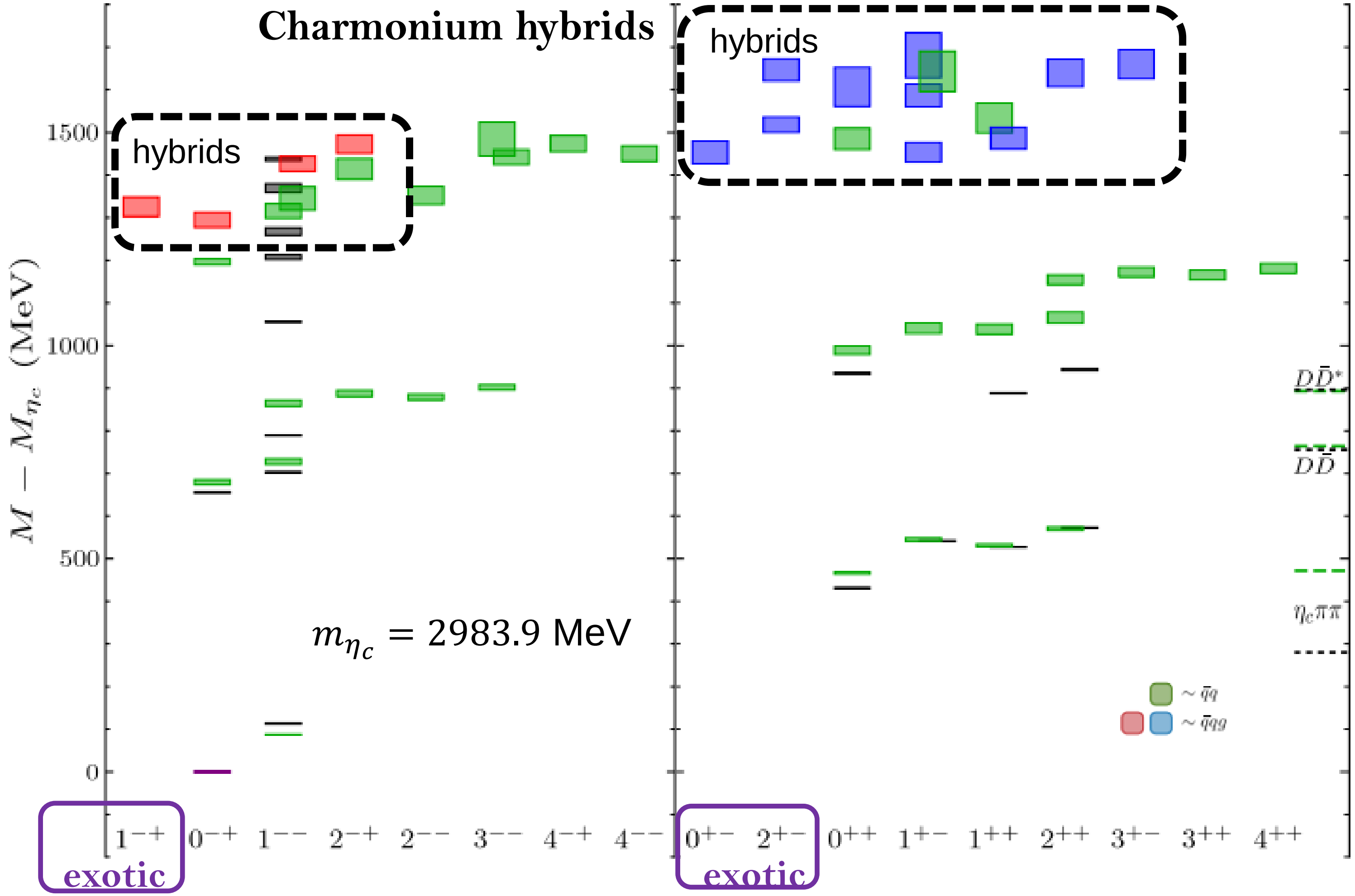
	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

Brambilla, Lai, AM, Vairo arXiv:2212.09187

BOEFT: Hybrids

- Lattice results for charm hybrids ($m_\pi \approx 240$ MeV) :

Results agree within error bars



Lattice data from
Hadron Spectrum collaboration JHEP 12 (2016) 89

Box represents uncertainties in lattice computations

- $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$ etc. are exotic quantum #'s

	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
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H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

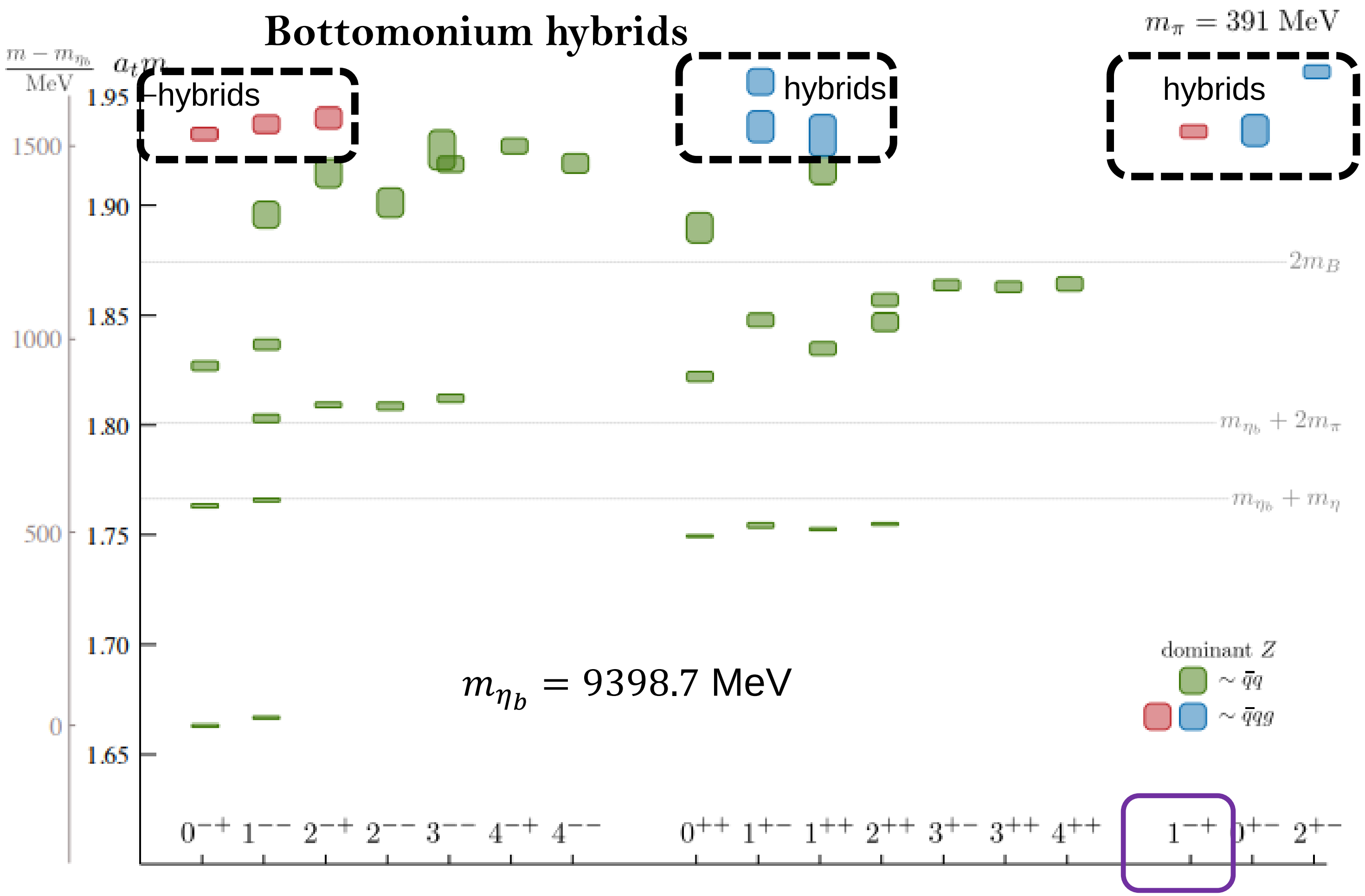
Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015)

Brambilla, Lai, AM, Vairo arXiv:2212.09187

BOEFT: Hybrids

- Lattice results for bottom hybrids ($m_\pi \approx 391$ MeV):

Results agree within error bars



Lattice data from
Hadron Spectrum collaboration JHEP 02 (2021) 214

Box represents uncertainties in lattice computations

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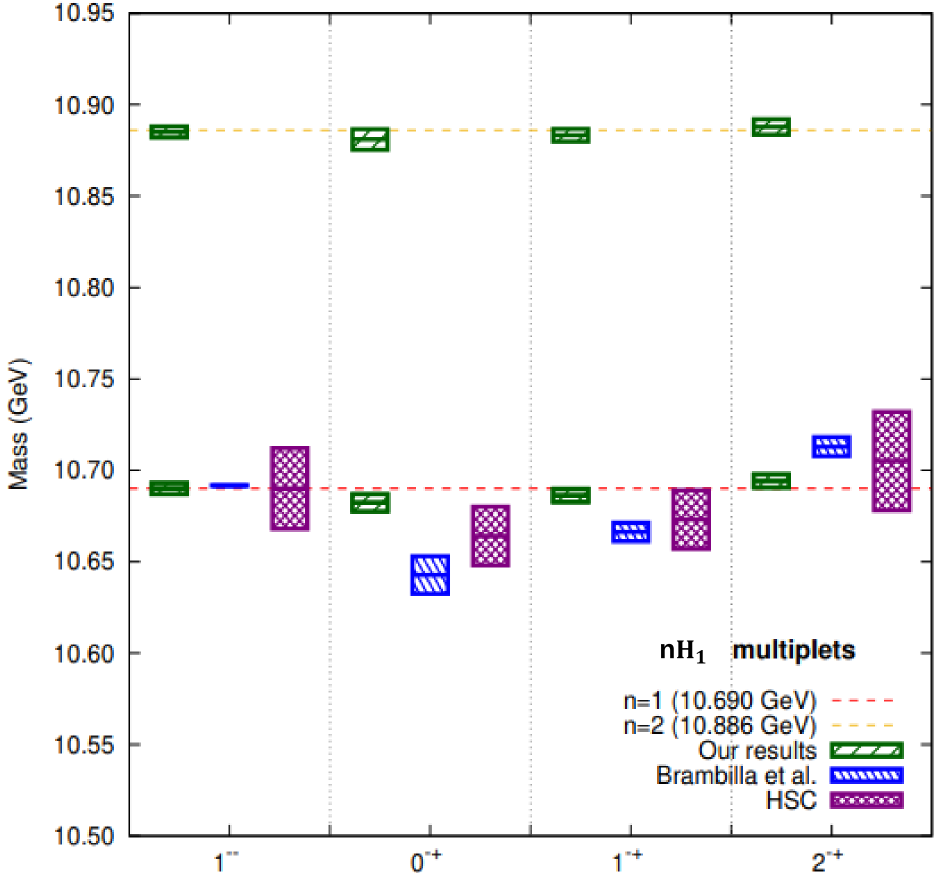
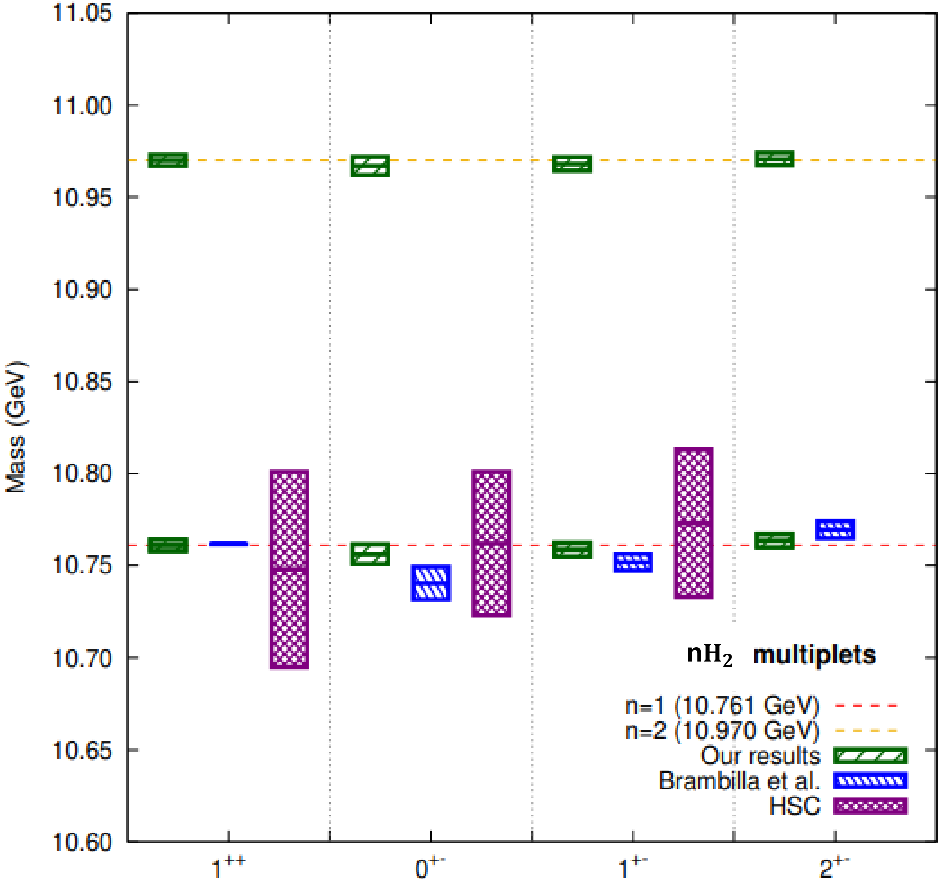
Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015)

Brambilla, Lai, AM, Vairo arXiv:2212.09187

BOEFT: Hybrids

- Including spin-dependent hybrid potentials:





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Our results refer to Soto & Valls arXiv 2302.01765

Brambilla, Lai, Segovia, Castellà, Vairo Phys. Rev. D. 101, (2020)

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Soto, Valls, arXiv 2302.01765

Lattice data from Hadron Spectrum collaboration JHEP 02 (2021) 214

Hybrid Decays

- Several exotic states discovered from decays to low-lying quarkonium.

- Consider the semi-inclusive process: $H_m \to Q_n + X$; H_m: low-lying hybrid, Q_n: low-lying quarkonium (states below threshold) and X: light hadrons.

Energy scale related to decay
$\Lambda_r^{-1} \equiv |\langle Q_n|\mathbf{r}|H_m\rangle|$

✓ ΔE: Large energy difference $\Rightarrow \Delta E \equiv E_{H_m} - E_{Q_n} \gtrsim 1$ GeV.

✓ Assume hierarchy of scales: $\Lambda_r \gg \Delta E \gg \Lambda_{QCD} \gg mv^2$

- In BOEFT, all energy scales above mv^2 are integrated out. So, scale ΔE must be integrated out. This gives imaginary contribution to hybrid potential:

Optical theorem: $$\sum_n \Gamma(H_m \to Q_n) = -2\,\mathrm{Im}\,\langle H_m|V|H_m\rangle$$

DISCLAIMER!!!
Decay to open-flavor threshold states not accounted here.

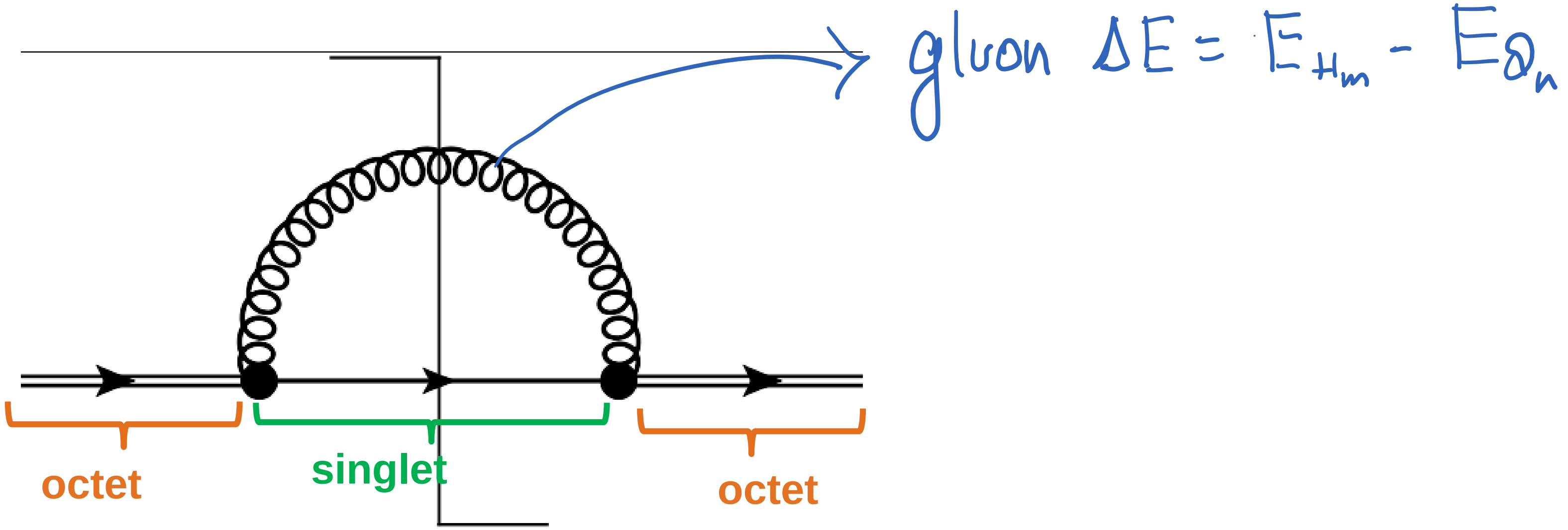
- Imaginary piece of hybrid potential: determined from matching **pNRQCD** and **BOEFT** effective theories.

Brambilla, Lai, AM, Vairo arXiv:2212.09187

Hybrid Decays

- $H_m \to Q_n + X$: ΔE (energy gap) >> Λ_{QCD}: gluon **resolves color configuration of** $Q\bar{Q}$ pair in hybrid and quarkonium:

Color configuration of $Q\bar{Q}$ pair:
Quarkonium - - - - -> Singlet
Hybrid - - - - -> Octet

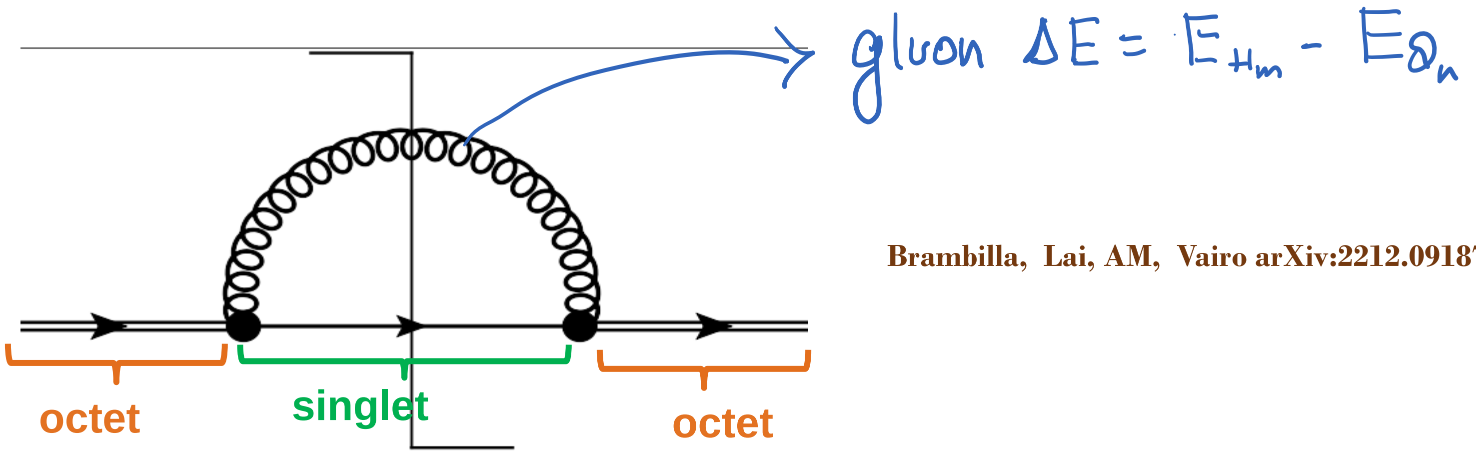


- For this work, assume: **quarkonium = singlet, hybrid = octet.**

Hybrid Decays

- $H_m \to Q_n + X$: ΔE (energy gap) >> Λ_{QCD}: gluon **resolves color configuration of** $Q\bar{Q}$ pair in hybrid and quarkonium:

Color configuration of $Q\bar{Q}$ pair:
Quarkonium ------> Singlet
Hybrid ------> Octet



Brambilla, Lai, AM, Vairo arXiv:2212.09187

- **Hybrid decays to heavy meson pair threshold states:** $\Delta E \lesssim \Lambda_{QCD}$

Selection rules: Hybrid decays to two S-wave mesons forbidden!

$$H_m \nrightarrow D^{(*)}\bar{D}^{(*)}$$

Kou & Pene, Phys Lett B 631 (2005) **Page, Phys Lett B 407 (1997)** **Farina, Tecocoatzi, Giachino, Santopinto & Swanson, Phys Rev D 102 (2020)**

Kou & Pene: Selection rule violated by 1/m dependent terms like hybrid-quarkonium mixing

Kou & Pene, Phys Lett B 631 (2005)

Computing decays of hybrid to threshold states in BOEFT framework ?

Hybrid Decays

- **pNRQCD Lagrangian**: d.o.f are perturbative singlet & octet fields and gluons of energy scale **mv²**.

$$\mathrm{S} = S\mathbb{I}_c/\sqrt{N_c}$$
$$\mathrm{O} = O^a T^a/\sqrt{T_F}$$

Weakly-coupled pNRQCD Lagrangian

$$L_{\mathrm{pNRQCD}} = \int d^3R \Big\{ \int d^3r \Big(\mathrm{Tr}\left[\mathrm{S}^\dagger \left(i\partial_0 - h_s\right)\mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - h_o\right)\mathrm{O}\right]$$
$$+ g\mathrm{Tr}\left[\mathrm{S}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,\mathrm{O} + \mathrm{O}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,\mathrm{S} + \frac{1}{2}\mathrm{O}^\dagger \boldsymbol{r}\cdot\{\boldsymbol{E},\mathrm{O}\}\right] + \frac{g}{4m}\mathrm{Tr}\left[\mathrm{O}^\dagger \boldsymbol{L}_{Q\bar{Q}}\cdot[\boldsymbol{B},\mathrm{O}]\right]$$
$$+ \frac{gc_F}{m}\mathrm{Tr}\left[\mathrm{S}^\dagger(\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\,\mathrm{O} + \mathrm{O}^\dagger(\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\,\mathrm{S} + \mathrm{O}^\dagger\boldsymbol{S}_1\cdot\boldsymbol{B}\,\mathrm{O} - \mathrm{O}^\dagger\boldsymbol{S}_2\mathrm{O}\cdot\boldsymbol{B}\right] - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\,a}\Big\}$$

- Connection with non-perturbative fields: quarkonium and hybrid in short-distance limit **r → 0**

Fields:

$$S\left(\mathbf{r},\mathbf{R},t\right) \to Z_\Psi^{1/2}(\mathbf{r})\,\Psi(\mathbf{r},\mathbf{R},t),$$
$$P^{i\dagger}_{\kappa\lambda}O^a\left(\mathbf{r},\mathbf{R},t\right)G^{ia}_\kappa(\mathbf{R},t) \to Z_\kappa^{1/2}(\mathbf{r})\,\Psi_{\kappa\lambda}(\mathbf{r},\mathbf{R},t)$$

Potentials:

$$E_{\Sigma_g^+}(r) = V_s(r) + b_{\Sigma_g^+}r^2 + \dots,$$
$$E_{\Sigma_u^-,\Pi_u}(r) = V_o(r) + \Lambda + b_{\Sigma,\Pi}r^2 + \dots$$

Brambilla, Lai, AM, Vairo arXiv:2212.09187

Hybrid Decays

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- **pNRQCD Lagrangian**: d.o.f are the perturbative singlet (S) and octet (O) fields and gluons of energy scale **mv²**.

$$\mathrm{S} = S\mathbb{I}_c/\sqrt{N_c}$$
$$\mathrm{O} = O^a T^a/\sqrt{T_F}$$

Weakly-coupled pNRQCD Lagrangian

$$L_{\mathrm{pNRQCD}} = \int d^3R \Big\{ \int d^3r \Big(\mathrm{Tr}\left[\mathrm{S}^\dagger\,(i\partial_0 - h_s)\,\mathrm{S} + \mathrm{O}^\dagger\,(iD_0 - h_o)\,\mathrm{O}\right]$$
$$+ g\mathrm{Tr}\left[\mathrm{S}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,\mathrm{O} + \mathrm{O}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,\mathrm{S} + \frac{1}{2}\mathrm{O}^\dagger \boldsymbol{r}\cdot\{\boldsymbol{E},\mathrm{O}\}\right] + \frac{g}{4m}\mathrm{Tr}\left[\mathrm{O}^\dagger \boldsymbol{L}_{Q\bar{Q}}\cdot[\boldsymbol{B},\mathrm{O}]\right]$$
$$+ \frac{gc_F}{m}\mathrm{Tr}\left[\mathrm{S}^\dagger(\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\,\mathrm{O} + \mathrm{O}^\dagger(\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\,\mathrm{S} + \mathrm{O}^\dagger\boldsymbol{S}_1\cdot\boldsymbol{B}\,\mathrm{O} - \mathrm{O}^\dagger\boldsymbol{S}_2\mathrm{O}\cdot\boldsymbol{B}\right] - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\,a}\Big\}$$

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Fields:

$$S(\mathbf{r},\mathbf{R},t) \to Z_\Psi^{1/2}(\mathbf{r})\,\Psi(\mathbf{r},\mathbf{R},t),$$
$$P^{i\dagger}_{\kappa\lambda}O^a(\mathbf{r},\mathbf{R},t)\,G^{ia}_\kappa(\mathbf{R},t) \to Z_\kappa^{1/2}(\mathbf{r})\,\Psi_{\kappa\lambda}(\mathbf{r},\mathbf{R},t)$$

G^{ia}_κ: Gluon fields

Potentials:

$$E_{\Sigma_g^+}(r) = V_s(r) + b_{\Sigma_g^+}r^2 + \dots,$$
$$E_{\Sigma_u^-,\Pi_u}(r) = V_o(r) + \Lambda + b_{\Sigma,\Pi}r^2 + \dots$$

V_s & V_o: singlet and octet potential

Λ: gluelump mass

Non-perturbative parameters

Hybrid Decays

- pNRQCD Lagrangian:

Weakly-coupled pNRQCD Lagrangian

$$L_{\text{pNRQCD}} = \int d^3R \Big\{ \int d^3r \Big(\text{Tr}\left[\text{S}^\dagger \left(i\partial_0 - h_s\right) \text{S} + \text{O}^\dagger \left(iD_0 - h_o\right) \text{O} \right]$$

$$+ g\text{Tr}\left[\text{S}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,\text{O} + \text{O}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,\text{S} + \frac{1}{2}\text{O}^\dagger \boldsymbol{r}\cdot\{\boldsymbol{E},\text{O}\} \right] + \frac{g}{4m}\text{Tr}\left[\text{O}^\dagger \boldsymbol{L}_{Q\bar{Q}}\cdot[\boldsymbol{B},\text{O}] \right]$$

$$+ \frac{gc_F}{m}\text{Tr}\left[\text{S}^\dagger (\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\,\text{O} + \text{O}^\dagger (\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\,\text{S} + \text{O}^\dagger \boldsymbol{S}_1\cdot\boldsymbol{B}\,\text{O} - \text{O}^\dagger \boldsymbol{S}_2 \text{O}\cdot\boldsymbol{B} \right] - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\,a} \Big\}$$

- Spin preserving decays $[O(r^2)]$
- Spin flipping decays $[O(1/m^2)]$

Brambilla, Lai, AM, Vairo arXiv:2212.09187

Hybrid Decays

- Spin-conserving decay due to $\boldsymbol{r} \cdot \boldsymbol{E}$ term :

$|S_H = 1 > ---\rightarrow |S_Q = 1 >$
$|S_H = 0 > ---\rightarrow |S_Q = 0 >$

$$\Gamma(H_m \to Q_n) = \frac{4\alpha_s(\Delta E)\, T_F}{3N_c}\, T^{ij}\,(T^{ij})^\dagger\, \Delta E^3$$

DISCLAIMER!!!
Decay to open-flavor threshold states not accounted here.

$$T^{ij} \equiv \langle H_m | r^j | Q_n \rangle = \int d^3\mathbf{r}\, \Psi^{i\dagger}_{(m)}(\mathbf{r})\, r^j\, \Phi^{Q\bar{Q}}_{(n)}(\mathbf{r})$$

$$\langle H_m | \boldsymbol{r} | Q_n \rangle = \sqrt{T^{ij}(T^{ij})^\dagger}$$

$\Psi^i_{(m)}$: Hybrid wf
Φ^Q_n : Quarkonium wf

R. Oncala, J. Soto, Phys. Rev. D96, 014004 (2017).
J. Castellà, E. Passemar, Phys. Rev. D104, 034019 (2021)

- Spin-flipping decay due to $\boldsymbol{S}.\boldsymbol{B}$ term:

$|\chi_H\rangle$: Hybrid spin wf
$|\chi_Q\rangle$: Quarkonium spin wf

$|S_H = 1 > ---\rightarrow |S_Q = 0 >$
$|S_H = 0 > ---\rightarrow |S_Q = 1 >$

$$T^{ij} \equiv \langle H_m | \left(S^j_1 - S^j_2\right) | Q_n \rangle = \left[\int d^3\mathbf{r}\, \Psi^{i\dagger}_{(m)}(\mathbf{r})\, \Phi^Q_{(n)}(\mathbf{r})\right] \langle \chi_H | \left(S^j_1 - S^j_2\right) | \chi_Q \rangle$$

- Depends on overlap of quarkonium and hybrid wavefunctions.
- Based on hierarchy: $\Lambda_r \gg \Delta E \gg \Lambda_{QCD}$

Brambilla, Lai, AM, Vairo arXiv:2212.09187

Results

Hybrid-to-Quarkonium transition decay rate
= **spin-conserving** + **spin-flipping** decay rates.

✓ **Decay to open threshold states not accounted !!!!**

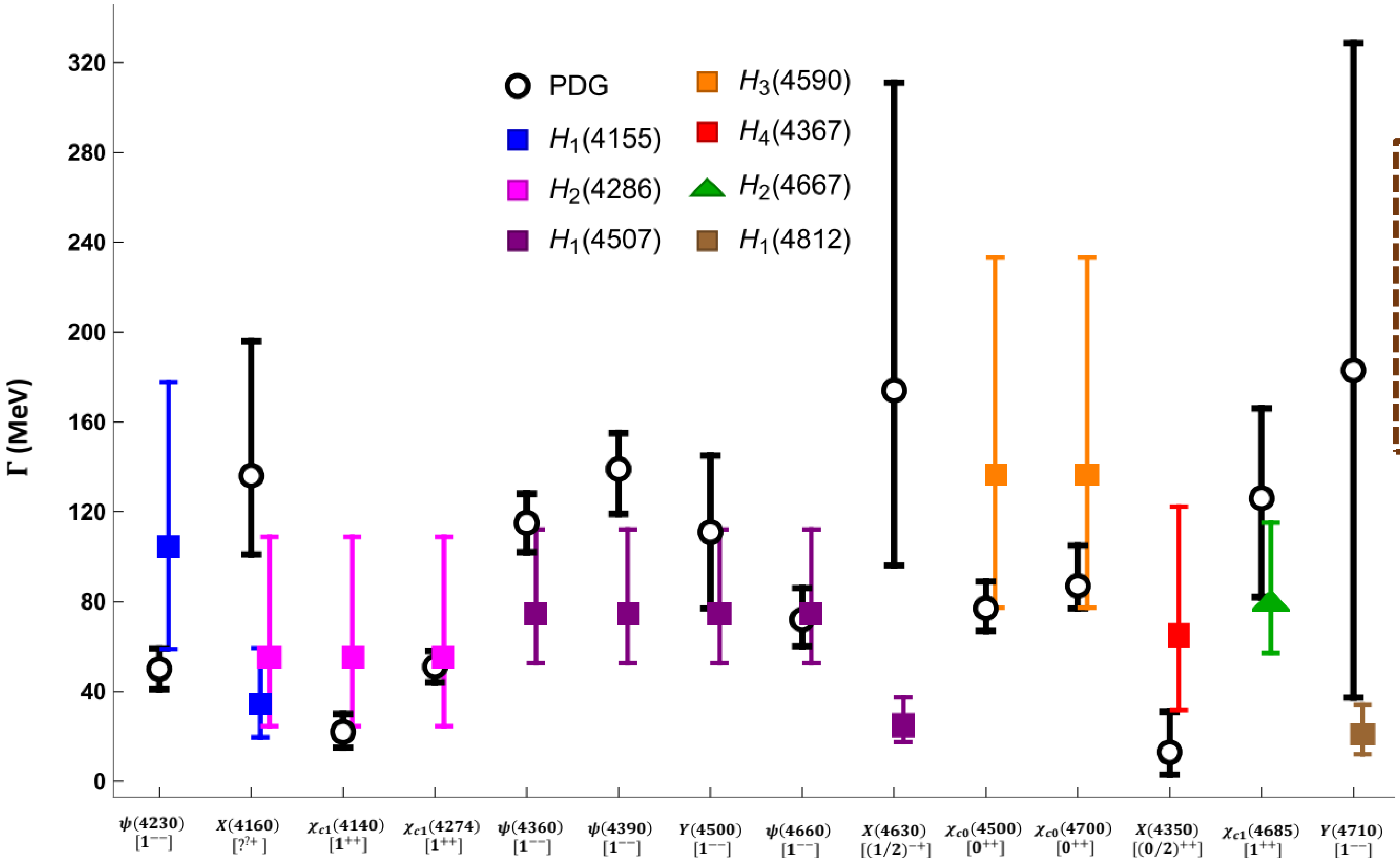
Our estimate of decay rate are lower-bounds for the **total width of hybrids**

For exotic states (which are possible hybrid candidates), our results sets **lower-bounds** on the inclusive widths of physical states.

Results

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- Comparison: charm exotic states with corresponding charmonium hybrid state:

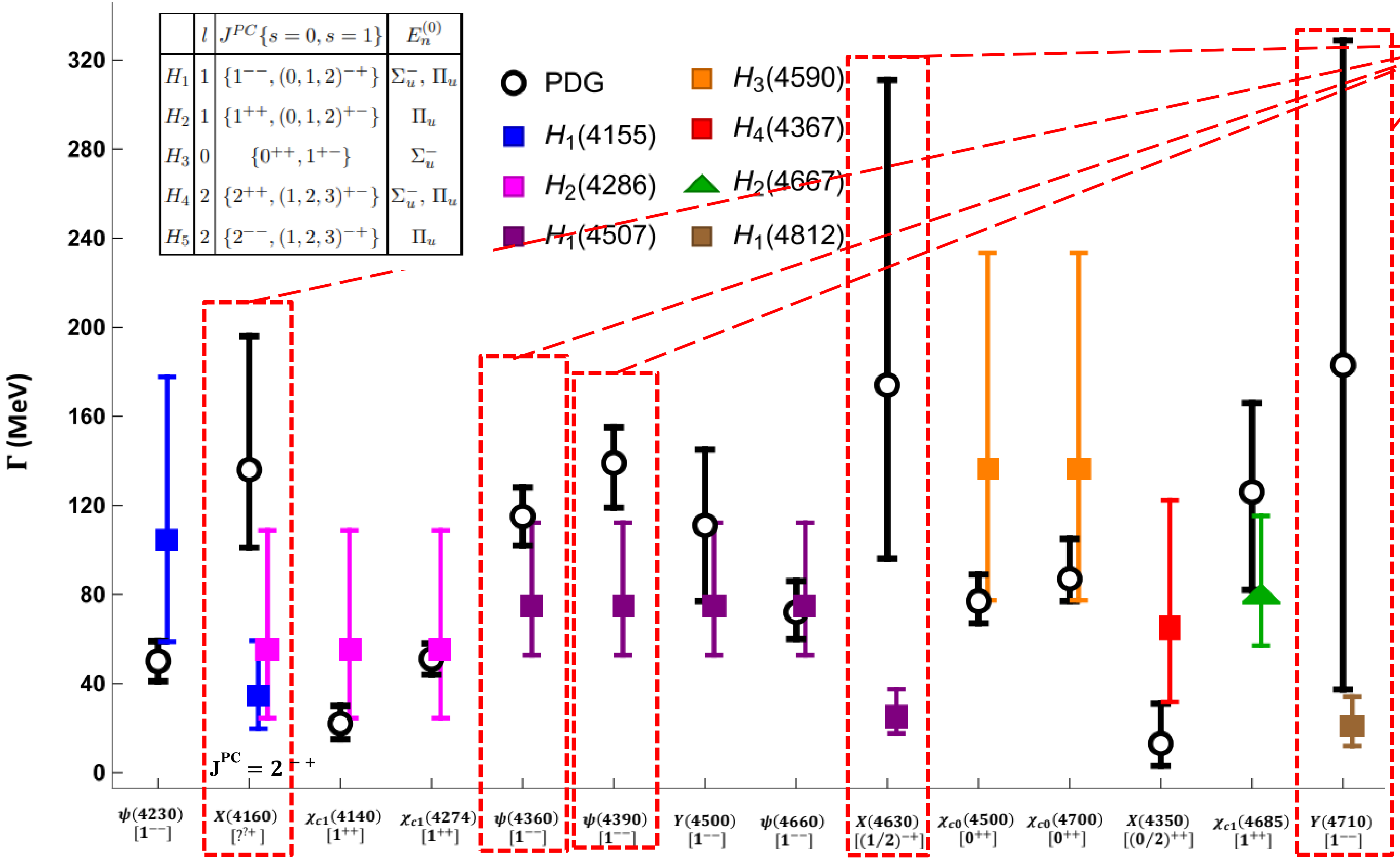


spin-conserving + spin-flipping decays
=
lower bound on the total decay widths of hybrids
which is compared with inclusive rate of physical states in PDG.

Results

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- Hybrid-to-quarkoniunium transition widths:



	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

Could have a significant hybrid component.

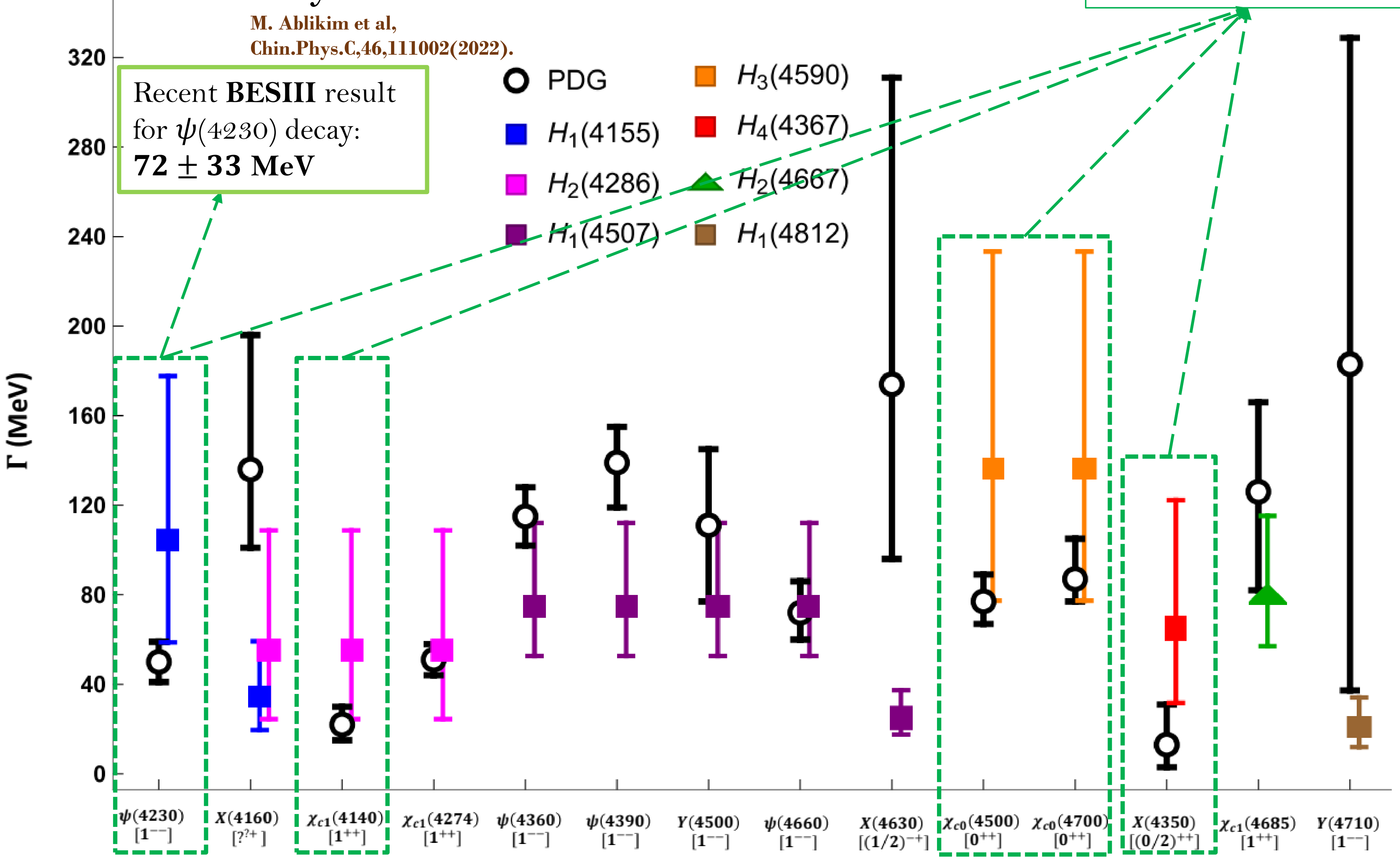
- ✓ Decay to open-threshold states could be significant or expected to be large (?).
- ✓ See Farina, Tecocoatzi, Giachino, Santopinto & Swanson, Phys Rev D 102 (2020) for 1^{--} hybrid decay to threshold states.
- ✓ No information on branching ratio (BR) for decays to threshold states in PDG.

Results

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- Comparison: charm exotic states with corresponding charmonium hybrid state:

Disfavors hybrid interpretation !



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H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

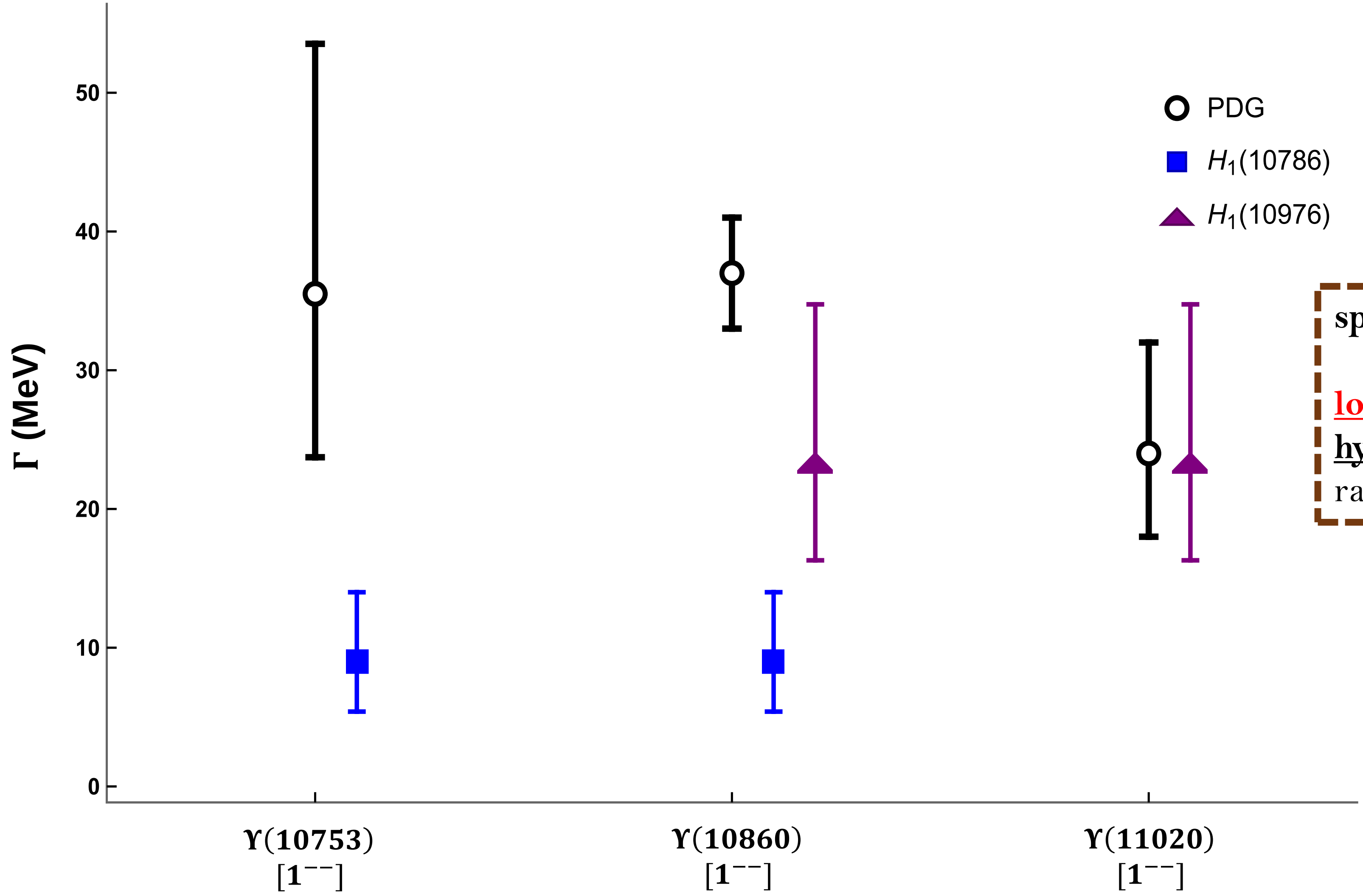
$\chi_{c0}(4500)$ & $\chi_{c0}(4700)$

Decay to $\chi_c(1P)$ not seen !!!. Major contribution to theoretical estimate from this decay channel.

Results

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- Comparison: bottom exotic states with corresponding bottomonium hybrid state:

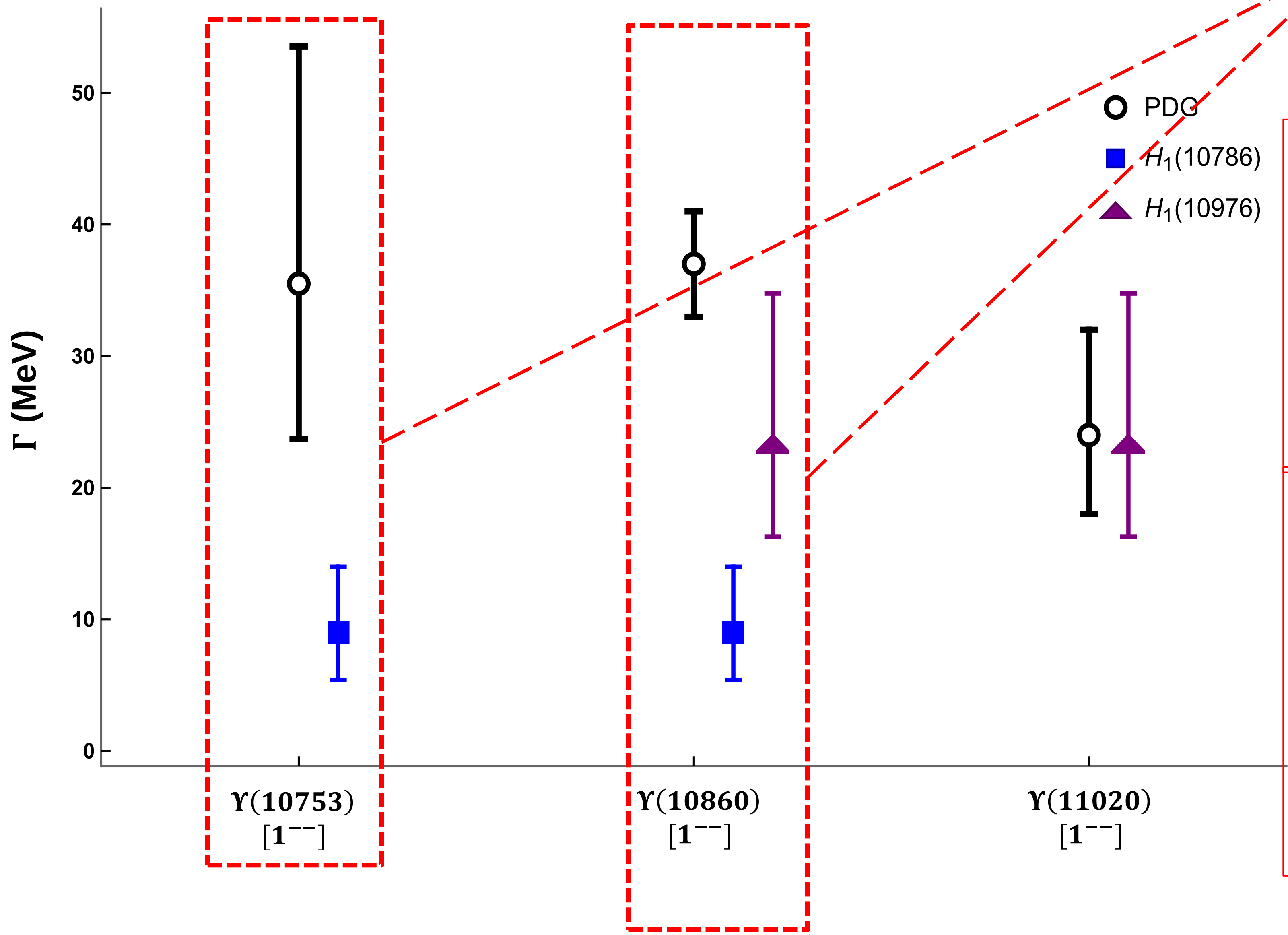


spin-conserving + spin-flipping decays
=
lower bound on the total decay widths of hybrids which is compared with inclusive rate of physical states in PDG.

Results

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- Hybrid-to-quarkoniunium transition widths:



Could have a significant hybrid component.

Υ(10860)

✓ Inclusive rate: **37 ± 4** MeV. **Branching fraction for decay to open-bottom mesons**: **76.6%**. Decay rate to quarkonium: $8.8^{+2.6}_{-1.8}$ MeV. Good agreement with our lower-bound estimate for $H_1[1^{--}](10786)$.

✓ Candidate for $\Upsilon(5S)$ state

Υ(11020) & Υ(10753)

Large branching fraction for decays to open-bottom mesons expected. Different quark model calculations predict large branching fraction. Need experimental input on branching fraction. See Hüsken, Mitchell & Swanson, Phys Rev D 106 (2022).

Υ(10753): Candidate for $\Upsilon(3D)$ state.

Results

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- Spin-flipping transitions: suppressed by powers of the heavy-quark mass due to the heavy-quark spin symmetry;

- Relative comparison between **spin-conserving** and **spin-flipping decays**: $H_m \to Q_n + X$:
  - ✓ Size of energy gap **ΔE**: final quarkonium states are different in both the decay process.
  - ✓ Depends on relative magnitude of matrix element (radial): $|\langle Q_n|r|H_m\rangle|$ & $|\langle Q_n|H_m\rangle|/m$

Ratio : $m\,|\langle Q_n|\mathbf{r}|H_m\rangle|/|\langle Q_n|H_m\rangle|$

No obvious hierarchy relation between the two-decay process.

  - ✓ For bottom hybrids: spin-flipping transitions are smaller compared to spin-conserving.
  - ✓ For charm hybrids: spin-flipping transitions are not necessarily small: $m\,|\langle Q_n|\mathbf{r}|H_m\rangle|/|\langle Q_n|H_m\rangle| \sim 1$

Spin-flipping ~ spin-conversing: indicating heavy-quark spin-symmetry violations!

- Quarkonium transition: $Q_m \to Q_n + X$:spin-flipping decay suppressed by $O(v)^2$ compared to spin- conserving.

$$\left(|\langle Q_n|Q_m\rangle|/m\,|\langle Q_n|\mathbf{r}|Q_m\rangle|\right)^2 \sim v^2 \ll 1$$

Hybrid-quarkonium mixing (in progress)

Brambilla, AM, Vairo,.... (in progress)

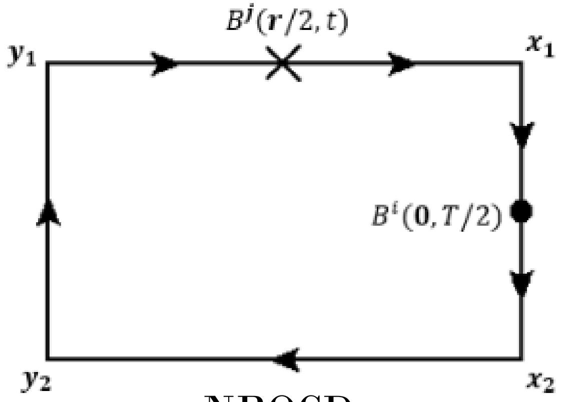
- Hybrid states in the same energy range and same quantum #'s as quarkonium can mix.
- Mixing impact spectrum and decay properties of hybrid. Implications on hybrid interpretation for exotics.

Oncala & Soto, Phys. Rev. D. 96, (2017)

Ex. $H_1\left[1^{--}\right](4155) \leftrightarrow c\bar{c}\left[1^{--}\right](3S)$

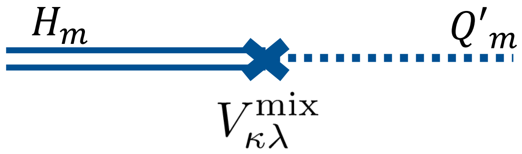
Effect on decay: $H_m \leftrightarrow Q_m^{'} \rightarrow (\eta_c, J/\psi, \cdots) + (\gamma, \cdots)$

- Hybrids with gluon quantum # $\boldsymbol{\kappa} = \mathbf{1}^{+-}$, mix with quarkonium through heavy-quark spin dependent operator. Mixing potential at $O(1/m)$ in BOEFT.
- Mixing potential $V_{\kappa\lambda}^{\text{mix}}$: determined from matching NRQCD and BOEFT at $O(1/m)$



NRQCD

=



BOEFT

Expression after matching:

$$V_{|\lambda|}^{\text{mix}} = -\frac{gc_F}{2m_Q}\, {}^{(0)}_{\lambda}\langle 1|B^j\left(\mathbf{r}/2,0\right)|0\rangle^{(0)} P_\lambda^j,$$

Above expression can be computed on lattice if we identify:

$$|0\rangle^{(0)} = |\Sigma_g^+\rangle$$

$$|1\rangle_{\lambda=0}^{(0)} = |\Sigma_u^-\rangle, |1\rangle_{|\lambda|=1}^{(0)} = |\Pi_u\rangle$$

Summary/Outlook

- BOEFT provides a model-independent & systematic way to study heavy quark hybrids (exotic) and decays.
- For the decay process, we assume the hierarchy of energy scales :

$$1/|\langle Q_n|\mathbf{r}|H_m\rangle| \gg \Delta E \gg \Lambda_{\rm QCD} \gg m_Q v^2$$

pNRQCD and BOEFT matching — Neglect hybrids of higher gluonic excitations and mixing.

- Our results for hybrid-to-quarkonium transition widths **sets lower-bounds** on the inclusive rate of physical exotic states, if interpreted as pure hybrid states .

Hybrid-to-Quarkonium transition decay rate = **spin-conserving** + **spin-flipping** decay rates.

- Our analysis disfavors: $\psi(4230)$, $\chi_{c1}(4140)$, $\chi_{c0}(4500)$, $\chi_{c0}(4700)$, and $X(4350)$ as pure hybrid states.
- Our analysis suggests:
  - $\boldsymbol{X(4160)}$: could be the **charm hybrid** $\boldsymbol{H_1[2^{-+}](4155)}$.
  - $\boldsymbol{X(4630)}$: could be the **charm hybrid** $\boldsymbol{H_1[(1/2^{-+})](4507)}$.
  - $\boldsymbol{\Upsilon(10753)}$: could be the **bottom hybrid** $\boldsymbol{H_1[(1^{--})](10786)}$.
  - $\boldsymbol{\psi(4390)}$: could be the **charm hybrid** $\boldsymbol{H_1[1^{--}](4507)}$.
  - $\boldsymbol{\psi(4710)}$: could be the **charm hybrid** $\boldsymbol{H_1[(1^{--})](4812)}$.
  - $\boldsymbol{\Upsilon(10860)}$: could be the **bottom hybrid** $\boldsymbol{H_1[(1^{--})](10786)}$.
  - Nothing conclusive can be said about other exotic states.

Ongoing/Future prospects

- Extending the BOEFT framework to include hybrid-quarkonium mixing.

  Hybrid states in the same energy range as quarkonium can mix (same quantum #'s). O (1/m) term in BOEFT.

  Impact on decay: $H_m \leftrightarrow Q'_m \rightarrow Q_n + X$; n & m denotes quantum #'s

- Computing hybrid decays to heavy-meson pair threshold

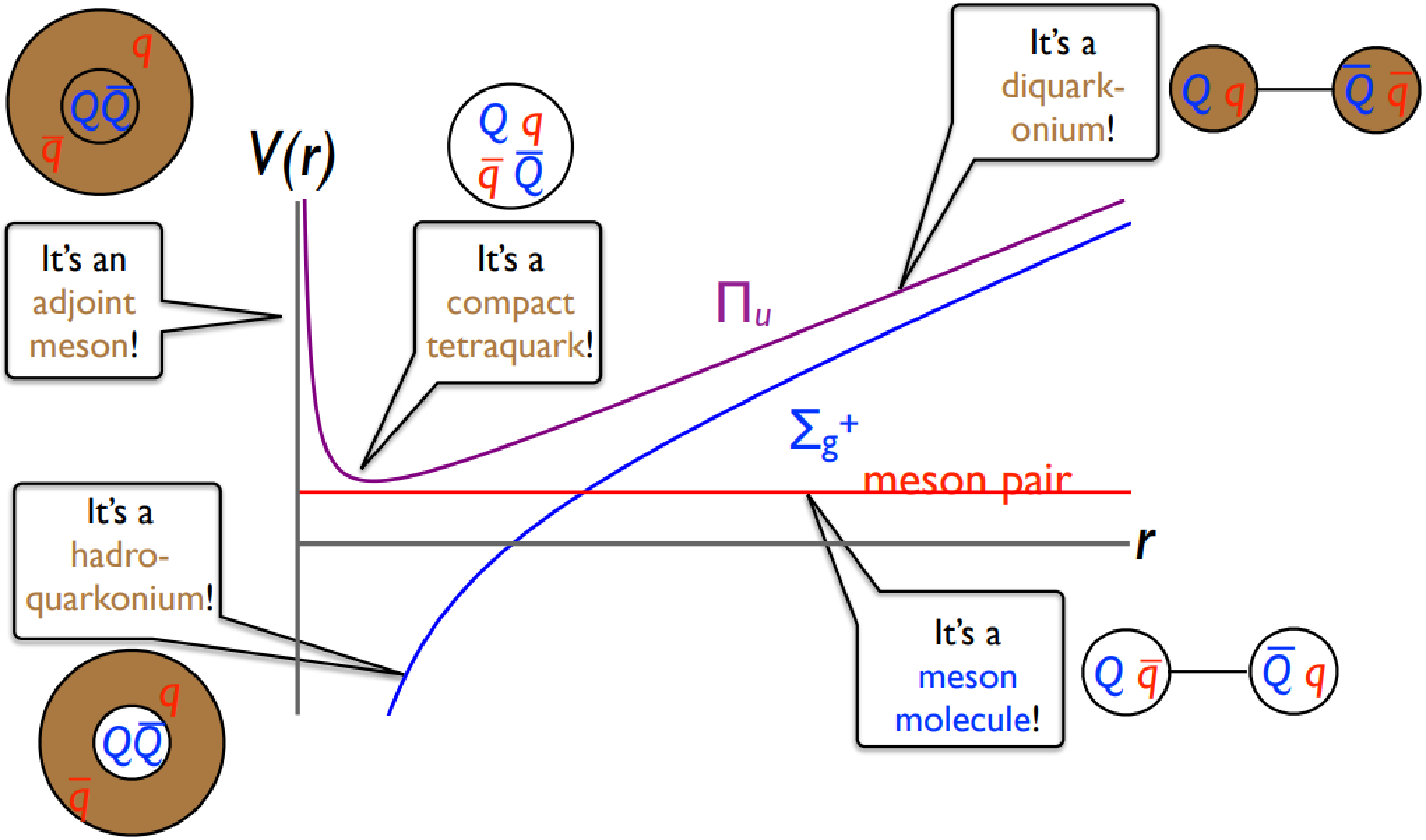
- **Extending BOEFT framework to study quarkonium tetraquarks** (in progress).

BOEFT framework: Aim is to have unified framework for XYZ exotics !!!.

Thank you!!

Backup Slides

What is an XYZ Meson?



Each model describes some region of the Born-Oppenheimer wavefunction

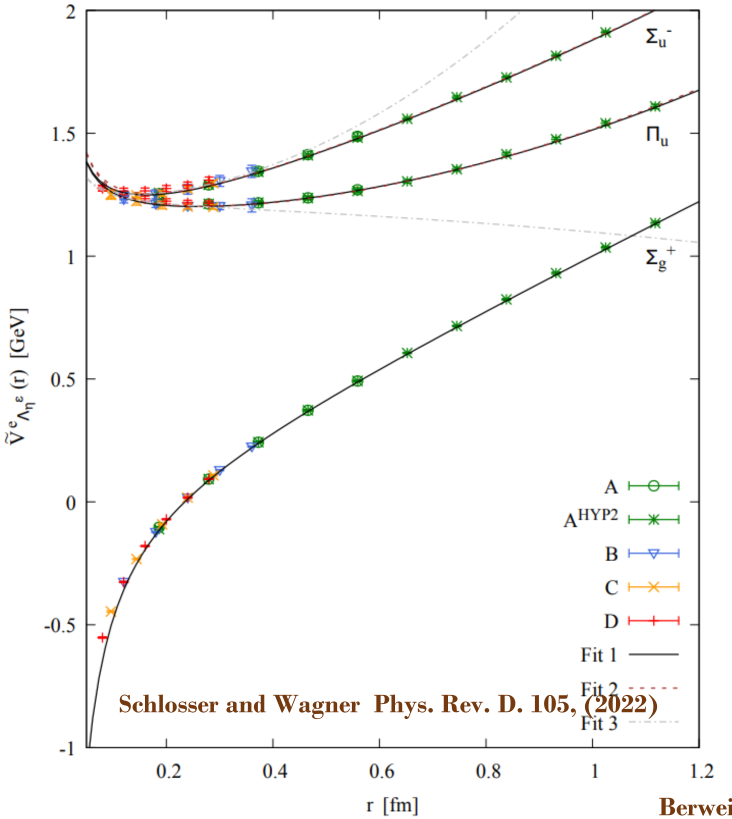
Figure from Eric Braaten talk: Charm 2020 conference

BOEFT

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- Quarkonium static potential: $V_\Psi(r) = E_{\Sigma_g^+}(r)$
- Hybrid static potential: $V_{10}(r) = E_{\Sigma_u^-}(r)\,,$ $V_{1\pm1}(r) = E_{\Pi_u}(r)$

Gluonic Static energies from lattice:



Schlosser and Wagner Phys. Rev. D. 105, (2022)

Quarkonium Potential:

$$V_{\Sigma_g^+}(r) = -\frac{\kappa_g}{r} + \sigma_g r + E_g^{Q\bar{Q}}$$

$m_c^{RS} = 1.477\,(40)$ GeV
$m_b^{RS} = 4.863\,(55)$ GeV

$\kappa_g = 0.489\,,\quad \sigma_g = 0.187\,\mathrm{GeV}^2 \qquad E_g^{c\bar{c}} = -0.254\,\mathrm{GeV},\ E_g^{b\bar{b}} = -0.195\,\mathrm{GeV},$

Hybrid Potential:

$$E_{\Sigma_u^-,\Pi_u}(r) = \begin{cases} V_o^{\mathrm{RS}}(\nu_f) + \Lambda_{\mathrm{RS}}(\nu_f) + b_{\Sigma,\Pi} r^2, & r < 0.25\ \mathrm{fm} \\ \frac{a_1^{\Sigma,\Pi}}{r} + \sqrt{a_2^{\Sigma,\Pi} r^2 + a_3^{\Sigma,\Pi}} + a_4^{\Sigma,\Pi}, & r > 0.25\ \mathrm{fm} \end{cases}.$$

$a_1^\Sigma = 0.000\,\mathrm{GeV fm}\,,\quad a_2^\Sigma = 1.543\,\mathrm{GeV}^2/\mathrm{fm}^2,\quad a_3^\Sigma = 0.599\,\mathrm{GeV}^2,\quad a_4^\Sigma = 0.154\,\mathrm{GeV}\,,$

$a_1^\Pi = 0.023\,\mathrm{GeV fm}\,,\quad a_2^\Pi = 2.716\,\mathrm{GeV}^2/\mathrm{fm}^2,\quad a_3^\Pi = 11.091\,\mathrm{GeV}^2,\quad a_4^\Pi = -2.536\,\mathrm{GeV}\,,$

$b_\Sigma = 1.246\,\mathrm{GeV/fm}^2,\quad b_\Pi = 0.000\,\mathrm{GeV/fm}^2$

Λ_{RS} : 0.87 (15) GeV

Gluelump mass definition:

$$\langle 0|G^{ia}_{1^{+-}}(\mathbf{R},T/2)\phi^{ab}(T/2,-T/2)G^{jb}_{1^{+-}}(\mathbf{R},-T/2)|0\rangle = \delta^{ij}e^{-i\Lambda T}$$

✓ Perturbative RS-scheme potentials V_o^{RS} upto order α_s^3.

Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015)

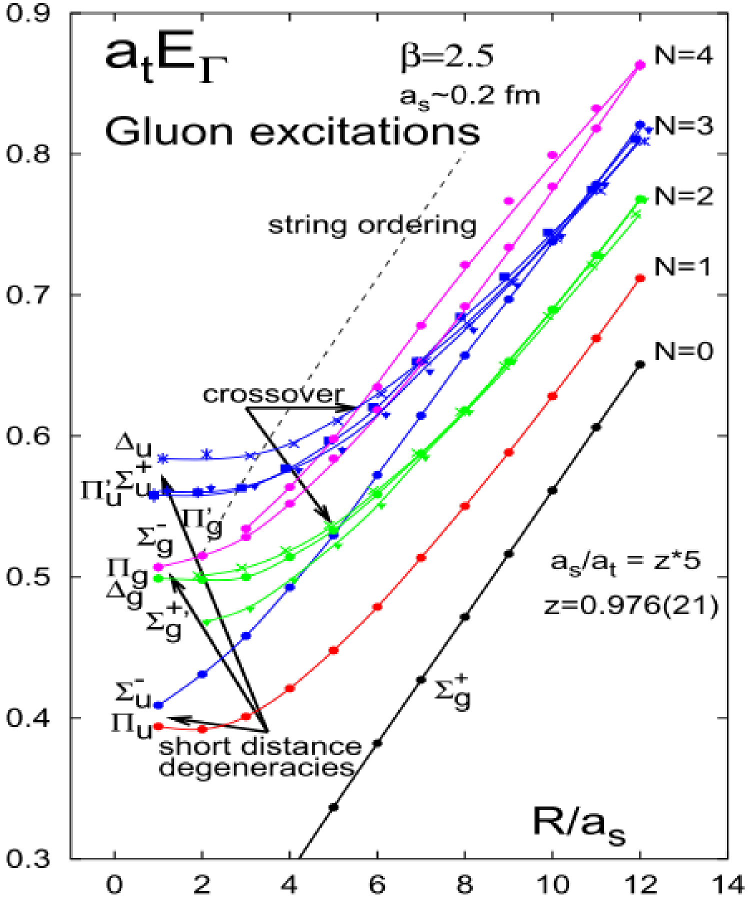
Bali and Pineda Phys. Rev. D. 69, (2004)

$Q\bar{Q}$ pair: Static Energies

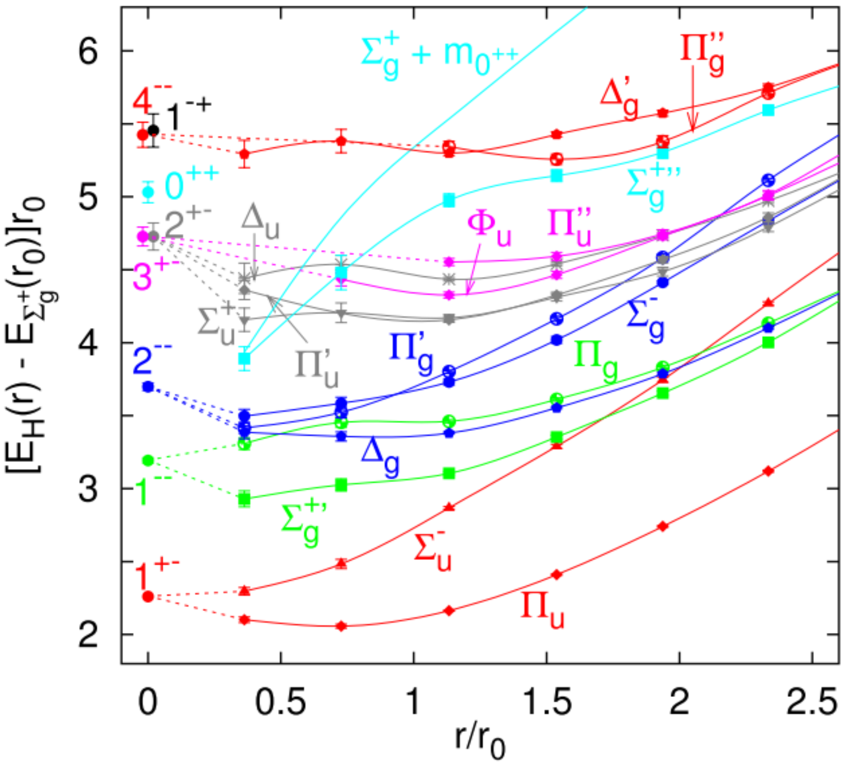
- **Static limit ($m \to \infty$):** heavy quarks are fixed in position. Interquark potential given by gluon configuration.

Results of **nonperturbative** Gluonic Static energies from lattice:

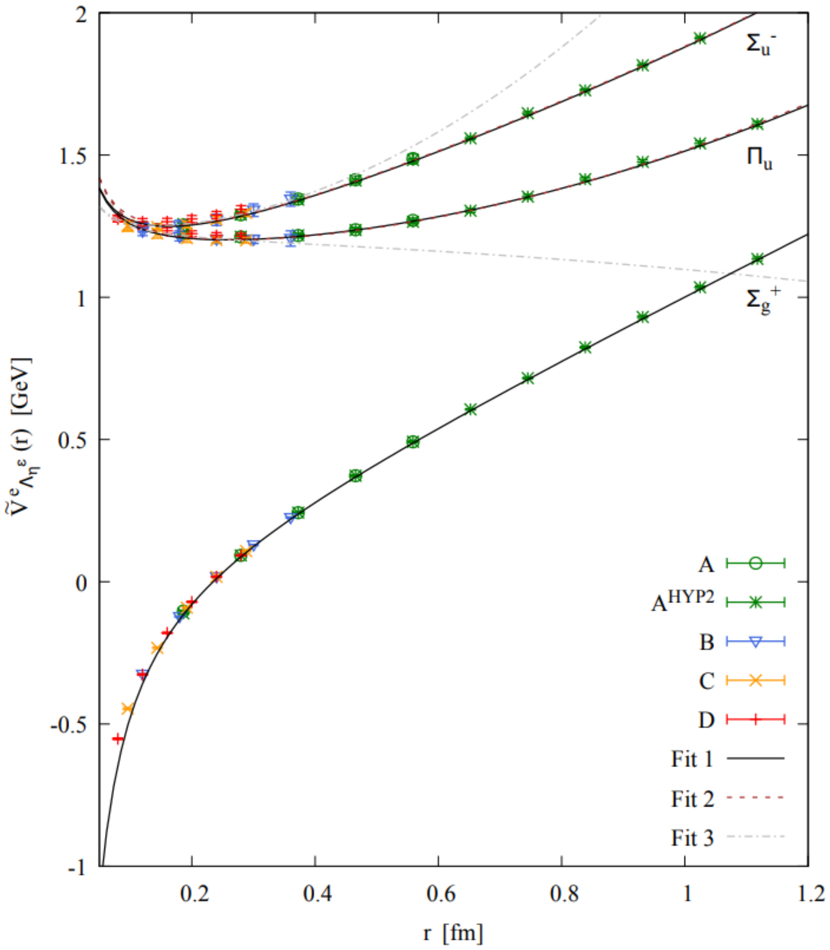
K. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. 90 (2003)



Bali and Pineda Phys. Rev. D. 69, (2004)



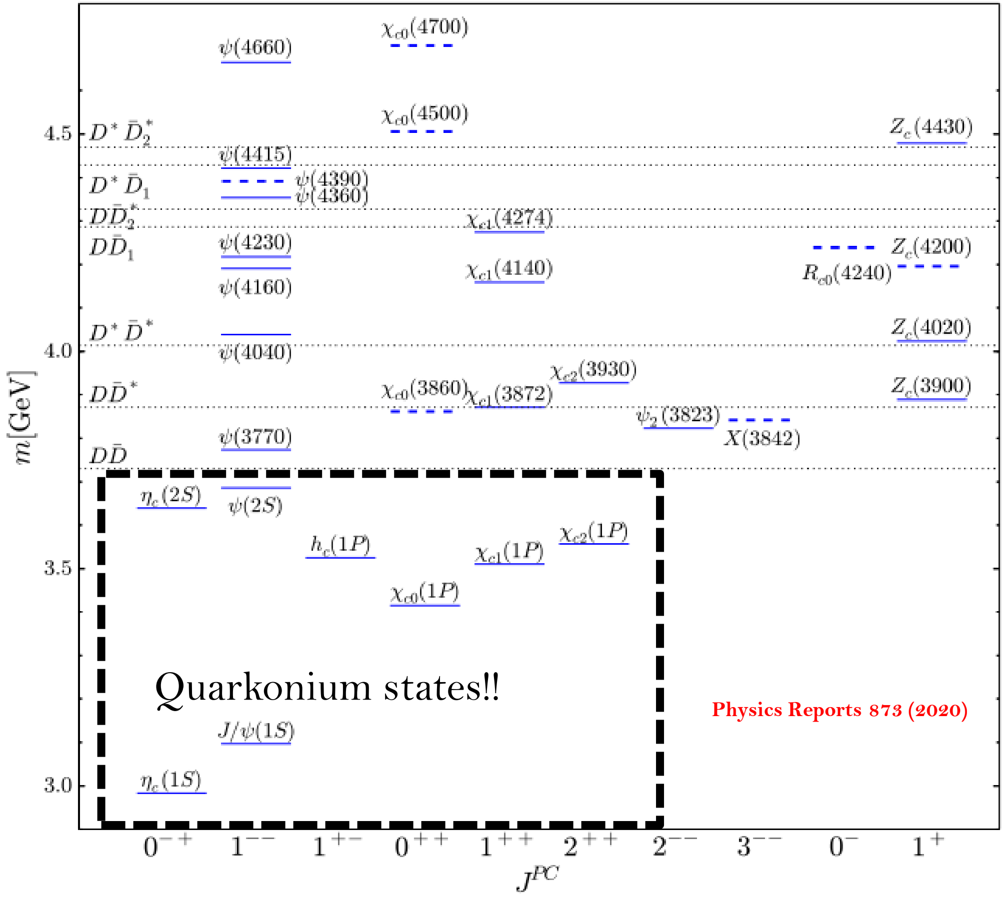
Schlosser and Wagner Phys. Rev. D. 105, (2022)



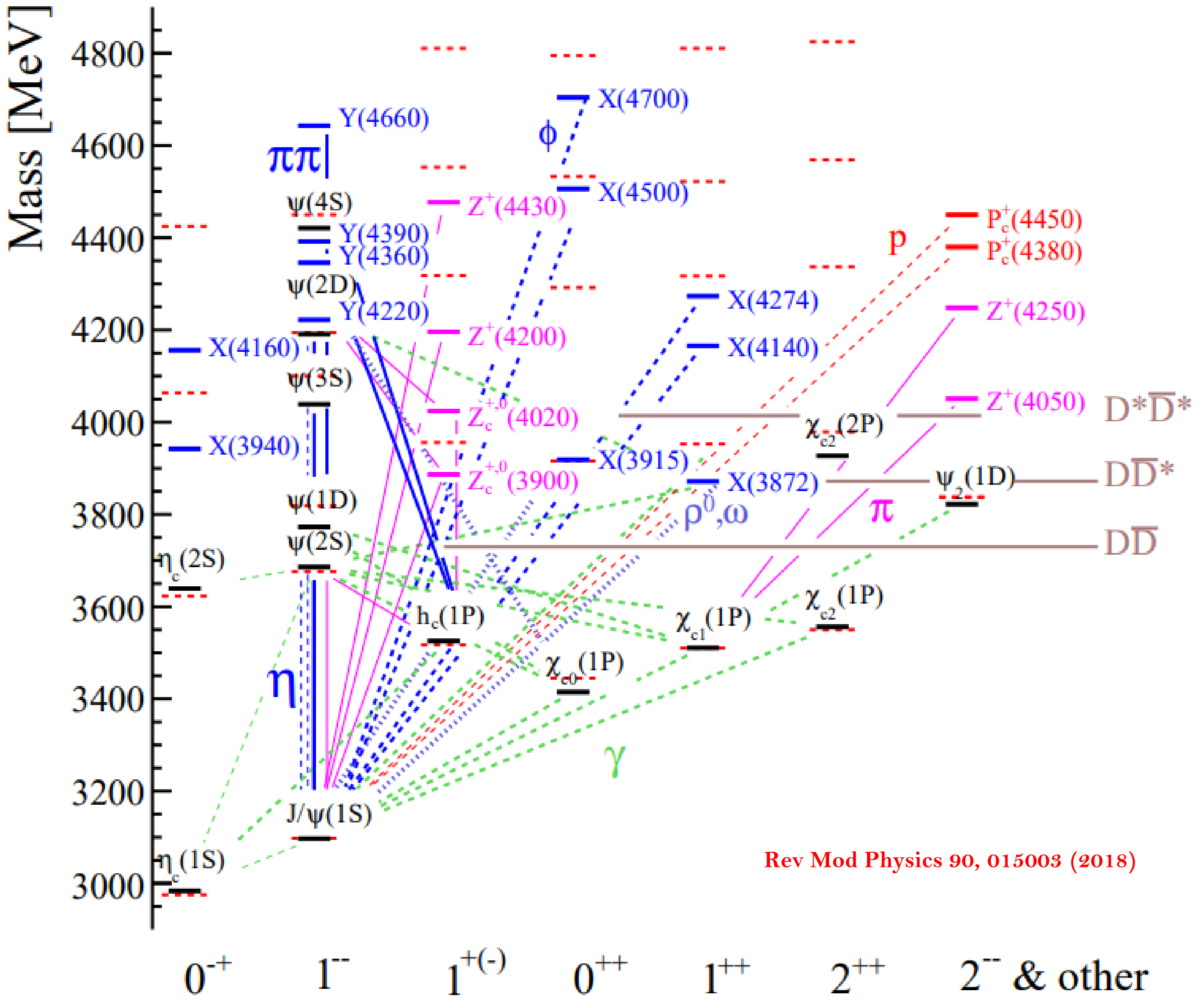
Exotic Hadrons: Charm

- All states below the lowest open-charm $D\bar{D}$ threshold are **conventional hadrons**
- Recently observed several new XYZ states

$T_{cc}^{+}(3875)$ $Y(4710)$ $Y(4500)$ $X(4630)$ $\chi_{c1}(4685)$....

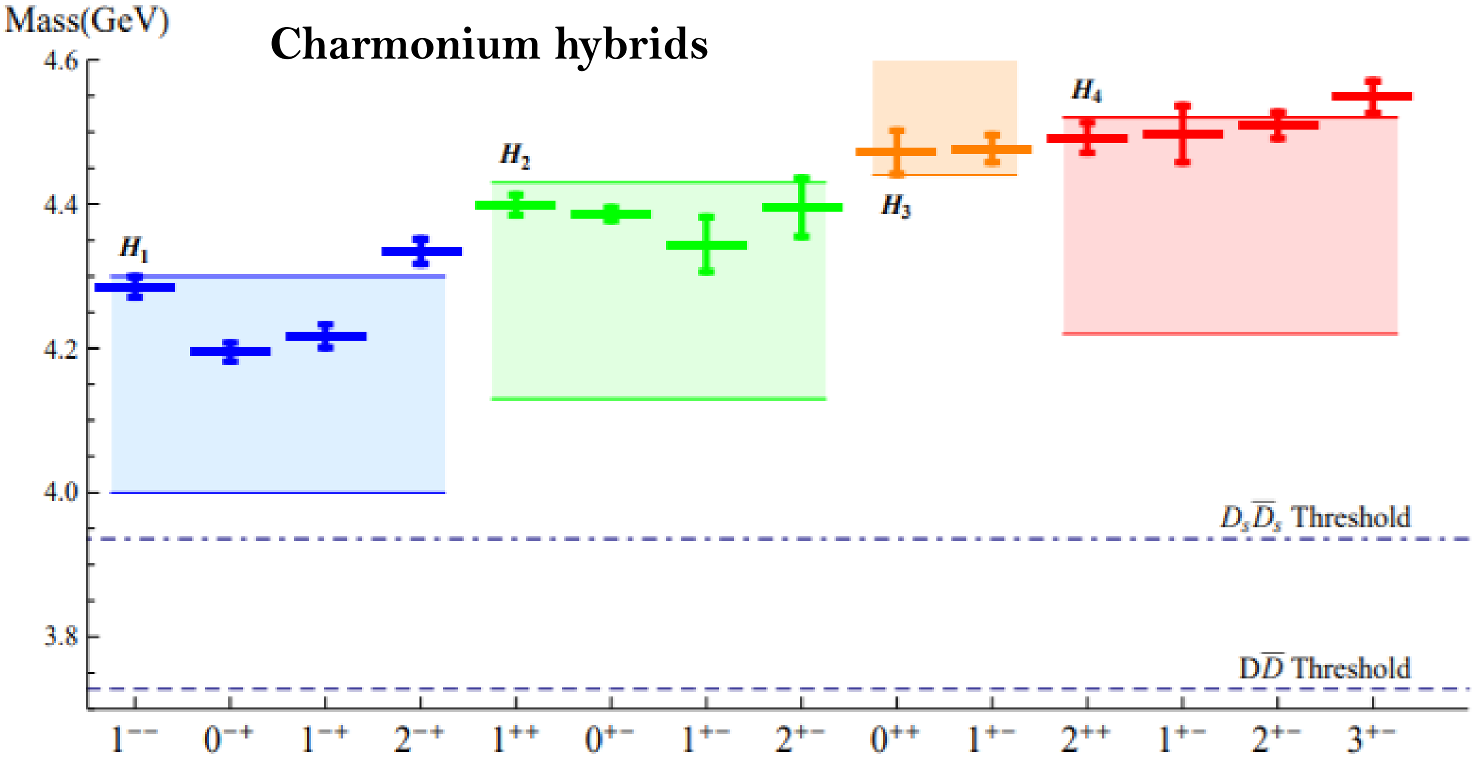


Hadrons in Charm sector:



BOEFT: Hybrids

- Comparison with lattice results ($m_\pi \approx 400$ MeV):



	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

- Λ- doubling: opposite parity states non-degenerate. Also confirmed by lattice

Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015)

Brambilla, Lai, AM, Vairo arXiv:2212.09187

Lattice data from
Hadron Spectrum collaboration JHEP 1207 (2012) 126

Hybrid Decays

- Connection with non-perturbative fields: quarkonium and hybrid in $\mathbf{r} \to \mathbf{0}$

Fields:

$$S(\mathbf{r},\mathbf{R},t) \to Z_\Psi^{1/2}(\mathbf{r})\,\Psi(\mathbf{r},\mathbf{R},t),$$

$$P^{i\dagger}_{\kappa\lambda} O^a(\mathbf{r},\mathbf{R},t)\,G^{ia}_\kappa(\mathbf{R},t) \to Z_\kappa^{1/2}(\mathbf{r})\,\Psi_{\kappa\lambda}(\mathbf{r},\mathbf{R},t)$$

Potentials:

$$E_{\Sigma_g^+}(r) = V_s(r) + b_{\Sigma_g^+} r^2 + \dots,$$

$$E_{\Sigma_u^-,\Pi_u}(r) = V_o(r) + \Lambda + b_{\Sigma,\Pi} r^2 + \dots$$

κ: denotes quantum numbers

Gluelump mass definition:

$$\langle 0|G^{ia}_{1^{+-}}(\mathbf{R},T/2)\phi^{ab}(T/2,-T/2)G^{jb}_{1^{+-}}(\mathbf{R},-T/2)|0\rangle = \delta^{ij} e^{-i\Lambda T}$$

$$\phi^{ab}(T/2,-T/2) = \mathrm{P}\exp\left\{-ig\int_{-T/2}^{T/2} dt\, A_0^{adj}(\mathbf{R},t)\right\}^{ab}$$

Eigenvalue of static NRQCD Hamiltonian

$$H_0 G^{ia}_\kappa(\boldsymbol{R},t)|0\rangle = \Lambda_\kappa G^{ia}_\kappa(\boldsymbol{R},t)|0\rangle\,,$$

$$H_0 = \int d^3R\,\frac{1}{2}\left[\boldsymbol{E}^a\cdot\boldsymbol{E}^a + \boldsymbol{B}^a\cdot\boldsymbol{B}^a\right].$$

Inclusive Decays

N. Brambilla, W.K. Lai, AM, A. Vairo (in progress)

- Spin-conserving:

$$\Gamma_{\rm Incl} = {\rm Re}\frac{2g^2}{3}\frac{T_F}{N_c}\int d^3\mathbf{r}\int_0^\infty dt\,\Psi^{i\dagger}_{(m)}(\mathbf{r})\left[e^{i\Lambda t}e^{ih_o t/2}r^k e^{-ih_s t}r^k e^{ih_o t/2}\int\frac{d^3\mathbf{k}}{(2\pi)^3}|\mathbf{k}|e^{-i|\mathbf{k}|t}\right]\Psi^i_{(m)}(\mathbf{r})\,.$$

Using complete set of octet and singlet states

$$\Gamma_{\rm Incl} = \sum_{n'}\Gamma_{m,n'} + \int\frac{d^3\mathbf{p}_s}{(2\pi)^3}\,\Gamma_{m,p_S}$$

$$\Gamma_{m,q} = \frac{4\alpha_s T_F}{3N_c}\int\frac{d^3\mathbf{l}}{(2\pi)^3}\int\frac{d^3\mathbf{l}'}{(2\pi)^3}\, f^i_{m\,\mathbf{l}}\; g^k_{\mathbf{l}\,q}\; g^{k\dagger}_{\mathbf{l}'\,q}\; f^{i\dagger}_{m\,\mathbf{l}'}\,(\Lambda + E^o_{\mathbf{l}}/2 + E^o_{\mathbf{l}'}/2 - E^s_q)^3$$

$$q = (n', p_s)$$

Inclusive Decays

N. Brambilla, W.K. Lai, AM, A. Vairo
(in progress)

Spin-preserving inclusive decay rate for $H_m \to Q_n + X$

$$\Gamma(H_m \to Q_n + X) = \frac{4\alpha_s T_F}{3N_c} \sum_{n'} |h_{nn'}|^2 \sum_{q,q'} \int dE \int dE' f^i_{mq}(E) g^j_{qn}(E) \times g^{j\dagger}_{q'n}(E') f^{i\dagger}_{mq'}(E') (\Lambda + E/2 + E'/2 - E^s_n)^3$$

Assumption:
$f^i_{mq}(E) \neq 0$ only for $E_m \approx E + \Lambda$

$h_{nn'} \approx 1$ and $E^Q_m \approx E^s_m$ (replace singlet with quarkonium)

Spin-preserving inclusive decay rate for $H_m \to Q_n + X$

$$\Gamma(H_m \to Q_n + X) = \frac{4\alpha_s T_F}{3N_c} (E_m - E^Q_n)^3 \, T^{ij} \, (T^{ij})^*$$

$$T^{ij} \equiv \int d^3r \, \Psi^{i\dagger}_m(\boldsymbol{r}) r^j \, \Phi^Q_n(\boldsymbol{r})$$

- Above result looks similar to the one in R. Oncala, J. Soto, Phys. Rev. D96, 014004 (2017). In general has **tensor structure T^{ij}** that agrees with J. Castellà, E. Passemar, arXiv:2104.03975.

Inclusive Decays

N. Brambilla, W.K. Lai, AM, A. Vairo (in progress)

- Spin-conserving decay due to $\boldsymbol{r}\cdot\boldsymbol{E}$ term :

$|S_H = 1 > \dashrightarrow |S_Q = 1 >$
$|S_H = 0 > \dashrightarrow |S_Q = 0 >$

$$\Gamma_{\rm Incl} = \sum_{n'} \Gamma_{m,n'} + \int \frac{d^3\mathbf{p}_s}{(2\pi)^3}\,\Gamma_{m,ps}$$

bound singlet states — continuum singlet states

$$\Gamma_{m,q} = \frac{4\alpha_s T_F}{3N_c}\int\frac{d^3\mathbf{l}}{(2\pi)^3}\int\frac{d^3\mathbf{l}'}{(2\pi)^3}\, f^i_{m\,\mathbf{l}}\; g^k_{\mathbf{l}\,q}\; g^{k\dagger}_{\mathbf{l}'\,q}\; f^{i\dagger}_{m\,\mathbf{l}'}\,(\Lambda + E^o_{\mathbf{l}}/2 + E^o_{\mathbf{l}'}/2 - E^s_q)^3$$

$q = (n', p_s)$

Depends on several Overlap functions:

$$f^i_{(m)\,\mathbf{l}} \equiv \langle H_m|\Phi^o_{\mathbf{l}}\rangle = \int d^3\mathbf{r}\,\Psi^{i\dagger}_{(m)}(\mathbf{r})\,\Phi^o_{\mathbf{l}}(\mathbf{r})\,,$$

$$g^k_{\mathbf{l}\,q} \equiv \langle\Phi^o_{\mathbf{l}}|r^k|\Phi^s_q\rangle = \int d^3\mathbf{r}\,\Phi^{o\dagger}_{\mathbf{l}}(\mathbf{r})\,r^k\,\Phi^s_q(\mathbf{r}),$$

$\Psi^i_{(m)}$: Hybrid wf
$\Phi^o_{\mathbf{l}}$: Octet wf
Φ^s_q : Singlet wf

✓ Cubic factor $(\Lambda + E^o_{\mathbf{l}}/2 + E^o_{\mathbf{l}'}/2 - E^s_q)^3 \sim \Delta E^3$

✓ Including continuum states can account for decay to meson-meson thresholds.

- For singlet wf : $V_s = -\,{}^{4\,\alpha_s(mv)}/_{3}$, where $v \sim 1/\sqrt{3}$ for charm and $\sim 1/\sqrt{10}$ for bottom

Semi-inclusive Decays

N. Brambilla, W.K. Lai, AM, A. Vairo (in progress)

- Spin-conserving decay due to $\boldsymbol{r}\cdot\boldsymbol{E}$ term :

$$\Gamma(H_m \to Q_n) = \sum_{n'} |w_{nn'}|^2 \Gamma_{m,n'} + \int \frac{d^3\mathbf{p}_s}{(2\pi)^3} |w_{np_s}|^2 \Gamma_{m,p_s}$$

bound singlet states — continuum singlet states

$$\Gamma_{m,q} = \frac{4\alpha_s T_F}{3N_c} \int \frac{d^3\mathbf{l}}{(2\pi)^3} \int \frac{d^3\mathbf{l}'}{(2\pi)^3} f^{i}_{m\,\mathbf{l}}\, g^{k}_{\mathbf{l}\,q}\, g^{k\dagger}_{\mathbf{l}'\,q}\, f^{i\dagger}_{m\,\mathbf{l}'} \left(\Lambda + E^o_{\mathbf{l}}/2 + E^o_{\mathbf{l}'}/2 - E^s_q\right)^3$$

$$q = (n', p_s)$$

Depends on several Overlap functions:

$$f^{i}_{(m)\,\mathbf{l}} \equiv \langle H_m | \Phi^o_{\mathbf{l}} \rangle = \int d^3\mathbf{r}\, \Psi^{i\dagger}_{(m)}(\mathbf{r})\, \Phi^o_{\mathbf{l}}(\mathbf{r}) \,,$$

$$g^{k}_{\mathbf{l}\,q} \equiv \langle \Phi^o_{\mathbf{l}} | r^k | \Phi^s_q \rangle = \int d^3\mathbf{r}\, \Phi^{o\dagger}_{\mathbf{l}}(\mathbf{r})\, r^k\, \Phi^s_q(\mathbf{r}),$$

$$w_{qn} = \int d^3\mathbf{r}\, \Phi^{s\dagger}_q(\mathbf{r}) \Phi^Q_{(n)}(\mathbf{r})$$

Quarkonium wf

✓ Significant overlap between quarkonium and continuum singlet states except for 1s quarkonium.

Decay rate **<u>different</u>** from R. Oncala, J. Soto, Phys. Rev. D96, 014004 (2017) and J. Castellà, E. Passemar, Phys. Rev. D104, 034019 (2021).

Inclusive Decays

N. Brambilla, W.K. Lai, AM, A. Vairo (in progress)

- Spin-flipping decay due to $\boldsymbol{S}.\boldsymbol{B}$ term:

$|S_H = 1 > \dashrightarrow |S_Q = 0 >$
$|S_H = 0 > \dashrightarrow |S_Q = 1 >$

$$\Gamma_{\mathrm{Incl}} = \sum_{n'} \Gamma_{m,n'} + \int \frac{d^3\mathbf{p}_s}{(2\pi)^3}\Gamma_{m,p_S}$$

bound singlet states — continuum singlet states

$$\Gamma_{m,q} = \frac{4\alpha_s T_F}{3N_c m_Q^2}\int \frac{d^3\mathbf{l}}{(2\pi)^3}\int \frac{d^3\mathbf{l}'}{(2\pi)^3} f^i_{m\,\mathbf{l}}\, g^k_{\mathbf{l}\,q}\, g^{k\dagger}_{\mathbf{l}'\,q}\, f^{i\dagger}_{m\,\mathbf{l}'}\left(\Lambda + E^o_{\mathbf{l}}/2 + E^o_{\mathbf{l}'}/2 - E^s_q\right)^3$$

$q = (n', p_s)$

Depends on several Overlap functions:

$$f^i_{(m)\,\mathbf{l}} \equiv \langle H_m|\Phi^o_{\mathbf{l}}\rangle = \int d^3\mathbf{r}\,\Psi^{i\dagger}_{(m)}(\mathbf{r})\,\Phi^o_{\mathbf{l}}(\mathbf{r})\,,$$

$$g^k_{\mathbf{l}\,q} \equiv \langle\Phi^o_{\mathbf{l}}|\left(S^k_1 - S^k_2\right)|\Phi^s_{n'}\rangle = \left[\int d^3\mathbf{r}\,\Phi^{o\dagger}_{\mathbf{l}}(\mathbf{r})\;\Phi^s_{n'}(\mathbf{r})\right]\langle\chi_o|\left(S^k_1 - S^k_2\right)|\chi_s\rangle$$

$|\chi_{o,s}\rangle$: Spin wf of octet and singlet

- $Q_m \to Q_n + X$ spin-flipping decays: Decay rate suppressed by additional $(\boldsymbol{r}\cdot\boldsymbol{E})^2 \sim v^2$ vertex factor.

Difficulties with continuum singlet states

W. Gordon, Ann. Phys. (Leipzig) 2, 1031 (1929)

A. Maquet, Phys. Rev. A 15, 1088 (1977)

- Dipole matrix element:

$$g^k_{\mathbf{k}_o\,\mathbf{p}_s} \equiv \langle \Phi^o_{\mathbf{k}_o} | \mathbf{r} | \Phi^s_{\mathbf{p}_s} \rangle = \sum_{l=0}^{\infty} \sum_{l'=0}^{\infty} \left[\int r^2 A_l^*(k_o, r)\, r\, B_{l'}(p_s, r)\, dr \right] \left[d\Omega\, P_l \left(\hat{\mathbf{k}}_o \cdot \hat{\mathbf{r}} \right) \hat{r}\, P_{l'} \left(\hat{\mathbf{p}}_s \cdot \hat{\mathbf{r}} \right) \right]$$

Continuum radial wf for Coulomb octet and singlet

After integrating over $d\Omega$: $l' = l + 1$ or $l' = l - 1$

✓ Radial matrix element:

$$R_{l,l+1}(k_o, p_s) = C_{l,l+1}(k_o, p_s)\, \mathcal{J}^{-2ip_s, -2ik_o, 1}_{l+2+i\eta_s, l+1+i\eta_o, 2l+4, 2},$$

$$R_{l,l-1}(k_o, p_s) = C_{l,l-1}(k_o, p_s)\, \mathcal{J}^{-2ik_o, -2ip_s, 1}_{l+1+i\eta_o, l+i\eta_s, 2l+2, 2}$$

$\eta_o = m_Q \alpha_s / 12 k_o$

$\eta_s = -4 m_Q \alpha_s / 6 p_s$

Smooth function of octet and singlet momentum k_o and p_s

Singular function: "Diagonal Singularity" for $k_o \to p_s$

Madajczyk, Trippenbach, J . Phys. A: Math. Gen. 22 2369 (1989)

Veniard, Piraux, Phys. Rev. A 41, 4019 (1989)

Difficulties with continuum singlet states

$$\mathcal{J}^{-2ip_s,-2ik_o,1}_{l+2+i\eta_s,l+1+i\eta_o,2l+4,2} = -\frac{(2l+3)!e^{-\frac{\pi m_Q}{4}\left(\frac{\alpha_s}{6k_o}+\frac{4\alpha_s}{3p_s}\right)}e^{-\frac{\pi m_Q}{4}\left(-\frac{\alpha_s}{6k_o}+\frac{4\alpha_s}{3p_s}\right)\mathrm{sgn}(p_s-k_o)}}{4\left(p_s-k_o\right)^{2l+4}p_s^2}\left|\frac{p_s-k_o}{k_o+p_s}\right|^{i\frac{m_Q}{2}\left(\frac{-\alpha_s}{6k_o}+\frac{4\alpha_s}{3p_s}\right)}\left({}_2F_1\left[l+2+i\eta_s,l+1+i\eta_o,2l+2,-\frac{4p_sk_o}{(p_s-k_o)^2}\right]\left(2ip_s+\frac{3m_Q\alpha_s}{2}\right)\right.$$
$$\left.+3m_Q\alpha_s\left(\frac{p_s-k_o}{k_o+p_s}\right){}_2F_1\left[l+1+i\eta_s,l+1+i\eta_o,2l+2,-\frac{4p_sk_o}{(p_s-k_o)^2}\right]+{}_2F_1\left[l+i\eta_s,l+1+i\eta_o,2l+2,-\frac{4p_sk_o}{(p_s-k_o)^2}\right]\left(-2ip_s+\frac{3m_Q\alpha_s}{2}\right)\left(\frac{p_s-k_o}{k_o+p_s}\right)^2\right)$$

$$\mathcal{J}^{-2ik_o,-2ip_s,1}_{l+1+i\eta_o,l+i\eta_s,2l+2,2} = \frac{(2l+1)!e^{\frac{-\pi m_Q}{4}\left(\frac{\alpha_s}{6k_o}+\frac{4\alpha_s}{3p_s}\right)}e^{-\frac{\pi m_Q}{4}\left(-\frac{\alpha_s}{6k_o}+\frac{4\alpha_s}{3p_s}\right)\mathrm{sgn}(p_s-k_o)}}{4\left(p_s-k_o\right)^{2l+2}k_o^2}\left|\frac{p_s-k_o}{k_o+p_s}\right|^{i\frac{m_Q}{2}\left(\frac{-\alpha_s}{6k_o}+\frac{4\alpha_s}{3p_s}\right)}\left({}_2F_1\left[l+1+i\eta_o,l+i\eta_s,2l,-\frac{4p_sk_o}{(p_s-k_o)^2}\right]\left(-2ik_o+\frac{3m_Q\alpha_s}{2}\right)\right.$$
$$\left.-3m_Q\alpha_s\left(\frac{p_s-k_o}{k_o+p_s}\right){}_2F_1\left[l+i\eta_o,l+i\eta_s,2l,-\frac{4p_sk_o}{(p_s-k_o)^2}\right]+{}_2F_1\left[l-1+i\eta_o,l+i\eta_s,2l,-\frac{4p_sk_o}{(p_s-k_o)^2}\right]\left(2ik_0+\frac{3m_Q\alpha_s}{2}\right)\left(\frac{p_s-k_o}{k_o+p_s}\right)^2\right)$$

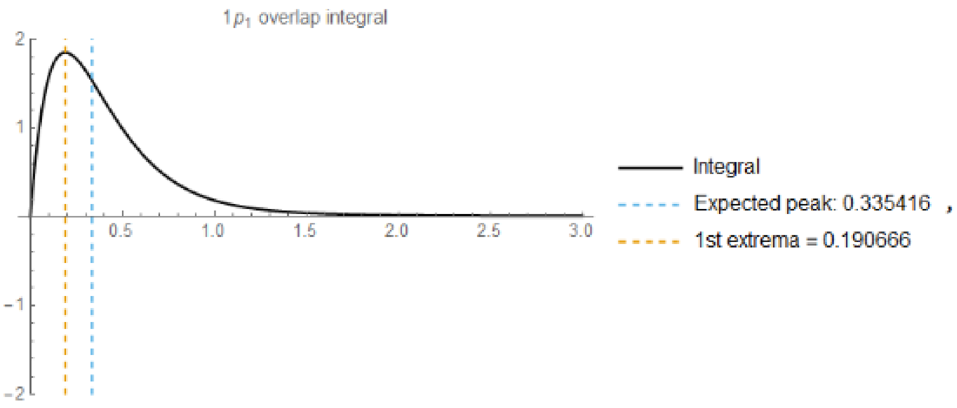
"Diagonal Singularity" for $k_o \to p_s$: Singular Gauss hypergeometric $2F_1$ function

Inclusive Decays

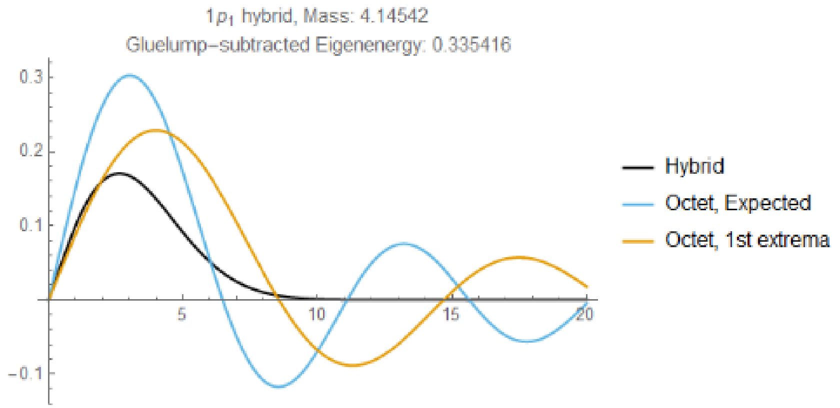
It is interesting to see how $f^i_{mq}(E) = \left[\int d^3r\, \Psi^{i\dagger}_m(\boldsymbol{r})\Phi^o_{E,q}(\boldsymbol{r})\right]$ looks like as a function of E:

N. Brambilla, W.K. Lai, AM, A. Vairo (in progress)

H_2-multiplet, $l = 1$, $J^{PC} = [1^{++}, (0,1,2)^{+-}]$
$H_2(4145)$:



Radial integral of $f^i_{mq}(E)$ vs E (GeV)



Radial hybrid wave function vs r (GeV^{-1})

- The actual peak is slightly off (at a lower E) from the expected peak at $E = E_m - \Lambda$.
- The peak is broad, with width ~ 1 GeV. The assumption that $f^i_{mq}(E)$ is nonzero only when $E_m \approx E + \Lambda$ is not true.

Singlet-Quarkonium overlap

2) Overlap of Coulomb singlet bound states & Quarkonium

$$w_{nn'} = \langle \Phi_n^{Q\bar{Q}} | \Phi_{n'}^{S} \rangle$$

$\Phi_n^{Q\bar{Q}}$ → Quarkonium wf

$\Phi_{n'}^{S}$ → Coulomb Singlet bound state wf.

3) Overlap of continuum Singlet & Quarkonium

$$\omega_n = \int \frac{d^3k}{(2\pi)^3} \left| \langle \Phi_n^{Q\bar{Q}} | \Phi_k^{S} \rangle \right|^2$$

Φ_k^{S} → continuum singlet states

Charm

n	$\sum_{n'} \lvert w_{nn'} \rvert^2$	ω_n
1s	0.73	0.26
2s	0.26	0.72
1p	0.18	0.76
2p	0.20	0.73

Bottom.

n	$\sum_{n'} \lvert W_{nn'} \rvert^2$	ω_n
1s	0.90	0.10
2s	0.42	0.55
1p	0.45	0.51
2p	0.37	0.59