Tetraquark bound states in quark potential models

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Based on PRD107(2023),054035 and papers in preparation

Together with Yan-Ke Chen, Yao Ma and Shi-Lin Zhu (PKU)

- Background
- Gaussian expansion method
- Resonating group method
- Diffusion Monte Carlo Method
- Summary

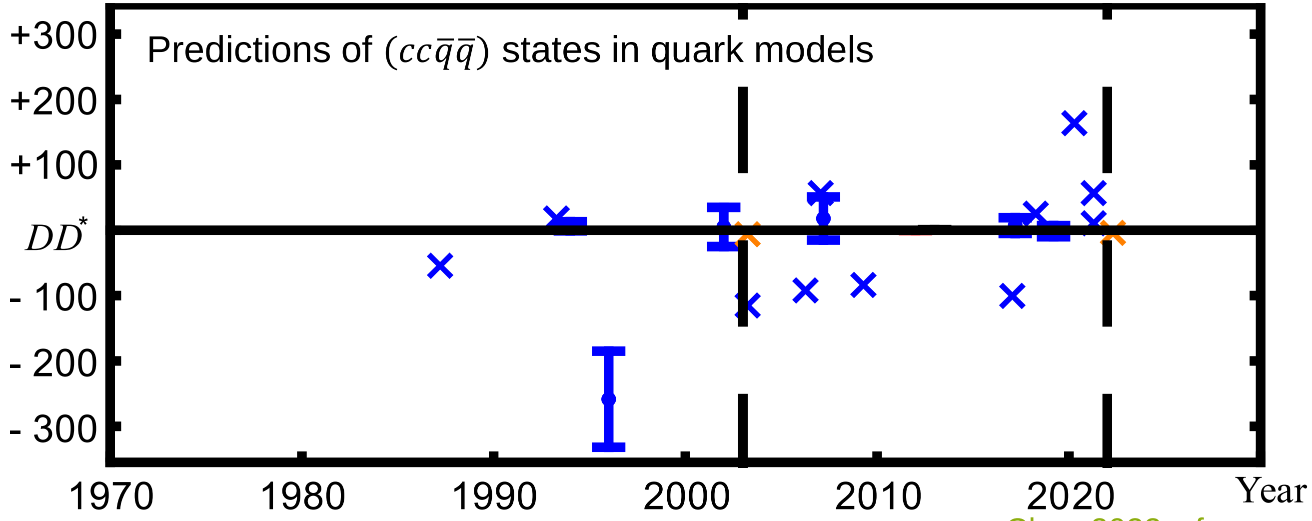
Tetraquark states

- Recently, more and more hadrons composed of at least four quarks were observed

New naming scheme: Gershon:2022xnn

$[cc\bar{c}\,\bar{c}]$ $X(6900)$	$[cs\bar{u}\,\bar{d}]$ $T_{cs1}(2900)$ $T_{cs0}(2900)$	$[cs\bar{u}\,\bar{d}]$ $Z_{cs}(3985)$ $Z_{cs}(4000)$	$[cc\bar{u}\,\bar{d}]$ $T_{cc}(3875)^+$	$[c\bar{s}u\bar{d}][c\bar{s}\bar{u}d]$ $T_{c\bar{s}0}(2900)^{++}$ $T_{c\bar{s}0}(2900)^0$
2006.16957 2304.08962 …	2009.00025 2009.00026	2011.07855 2103.01803	2109.01038 2109.01056	2212.02716 2212.02717

- Different quark models predicted different results
  - ▶ Example: T_{cc} states



Chen:2022asf

- What is responsible for variations?

InteractionS + few-body methodS

- Benchmark calculations

(AL1,AP1,SLM)$\otimes$(GEM,RGM,DMC)

Quark potential models

- Cornell model: One-gluon-exchange+Confinement

$$V_{ij}(r) = \left[\frac{\alpha_s}{r} + \left(-\frac{3b}{4}r + V_c\right) - \frac{8\pi\alpha_s}{3m_i m_j}\frac{\tau^3}{\pi^{3/2}}e^{-\tau^2 r^2}\frac{\lambda_i}{2}\frac{\lambda_j}{2}\boldsymbol{s}_i \cdot \boldsymbol{s}_j\right]\frac{\lambda_i \cdot \lambda_j}{4}$$

- Semay-Silvestre-Brac Models

Semay:1994ht, Silvestre-Brac:1996myf

$$V_{ij}(r) = \left[-\frac{\kappa}{r} + \lambda r^p - \Lambda + \frac{2\pi}{3m_i m_j}\kappa' \frac{1}{\pi^{3/2} r_0^3} e^{(-r^2/r_0^2)}\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\right]\lambda_i \cdot \lambda_j$$

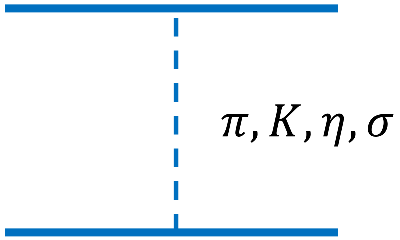
AL1: $p = 1$ and AP1: $p = 2/3$

- Chiral quark models [e.g Salamanca model (SLM)]

Vijande:2004he, Gonzalez:2012gka

$$V_{ij}(r) = \left[\frac{\alpha_s}{4}\left(\frac{1}{r} - \frac{1}{6m_i m_j}\frac{e^{-r/r_0}}{r_0^2 r}\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\right) + \underbrace{\left(-a_c(1 - e^{-\mu_c r}) + \Delta\right)}_{\text{Screened confinement}}\right]\lambda_i \cdot \lambda_j$$

$$+V_\pi + V_K + V_\eta + V_\sigma$$



- In this work, we use AL1, AP1 and SLM

Gaussian Expansion Method

Gaussian Expansion Method

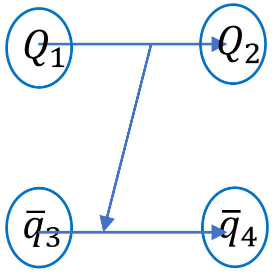
- Color functions

$$\begin{cases}[(Q_1Q_2)_{\bar{3}}(\bar{q}_3\bar{q}_4)_3]_1\\ [(Q_1Q_2)_6(\bar{q}_3\bar{q}_4)_{\bar{6}}]_1\end{cases} \quad \text{Or} \quad \begin{cases}[(Q_1\bar{q}_3)_1(Q_2\bar{q}_4)_1]_1\\ [(Q_1\bar{q}_4)_1(Q_2\bar{q}_3)_1]_1\end{cases} \quad \text{Or} \quad \begin{cases}[(Q_1\bar{q}_3)_1(Q_2\bar{q}_4)_1]_1\\ [(Q_1\bar{q}_4)_8(Q_2\bar{q}_3)_8]_1\end{cases}$$

- Spin wave function

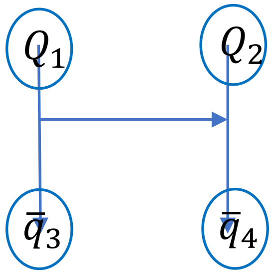
$$S_{12} = 0,1;\ S_{34} = 0,1 \quad S_{12}\otimes S_{34} \to J \qquad \text{Or} \qquad S_{13} = 0,1;\ S_{24} = 0,1 \quad S_{13}\otimes S_{24} \to J \qquad \text{Or} \qquad S_{14} = 0,1;\ S_{23} = 0,1 \quad S_{14}\otimes S_{23} \to J$$

- Spatial wave functions



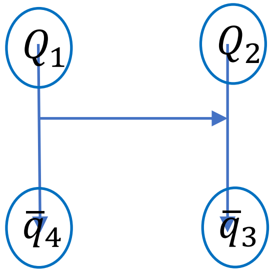
Diquark-antidiquark

And



Di-meson

And



Di-meson

$$\phi_{nlm}(\boldsymbol{r}) = N_{lm} r^l e^{-\frac{r^2}{r_n^2}} Y_{lm}(\hat{r})$$

Geometric progression

$$r_n = r_0 a^{n-1}$$

Hiyama:2003cu

Embed both long- and short-range correlations

- Antisymmetrization (e.g. $Q_1 = Q_2$ and $q_3 = q_4$) :

$$\psi = \mathcal{A}[\psi_{color} \otimes \psi_{spin} \otimes \psi_{spatial} \otimes \psi_{flavor}], \qquad \mathcal{A} = (1 - P_{12})(1 - P_{34})$$

Tetraquark systems

- Fully heavy tetraquark states ($QQ\bar{Q}\bar{Q}$)
- Triply heavy tetraquark states ($QQ\bar{Q}\bar{q}$)
- Doubly heavy tetraquarks states ($QQ\bar{q}\bar{q}$)
- Single heavy strange states ($Qs\bar{q}\bar{q}$, $Q\bar{s}q\bar{q}$)

$q = u, d, s;\ \ Q = b, c$

- $J^P = 0^+, 1^+, 2^+$
  - ▶ Only S-wave

Over 150 states

- In this work, we only focus on bound states

	$QQ\bar{Q}\bar{Q}$	$QQ\bar{Q}\bar{q}$	$QQ\bar{q}\bar{q}$	$Qs\bar{q}\bar{q}$	$Q\bar{s}q\bar{q}$
$J^P = 0^+$	No bound	No bound	😊	😊	No bound
$J^P = 1^+$	No bound	No bound	😊	😊	No bound
$J^P = 2^+$	No bound	No bound	😊	😊	No bound

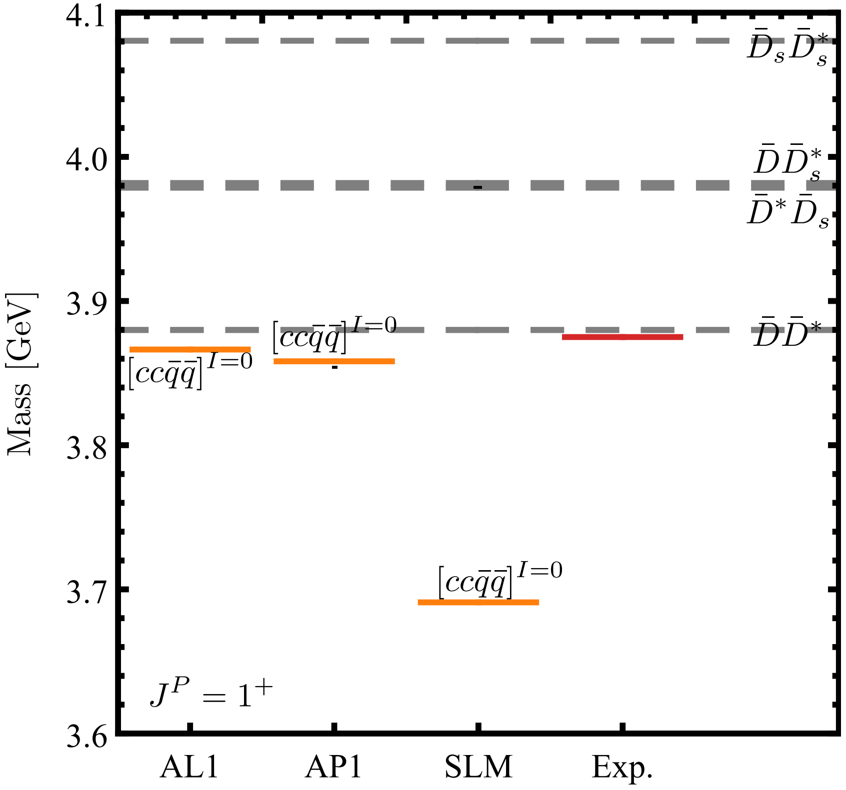
- Masses are shifted to align the theoretical thresholds with the physical ones.

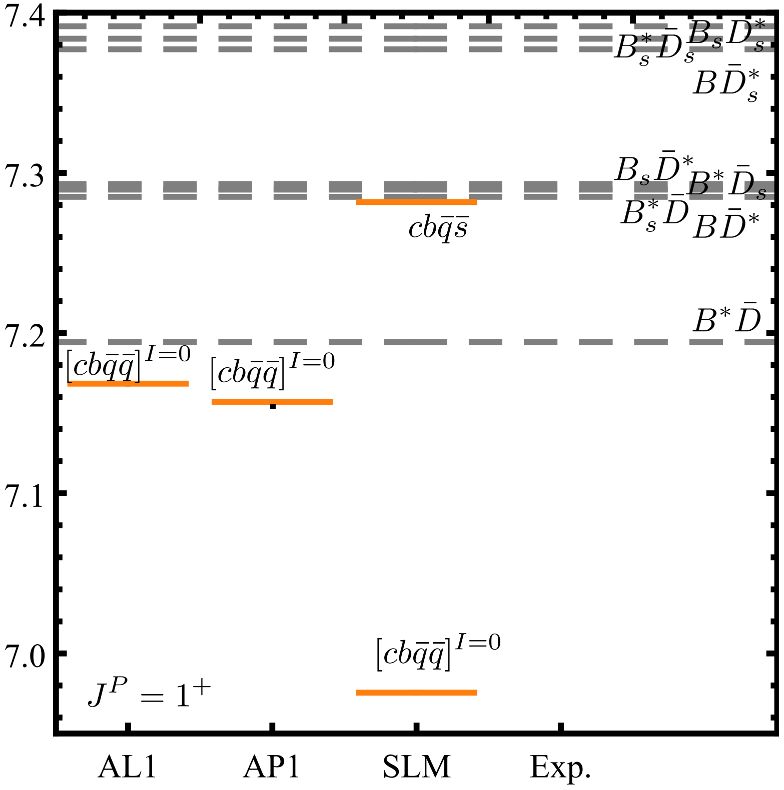
$QQ\bar{q}\,\bar{q}$ with $J^P = 1^+$

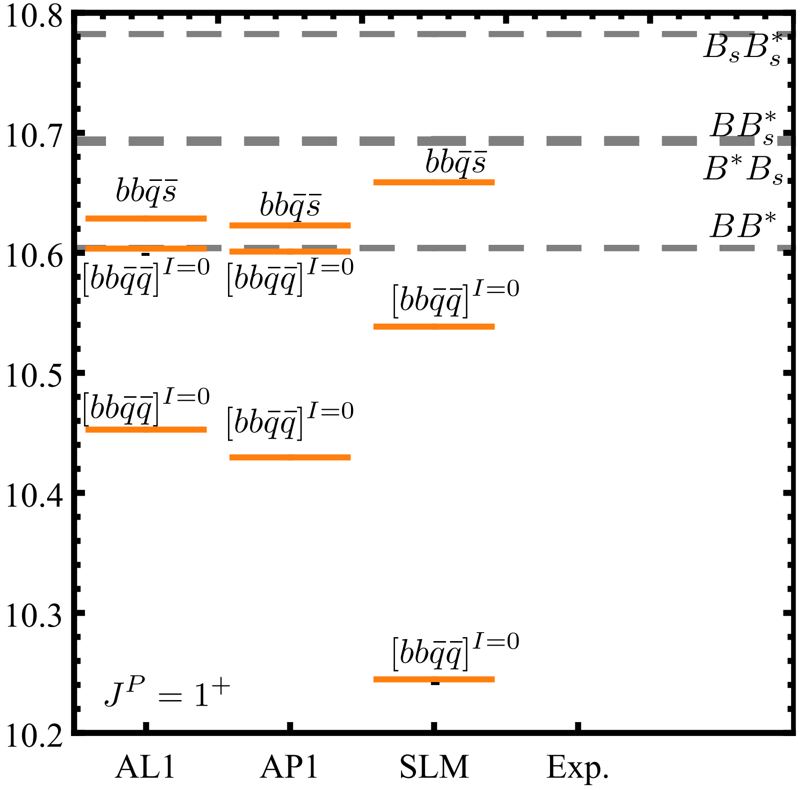
Preliminary

- Points of agreement
  - ▶ $[QQ\bar{q}\,\bar{q}]^{I=0}$ ($QQ = cc$ or bb or bc) bound states ; $[bb\bar{q}\bar{s}]$ bound states
  - ▶ For $[bb\bar{q}\,\bar{q}]^{I=0}$ systems, the 1st excited states are bound states
  - ▶ No $[QQ\bar{q}\,\bar{q}]^{I=1}$ states
- SLM

  (1) $[cc\bar{q}\bar{q}]^{I=0}$ are too deep compared with ex. (200MeV VS 200 keV); (2) $[cb\bar{q}\bar{s}]$ bound states



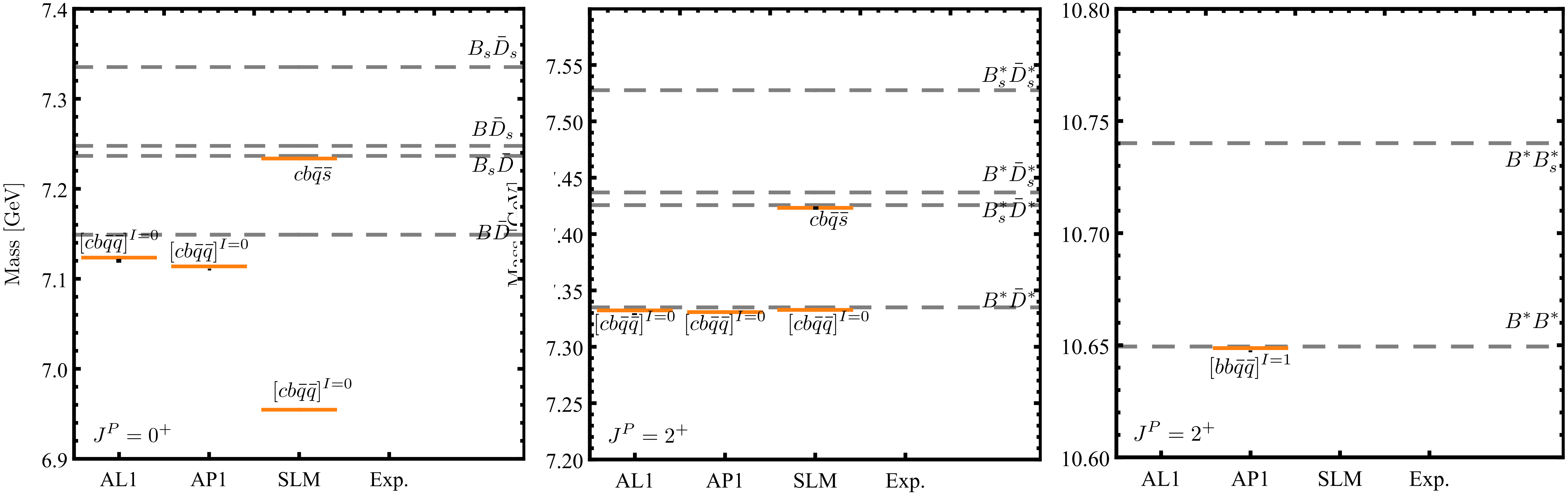




$QQ\bar{q}\,\bar{q}$ with $J^P = 0^+, 2^+$

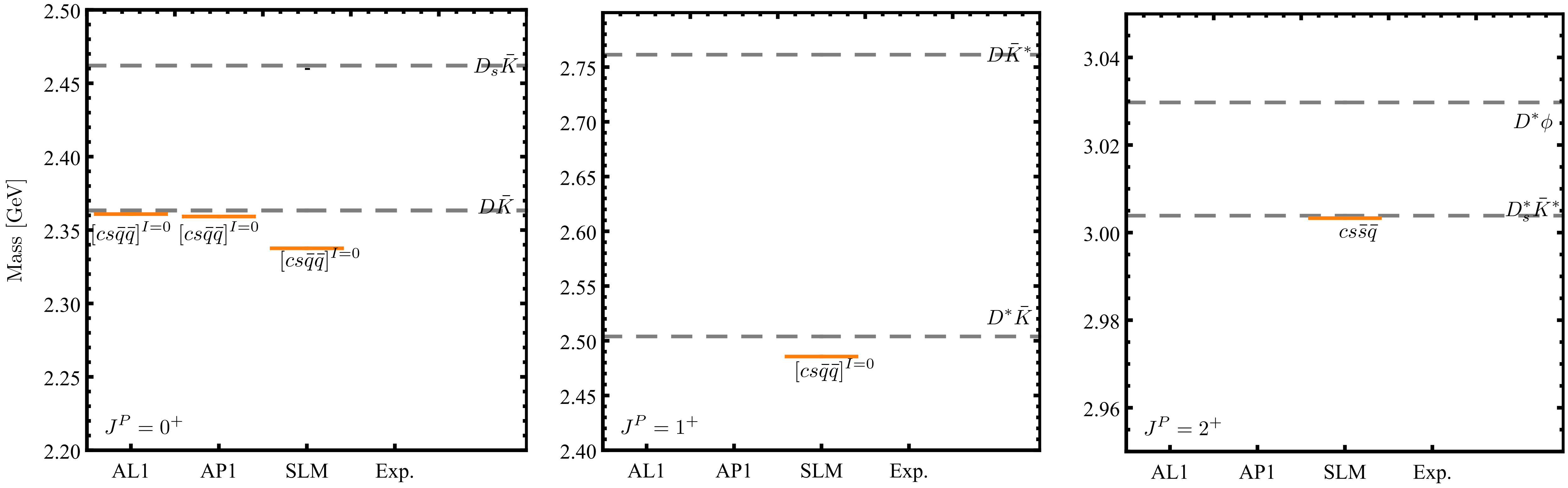
Preliminary

- Points of agreement
  - $[cb\bar{q}\,\bar{q}]^{I=0}$ bound states for $J^P = 0^+, 2^+$
- SLM: $cb\bar{q}\bar{s}$ bound states for $J^P = 0^+, 2^+$
- AP1: $bb\bar{q}\bar{s}$ bound states for $J^P = 2^+$



$cs\bar{q}\bar{q}$ systems with $J^P = 0^+, 1^+, 2^+$

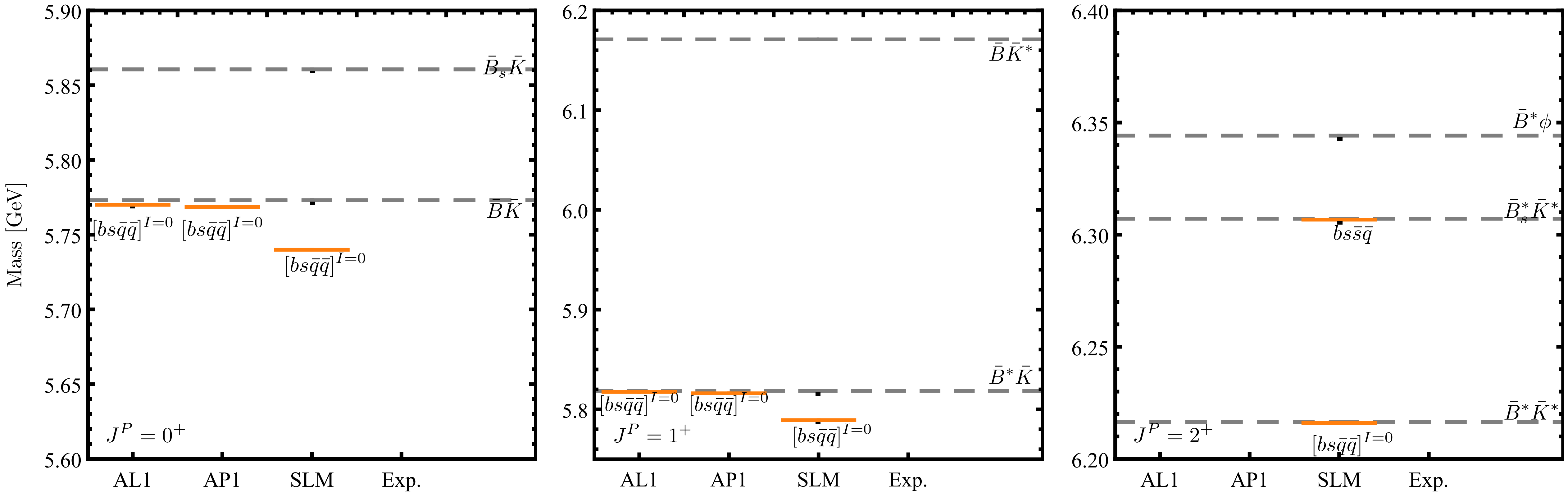
- Points of agreement
  - ▶ $[cs\bar{q}\,\bar{q}]^{I=0}$ bound states for $J^P = 0^+$
- SLM:
  - ▶ $[cs\bar{q}\bar{q}]^{I=0}$ for $J^P = 1^+$and $cs\bar{s}\bar{q}$ for $J^P = 2^+$ bound states
- Note: The experimental $T_{cs0}(2900)$ and $T_{cs1}(2900)$ are close to $D^*\bar{K}^*$ thresholds, resonances



$bs\bar{q}\bar{q}$ systems with $J^P = 0^+, 1^+, 2^+$

- Points of agreement
  - $[bs\bar{q}\,\bar{q}]^{I=0}$ bound states for $J^P = 0^+, 1^+$
- SLM:
  - $[bs\bar{q}\bar{q}]^{I=0}$ and $bs\bar{s}\bar{q}$ for $J^P = 2^+$ bound states

SLM tends to predict extra states



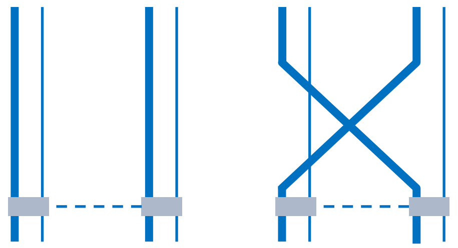
Resonating Group Method

Resonating Group Method

- Dimeson-wave function

$$\psi_{AB}(\boldsymbol{P}) = \mathcal{A}[\phi_A(\boldsymbol{p}_A)\phi_B(\boldsymbol{p}_B)\chi(\boldsymbol{P})\chi_{AB}^{CST}]$$

  - ▶ ϕ_A and ϕ_B are meson wave functions
  - ▶ We use GEM to get the meson wave functions
  - ▶ $\mathcal{A}$ represents antisymmetriziation operator of identical quarks



Entem:2000mq, Ortega:2022efc

- Schrodinger equation of RGM

$$\left(\frac{\boldsymbol{P}'^2}{2\mu} - E\right)\chi(\boldsymbol{P}') + \int d^3\boldsymbol{P}\left(V_D(\boldsymbol{P}',\boldsymbol{P}) + K_{Ex}(\boldsymbol{P}',\boldsymbol{P})\right)\chi(\boldsymbol{P}) = 0$$

  - ▶ V_D direct interaction, K_{Ex} the exchange kernel

- Compared with GEM
  - ▶ The spin-color-flavor wave functions are complete as well
  - ▶ The RGM neglecting the distortion of the meson wave functions in the tetraquark system
  - ▶ Only the di-meson-type spatial correlations are included
  - ▶ The trial functions are not as general as GEM

$$E_{RGM} \gtrsim E_{GEM}$$

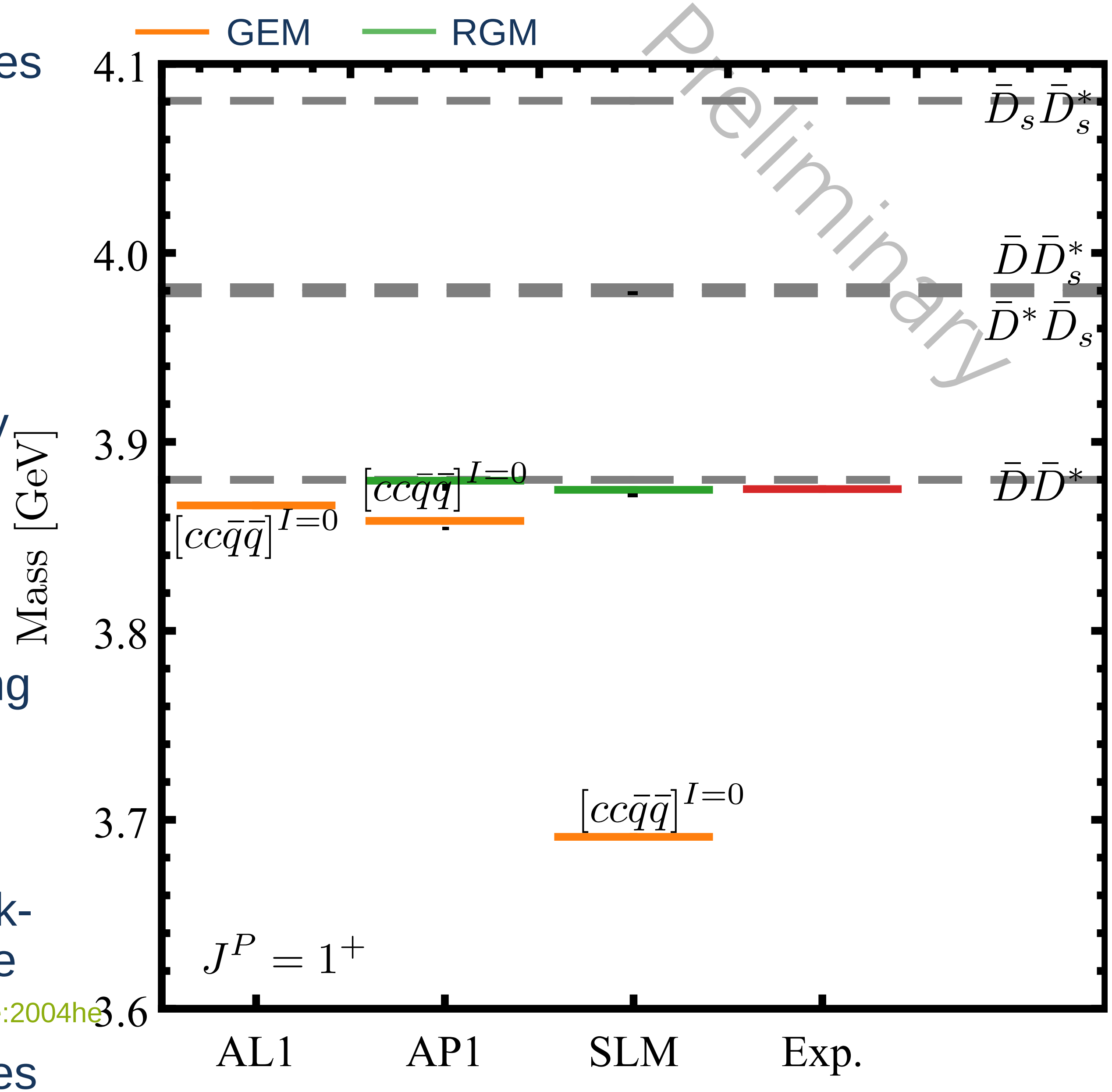
RGM results

- The RGM gives the smaller binding energies
  - ▶ Without the diquark-antidiquark-type correlation
- The RGM results agree with the GEM neglecting diquark-antidiquark correlation
  - ▶ Not general enough trail wave function
  - ▶ Cannot get the ground state accurately
  - ▶ A drawback as a few-body method

However…

- Some quark models (e.g. SLM) constraining the para. using NN phase shifts with RGM
  - ▶ The spatial correlations other than di-hadron types are neglected from birth
  - ▶ Perhaps, it is misleading to use diquark-antidiquark type trial functions for these models
  - ▶ Otherwise, deeper or extra bound states

Entem:2000mq, Vijande:2004he



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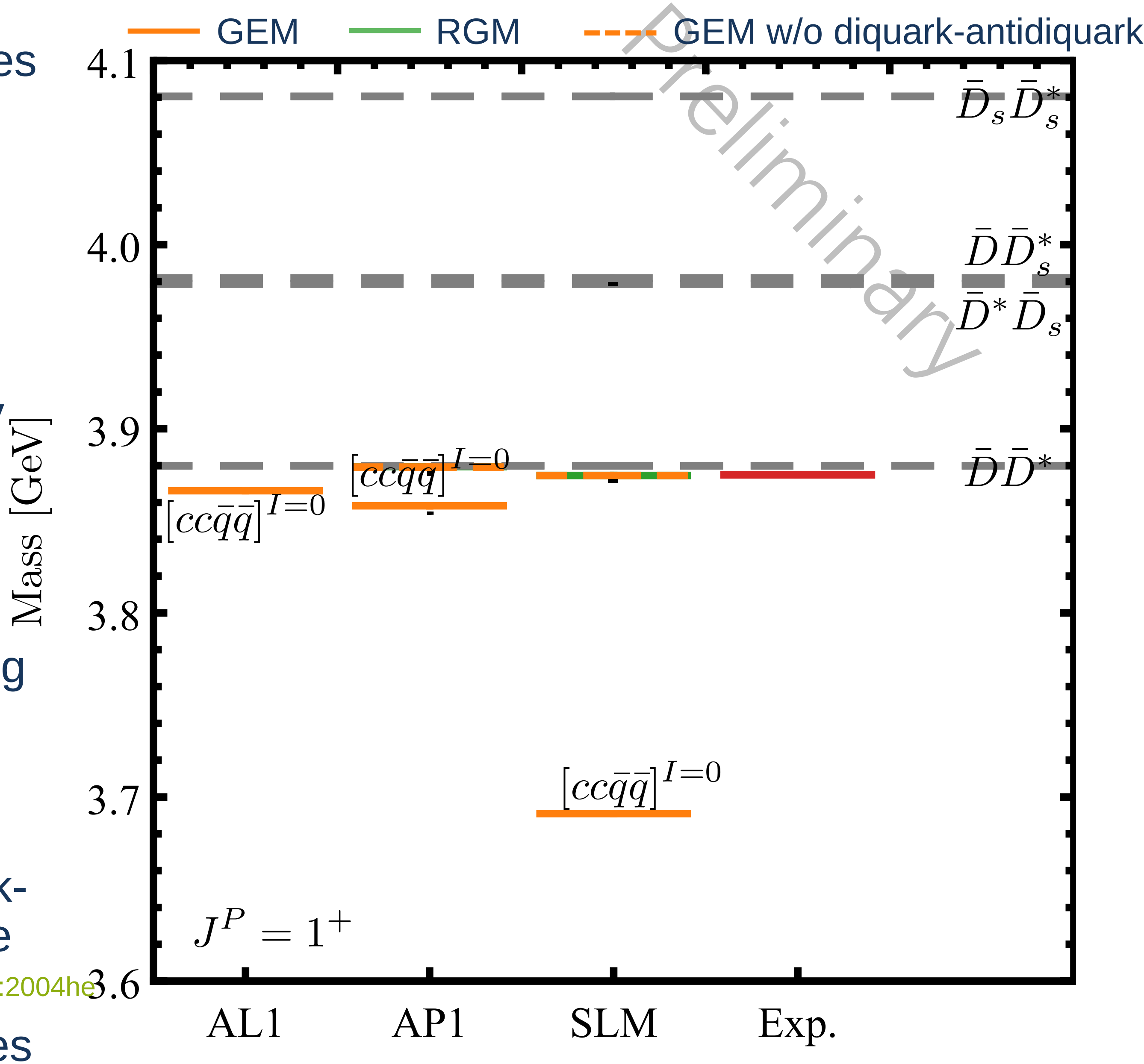


Diffusion Monte Carlo method

Diffusion Monte Carlo

- Imaginary Schrödinger equation

$$-\frac{\partial\Psi(\boldsymbol{R},t)}{\partial t}=[H-E_R]\Psi(\boldsymbol{R},t),\quad \Psi(\boldsymbol{R},t)=\sum_i c_i\Phi_i(\boldsymbol{R})e^{-[E_i-E_R]t}$$

  - ▶ If we take $E_R\to E_0$, the $\Psi(\boldsymbol{R},t)$ will approach to the ground state when $t\to\infty$
- The wave function is sampled by walkers
  - ▶ The distribution of the walkers with $\boldsymbol{R}$ represent $\Psi(\boldsymbol{R},t)$
- Importance sampling: $f(\boldsymbol{R},t)=\Psi(\boldsymbol{R},t)\psi_T(\boldsymbol{R})$ ⇒Convection–diffusion equation
- Drift, Diffusion, Birth-Death and repeating…

$$-\frac{\partial f(\boldsymbol{R},t)}{\partial t}=\underbrace{-\sum_{i=1}^{m}\frac{1}{2m_i}\boldsymbol{\nabla}^2_{\boldsymbol{r}_i}f(\boldsymbol{R},t)}_{\text{Random walk}}+\underbrace{\sum_{i=1}^{m}\frac{1}{2m_i}\boldsymbol{\nabla}_{\boldsymbol{r}_i}(\boldsymbol{F}_i(\boldsymbol{R})f(\boldsymbol{R},t))}_{\text{Drift}}+\underbrace{[E_L(\boldsymbol{R})-E_R]f(\boldsymbol{R},t)}_{\text{Sink or source}},$$

  - ▶ $E_L(\boldsymbol{R})=\psi_T(\boldsymbol{R})^{-1}\widehat{H}\psi_T(\boldsymbol{R})$ and $\boldsymbol{F}_i(\boldsymbol{R})=2\psi_T(\boldsymbol{R})^{-1}\nabla_{r_i}\psi_T(\boldsymbol{R})$
- No presumed clustering
- Milder increase computational cost as particles numbers
- High precision (in molecular physics and nuclear physics)

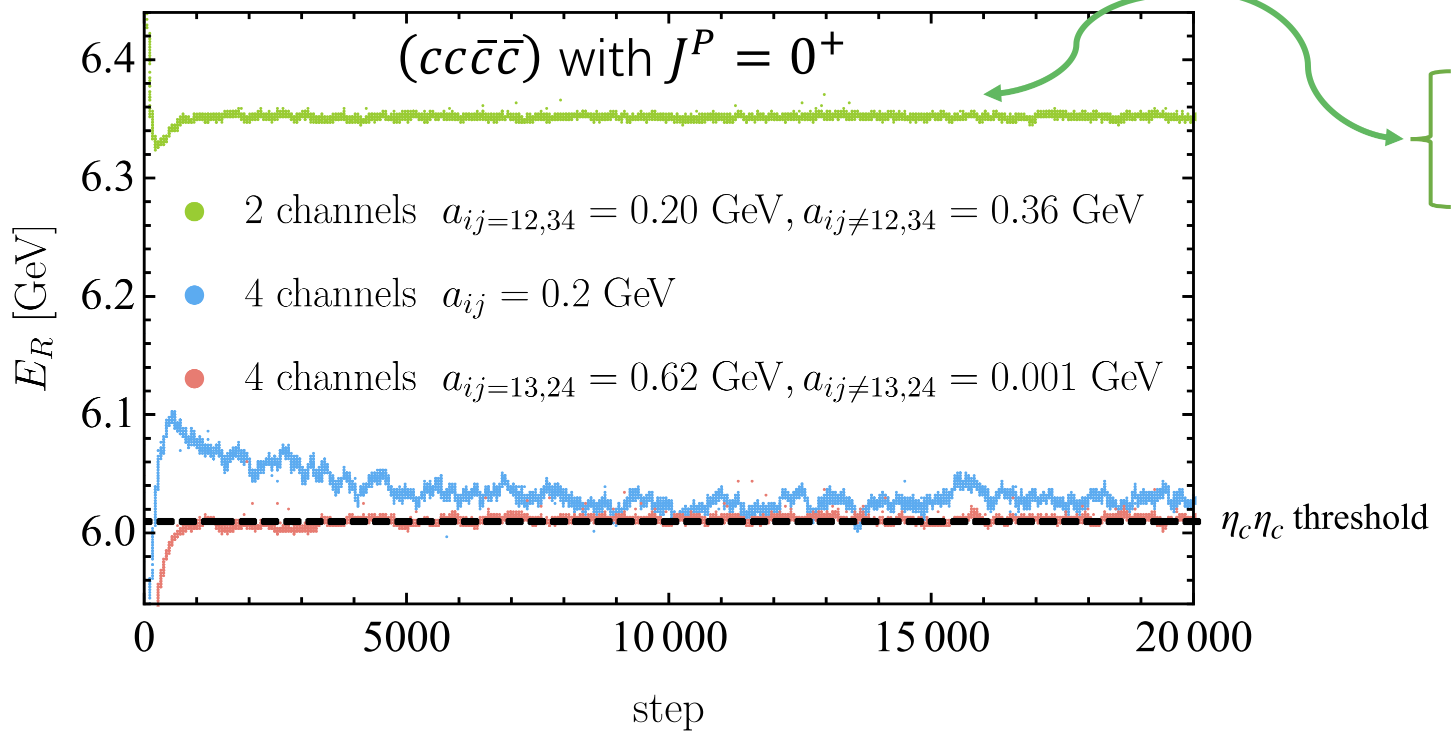
Advantages

Hjorth-Jensen:2017gss, Bai:2016int, Gordillo:2020sgc, Ma:2022vqf

DMC in quark models

- Unique features of multiquarks: complicate color structures and confinement
- In literature, it was proposed a method to deal with coupled channels Gordillo:2020sgc
  - ▶ Cannot get the di-meson thresholds (real ground state) for the systems w/o bound states
  - ▶ The four-quark threshold makes no sense due to confinement
- Our advancement: including the extra two channels Ma:2022vqf
  - ▶ Obtain the di-meson thresholds independent of the importance functions

Gordillo:2020sgc



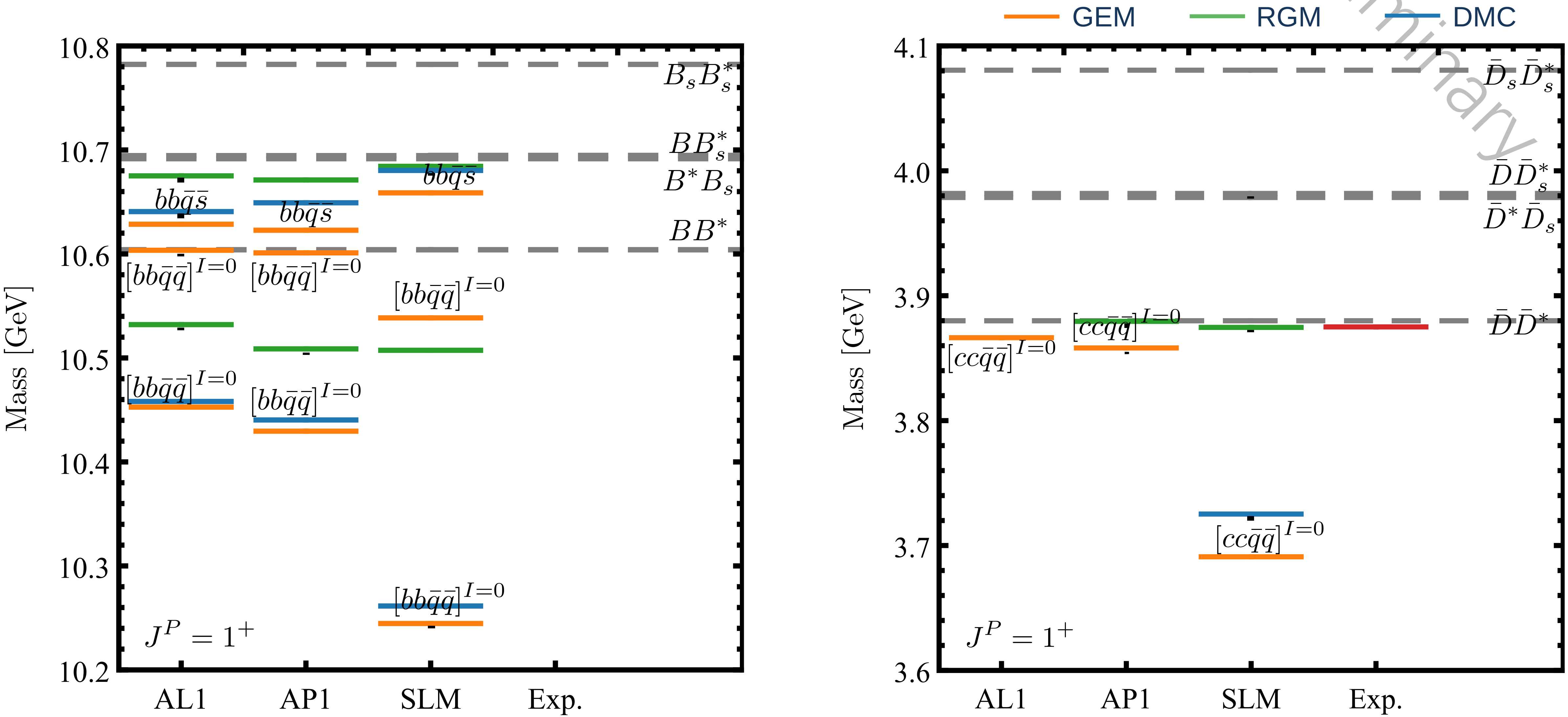
$$
\begin{aligned}
|T1\rangle &= \left[(12)^{0_s}_{\bar{3}_c}(34)^{0_s}_{3_c}\right]^{0_s}_{1_c}\psi_1^{SS}(12;34),\\
|T2\rangle &= \left[(12)^{1_s}_{6_c}(34)^{1_s}_{\bar{6}_c}\right]^{0_s}_{1_c}\psi_2^{SS}(12;34)\\
|T3\rangle &= \left[(12)^{1_s}_{\bar{3}_c}(34)^{1_s}_{3_c}\right]^{0_s}_{1_c}\psi_3^{AA}(12;34),\\
|T4\rangle &= \left[(12)^{0_s}_{6_c}(34)^{0_s}_{\bar{6}_c}\right]^{0_s}_{1_c}\psi_4^{AA}(12;34)
\end{aligned}
$$

Ours

$$\psi_T(\boldsymbol{R}) = \prod_{i<j} e^{-a_{ij}r_{ij}}\ .$$

Results from DMC

- The DMC with the present coupled-channel strategy give the higher energy than GEM
- The coupled-channel strategy need to be improved in the future



Summary and outlook

- Investigate the tetraquark bound states with (AL1,AP1,SLM)⊗(GEM,RGM,DMC)
  - ► $(QQ\bar{Q}\bar{Q})$, $(QQ\bar{Q}\bar{q})$, $(QQ\bar{q}\bar{q})$, $(Qs\bar{q}\bar{q})$, $(Q\bar{s}q\bar{q})$
- Recommended tetraquark states below di-meson thresholds (consistent predictions of 3 models)

$J^P=1^+$	$[cc\bar{q}\bar{q}]^{I=0}$	$[bb\bar{q}\bar{q}]^{I=0}$	$[bc\bar{q}\bar{q}]^{I=0}$	$bb\bar{q}\bar{s}$	$[bs\bar{q}\bar{q}]^{I=0}$
$J^P=0^+$	$[cb\bar{q}\bar{q}]^{I=0}$	$[cs\bar{q}\bar{q}]^{I=0}$	$[bs\bar{q}\bar{q}]^{I=0}$		
$J^P=2^+$	$[cb\bar{q}\bar{q}]^{I=0}$				

- The trial functions of RGM are not general enough to give the ground state
  - ► For quark models born with RGM, it is inconsistent to include diquark-antidiquark correlations
- DMC: improved to give the di-meson threshold
  - ► By now, has no advantages for tetraquark bound states compared with GEM
- Outlook:
  - ► Resonances and virtual states (on-going)

    E.g. T_{cs} and $T_{c\bar{s}}$ states, HQSS partner of T_{cc} close to $D^*D^*, J=1$ Albaladejo:2021vln,
  - ► DMC: promising

    Auxiliary field diffusion Monte Carlo Gandolfi:2007ed

    Flux-tube confinement potentials

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    Auxiliary field diffusion Monte Carlo

    Flux-tube confinement potentials

Thanks for your attention!

Albaladejo:2021vln,

Gan

Extracting V from Ψ, HALQCD related talk ,
09:25, 9th June, DAD - Room 5H
Speaker: Lu Meng

Backup

Benchmark test calculation of a four-nucleon bound state

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W. Leidemann and G. Orlandini
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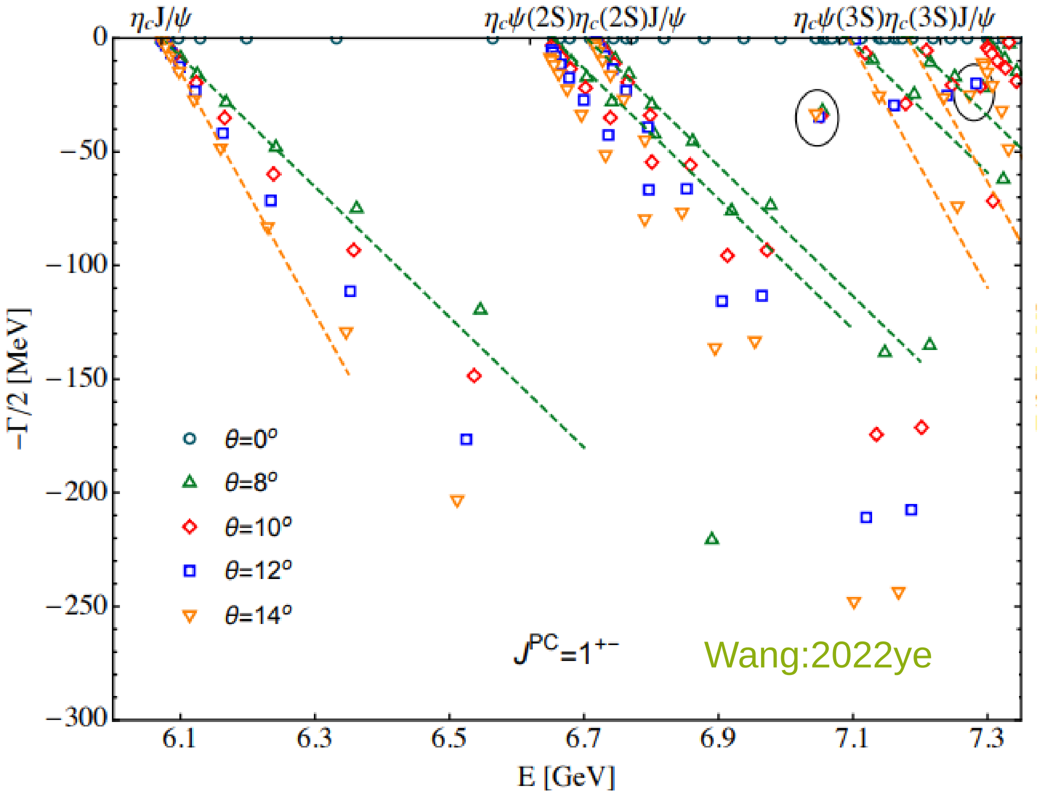
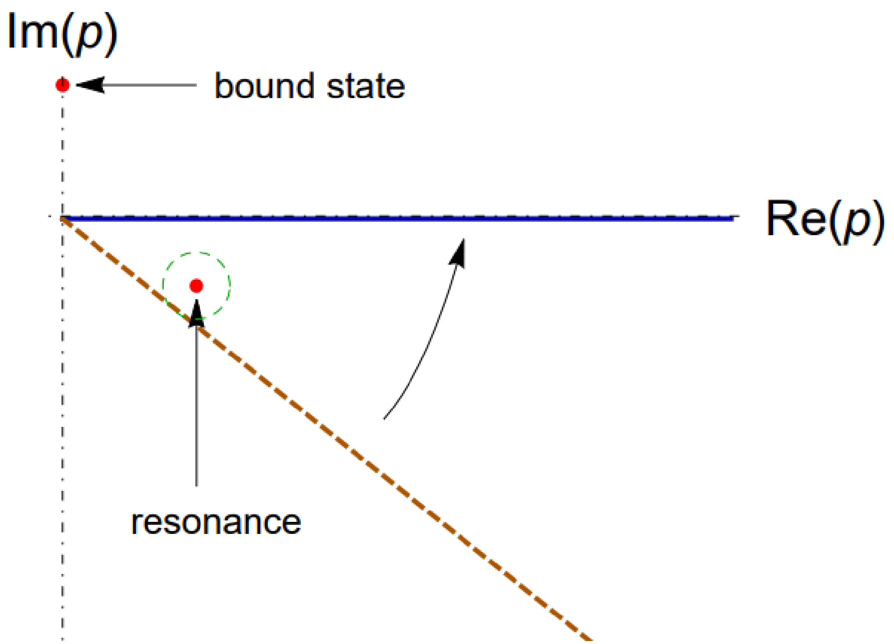
(Received 20 April 2001; published 27 August 2001)

Methods to obtain resonance and virtual states

- Complex scaling methods with GEM
  - ▶ It is hard to detect the higher states
  - ▶ The unclear relation with Riemann surface
  - ▶ The tetraquark resonance: two-body scattering problems (confinement)
- RGM + Complex Scaling in coupled-channel two-body problem



Solving Freedholm determinant⇒ Eigenvalue problem



Comparison

TABLE VI. Mass and binding energy (in MeV/c^2) and probabilities of each channel (in %) for the $J^P = 1^+$ T_{bb} states predicted in this work.

Mass	E_B	$\mathcal{P}_{B^0B^{*+}}$	$\mathcal{P}_{B^+B^{*0}}$	$\mathcal{P}_{B^{*+}B^{*0}}$	$\mathcal{P}_{I=0}$	$\mathcal{P}_{I=1}$
10582.2	21.9	47.8	50.0	2.2	99.99	0.01
10593.5	10.5	51.0	48.6	0.4	0.02	99.98

Our results: there is no isospin vector states

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10582.2	21.9	47.8	50.0	2.2	99.99	0.01
10593.5	10.5	51.0	48.6	0.4	0.02	99.98

TABLE VII. Properties of the T_{bb} candidates as $B^{(*)}B^{(*)}$ molecules in the $J^P = 0^+$ and 2^+ sectors obtained in this work. Masses, widths, binding energies and partial widths are shown in MeV/c^2.

J^P	I	Mass	Width	E_B	$\mathcal{P}_{BB}$	$\mathcal{P}_{B^*B^*}$	Γ_{BB}	$\Gamma_{B^*B^*}$
0^+	0	10553.0	0	6.0	92%	8%	0	0
		10640.7	2.8	8.7	76%	24%	2.8	0
	1	10545.9	0	13.1	93%	7%	0	0
		10672.6	72.0	−23.2	39%	61%	30.7	41.3
2^+	1	10642.3	0	7.1	-	100%	-	0

The S-wave BB states can not be $J^P(I) = 0^+(1)$

Coupled-channel in DMC

- Coupled channels

$$\Psi(\boldsymbol{R},t)=\sum_{\alpha}\Psi_{\alpha}(\boldsymbol{R},t)\chi_{\alpha}\,,$$

$$-\frac{\partial\Psi_{\alpha'}}{\partial t}=\sum_{\alpha}\hat{H}_{\alpha'\alpha}\Psi_{\alpha}-E_R\Psi_{\alpha'}\,.$$

- Sampling $\mathcal{F}$

$$f_{\alpha}(\boldsymbol{R},t)\equiv\psi_T(\boldsymbol{R})\Psi_{\alpha}(\boldsymbol{R},t)\,,$$

$$\mathcal{F}(\boldsymbol{R},t)\equiv\sum_{\alpha}f_{\alpha}(\boldsymbol{R},t)\,.$$

- Assuming $\mathcal{F}$ is positive such that can be sampled by distribution of walkers

DMC results from [Gordillo:2020sgc]

- Results from [Gordillo:2020sgc]

	$n^{2S+1}L_J$	J^{PC}	DMC
η_c	1^1S_0	0^{-+}	3005
J/ψ	1^3S_1	1^{--}	3101
B_c	1^1S_0	0^{-+}	6292
B_c^*	1^3S_1	1^{--}	6343
η_b	1^1S_0	0^{-+}	9424
$\Upsilon(1S)$	1^3S_1	1^{--}	9462

$cc\bar{c}\bar{c}$

J^{PC}	DMC
0^{++}	6351
1^{+-}	6441
2^{++}	6471

- The mass of $T_{cc\bar{c}\bar{c}}$ is about several hundreds MeV above the related di-meson thresholds

Statistical uncertainties of DMC

- Jackknife resampling method

$$\sigma[\bar{X}] = \sqrt{\frac{1}{R(R-1)}\sum_i^R (X_i^{(1)} - \bar{X})^2}$$
$$= \sqrt{\frac{R-1}{R}\sum_i^R (\bar{X}_{(i)\mathrm{jack}} - \bar{X}_{\mathrm{jack}})^2}\,.$$

- Statistical uncertainties: less than 1 MeV